A Rheological Study of Cohesive Soils

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MEMORANDUM TO: J. R. Harbison
State Highway Engineer
Chairman, Research Committee

SUBJECT: Research Report 337; "A Rheological Study of Cohesive Soils;"
KYHPR-65-38; HPR-1(8), Part II

The interim progress (Phase II) report enclosed here presents a scholarly treatise on the classical behavior of cohesive soils. This type of study has direct bearing on the design and performance of embankments, pavement foundations, and pilings. The creep phenomenon is evident throughout the Eden soils belt surrounding the Inner Bluegrass area.

A material consisting only of granular material (mineral aggregate or sand) possesses friction only. Tan φ is the coefficient of friction, and φ is the natural angle of repose. The force to overcome friction is $F = n \tan \phi = WF$. When the force exceeds the resistance, slippage occurs, and movement accelerates. Only fluid damping can prevent acceleration; fluid damping (rate of flow proportional to force) can bring forces into balance and produce a steady-state velocity. Before slip occurs, we have retarded elasticity.

Perhaps a better analogy is found in the general equation for linear translational motion:

$$\frac{Wa}{g} + \eta \frac{dv}{dt} A + fW - F = 0,$$

or in forced vibration,

$$\frac{Wa}{g} + \eta \frac{dv}{dt} A + fW = F_0 (\sin \omega t).$$

Indeed, the total force is equal to the sum of the parts. Stresses add in the same way. Neglecting inertia and substituting stress for the "body force", $W$, we have an equation for a plastic material:

$$\tau = \eta \frac{dv}{dt} + \sigma_n \tan \phi,$$

or where fluid pressure enters,

$$\tau = \eta \frac{dv}{dt} + (\sigma_n - \sigma_p) \tan \phi.$$

That portion of the shear stress given by $(\sigma_n - \sigma_p) \tan \phi$ is equal to $G'$, the shear modulus, times the shear strain. So, dividing the above equation by strain yields a complex shear modulus,

$$G^* = G' + iG''.$$

The models shown in Figs. 14 and 56 of the report may have the same stiffness (rigidity), but the deformations differ unless the time of loading is very short. In the theory presented, deformation
is assumed to be continuous with time. Stresses, rates of deformation and total deformation can be predicted ad infinitum assuming the material does not fail in the meantime. Thus, separate equations (empirical or phenomenological) stating limiting stresses and (or) strains are needed to define failure criteria. These criteria were considered in an antecedent report but will be brought into issue again in the third and final report of the study -- now in preparation. It appears possible now to establish a failure envelope in a valid way from a simple triaxial specimen and to determine other essential rheological parameters at the same time.

Respectfully submitted,

[Signature]

Jas. H. Havens
Director of Research

cc's: Research Committee
A Rheological Study of Cohesive Soils

An attempt has been made to describe the mechanistic behavior of remolded and undisturbed soils by application of the principles of rheology. This was accomplished by approximating and comparing the data obtained from transient creep tests and relaxation tests to the mathematical behavior of mechanical impedance models. The data were also analyzed by transforming static moduli and compliances into dynamic values by the application of Fourier transforms. Distribution functions of relaxation and retardation times, obtained by the application of Laplace transforms, are presented.

All soils tested exhibited similar dynamic moduli and compliance curves indicating that soil type had little effect on the general shape of these curves. A generally direct, linear relationship existed between magnitudes of the dynamic moduli and confining pressure for the remolded soils. Distribution functions proved to be useful indicators of mechanical behavior; however, they were not consistently affected by either confining pressure or static stress level.

Distribution Functions, Dynamic Compliance, Dynamic Modulus, Soils, Strain Retardation, Stress Relaxation

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A RHEOLOGICAL STUDY OF COHESIVE SOILS

Interim Report
KYHPR-65-38; HPR-1(8), Part II

by

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US DEPARTMENT OF TRANSPORTATION
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The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Kentucky Department of Highways and Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

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INTRODUCTION

Interaction of the distinctly different components of soils – solid particles, water and air – brings into play forces which account for its difficult-to-describe behavior. These forces often take the form of frictional resistance between particle surfaces, electrochemical attractions between particles or between water and particles, viscous resistance of water, and air-water solubility. Investigators have used various methods for explaining and relating these forces; none have been entirely satisfactory. Consequently, design of soil structures continues to be somewhat empirical.

A known phenomenon of soil behavior is the tendency of the mechanical properties to change with time, i.e., the phenomenological behavior of soils is time dependent. To more accurately formulate this behavior, a number of researchers have successfully applied the concepts of rheology. Weyl and Ormsby (16) wrote concerning the atomistic approach to rheology of a clay-water mixture. Tan (13) used rheological models to represent creep in long-term consolidation tests. Others (10, 11) have used linear viscoelastic theory to represent behavior of other materials, such as bituminous materials. Pagen and Jagannath (12) applied rheological models to kaolin clay prepared at different moisture contents and compactive efforts, Saada (13) has used a simplified, four-parameter empirical model to represent creep data for saturated clays.

To investigate the rheological response of soils in the study reported herein, a large number of relaxation and creep tests were performed. According to Gross (3), data obtained from such transient tests can be transformed into dynamic parameters by Fourier transforms, permitting an approximation of the complex modulus or complex compliance and their components. This method has been applied in this study. However, Krizek (7) warns this technique is theoretically valid only for linear viscoelastic materials. Because of the nonlinear behavior of most soils, caution is recommended regarding unqualified use of such procedures. Another major objection to this method is the inability to measure instantaneous or peak relaxation modulus. However, there is a method for calculating this modulus, and it will be discussed in greater detail later. In spite of these objections, several investigators have employed this method in conjunction with actual dynamic tests and have obtained excellent correlation between the two methods.

Significance of Distribution Functions

In a somewhat analogous manner, Gross (3) and Macey (9) reported that the application of an inverted LaPlace transform to creep and relaxation functions (with some modifications) will produce the distribution function for the particular test specimen and experimental conditions. Distribution functions have been employed in the analyses herein.

Distribution functions have, for many years, been used for the interpretation of creep and relaxation test data. The form of the distribution function is a characteristic of the material tested, i.e., it is another means of defining viscoelastic properties of materials. There are two distinctly different distribution functions: 1) the distribution of relaxation times (relaxation spectrum) and 2) the distribution of retardation times (retardation spectrum). The relaxation spectrum is associated with strain-controlled experiments where strain is the dependent variable; the retardation spectrum is associated with stress-controlled experiments wherein strain is the dependent variable. In strain-controlled experiments on viscoelastic materials, strain retardation occurs. It is, in fact, the observed phenomena of stress relaxation and strain retardation which differentiates a viscoelastic material from an ideally plastic or ideally elastic one and necessitates a viscoelastic theory.

Superposition Principle

Elementary mathematical models which superimposed elastic and viscous effects were first employed to represent viscoelastic behavior. These models were represented by simple exponential laws for stress decay and strain retardation. When experiments indicated a more complex behavior, the simple models were extended to a series of exponential functions, each characterized by its amplitude and time constant. Further generalization resulted in a continuous set of exponential functions (the distribution functions) with time constants distributed continuously over an interval.

