Modifications to Chevron N-Layer Computer Program

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MODIFICATIONS TO
CHEVRON N-LAYER COMPUTER PROGRAM

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ABSTRACT

This report documents changes made to the Chevron N-layer computer program to:

1. Include superposition principles.
2. Calculate strain energy density (or work) at specified locations within the pavement structure.
3. Analyze pavement response at specified radii from one circularly loaded area to permit comparison of analyses by the program as originally written with results incorporating superposition principles.
4. Evaluate pavement response to any combination of loads on circular areas defined by XY coordinates on the surface. Loads and contact pressures are permitted to be different from one loaded area to another, but must be constant for any one loaded area.
5. Simulate dynamic loads as the difference between the root mean squares of the maximum and minimum dynamic loads. This analysis is appropriate for constant vibratory testers such as Road Raters and Dynaflects. Moduli of asphaltic concrete must be adjusted for frequency effects.
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INTRODUCTION

The Chevron N-Layer computer program (1) was received in 1968 from the Chevron Research Company, Chevron Oil Corporation. Appendix A contains the background documents as written and distributed by Chevron Research Company. That version permitted analyses of one circular load applied to the surface, and output was requested at specified radii from the center of the loaded area. The program was modified to permit analyses of multiple loads located at coordinates in the X-Y plane, and output for specified locations within the X-Y-Z coordinate system may be requested. Superposition principles were applied to each component of strain and stress to resolve them into a value compatible with the X-Y-Z coordinate system. Appropriate equations are shown in Figure 1. The Kentucky modified Chevron N-layer program is listed in Appendix B. Appendix A also contains the letter from Chevron granting permission to publish the listed program as modified herein.

The original Chevron N-layer program contained a subroutine called COE5 that limited the number of layers to be analyzed to five. Shortly thereafter, another subroutine was written by Chevron called C015 that expanded the capability to analyze a pavement having 15 layers. Some users noted that slightly different answers for locations near the surface of the pavement were obtained, depending upon whether subroutine COE5 or C015 was being used. Several years later, these two subroutines were replaced with a subroutine called COFE that resolved the problem, and this subroutine is used in the listing given in Appendix B. The number of layers is still limited to 15.

For those having an original Chevron N-layer program, one additional change will be noted. In Appendix B, Subroutine CALCIN has been split into two sections. The part listed herein as Subroutine CALCIN is the first half of the original subroutine. The output portion of the original subroutine was separated to form Subroutine PRN7.

STRAIN ENERGY

The "work" done by a force when its point of application is displaced is the product of that force (parallel to the direction of movement) and the displacement. When work is done on some systems, the internal geometry is altered in such a way that there is a potential to "give back" work when the force is removed, and the system returns to its original configuration. This stored energy is defined as strain energy. Strain energy per unit volume at a given point in the body is the strain energy density at that point.

Strain energy density is a function of the Young's modulus of elasticity and Poisson's ratio of the material and the nine strain (or stress) components; however, it is independent of the coordinate system. Stress and strain components, referenced to a local cylindrical coordinate system, for each load are calculated by the Chevron program (1). The classical equation for strain energy density derived by Sokolnikoff (2) is as follows:
\[ W = \frac{1}{2} \lambda \sum_{i} \varepsilon_{i}^{2} + G \varepsilon_{ij} \varepsilon_{ij} \]
\[ = \frac{1}{2} \lambda \sum_{i} \varepsilon_{i}^{2} + G(\varepsilon_{11}^{2} + \varepsilon_{22}^{2} + \varepsilon_{33}^{2} + \varepsilon_{12}^{2} + \varepsilon_{23}^{2} + \varepsilon_{13}^{2}), \]  
(1)

in which \( W \) = strain energy density, or energy of deformation per unit volume;

\( \varepsilon_{ij} \) = \( i, j \)th component of the strain tensor;

\( G = \frac{E}{2(1 + \mu)} \), the "modulus of rigidity" or the "shear modulus";

\( E \) = Young's modulus;

\( \mu \) = Poisson's ratio;

\( \lambda = \frac{E \mu}{(1 + \mu)(1 - 2\mu)} \); and

\( \gamma = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \).

Strain energy density may be calculated using stress components by the equation

\[ W = -\mu \theta^{2}/2E + ((1 + \mu)/2E)(\sigma_{11}^{2} + \sigma_{22}^{2} + \sigma_{33}^{2}) + \\
\]
\[ (4(1 + \mu)/E)(\sigma_{12}^{2} + \sigma_{23}^{2} + \sigma_{31}^{2}), \]  
(2)
in which \( \theta = \sigma_{11} + \sigma_{22} + \sigma_{33} \) and

\( \sigma_{ij} \) = \( i, j \)th component of the stress tensor.

Equation 2 differs from Sokolnikoff's (2) Equation 26.17 by a factor of "4" in the third term. Shear stress is usually written as \( 2(1 + \mu) \varepsilon_{ij}/E \).

Sokolnikoff (2, p 25) states, "The factor \( 1/2 \) was inserted in the formulas (7.5) in order that the set of quantities may transform according to the Tensor Laws". It has been verified that the Chevron N-Layer Program computes shear stresses in a manner that includes the "4" as shown in Equation 2.

Inspection of Equation 1 shows that the terms \( E \) & \((1 + \mu)\) are contained by means of the terms \( \lambda \) and \( G \). Also, it is noted that the strain components are squared. Having calculated strain energy density, "work strain" (3) may be obtained from

\[ \varepsilon_{w} = (2 W/E)^{0.5} \]  
(3)
in which \( \varepsilon_{w} \) = work strain. The associated "work stress" is given by \( EE_{w} \).

INTERPRETATIONS OF WORK STRAIN

Admittedly, work strain is not a true strain because Poisson's ratio has not been eliminated prior to taking the square root; however, it is of the same order of magnitude as any of the strain components. Calculating the work strain is a minor effort since all terms of the equations are either required input to, or calculated output of, the Chevron N-layer (1, 4) program. Work strain is also the composite, or net effect, of all strain components and thus
is an indicator of the total strain behavior. Figure 1 illustrates there is a direct correlation between a strain component and work strain.

USES FOR WORK STRAIN

Some thickness design systems for flexible pavements are based partially upon tensile strain criteria at the bottom of the asphaltic concrete layer. Kentucky's system (5-10) is based in part upon the tangential strain component. The tangential component is generally the largest in magnitude, but the radial component often is nearly as large. Only the tangential component has been utilized because laboratory test data yield one component of tensile strain. The net effect of all components of strain (work strain) can be correlated with any component of strain. Thus, design systems based upon one component of strain may be converted to a design system that utilizes the net effect of all component strains. The load-damage factor relationships presented in this paper are based on work strain. All comments concerning component strains also apply to component stresses.

The concept of work strain has been used as the basis for the development of thickness design systems for pavements consisting of portland cement concrete or pozzolanic base courses under asphaltic concrete (11-13).

MODIFICATIONS TO ORIGINAL CHEVRON N-LAYER PROGRAM

GENERAL COMMENTS

The first data card, or line of input data, contains two variable names — OPTN and OUTPUT. OPTN determines which type of analyses is to be performed. OUTPUT specifies a hard copy form and/or storage of output in the computer's memory for OPTN = 2 or 3 only. For OPTN = 1, OUTPUT has no meaning. A value of 1 for OUTPUT produces a hardcopy and stores the output in the computer memory. A blank or 0 produces only a hardcopy. A value of 2 stores the output in computer memory only.

OPTN = 1

A value of "1" causes the program to run according to the original Chevron N-Layer program. This version restricts the input data to one circular load and output locations are specified as radial distances from center of the load.

OPTN = 2

A value of "2" permits the entry of as many as 99 loads at locations in the form of X-Y coordinates, and up to 99 desired output locations may be specified as X-Y coordinates and up to 99 depths (Z) for each X-Y coordinate. As many as 15 layers of materials with their respective input data for thickness, modulus, and Poisson's ratio may be investigated in each computer run.

OPTN = 3

Kentucky research has shown that dynamic deflection testing may be simulated utilizing the Chevron N-layer computer program with static loads as
input. Two adjustments must be made to make the simulation match measured values. First, the modulus of the asphaltic concrete layers is frequency dependent and must be adjusted to an equivalent value appropriate to the testing frequency. The second adjustment involves the oscillating load. While the actual loading is applied as a sine wave, the sensors measure either velocity waves or accelerations and the electronic circuits integrate, or double integrate, the signals respectively to obtain the deflection readings shown on the meters. Thus, the deflections are equivalent to static deflections. Therefore, the dynamic loading applied as a true sine wave may be considered as a static loading by converting the sine wave to an equivalent square wave by

\[
MXSL = \text{MEAN} + 0.707 \times (MXPL - \text{MEAN}) \quad (4)
\]

and

\[
MNSL = \text{MEAN} - 0.707 \times (\text{MEAN} - MNPL), \quad (5)
\]

in which

- \( MXSL = \) maximum static load,
- \( MNSL = \) minimum static load,
- \( MXPL = \) maximum dynamic load,
- \( MNPL = \) minimum dynamic load, and
- \( \text{MEAN} = \) dead load about which the dynamic loading oscillates.

Correlation testing of in-place pavements has shown that the measured deflection as displayed on the meters is identical to the calculated values for each sensor location by

\[
DEFL = \text{DEFMX} - \text{DEFMN} \quad (6)
\]

in which

- \( DEFL = \) deflection displayed on the meter for that sensor,
- \( \text{DEFMX} = \) calculated deflection corresponding to the adjusted load \( MXSL \), and
- \( \text{DEFMN} = \) calculated deflection corresponding to adjusted load \( MNSL \).

Thus, for \( \text{OPTN} = 3 \), the calculated deflections, stresses, and strains are obtained by storing the calculated values for the equivalent maximum square-waved loads, then subtracting the calculated values for the minimum square-waved loads and printing the resulting values. This procedure has been used most successfully for both the Road Rater (14-16) and for limited data obtained with the Dynaflect. It has not been attempted for a falling weight tester.
TYPICAL PROBLEMS INVESTIGATED

WHEEL LOADS

The original Chevron N-layer program permitted behavioral analyses of a pavement due to the application of one load. Such analyses corresponded to the use of OPTN = 1. These analyses did provide a major breakthrough at that time. However, it became evident very quickly that analyses were needed for multiple wheel loads and axles.

AXLELOAD ANALYSES

Actual tire loadings and configurations required modifications to be made to the program. The first modifications involved those found in OPTN = 2. Other modifications were made and reported (3). However, the version contained in Appendix B covers all those options. Except for the original version and the simulation of dynamic loadings, the other options primarily were written to choose which variables were to be output. One exception to the above was an attempt to analyze axleloads for each group of axles on a given vehicle. While this type of analysis is possible, analyses may be made by inputting all groups of axles as separate problems using OPTN = 2 and summing the results manually.

SIMULATION OF DYNAMIC DEFLECTIONS

As stated earlier, OPTN = 3 should be used to simulate dynamic deflection tests. Under this option, the stresses, strains, and deflections are calculated for the equivalent maximum and minimum dynamic loads when expressed as the square wave of the applied sinusoidal loading. These deflections have correlated with the measured values as shown on the meters of either the Road Rater or the Dynaflect testers.

EXAMPLES

The input and output data for an example problem for each method of analyses is presented in Appendix C.

RECOMMENDATIONS

The modified Chevron N-Layer Computer Program is recommended for analyzing pavements using a single wheel load, multiple wheelloads on one axle, or multiple axles (up to a total of 99 wheelload locations), and for simulation of dynamic testers such as the Dynaflect and various models of Road Raters.

REFERENCES


SUPERPOSITION EQUATIONS

\[ \sigma_x = \sigma_R \cos^2 \theta - 2\tau_{RT} \cos \theta \sin \theta + \sigma_T \sin^2 \theta \]

\[ \tau_{xy} = \sigma_R \cos \theta \sin \theta + \sigma_{RT} (\cos^2 \theta - \sin^2 \theta) - \sigma_T \sin \theta \cos \theta \]

\[ \tau_{xz} = \tau_{RZ} \cos \theta - \tau_{TZ} \sin \theta \]

\[ \sigma_y = \sigma_R \sin^2 \theta + 2\tau_{RT} \sin \theta \cos \theta + \sigma_T \cos^2 \theta \]

\[ \tau_{yz} = \tau_{RZ} \sin \theta + \tau_{TZ} \cos \theta \]

\[ \sigma_z = \sigma_z \]

\[ \tau_{yx} = \tau_{xy} \]

\[ \tau_{zx} = \tau_{xz} \]

\[ \tau_{zy} = \tau_{yz} \]

Figure 1. Basic Equations by Superposition Principles.
APPENDIX A

ORIGINAL CHEVRON DOCUMENTATION
April 18, 1978

Mr. Herbert F. Southgate
Chief Research Engineer
Kentucky Department of Transportation
Division of Research
533 South Limestone
Lexington, Kentucky 40508

Dear Mr. Southgate:

You requested in your letter of April 4, 1978, our permission to publish modifications to our layered elastic computer program used for pavement design. We are delighted to grant you this permission.

Our current policy is to supply upon request the layered elastic program to universities and public agencies at no charge. With this in mind, you should not feel restricted in the use of the program. We understand you will give appropriate reference and acknowledgement in your publications to Chevron Research Company for providing the original programs. We greatly appreciate the recognition you have given us in the past.

Your proposed modifications to the program sound interesting. Your interim report on a rational overlay design method is particularly timely. I would appreciate a copy of the report, when available, and a copy of the modified elastic layered program.

You and your associates at the Kentucky Department of Transportation have done an excellent job of pioneering the field of rational pavement design. We are delighted that the Chevron program provided some assistance in accomplishing that task. We look forward to your effort on a rational overlay design method.

Sincerely,

Larry Santucci
L. E. Santucci

LES:cic
NUMERICAL COMPUTATION OF STRESSES AND STRAINS IN A MULTIPLE-LAYERED ASPHALT PAVEMENT SYSTEM

September 24, 1963

By

H. Warren and W. L. Dieckmann
The program numerically carries out the solutions given by Mr. J. Michelow in his analysis of multilayered asphalt pavement systems. The present program computes vertical, tangential, radial, and shear stress together with vertical and radial strain at any point in the system for a given load on a circular area of the surface.

At interfaces between two layers, the stresses are computed only for the lower layer. The corresponding radial and tangential stresses for the upper layer are proportional to those given for the lower, with ratios the same as the ratio of the moduli of elasticity of the two layers.

Input

The order and format of the data cards are given in the attached input sheet. Each system to be analyzed is represented by six cards in the data deck. Any number of systems may be processed on the same run by placing sets of cards one after the other.

The stress and strain solutions may be obtained in either absolute terms (e.g., psi and inches) or relative to a unit pressure. In the latter case, the input total load and tire pressure are used only to define the radius of the loaded area. The type of solution desired (absolute or relative) is indicated on the second input card in Column 36 by a 0 (or blank) for absolute or a 1 for relative output.

Output

The input heading supplied by the user is printed out at the top of each page. The results and a list of the parameters are printed out in columns below the heading.

The Organization of the Computation

The final answers are expressed as integrals of the form

\[ \int_0^\infty T(m)dm \quad (1) \]

For purposes of numerical evaluation (1) is replaced by

\[ \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} T(m)dm \quad (2) \]

Each of the integrals in (2) is evaluated using four-point Legendre-Gauss quadrature [1]. To maximize the accuracy of

Encl. - Input Sheet
Sample Output
these evaluations, the set of $x_i$'s are chosen from the zeros of $T(m)$, such that in the interior of any interval $(x_{i-1}, x_i)$ there is at most one zero of $T(m)$.

Two criteria determine the value of $n$ - that is, when the integration will stop. One is the size of the integrand; the other is the number of intervals already integrated over. Specifically, the integration is terminated at $x_n$ if either condition a or b is satisfied.

a. The integrals over $(x_{n-2}, x_{n-1})$ and $(x_{n-1}, x_n)$ both have magnitude less than $10^{-4}$.

b. $n$ reaches its maximum assigned value of 46.

Because $T(m)$ is a product of two Bessel functions times a term which decays as $K m^{-p} e^{-zm}$, condition (a) is an approximation to

$$\int_{x_n}^{\infty} T(m) \, dm \leq 10^{-4}. \tag{3}$$

$p$ is an integer greater than one and $K$ is only a function of $r$. and $z$; where $r$ is the radial coordinate and $z$ is the vertical coordinate of a point in the elastic system. Equation (3) will be satisfied for $m$ greater than one if

$$K e^{-zm_n} < 10^{-4}$$

which is equivalent to

$$zm_n > 4 \log 10 - \log K.$$

If $M = \max(a, r)$ where $a$ is the load radius, then $m_{zc}$ is approximately equal to $71/M$. It follows that when the integration stops, condition (a) will hold if

$$\frac{z}{M} > \frac{4 \log 10 - \log K}{71}. \tag{4}$$

Conversely, condition (a) will not be satisfied if $z$ is too small, or either $a$ or $r$ is too large. In concrete terms, if the point under consideration is (i) too near or on the surface of the system, or (ii) too far from the center of the load area, or (iii) if the load area is too large, then the convergence of Equation (2) to (1) is slow, and the computations of the present program are more subject to error.

