A Review and Analysis of Pile Design

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**Abstract**

A review of lateral load design of piles is presented. It appears many different methods and allowable lateral loads are permitted by the states. One particular method is discussed in detail, and from that method two charts are presented that relate pile deflection to lateral load. Recommendations are given on design criteria for lateral loads.

A review of allowable axial stresses is also presented. The results of a brief finite element analysis of pile tip settlement versus load for various conditions are also presented. These results are compared to data obtained from the literature. Recommendations on maximum allowable axial stress are given.

**Key Words**

- Piles
- Bridge Abutments
- Design

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A REVIEW AND ANALYSIS OF PILE DESIGN

by

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in cooperation with
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Commonwealth of Kentucky

and

Federal Highway Administration
U.S. Department of Transportation

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INTRODUCTION

Steel H-piles often are used in Kentucky to support bridge end-bents. These piles are usually point-bearing and are most often driven through specially prepared earth cores, as illustrated in the Kentucky Department of Highways' Standard Drawings RGX-100 and RGX-105.

Lateral forces on end-bent piles can be produced by earth pressures, impact of moving vehicles, braking forces, and eccentric axial loads. Vertical loads on these piles are derived from dead load of the structure, dynamic forces from traffic, and downdrag.

Kentucky's current design procedure for piles permits a maximum allowable lateral load of 2 kips per pile for the dead load condition, regardless of pile size (Section 66-05.0345, Guidance Manual, Division of Bridges). The maximum allowable axial load for point-bearing steel H-piles is 9,000 pounds per square inch times the end area of the pile (Section 66-05.0312, Guidance Manual, Division of Bridges). This is in accordance with Section 4.3.4.7 of AASHTO's Standard Specification for Highway Bridges, 1983 edition.

A study of the final report on "Foundation Engineering Management Reviews" by the Federal Highway Administration (1) shows there is considerable variation in methods of analysis used by the states to determine lateral loads. Of the states that responded to the survey by FHWA, approximately 22 indicated that lateral loads were not analyzed and that battered piles were used to resist any lateral loads that might be present. Two states used the method presented in NAVFAC DM 7.2 (2), which is based largely upon an analysis method published by Reese and Matlock (3). Three states used a method reported by Broms (4, 5). Also, there is a wide range in maximum allowable lateral loads permitted by the states.
Allowable loads range from 2 kips per pile to 10 kips per pile. Other states use a deflection criterion to determine allowable lateral load. NAVFAC DM 7.2 (2) states that 0.25 inch is often used as the maximum allowable deflection at the top of the pile. However, this criterion would depend largely upon the structure and the amount of deflection it could tolerate.

The maximum allowable axial load permitted by the states for steel H-piles appears to be more uniform than for lateral loads. Approximately 40 of the respondents to FHWA's survey indicated the previously mentioned AASHTO specification (9,000 psi) is used. However, some states permit 12,000 psi on a site-by-site basis. Other states allow 12,000 psi when load tests are performed. The state of Texas sets no maximum allowable stress, but "loads are determined from information furnished by the soils laboratory."

From discussions with bridge design engineers in other states, and from a review of literature, it appears there is no clear rationale for the AASHTO specification on maximum allowable axial load. However, it is apparently related to concern for the strength of the bearing stratum and the inability to always obtain reliable material properties of the stratum. Consequently, it appears there is a considerable degree of uncertainty as to the appropriate value to be used in design.

The purpose of this report is to present a more analytical approach to lateral load design than is presently available in the literature, and from this, present in graphical form information relating pile deflection to lateral load. Also, a discussion of maximum allowable axial loads is given, and a finite element analysis has been performed to estimate the effects of a point-bearing pile on the bearing stratum. Results of that
analysis are presented along with recommendations for both lateral load design and maximum allowable axial loads.

LATERAL LOAD DESIGN

The Federal Highway Administration has recently published a report by Reese entitled "Handbook on Design of Piles and Drilled Shafts Under Lateral Load" (6). In that report the following statement appears: "The application of a lateral load to the top of a pile must result in the lateral deflection of the pile. The reactions that are generated in the soil must be such that the equations of static equilibrium are satisfied, and the reactions must be consistent with the deflections. Also, because no pile is completely rigid, the amount of pile bending must be consistent with the soil reactions and the pile stiffness.

"Thus, the problem of the laterally loaded pile is a 'soil-structure-interaction' problem."