Linear viscoelastic theory is based on the principle of superposition (8). Mechanical behavior of materials has been explored using various static and dynamic methods. Tests yield certain constants and functions of time and frequency which represent behavior of the material under the test conditions. The superposition principle serves to correlate results of different types of tests systematically (2).
If a strain \( e_0 \) (see APPENDIX A for a listing and definition of symbols) is applied at time \( t = 0 \), from the definition of the relaxation modulus (\( G(t) \)), stress as a function of time is

\[
\sigma(t) = e_0 G(t)
\]

If a larger strain \( (e_0 + e_i) \) is applied at \( t = 0 \), the stress will be proportionately larger, that is

\[
\sigma(t) = (e_0 + e_i) G(t).
\]

But if \( e_0 \) is applied at \( t = 0 \) and \( e_i \) is added at a later time \( t = t_i \), the total stress according to the superposition principle is

\[
\sigma(t) = e_0 G(t) + e_i G(t - t_i),
\]

that is the linear superposition of the two stresses at their respective elapsed times. There are an infinite number of possible variations of loading, the effect of which can be generalized by

\[
\sigma(t) = \sum_{i=1}^{n} e_i G(t - u_i)
\]

where \( u_i \) is the time at which the strain \( e_i \) is applied. In terms of strain varying continuously,

\[
\sigma(t) = G(\varepsilon(t)) + \int_0^\varepsilon(t) \frac{dG(s)}{ds} ds
\]

where \( s = t - u \).

**Methodology**

To determine and evaluate fundamental rheological properties of some cohesive soils, samples were obtained from four Kentucky soils — Baxter, Calloway, Eden and Maury soil series. These soils, in addition to a commercial kaolin clay, were remolded by means of a vacuum extrusion machine after distilled water had been added to obtain the desired moisture content. Relaxation and creep tests were later performed. Relaxation tests were also performed on some undisturbed samples from various highway embankments located throughout the state.

Results of engineering classification tests on the remolded and undisturbed soils are listed in APPENDIX B. For detailed descriptions of the testing equipment, sample preparation (both remolded and undisturbed), and procedures for performing the relaxation tests, the reader is referred to the first report on this research program (14). The creep test procedure used in this study is described in APPENDIX C.

**RELAXATION FUNCTION**

Application of a constant strain \( \varepsilon \) produces a stress (see Figure 1) given by

\[
\sigma = [G(\varepsilon(t)) - G(\varepsilon)] \varepsilon
\]

where \( G \) is the static or equilibrium elastic shear modulus,

\[
G = G(\varepsilon) + \varepsilon \Psi(t)
\]

is the instantaneous elastic modulus, and

\[
\Psi(t)
\]

is the relaxation function with \( \Psi = 0 \).

Stress, therefore, decreases from an initial value of \( G_0 \varepsilon \) to a final value of \( G - \varepsilon \). The static elastic modulus may be zero, in which case the stress will decay to zero. The instantaneous modulus is accessible experimentally only if the specimen can be strained and the reading taken in so short a time that stress decay can be neglected.

The relaxation shear modulus can be calculated from

\[
G = S/2\varepsilon
\]

where \( S = \) deviator stress and

\( \varepsilon = \) deviator strain.

The mean normal stress \( \sigma \) is given by \((\sigma_1 + \sigma_2 + \sigma_3)/3\), where \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are the principal normal stresses. Since \( \sigma_2 = \sigma_3 \) in the conventional triaxial test,

\[
\sigma = (\sigma_1 + 2\sigma_3)/3.
\]

For the triaxial test, \( \sigma_1 = \sigma_2 + \sigma_3 = (P/A) + \sigma_3 \). Thus the mean normal stress becomes

\[
\sigma = (P/3A) + \sigma_3.
\]

The deviator stress is defined as

\[
S = \sigma_1 - \sigma.
\]

Substituting for \( \sigma_1 \) and \( \sigma \), Equation 10 becomes

\[
S = 2P/3A.
\]

For a no-volume-change condition, as assumed in the undrained triaxial test, the mean normal strain \( \varepsilon \) is zero. The deviator strain for the undrained test is then defined as

\[
\varepsilon = \varepsilon_1 - \varepsilon = \varepsilon_1
\]
Substituting Equations 11 and 12 into Equation 7,

\[ G = \frac{P}{3A\varepsilon_1} = KP \]

where \( K = 1/3 \ A\varepsilon_1 \).

Relaxation modulus curves, identified according to confining pressure, calculated as above are shown for the remolded soils in Figures 2 through 13. Curves for the undisturbed soils are not shown. The relaxation function \( \Psi(t) \) is \( G(t) - G_\infty \). Test durations of one hour were not sufficient to permit calculation of \( G_\infty \) in most cases. Some tests of 24-hour duration were also performed and indicated continuing relaxation. It was therefore assumed for the soils investigated that \( G_\infty = 0 \) and \( \Psi(t) = G(t) \).

Preliminary analysis indicated a large part of the rigidity of the soils investigated is associated with relaxation times considerably less than the strain application time of about one-fifth second. The apparent instantaneous modulus was less than the actual instantaneous modulus due to partial relaxation of stress which occurred during application of strain. Attempts were made, unsuccessfully, to eliminate this effect by applying strain nearly instantaneously. Nearly instantaneous application of strains on the order of 0.3 percent caused sample failure, and application of smaller strains reduced precision of measurements below acceptable limits due to seating errors and strain of the test equipment.

There is, however, a method for calculating instantaneous modulus from the stress-time loading curve if it is assumed that deformation is produced at a constant rate of strain. If the strain is applied at a constant rate \( \dot{\varepsilon} \), its value at time \( t \) will be \( \varepsilon = \dot{\varepsilon} t \), and the stress will be the result of the superposition of a series of partially relaxed stresses up to that time. In terms of the relaxation distribution function (2),

\[ \sigma = \varepsilon \int_{0}^{t} \beta F(t) \exp (-t/r) dr + G_\infty \dot{\varepsilon} t \]

where \( r \) and \( u \) are time variables. A plot of \( \dot{\sigma} \) versus \( \varepsilon \) yields a stress-strain curve for a constant rate of strain. Differentiation of the stress-strain curve gives the relaxation modulus \( d\sigma/d\varepsilon \) as

\[ d\sigma/d\varepsilon = (1/\dot{\varepsilon}) (d\dot{\sigma}/dt) = \int_{0}^{t} \beta F(t) \exp (-t/r) dr + G_\infty \dot{\varepsilon} \]

since

\[ \int_{0}^{t} \beta F(t) \exp (-t/r) dr = \Psi(t) \]

is the relaxation function as indicated by Gross (3).\( \beta \)

is a normalization factor such that

\[ \int_{0}^{\infty} F(t) dt = 1, \]

which is the same as

\[ \beta = \Psi(0). \]