It should be noted, however, that (ii) and (iii) are problem caused only because (i) is being evaluated numerically. The zeros of $T(m)$ are approximately inversely proportional to $a$ and $r$; hence increasing them shortens the intervals $(x_{i-1}, x_i)$.
The remedy for this problem is to take a great number of intervals; but to limit computing time some number $u$ must be set as the maximum number of intervals ($u = 3$ is used for $u$ in this program).

The main work of the program lies in evaluating $T(m)$. The parameters of $T$ are the coordinates of the point $(r, z)$ and all the physical constants describing the system. As shown in Michelow's report, a large part of the work of the evaluation of $T(m)$ goes into evaluating the functions $A_j(m)$, $B_j(m)$, $C_j(m)$, and $D_j(m)$, which do not include the parameter $z$ but depend on $r$. Hence, these evaluations are made once for each $r$, and the computation then proceeds to finding the solutions for the different $z$'s. A further saving would be made by grouping all points $(r, z)$ such that $r < a$, since in this event $M = 0$, and the same subdivision $\{x_i\}$ applies to all these points (although the cutoff point, $x_n$, may differ). This refinement has not been incorporated into the present program.

The evaluation of $A_j(m)$, $B_j(m)$, $C_j(m)$, and $D_j(m)$ is carried out by one of two subroutines, depending on the size of $m$. For large $m$, $A_j$ and $B_j$ decrease exponentially and may be assumed to be zero, in which case the computation of $C_j$ and $D_j$ can be proceed independently. It can be shown that $C_j$ and also $D_j$ asymptotically approach multiples of $m^{-3}$ or $m^{-2}$, depending on the parameters of the system. Using asymptotic approximations to these variables should be considered if it is desirable to continue the integration in slowly convergent cases of type (i).

**Comparison with Published Results**

No known published results exist for the upper one third of the top layer. In other regions the results obtained with this program compare almost identically with those published by Jones [2] and by Mehta [3]. We have gotten good results in the difficult upper region but have experienced slow convergence as the $z$ coordinate (depth below surface) goes to zero. The extreme case, at $z = 0$, where the theoretical results are known, is considered below.

**Error Estimation in Slowly Convergent Cases**

Whenever the integration of (1) is terminated by condition $b$ ($n = 46$), the program writes the comment "SLOW CONVERGENCE" for that printed line of output. In computing the deflections, damping factors other than the exponential make it unlikely that slow convergence of type (i) will produce any error over 5%. Further, the vertical and shear stresses for $z = 0$ - that is, at the surface - are known from the boundary conditions, namely:
vertical stress = \begin{cases} 
-1, & r < a \\
-0.5, & r = a \\
0, & r > a 
\end{cases}

shear stress = 0 for all r

Comparing the computed values with these boundary conditions gives a good idea of the quality of convergence, which may well give results good to 1% even though the "SLOW CONVERGENCE" message is written out.

The results from the analysis of the following two systems which converged slowly illustrate this conclusion:

System 1

System 1 consists of two layers with a load radius of 1.5. The layers have the following characteristics:

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<th>Thickness</th>
<th>Modulus of Elasticity</th>
<th>P's Ratio</th>
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<tr>
<td>1</td>
<td>3.0</td>
<td>1000</td>
<td>0.25</td>
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<tr>
<td>2</td>
<td>2</td>
<td>2000</td>
<td>0.25</td>
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System 2

System 2 consists of four layers with a load radius of 14.82. The layers have the following characteristics:

<table>
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<th>Modulus of Elasticity</th>
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<td>1</td>
<td>2.0</td>
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<td>0.75</td>
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<td>2</td>
<td>3.0</td>
<td>15,000</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>4.0</td>
<td>12,000</td>
<td>0.50</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>10,000</td>
<td>0.45</td>
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</table>

On the surface (z = 0), at radial distance (r), vertical stress was computed to be
The shear stresses were all computed to be less than $10^{-6}$.

Thus, we see that the computed values for System 1 are close to the theoretical values of the vertical stress which is $-1$ for values of $r$ less than 1.5, $-0.5$ at $r$ equal 1.50, and zero for values of $r$ greater than 1.5. None of the computed shear stresses exceeded $10^{-6}$ which is close to the theoretical value of zero for all radii.

For System 2, the computed values are very close to the theoretical value of $-1$. In this system, $r$ does not exceed the load radius.

Survey of the Component Routines of the Program

Main Routine

Handles input and output (except for writing the final answers) and calls the subroutines, which do all the arithmetic operations.

PART

Calculates the partition $\{x_i\}$ and the four points in each interval $(x_{i-1}, x_i)$ at which $T(m)$ is to be evaluated for integration.

COEB and COHIGH

Computes $A_1(m)$, $B_1(m)$, $C_1(m)$, and $D_1(m)$. COHIGH works in the range where $A_1$ and $B_1$ are effectively zero.
**BESSER**

Computes either $J_0(m)$ or $J_1(m)$ to within an absolute tolerance of less than $10^{-6}$. This routine could be used in any program requiring evaluation of zero and first-order Bessel functions.

**CALCIN**

Completes the evaluation of $T(m)$ and performs the integration of (2). It writes out the final results.

**HIGHM**

As now compiled, just prints out the message "SLOW CONVERGENCE." There is, however, a FORTRAN deck for an alternate version of HIGHM which would effectively raise the value of $u$ on a point-by-point basis - that is, continue the integration indefinitely, or until some criterion such as an error bound or time limit was met. This alternate version of HIGHM would be used with the subroutines RANGE2-346 and CDHIGH 338, which are already compiled and fulfill the functions of CALCIN and COHIGH, respectively.

References


ANALYSIS OF STRESSES AND DISPLACEMENTS IN AN n-LAYERED ELASTIC SYSTEM UNDER A LOAD UNIFORMLY DISTRIBUTED ON A CIRCULAR AREA

September 24, 1963

By

J. Michelow
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I. Introduction

Asphalt pavements may be idealized as a semi-infinite solid with three or four layers of substances whose elastic characteristics are different from one layer to the other.

In view of this, a computer program for the IBM 7090 is being prepared at the request of the Asphalt Development Group. This program will give the distribution of stresses and deformations of such an n-layered system when subject to a load uniformly distributed on a circular area on the free surface of the semi-infinite solid. The interfaces between layers are considered to be "rough."

Classical papers in this field were consulted [1;2;3]. None of these papers was considered satisfactory for several reasons; hence, we conducted our own analysis of the problem. This analysis constitutes the content of this report and possesses the following desirable characteristics:

1. The system may have any number of layers.
2. The elastic coefficients in each layer can assume any value (where the Poisson modulus $\nu$ is different from one).
3. The mathematical analysis is straightforward and self-contained.
4. The constants of integration are given explicitly and can be found by solving a system of two linear equations with two unknowns.
5. The results are given in a form convenient for a digital computer to handle.

The major contribution of this paper is that of Items 3, 4, and 5. Mehta and Veletsos$^2$ had also considered systems satisfying 1 and 2.

II. Description of the System

A. Geometry of the System

Consider an orthogonal system of coordinates in three dimensional space, where the $x$ and $y$ axes are in a horizontal plane, and the $z$ axis, therefore vertical, is pointing downward.

Consider the semi-infinite solid consisting of all points $(x, y, z)$ where $z > 0$. This solid will be divided in $n$ "layers" $L_i$, $i = 1, 2, \ldots, n$ by the collection of numbers

$$h_0 = 0 < h_1 < h_2 < \ldots < h_{n-1}.$$  (1)
The layer $L_i$ will consist of all points $(x, y, z)$ such that

$$h_{i-1} \leq z < h_i,$$  \hspace{1cm} (2)

for $i = 1, 2, \ldots, n-1$. The layer $L_n$ will consist of all points $(x, y, z)$ such that

$$h_{n-1} \leq z.$$  \hspace{1cm} (3)

The plane $z = h_i$ ($i = 0, 1, 2, \ldots, n-1$) will be called the $i^{th}$-interface.

B. Elastic Characteristics of the System

The semi-infinite solid will be assumed to be perfectly elastic, homogeneous, etc.

$E_i$ will denote the modulus of Young and $v_i$, the modulus of Poisson for layer $L_i$.

C. Description of the Load

We will consider a load $f(x, y)$ normal to the $0^\text{th}$-interface given by the following equation.

$$f(x, y) = \begin{cases} 1 & \text{whenever } x^2 + y^2 < a^2 \\ \frac{1}{2} & \text{whenever } x^2 + y^2 = a^2 \\ 0 & \text{whenever } x^2 + y^2 > a^2 \end{cases}.$$  \hspace{1cm} (4)

Later on we will see why we define $f(x, y)$ to be $1/2$, when $x^2 + y^2 = a^2$, and not one.

We will also consider a load $f(x, y)$ normal to the $0^\text{th}$-interface given by the equation

$$f(x, y) = p(m) J_0 \left[ m (x^2 + y^2)^{1/2} \right],$$  \hspace{1cm} (5)

where $p(m)$ is an arbitrary function of the parameter $m$, and $J_0(t)$ denotes the Bessel function of the first kind of order zero.

The load described by equation (4) will be called Load A, and the one described by equation (5) will be called Load B.

D. Description of the Boundary Conditions

For each layer $L_i$ the boundary conditions are the conditions that have to be satisfied at the $i^{th}$-interface and at the $(i-1)^{th}$-interface.
We will consider the case when the 1st-interface is "rough", that is, the components of stress $\sigma_r$ and $\tau_{r\theta}$ as well as the components of the displacement $u$ and $w$ are continuous at $z = h_i \ (i = 1, 2, \ldots, n-1)$. (See IIIA below.)

At the 0th-interface, sometimes called the free surface, $\sigma_z$ has to be equal to the load and $\tau_{z\theta}$ has to be zero.

E. Description of the Problem

To determine the components of stress and displacement at any point $(x, y, z)$ in a system with a geometry given by Section IIA, with elastic characteristics as the ones given in Section IIB, with boundary conditions as described in Section IID, when subject to one of the loads defined in Section IIC.

We will consider then two different problems that we will call A and B, depending on which of the loads of Section IIC is acting on the system.

As we will see later on (Section VI), the solution of Problem B leads to the solution of Problem A by means of superposition. In the following sections we will consider it advantageous to adopt a cylindrical system of coordinates: Each point is defined by the coordinates $(r, \theta, z)$ where

$$r = (x^2 + y^2)^{1/2},$$
$$\cos \theta = x/r, \sin \theta = y/r,$$
and
$$z = z. \quad (6)$$

The reason for this is that the $z$ axis is an axis of symmetry of the geometry of the system as well as of the load.

III. Basic Equations

A. For a cylindrical system of coordinates, the components of stress are:

$$\sigma_z, \sigma_r, \sigma_\theta, \tau_{z\theta}, \tau_{r\theta}, \tau_{r\theta};$$

and the components of displacement are:

$u$, the radial displacement,
$v$, the tangential displacement, and
$w$, the vertical displacement.

Because of the symmetry of the system under consideration

$$\tau_{r\theta} = \tau_{z\theta} = v = 0. \quad (7)$$
We have, then, to determine six functions: four components of stress, \( \sigma_r, \sigma_\theta, \sigma_z, \tau_{rz}, \) and two components of displacement, \( u, v. \)

B. The four components of stress have to satisfy two sets of equations called the equilibrium conditions and the compatibility conditions, respectively.

The equilibrium conditions are:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \quad \text{and} \quad \frac{\partial \sigma_\theta}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0. \tag{8}
\]

The compatibility conditions are:

\[
\nabla^2 \sigma_r - \frac{2}{r^2} (\sigma_r - \sigma_\theta) + \frac{1}{1 + \nu} \frac{\partial^2 Q}{\partial r^2} = 0, \tag{10}
\]

\[
\nabla^2 \sigma_\theta + \frac{2}{r^2} (\sigma_r - \sigma_\theta) + \frac{1}{1 + \nu} \frac{1}{r} \frac{\partial Q}{\partial r} = 0, \tag{11}
\]

\[
\nabla^2 \tau_{rz} + \frac{1}{1 + \nu} \frac{\partial^2 Q}{\partial z^2} = 0, \quad \text{and} \quad \frac{\partial^2 Q}{\partial r \partial z} = 0. \tag{12}
\]

\[
\nabla^2 \tau_{rz} - \frac{1}{r^2} \tau_{rz} + \frac{1}{1 + \nu} \frac{\partial^2 Q}{\partial r \partial z} = 0. \tag{13}
\]

Here

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}, \tag{14}
\]

\[
Q = \sigma_r + \sigma_\theta + \sigma_z, \quad \text{and} \quad \nu \text{ is Poisson's modulus.} \tag{15}
\]

C. We can simplify the problem by expressing the four components of stress and the two components of displacement in the following way:

\[
\sigma_r = \frac{3}{\partial z} \left[ \nu \nabla^2 \phi - \frac{\partial^2 \phi}{\partial r^2} \right], \tag{16}
\]

\[
\sigma_\theta = \frac{3}{\partial z} \left[ \nu \nabla^2 \phi - \frac{1}{r} \frac{\partial \phi}{\partial r} \right], \tag{17}
\]

\[
\sigma_z = \frac{3}{\partial z} \left[ (2 - \nu) \nabla^2 \phi - \frac{\partial^2 \phi}{\partial z^2} \right], \tag{18}
\]

\[
\tau_{rz} = \frac{3}{\partial r} \left[ (1 - \nu) \nabla^2 \phi - \frac{\partial^2 \phi}{\partial z^2} \right], \tag{19}
\]

\[
\nu \text{ is Poisson's modulus.} \tag{23}
\]
\[ u = -\frac{1 + \nu}{E} \frac{\varepsilon^2 \phi}{\delta r \delta z}, \text{ and} \]

\[ w = \frac{1 + \nu}{E} [2(1 - \nu) \nabla^2 \phi - \frac{\partial^2 \phi}{\partial z^2}], \quad (21) \]

where \( \phi(r, z) \) is a so-called Airy's function.

By expressing \( \sigma_r, \sigma_\theta, \sigma_z, \) and \( \tau_{rz} \) in this way, we automatically satisfy equation (8). Moreover, equations (9) through (13) are also satisfied whenever \( \phi \) is a solution of

\[ \nabla^4 \phi = 0. \quad (22) \]

The problem has been reduced to finding one function \( \phi \) that satisfies equation (22) and the boundary conditions of the problem under consideration.

D. It is easy to check that equation (22) is satisfied by the function

\[ \phi(r, z, m) = J_0(mr)[(A + Bz)e^{mz} + (C + Dz)e^{-mz}], \quad (23) \]

where \( J_n(t) \) is the Bessel function of the first kind of order \( n \), and \( m \) is a parameter. \( A, B, C, \) and \( D \) are constants of integration; hence they do not depend on \( r \) and \( z \), but they may be functions of the parameter \( m \).

IV. Boundary Conditions

A. It follows from the considerations of the previous section that in the layer \( L_1 \), the solution of the problem will be given by the Airy's function:

\[ \phi_1(r, z, m) = J_0(mr)[(A_1 + B_1z)e^{mz} + (C_1 + D_1z)e^{-mz}], \quad (24) \]

where the constants \( A_1, B_1, C_1, \) and \( D_1 \) should be determined in such a way that the boundary conditions are satisfied.

The condition of continuity at the interfaces is not difficult to fulfill. [See equation (31).]

B. Equation (24), together with equations (16) through (21), give:
\[ j_{2 \downarrow} = m^2 J_0(mr) \left\{ (1-2v_1)[D_1 e^{mz} + D_1 e^{-mz}] - m[(A_1+B_1z)e^{mz} - (C_1+D_1z)e^{-mz}] \right\}, \]

\[ \tau_{rz} = m^2 J_1(mr) \left\{ 2v_1[D_1 e^{mz} - D_1 e^{-mz}] + m[(A_1+B_1z)e^{mz} + (C_1+D_1z)e^{-mz}] \right\}, \]

\[ u^\downarrow = \frac{1+v_1}{E_1} m J_1(mr) \left\{ B_1 e^{mz} + D_1 e^{-mz} + m[(A_1+B_1z)e^{mz} - (C_1+D_1z)e^{-mz}] \right\}, \]

\[ w^\downarrow = \frac{1+v_1}{E_1} m J_0(mr) \left\{ 2(1-2v_1)[B_1 e^{mz} - D_1 e^{-mz}] - m[(A_1+B_1z)e^{mz} + (C_1+D_1z)e^{-mz}] \right\}, \]

Continuity conditions at the \( i \)th-interface can be expressed by

\[ S_{i}(h_1 - 0) = S_{i+1}(h_1), \quad i = 1, 2, \ldots, n-1. \]
V. Solution of Problem D

A. To solve Problem B, we need only to determine the constants $A_i, B_i, C_i,$ and $D_i$ ($i = 1, 2, \ldots, n$) that fulfill the conditions given by equation (31) and that are compatible with Load B. See last paragraph of Section IID.

We have $4n$ constants to determine; equation (31) gives us $4n-4$ equations. The compatibility of the stresses with Load B gives us two more equations. The fact that at infinity the stresses should be zero allows us to conclude that $A_n$ and $B_n$ should be zero; this gives us the last two equations needed.