The problem is necessarily a difficult one, and in the past, numerous approaches have been used to solve it. One of the better known analytical methods was published by Broms (4, 5). In recent years, others, including those listed in References 7 through 22, have contributed to the development of more rational methods that are generally based upon more complete data.

In Reference 6, Reese details a non-dimensional method of analysis based on his work and the work done by some of the authors in the previously mentioned references. A summary of that method is presented here, along with equations and assumptions. (Derivations of all equations may be found in the listed references and will not be given here.)

Because piles that support bridge end-bents are often driven into clay cores that are generally above the water table, that particular case is
discussed and illustrated in this report. Also, it is assumed the pile heads are fixed against rotation. In reality, a non-dimensional analysis is valid only for piles that have a constant stiffness and no axial load. However, computer solutions have shown that these parameters only moderately influence pile deflections. Therefore the method can be used for estimating purposes.

To calculate pile deflections, the resistance of the soil to pile movement must be known. The ultimate soil resistance per unit length of pile, $p_u$, can be calculated from one of the following two equations, and the smaller value for $p_u$ is always used.

\[
p_u = (3 + (w'/c)x + (J/b)x)c b
\]

or

\[
p_u = 9cb
\]

where $x =$ distance from ground surface to point on pile where calculation is being made (in.),

$w'$ = average unit weight from ground surface to depth $x$ (lb/cu in.),

$c =$ shear strength at depth $x$ (lb/sq in.),

$b =$ width of pile (in.), and

$J =$ empirical parameter (generally a value of 0.5 is used).

Next, it is necessary to develop a soil-resistance-versus-pile-deflection curve (called a p-y curve) for various depths on the pile. The depths chosen should be more closely spaced near the top of the pile where the deflection is greatest. To develop the p-y curves, the soil deflection at one-half the ultimate soil resistance is calculated from the following:

\[
y_{50} = 2.5e_{50}b
\]

where $y_{50} =$ deflection at one-half ultimate soil resistance (in.).
\[
e_{50} = \text{strain at one-half the maximum principal stress difference from the triaxial test (if no test data are available, 0.01 is often used for medium to stiff clays).}
\]

From Equation 3, a general p-y curve for the particular soil and pile under consideration may be calculated using the following equation:

\[\frac{p}{p_u} = 0.5 \left(\frac{y}{y_{50}}\right)^{0.25}.\]  (4)

Beyond \(y = 16y_{50}\), \(p\) equals \(p_u\) for all values of \(y\). An example of a generalized p-y curve is shown in Figure 1.

From Equation 4, individual p-y curves for each chosen depth on the pile may be generated. A number of arbitrarily chosen \(y\) values are used in Equation 4 to obtain a ratio of some soil resistance value to the ultimate soil resistance value, \(p_u\) (calculated from Equations 1 or 2). This ratio is multiplied by \(p_u\) for the particular depth in question, and the resultant value is plotted as a function of \(y\). Example p-y curves for various depths are shown in Figure 2.

The relative stiffness factor between the pile and the soil is then computed:

\[T = (EI/k)^{0.20}\]  (5)

where \(EI\) = flexure rigidity of the pile and

\(k\) = constant that relates secant modulus of soil reaction to depth \((E_s = kx)\).

However, \(k\) is not known. Therefore, \(T\) must be assumed and an iteration procedure is used to solve for pile deflections. In addition, the depth coefficient, \(Z_{\text{max}}\), must be calculated:

\[Z_{\text{max}} = \frac{x_{\text{max}}}{T_{\text{assumed}}}\]  (6)

A \(Z_{\text{max}}\) value of 5 or greater indicates the pile is a "long" pile, as opposed to a "short" or "intermediate" pile. Therefore, the pile will fail
with two plastic hinges -- one near the top where the pile is constrained, and the other at some critical depth.

The pile deflection may be calculated at each point on the pile for which a p-y curve has been constructed:

\[ y_f = F_y (P_t/EI) \]  \hspace{1cm} (7)

where \( y_f \) = pile deflection (in.),

\( F_y \) = deflection coefficient, and

\( P_t \) = lateral load.

\( F_y \) may be obtained from Figure 3, which is from Reference 6. After computing a value of \( y_f \) for each depth on the pile, a value of soil resistance, \( p \), is selected that corresponds to the pile deflection \( y_f \) at the depth of the p-y curve that is available.