\( F(t) \) is the relaxation spectrum associated with relaxation times in the range between \( r \) and \( r + dr \).
Figure 2. Relaxation Modulus vs Log Time – Kaolinite (0.3 Percent Strain).
Figure 3. Relaxation Modulus vs Log Time — Kaolinite (0.3 Percent Strain).
Figure 4. Relaxation Modulus vs Log Time -- Kaolinite (0.6 Percent Strain).
Figure 5. Relaxation Modulus vs Log Time — Kaolinite (0.6 Percent Strain).
Figure 6. Relaxation Modulus vs Log Time – Baxter Soil Series.
Figure 7. Relaxation Modulus vs Log Time -- Baxter Soil Series.
Figure 8. Relaxation Modulus vs Log Time -- Calloway Soil Series.
Figure 9. Relaxation Modulus vs Log Time – Calloway Soil Series.
Figure 10. Relaxation Modulus vs Log Time – Eden Soil Series.
Figure 11. Relaxation Modules vs Log Time – Eden Soil Series.
Figure 12. Relaxation Modulus vs Log Time – Maury Soil Series.
Figure 13. Relaxation Modulus vs Log Time – Maury Soil Series.
DISTRIBUTION FUNCTION OF RELAXATION TIMES

The integral of Equation 16 can be transformed into the LaPlace integral,

\[ \tilde{\Psi}(t) = \int_0^\infty \tilde{N}(c) \exp(-tc) \, dc \]

by introducing a relaxation frequency \( c = 1/\tau \) and a frequency function \( \tilde{N}(c) \, dc \), where

\[ \tilde{N}(c) = \frac{\beta F}{(1/c)c^2}. \]

Therefore, if \( \tilde{\Psi}(t) \) can be expressed as an analytical expression, the relaxation distribution function can be determined by standard methods for inversion of the LaPlace integral. The relaxation function can be represented to any degree of accuracy by a function of the form

\[ \tilde{\Psi}(t) = \sum \gamma_i \exp(-t/\tau_i). \]

which is the well known general Maxwell model (Figure 14) where \( \tau_i = \eta_i/G_i \). However, Equation 21 does not allow for the inversion of the LaPlace transform in analytical form because it cannot be found in the tables of LaPlace transforms. On the other hand, a mathematical expression very similar to Equation 21,

\[ \Psi(t) = \sum \gamma_i \exp(-b_1\sqrt{t}), \]

is the LaPlace transform of known function

\[ a_i b_1/2\sqrt{\pi c^3} \exp(-b_1^2/4c). \]

Thus, if the experimental data can be fitted to Equation 22, Equation 23 is the distribution function, and the problem is solved.

The graphical curve-fitting procedure was simple. \( \log G(t) \) was plotted against \( \sqrt{t} \), and the constants \( a_i \) and \( b_1 \) were determined from a straight line drawn through the points for large values of time. The first term was then evaluated and subtracted from \( G(t) \) and the plotting repeated. For the data in this study, three or four terms were found to be sufficient to represent the data. A more detailed description of this curve-fitting procedure was given in a previous report [10]. Equation 23 gives the spectrum of relaxation times for each term or "element" of the relaxation test and consequently describes the behavior of that element for the entire range of time which the data represents. In the normalized (dimensionless) distribution function, the relative amount of time-dependent stress which decays to a fraction \((1/e)\) of the initial value within the interval \( \tau \) and \( \tau + d\tau \) is associated with a frequency or time period. Therefore, the behavior (the distribution function) of the test sample can be found by summing the corresponding number of terms or elements of the form of Equation 23 for that particular test. This is clearly demonstrated in Figure 15.

It was determined that Equation 22 represented the experimental data as well as the Maxwell Model, Equation 21, to any degree of precision desired within experimental error. The graphical curve-fitting required approximately one-half to one hour for three or four terms. The number of terms required for a fit of the desired precision was immaterial as they were recombined in the final curve. Errors in determination of the constants of individual terms tended, for the same reason, to be compensatory. The transform, Equation 22, is shown in Figure 16 as a solid curve, and the experimental data are shown as points. This excellent agreement between the data and Equation 22 indicates that Equation 23 is an accurate distribution. The distribution functions \( N(c) \) are shown in Figures 17 through 27.

Figure 14. Maxwell Element.
Figure 15. Distribution Function as the Sum of the Relaxation Spectrums of Individual Elements.
Figure 16. Comparison of Observed and Computed Relaxation Functions.
Figure 17. Relaxation Distribution Functions - Kaolinite (0.3 Percent Strain).
Figure 18. Relaxation Distribution Functions – Kaolinite (0.6 Percent Strain).
Figure 19. Relaxation Distribution Functions -- Baxter Soil Series.
Figure 20. Relaxation Distribution Functions - Calloway Soil Series.
Figure 21. Relaxation Distribution Functions - Eden Soil Series.
Figure 22. Relaxation Distribution Functions – Maury Soil Series.
Figure 23. Relaxation Distribution Functions -- Western Kentucky Parkway, Milepost 96, Layer 1.
Figure 24. Relaxation Distribution Functions – Western Kentucky Parkway, Milepost 96, Layer 2.
Figure 25. Relaxation Distribution Functions – Western Kentucky Parkway, Milepost 96, Layer 3.
Figure 26. Relaxation Distribution Functions – Western Kentucky Parkway, Milepost 96, Layer 4.
Figure 27. Relaxation Distribution Functions – I 64, Milepost 44.
As demonstrated earlier, small "peaks" in the distribution functions occurred when the relaxation spectrum of each element reached a maximum and began to drop toward zero. Factors controlling the location and magnitude of these peaks can be seen from observation of Figures 28 and 29. In Figure 28, the term $a_i$ of Equation 23 was held constant and $b_i = 1/r_i$ was allowed to vary. There was an inverse relationship between the magnitude of $b_i$ and the peak magnitude of the relaxation spectrum, and there was a direct relationship between $b_i$ and the frequency at which the peak occurred. In Figure 29, $b_i$ was held constant and $a_i$ varied. A direct relationship existed between $a_i$ and the magnitude of the peak; however, $a_i$ had no effect on the location of the peak.

The relationship between the maximum in the distribution function and the frequency at which it occurred is shown in Figure 30. The relationship is linear on log-log plots and has a logarithmic slope of approximately -1. The first term of the relaxation function, for all practical purposes, always determines the location and magnitude of the maximum in the distribution function. Therefore, the scatter of points evident in these figures is produced by differences in magnitude of the constant $a_i$ in the first term of the relaxation function (Equation 22).

Distribution functions yield considerable information concerning the mechanical behavior of soils. For instance, when the maximum in the distribution function is small and is shifted toward high frequencies, the retarding or viscous damping portion of the soil response is small in comparison to the elastic portion. Stress relaxation in this type of soil will be a rapid process with most relaxation occurring at "short times" However, if the ratio of the elastic mechanism to the viscous is small, the peak in the distribution function will have a larger magnitude and be shifted toward the low frequency end. Most relaxation will then occur slowly over long time periods. When two samples have distribution functions which peak at the same location but one has greater magnitude than the other, the ratios of the viscous to elastic mechanisms of the two samples are equal. However, the magnitude of both the elastic and viscous mechanisms of the sample with the larger distribution function will be greater than in the other sample. Therefore, relaxation processes will occur simultaneously but in different amounts.

All of the remolded soils series gave distribution functions similar in appearance, with possibly one exception -- the Baxter series. In this case, many of the distribution functions peaked at higher frequencies (one to two cycles) than did the other series. In spite of this small exception, it was possible that similarities between all of the series may have been caused by the process by which the samples were prepared -- that is, preconditioning all samples to very nearly the same mechanical behavior.

