Some Uncertainties of Slope Stability Analyses

Robert C. Deen*   Tommy C. Hopkins†
David L. Allen‡

*Kentucky Department of Transportation
†Kentucky Department of Transportation
‡Kentucky Department of Transportation, dallen@engr.uky.edu

This paper is posted at UKnowledge.
https://uknowledge.uky.edu/ktc_researchreports/1065
SOME UNCERTAINTIES OF SLOPE STABILITY ANALYSES

by
Robert C. Deen
Assistant Director

Tommy C. Hopkins
Research Engineer Chief

and

David L. Allen
Research Engineer Principal

Division of Research
Bureau of Highways
DEPARTMENT OF TRANSPORTATION
Commonwealth of Kentucky

offered for publication to the
Transportation Research Board

April 1977
SOME UNCERTAINTIES OF
SLOPE STABILITY ANALYSES

by

R. C. Deen, T. C. Hopkins, and D. L. Allen

ABSTRACT

Some practical limitations of total stress and effective stress analyses are discussed. For clays having a liquidity index of 0.36 or greater, \( \phi \)-equal-zero analyses based on laboratory undrained shear strengths give factors of safety close to the actual factor of safety. However, based on field vane strengths, \( \phi \)-equal-zero analyses may yield factors of safety which may be too high. The difference between field vane and calculated shear strengths increased as the plasticity index increased. For clays having a liquidity index less than 0.36, \( \phi \)-equal-zero analyses using laboratory undrained shear strengths give factors of safety that are much too high; but the strength parameters can be corrected by the empirical relationship presented herein. An empirical relationship for correcting vane shear strength is also presented. A method is proposed for predicting the probable success of a \( \phi \)-equal-zero analysis.

Data suggest that overconsolidated clays and clay shales or clays having a liquidity index less than 0.36 pose the greatest slope design dilemma. An effective stress analysis based on peak triaxial shear strength parameters generally yields factors of safety which are too high; residual shear strength parameters frequently yield factors of safety which are too low. To approximate the theoretical strength of an overconsolidated clay which has undergone a process of softening, the effective stress parameters might be obtained from triaxial tests performed on remolded, normally consolidated clay. It is suggested the soil be remolded to a moisture content equal to the plastic limit plus the product of 0.36 and the plasticity index.
SOME UNCERTAINTIES OF SLOPE STABILITY ANALYSES

by

Robert C. Deen
Assistant Director

Tommy C. Hopkins
Research Engineer Chief

and

David L. Allen
Research Engineer Principal

INTRODUCTION

Two limiting conditions (2) are generally recognized when designing a cutting in a clay or an embankment on a clay foundation against a "first-time" failure (no pre-existing shear plane). The first condition is the short-term or end-of-construction case in which the water content of the clay does not change. Excess pore pressures are controlled by the magnitude of the stresses acting in the clay or tending toward instability. Significant pore pressure dissipation does not occur. Prediction of the excess pore pressures is difficult. Consequently, the short-term design is made using the \( \phi \)-equal-zero analysis and the undrained shear strength obtained from unconsolidated-undrained (UU) triaxial tests, unconfined compression (U) tests, field vane shear (FV) tests, or a combination of these tests.

The second limiting condition is the long-term, steady-seepage case. Pore pressures do not depend on the magnitude of total stresses and are controlled by the flow pattern of underground water or the ground-water level. Excess pore pressure dissipation has occurred and the clay exists in a drained state. Long-term design is performed in terms of effective stress and the drained shear strength parameters, \( \phi' \) and \( c' \), conventionally obtained from consolidated isotropically, drained triaxial (CID) tests; consolidated isotropically, undrained triaxial tests with pore pressure measurements (CIU); consolidated-drained, direct shear (slow) (CDS) tests; or a combination of these tests.
For a cutting in a clay, the long-term stability is considered more critical since pore pressures are initially small or negative and gradually increase toward steady-seepage pore pressures. The increase of pore pressures causes a decrease in the shear strength of the clay since there is a reduction in effective stresses. In the case of an embankment on a clay foundation, the short-term stability is considered the more critical case. For this case, pressures steadily increase to maximum values during construction and gradually decrease thereafter toward the initial pore pressures, increasing shear strengths with time.