The secant modulus of soil reaction, \( E_s \), is computed for each depth from the following:

\[ E_s = p/y_f. \]  \hspace{1cm} (8)

\( E_s \) is plotted as a function of depth, \( x \). A linear fit is drawn through the points, and the slope of the line is equal to the soil constant, \( k \):

\[ k = E_s/x. \]  \hspace{1cm} (9)

A new value of the relative stiffness factor \( T \) is computed using the value of \( k \) determined from Equation 9. The new value of \( T \) is compared to the original assumed value. Then, all calculations, beginning at Equation 5, are repeated assuming a new value of \( T \). The procedure is repeated until the assumed value of \( T \) equals the calculated value of \( T \) (converges). The \( y_f \) values calculated in the last iteration are the correct deflection values for the pile. A detailed numerical example of this procedure is given in the Appendix. Figures 4 and 5 illustrate the relationship between pile deflection at the groundline, calculated from the procedure as
described, and lateral load. Figure 4 is for a 12x53 pile, and Figure 5 is for a 14x73 pile. Deflections for four different soil strengths (c = undrained shear strength) also are shown.

From these two figures, it is evident the two piles under consideration can withstand considerably greater loads than the 2 kips presently allowed in Kentucky. Consequently, it is recommended that the 2-kip criterion be abandoned, and a limiting deflection criterion be adopted. The magnitude of the limiting deflection would depend largely upon the amount of movement the structure can tolerate without damage. However, as stated earlier, NAVFAC DM 7.2 (2) states that a limiting deflection value of 0.25 inch is often used.

Figures 4 and 5 also could be used as design aids. After calculating the lateral loads, the designer simply refers to the appropriate figure (assuming the undrained shear strength of the earth core is known or may be estimated) and checks to see if the calculated load produces deflections less than or equal to the chosen limiting deflection.

AXIAL LOAD CAPACITY

In December 1983, the Federal Highway Administration issued a report entitled "Allowable Stresses in Piles" (23). In that report, a review of allowable stresses used by various agencies, states and foreign countries is presented. The rationale behind and the explanation for the allowable stresses in use generally are not available.

A number of factors that can contribute to the bearing capacity of a pile being exceeded are listed and discussed in that report and in other reports (24, 25, 26):

1. Overloading the bridge (such as permitting overloaded trucks),
2. Negative skin friction,
3. Inadequate design or calculation errors that result in underestimated loads,
4. Group behavior -- all piles in a group normally do not carry the same load,
5. Pile mislocation,
6. Differential settlement,
7. Construction activities,
8. Variation in material strength or size,
9. Driving damage,
10. Soil heave,
11. Poor inspection, and

In Reference 23, a parameter identified as the Hidden Defect Factor (HDF) is introduced. This parameter is an allowable stress reduction factor that accounts for the environment into which the pile is driven. For example, a soft soil having no particles larger than gravel size would be considered ideal conditions and the stress reduction factor would be 1.0. If the pile is penetrating material having large cobbles or boulders, or is penetrating soft rock, the conditions would be considered severe, and the stress reduction factor would be 0.7. For normal conditions, the stress reduction factor would 0.85. However, no particular rationale for choosing the magnitudes for the factor is provided.

The same report also recommended other stress reduction factors. The first is the $\phi$-factor with a recommended value of 0.85. This factor is used because there are residual stresses in the pile cross section that result from differential cooling during and after the mill roll operation. The second factor is the eccentricity factor (ecc). This is introduced to
account for off-axis or eccentric loading. The recommended value for this factor is 0.70.

From the previous information, the following equation to calculate the allowable stress in a steel H-pile was proposed:

\[ f_a = (\phi) (\text{ecc}) (\text{HDF}) (F_y) / \text{LF} \]  

(10)

where \( f_a \) = allowable stress,
\( \phi = 0.85 \),
\( \text{ecc} = 0.70 \),
\( \text{HDF} = \) either 1.0, 0.85, or 0.70
\( F_y = \) yield stress of steel (lb/sq in.), and
\( \text{LF} = \) Load Factor (2.00).

Therefore,

\[ f_a = 0.2975 (\text{HDF}) (F_y). \]  

(11)

For ideal conditions, an allowable stress of 0.30 \( F_y \) (10.8 ksi for 36-ksi steel) was recommended. For severe conditions, the recommended stress would be 0.20 \( F_y \) (7.2 ksi for 36-ksi steel).