The undisturbed soils gave distributions somewhat different; small peaks were generally more pronounced than in the remolded soils. Many of the undisturbed samples yielded distributions which were shifted toward higher frequencies -- consequently, relaxing in much shorter times than did the remolded soils.

The confining pressure appeared to have little or no consistent effect upon the location or magnitude of the maximum in the distribution function.
Figure 28. Effect of $b_1$ on Relaxation Distribution Function.
Figure 29. Effect of $\alpha_1$ on Relaxation Distribution Function.
Figure 30. Relationship between Maximum of Distribution Function and Frequency at Which it Occurs.
**COMPLEX MODULUS FUNCTION**

Application of an alternating strain, \( e = e_0 \exp(\text{i} \omega t) \), produces, after transient effects have disappeared, a sinusoidal stress, \( \sigma = G^* (\text{i} \omega) e \), partly in phase with the strain and partly out of phase. \( G^*(\text{i} \omega) \) is the complex shear modulus. Decomposition into real and imaginary components yields

\[
G^*(\text{i} \omega) = G_w + G_1(\omega) + \text{i} G_2(\omega) \tag{24}
\]

where \( G_w \) is the static or equilibrium modulus,
\( G_1 \) is the storage modulus,
\( G_2 \) is the loss modulus, and
\( \text{i} \) indicates imaginary root.

It can be seen from Figure 31 that \( G_1(\omega) + \text{i} G_2(\omega) \) is that part of the complex modulus associated with stress relaxation. In an alternating strain (dynamic) test,

\[
G^* = \sqrt{G_1^2 + G_2^2} \tag{25}
\]

is the ratio of peak stress to peak strain, and the phase angle \( \delta \) is given by

\[
\tan \delta = G_2/G_1. \tag{26}
\]

The frequency \( \omega = 2 \pi v \) in radians per minute if \( v \) is the frequency in cycles per minute.

With the aid of superposition, the complex modulus can be expressed in terms of the relaxation function,

\[
G^*(\text{i} \omega) = \omega \int_0^\infty \exp(-\text{i} \omega t) \overline{\Phi}(t) \, dt + G_\omega, \tag{27}
\]

or separated into its real and imaginary components,

\[
G_1(\omega) = \omega \int_0^\infty \overline{\Phi}(t) \sin \omega t \, dt \tag{28}
\]

and

\[
G_2(\omega) = \omega \int_0^\infty \overline{\Phi}(t) \cos \omega t \, dt. \tag{29}
\]

These integrals permit calculation of the complex modulus or the storage and loss moduli from the relaxation function.

Substituting the general Maxwell model form of the relaxation function, Equation 21, into Equations 28 and 29 yields

\[
G_1(\omega) = \omega \int_0^\infty \sum \sigma_i \exp(-t_i) \sin \omega t \, dt \tag{30}
\]

and

\[
G_2(\omega) = \int_0^\infty \sum \sigma_i \exp(-t_i) \cos \omega t \, dt \tag{31}
\]

The integrals of Equations 30 and 31 are easily evaluated analytically by parts, yielding

\[
G_1(\omega) = \sum \sigma_i \omega^2 t_i^2 / (1 + \omega^2 t_i^2) \tag{32}
\]

and

\[
G_2(\omega) = \sum \sigma_i \omega t_i / (1 + \omega^2 t_i^2). \tag{33}
\]

\( G_1(\omega) \) and \( G_2(\omega) \) were calculated from Equations 32 and 33 for the six sets of stress relaxation data and are shown in Figures 32 through 42. \( G^*(\omega) \) and \( \tan \delta \) were calculated from Equations 25 and 26 and are also shown in the same figures.

**Storage Modulus Curves**

The storage modulus \( G_1(\omega) \) is defined in Figure 31 and in Equations 28, 30, and 32. Under sinusoidal deformation, it can be described as the stress in phase with the strain divided by the strain. It is the quantity of energy stored or recovered in each cycle of deformation. Both the storage modulus and the relaxation modulus measure the amount of elastic energy stored; therefore, curves of these two functions can be compared quite successfully. In the case of the soils tested, the relaxation modulus goes to zero at very long times as does \( G_1(\omega) \) at very low frequencies (long times). Also, at time \( t = 0 \) (high or infinite frequency), the relaxation modulus \( G_{\text{max}} \) is approximately equal to the "glassy" modulus (horizontal portion) of the \( G_1(\omega) \) curves.

At very low frequencies, the storage modulus increases rapidly from near zero on a logarithmic slope of 2. As the frequency continues to decrease, \( G_1(\omega) \) will asymptotically approach zero, indicating that as the frequency decreases, the phase angle \( \delta \) between stress and strain approaches \( \pi/2 \) radians (the ratio of energy stored for each cycle of strain to that dissipated as heat becomes extremely small). This portion of the curve has often been described as the "zone of flow" where particles of soil undergo large displacements relative to their initial positions. These displacements are related to "times of adjustment". At low frequencies, the strain is maintained for a sufficiently long period to "break down" the various forces binding particles together, thus allowing the particles to move and adjust their positions. Consequently, this movement and rearrangement of particles causes the soil sample to act as a plastic and to undergo a large amount of flow. Therefore, in the zone of flow, the amount of elastic energy stored is small but increases rapidly with a small increase in frequency.
Figure 31. Complex Modulus.
Figure 32. Complex Modulus and Components – Kaolinite Series (0.3 Percent Strain).
Figure 33. Complex Modulus and Components -- Kaolinite Series (0.6 Percent Strain).
Figure 34. Complex Modulus and Components – Baxter Series.
Figure 35. Complex Modulus and Components – Calloway Series.
Figure 36. Complex Modulus and Components -- Eden Series.
Figure 37. Complex Modulus and Components -- Maury Series.
Figure 38. Complex Modulus and Components -- Western Kentucky Parkway, Milepost 96, Layer 1.
Figure 39. Complex Modulus and Components -- Western Kentucky Parkway, Milepost 96, Layer 2.
Figure 40. Complex Modulus and Components -- Western Kentucky Parkway, Milepost 96, Layer 3.
Figure 41. Complex Modulus and Components – Western Kentucky Parkway, Milepost 96, Layer 4.
Figure 42. Complex Modulus and Components – I 64, Milepost 44.
As the frequency increases to approximately 0.01 radians per minute, slopes of the $G_1(\omega)$ curves begin to decrease rapidly and continue this change over four logarithmic decades to approximately 100 radians per minute. These four decades are termed the "transition zone". Here the sample begins to act more elastically as the particles have less time to adjust to the cyclic strain. The storage modulus reaches a maximum at the end of the transition zone.

The term "glassy zone" is used to describe the horizontal portion of the curve. From the end of the transition zone, the storage modulus remains constant and at its maximum. The sample acts more as an elastic body as a result of higher frequencies or extremely short periods of oscillatory deformation. Soil particles are not able to adjust themselves to the stress in such short times and remain somewhat rigid or fixed relative to each other. It is this fixing of particle positions which produces the elastic effect of the sample and causes the storage modulus to be at its highest value.