LIMITATIONS OF TOTAL STRESS ANALYSES

Application of the first limiting condition to the design of embankments founded on clay foundations or to cut slopes without regard to the stress history and moisture state of the clays in the foundation or slope may lead to erroneous conclusions concerning the safety factor \( (11) \); that is, the undrained shear strengths obtained from laboratory or field tests may be larger than the actual (back-computed) shear strengths existing at failure.

Long-Term Stability of Cut and Natural Slopes -- Bishop and Bjerrum (2), summarizing results of a number of failures in natural slopes and cuts, showed that application of the \( \phi \)-equal-zero analysis to slopes where pore pressure and water content equilibrium have been attained is unreliable. In these cases, the \( \phi \)-equal-zero analysis gave safety factors ranging from 0.6 for sensitive soils to 20 for heavily overconsolidated soils. Two reasons for the differences between the in situ shear strength and the shear strength obtained from the undrained test are differences between field and laboratory pore pressures and migration of water to the failure zone of a slide in overconsolidated clays \((9, 18)\). Lo, et al. \((13)\) has also shown that the effect of sample size in stiff fissured clays is an important factor in stability analyses. The shear strength of large samples is less than that of small specimens.

Examination of case records of long-term failures in cuts and natural slopes revealed that large safety factors are associated with low to negative values of liquidity index while low safety factors are associated with high values of liquidity index. In data cited by Bishop and Bjerrum, there were four cases where the safety factor was near one; the liquidity indices ranged from 0.20 to 1.09. In the other cases, the liquidity indices ranged from about 0.19 to -0.36 while the safety factors ranged from 1.9 to 20.

Short-Term Stability of Loads on Soft Foundations -- Bjerrum \((3, 5)\) assembled a number of case records which showed that procedures normally used to determine the short-term stability of embankments, footings, and load tests on soft clay foundations are unsatisfactory. In those cases, the \( \phi \)-equal-zero analysis using undrained shear strengths from field vane shear tests overestimated the safety factor (open points in Figure 1) for soils having liquid limits and plasticity indices in excess of
approximately 80 and 30 percent, respectively. Also, the difference between field vane, \((S_u)_{\text{vane}}\), and corrected shear strengths, \((S_u)_{\text{corrected}}\), increased as the plasticity index, PI, and the liquid limit of the clay increased. Assuming a linear relationship between safety factor and plasticity index, the corrected shear strength may be expressed as

\[
(S_u)_{\text{corrected}} = (S_u)_{\text{vane}}/[0.84 + 0.0082 \text{ PI}] \pm 0.12].
\]

Data (dark points) assembled by Bishop and Bjerrum \((2)\) representing end-of-construction failures of footings, fills, and excavations on saturated clay foundations are compared to Bjerrum's data (open points) in Figure 1. Liquidity indices of the former data ranged from about 0.25 to 1.44. The undrained strengths of the soils in these analyses were obtained primarily from unconsolidated-undrained tests. While Bjerrum's data showed that the difference between vane and back-computed shear strengths increased as the plasticity index of the clay increased, Bishop and Bjerrum's data, in marked contrast, showed that the computed shear strength and laboratory shear strength were almost equal.

**Short-Term Stability of Embankments Founded on Overconsolidated Clays and Clay Shales** -- A number of short-term failures of embankments on overconsolidated soils have occurred, even though the \(\phi\)-equal-zero analysis indicated the embankment slopes should have been stable. Some examples include case histories by Beene \((1)\), Wright \((20)\), Peterson, et al. \((16)\), and Hopkins and Allen \((10)\). Safety factors from \(\phi\)-equal-zero analyses ranged from 1.23 to 4.0 for these cases; all had liquidity indices less than 0.36.