B. From equations (25) through (28), we see that the matrix $S_1(z)$ can be expressed in the following form:

$$S_1(z) = K(v_1, E_1)M(z, v_1)D(z)\begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix}, \quad (32)$$

where

$$K(v, E) = \begin{bmatrix} -m^2 J_0(mr) \\ m^2 J_1(mr) \end{bmatrix}, \quad (33)$$

$$M(z, v) = \begin{bmatrix} 1 & mz + 2v - 1 & -1 & -mz + 2v - 1 \\ 1 & mz + 2v & 1 & mz - 2v \\ 1 & mz + 1 & -1 & -mz + 1 \\ 1 & mz + 4v - 2 & 1 & mz - 4v + 2 \end{bmatrix}, \quad (34)$$

and

$$D(z) = \begin{bmatrix} me^{mz} \\ e^{mz} \\ me^{-mz} \\ e^{-mz} \end{bmatrix}. \quad (35)$$

C. Therefore, equation (31) can be written as:

$$K(v_1, E_1)K(h_1, v_1)D(h_1)\begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} = K(v_{i+1}, E_{i+1})M(h_1, v_{i+1})D(h_1)\begin{bmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \end{bmatrix}. \quad (36)$$
Hence, we can express $A_i$, $B_i$, $C_i$, and $D_i$ as functions of $A_{i+1}$, $B_{i+1}$, $C_{i+1}$, and $D_{i+1}$ by

$$
\begin{pmatrix}
A_i \\
B_i \\
C_i \\
D_i \\
\end{pmatrix} = D^{-1}(h_1)M^{-1}(h_1,\nu_1)K^{-1}(\nu_1,E_1)K(\nu_{i+1},E_{i+1})M(h_1,\nu_{i+1})D(h_1)
$$

Note that

$$
K^{-1}(\nu_1,E_1)K(\nu_{i+1},E_{i+1}) = \begin{pmatrix} 1 & 1 \\ L_i & L_i \end{pmatrix},
$$

where

$$
L_i = \frac{E_i}{E_{i+1}} \frac{1 + \nu_{i+1}}{1 + \nu_i}.
$$

It is easy to check that

$$
M^{-1}(z,\nu_1) = \frac{1}{4(\nu_1-1)} \begin{pmatrix}
-mz-1 & mz+4\nu_1-2 & mz+2\nu_1-1 & -mz-2\nu_1 \\
1 & -1 & -1 & 1 \\
-mz+1 & -mz+4\nu_1-2 & mz-2\nu_1+1 & mz-2\nu_1 \\
1 & 1 & -1 & -1 \\
\end{pmatrix}.
$$

Let us define the matrix $X_i$ by

$$
X_i = 4M^{-1}(h_1,\nu_1)K^{-1}(\nu_1,E_1)K(\nu_{i+1},E_{i+1})M(h_1,\nu_{i+1})(\nu_1-1),
$$

thus
\[
\begin{array}{c|c|c}
2(1-L_4) & (2m_{1,1-v,1}(1-L_4)+1) & (4v_{1}-3) - L_4 \\
(2m_{1,1-v,1}(1-L_4)+1) & (2m_{1,1-v,1}(1-L_4)+1) & (2m_{1,1-v,1}(1-L_4)+1) \\
0 & (4v_{1}-3) - L_4 & (2m_{1,1-v,1}(1-L_4)+1) \\
L_4(v_{1}-4v_{1}+1) & -L_4 p(-h_{1},1,1+1) & L_4 p(h_{1},1,1+1) \\
\end{array}
\]
where
\[ p(h, v, \mu) = m^2 (2h^2) + 4mh(v-\mu) + (1-8v_1 \mu + 2\mu) \]
and
\[ q(h, v, \mu) = 2mh(2v_1-1) - (1+8v_1 \mu - 6\mu) \]  
Finally, we can write equation (37) as
\[
\begin{pmatrix}
A_1 \\
B_1 \\
C_1 \\
D_1
\end{pmatrix} = D^{-1}(h) \begin{pmatrix}
X_1 \\
D(h_1)
\end{pmatrix} = \begin{pmatrix}
A_{i+1} \\
B_{i+1} \\
C_{i+1} \\
D_{i+1}
\end{pmatrix}
\]
We can, therefore, by repeated use of equation (45), express the set of constants \(A_i, B_i, C_i,\) and \(D_i\) as functions of \(C_n\) and \(D_n\) for all \(i = 1, 2, \ldots, n-1.\) We thus obtain
\[
\begin{pmatrix}
A_1 \\
B_1 \\
C_1 \\
D_1
\end{pmatrix} = \frac{n-1}{j=1} \begin{pmatrix}
D^{-1}(h_j) X_j D(h_j)
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
C_n \\
D_n
\end{pmatrix}
\]
D. From equation (26), using the condition \((\tau^2_{rz})_{z=0} = 0,\) we obtain
\[ m A_1 + 2v_1 B_1 + m C_1 - 2v_1 D_1 = 0 \]  
From equation (25), imposing the condition that
\[ (\sigma^1_z)_{z=0} = -p(m)J_0(mr) \]
we obtain
\[ m A_1 + (2v_1-1)B_1 - m C_1 + (2v_1-1)D_1 = p(m)/m^2 = K(m) \]
Therefore, we can write the matrix equation
\[
\begin{pmatrix}
0 \\
K(m)
\end{pmatrix} = \begin{pmatrix}
m & 2v_1 & m & -2v_1 \\
m & 2v_1-1 & -m & 2v_1-1
\end{pmatrix} \begin{pmatrix}
A_1 \\
B_1 \\
C_1 \\
D_1
\end{pmatrix}
\]
E. The constants \(C_n\) and \(D_n\) can be determined by combining equations (50) and (46). We thus obtain
\[
\begin{pmatrix}
0 \\
K(m)
\end{pmatrix} = \begin{pmatrix}
m & 2v_1 & m & -2v_1 \\
m & 2v_1-1 & -m & 2v_1-1
\end{pmatrix} \frac{n-1}{j=1} \begin{pmatrix}
D^{-1}(h_j) X_j D(h_j)
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
C_n \\
D_n
\end{pmatrix}
\]
This gives a system of two linear equations in two unknowns whose solution is immediate. (We do not expect the determinant of the system to be zero.)

F. In summary, Problem B is solved by equations (25) through (30), where \( C_n \) and \( D_n \) are determined from equation (51) and \( A_1, B_1, C_1, \) and \( D_1 \) are given by equation (46), \( (i = 1, 2, \ldots, n-1) \).

VI. Solution of Problem A

A. From equation (51) we see that \( C_n \) and \( D_n \) are proportional to \( p(m) \); therefore, equation (46) tells us that \( A_i, B_i, C_i, \) and \( D_i \) are proportional to \( p(m) \) for all \( i = 1, 2, \ldots, n-1 \). We can conclude from equations (25) through (30) that \( \sigma_{i2}, \tau_{iz}, \sigma_{i3}, \tau_{iz}, u_i, \) and \( w_i \) are proportional to \( p(m) \) for \( i = 1, 2, \ldots, n \).

Therefore, we can put \( K(m) = 1/m^2 \) in equation (49) and write the factor \( p(m) \) in front of equations (25) through (30).

B. Because equations (16) through (21), as well as equation (22), are linear in \( \phi \), whenever the Airy's function \( \phi(r, z, m_j) \) solves Problem B, then the function

\[
\phi(r, z) = \sum_{j=1}^{n} \Delta j \phi(r, z, m_j)
\]

solves the problem of finding stresses and displacement when the system is subject to a load

\[
\sum_{j=1}^{n} \Delta j p(m_j)J_0(m_j \rho).
\]

Therefore, the function

\[
\phi(r, z) = \int_0^\infty \phi(r, z, m) dm,
\]

will solve the problem of finding stresses and displacements when the system is subject to a load.

\[
\int_0^\infty p(m)J_0(m \rho) dm.
\]

C. The inversion formula for the Hankel transform of order zero is:

\[
\frac{1}{2} [g(z-0) + g(z+0)] = \int_0^\infty m \int_0^\infty r g(r)J_0(mr) dr J_0(r \rho) dm,
\]

provided \( g(r) \) satisfies a certain set of conditions that we will call Condition C [6, p. 52].
In particular we see that if we chose
\[ p(m) = m \int_0^\infty f(r) J_0(mr) dr , \]  
where \( f(r) \) is the function defined by equation (4), then equation (55) gives Load A. This is permissible because the function \( f(r) \) satisfies Condition C.

We see now why equation (4') takes that rather peculiar form at \( r = a \). Integrating equation (57) we get.
\[ p(m) = a J_1(ma) . \]  

D. In summary, the chain of reasoning is the following:

1. We chose an Airy's function \( \phi(r, z, m) \) as the one given by equation (23).

2. We chose a Load B as the one given in equation (5) where \( p(m) \) is defined by equation (58).

3. By the last paragraph of Section VIA, we see that the stresses and displacements are given by equations (25) through (30) with a factor \( a J_1(ma) \) in front of each equation.

4. The constants \( A_1, B_1, C_1, D_1, (i = 1, 2, \ldots, n-1) \) \( C_n \) and \( D_n \) are given as functions of the parameter \( m \); and \( A_n = B_n = 0 \), by equations (51) and (46), where \( K(m) = 1/m^2 \).

5. Then we chose an Airy's function given by
\[ \phi(r, z) = \int_0^\infty \phi(r, z, m) dm , \]
where \( \phi(r, z, m) \) is the function chosen in Step 1.

6. Sections VIB and VIC tell us that the Airy's function \( \phi(r, z) \) solves Problem A.

7. The stresses and displacements defined by the function \( \phi(r, z) \) are:
\[ \bar{\sigma}_z = a \int_0^\infty J_1(ma) \sigma_z^1 dm , \]  
\[ \bar{\sigma}_r = a \int_0^\infty J_1(ma) \sigma_r^1 dm , \]  
\[ \bar{\sigma}_\theta = a \int_0^\infty J_1(ma) \sigma_\theta^1 dm , \]  
\[ \bar{\tau}_{rz} = a \int_0^\infty J_1(ma) \tau_{rz}^1 dm , \]  
\[ \bar{u}_r = a \int_0^\infty J_1(ma) u_r^1 dm , \]  

and
\[ \bar{u}_z = a \int_0^\infty J_1(ma) u_z^1 dm , \]
\[
\overline{w} = a \int_0^\infty J_1(ma) \overline{w} dm, \quad (1 = 1, 2, \ldots, n).
\] (54)

\(c_1, c_2, c_3, \tau_1, u_1, w_1\) are given by equations (25) through (30). The constants being determined as in Step 4. This finally solves Problem A.
VII. Bibliography


May 9, 1980

Chevron N-Layer Computer Program - Improved Accuracy

Mr. Herbert F. Southgate
Chief Research Engineer
Kentucky Department of Transportation
533 South Limestone
Lexington, KY 40508

Dear Mr. Southgate:

Attached is a report by Mr. L. J. Painter of our staff describing a new subroutine COFE for the Chevron N-Layer Elastic System computer program. This new routine, which completely replaces the two earlier routines, COE5 and CO15, makes accurate calculations of all stress, strain, and deflection parameters for any number of layers, up to the maximum program dimension of 15 layers.

Additionally, the new subroutine will provide computer run time savings of as much as 45%.

If you have previously received a FORTRAN source deck of the program from us, you will be sent a source deck for the new subroutine.

Very truly yours,

A.L. Grossberg

Encl. - Report, above subject and date
Summary

This report describes a new subroutine COFE to replace the two subroutines COE5 and CO15. The new routine eliminates the serious numerical errors in CO15 (used for more than 5 layers) which have hampered the use of the program in research on asphalt pavement systems. The results using the new subroutine are every bit as accurate as those obtained from the older subroutine COE5 used for 5 or fewer layers, which is excellent. A listing of the new subroutine is attached.

To use the new routine, it is only necessary to remove the original routines COE5 and CO15, replacing them by the single new routine COFE. COFE has two entry points, COE5 and CO15, so that no change is necessary in the calling (MAIN) program.

A further benefit of the new routine is that typical execution times will be reduced as much as 45%.

Background

The original version of the Chevron n-layer program as developed in 1963 was limited to 5 or fewer layers. An ingenious method of scaling the calculations was used to avoid overflows (generation of numbers larger than can be handled by the computer). This required, however, separate sections of computer code in subroutine COEE for each possible layer. [Subroutine COEE computed the coefficient matrices A(I,K), B(I,K), C(I,K), and D(I,K) for the K-th layer at the I-th level of the variable of integration AZ(I).]

In 1967 when the program was expanded to handle up to 15 layers, it was found impractical to continue this programming scheme. Hence, a separate and much simpler subroutine CO15 was written to handle the general n-layer case. Unfortunately, numerical difficulties arose because numbers in excess of e\textsuperscript{174.67} were generated in the calculation of the intermediate product matrices PM, leading to overflow. The power of e as used in parts of the calculations is 2*AZ(I)*H(N); AZ

Encl. - Table I
Program Listing
could be as high as \( \pi \cdot 23/\text{radius of the loaded area} \) and \( H(N) \) is the depth of the lowest interface between layers (in some research problems, as much as 300 inches). With a load radius of about 10 inches, this could give exponents of \( 2 \pi \cdot 23 \cdot 300/10 = 4335 \), far in excess of the (IBM) computer capacity when exponentiated.

Several reviews of the computer algorithm in CO15 were unsuccessful in solving the problem, so a simple check was made on the magnitude of the offending value and if it approached the critical level, further calculation was halted and the remaining \( A, B, C, \) and \( D \) values were set equal to zero; hopefully they would have been small anyway. This truncation of the series to be integrated gave rise to numerical errors which were at least annoying for pavement structures of moderate maximum depth. But for deep structures, such as in studies of the effect of subsoil elastic properties changing with depth, the errors could be disastrous.

The original 5-layer routine (renamed COE5) was kept to preserve accuracy for systems of 5 or fewer layers, with CO15 being used only for more than 5 layers.

**Solution of Problem**

The calculation of the product matrices \( PM \) has been reorganized so as to include the scaling by \( e^{-AZ[H(N) + H(K)]} \) from the very start. This led to a simplification of the exponential multipliers required.

After initialization of \( PM(N,M,J), M = 1,4 \) and \( J = 3,4 \), we obtain:

For \( M = 1 \) or \( 2 \),

\[
PM(K,M,J) = T1 \cdot e^0 + T3 \cdot e^{-2P \cdot H(K)};
\]

for \( M = 3 \) or \( 4 \),

\[
PM(K,M,J) = T1 \cdot e^{2P \cdot H(K)} + T3 \cdot e^0,
\]

with

\[
T1 = X(K,M,1) \cdot PM(K+1, 1, J) + X(K,M,2) \cdot PM(K+1, 2, J),
\]

\[
T3 = X(K,M,3) \cdot PM(K+1, 3, J) + X(K,M,4) \cdot PM(K+1, 4, J),
\]

and \( P = AZ(I) \).

Detailed study of the course of the calculations has shown that by the time \( e^{-2P \cdot H(K)} \) becomes unmanageable, \( T1 \) has been driven to zero.
Judicious checking of the magnitude of the exponent makes it possible to keep the calculations under control at all times.

As a result, it is possible to substitute the new routine COFE for the two older routines COE5 and CO15. Use of entry points COE5 and CO15 in COFE obviates the need for modifications to the MAIN (calling) program.

Timing

Computer runs on a 5-layer problem using the new COFE and the old COE5 routines show a 45% reduction in CPU time (1.15 seconds versus 2.08 seconds). No meaningful comparisons can be made with CO15 because that routine was effectively short-circuited as soon as potential overflow occurred.

Verification of Accuracy

A check on the accuracy has been made between a 4-layer solution (known to be accurate) and a 6-layer problem artificially constructed by splitting one layer into three with identical elastic properties.

Basic problem parameters were: 30,000 lb load, 105 psi tire pressure.

<table>
<thead>
<tr>
<th>Layer, i</th>
<th>E1</th>
<th>ni</th>
<th>Thickness, In.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150,000</td>
<td>0.35</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>17,800</td>
<td>0.40</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>6,000</td>
<td>0.40</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>555,555</td>
<td>0.40</td>
<td>Semi-infinite</td>
</tr>
</tbody>
</table>

The 6-layer problem split layer 3 above into 3 layers of 100-inches thickness each. The calculated values of several stresses and strains are compared in Table I. In general, the new COFE routine agrees with the 4-layer solution to about 5 significant digits, compared to discrepancies by a factor of almost 2 for the old program at the surface.