In all of the tests, moisture content and density were held constant (within limits of experimental error). Therefore, the effect of these quantities on storage modulus could not be studied. The strain was constant for all remolded series and undisturbed soils (except the kaolinite series which was strained 0.6 percent instead of the usual 0.3 percent). In the kaolin samples which were strained 0.6 percent, the maximum in the curves was smaller, in most cases, than it was in the 0.3 percent samples.

The only other variable involved in testing was the confining pressure. As was expected, there was a general increase in magnitude of the storage modulus throughout the entire range of frequencies as the confining pressure was increased (Figures 43 through 45). However, there was very little correlation between storage modulus and confining pressure for the undisturbed soils.

In most cases, the maximum portion of the storage modulus curves was smaller in magnitude for the undisturbed soils than for the remolded soils, the exception being the kaolinite series (0.6 percent strain) which was comparable in magnitude to the undisturbed soils.

The assumption that stress relaxed to zero, or that $G_\infty = 0$, did not hold true in some cases (especially for the undisturbed soils). In these cases, the storage modulus did not asymptotically approach zero but decreased to a value equal to the relaxation modulus at infinity.

Loss Modulus Curves

Under a sinusoidal deformation, stress which is 90° out of phase with the strain is defined as the loss modulus $G_2(\omega)$. It is a measure of the energy dissipated as heat per cycle of strain. It is defined mathematically in Equations 29, 31, and 33.

An examination of the $G_2(\omega)$ curves in Figures 32 to 42 again reveal three distinct zones. At very low frequencies, the loss modulus rises from near zero on a logarithmic slope of 1 to a maximum which generally occurs somewhere within the logarithmic cycle of 0.001 radians per minute. This zone corresponds to the flow zone of the storage modulus. In this zone, energy which is dissipated as heat decreases with longer times because the mechanical energy being supplied to the sample by means of the cyclic strain is being transformed into heat energy, accounting for energy losses; however, this is a partially reversible process. Some heat energy in the sample is continuously being transformed back into mechanical energy. Therefore, the difference between these two processes equals the magnitude of the loss modulus; and as the frequency decreases, this difference approaches zero (6).

The general Maxwell model, Figure 14, used to represent the relaxation data could be expected to act almost perfectly elastically at high frequencies because the motion of the dashpot becomes negligible compared to that of the springs (2). Thus, the loss modulus curves show $G_2(\omega)$ approaching zero on a logarithmic slope of -1. This corresponds to the "glassy zone" of the storage modulus where there is an absence of particle adjustments capable of dissipating energy within the period of cyclic strain.

All loss modulus curves appeared to be very similar regardless of soil type. There appeared to be a fairly linear relationship between $G_1(\omega)$ and $G_2(\omega)$ when compared at the same frequencies (Figures 46 through 49). In the plastic or flow zone, there is less scatter of points than in the elastic or "glassy zone". In special relaxation tests, the sample received 0.01 inch strain at a confining pressure of 30 psi. After completion of the relaxation period (60 minutes), the confining pressure was increased to 50 psi and the sample allowed to consolidate overnight; the next day the specimen was subjected to a second strain of 0.01 inch. It appears particularly significant that the ratio for $G_2(\omega)$ to $G_1(\omega)$ is smaller for the special relaxation tests where the confining pressure $\sigma_c$ was 50 psi than for the other relaxation tests. This ratio is smaller because, at 50 psi confining pressure, the specimen had been strained previously and consequent began to act more elastically with a corresponding decrease in the internal friction and consequent heat loss at higher frequencies.

In the kaolinite series, which was strained 0.6 percent, the ratio of the first maximum to the second is much smaller, in most instances, than it was in the other remolded series. However, in nearly all tests on the undisturbed soils, the ratio of the first maximum to the second was greater than one; the reverse was true on the remolded soils.
Figure 43. Maximum Storage Modulus vs Confining Pressure - Remolded Soils.
Figure 44. Maximum Storage Modulus vs Confining Pressure – Western Kentucky Parkway, Milepost 96.
Figure 45. Maximum Storage Modulus vs Confining Pressure – I 64, Milepost 44.
Figure 46. Storage Modulus vs Loss Modulus (Frequency = 0.001 Radians Per Minute) - Remolded Soils.
Figure 47. Storage Modulus vs Loss Modulus (Frequency = 0.001 Radians Per Minute) – Western Kentucky Parkway, Milepost 96.
Figure 48. Storage Modulus vs Loss Modulus (Frequency = 1000 Radians Per Minute) -- Remolded Soils.
Figure 49. Storage Modulus vs Loss Modulus (Frequency = 1000 Radians Per Minute) -- Western Kentucky Parkway, Milepost 96.
**Loss Tangent Curves**

The loss tangent is a dimensionless quantity defined as the ratio of energy lost to energy stored in each cycle of deformation (Equation 26). For all soils tested, the three zones previously discussed were present in the loss tangent curves. In the flow zone (low frequencies), the loss tangent was a decreasing function with a logarithmic slope of \(-1\). This is reasonable because \(G_1(\omega)\) decreases much faster than does \(G_2(\omega)\) as the frequency decreases; consequently, the ratio of \(G_2(\omega)/G_1(\omega)\) would increase.

The transition zone, as in the case of the other functions, extends from approximately 0.001 to 100 radians per minute. In this zone, the loss tangent decreases sharply with increasing frequency and then levels to a small plateau region with one or two small maxima present.

Finally, as frequency increases beyond 100 radians per minute, the loss tangent decreases toward zero. This is the zone where the storage modulus is essentially constant and where the loss modulus is decreasing; therefore, the ratio \(G_2(\omega)/G_1(\omega)\) will decrease on the same logarithmic slope as \(G_2(\omega)\). All soils presented essentially the same trends, i.e., only small differences in the transition zone. There were no discernible trends in these differences.

In the case of the exceptions which were discussed under the storage modulus curves, the loss tangent approaches zero at low frequencies. The storage modulus is constant, but the loss modulus is decreasing; therefore, the ratio \(G_2(\omega)/G_1(\omega)\) will also approach zero.

**Complex Modulus Curves**

Equation 24 defines the complex modulus as the vector sum of \(G(\omega)=\text{equilibrium modulus} \text{ assumed zero for the soils tested}\), the storage modulus and the loss modulus. It becomes apparent that the complex modulus curve will have the same configuration and very nearly the same magnitude as the component modulus which happens to be larger at that particular frequency. The complex modulus curves in Figures 32 through 42 clearly demonstrate this. As the frequency decreases from approximately 0.01 radians per minute, the loss modulus becomes the larger and, consequently, the dominating component of modulus. Therefore, the shape of the complex modulus curve is nearly identical to the loss modulus curve at low frequencies.

As the frequency increases from 0.01 radians per minute, the larger component is the storage modulus and, as a result, dominates the complex modulus. Thus, for all practical purposes, from 0.1 radians per minute through higher frequencies, the complex modulus curve is a repeat of the storage modulus curve.

**CREEP FUNCTION**

Application of a constant stress \(\sigma\) produces a strain (Figure 50) given by

\[
\varepsilon = \varepsilon_0 + t/\eta_\infty + \Psi(t)\sigma
\]

where \(\varepsilon_0\) = the instantaneous component of strain
\(\eta_\infty\) = a Newtonian viscosity coefficient,
\(\Psi(t)\) = the creep function with \(\Psi(0) = 0\), and
\(J_\infty\) = equilibrium compliance.