**Short-Term Stability of a Cut or Excavated Slope in Overconsolidated Clays and Clay Shales** -- Because the short-term safety factor is usually a maximum during or near the end of construction, the \(\phi\)-equal-zero analysis is oftentimes used to determine the short-term stability of a cut or excavated slope. However, stability of cuts in overconsolidated clays and clay shales may not always conform to this concept. For instance, Skempton and Hutchinson \((19)\) described two slides in a stiff overconsolidated London clay. Based on a \(\phi\)-equal-zero analysis and undrained shear strengths, the short-term safety factors were about 1.8.

**Proposed Method of Predicting Success in a \(\phi\)-Equal-Zero Analysis** -- Peck and Lowe \((15)\) presented a portion of Bishop and Bjerrum's data (long-term failures in cuts and natural slopes) which showed that the computed safety factor of failed slopes, obtained from a \(\phi\)-equal-zero analysis using undrained strengths, was apparently a function of the liquidity index. Peck and Lowe suggested the possibility of using that empirical relationship to determine correction factors for laboratory undrained strength parameters.

Plotting additional portions of Bishop and Bjerrum's data \((2)\) and Bjerrum's data \((5)\) (safety factor as a function of liquidity index), a distinctive division can be observed. All data in Figure 2 represent
failures where the $\phi$-equal-zero analysis was performed using undrained shear strengths obtained from UU, U, or FV tests. In failures where the soils had a liquidity index equal to or greater than approximately 0.36, safety factors estimated from a $\phi$-equal-zero analysis and UU or U strengths should have an accuracy within $\pm 15$ percent (see Figure 2), and design safety factors as low as 1.3 may be justified in many routine designs. Where the undrained strength is obtained from in situ vane shear tests, the vane strength should be corrected.

In failures where the soils had a liquidity index less than about 0.36, the $\phi$-equal-zero analysis using UU or U strengths gave safety factors which were much too high; in situ shear strengths were greatly overestimated by laboratory tests. For soils having liquidity indices less than 0.36, the safety factor appears to be a function of the liquidity index, LI:

$$F = (3.98)(0.0192)^{LI}.$$  

Since the safety factor can be expressed as

$$F = \frac{S_u}{S_s},$$  

where $S_u$ is the laboratory undrained shear strength, the corrected laboratory or "softened" shear strength may be expressed in terms of the standard error as

$$S_s \approx (0.252) S_u (0.0192)^{LI} (10^{0.24});$$  

and the error in the corrected shear strength may be as large as 70 percent.

**LIMITATIONS OF EFFECTIVE STRESS ANALYSES**

Uncertainties in the application of the effective stress approach to the design of earth slopes arise in the selection of shear strength parameters, $\phi'$ and $c'$, and the evaluation of pore pressures. Although the effective stress method has been successfully applied to normally consolidated and very lightly overconsolidated clays and silty clays having an intact structure (free of fissures or joints), the method is not successful when applied to the design of slopes composed of overconsolidated clays and clay shales. Although much research (4, 17, 18) has been directed toward understanding the characteristics of those soils, overconsolidated soils still pose the greatest design dilemma to engineers.

**Shear Strength Characteristics** - Typical stress-strain curves for normally consolidated and overconsolidated clays, tested similarly under drained conditions, show that both reach a peak strength. As the overconsolidated soil is strained beyond the peak strength, the shear resistance decreases until at large strains the strength falls to a (nearly) constant value. This lower limit of resistance is referred to as the "residual" or "ultimate" strength (17, 18, 19). After the peak strength has been attained, the shear resistance of the normally consolidated clay may fall only slightly. After large strains, the shear resistance of the overconsolidated and normally consolidated clays coincide. In heavily
overconsolidated plastic clays, there is a large difference in the peak and residual strengths. In silty clays
and soils of low plasticity, this difference is very small. With an increase in clay content, this difference
increases even in normally consolidated clays, although not as much as in overconsolidated clays.

The "softened" shear strength of an overconsolidated clay (as obtained from Equation 4) may be
defined as the intersection of a horizontal line projected from the peak strength of the normally
consolidated clay with the stress-strain curve of the overconsolidated clay (17). The softened strength
is intermediate to the peak and residual strengths and probably occurs at much lower strains (representing
a condition wherein a number of small, independent shear planes exist) than the residual strength (wherein
the shear planes have joined to form a well defined failure plane).