References

### TABLE I
COMPARISONS OF NEW AND OLD CALCULATIONS IN LAYERED ELASTIC SYSTEMS

**Vertical Stress Values**
6-Layer

<table>
<thead>
<tr>
<th>R</th>
<th>z</th>
<th>COE5</th>
<th>COFE</th>
<th>C015</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-105.00</td>
<td>-105.00</td>
<td>-63.29</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>-96.72</td>
<td>-96.72</td>
<td>-57.19</td>
</tr>
<tr>
<td>0</td>
<td>38</td>
<td>-5.63</td>
<td>-5.63</td>
<td>-5.19</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>0.47</td>
<td>0.47</td>
<td>-7.33</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>-0.82</td>
<td>-0.82</td>
<td>-8.27</td>
</tr>
<tr>
<td>19</td>
<td>38</td>
<td>-3.71</td>
<td>-3.71</td>
<td>-3.73</td>
</tr>
</tbody>
</table>

**Tangential Strain**

<table>
<thead>
<tr>
<th>R</th>
<th>z</th>
<th>COE5</th>
<th>COFE</th>
<th>C015</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-968.85</td>
<td>-968.83</td>
<td>-491.14</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>794.50</td>
<td>794.47</td>
<td>331.02</td>
</tr>
<tr>
<td>0</td>
<td>38</td>
<td>408.44</td>
<td>408.44</td>
<td>402.34</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>-201.35</td>
<td>-201.35</td>
<td>-252.48</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>67.69</td>
<td>67.68</td>
<td>120.22</td>
</tr>
<tr>
<td>19</td>
<td>38</td>
<td>314.83</td>
<td>314.84</td>
<td>314.99</td>
</tr>
</tbody>
</table>

**Vertical Deflection**

<table>
<thead>
<tr>
<th>R</th>
<th>z</th>
<th>COE5</th>
<th>COFE</th>
<th>C015</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.1013</td>
<td>0.1013</td>
<td>0.0846</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0.1003</td>
<td>0.1003</td>
<td>0.0841</td>
</tr>
<tr>
<td>0</td>
<td>38</td>
<td>0.0425</td>
<td>0.0425</td>
<td>0.0424</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>0.0408</td>
<td>0.0408</td>
<td>0.0443</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>0.0410</td>
<td>0.0410</td>
<td>0.0444</td>
</tr>
<tr>
<td>19</td>
<td>38</td>
<td>0.0335</td>
<td>0.0335</td>
<td>0.0335</td>
</tr>
</tbody>
</table>
SUBROUTINE COF(KIN)
USE FOR ALL PROBLEMS, UP TO MAX DIMENSION OF 15 LAYERS
REPROGRAMMED 1 MAY 1980 BY L.J. PAINTER - EXCELLENT ACCURACY
NOTE DOUBLE ENTRIES FOR COE1 & COE2 PREVIOUSLY USED
COMM COF0000

COMMON /RNC0Y/RR(99), ZE(97), E(15), V(15), HH(14), COF00070
1 H(10), A(36), M(36), B(36,15), C(36,15), COF00003
2 T(36,15), A(36), R(36), RU(36), T1(36), T1(36), TITLE(20), COF00090
3 TEST(11), BZ1(100), M(36,4), DD(14), FN(4), COF00100
4 PH(14,4), R, D, AP, M3, COF00110
5 N, L, ITH, RZZ, RST, COF00110
6 RC, R, SF, CSZ, CST, COF00120
7 CSM, CTR, CMU, CMU, CST, FSI, COF01440
8 NLINE, NOUTP, NTEST, I, ITN, COF00150
9 K, LC, JT, TZZ, PR, COF00140
A PA, P, T, TP, T1, T1, COF00170
B T1, T2, T3, T4, T5, T6, COF00190
C T6, TLP, TCM, W, DJl, COF00190
D BJO, ZF, C1, SIZ, S1, COF00210
E SCZ, PH, PH2, VB2, VB2, COF00210
F VK4, VKF4, VKG, RST, RDS COF00220
REAL*8 R(1,2)
ENTRY COF(KIN)
ENTRY COF0000
ENTRY COLBRED

LC = KIN

c5-nx
SET UP MATRIX K #OIN=I+K#N#V
C COOMUTE THE MATRICES X(K)
D O 19 K101.
1 T1=T1/(1.0+V(K+1))/(C(K+1)=(1.0+V(K)))
2 X=0.1
3 R=2.0+WK1
4 V=2.0+WK1
5 W=2.0+WK2
6 V=2.0+WK2
7 X=V+X+V(K+1) COF00270

C X(K+1,1)=V(K)-3.0-T1
X(K+1,1)=0.0 COF00420
X(K+1,1)=X(K+1,1)+PH2-V(K+1.0)
X(K+1,1)=X(K+1,1)+PH3-V(K+1.0)

C T3=T3/(V(K)-1.0)
T4=T4/(V(K)-1.0)
T5=T5/(V(K)-1.0)
T6=T6/(V(K)-1.0)

C X(K+1,2)=T3+T4+T5+T6 COF00750

C X(K+1,4)=X(K)+V(K)-3.0+1.0 COF00810

C X(K+1,6)=X(K)+V(K)-3.0+1.0 COF00970

40
CCFE FORTRAN P ID=3617 10.06.70 FRIDAY 2 MAY 1980  PAGE 2

C T3=CN+PH-VXK8+1.0
  T4=VXK2-VXK2
C
C X(K,1,4)=(T3+T4+VXK2-T2*(T3+T4+VX/2))/P
X(K,3,2)=- (T3+T4+VXK2+1*(T3+T4+VX/2))/P
C
C X(K,1,3)=TIM*(1.0-PHC-VXK4)
X(K,2,3)=2.0*TIM*P
X(K,3,3)=VXK4-3.0-T1
X(K,4,3)=0.0
C
C X(K,2,4)=TIM*(PH2-VX+1.0)
X(K,4,4)=T1*(VXP4+3.0)-1.0
C
K = K
10 CONTINUE
C COMPUTE THE PRODUCT MATRICES FM
SCN(N)=4.0*(VIN)-1.0
IF (N-C) 13,11,11
11 CD 12 K=2.0
CD 13 SCN=1
CD 14 SCN=SC(N+1)*4.0*(VIN)-1.0
10 CONTINUE
13 CONTINUE
C
7 Q(1,1)=1.
Q(2,2)=1.
Q(1,2)=0.
C2 =PN2.=W(K)
IF (T7=170.1 15,15,16
15 CONTINUE
Q(1,2)=EXP(-C3)
C C(1,1) IS NOT NEEDED FOR INITIALIZING THE FM MATRIX
16 CONTINUE
C CD LCP Initializes FM(k,j)
CD C0 H=1,4
LL=H+1/2
CD C9 J=3,4
FM(K,J)= X(K,H,J) + Q(LL,C)
C0 CONTINUE
CD C6 K=2,N
D=K+1
K=K+1
C3 =PH2.*W(K)
IF (C7=170.1 22,CD,03
C0 CONTINUE
'Y(K,2)=W(K)
Q(I,2)=1./Q(2,1)
CD TO 24
C3 CONTINUE
C0 C1=0.
C4 I=1.2=1.20
C0 CONTINUE
CD C5 H=1,4
LL=I+1/2
CD C5 J=3,4

```
COFE    FETRAN P ID=361V  10.06.30 FRIDAY 2 MAY 1980  PAGE 3

C  PKK, M, J) = ( X(M, M, J) * PKK, M, J)
2  +X(M, M, J) * PKK, M, J) * Q(II, L, J)
3  +X(M, M, J) * PKK, M, J) * Q(II, L, J)
4  +X(M, M, J) * PKK, M, J) * Q(II, L, J)
25 CONTINUE
26 CONTINUE
C  SOLVE FOR (HS) AND D(HS)

T3 = 2.0 * (1)
T4 = T3 - 1.0
F(1) = P*(F(1,1,3)+F(1,1,3)) + T3*(F(1,1,3)+F(1,1,3))
F(2) = P*(F(1,1,4)+F(1,1,4)) + T3*(F(1,1,4)+F(1,1,4))
F(4) = P*(F(1,1,4)+F(1,1,4)) + T3*(F(1,1,4)+F(1,1,4))
DFAC = C(1)*(F(1)*F(4)-F(3)*F(2)) + P
A(LC, NS) = 0.0
B(LC, NS) = 0.0
C(LC, NS) = -F(3)-DFAC
D(LC, NS) = F(1)-DFAC
C  EACH SOLVE FOR THE OTHER A, B, C, D

DO 91 K1=1,N
A(LC, K1) = F(K1, L, 1, 3) * C(LC, NS) + F(K1, L, 1, 4) * D(LC, NS) / SC(K1)
B(LC, K1) = F(K1, L, 2, 3) * C(LC, NS) + F(K1, L, 2, 4) * D(LC, NS) / SC(K1)
C(LC, K1) = F(K1, L, 3, 3) * C(LC, NS) + F(K1, L, 3, 4) * D(LC, NS) / SC(K1)
D(LC, K1) = F(K1, L, 4, 3) * C(LC, NS) + F(K1, L, 4, 4) * D(LC, NS) / SC(K1)
91 CONTINUE
100 RETURN
END
```
APPENDIX B

KENTUCKY VERSION OF
CHEVRON N-LAYER COMPUTER PROGRAM
FILE: CHEVMODB PGM A1 University of Kentucky Computing Center

C$JOB
.List_T=6000
MODIFIED CHEVRON N-LAYERED PROGRAM
AS OF 3/17/87
MODIFIED BY H. F. SOUTHGATE
THIS VERSION USES COEF IN PLACE OF COES AND C015
THIS VERSION HAS 3 OPTIONS ONLY.

C19IMN ****** MAIN ROUTINE - N-LAYER ELASTIC SYSTEM ******
COMMON/CALMAN/RJ1(396), RJ0(396), AJ(396), TZZ, L Z
COMMON/CALPRN/CSZ, CST, CSR, CTR, COM, RDS, RDT, RDZ, SST, JT
COMMON/CAMAPA/AZ(396), AR, ITN, ITN4, NTEST, TEST(99)
COMMON/CAPP7M/BZ(100), NLINE, R
COMMON/COMAIN/H(15), P
COMMON/DON/, NPAGE
COMMON/GARY/IM(15), IT(15), IMH(15)
COMMON/HERB1/WORK(99), STENDN(99), ASTEN(99), WSTRS(99), ZZZ(99)
COMMON/HERB2/ T, ETA(99)
COMMON/HERB3/ ITZI, CSXX(99), CSXY(99), CSXZ(99), CSYY(99), CXYZ(99),
1 CSZZ(99), CSOM(99) CC6M(99)
COMMON/HERB4/RCOM(99), IR(99)
COMMON/HERBS/CRXX(99), CRXY(99), CRZZ(99), CRXZ(99), CRXY(99), CRYZ(99)
COMMON/JESSE1/ RSTB(7.99), RSTT(7.99), RLB(7.99), RLS(7.99), DFB(7.99),
1 DFS(7.99)
COMMON/JESSE2/REPB(2), REPS(2), STWB(2), STWS(2)
COMMON/JESSE3/AJX, AJY
COMMON/RDA/NS, IZ, E(15), V(15), HH(15), ZZ(99), N, TITLE(20), LLLL(99)
COMMON/RD1/OPTN
COMMON/RD1/PSI, WGT, RR(99), IR
COMMON/RD2/I2B, IB3, IA, IMA, AJXX(99), AJYY(99)
COMMON/RD3/RX(99), RY(99), BPSI(99), BWGT(99), JRUN, KST
COMMON/RD4/IEQ5, IEQS, JZLB, JZLS, JAI, Berokee, KA1
COMMON/JESSE5/ JA
INTEGER OPTN, OUTPUT
DIMENSION IE(15)
DATA ASTER, PERD/4H****, AH:.../
COMMON STATEMENTS 'RDA', 'RDO', 'RD1', 'RD2', 'RD3', AND 'RD4'
ARE USED IN SUBROUTINES 'READ1', 'READ2', 'READ3', 'READ4', AND
'READS'. SUBROUTINE 'READS' ALSO REQUIRES COMMON STATEMENTS
'HERB2', 'JESSE2', AND 'JESSE3' FOR ADDITIONAL ARGUMENTS.
COMMON STATEMENTS 'DON', 'GARY', 'HERB1', 'HERB2', 'HERB3'
'HERB4', 'HERBS', 'JESSE1', 'JESSE2', 'JESSE3', 'RDA', 'RDO'
'REDI', 'RDO', 'RD3', AND 'RD4' CONTAIN ARGUMENTS USED IN 'WRITE'
STATEMENTS.
COMMON STATEMENT 'COMAIN' CONTAINS ARGUMENTS USED IN MAIN AND
SUBROUTINES COES AND C015.
COMMON STATEMENT 'CALCM' CONTAINS ARGUMENTS USED IN SUBROUTINES
CALCIN, COES, AND C015.
COMMON STATEMENT 'CALMAN' CONTAINS ARGUMENTS USED IN MAIN AND
SUBROUTINE CALCIN.

COMMON STATEMENT 'CALPRN' CONTAINS ARGUMENTS USED IN SUBROUTINES CALCIN AND PRN7.

COMMON STATEMENT 'CAPP7M' CONTAINS ARGUMENTS USED IN MAIN AND SUBROUTINES CALCIN, PART, AND PRN7.

COMMON STATEMENT 'CAMAPA' CONTAINS ARGUMENTS USED IN MAIN AND SUBROUTINES CALCIN AND PART.

SUBROUTINE 'CALCIN' CALCULATES STRESSES 'CSZ' ETC. AND STRAINS 'CRZ' ETC. AT DEPTH 'IZT' AND DESIRED ANSWER LOCATION 'AJX,AJY'.

DUE TO INDIVIDUAL LOAD 'BWGT' LOCATED AT IT'S 'RX,RY'.

'COMMON/RDA/...' CONTAINS INPUT VARIABLES USED IN ALL OPTIONS.

'COMMON/RDI/...' CONTAINS INPUT VARIABLES USED WHEN OPTN = 1.

'COMMON/RD2/...' CONTAINS INPUT VARIABLES USED WHEN OPTN=2-7.

'COMMON/RD3/...' CONTAINS INPUT VARIABLES USED WHEN OPTN=7 OR 8.

AJX=X COORDINATE OF POSITION OF DESIRED ANSWERS--INPUT ARGUMENT

AJY=Y COORDINATE OF POSITION OF DESIRED ANSWERS--INPUT ARGUMENT

AR=CALCULATED RADIUS OF LOADED AREA.

ASTEN=SQUARE ROOT OF THE QUANTITY STENDN/(1/2 MODULUS) OF THAT LAYER AND CALLED 'WORK STRAIN'.

BPSI=APPLIED PRESSURE UNDER LOAD 'BWGT' WHEN OPTN = 2.

BWGT(JA)= GIVES THE OPTION OF DIFFERENT TIRE LOADS WITHIN THE LOAD GROUP FOR OPTN = 2.

CRZZ(IZT)=SUPERPOSITIONED STRAIN(ZZ) AT DEPTH IZT ETC. FOR OTHER COMPONENTS

CSZZ(IZT)=SUPERPOSITIONED STRESS(ZZ) AT DEPTH IZT ETC. FOR OTHER COMPONENTS

E(I)=YOUNG'S MODULUS OF ELASTICITY FOR ITH LAYER.

HH(I)=THICKNESS OF ITH LAYER

IA=NO. OF POSITIONS FOR WHICH THE SUPERPOSITIONED STRESSES AND STRAINS ARE DESIRED.

IR=NUMBER OF HORIZONTAL RADII FOR WHICH ANALYSES ARE DESIRED.

IREA=NUMBER OF LOADS (BWGT) WITHIN LOAD GROUP FOR OPTN = 2 & 3.

'RX' AND 'RY' = 'X' & 'Y' COORDINATES OF INDIVIDUAL LOADS WITHIN A LOAD GROUP AND USED WHEN OPTN=1.

IZ=NUMBER OF VERTICAL POINTS FOR WHICH ANALYSES ARE DESIRED.

IZI=COUNTER USED TO DETERMINE THE LAYER NO. AT THE INTERFACE BETWEEN LAYERS.

IZT=COUNTER USED TO DETERMINE I'TH LAYER.

JA= COUNTER OF POINTS FOR WHICH ANSWERS ARE DESIRED, 1 TO IA.

MAXSTR= MAXIMUM TENSILE STRAIN

NS=NUMBER OF LAYERS ABOVE THE BOTTOM LAYER -- SUBGRADE.

OPTN= CODE NUMBER TO TELL CHEVRON WHICH TYPE OF ANALYSES IS WANTED

OPTN=1 IS ORIGINAL CHEVRON N-LAYER PROGRAM WITHOUT MODIFICATIONS.

OPTN=2 IS ANALYSES OF SIMULATED DYNAMIC LOADING (ROAD RATER, ETC.)

OUTPUT = 0 : OUTPUT IS TO BE PRINTED ONLY.

OUTPUT = 1 : OUTPUT IS PRINTED AND ALSO SENT TO DEVICE 8 (SPECIFIED DEVICE. OS DISK, OR TERMINAL).

OUTPUT = 2 : OUTPUT IS SENT TO DEVICE 8 ONLY.

PSI=APPLIED UNIT PRESSURE UNDER THE LOAD.

R=HORIZONTAL RADIUS FROM CENTER OF APPLIED LOAD.

RR(I)=RADIAL DISTANCE FOR ITH RADIUS.
STENDN = THE STRAIN ENERGY DENSITY CAUSED BY ALL THE LOADS WITHIN THE "LOAD GROUP" AND AT THE PARTICULAR DEPTH "Z".

THETA = ANGLE IN R-V PLANE MEASURED FROM R AXIS; POSITIVE IS CLOCKWISE FROM R AXIS.

TR(I) = CALCULATED RADIUS FROM CENTER OF LOAD TO DESIRED ANSWER PT.