However, BP 12x53 and BP 14x73 steel sections do not meet requirements for a non-compact section as defined in AASHTO's Standard Specifications for Highway Bridges (1977 Edition), Section 1.7.59 (B). Therefore, these sections are subject to even further stress reductions of 0.75 and 0.70, respectively. Applying these reductions to Equation 11 would yield an allowable stress of 7.9 ksi for ideal conditions and 5.4 ksi for severe conditions. This appears to be one of the most conservative recommendations in the literature. Also, from experience with piles in Kentucky, this would appear to be overly conservative.

The American Iron and Steel Institute indicates that 50 percent of the specified yield strength of the steel may be used in design. Dismuke (27)
states that "experience with the steel pile stress level of $0.5 F_y$ is considered to be sufficient for inclusion (in codes) as an allowable stress." Also, the building code of New Orleans and Germany permit 50 percent of the yield strength.

However, a number of authors, including Davisson (24, 28), Fuller (25), and Swiger (26) indicate that 50 percent of the yield stress is too high. A number of reports (29, 30, 31) have presented data that show steel H-piles may often fail at stresses well below the stated yield stress of the steel. Williams (31) reported steel H-piles bearing on rock failed at stress levels as low as 73 percent of the yield strength. In that particular case, if an allowable stress of 50 percent of the yield strength had been used in design, a factor-of-safety of only 1.46 would have been the result. This is well below the generally accepted factor of safety of 2.0. It appears most building codes permit maximum allowable stresses of 12.0 ksi or $0.35 F_y$ (12.6 ksi for steel with 36-ksi yield strength). These building codes include the Canadian Standard Association; the Basic Building Code; the Standard Building Code; the Uniform Building Code; the U.S. Navy; and cities of Chicago, Los Angeles, and New York. Davisson (28) and Fuller (25) indicate that a maximum allowable stress of 12.0 ksi appears to be reasonable and appropriate to use in design. Their conclusions were based upon experience and data.

Fellenius (32) states that not only should the structural capacity of the pile be considered, but the pile-soil system should be considered. He further states that the geotechnical capacity of a pile is independent of the yield strength above 36 ksi, but very dependent on the hammer-soil-pile combination used in any particular case. Thompson and Thompson (29) report data that suggest a relationship between the ultimate bearing capacity of
an end-bearing pile and the impact driving stress near the top of the pile. Based upon approximately 24 cases, they conclude the ultimate bearing capacity is 1.2 times greater than the impact driving stress at the pile head. One standard deviation of that data was ±20 percent. Based partly upon the data by Thompson and Thompson and on other sources, Fellenius proposes the following relationship to calculate the allowable stress:

\[ f_a = \frac{FE}{c} \]  

where \( F = \) F-factor (equal to 7.0 ft/sec for steel and concrete piles), \( E = \) modulus of elasticity (30,000 ksi for steel), and \( c = \) speed of the driving wave in the pile (16,800 ft/sec for steel).

From Equation 12, the allowable stress in a steel pile would be

\[ f_a = (7) \frac{(30,000)}{(16,800)} = 12.5 \text{ ksi}. \]

From the previous discussion and results from Equation 12, it appears the most appropriate value of allowable stress to use in design is 12,000 psi. For a BP 12x53 steel H-pile, this is a total load of approximately 94 tons.

To estimate the effect of this increased load on the bearing stratum, a finite element analysis was performed on a model soil-pile system. A number of cases were analyzed and are listed in Table 1. In all cases, it was assumed that 100 percent of the load was carried by the pile tip and none by the shaft. The results listed in the table show only small settlements, even for loads of 200 tons on weathered shale. Figures 6 through 8 show the distribution of settlement for Cases 2, 3, and 4, respectively. Those figures show that most of the settlement and, consequently, stress occur in the top 5 feet of the bearing stratum. Therefore, it appears that during subsurface exploration, coring to a depth of 10 feet should be sufficient in nearly all cases, unless obvious site
conditions dictate otherwise. To help verify the validity of the finite
element results, comparisons were made with load test data published by
Brierley, et al. (33) on end-bearing piles driven to bedrock. Figure 9
illustrates that reasonably good correlations were obtained, with Case 5
yielding a value identical to load test data. It should be noted that
Brierley and his co-authors did not describe the type or condition of the
bedrock to which their piles were driven.