All components are not always present; sometimes one or more may be equal to zero. However, if all components are present, application of a stress will produce an instantaneous strain. The deflection will continue to increase with time at a continuously decreasing rate until a constant or equilibrium rate of strain is reached.

The creep compliance, \(J\), can be found from

\[J = 2\varepsilon/S.\]

Substituting from Equations 11 and 12, it is found that

\[J = 3A\varepsilon_1/P.\]

For the undrained triaxial test, the creep compliance \(J\) is the inverse of the relaxation modulus \(G\). This becomes obvious upon analysis of Equations 13 and 36. Creep compliance curves calculated from the above equations are shown in Figures 51 through 55. Curves have been identified by confining pressure and stress level expressed as a percentage of maximum \((\sigma_1 - \sigma_3)\) determined in Phase 1 of this study (14).

![Figure 50. Strain under Constant Stress.](image)
Figure 51. Creep Compliance vs Time -- Kaolinite Series.
Figure 52. Creep Compliance vs Time -- Baxter Series.
Figure 53. Creep Compliance vs Time -- Calloway Series.
Figure 54. Creep Compliance vs Time - Eden Series.
Figure 55. Creep Compliance vs Time -- Maury Series.
In all soils studied, the instantaneous component of strain was not experimentally observable because the method employed did not permit a strain reading at zero time. Therefore, total strain was assumed to be the sum of only two components – the delayed or reversible component which is the creep function $\Psi(t)$ and a plastic component $t/\eta$. Equation 34 then becomes

$$e = \left[\frac{t}{\eta} + \Psi(t)\right] \sigma.$$  \hfill{37}

In nearly all cases, a test duration of 24 to 30 hours was sufficient time for the sample to reach an equilibrium rate of strain, thereby facilitating computation of the plastic component of strain $(t/\eta)$. A few specimens remained under constant stress for a period of 72 hours. Data from these tests also supported the conclusion that the rate of strain did not change perceptibly beyond 30 hours.

The creep function $\Psi(t)$ has been defined by Gross (3) as a continuous, uniformly increasing function. Ferry (2) and others (1, 3) suggest that the Voigt model (often referred to as the Kelvin model) exhibits properties of the creep function and is defined mathematically (see Figure 56) according to

$$\Psi(t) = J_1 \left[1 - \exp\left(-\frac{t}{\tau_1}\right)\right].$$  \hfill{38}

where $J_1$ = creep compliance and $\tau_1 = \eta_1/G_1$ = ratio of viscosity (dashpot) to shear rigidity (spring).

A group of Voigt elements in series will represent (as is generally the case with experimental data) an individually distinct spectrum of retardation times where each $\tau_1$ will be associated with a magnitude of compliance $J_1$. Since strains are additive in such a case, $\Psi(t)$ can be found by summing Equation 38 over the entire series of elements (2). Therefore,

$$\Psi(t) = \sum_{i=1}^{n} J_1 \left[1 - \exp\left(-\frac{t}{\tau_i}\right)\right].$$  \hfill{39}

However, for a continuous spectrum of retardation times, the creep function can be written in terms of the distribution of retardation times as defined by Gross (3):

$$\Psi(t) = \int_{0}^{\infty} \beta F(\tau) \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] d\tau.$$  \hfill{40}

where, as in the case of the relaxation function, $\beta$ is a normalization factor chosen such that

$$\int_{0}^{\infty} F(\tau) \, d\tau = 1.$$  \hfill{41}

This gives

$$\beta = \Psi(\infty);$$  \hfill{42}

and again, $F(\tau)d\tau$ is the retardation spectrum associated retardation times in the range between $\tau$ and $\tau + d\tau$. 

---

![Figure 56. Voigt Element.](image-url)
DISTRIBUTION FUNCTION OF RETARDATION TIMES

The distribution function of retardation times (like the relaxation spectrum) can be found by inversion of the LaPlace integral. However, the creep function (unlike the relaxation function) is not a LaPlace integral. For this reason, the rate-of-creep function, the derivative of the creep function, is introduced. Differentiating Equation 40 gives

\[
\frac{d\Psi(t)}{dt} = \int \beta F(\tau) \exp(-t/\tau) d\tau.
\]

43

This is the rate-of-creep function which can be transformed into a LaPlace integral by introduction of the frequency distribution

\[
N(c) = \beta F \left( \frac{1}{c} \right) c^{-2}.
\]

44

Substituting (3, 4, 5, 9) into Equation 43 gives

\[
\frac{d\Psi(t)}{dt} = \int \frac{cN(c) \exp(-tc)}{c} dc.
\]

45

Differentiating Equation 38, the equation of the Voigt model representing the experimental creep data, gives

\[
\frac{d\Psi(t)}{dt} = J_1 \exp(-t/\tau_1)/\tau_1.
\]

46

Equation 46 does not lend itself to LaPlace inversion. Therefore, the experimental data (Figure 57) were fitted to

\[
\Psi(t) = J_1 \left[ 1 - \exp(-b_1 \sqrt{t}) \right].
\]

47

Taking the derivative of this equation gives

\[
\frac{d\Psi(t)}{dt} = J_1 b_1 \exp(-b_1 \sqrt{t})/2\sqrt{t}.
\]

48

Equation 48, the rate of creep function, is the LaPlace transform of a known function,

\[
J_1 b_1 \exp(-b_1^2/4c)/2\sqrt{\pi c}.
\]

49

Therefore, Equation 49 is the distribution function of retardation times. Equation 48 is equivalent to the left-hand side of Equation 45 and Equation 49 is equal to cN(c) of Equation 45. Therefore, the distribution function is given by summing the corresponding terms of the form of Equation 49. Retardation distribution functions are shown in Figures 58 through 62.

The graphical curve-fitting procedure to find the equation of the creep curve is basically the same as for the relaxation curve. Log J(t) is plotted against \( \sqrt{t} \). The equilibrium rate of strain (plastic flow) is then subtracted from J(t) at each point in order to find the retarded portion of strain. The remainder of the fitting procedure is the same.

Figure 57. Comparison of Observed and Computed Creep Functions.
Figure 58. Retardation Distribution Functions -- Kaolinite Series.
Figure 59. Retardation Distribution Functions - Baxter Series.
Figure 60. Retardation Distribution Functions – Calloway Series.
Figure 61. Retardation Distribution Functions -- Eden Series.
Figure 62. Retardation Distribution Functions -- Maury Series.
Figure 63 demonstrates clearly that when \( b_i \) is held constant and \( J_i \) varies, only the magnitude of the peak of the retardation spectrum is affected; consequently, \( J_i \) does not affect the location of the peak. In the relaxation spectrum, when \( b_i \) varied, both the magnitude and location of the peak are affected. The effect of \( b_i \) on the retardation spectrum is shown in Figure 64.

There appeared to be no predictable trends in the distribution functions produced by either confining pressure, stress level or soil type.