V(I) = POISSON'S RATIO FOR 2TH LAYER.

WGT = APPLIED LOAD, POUNDS.

WKSTRS = 'WORK STRAIN' MULTIPLIED BY MODULUS OF LAYER AND CALLED 'WORK STRESS'.

Z = VERTICAL DEPTH FROM SURFACE AT WHICH ANALYSIS IS DESIRED.

TYPICAL POINTS ARE SURFACE, INTERFACES OF LAYERS, AND DOWN INTO SUBGRADE.

 ZZ(I) = VERTICAL DEPTHS AT WHICH ANALYSES ARE DESIRED.

** COMPUTE ZEROS OF J1(X) AND JO(X). SET UP GAUSS CONSTANTS **

BZ(1) = 0.0
BZ(2) = 1.0
BZ(3) = 2.4048
BZ(4) = 3.8317
BZ(5) = 5.5201
BZ(6) = 7.0158
DO 1395 I = 7, 100
BZ(I) = 0.0

1395 CONTINUE
ITN = 46
ITN = ITN + 1
K = ITN + 1
DO 2 I = 7, K, 2
2 T = 1/2
DO 3 I = 8, ITN, 2
3 T = (I - 2)/2
DO 1 END = 4.0*T - 1.0
1 BZ(I) = 3.1415927*(T - 0.25 + 0.050661/TD**3 + 0.262051/TD**5)
DO 3 END = 4.0*T + 1.0
3 BZ(I) = 3.1415927*(T + 0.25 - 0.151982/TD**3 - 0.245270/TD**5)

C *** END ROUTINE SETTING GAUSS CONSTANTS. ****

C READ IN THE VALUE FOR OPTION AND THE NUMBER OF LOAD GROUPS.
READ (11, 315) OPTN, OUTPUT

315 FORMAT (2I5)
KA = I
JA = I
NPAGE = 1
10 CONTINUE
IF (OPTN.NE.1) CALL READ1(IEND)
IF (OPTN.NE.2) CALL READ2(IEND)
IF (IEND.NE.1) GO TO 9999
MS*NS
IF (OPTN.NE.1) GO TO 2001
C SUBROUTINES INIT1 & INIT2 INITIALIZE COMPUTATIONAL VARIABLES TO 0.
CALL INIT1
12 CONTINUE
2007 CALL INIT2
2008 CONTINUE
FILE: CHEVMODB PGM

A1 University of Kentucky Computing Center

C CALCULATE RADIUS AND ANGLE FROM X-AXIS
AJX=AJXX(JA)
AJY=AJYY(JA)
IR=IRA
C CALCULATE RADIAL DISTANCE FROM LOCATION OF DESIRED ANSWER TO EACH
C LOAD IN THE LOAD GROUP.
DO 2003 I=1,IR
RR(I)=SQRT((RX(I)-AJX)**2+(RY(I)-AJY)**2)
IF(RR(I).EQ.0.)GO TO 625
COSTH=(RX(I)-AJX)/RR(I)
GO TO 626
625 COSTH=0.
626 THETA(I)=ARCOS(COSTH)
2003 CONTINUE
TR(JA)=RR(I)
C *** END CALCULATION OF RADIUS AND ANGLE FROM X-AXIS. ****
2001 CONTINUE
DO 372 I=1,NS
IE(I)=E(I)
IH(I)=IE(I)-IE(I)/1000*1000+1000
IM(I)=IE(I)/1000000
IT(I)=IE(I)/1000-IM(I)*1000+1000
372 CONTINUE
IF(OPTN.GE.2)GO TO 374
NPAGE = 1
IF(OPTN.EQ.1)CALL PRN1
C SUBROUTINE PRN1 PRINTS TITLE AND COLUMN HEADINGS FOR OPTN=1, I.E.
C THE ORIGINAL CHEVRON N-LAYER PROGRAM. CONTROL OF OUTPUT OCCURS
C IN SUBROUTINES CALCIN AND PRN7
C SUBROUTINE PRN2 PRINTS HEADINGS AND PROBLEM INPUT DATA OPTN=2.
C *** ADJUST LAYER DEPTHS ***
374 H(I)=HH(I)
IRT=0
IF(N.LE.1)GO TO 100
DO 25 I=2,N
25 H(I)=H(I-1)+HH(I)
C ** START ON A NEW R **
100 IRT=IRT+1
IF(OPTN.GE.2)GO TO 50
IF(OPTN.EQ.1)WRITE(6,128)
128 FORMAT(/)
C *** IF IRT-IRT <= 1. THE CONTROL OF THE PROGRAM WILL TAKE ONE OF TWO
C MAJOR ROUTES. IF OPTN = 1, THE PROGRAM IS ROUTED TO READ
C A NEW PROBLEM. IF OPTN >= 2, THE STRESSES AND STRAINS OBTAINED
C IN THE SUBROUTINE "SUPER" (THE ACCUMULATED VALUES BY SUPER-
C POSITION PRINCIPLES) ARE PRINTED.
C IF (IRT-IRT) 105,105,1010
50 IF(IRT.GT.IRA)GO TO 1010
105 R=RR(IRT)
IF(OPTN.GE.2)WGT=WGT(IRT)
IF(OPTN.GE.2)PSI=PSI(IRT)
AR = SQRT(WGT/(3.14159*PSI))
DO 31 I=1,IZ
DO 31 J=1,N
TZ = H(J) - ZZ(I)
31 CONTINUE
ATZ = ABS(TZ)
C ** NEXT STATEMENT IS CHECKING TO SEE IF "ZZ" IS AT AN INTERFACE. ***
32 IF (ATZ .LE. .0001) 32,32,31
31 CONTINUE
C *** NLINE IS A COUNTER FOR PRINTOUT CONTROL. ****
32 NLINE=NLINE+1
C ** CALCULATE THE PARTITION **
33 CALL PART
C ** CALCULATE THE COEFFICIENTS **
DO 125 I=1,ITN4
   P=AZ(I)
   107 CONTINUE
   CALL COFE(I)
   109 IF (R) 115,115,110
   110 PR = P*R
C ***SUBROUTINE "BESSEL" CALCULATES THE "BESSEL FUNCTIONS". ***
   CALL BESSEL (0,PR,Y)
   RJ0(I) = Y
   CALL BESSEL (1,PR,Y)
   RJ1(I) = Y
   115 PA=P*AR
   CALL BESSEL (1,PA,Y)
   AJ(I)=Y
   125 CONTINUE
195 IZT=O
IZI=O
C ** START ON A NEW Z **
200 IZT=IZT+1
IZI=IZI+1
C *** IF IZT-IZ IS GREATER THAN 0., THE LAST "Z" HAS BEEN INVESTIGATED
C AND THE PROGRAM INCREMENTS TO THE NEXT DESIRED "RADIUS". ****
205 IF (IZT-IZ) 205,205,100
   IF(OPTN .GE.2) GO TO 207
   IF (NLINE -54 ) 207,206,206
   206 NPAGE = NPAGE + 1
   NLINE = 8
C SUBROUTINE PRN3 PRINTS TITLE FOR PROBLEM S ON SUCCEEDING PAGES OF
C SAME PROBLEM.
   CALL PRN3
   CALL PRN6
207 CONTINUE
C ** FIND THE LAYER CONTAINING Z **
   TZZ = 0.0
   DO 210 J1=1,N
      J-NS-J1
      TZ = Z-H(J)
      IF(TZ+.001) 210,215,215
   210 CONTINUE
   L = 1
   GO TO 34
   215 L=J+1
   IF (ZZ(IZT)) 33,34,34
33 L = J
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TZZ = 1.0

34 CONTINUE
C ****************************SUBROUTINE "CALCIN" CALCULATES THE
C  DEFLECTIONS AND THE NINE
C  COMPONENTS OF STRESS AND STRAIN. IF OPTN = 1, THE OUTPUT
C  STATEMENTS AND CONTROL REMAIN IN "CALCIN". FOR OPTN = 2,
C  THE CONTROL OF THE OUTPUT IS RETURNED TO "MAIN". ****************************
C LLLL(IIZI) = TEMPORARY STORAGE LOCATION.
LLL(IIZI) = L
CALL CALCIN
IF(OPTN .EQ. 1) CALL PRN7
ZZZ(IIZI) = ZZ(IIZT)
IF (TZZ .LT. 5) 36, 36, 35
ZZZ(IIZT) = ZZZ(IIZT)
IZT = IZT - 1
36 CONTINUE
IF (OPTN .EQ. 1) GO TO 200
RDS = RDS * 1.0E-06
RDT = RDT * 1.0E-06
RDZ = RDZ * 1.0E-06
SST = SST * 1.0E-06
COM = COM
IF (OPTN .LT. 3) GO TO 37
IF (IRT .EQ. 1 .OR. IRT .EQ. 3) GO TO 37
40 RDS = -RDS
RDT = -RDT
RDZ = -RDZ
SST = -SST
COM = -COM
CSR = -CSR
CTR = -CTR
CSZ = -CSZ
CST = -CST
37 CSOM(IIZI) = CSOM(IIZI) + COM
C ****************************THE PROGRAM NOW CALLS SUBROUTINE "SUPER"
C  TO RESOLVE AND ACCUMULATE THE STRESS COMPONENTS BY SUPERPOSITION
C  PRINCIPLES.
CALL SUPER(CSR, CTR, CST, CSZ, THETA(IRT), CSXX(IIZI), CSXY(IIZI),
CSZ(IIZI), CSYY(IIZI), CSYY(IIZI), CSZZ(IIZI))
C ****************************THE PROGRAM NOW CALLS SUBROUTINE "SUPER"
C  TO RESOLVE AND ACCUMULATE THE STRAIN COMPONENTS BY SUPERPOSITION
C  PRINCIPLES.
CALL SUPER(RDS, SST, RDT, RDZ, THETA(IRT), CRXX(IIZI), CRXY(IIZI),
CRXZ(IIZI), CRYY(IIZI), CRYY(IIZI), CRZZ(IIZI))
C NOW INVESTIGATE THE NEXT SPECIFIED DEPTH 'ZZZ(I)'.
GO TO 200
1010 CONTINUE
IZI = IZI - 1
C IF OPTN = 1, ANALYSIS IS BY ORIGINAL CHEVRON N-LAYER PROGRAM.
C PROCEED TO READ DATA FOR NEXT PROBLEM.
IF (OPTN .EQ. 1) GO TO 10
C START PRINTOUT OF SUPERPOSITIONED STRESSES, STRAINS, AND DEFLECTIONS
599 CONTINUE
IF (JA .EQ. 1 .AND. NPAGE .EQ. 1 .AND. OUTPUT .LT. 2) CALL PRN2
JLINE = 8 + 2 * IZI
IF (JA .EQ. 1) MLINE = 6 + MLINE
MLINE = MLINE + JLINE.
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597 IF (MLENE.LE.61) GO TO 598
598 NPAGE=NPAGE+1
MLINE=8+NS+IRA
CALL PRN2
C
C SUBROUTINES PRN4 & PRN5 PRINT COLUMN HEADINGS ONLY AND DIFFER
C ACCORDING TO FORMAT OF OUTPUT DATA.
C
C SUBROUTINE 'STRNEN' USES THE LAYER MODULI AND POISSON'S RATIOS
C TOGETHER WITH THE COMPONENT STRAINS RESOLVED THROUGH SUPERPOSI-
C TION PRINCIPLES IN SUBROUTINE 'SUPER' TO CALCULATE STRAIN
C ENERGY DENSITY (ARGUMENT STENDN), 'WORK STRAIN' (ARGUMENT
C ASTEN), AND 'WORK STRESS' (ARGUMENT WKSTRE). IF (OUTPUT.EQ.0) GO TO 600
C
C *** WRITE STATEMENTS USING "UNIT=7" ARE STORING THE OUTPUT IN COMPUTER
C *** MEMORY. WRITE STATEMENTS USING "UNIT=6" ARE WRITING THE OUTPUT
C *** TO A PRINTER TO OBTAIN A HARD COPY.
WRITE(8,7772)
7772 FORMAT('0')
WRITE(8,7774) (I,E(I),V(I),HH(I),I=1,N)
7774 FORMAT('1','13,F8.0,F5.3,F6.2')
WRITE(8,7773) NS,E(NS),V(NS)
7773 FORMAT('1','13,F8.0,F5.3,'999.99')
WRITE(8,7775) (I,XX(I),YY(I),BWGT(I),BPSI(I),I=1,IRA)
7775 FORMAT('2','13,F2.2,F8.2,F6.2')
WRITE(8,7776)
7776 FORMAT('2','0')
C
600 CALL STRNEN
IF (OUTPUT.LE.2) CALL PRN4
DO 601 J=1,IZI
IF (OUTPUT.LE.2) WRITE(6,586) AJX,AJY,ZZZ(J),CSXX(J),CSYY(J),CSZZ(J),CSXY(J),CSXZ(J),CSYX(J),CSYZ(J),CSOM(J)
586 FORMAT('1X,F8.2,2X,F8.2,2X,F6.2,E14.6,E15.6,F5.1,E15.6')
IF (OUTPUT.EQ.0) GO TO 601
WRITE(8,7770) AJX,AJY,ZZZ(J),CSXX(J),CSYY(J),CSZZ(J),CSXY(J),CSXZ(J),CSYX(J),CSYZ(J),CSOM(J)
7770 FORMAT('3','3F6.2,2F8.2,E12.6')
WRITE(8,7771) AJX,AJY,ZZZ(J),CRXX(J),CRXY(J),CRYX(J),CRXX(J),CRYY(J),CRZ(J),CSZZ(J),STENDN(J)
7771 FORMAT('4','3F6.2,2F8.2,E12.6')
WRITE(8,7772) AJX,AJY,ZZZ(J),CRXX(J),CRXY(J),CRYX(J),CRXX(J),CRYY(J),CRZ(J),STENDN(J),ASTEN(J)
7772 FORMAT('4','13,F8.2,2X,F6.2,E14.6,E15.6,F5.1,E15.6')
601 CONTINUE
IF (OUTPUT.EQ.0) GO TO 610
CALL PRN5
DO 602 J=1,IZI
WRITE(6,587) AJX,AJY,ZZZ(J),CRXX(J)
587 FORMAT('1X,F8.2,2X,F6.2,2X,F6.2,E14.6,E15.6,F5.1,E15.6')
602 CONTINUE
610 JA=JA+1
IF (JA.LE.IA) GO TO 12
615 JA=1
C
C *** NOW THE PROGRAM CHECKS TO SEE IF THERE IS ANOTHER PROBLEM. ***
NPAGE=1
GO TO 10

50
CONTINUE
STOP
END

SUBROUTINES ARE ASSEMBLED IN ALPHABETICAL ORDER.