CONCLUSIONS AND RECOMMENDATIONS

1. It appears the finite element analysis approximates actual load
test data reasonably well.

2. Under the conditions analyzed, most settlements occur in the top 4
to 5 feet of the bearing stratum.

3. It is recommended that the 2-kip maximum allowable lateral load per
pile be abandoned, and a limiting deflection criterion be adopted.

4. A maximum limiting deflection of 0.25 inch is recommended.

5. It is recommended that the maximum allowable axial stress be
12,000 psi. This recommendation must be qualified and limited when the
following conditions are considered. (1) This recommendation applies only
to 12x53 and 14x73 steel H-piles. (2) If boulders, outliers, or other
conditions are present that might be expected to damage the pile during
driving, 9,000 psi should be the maximum allowable axial stress. (3) Also,
if such conditions exist, a reinforced driving tip is recommended. (4) A
dynamic pile driving analysis should be made to insure that driving
stresses are not excessive.

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Research Report 244-2F, The University of Texas, Austin, Texas, May 1982.


21. Reese, L. C., and Nyman, K. J., "Field Load Tests of Instrumented


<table>
<thead>
<tr>
<th>CASE NO.</th>
<th>DESCRIPTION</th>
<th>LOAD (TONS)</th>
<th>SETTLEMENT (INCH)</th>
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<tbody>
<tr>
<td>1</td>
<td>30 Feet of Competent Limestone</td>
<td>140</td>
<td>0.040</td>
</tr>
<tr>
<td>2</td>
<td>30 Feet of Unweathered Shale</td>
<td>140</td>
<td>0.084</td>
</tr>
<tr>
<td>3</td>
<td>Top 3 Feet of Weathered, Fractured Shale; Bottom 27 Feet Unweathered Shale</td>
<td>140</td>
<td>0.140</td>
</tr>
<tr>
<td>4</td>
<td>Top 3 Feet Weathered, Fractured Shale; Bottom 27 Feet Unweathered Shale</td>
<td>200</td>
<td>0.199</td>
</tr>
<tr>
<td>5</td>
<td>Top 4 Feet Interbedded Layers of Unweathered Shale and Weathered Shale</td>
<td>140</td>
<td>0.110</td>
</tr>
</tbody>
</table>
Figure 1. Ratio of Soil Resistance to Peak Soil Resistance as a Function of the Ratio of Soil Deflection to Deflection at 50 Percent of Peak Soil Resistance.
Figure 2. Soil Resistance versus Soil Deflection Curves.
Figure 3. Deflection Coefficients for Pile Fixed Against Rotation at Groundline. [Ref. 6]
Figure 4. Lateral Deflection at Top of Pile versus Lateral Load for BP12x53 Pile.
Figure 5. Lateral Deflection at Top of Pile versus Lateral Load for BP14x73 Pile.
Figure 7. Settlement Contours (inches) for Case III.
Figure 8. Settlement Contours (inches) for Case IV.
Figure 9. Settlement Predictions from Finite Elements Compared to Data from Brierley et al. (33).
APPENDIX

EXAMPLE PROBLEM
APPENDIX

GIVEN: Pile Section - BP 14x73

Pile Length - 40 ft
Pile Head - Restrained
Soil Type - Clay (above water table)
Soil Strength - 1.0 tsf
Soil Unit Weight - 110 lb/cu ft
Lateral Load - 30 kips
 Depths of Calculations - 0, 10, 20, 30, 40, 70, 100, 180 in.

COMPUTE: Deflection of Pile at Groundline.

STEP 1. Compute the ultimate soil resistance per unit length of pile at each depth.

Depth = 0 inch
\[ p_u = (3 + (w'/c)x + (J/b)x)cb \]
\[ = (3 + (0.0637 \text{ lb/cu in.} / 13.89 \text{ lb/sq in.})0 \text{ in.} + \]
\[ (0.5 / 14.6 \text{ in.})0 \text{ in.})(13.89 \text{ lb/sq in.} \times 14.6 \text{ in.}) \]
\[ p_u = 607.8 \text{ lb/in.} \]

Depth = 10 inches
\[ p_u = (3 + (0.0637 \text{ lb/cu in.} / 13.89 \text{ lb/sq in.})10 \text{ in.} + \]
\[ (0.5 / 14.6 \text{ in.})10 \text{ in.})(13.89 \text{ lb/sq in.} \times 14.6 \text{ in.}) \]
\[ p_u = 686.8 \text{ lb/in.} \]