The critical state of a normally consolidated clay can be defined (17) as the state (in a drained
condition) in which any further increment in shear distortion will not result in any change in water
content. The water content at the critical state is equal to that ultimately attained by an overconsolidated
clay due to expansion during shear.

**Peak and Residual Shear Strengths -- Bjerrum (4)** assembled shear strength data on a number of
"first-time" failures of natural and cut slopes in overconsolidated clays and clay shales which showed
that the average shear stress along the failure surface was much smaller than the shear strength measured
from laboratory triaxial tests. In each case, the peak shear strength parameters, $\phi_p$ and $c'_p$, were larger
than the back-computed parameters, $\phi'_c$ and $c'_c$ (assumed zero), and therefore, the safety factors were
too large. The liquidity indices of these clays ranged from -0.51 to 0.25. Discrepancies between the
field (back-computed) and laboratory strengths are illustrated in Figure 3. The back-computed effective
stress angle of shearing resistance is plotted as a function of the peak effective stress parameter obtained
from triaxial tests. Even neglecting the cohesion, the data plots below the line of equality. If residual
shear strengths are used, there is better agreement between the computed shear strengths and those
determined by direct shear tests.

Use of the residual shear strength parameters, $\phi'_r$ and $c'_r$, in effective stress analyses does not
necessarily yield safety factors which are in agreement with the actual safety factor at failure, although
the error in the safety factor based on residual strength is generally smaller than the error in the safety
factor based on peak strength. In Table 1, a number of well-documented embankment, cut slope, and
natural slope failures based on the effective stress analysis have been summarized and arranged according
to increasing values of liquidity indices. Except for the case by D'Appolonia, et al. (7), all cases are
"first-time" failures. Those case records clearly show that the effective stress analysis based on residual
strength generally gives safety factors which are less than one. All of those cases, except for the last
two failures listed in the table, involve soils which have liquidity indices less than 0.36; and the effective
stress analysis based on peak strength yields safety factors which are too high and may be as much as 100 percent in error. Additionally, the use of $c'_p$ equal to zero and $\phi'_p$ does not always yield the correct safety factor.

**Evaluation of Pore Pressures** -- If the excess and initial pore pressures are known when designing a cutting in clay or an embankment on a clay foundation, the stability of these earth structures may be determined during or at any time after construction from an effective stress analysis using total pore pressures. However, methods of estimating excess pore pressures are particularly difficult, and the results obtained from such methods are highly questionable (see Moh, et al. (14)). Additionally, to determine the stability of the cut or embankment at any time, dissipation of excess pore pressures must be estimated, generally based on the results of consolidation tests which may be very inaccurate. Unless piezometers are installed to obtain the necessary pore pressure data, the effective stress analysis is limited to analyzing the long-term stability of cuts and embankments. For this condition, the excess pore pressures are assumed equal to zero. In the case of a cutting in clay, the pore pressures are obtained from a prediction of the steady seepage pore pressures. In the case of an embankment, the pore pressures are usually obtained from groundwater level observations in boreholes. Where large fluctuations in groundwater levels may exist, inaccurate pore pressure data may be obtained. Where the embankment is located on a sloping foundation and "damming" of the groundwater may occur, prediction of the steady seepage pore pressures is very difficult, especially where large fluctuations of the groundwater level may occur.

To make a valid comparison between field and laboratory shear strengths in terms of effective stress, accurate values of pore pressures existing at the time of failure must be known. The "back-computed" shear strength parameters, $\phi'_c$ and $c'_c$, are particularly sensitive to the magnitude of the pore pressures used in the computation. Inaccurate pore pressures may produce an error of several degrees in the computed parameter, $\phi'_c$. An accurate determination of the pore pressures in a landslide at failure poses certain difficulties. Even when piezometers are installed, measurements obtained may not correspond to the pore pressures existing at the time of failure, particularly when the failure is preceded by a heavy rainfall and field personnel may not be present at the time of failure. In delayed failures where several years may be required for the pore pressures to reach the steady-state values, use of measured pore pressures obtained before pore pressure equilization has occurred will lead to computed parameters, $\phi'_c$ and $c'_c$, which cannot validly be compared to laboratory shear strength parameters.