**SUBROUTINE BESSEL(NI, XI, Y)**

******SUBROUTINE BESSEL - N-LAYER ELASTIC SYSTEM ******

REAL*8 PZ(6)/1.0D0,-1.125D-4,2.8710938D-7,-2.3449658D-9,
A3.9806841D-11,-1.1536133D-12/.
QZ(6)/-5.0D-3,4.6875D-6/.
B2.3255859D-8,2.8307087D-10,-6.3912098D-12,2.3124704D-12/.
CP1(6)/1.0D0,-1.875D-4,-3.6914063D-7,2.7713223D-9,
-6.5629D-8/.
D-4.5144212D-11,1.2750463D-12/.
Q1(6)/1.5D-2,-6.5625D-6,2.8423823D-8,-3.2662024D-10,7.1431166D-12,-2.5327056D-13/.
F PI/3.1415927/.
D(20)

C

9 N = NI
10 X = XI
IF (X - 7.0) 10,10,160
10 X2 = X/2.0
FAC = X2*X2
IF (N) 11,11,14
11 C=1.0
12 Y=Y+C
DO 13 I=1,34
T=I
C=FAC*C/(T*T)
TEST=ABS (C) - 10.0**(-8)
IF (TEST) 17,17,12
12 Y=Y+C
13 CONTINUE
14 C=X2
Y=C
DO 16 I=1,34
T=I
C=FAC*C/(T*(T+1.0))
TEST=ABS (C) - 10.0**(-8)
IF (TEST) 17,17,15
15 Y=Y+C
16 CONTINUE
17 RETURN
160 IF (N) 161,161,164
161 DO 162 I=1,6
D(I) = PZ(I)
D(I+10) = QZ(I)
162 CONTINUE
GO TO 163
164 DO 165 I=1,6
D(I) = P1(I)
D(I+10) = Q1(I)
165 CONTINUE
163 CONTINUE
T1 = 25.0/X
T2 = T1*T1
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P = D(6)*T2+D(5)
DO 170 I=1,4
J = 5-I
P = P*T2+D(J)
170 CONTINUE
Q = D(16)*T2+D(15)
DO 171 I=1,4
J = 5-I
Q = Q*T2+D(J+10)
171 CONTINUE
Q = Q*T1
T4 = DSORT(X*PI)
T6 = SIN(X)
T7 = COS(X)
IF (N) 180,180,185
180 T5 = ((P-Q)*T6 + (P+Q)*T7)/T4
GO TO 99
185 T5 = ((P+Q)*T6 - (P-Q)*T7)/T4
99 Y = T5
RETURN
END

SUBROUTINE CALCIN

******SUBROUTINE CALCIN - N-LAYER ELASTIC SYSTEM ******

COMMON/CALCO/A(396,15),B(396,15),C(396,15),D(396,15)
COMMON/CALM/R1(396),RJ0(396),AJ(396),TZZ,L,Z
COMMON/CALPR/CNZ,CST,CSR,CTR,COM,RSR,RTZ,STJ
COMMON/CAMAP/AZ(396),AR,ITN,ITION,TEST(99)
COMMON/CAPP7M/BZ(100),NLINE,N
COMMON/RDA/NS,I,IZ,E(15),N,TTLE(20),LLL(99)
COMMON/RD1/PSI,WR,RR(99),IR
REAL*8 W(4),0.34785485,2*0.65214515,0.34785485/
REAL MARY, MAN, JUNIOR, MAXSTR

2 VL=2.0*V(L)
EL=(1.0+V(L))/E(L)
VL1=1.0-VL
CSZ=0.0
CST=0.0
CSR=0.0
CTR=0.0
COM=0.0
NTS1 = NTEST + 1
ITS = 1
JT = 0
ARP = AR*PSI
10 DO 40 I=1,ITN

C INITIALIZE THE SUB-INTEGRALS
RSZ=0.0
RST=0.0
RSR=0.0
RTR=0.0
ROM=0.0

C COMPUTE THE SUB-INTEGRALS
K = 4*(I-1)
DO 30 J=1,4
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J1 = K + J
P=AZ(J1)
EP=EXP (P*Z)
T1=B(J1,L)*EP
T2=D(J1,L)/EP
T1P=T1+T2
T1M=T1-T2
T1P=(A(J1,L)*B(J1,L)*Z)*EP
T2P=(C(J1,L)+D(J1,L)/EP
T2M=(T1P+T2P)
WA=AZ(J1)*W(J)

IF (R) 20,20.15
BJ1=RJ1(J1)*P
BJ0=RJ0(J1)*P
RSZ=RSZ+WA*BJ0*(VL1*T1P-T2M)
ROM=ROM+WA*EP*(VL1*T1M-T2P)
RTR=ROM+WA*EP*(VL1*T1M+T2P)
RTR=ROM+WA*EP*(VL1*T1P+T2M)-BJ1*(T1P+T2M)/R
RSR=RST=WA*(VL*P*BJ0*T1P+BJ1*(T1P+T2M)/R)
GO TO 30

C SPECIAL ROUTINE FOR R = ZERO
PP=P*P
RSZ=RSZ+WA*PP*(VL1*T1P-T2M)
ROM=ROM+WA*EL*PP*(2.0*VL1*T1M-T2P)
RST=RST+WA*PP*(2.0*VL1*T1M+T2P)
RSR=RST

CONTINUE
SF=(AZ(K+4)-AZ(K+1))/1.7222726
CSZ=CSZ+RSZ*SF
CST=CST+RST*SF
CSR=CSR+RSR*SF
CTR=CTR+RTR*SF
COM=COM+ROM*SF
RSZ=2.0*RSZ*AR*SF
TESTH=ABS(RSZ-10.0**(-4))
IF (ITS-NTS1) 31,32,32
31 CONTINUE
TEST(ITS) = TESTH
ITS = ITS+1
GO TO 40
32 CONTINUE
TEST(NTS1) = TESTH
DO 33 J = 1,NTS
IF (TESTH-TEST(J)) 35,36,36
35 CONTINUE
TESTH = TEST(J)
36 CONTINUE
TEST(J) = TEST(J+1)
33 CONTINUE
IF (TESTH) 50,50.40
40 CONTINUE
JT = 1
50 CPS=CPS*ARP
CST=CST*ARP

53
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CRT=CTR*ARP
CSR=CSR*ARP
COM=COM*ARP
IF (TZ-5) 72,72,71
71 Z=Z
72 CONTINUE
RDZ=((1000000./E(L))*CSZ-V(L)*(CSR+CST))
RDT=((1000000./E(L))*CST-V(L)*(CSZ+CSR))
RDS=((10000000./E(L))*(CSR-V(L)*(CSZ+CST))
SST=2000000./E(L)*(1.0+V(L))*CTR
99 CONTINUE
RETURN
END

******************************************************************************
C THIS SUBROUTINE COPIED TO PROGRAM BY DHC, 5/08/84
******************************************************************************

SUBROUTINE COFE(KIN)
COMMON/CALCO/A(396,15),B(396,15),C(396,15),D(396,15)
COMMON/COIN/H[15],P
COMMON/RDA/NS,I,E(15),V(15),ZZ(99),N,TITLE(20),LLLL(99)
COMMON/SPSCOM/X(15,4,4),SC(14),PM(14,4,4),FM(4)
SUBROUTINE COFE(KIN,PSI)
COMMON/SPSCOM/E(15),V(15),H(15),A(396,15),B(396,15),C(396,15),D(396,15)
COMMON/SPSCOM/AA(15),BB(100),X(15,4,4),SC(14),FM(4),PM(14,4,4),
2Z,AR,NS,N,L,ITN,RSZ,CSZ,SRZ,SGZ,SG2,PH,PH2,VK2,VK4,VK5,VK6,VK8,
4HH(15),SIGMA3(15),W(15),MATYP(15),ANSDPT(15)
REAL*4 Q(2,2)
LC=KIN
1 DO 10 K=1,N
10 E(K)=1.0+V(K+1)/(E(K+1)*(1.0+V(K)))
TIME=T1-1.0
PH=P*H(K)
PH2=PH*2.0
VK2=2.0*V(K)
VKP2=2.0*V(K+1)
VK4=2.0*V(K)
VKP4=2.0*V(K2)
VKK8=8.0*V(K)*V(K+1)
C X(K,1,1)=VK4-3.0-T1
X(K,2,1)=TIME*(PH2-VK4+1.0)
X(K,3,1)=-2.0*TIME*P
C T3=PH2*(VK2-1.0)
T4=VKK8+1.0-3.0*VKP2
T5=PH2*(VKP2-1.0)
T6=VKK8+1.0-3.0*VKK8
C X(K,1,2)=(T3+T4-T1*(T5+T6))/P
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\[
X(K, 2, 2) = T_1 \times (VKP4 - 3.0) / 1.0 \\
X(K, 4, 2) = T_1 \times (1.0 - PH2 - VKP4) \\
X(K, 3, 4) = (T_3 - T_4 - T_1 \times (T_5 - T_6)) / P \\
T_3 = PH2 \times PH - VKK8 + 1.0 \\
T_4 = PH2 \times (VK2 - VKP2) \\
X(K, 1, 4) = (T_3 + T_4 + VKP2 - T_1 \times (T_3 + T_4 + VK2)) / P \\
X(K, 3, 2) = \frac{(- T_3 + T_4 - VKP2 + T_1 \times (T_3 - T_4 + VK2))}{P} \\
X(K, 1, 3) = T_1 \times (1.0 - PH2 - VK4) \\
X(K, 2, 3) = 2.0 \times T_1 \times P \\
X(K, 3, 3) = VK4 - 3.0 - T_1 \\
X(K, 4, 3) = 0.0 \\
X(K, 2, 4) = T_1 \times (PH2 - VKP4 + 1.0) \\
X(K, 4, 4) = T_1 \times (VKP4 - 3.0) / 1.0 \\
\]

10 CONTINUE

C COMPUTE THE PRODUCT MATRICES PM
SC(N) = 4.0 \times (V[N] - 1.0)
IF (N-2) 13, 11, 11

11 DO 12 K1=2, N

M=NS-K1
SC[M] = SC[M+1] \times 4.0 \times (V[M] - 1.0)

12 CONTINUE

13 CONTINUE

C Q(1,1) = 1.0 \\
Q(2,2) = 1.0 \\
Q(1,2) = 0.0 \\
QQ = P \times 2.0 \times H(N)
IF (QQ-172.0) 15, 15, 16

15 CONTINUE

Q(1,2) = EXP(-QQ) \\
Q(2,1) IS NOT NEEDED FOR INITIALIZING THE PM MATRIX

16 CONTINUE

C 20 LOOP INITIALIZES PM(N,,)
DO 20 M=1, 4
LL=(M+1)/2 \\
DO 20 J=3, 4
PM(N, M, J) = X(N, M, J) \times Q(LL, 2)

20 CONTINUE

C 26 CONTINUE
K=NS-K1 \\
K=K+1 \\
QQ = P \times 2.0 \times H(K)
IF (QQ-172.0) 22, 22, 23

22 CONTINUE

Q(2,1) = EXP(QQ) \\
Q(1,2) = 1.0 / Q(2,1) \\
GO TO 24

23 CONTINUE

Q(1,2) = 0.0
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Q(2.1)=1E20

DO 25 M=1,4
   LL=(M+1)/2
   DO 25 J=3,4
      PM(K,M,J)= ( X(K,M,1) * PM(KK,1,J) 
      + X(K,M,2) * PM(KK,2,J) ) * Q(LL,1)
      +( X(K,M,3) * PM(KK,3,J) 
      + X(K,M,4) * PM(KK,4,J) ) * Q(LL,2)

25 CONTINUE

26 CONTINUE

C SOLVE FOR C(NS) AND D(NS)

T3=2.0*V(1)
T4=T3-1.0

FM(1)= P*(PM(1,1,3)+PM(1,1,3)) + T3*(PM(1,2,3)-PM(1,4,3))
FM(2)= P*(PM(1,1,3)-PM(1,3,3)) + T4*(PM(1,2,3)+PM(1,4,3))
FM(3)= P*(PM(1,1,4)+PM(1,3,4)) + T3*(PM(1,2,4)-PM(1,4,4))
FM(4)= P*(PM(1,1,4)-PM(1,3,4)) + T4*(PM(1,2,4)+PM(1,4,4))

DFAC=SC(1)/((FM(1)*FM(4)-FM(3)*FM(2))*P*P)

A(LC,NS) = 0.0
B(LC,NS) = 0.0
C(LC,NS) = -FM(3)*DFAC
D(LC,NS) = FM(1)*DFAC

C BACKSOLVE FOR THE OTHER A,B,C,D

DO 91 Kl=1,N
   A(LC,Kl)=(PM(Kl,1,3)*C(LC,NS)+PM(Kl,1,4)*D(LC,NS))/SC(K1)
   B(LC,Kl)=(PM(Kl,2,3)*C(LC,NS)+PM(Kl,2,4)*D(LC,NS))/SC(K1)
   C(LC,K1)=(PM(Kl,1,3)*C(LC,NS)+PM(Kl,1,4)*D(LC,NS))/SC(K1)
   D(LC,K1)=(PM(Kl,4,3)*C(LC,NS)+PM(Kl,4,4)*D(LC,NS))/SC(K1)

91 CONTINUE

RETURN

END

**********************************************************************

SUBROUTINE INIT1

COMMON/HERB4/RRCOM(99),TR(99)
COMMON/JESSE1/RRSB(7,99),RSTS(7,99),RLB(7,99),RLS(7,99),DFB(7,99),
   DFS(7,99)
COMMON/JESSE2/REPB(2),REPS(2),STWB(2),STWS(2)
COMMON/RD2/IB2,IB3,IA,IRA,AJXX(99),AJYY(99)

C** INITIALIZE THE ACCUMULATING VARIABLES FOR STRAIN ENERGY DENSITY

C CALCULATIONS.  ******

DO 414 I=1,7
   DFB(I,J)=0.
   DFS(I,J)=0.
   RSB(I,J)=0.
   RSTS(I,J)=0.
   RLB(I,J)=0.
   RLS(I,J)=0.

414 CONTINUE

RETURN
END
SUBROUTINE INIT2
COMMON/HERB1/WORK(99),STENDN(99),ASTEN(99),WKSTRS(99),ZZZ(99)
COMMON/HERB2/THETA(99)
COMMON/HERB3/IZI,CSEX(99),CSXY(99),CSXZ(99),CSXY(99),CSY(99),CSY(99)
COMMON/HERB5/CRXX(99),CRY(99),CRZZ(99),CRXY(99),CRXZ(99),CRYZ(99)
COMMON/RDZ,IB2,IB3,IA,IRA,AJXX(99),AJYY(99)
DO 11 I=1,99
CSXX(I)=0.
CSYY(I)=0.
CSZZ(I)=0.
CSXY(I)=0.
CSXZ(I)=0.
CSY(I)=0.
CRXX(I)=0.
CRYY(I)=0.
CRZZ(I)=0.
CRXY(I)=0.
CRXZ(I)=0.
CRYZ(I)=0.
ASTEN(I)=0.
STENDN(I)=0.
11 CONTINUE
RETURN
END
SUBROUTINE PART
C ******SUBROUTINE PART - N-LAYER ELASTIC SYSTEM ******
C
COMMON/CAMAPA/АЗ(396),AR,ITN,ITN4,NTEST,TEST(99)
COMMON/CAPP7M/BZ(100),NLINER
REAL*8 G1/0.85113531/,G2/0.33998104/
4 ZF = AR
NTEST = 2
IF (R) 8, 8, 9
9 CONTINUE
NTEST = AR/R + .0001
IF (NTEST) 6, 6, 5
6 CONTINUE
NTEST = R/AR + .0001
ZF = R
5 CONTINUE
NTEST = NTEST + 1
IF (NTEST-10) 8, 8, 7
7 CONTINUE
NTEST = 10
8 CONTINUE
C ******SUBROUTINE PART - N-LAYER ELASTIC SYSTEM ******
C
DO 28 I=1,ITN
SZ1 = SZ2
SZ2 = BZ(I+1)/ZF
28 CONTINUE

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SF = SZZ - SZ1
PP = SZZ + SZ1
SG1=SF*G1
SG2=SF*G2
AZ(K)=PP*SG1
AZ(K+1)=PP*SG2
AZ(K+2)=PP*SG1
K = K + 4

28 CONTINUE
40 RETURN

END

SUBROUTINE PRN1
COMMON/CAMAPA/AR(396),AR(396).AR(396).ITN.ITN4.NTEST.TEST(99)
COMMON/CAPP7M/BZ(100),NLINE,R
COMMON/GARY/IM(15).IT(15).IH(15)
COMMON/RDA/NS.IZ.E(15),V(15).HH(15),ZZ(99),N,TITLE(20),LLLL(99)
COMMON/RD1/PSI,WGT,RR(99),IR
DATA ASTER/4H****/
C *** IF OPTN 1, THIS NEXT SECTION PRINTS TITLE AND HEADINGS. ****
C *** THESE HEADINGS AND VALUES CORRESPOND TO THE ORIGINAL N-LAYER PROGR

CALL PRN2
WRITE(6,351)WGT,PSI,AR,(I,IM(I),IT(I),IH(I),V(I),HH(I),I=1,N)
351 FORMAT(1HO,4X,28HTHE PROBLEM PARAMETERS ARE/
14HO,20X,12HTOTAL LOAD...,3X,F10.2,5HLBS/ 2
1HO,20X,12HTIRE PRESSURE...,5X,F10.2,5HPSI/
3
1HO,20X,12LOAD RADIUS...,7X,F10.2,5HIN./IH/
4
1H,20X,12SHELMERYERTER,13,14THAS MODULUS,314
6
18H POISSONS RATIO , F5.3,17H AND THICKNESS , F6.2,
7
4IN. T48.1H,T52.1H,)
WRITE(6,354)NS,IM(NS).IT(NS).IH(NS).V(NS)
354 FORMAT(1H,20X,12SHAS MODULUS,314
118H POISSONS RATIO , F5.3,24H AND IS SEMI-INFINITE.