**COMPLEX COMPLIANCE FUNCTION**

With the application of an alternating stress \( \sigma = \sigma_0 \exp(i\omega t) \), a sinusoidal strain \( \varepsilon = J^*(i\omega)\sigma \), is produced, part of which is in phase and part out of phase. \( J^* \) is defined as the complex shear compliance, which can be resolved into its real and imaginary constituent vectors as shown in Figure 65.

The vector sum of \( J_1(\omega) \), the storage compliance, and \( J_2(\omega) \), the loss compliance, is that part of the compliance associated with the creep function and can be written as \( J(\omega) = J_1(\omega) + ij_2(\omega) \). In the alternating stress test, the phase angle \( \delta \) can be defined as the ratio of the loss compliance to the storage compliance,

\[
\tan \delta = \frac{J_2(\omega)}{J_1(\omega)}.
\]

The frequency \( \omega \) (radians per minute) is defined in like manner as for the complex modulus function.

As in the case of the complex modulus and the relaxation function, the complex compliance can be expressed in terms of the rate-of-creep function (3),

\[
J^* = \varepsilon_0 + \frac{1}{\omega \eta} + \int \exp(i\omega t) \frac{d\Psi(\tau)}{d\tau} d\tau.
\]

Separating \( J^* \) into its real and imaginary components gives

\[
J_1(\omega) = \int_0^\infty \frac{d\Psi(\tau)}{d\tau} \cos \omega \tau \; d\tau
\]

and

\[
J_2(\omega) = \int_0^\infty \frac{d\Psi(\tau)}{d\tau} \sin \omega \tau \; d\tau.
\]

However, for a finite number of Voigt elements, the storage and loss compliances, \( J_1(\omega) \) and \( J_2(\omega) \), can be found from the following equations (2):

\[
J_1(\omega) = J_g + \sum_{i=1}^{n} J_i(1 + \omega^2 \tau_i^2)
\]

and

\[
J_2(\omega) = \sum_{i=1}^{n} J_i \omega \tau_i(1 + \omega^2 \tau_i^2)
\]

where \( J_g \) equals the compliance produced by the instantaneous component of strain \( \varepsilon_0 \).
Because $\epsilon_0$ was not observed experimentally, the complex compliance $J^*$ is merely the vector sum of $J_1(\omega), J_2(\omega)$, and $1/\omega \eta$ (Figure 66) and is given by

$$J^* = \sqrt{(J_2(\omega) + 1/\omega \eta)^2 + J_1(\omega)^2}.$$ 56

$J_1(\omega), J_2(\omega), J^*$ and tan $\delta$ are shown in Figures 67 through 71 where $J_1(\omega)$ and $J_2(\omega)$ were calculated from Equations 54 and 55. $J^*$ and tan $\delta$ were calculated from Equations 56 and 50, respectively.
Figure 67. Complex Compliance and Components -- Kaolinite Series.
Figure 68. Complex Compliance and Components – Baxter Series.
Figure 69. Complex Compliance and Components – Calloway Series.
Figure 70. Complex Compliance and Components -- Eden Series.
Figure 71. Complex Compliance and Components -- Maury Series.
Storage Compliance Curves

Under a sinusoidal deformation, the strain in phase with the stress divided by the stress is defined as the storage compliance (Equations 52 and 54). It, like the storage modulus, is a measure of energy stored or regained during each cycle of stress. For soils, it is necessary to subtract the plastic flow \( i/\eta \) from the total compliance to obtain the storage compliance.

For the soils tested, the lower frequencies (up to 0.1 radians per minute) produced a horizontal line equal in magnitude to the equilibrium compliance \( J_m \). In the transition zone (from 0.01 to 100 radians per minute), the storage compliance changes rapidly from a horizontal line to a logarithmic slope of -2. At these higher frequencies, the storage compliance decreases to zero on the previously mentioned slope. This is as expected since compliance is a measure of "ability" to strain in response to stress. As the sample becomes more elastic or rigid with increasing frequency, the compliance must therefore approach zero.

As in the case of the complex modulus, the different soil series produced similar curves. The most striking difference was noted when comparing samples with the same confining pressure but with different stress levels. For most samples, application of a higher percentage of ultimate failure stress produced a higher equilibrium compliance (horizontal portion). There was also a general trend for the curves with higher equilibrium compliances to "break" and begin dropping toward zero at lower frequencies than did the samples with lower equilibrium compliances.

Loss Compliance Curves

The loss compliance, defined by Equations 53 and 55, is the strain 90° out of phase with the stress divided by the stress. It is another function by which energy dissipation may be measured. Curves for the test specimens were very similar to the loss modulus curves; the major difference between the two functions is in the magnitude and units. Examination of Equations 33 and 55 shows the two functions to be identical, except \( G_i \) is used to calculate the loss modulus and \( J_1 \) is used in calculating the loss compliance.

When compared to loss modulus curves, the maxima in the loss compliance curves were shifted toward the high frequency end of the time spectrum. The first maximum generally occurred between 0.01 to 0.1 radians per minute, and the second usually occurred at approximately 1000 radians per minute. However, there were several notable exceptions. These exceptions presented curves with very narrow transition zones; and, in some cases, only one maximum was present. For such samples, the maximum or maxima were likely to occur at any frequency. These unusual tests occurred in any soil, at any stress level, and at any confining pressure.

Unlike the complex modulus function, a comparison of loss compliance to storage compliance at the same frequency did not produce a linear relationship.

Loss Tangent Curves

When considering the complex compliance function, the loss tangent is defined as \( J_2(\omega)/J_1(\omega) \) (Equation 50). Loss tangent curves for the complex compliance are similar in shape to loss tangent curves for the complex modulus, but the curves are "inverted". The loss tangent rises from zero on a logarithmic slope of 1 to the beginning of the transition zone, which usually occurs near 0.01 radians per minute. From the end of the transition zone, the loss tangent again rises on a logarithmic slope of 1 as the frequency continues to increase.

There are four functions which measure energy dissipation or heat loss in a dynamic test: loss modulus, loss compliance, loss tangent using \( G_2/G_1 \) and loss tangent using \( J_2/J_1 \). A quick observation of these four functions shows that the frequency at which the most energy is lost depends upon which function is under consideration (2).

Complex Compliance Curves

Equation 56 defines the complex compliance as the vector sum of the storage compliance and the loss compliance plus the additional plastic component of strain, \( 1/\omega \eta \). For soils, \( \eta \) is extremely large, usually on the order of \( 10^6 \). With such a large denominator, the plastic component will have, in higher ranges of frequency, a very small magnitude. As a consequence, the plastic component will not affect the complex compliance to any large degree. However, as the frequency becomes very low (\( 10^5 \) and below), \( 1/\omega \eta \) becomes a fairly sizeable number and thus the dominate component of strain.

As in the case of the complex modulus, complex compliance curves take the shape and approximately the magnitude of its largest component at the particular frequency range. At higher frequencies, the curve decreases on a logarithmic slope of -1, and so conforms to the loss compliance component. The complex compliance is horizontal and approximately equal to the equilibrium compliance on the low frequency end. For some of the samples, as the frequency decreased below \( 10^5 \) radians per minute, the complex compliance deviated from the horizontal and began to increase slightly. This was due to the plastic component \( 1/\omega \eta \) becoming the largest component and, consequently, controlling the magnitude of the complex compliance.
DISCUSSION

The hydraulic system used to apply a static and constant load to the creep test specimen had a tendency to lose minute amounts of pressure over a period of several hours. This effect, which would tend to make the strain rate decrease more rapidly than normal, doubtless influenced the results of calculations of the plastic component of strain. However, the decrease was relatively small – the maximum being approximately two percent of the applied load.