Depth = 20 inches
\[ p_u = (3 + (0.0637 \text{ lb/cu in.} / 13.89 \text{ lb/sq in.})20 \text{ in.} + \]
\[ (0.5 / 14.6 \text{ in.})20 \text{ in.})(13.89 \text{ lb/sq in.} \times 14.6 \text{ in.}) \]
\[ p_u = 765.8 \text{ lb/in.} \]
Depth = 30 inches

\[ p_u = (3 + (0.0637 \text{ lb/cu in.} / 13.89 \text{ lb/sq in.}) \times 30 \text{ in.}) + (0.5 / 14.6 \text{ in.}) \times 30 \text{ in.}) \times (13.89 \text{ lb/sq in.} \times 14.6 \text{ in.}) \]

\[ p_u = 844.8 \text{ lb/in.} \]

Depth = 40 inches

\[ p_u = (3 + (0.0637 \text{ lb/cu in.} / 13.89 \text{ lb/sq in.}) \times 40 \text{ in.}) + (0.5 / 14.6 \text{ in.}) \times 40 \text{ in.}) \times (13.89 \text{ lb/sq in.} \times 14.6 \text{ in.}) \]

\[ p_u = 923.8 \text{ lb/in.} \]

Depth = 70 inches

\[ p_u = (3 + (0.0637 \text{ lb/cu in.} / 13.89 \text{ lb/sq in.}) \times 70 \text{ in.}) + (0.5 / 14.6 \text{ in.}) \times 70 \text{ in.}) \times (13.89 \text{ lb/sq in.} \times 14.6 \text{ in.}) \]

\[ p_u = 1,160.8 \text{ lb/in.} \]

Depth = 100 inches

\[ p_u = (3 + (0.0637 \text{ lb/cu in.} / 13.89 \text{ lb/sq in.}) \times 100 \text{ in.}) + (0.5 / 14.6 \text{ in.}) \times 100 \text{ in.}) \times (13.89 \text{ lb/sq in.} \times 14.6 \text{ in.}) \]

\[ p_u = 1,397.8 \text{ lb/in.} \]

Depth = 180 inches

\[ p_u = (3 + (0.0637 \text{ lb/cu in.} / 13.89 \text{ lb/sq in.}) \times 180 \text{ in.}) + (0.5 / 14.6 \text{ in.}) \times 180 \text{ in.}) \times (13.89 \text{ lb/sq in.} \times 14.6 \text{ in.}) \]

\[ p_u = 2,029.8 \text{ lb/in.} \]

STEP 2. Compute the deflection, \( y_{50} \), at one-half the ultimate soil resistance (\( e_{50} = 0.010 \)).

\[ y_{50} = 2.5(e_{50})b \]

\[ = 2.5(0.010)14.6 \]
STEP 3. Compute the points describing the generalized p-y curve.

\[
p/p_u = 0.5(y/y_{50})^{0.25}
\]

for \(y/y_{50} = 1; \ p/p_u = 0.50\)

\(y/y_{50} = 2; \ p/p_u = 0.60\)

\(y/y_{50} = 5; \ p/p_u = 0.74\)

\(y/y_{50} = 10; \ p/p_u = 0.89\)

\(y/y_{50} = 16; \ p/p_u = 1.00\)

(This is Figure 1 in the report.)

STEP 4. Construct a p-y curve for each depth.

**Depth = 0 inches**

When the deflection, \(y\), equals 1.0 times \(y_{50}\) (obtained from STEP 2), then the soil resistance, \(p\), will equal 50 percent of the ultimate soil resistance, \(p_u\), for that depth (obtained from STEP 1).

\(y_1/y_{50} = 1\)

\(y_1 = 1.0(y_{50})\)

\(y_1 = 1.0(0.365)\)

\(y_1 = 0.365\)

therefore:

\(p_1 = 0.5(607.8)\)

\(= 304\)

in like manner:

\(y_2/y_{50} = 2\)

\(y_2 = 2.0(0.365)\)
\[ y_2 = 0.73 \]

and

\[ p_2 = 0.6(607.8) = 365 \]

This procedure is repeated for three additional points.

\[ y_3 = 1.1 \quad y_4 = 1.5 \quad y_5 = 1.8 \]
\[ p_3 = 401 \quad p_4 = 425 \quad p_5 = 450 \]

The corresponding \( y \) and \( p \) values are plotted to create a load versus deflection curve (p-y curve) at depth 0, as shown in Figure 2 of the report. The p-y curves are now developed for the remaining depths. The values are listed below.