7
2T48.1H,T52.1H,)
WRITE(6,352)
1HO,4X,8LOCATION,2X,1H*,14X,15HS-T-R-E-S-S-E-S-S,14X,12H*DEF
1ECTION*,21X,13HS-T-R-A-I-N-S,21X,8H*ANGLE/15X,1H*,19X,3HPSI,21X,
21X,2X,6HINCHES/INCH,19X,1HX,1X.3MDEG/
315X.1HX,43X.1HX,10X,1HX,55X.1HX,5X.1HX,6X,1HX,2X,1HX,2X.8HVERTICAL
43X,10HTANGENTIAL,2X,6HRADIAL,5X,5HSHEAR,2X,1HX,1X.8HVERTICAL
43X,1HX
52X,8HVERTICAL,3X,10HTANGENTIAL,2X,6HRADIAL,2X,8HSHEAR IN,2X,
51HMAX PRIN IN,1X,1HX,1X.4HWITH/
615X.1HX,43X,1HX,10X,1HX,32X,10HMICRO RAD...,1X,
71HTENSILE DIR.*R AXIS/IH)
RETURN
END

SUBROUTINE PRN2
COMMON/CAPP7M/BZ(100).NLINE.R
COMMON/GARY/IM(15).IT(15).IH(15)
COMMON/HERB2/ THETA(99)
COMMON/RDA/NS.IZ.E(15),V(15).HH(15),ZZ(99),N,TITLE(20).LLLL(99)
COMMON/RD2/IB2,IB3,IA,INA,AAJX(99),AJYY(99)

58
FILE: CHEVMODB PGM

COMMON/RD3/RX(99),RY(99),BPSI(99),BWGT(99),JRURN,KST

DATA ASTER/4H****/

C

///////////////////////////////////////////////////////////////////////////////

NL INE = 13+NS
CALL PRN3
WRITE(6,370)
370 FORMAT(1'I,120('*)&
WRITE(6,371)
371 FORMAT(1'I,41('*)&,'THE ANSWERS BELOW ARE BY SUPERPOSITION', & &41('*)&)
WRITE(6,381)
381 FORMAT(I,IM(I),IT(I),IH(I),V(I),HH(I),I=1,N)
WRITE(6,354) NS,IM(NS),IT(NS),IH(NS),V(NS)
354 FORMAT(1'H, 20X, 5HLAYER, I3, 14H, HAS MODULUS, 314, & 18H, POISCONS RATIO, F5.3, 24H, AND IS SEMI-INFINITE, , & 2T48, IH, T52, IH,)
WRITE(6,580)
580 FORMAT(1'O, 39X,'COORDINATES OF THE LOAD POINTS AND LOAD VALUES A &RE)
373 CONTINUE
DO 603 I=1,IRA
WRITE(6,582) I,RX(I),I,RY(I),I,BWGT(I),BPSI(I)
582 FORMAT(13X,'X(' ,I2,')=',F6.2,'Y(' ,I2,')=',F6.2,'P(' ,I2,')=',' & F8.2,' 'TIRE PRESSURE=',F6.2,'PSI')
603 CONTINUE
C

END PRINTING HEADINGS FOR OPTN = 2

C

CONTRINUE
RETURN
END
SUBROUTINE PRN3
COMMON/DON/ NPAGE
COMMON/RDA/NS,I2,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
COMMON/ASTER/4H****/
WRITE(6,350) [ASTER,I=1,5),(TITLE(I),I=1,20),(ASTER,I=1,5),NPAGE
350 FORMAT(1'H1, 5A4,1X,26A4,1X,5A4,6H PAGE,I3)
RETURN
END
SUBROUTINE PRN4
WRITE(6,552)
552 FORMAT(1*X, /2X,'COORDINATES DEPTH', 29X,'S-T-R-E-S-S-E-S', ./,4X & 'X', /7X,'Y', /7X,'Z', /8X,'XX', /13X,'XY', /8X,'XZ', /8X,'YY', /8X 'YZ', /8X,'ZZ', /10X,'DEFLECTION')
RETURN
END
SUBROUTINE PRN5
WRITE(6,553)
553 FORMAT(1*X, /2X,'COORDINATES DEPTH', 30X,'S-T-R-A-I-N-S', &29X,'STRAIN ENERGY', 5X,'WORK', ./,4X,'X', /7X,'Y', /7X,'Z', /8X,'XX', /13X, & 'XY', /8X,'XZ', /8X,'YY', /8X,'YZ', /8X,'ZZ', /11X,'DENSITY', /8X,'STRAIN')
END
SUBROUTINE PRN6
WRITE(6,352)
352 FORMAT(lH0,4X,8HLOCATION,2X,1H*,14X,15HS-T-R-E-S-E-S-E-S,14X,12H*DEF
ILECTION*,21X,12H$-R-A-I-N-S,21X,6H$ANGLE/15X,1H*,19X,3H$PSI,21X,
2H*2X,6H$INCHES,2X,1H*,2X,16H$MICRONCHES/INCH,19X,1H*,1X,3H$DEG/
315X,1H*,43X,1H*,10X,1H*,55X,1H*,5X,1H*,8X,1H*,2X,1H*,2X,8HVERTICAL
4.2X,10HTANGENTIAL,2X,6HRADIAL,5X,5HSHEAR,2X,1H*,1X,8HVERTICAL,
41X,1H*,52X,3HVERTICAL,3X,LOHTANGENTIAL,2X,6HRADIAL,2X,8HSHEAR IN,2X,
511HVMAX PRIN IN,1X,1H*,1X,4HWITH/
815X,1H*,43X,1H*,10X,1H*,32X,10HMICRO RAD,1X,
719HTENSILE DIR.*R AXIS/IN )
RETURN
END
SUBROUTINE PRN7

THIS SUBROUTINE IS USED TO PRINT OUTPUT WHEN OPTN=1 ONLY. IT
IS ACTUALLY THE LAST HALF OF SUBROUTINE CALCIN IN THE ORIGINAL.

CHEVRON N-LAYER PROGRAM.

COMMON/CALMAN/RJ1(396),RJ0(396),AJ(396),IZZ,L,Z
COMMON/CALPRN/CSZ,CST,CSR,CTR,COM,ROS,ROT,ROZ,SST,HT
COMMON/CAPP7M/BZ(100),NLINE,R
COMMON/RDA/NS,IZ,E(15),V(15),H(15),ZZ(99),N,TITLE(20),LLLL(99)
REAL MARY,MAN,JUNIOR,MAY

CALCULATE MAXIMUM PRINCIPAL STRAIN IN TENSILE DIRECTION
AND ITS ANGLE OR ANGLES WITH SPATIAL AXES T.R, AND V.
IN PRINTOUT: A NUMERICAL ANGLE IS DIRECTION OF THIS STRAIN
WITH R-AXIS IN THE R-V PLANE AND MINUS IS COUNTERCLOCKWISE.
IN PRINTOUT: A COMBINATION DIRECTION DEFINES THE PLANE OR
PLANES IN WHICH THIS STRAIN IS CONSTANT.

BSC = ABS(CTR) - 0.0009

IF (BSC.GT.0.0) GO TO 500

WHEN SHEAR STRESS IS ZERO, T.R, & V ARE PRINCIPAL AXES.

TMPMX1 = (1000000./E(L))*((CSR-V(L)*((CSR+CST)))
TMPMX2 = (1000000./E(L))*((CSZ-V(L)*((CSR+CST)))
TMPMX3 = (1000000./E(L))*((CST-V(L)*((CSR+CSZ))
MARY = ((CST-CSR)
THOMP = ABS(MARY) - 0.0009
SUTTON = (CSR-CSZ)
JUNIOR = ABS(SUTTON) - 0.0009
SAM = (CST-CSZ)
SAMPAT = ABS(SAM) - 0.0009
IF ((JUNIOR.LT.0.0).AND.(THOMP.LT.0.0)) GO TO 530
GO TO 531

530 MAXSTR = TMPMX1
GO TO 501

531 CONTINUE
IF ((JUNIOR.LT.0.0).AND.(MARY.LE.-0.0009)) GO TO 534
GO TO 535

534 MAXSTR = TMPMX1
GO TO 503

535 CONTINUE
IF (SAMPAT.LT.0.0).AND.(MARY.GE.0.0009)) GO TO 532
GO TO 533

532 MAXSTR = TMPMX1
GO TO 504

534 MAXSTR = TMPMX1
GO TO 505

535 CONTINUE
IF (SAMPAT.LT.0.0).AND.(MARY.GE.0.0009)) GO TO 532
GO TO 533
FILE: CHEVMODB PGM

532 MAXSTR = TMPMX2
GO TO 505
533 CONTINUE
IF((THOMP LT 0.0) .AND. (SUTTON GE 0.0009)) GO TO 536
GO TO 507
536 MAXSTR = TMPMX3
GO TO 507
537 CONTINUE
IF((MARY GE 0.0009) .AND. (SAM GE 0.0009)) GO TO 538
GO TO 509
538 MAXSTR = TMPMX3
GO TO 509
539 CONTINUE
IF((MARY LE -0.0009) .AND. (SUTTON GE 0.0009)) GO TO 5400
GO TO 5401
5400 MAXSTR = TMPMX1
GO TO 511
5401 CONTINUE
MAXSTR = TMPMX2
GO TO 513
501 WRITE(6,502) R,Z,C SZ,C ST,C SR,C TR ,C OM ,RDZ, RDT, RDS, SST, MAXSTR
502 FORMAT(IH,IX,F6.2,1X,F6.2,1HX,F10.4,1X,F10.4,1X,F10.4,
11HX,F10.6,1HX,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
21X,7H*TRV)
GO TO 557
503 WRITE(6,504) R,Z,C SZ,CSR,CTR,COM,RDZ, RDT, RDS, SST, MAXSTR
504 FORMAT(IH,IX,F6.2,1X,F6.2,1HX,F10.4,1X,F10.4,1X,F10.4,
11HX,F10.6,1HX,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
21X,6H*RV)
GO TO 557
505 WRITE(6,506) R,Z,C SZ,CST,CTR,COM,RDZ, RDT, RDS, SST, MAXSTR
506 FORMAT(IH,IX,F6.2,1X,F6.2,1HX,F10.4,1X,F10.4,1X,F10.4,
11HX,F10.6,1HX,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
21X,6H*TV)
GO TO 557
507 WRITE(6,508) R,Z,C SZ,CST,CTR,COM,RDZ, RDT, RDS, SST, MAXSTR
508 FORMAT(IH,IX,F6.2,1X,F6.2,1HX,F10.4,1X,F10.4,1X,F10.4,
11HX,F10.6,1HX,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
21X,6H*T R)
GO TO 557
509 WRITE(6,510) R,Z,C SZ,CST,CTR,COM,RDZ, RDT, RDS, SST, MAXSTR
510 FORMAT(IH,IX,F6.2,1X,F6.2,1HX,F10.4,1X,F10.4,1X,F10.4,
11HX,F10.6,1HX,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
21X,7H*T DIR)
GO TO 557
511 WRITE(6,512) R,Z,C SZ,CSR,CTR,COM,RDZ, RDT, RDS, SST, MAXSTR
512 FORMAT(IH,IX,F6.2,1X,F6.2,1HX,F10.4,1X,F10.4,1X,F10.4,
11HX,F10.6,1HX,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
21X,7H*R DIR)
GO TO 557
513 WRITE(6,514) R,Z,C SZ,CSR,CTR,COM,RDZ, RDT, RDS, SST, MAXSTR
514 FORMAT(IH,IX,F6.2,1X,F6.2,1HX,F10.4,1X,F10.4,1X,F10.4,
11HX,F10.6,1HX,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
21X,7H*V DIR)
GO TO 557

61
WHEN SHEAR STRESS IS NOT ZERO, PRINCIPAL STRAIN MAY BE AN ANGLE IN R-V PLANE OR IN T-DIRECTION OR MAY BE IN A COMBINATION OF THESE PLANES.

\[
\begin{align*}
\text{TEMP} & = 0.5 \times \text{SORT}[(\text{CSR} - \text{CSZ}) \times (\text{CSR} - \text{CSZ}) + 4.0 \times \text{CTR} \times \text{CTR})] \\
\text{CS1TEN} & = 0.5 \times (\text{CSZ} + \text{CSR}) + \text{TEMP} \\
\text{CS2TEN} & = \text{CS1TEN} - 2.0 \times \text{TEMP} \\
\text{CS3TEN} & = \text{CS1TEN} - \text{CS3TEN} \\
\text{TUT} & = \text{ABS}[(\text{GEORGE}) - 0.0009] \\
\text{IF}(\text{GEORGE}, \text{LE}, -0.0009) & \text{ GO TO 554} \\
\text{MAN} & = 1000.0 \\
\text{COFF} & = \text{CS1TEN} - \text{CS2TEN} \\
\text{IF}(\text{COFF}, \text{LE}, -0.0009) & \text{ MAN} = 0.0 \\
\text{SAM} & = \text{ABS}[(\text{COFF}) - 0.0009] \\
\text{IF}(\text{SAM}, \text{GT}, 0.0) & \text{ GO TO 520} \\
\text{MINUS 45.0 DEGREES WITH R-AXIS.} \\
\text{IF}(\text{CTR}, \text{GE}, 0.0009) & \text{ THETA} = 45.0 \\
\text{IF}(\text{CTR}, \text{LE}, -0.0009) & \text{ THETA} = -45.0 \\
\text{GO TO 540} \\
\end{align*}
\]

WHEN STRESSES IN R AND V DIRECTIONS ARE EQUAL, THETA IS PLUS OR MINUS 45.0 DEGREES WITH R-AXIS.

\[
\begin{align*}
\text{THETA} & = 0.5 \times \text{ARSIN}(\text{CTR}/\text{TEMP}) \\
\text{THETA} & = (180.0/3.1415927) \times \text{THETA} \\
\text{IF}(\text{MAN}, \text{EQ}, 0.0) \text{ AND } (\text{CTR}, \text{GE}, 0.0009) & \text{ THETA} = 90.0 - \text{THETA} \\
\text{IF}(\text{MAN}, \text{EQ}, 0.0) \text{ AND } (\text{CTR}, \text{LE}, -0.0009) & \text{ THETA} = -(90.0 - \text{ABS}(\text{THETA})) \\
\end{align*}
\]

IN PRINTOUT: THETA CAN VARY FROM PLUS OR MINUS ZERO TO NINETY DEGREES (WITH R-AXIS).

\[
\begin{align*}
\text{CS1} & = \text{CS1TEN} \\
\text{IF}(\text{CS3TEN} - \text{CS2TEN}), \text{GE}, 0.0009) \text{ GO TO 545} \\
\text{CS2} & = \text{CS2TEN} \\
\text{CS3} & = \text{CS3TEN} \\
\text{GO TO 550} \\
\text{CS2} & = \text{CS3TEN} \\
\text{CS3} & = \text{CS2TEN} \\
\text{MAXSTR} & = (1000000.0/E(L)) \times (\text{CS1} - V(L)) \times (\text{CS2} + \text{CS3}) \\
\text{IF}(\text{TUT}, \text{LT}, 0.0) \text{ GO TO 551} \\
\end{align*}
\]

WHEN TANGENTIAL STRESS IS MAJOR PRINCIPAL TENSILE STRESS, MAXIMUM TENSILE STRAIN IS IN TANGENTIAL DIRECTION, ONLY.
FILE: CHEVMODB PGM A1 University of Kentucky Computing Center

MAXSTR = (1000000.0) / (E(L)) * (CS1-V(L)) * (CS2+CS3)
WRITE(6, 555) R, 2, CSZ, CST, CSR, CTR, COM, RDZ, RDT, RDS, SST, MAXSTR
21X, 7H* T DIR)
556 NLINE=NLINE+1
60 IF (OPTN.EQ.2) GO TO 99
99 RETURN
END

SUBROUTINE READ1 (IEND)

COMMON/CAMAPA/ AZI396, AR, ITN, ITN4, NTEST, TEST(99)
COMMON/RDA/NS, IZ, EILS, VILS, HH15, ZZ99, N, TITLE(I), LLLL99)
COMMON/RD1/PSI, WGT, RR99, IR
IEND=0
READ(I11, 310) END=9999) (TITLE(I), I=1,20)
310 FORMAT (20A4)
READ(I11, 311) NS, IZ, IRA, PSI, WGT
314 FORMAT (315, 2F10.3)
AR=SQRT(WGT/(3.14159*PSI))
READ(I11, 302) (E(I), V(I), I=1, NS)
302 FORMAT (5(F7.0, F6.5))
READ(I11, 313) (RR(I), I=1, IR)
313 FORMAT (10F8.3)
N=NS-1
READ(I11, 313) (HH(I), I=1, N)
READ(I11, 313) (ZZ(I), I=1, IZ)
GO TO 9998
9998 CONTINUE
RETURN
END

SUBROUTINE READ2 (IEND)

COMMON/RDA/NS, IZ, EILS, VILS, HH15, ZZ99, N, TITLE(I), LLLLL99)
COMMON/RD2/I2, I3, IA, IRA, AJX(I), AJY(I)
COMMON/RD3/RX(I), RY(I), BPSI(I), BWGT(I), JRUN, KST
COMMON/JESSES/JA
IEND=0
READ(I11, 310) END=9999) (TITLE(I), I=1,20)
310 FORMAT (20A4)
READ(I11, 311) NS, IZ, IA, IRA
315 FORMAT (4I5)
READ(I11, 302) (E(I), V(I), I=1, NS)
302 FORMAT (5(F7.0, F6.5))
N=NS-1
READ(I11, 313) (HH(I), I=1, N)
313 FORMAT (10F8.3)
READ(I11, 313) (ZZ(I), I=1, IZ)
GO TO 9998
9998 CONTINUE
RETURN
END

C ****READ IN THE LOCATIONS OF THE DESIRED ANSWER POINTS AS X,Y COORDINATES
C
READ(I11, 317) (AJX(I), AJY(I), I=1, IA)
FILE: CHEVMODB PGM  Al University of Kentucky Computing Center

317 FORMAT(16F5.2)
 **** READ IN THE X, Y COORDINATE LOCATIONS AND RESPECTIVE LOADS.
 WHEN OPTN=2, READ MAGNITUDE OF LOAD FOR EACH LOAD LOCATION
 WHEN OPTN=3, ANALYSES SUBTRACTS CALCULATED ANSWERS FOR MINIMUM
 LOADS FROM ANSWERS FOR MAXIMUM LOADS.
 WHEN OPTN=3, LOADS MUST BE READ IN AS THE MAXIMUM, THEN MINIMUM
 FOR FIRST LOADED AREA FOLLOWED BY THE MAXIMUM THEN MINIMUM FOR
 OTHER LOADED AREA.
 BWGT(1)=MAXIMUM LOAD ON LOADED AREA(1).
 BWGT(2)=MINIMUM LOAD ON LOADED AREA(1).
 BWGT(3)=MAXIMUM LOAD ON LOADED AREA(2).
 BWGT(4)=MINIMUM LOAD ON LOADED AREA(2).
 IF (JAGT1) GO TO 2008
 403 READ (11,316) (RX(I),RY(I),BPSI(I),BWGT(I),I=1,IRA)
 316 FORMAT(4(2F5.2,F4.1,F6.1))
 2008 CONTINUE
 GO TO 9998
 9999 IEN=1
 9998 CONTINUE
 RETURN
END