In calculating and plotting the complex modulus and the complex compliance and their components, calculations were extrapolated approximately two logarithmic cycles beyond the range of data at both ends of the time spectrum. This was done for the purposes of computer calculations. Consequently, the data in the first two and the last two logarithmic cycles must be treated as extrapolated data and should be credited with little further significance other than indications of general behavior.

It is possible that the similarity of the behavior of the different remolded soils could have been due to the method of preparation. It was noticed in working with remolded samples that all had a tendency to break in a helical fashion, reminiscent of the way in which they were molded. Intuitively, this could be said to have influenced their behavior; however, the data, though similar, do not necessarily support this conclusively.

A fairly large error would be introduced if the sample was not properly placed in the test chamber. This would induce side friction between the loading piston and the top end cap. If samples were not placed directly under the loading piston, small eccentricities in the loading pattern would develop. If sample ends were not trimmed perfectly flat and perpendicular to the longitudinal axis, an incorrect pattern of stress would result. These possible sources of error were hopefully kept to a minimum by extremely cautious experimental techniques. It is believed that a small amount of inertial effect was present in the loading apparatus; however, due to the somewhat preliminary status of this phase of research, these effects were not studied.

CONCLUSIONS AND RECOMMENDATIONS

Data in both the relaxation and creep tests fit the linear models sufficiently well to support the use of the principle of linear superposition.

All relaxation samples which were strained (loaded) more than once acted more elastically under each subsequent load. This is in good agreement with the findings of other researchers.

It can be said that, in general, an increase in confining pressure also produced an increase in the magnitude of the complex modulus for remolded soils. However, there was little correlation amongst undisturbed soils.

There appeared to be a direct, linear relationship between the magnitudes of the storage modulus and the loss modulus (when compared at the same frequency). The correlation was considerably better at the low frequency end (0.001 radians per minute) than at higher frequencies (1000 radians per minute). This relationship is associated with the total energy input necessary to produce equivalent strains in different specimens. The most notable exception to this was the kaolinite series which was strained 0.6 percent. In this case, it was believed that the almost instantaneous strain of such magnitude failed a number of samples. (For a more detailed discussion, the reader is again referred to the first report on this research study (14).)

An increase in static stress level in a creep test produced a corresponding increase in the magnitude of the storage compliance and consequently an increase in the complex compliance. The generally linear association between the magnitudes of the storage and loss moduli was not evident between the storage and loss compliances.

Distribution functions were useful in describing rheological behavior of stress relaxation and strain retardation samples. Retardation distribution functions gave broader distributions due to the difference in the mathematical definitions of the creep and relaxation functions.

In the relaxation distribution function, a peak with a large magnitude shifted toward the low frequency end of the time spectrum indicated that most relaxation would occur slowly over long periods of time; the converse was true of a low magnitude peak shifted toward high frequencies.

Confining pressure had little or no consistent effect on the relaxation and retardation distribution functions. Static stress level apparently had no consistent effect on the retardation distribution functions.

The following recommendations are offered:

1. Further testing should be done allowing other parameters such as strain amplitude, density and moisture content to vary.
2. Dynamic tests should be performed and the results compared to results reported herein.
3. A refinement of the hydraulic loading system should be made to eliminate pressure drops over long periods of time.


NOMENCLATURE

A = Area of sample
\( a_i, b_i \) = Constants of the rheological model
c = Relaxation or retardation frequency = \( 1/\tau \)
e = Deviator strain
\( \tilde{F} \) = Relaxation spectrum
F = Retardation spectrum
G = Relaxation shear modulus
\( G_0 \) = Instantaneous elastic modulus
\( G^* \) = Static or equilibrium elastic modulus
\( G_1 \) = Storage modulus
\( G_2 \) = Loss modulus
J = Creep compliance
\( J_e \) = Equilibrium creep compliance
\( J^* \) = Complex compliance
\( J_1 \) = Storage compliance
\( J_2 \) = Loss compliance
K = \( 1/3 \) \( \sigma \epsilon_1 \)
\( N(\epsilon) \) = Distribution function
P = Applied load
S = Deviator stress
s = t - u
t = Time
u, \( \tau \) = Time variables
\( \beta \) = Normalization factor chosen such that \( \int \tilde{F}(\tau)d\tau = 1 \)
\( \beta^* \) = Normalization factor chosen such that \( \int F(\tau)d\tau = 1 \)
b = Frequency in cycles per minute
\( \delta \) = Phase angle
e = Strain rate
e = Mean normal strain
e_0 = Instantaneous strain (t = 0)
e_i = Strain at time \( t_1 \)
e_I = Strain in the first principal stress direction
\( \eta \) = Newtonian viscosity coefficient
\( \sigma \) = Mean normal stress
\( \sigma_1, \sigma_2, \sigma_3 \) = Principal normal stresses
\( \sigma_a \) = Applied load/area of sample (P/A)
\( \sigma_c \) = Confining pressure = \( \sigma_3 \)
\( \tilde{\sigma} \) = Relaxation stress
\( \tilde{\Psi}(t) \) = Relaxation function
\( \Psi(t) \) = Creep function
\( \omega \) = Frequency in radians per minute = \( 2\pi b \)
### Grain Size Analysis (Percent Finer)

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### Classification

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CREEP TEST PROCEDURE

Preparation of remolded creep specimens was the same as that used in the relaxation test samples previously described in the first report on this study (14). A detailed description of the testing equipment and the procedure used in setting up the creep tests can also be found in the same reference.

After the sample was allowed to consolidate overnight in the triaxial chamber, the desired load was applied. Loading was accomplished by mounting an inverted load cell, normally used in consolidation tests, atop the triaxial chamber. Air pressure was applied to the cell and controlled by a control panel.

After firm contact was made between the loading piston and the sample end cap, the desired load was then set on the control panel and applied instantaneously to the sample by means of an "on-off" toggle valve. Strain was monitored by a 0.001-inch dial extensometer. The extensometer moved so rapidly in the first few seconds after loading that visual-manual recording was impossible. Therefore, a Bolex-H 16 reflex movie camera, running 64 frames per second, was used to record the strain for the first ten seconds. Subsequent readings were made visually at 15, 30, 45 and 60 seconds. Readings were made thereafter at double the previous time interval. Most tests continued for 24 hours; a few were continued for 72 hours.

Creep specimens were preconsolidated to three different confining pressures -- 10, 30 and 50 pounds per square inch. Four tests were performed at each confining pressure, each being subjected to a different stress level. The ultimate strength of a particular soil series under a particular confining pressure was determined from Phase I of this study (14). Therefore, the four stress levels which were applied to the samples were equal to 25, 40, 55 and 70 percent of the ultimate strength of the soil at that particular confining pressure. The results were analyzed according to equations presented under the section entitled "Creep Function".