**Depth = 10 inches**

<table>
<thead>
<tr>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>( y_3 )</th>
<th>( y_4 )</th>
<th>( y_5 )</th>
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<td>0.37</td>
<td>0.73</td>
<td>1.1</td>
<td>1.5</td>
<td>1.8</td>
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<tr>
<td>( p_1 )</td>
<td>( p_2 )</td>
<td>( p_3 )</td>
<td>( p_4 )</td>
<td>( p_5 )</td>
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<tr>
<td>343</td>
<td>412</td>
<td>453</td>
<td>481</td>
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**Depth = 20 inches**

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<th>( y_3 )</th>
<th>( y_4 )</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>0.73</td>
<td>1.1</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>( p_2 )</td>
<td>( p_3 )</td>
<td>( p_4 )</td>
<td>( p_5 )</td>
</tr>
<tr>
<td>383</td>
<td>459</td>
<td>505</td>
<td>536</td>
<td>567</td>
</tr>
</tbody>
</table>

**Depth = 30 inches**

<table>
<thead>
<tr>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>( y_3 )</th>
<th>( y_4 )</th>
<th>( y_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>0.73</td>
<td>1.1</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>( p_2 )</td>
<td>( p_3 )</td>
<td>( p_4 )</td>
<td>( p_5 )</td>
</tr>
<tr>
<td>422</td>
<td>507</td>
<td>558</td>
<td>591</td>
<td>625</td>
</tr>
</tbody>
</table>

**Depth = 40 inches**

<table>
<thead>
<tr>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>( y_3 )</th>
<th>( y_4 )</th>
<th>( y_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>0.73</td>
<td>1.1</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>( p_2 )</td>
<td>( p_3 )</td>
<td>( p_4 )</td>
<td>( p_5 )</td>
</tr>
<tr>
<td>461</td>
<td>554</td>
<td>610</td>
<td>647</td>
<td>684</td>
</tr>
</tbody>
</table>

**Depth = 70 inches**

<table>
<thead>
<tr>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>( y_3 )</th>
<th>( y_4 )</th>
<th>( y_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>0.73</td>
<td>1.1</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>( p_2 )</td>
<td>( p_3 )</td>
<td>( p_4 )</td>
<td>( p_5 )</td>
</tr>
<tr>
<td>580</td>
<td>696</td>
<td>766</td>
<td>812</td>
<td>859</td>
</tr>
</tbody>
</table>
Depth = 100 inches
\[ y_1 = 0.37 \quad y_2 = 0.73 \quad y_3 = 1.1 \quad y_4 = 1.5 \quad y_5 = 1.8 \]
\[ p_1 = 699 \quad p_2 = 839 \quad p_3 = 922 \quad p_4 = 978 \quad p_5 = 1034 \]

Depth = 180 inches
\[ y_1 = 0.37 \quad y_2 = 0.73 \quad y_3 = 1.1 \quad y_4 = 1.5 \quad y_5 = 1.8 \]
\[ p_1 = 1015 \quad p_2 = 1218 \quad p_3 = 1340 \quad p_4 = 1421 \quad p_5 = 1502 \]

These curves also are shown in Figure 2 of the report.

STEP 5. Assume a value for the relative stiffness factor, T (Try T = 100)

\[ T = (EI/k)^{0.20} \]

\[ 100 = ((3.0 \times 10^7)(734)/k)^{0.20} \]

\[ k = 2.20 \]

STEP 6. Compute depth coefficient, \( Z_{\text{max}} \):

\[ Z_{\text{max}} = \frac{x_{\text{max}}}{T} \]

\[ = \frac{480 \text{ in.}}{100} \]

\[ = 4.8 \]

STEP 7. Use \( Z_{\text{max}} \) curve labelled 5 in Figure 3 of the report to get \( F_y \) coefficients.