SUBROUTINE STRNEN
COMMON/H3/HERB1(WORK(99),STENDN(99),ASTEN(99),WKSTRS(99),ZZZ(99)
COMMON/H3/HERB3/IZI,CSXX(99),CSXY(99),CSXZ(99),CSYY(99),CSYZ(99)
1,CSZZ(99),CSOM(99),CCOM(99)
COMMON/H3/HERB5/CRXX(99),CRYY(99),CRZZ(99),CRYX(99),CRYX(99)
CRXZ(99),CRYZ(99)
COMMON/R3/NS,NZ,E(15),V(15),HH(15),ZZ(99),N,TITLE[20],LLL(99)
COMMON/R3/IB2,IB3,IRA,AJXX(99),AJYY(99)
DO 602 IZT=1,IZI
C ** START THE CALCULATION OF THE STRAIN ENERGY DENSITY. *****
 L=LLL(IZT)
 ALAMB=E(L)/(1.+V(L))**2
 AMU=E(L)/(2.*(1.+V(L)))
 PHI=CRXX(IZT)+CRYX(IZT)+CRYY(IZT)
 STENDN(IZT)=SQRT(STENDN(IZT)**2)/(1.-2.*V(L))
 AMU=E(L)/(2.*(1.+V(L)))
 PHI=CRXX(IZT)+CRYX(IZT)+CRYY(IZT)
 CONTINUE
 ASTEN(IZT)=SQRT(STENDN(IZT)**2)/(1.-2.*V(L))
 609 CONTINUE
 602 CONTINUE
 RETURN
END

SUBROUTINE SUPER(RR,RT,TT,ZZ,THETA,XXT,XYT,XZT,YYT,YZT,ZZT)
 CO=COS(THETA)
 SI=SIN(THETA)
 SICO=SI*CO
 COSQ=CO*CO
 SISQ=SI*SI
 RZ=0
 TZ=0
 XX=COSQ*RR-2*SICO*RT+SI*SISQ*TT
 XY=SICO*RR+(COSQ-SICO)*RT-SICO*TT
 XZ=CO*RZ-SI*TZ
 YY=SISO*RR+2*SICO*RT+COSQ*TT
 YZ=SI*RZ+CO*TZ
ZZ=ZZ
XXT=XXT+XX
XYT=XYT+XY
XZT=XZT+XZ
YYT=YYT+YY
YZT=YZT+YZ
ZZT=ZZT+ZZ
CONTINUE
RETURN
END
APPENDIX C

EXAMPLE PROBLEMS
FILE: CHOPTN1  SCALE  A1  University of Kentucky Computing Center

|   +   |   +   |   +   |   +   |   +   |   +   |   +   |   +   |

1  
OPTN = 1  (OLD CHEVRON)  
3 3 1 285 4500  
550000 400 11350 400 3000 450  
65 7 16  
0 7 23  
OPTN = 1  (OLD CHEVRON)  
2 2 1 285 4500  
200000 400 12000 450  
65 1825  
0 1825  
|   +   |   +   |   +   |   +   |   +   |   +   |   +   |   +   |
**THE PROBLEM PARAMETERS ARE**

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**NOTE:**
- The problem parameters include the total load, tire pressure, and load radius.
- Layer 1 has modulus 0.200,000 and Poisson's ratio 0.400, with thickness 18.25 inches.
- Layer 2 has modulus 0.012,000 and Poisson's ratio 0.450, and is semi-infinite.
FILE: CHOPTN2 SCALE A1 University of Kentucky Computing Center

|...+...1...+...2...+...3...+...4...+...5...+...6...+...7...+...8

   2
OPTN = 2
   3
    3
   2 4
480000 400 30000 400 4500 450
   8 16
   0 8 24
   2000 2927 2000 3150
   2000 2000 80 4500
   2000 2000 80 4500
   2000 3350 80 4500
   2000 9430 80 4500
   2000 10780 80 4500

OPTN = 2 (TANDEM)
   4
   4
   4
   2 8
480000 400 30000 400 24500 400 4500 450
   8 6 16
   0 8 14 30
   2000 2927 2000 3150
   2000 2000 80 4500
   2000 3350 80 4500
   2000 9430 80 4500
   2000 10780 80 4500
   7000 2000 80 4500
   7000 3350 80 4500
   7000 9430 80 4500
   7000 10780 80 4500

|...+...1...+...2...+...3...+...4...+...5...+...6...+...7...+...8
### Problem Parameters

Layer 1 has modulus 0.480,000, Poisson's ratio 0.400, and thickness 8.00 in.

Layer 2 has modulus 0.030,000, Poisson's ratio 0.400, and thickness 16.00 in.

Layer 3 has modulus 0.004,500, Poisson's ratio 0.450, and is semi-infinite.

### Coordinates of the Load Points and Load Values

- **LOADED POINT 1:**
  - \( X = 20.00 \) in
  - \( Y = 20.00 \) in
  - \( P = 4500.00 \) tire pressure

- **LOADED POINT 2:**
  - \( X = 20.00 \) in
  - \( Y = 33.50 \) in
  - \( P = 4500.00 \) tire pressure

- **LOADED POINT 3:**
  - \( X = 20.00 \) in
  - \( Y = 94.30 \) in
  - \( P = 4500.00 \) tire pressure

- **LOADED POINT 4:**
  - \( X = 20.00 \) in
  - \( Y = 107.80 \) in
  - \( P = 4500.00 \) tire pressure

### Coordinates Depth Stresses S-T-R-E-S-E-S

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### Coordinates Depth Strains S-T-R-A-I-N-S

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### Tables of Stresses and Strains for Different Points

For each point, the tables provide detailed stress and strain values at various depths, along with deflection values.
### THE ANSWERS BELOW ARE BY SUPERPOSITION

#### THE PROBLEM PARAMETERS ARE

- **Layer 1**: Modulus 0.480,000, Poisson's Ratio 0.400, and Thickness 8.00 in.
- **Layer 2**: Modulus 0.030,000, Poisson's Ratio 0.400, and Thickness 6.00 in.
- **Layer 3**: Modulus 0.024,500, Poisson's Ratio 0.400, and Thickness 16.00 in.
- **Layer 4**: Modulus 0.004,500, Poisson's Ratio 0.450, and is semi-infinite.

#### COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

- **Load Point 1**: 
  - $X = 20.00$ in, 
  - $Y = 20.00$ in, 
  - $P = 4500.00$ psi, 
  - Tire Pressure = 80.00 psi
- **Load Point 2**: 
  - $X = 20.00$ in, 
  - $Y = 33.50$ in, 
  - $P = 4500.00$ psi, 
  - Tire Pressure = 80.00 psi
- **Load Point 3**: 
  - $X = 20.00$ in, 
  - $Y = 94.30$ in, 
  - $P = 4500.00$ psi, 
  - Tire Pressure = 80.00 psi
- **Load Point 4**: 
  - $X = 70.00$ in, 
  - $Y = 20.00$ in, 
  - $P = 4500.00$ psi, 
  - Tire Pressure = 80.00 psi
- **Load Point 5**: 
  - $X = 70.00$ in, 
  - $Y = 33.50$ in, 
  - $P = 4500.00$ psi, 
  - Tire Pressure = 80.00 psi
- **Load Point 6**: 
  - $X = 70.00$ in, 
  - $Y = 94.30$ in, 
  - $P = 4500.00$ psi, 
  - Tire Pressure = 80.00 psi
- **Load Point 7**: 
  - $X = 70.00$ in, 
  - $Y = 107.80$ in, 
  - $P = 4500.00$ psi, 
  - Tire Pressure = 80.00 psi
- **Load Point 8**: 
  - $X = 70.00$ in, 
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  - Tire Pressure = 80.00 psi

#### DEPTH-STRESS-VALUE

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#### DEPTH-STRAIN ENERGY

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### Coordinates of the Load Points and Load Values

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- 1) 20.00
- 2) 20.00
- 3) 20.00
- 4) 20.00
- 5) 70.00
- 6) 70.00
- 7) 70.00
- 8) 70.00

**Y**
- 1) 20.00
- 2) 33.50
- 3) 94.30
- 4) 107.80
- 5) 20.00
- 6) 33.50
- 7) 94.30
- 8) 107.80

**Tire Pressure**
- 80.00 PSI

### Depth-Strains

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<td>73</td>
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<td></td>
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</tbody>
</table>
THE ANSWERS BELOW ARE BY SUPERPOSITION

The problem parameters are:

- Layer 1 has modulus 1,200,000, Poisson's ratio 0.400, and thickness 7.00 in.
- Layer 2 has modulus 0.011,850, Poisson's ratio 0.400, and thickness 16.00 in.

Coordinates of the load points and load values are:

<table>
<thead>
<tr>
<th>Coordination</th>
<th>Depth</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>X2</th>
<th>Y2</th>
<th>Z2</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.25</td>
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<td>-7</td>
<td>-0.729191E+01</td>
<td>0.857135E-05</td>
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<td>0.143685E-01</td>
<td>0.139009E-02</td>
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<tr>
<td>15.25</td>
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<td>-0.800193E-01</td>
<td>0.981052E-01</td>
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<td>0.172346E-00</td>
<td>0.138950E-02</td>
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<td>0.172338E-00</td>
<td>0.138951E-02</td>
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<td>-0.874502E-01</td>
<td>0.491461E-02</td>
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<td>0.558273E-01</td>
<td>0.122256E-02</td>
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<td>0.122256E-02</td>
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Coordinates of the strain energy and work density are:

<table>
<thead>
<tr>
<th>Coordination</th>
<th>Depth</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>X2</th>
<th>Y2</th>
<th>Z2</th>
<th>Strain Energy</th>
<th>Density</th>
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<tbody>
<tr>
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<td>0.754050E-05</td>
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<tr>
<td>15.25</td>
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<td>-0.480404E+01</td>
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<td>0.405424E-06</td>
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The tire pressure is 26.50 psi.
### OPTN = 3 [ROAD RATER]

--------------- THE ANSWERS BELOW ARE BY SUPERPOSITION -------------------------------

**THE PROBLEM PARAMETERS ARE**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Modulus</th>
<th>Poisson's Ratio</th>
<th>Thickness</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1,200,000</td>
<td>0.400</td>
<td>7.00 in</td>
</tr>
<tr>
<td>2</td>
<td>0.011,850</td>
<td>0.400</td>
<td>16.00 in</td>
</tr>
<tr>
<td>3</td>
<td>0.003,000</td>
<td>0.450</td>
<td>SEMI-INFINITE</td>
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</table>

**COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>P</th>
<th>Tire Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.00</td>
<td>10.00</td>
<td>1</td>
<td>941.00 PSI</td>
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<tr>
<td>20.50</td>
<td>10.00</td>
<td>3</td>
<td>729.00 PSI</td>
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**COORDINATES**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
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</thead>
<tbody>
<tr>
<td>15.25</td>
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<td>-7</td>
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<tr>
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**DEPT**

<table>
<thead>
<tr>
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<tbody>
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**XZ YY**

<table>
<thead>
<tr>
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<th>ZZ</th>
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</thead>
<tbody>
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<td>0</td>
<td>0.108245E-02</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.109182E-02</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.368380E-01</td>
<td></td>
</tr>
</tbody>
</table>

**S-T-R-E-S-S-E-S**

<table>
<thead>
<tr>
<th>X</th>
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<th>ZZ</th>
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<tbody>
<tr>
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<tr>
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**S-T-R-A-I-N-S**

<table>
<thead>
<tr>
<th>X</th>
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<th>ZZ</th>
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<tbody>
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**DEFLECTION**

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</table>

**STRAIN ENERGY**

<table>
<thead>
<tr>
<th>X</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.108245E-02</td>
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<td>0.0</td>
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<tr>
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<td>0.368380E-01</td>
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</table>

**STRAIN ENERGY**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>ZZ</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.108245E-02</td>
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<tr>
<td>0.0</td>
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</tr>
<tr>
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</tbody>
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**DENSITY**

<table>
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<tr>
<th>X</th>
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<th>ZZ</th>
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</thead>
<tbody>
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<td>0.0</td>
<td>0.108245E-02</td>
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<td>0.0</td>
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</tr>
<tr>
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<td>0.368380E-01</td>
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</tbody>
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**STRAIN**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>ZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.108245E-02</td>
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<tr>
<td>0.0</td>
<td>0.109182E-02</td>
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</tr>
<tr>
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<td></td>
</tr>
</tbody>
</table>

**WORK**

<table>
<thead>
<tr>
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<th>Y</th>
<th>ZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.108245E-02</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.109182E-02</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.368380E-01</td>
<td></td>
</tr>
</tbody>
</table>
*THE ANSWERS BELOW ARE BY SUPERPOSITION*

**THE PROBLEM PARAMETERS ARE**

- **Layer 1** has modulus 2,000,000, Poisson's ratio 0.400, and thickness 7.00 IN.
- **Layer 2** has modulus 1,11,850, Poisson's ratio 0.400, and thickness 6.00 IN.
- **Layer 3** has modulus 1,000,000,000,000, Poisson's ratio 0.450, and thickness 16.00 IN.
- **Layer 4** has modulus 0,007,500, Poisson's ratio 0.450, and is semi-infinite.

**COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>Deflection</th>
<th>Strain Energy</th>
<th>Work Density</th>
<th>Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.25</td>
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<td>0.697501E-01</td>
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<td>0.167153E-00</td>
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### THE ANSWERS BELOW ARE BY SUPERPOSITION

<table>
<thead>
<tr>
<th>Layer</th>
<th>Modulus (ksi)</th>
<th>Poisson's Ratio</th>
<th>Thickness (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,000,000</td>
<td>0.400</td>
<td>7.00</td>
</tr>
<tr>
<td>2</td>
<td>0.111,850</td>
<td>0.400</td>
<td>6.00</td>
</tr>
<tr>
<td>3</td>
<td>0.027,500</td>
<td>0.450</td>
<td>16.00</td>
</tr>
<tr>
<td>4</td>
<td>0.007,500</td>
<td>0.450</td>
<td>Semi-Infinite</td>
</tr>
</tbody>
</table>

### COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

<table>
<thead>
<tr>
<th>X (1)</th>
<th>Y (1)</th>
<th>Z (1)</th>
<th>P (1)</th>
<th>Tire Pressure</th>
</tr>
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<tbody>
<tr>
<td>10.00</td>
<td>1.00</td>
<td>0.00</td>
<td>9.41</td>
<td>100 PSI</td>
</tr>
<tr>
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<td>2.00</td>
<td>0.00</td>
<td>2.50</td>
<td>26.5 PSI</td>
</tr>
<tr>
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<td>3.00</td>
<td>0.00</td>
<td>9.41</td>
<td>35.5 PSI</td>
</tr>
<tr>
<td>20.50</td>
<td>4.00</td>
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<td>7.29</td>
<td>26.5 PSI</td>
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### STRAIN ENERGY WORK

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density (psi)</th>
<th>Strain Energy (in-lbf)</th>
<th>Work (in-lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>549,936</td>
<td>0.113,411</td>
<td>0.776558</td>
</tr>
<tr>
<td>2</td>
<td>570,633</td>
<td>0.349,316</td>
<td>0.349,316</td>
</tr>
<tr>
<td>3</td>
<td>689,930</td>
<td>0.411,876</td>
<td>0.411,876</td>
</tr>
<tr>
<td>4</td>
<td>549,936</td>
<td>0.113,411</td>
<td>0.776558</td>
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### STRAIN WORK

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density (psi)</th>
<th>Strain (in)</th>
<th>Strain Energy (in-lbf)</th>
<th>Work (in-lbf)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>549,936</td>
<td>0.113,411</td>
<td>0.776558</td>
<td>0.349,316</td>
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<tr>
<td>2</td>
<td>570,633</td>
<td>0.349,316</td>
<td>0.349,316</td>
<td>0.349,316</td>
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<td>3</td>
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<td>0.349,316</td>
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THE PROBLEM PARAMETERS ARE

<table>
<thead>
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<th>Layer</th>
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<th>Thickness</th>
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<tr>
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<td>6.00 IN</td>
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<td>16.00 IN</td>
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<td>0.067.50</td>
<td>0.450</td>
<td>AND IS SEMI-INFINITE.</td>
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COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.25</td>
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<td>729.00</td>
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<tr>
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<tr>
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<td>33.50</td>
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COORDINATES

<table>
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<tr>
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<th>Z</th>
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<tbody>
<tr>
<td>15.25</td>
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<td>34.00</td>
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<tr>
<td>15.25</td>
<td>34.00</td>
<td>29.00</td>
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DEPTH

<table>
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<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
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<tbody>
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<td>15.25</td>
<td>34.00</td>
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<tr>
<td>15.25</td>
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COORDINATES

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
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