**Slope Design Dilemma** -- Observations (4) suggest the rate of development of a continuous sliding surface in a clay slope prior to failure varies from one type of clay to another. In the stiffer clays, the rate may be very small; delay of the failure may be on the order of years. Data in Figure 4 suggest that, for clay soils having liquidity indices less than approximately -0.1 to -0.2 (very stiff clays), the
failure delay may be several years. In slopes where the liquidity indices are greater, the delay in failure may be very short.

Since the critical-state shear strength of overconsolidated clays cannot readily be determined, a practical approximation to the critical state might be obtained from triaxial tests performed on normally consolidated samples remolded at a water content given by

$$w_c = (0.36) P_l + P_l,$$

where $P_l$ is the plastic limit and the constant 0.36 is the liquidity index at the break point in Figure 2.

**SUMMARY AND CONCLUSIONS**

The following summary and tentative conclusions are based on observations and data analyses of results of several published, well-documented, landslide case histories:

1. Application of the $\phi$-equal-zero analysis to the design of an embankment located on a clay foundation or to a slope cut in a clay without regard to the stress history and moisture state of the clay may lead to erroneous conclusions concerning the stability of the slope. For clays having a liquidity index equal to or greater than approximately 0.36, the $\phi$-equal-zero analysis, based on laboratory undrained strengths, will yield fairly reliable safety factors, provided the liquid limit and plasticity index of the clay are equal to or below values of about 80 and 30 percent, respectively. For clays having a liquidity index below a value of about 0.36, the $\phi$-equal-zero analysis will probably yield safety factors which are much too high. The reliability of the high safety factors may depend on the liquidity index of the clay. For clays having a liquidity index less than about 0.36, the time to failure may vary from a few days or months to several years. If high safety factors are obtained from a $\phi$-equal-zero analysis, then Figure 2 should be reviewed to evaluate the probable success of the slope design. The stability of the slope might be checked using the corrected undrained shear strength given by the empirical relationship of Equation 4.

2. The use of uncorrected vane shear strength to determine the stability of an embankment on a soft foundation, cut slopes, footings, and loading tests may yield unreliable results. The vane shear strength should be corrected by the empirical relationship in Equation 1.

3. The liquidity index appears to be a general indicator of the stress history of a clay. Clays having a liquidity index less than about 0.36 might be considered to be "overconsolidated" while clays having a liquidity index greater than 0.36 might be considered "normally consolidated."

4. The use of residual shear strength may be too conservative and expensive in many slope design problems involving overconsolidated clay, especially in cases where temporary cuts are made. However,
the use of peak shear strength in such soils may be unreliable and unsafe. The intermediate shear strength obtained from triaxial tests performed on normally consolidated clays remolded to a water content given by Equation 5 might provide a practical value to use in the design of slopes against "first-time" failures.

REFERENCES


13. Lo, K. Y.; Adams, J. I.; and Seychuk, J. L.; The Shear Behavior of a Stiff Fissured Clay,
Deen, Hopkins, and Allen

Geotechnique, 1970.


LIST OF FIGURES

Figure 1. Factor of Safety as a Function of Plasticity Index (data from Table I and Tables IV, V, and VII of Bjerrum's (3, 5) papers and Table II of Bishop and Bjerrum's (2) paper). All slopes represented in the diagram failed, i.e., $F = 1.0$. All curves are by the authors (11).

Figure 2. Factor of Safety as a Function of Liquidity Index (by the authors; the data are from Tables II, III, and V of Bishop and Bjerrum's paper (2), Peterson, et al. (16), and Kentucky DOT Division of Research (11)).

Figure 3. Back-Computed Shear Strength Parameter as a Function of Peak Shear Strength Parameter from Triaxial Tests and Residual Shear Strength Parameter from Consolidated-Drained, Direct Shear Tests (data from Bjerrum (4), Skempton (17), and D'Appolonia, et al. (7)).