Depth coefficient, \( Z \):
\[ Z = \frac{x}{T} \]

Therefore:
\[ Z_0 = 0 \quad Z_{20} = 0.2 \quad Z_{40} = 0.4 \quad Z_{100} = 1.0 \]
\[ Z_{10} = 0.1 \quad Z_{30} = 0.3 \quad Z_{70} = 0.7 \quad Z_{180} = 1.8 \]

From Figure 3:
\[ F_y(0) = 0.94 \quad F_y(20) = 0.92 \quad F_y(40) = 0.86 \quad F_y(100) = 0.64 \]
\[ F_y(10) = 0.93 \quad F_y(30) = 0.90 \quad F_y(70) = 0.75 \quad F_y(180) = 0.24 \]
STEP 8. Calculate \( y_f \) values for each depth.

\[
y_f = \frac{F_y (P_t T^3/EI)}{
\]

Therefore:

\[
y_f(0) = 0.94(1.36) = 1.28 \\
y_f(10) = 0.93(1.36) = 1.27 \\
y_f(20) = 0.92(1.36) = 1.25 \\
y_f(30) = 0.90(1.36) = 1.23 \\
y_f(40) = 0.86(1.36) = 1.17 \\
y_f(70) = 0.75(1.36) = 1.02 \\
y_f(100) = 0.64(1.36) = 0.87 \\
y_f(180) = 0.24(1.36) = 0.32 \\
\]

If \( T = 100 \), the preceding \( y_f \) values would be the pile deflections at the corresponding depths. However, since \( T \) was assumed, calculations must continue to find the correct \( T \).

STEP 9. From the p-y curves in Figure 2, select the value of soil resistance that corresponds to the \( y_f \) value calculated for each depth above. Compute the secant modulus of soil reaction \( (E_s = p/y) \). Plot the \( E_s \) values versus depth (see Figure A1).

STEP 10. From the \( E_s \) versus depth curve, compute \( k \) \( (k = E_s /x) \).

From Figure A1:

\[
k = 11.7 \\
\]

STEP 11. Recompute \( T \) from the new \( k \).

\[
T = (2.202 \times 10^{10}/11.7)^{0.20} \\
\]
This does not equal the assumed T of 100.

STEP 12. Assume a new T (Try T = 40).

(a) \[ 40 = (2.202 \times 10^{10}/11.7)^{0.20} \]
\[ k = 215 \]

(b) \[ Z_{\text{max}} = x_{\text{max}}/40 \]
\[ = 480/40 \]
\[ = 12 \]

(c) Use \( Z_{\text{max}} \) curve labelled 10 in Figure 3.

(d) Depth coefficient, Z:
\[ Z(0) = 0/40 = 0 \]
\[ Z(10) = 10/40 = 0.25 \]
\[ Z(20) = 20/40 = 0.50 \]
\[ Z(30) = 30/40 = 0.75 \]
\[ Z(40) = 40/40 = 1.00 \]
\[ Z(70) = 70/40 = 1.75 \]
\[ Z(100) = 100/40 = 2.50 \]
\[ Z(180) = 180/40 = 4.50 \]

(e) From Figure 3 (in the report):
\[ F_y(0) = 0.94 \]
\[ F_y(10) = 0.91 \]
\[ F_y(20) = 0.84 \]
\[ F_y(30) = 0.73 \]
\[ F_y(40) = 0.64 \]
\[ F_y(70) = 0.30 \]
\[ F_y(100) = 0.08 \]
\[ F_y(180) = -0.02 \]

(f) \( y_f \) values for each depth:
\[
y_f = F_y \left( \frac{30,000 \times 64,000}{2.202 \times 10^{10}} \right) 
\]
\[ y_f(0) = 0.94(0.087) = 0.082 \]
\[ y_f(10) = 0.91(0.087) = 0.079 \]
\[ y_f(20) = 0.84(0.087) = 0.073 \]
\[ y_f(30) = 0.73(0.087) = 0.064 \]
\[ y_f(40) = 0.64(0.087) = 0.056 \]
\[ y_f(70) = 0.30(0.087) = 0.026 \]
\[ y_f(100) = -0.08(0.087) = -0.007 \]
\[ y_f(180) = -0.02(0.087) = -0.002 \]

(g) Repeating STEPS 9 and 10 gives:
\[ k = 138. \]

(h) Repeating STEP 11 gives:
\[ T = 43.7 \]

**STEP 13.** Plot \( T_{assumed} \) versus \( T_{obtained} \) as shown in Figure A2. Where the line of Equality crosses the line drawn between the two data points indicates the correct \( T \) value.

Use \( T = 47 \), and repeat STEPS 5 through 11.
This yields the correct deflection at the top of the pile.
\[ y_f(0) = 0.94(0.141) = 0.133 \text{ in.} \]
FIGURE A1.
FIGURE A2.