Figure 4. Time to Failure of Overconsolidated Clays and Shales as a Function of Liquidity Index (data from Cassel (6), Skempton (18), Henkel (8), Beene (1), Skempton and Hutchinson (19), and Kentucky DOT Division of Research (11)).
<table>
<thead>
<tr>
<th>LOCATION</th>
<th>PEAK</th>
<th>c = 0</th>
<th>RESIDUAL</th>
<th>REFERENCE</th>
<th>TYPE OF SLOPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>bluegrass Pky, MP 21</td>
<td></td>
<td></td>
<td></td>
<td>1.46</td>
<td>Filla</td>
</tr>
<tr>
<td>West Ky Pky, MP 96</td>
<td></td>
<td></td>
<td></td>
<td>1.94</td>
<td>Filla</td>
</tr>
<tr>
<td>Bluegrass Pky, MP 43</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>Filla</td>
</tr>
<tr>
<td>Solder</td>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>Filla</td>
</tr>
<tr>
<td>164, MP 118</td>
<td></td>
<td></td>
<td></td>
<td>1.28</td>
<td>Filla</td>
</tr>
<tr>
<td>Weirton, W. Va.</td>
<td></td>
<td></td>
<td></td>
<td>1.51</td>
<td>Cut</td>
</tr>
<tr>
<td>Northolt</td>
<td></td>
<td></td>
<td></td>
<td>1.63</td>
<td>Cut</td>
</tr>
<tr>
<td>Jackfield</td>
<td></td>
<td></td>
<td></td>
<td>2.06</td>
<td>Cut</td>
</tr>
<tr>
<td>Kensee Green</td>
<td></td>
<td></td>
<td></td>
<td>1.60</td>
<td>Cut</td>
</tr>
<tr>
<td>Suhrurry Hill</td>
<td></td>
<td></td>
<td></td>
<td>2.27</td>
<td>Cut</td>
</tr>
<tr>
<td>164, MP 44</td>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>Filla</td>
</tr>
<tr>
<td>Amooy</td>
<td></td>
<td></td>
<td></td>
<td>2.27</td>
<td>Filla</td>
</tr>
<tr>
<td>US 119</td>
<td></td>
<td></td>
<td></td>
<td>1.13</td>
<td>Filla</td>
</tr>
<tr>
<td>Seven Sisters, S-6</td>
<td></td>
<td></td>
<td></td>
<td>1.07</td>
<td>Filla</td>
</tr>
<tr>
<td>Loudon</td>
<td></td>
<td></td>
<td></td>
<td>1.07</td>
<td>Filla</td>
</tr>
<tr>
<td>Danmen</td>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>Filla</td>
</tr>
</tbody>
</table>

*Fills located on sloping foundations

**NS**. Natural slope

*Details contained in Ky. D.O.T. Division of Research Reports
Figure 1. Factor of Safety as a Function of Plasticity Index (data from Table I and Tables IV, V, and VII of Bjerrum’s (3, 5) papers and Table II of Bishop and Bjerrum’s (2) paper). All slopes represented in the diagram failed, i.e., $F = 1.0$. All curves are by the authors (11).
Figure 2. Factor of Safety as a Function of Liquidity Index (by the authors; the data are from Tables II, III, and V of Bishop and Bjerrum's paper (2), Peterson, et al. (16), and Kentucky DOT Division of Research (11)).
Figure 3. Back-Computed Shear Strength Parameter as a Function of Peak Shear Strength Parameter from Triaxial Tests and Residual Shear Strength Parameter from Consolidated-Drained, Direct Shear Tests (data from Bjerrum [4], Skempton [17], and D'Appolonia, et al. [7]).
Figure 4. Time to Failure of Overconsolidated Clays and Shales as a Function of Liquidity Index (data from Cassel (6), Skempton (18), Henkel (8), Beene (1), Skempton and Hutchinson (19), and Kentucky DOT Division of Research (11)).