Gray Code Composite Pattern Structured Light Illumination

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Abstract of Thesis

Gray Code Composite Pattern Structured Light Illumination

Structured light is the most common 3D data acquisition technique used in the industry. Traditional Structured light methods are used to obtain the 3D information of an object. Multiple patterns such as Phase measuring profilometry, gray code patterns and binary patterns are used for reliable reconstruction. These multiple patterns achieve non-ambiguous depth and are insensitive to ambient light. However their application is limited to motion much slower than their projection time. These multiple patterns can be combined into a single composite pattern based on the modulation and demodulation techniques and used for obtaining depth information. In this way, the multiple patterns are applied simultaneously and thus support rapid object motion.

In this thesis we have combined multiple gray coded patterns to form a single “Gray code Composite Pattern”. The gray code composite pattern is projected and the deformation produced by the target object is captured by a camera. By demodulating these distorted patterns the 3D world coordinates are reconstructed.

KEYWORDS: Data acquisition, Composite pattern, Gray code, Phase, 3D reconstruction.
GRAY CODE COMPOSITE PATTERN STRUCTURED LIGHT ILLUMINATION

By

Pratibha Gupta

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Director of Thesis

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Director of Graduate Studies

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Date
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THESIS

PRATIBHA GUPTA

The Graduate School
University of Kentucky
2007
GRAY CODE COMPOSITE PATTERN STRUCTURED LIGHT ILLUMINATION

THESIS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the College of Engineering at the University of Kentucky

By

Pratibha Gupta
Andhra Pradesh, India

Director: Dr. Laurence G. Hassebrook, Department of Electrical Engineering
Lexington, Kentucky
2007
Dedication

To my Family, Teachers and friends
Acknowledgements

It has been a privilege for me to work under Dr. Laurence Hassebrook for my Master’s thesis. I would like to acknowledge him gratefully for being my advisor.

I am thankful to Dr. Veera Ganesh Yalla for suggesting and helping me in my work. I would also like to thank the people at Center for Visualization and Virtual Environments for giving me time and supporting me. I would also like to thank Dr. Daniel Lau and Dr. Donohue for their time and for serving as members for my defense committee.

Finally, to my parents who always supported and motivated me to reach greater heights in life and achieve my goals.
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Chapter 1
INTRODUCTION

1.1 Thesis overview

Over the past few decades research on 3D shape measurement and reconstruction has become increasingly active. Many applications require the non-contact measurement of surface topology [1]. The idea behind non-contact surface measurement is that the patterns tend to be modified when projected onto an object in such a way that the modification is related to the object surface topology. The modified patterns are then analyzed to measure a particular topology of the object. The various devices that are used for non-contact measurement are cameras, projectors, computer etc. New developments in the field of CCD cameras, Optics, laser range systems and computer software have made 3D depth data acquisition more practical and of higher performance.

Many depth scanning techniques have evolved and many of these use Structured Light Illumination. Structured light illumination (SLI) is an optical, non-contact scanning technique that is used for 3D data acquisition and has gained wide acceptance in industry and scientific applications. The underlying concept that makes the depth measurement by SLI possible is the Optical Triangulation [2]. The basic SL system consists of a single camera and a single LCD projector which is called the “active stereo pair”. The projector projects a light pattern which can be single stripe, multi stripe, single spot, binary, gray code, sine wave or any complex pattern, on a target surface. By measuring the distortion between the captured and the reflected image, the depth information can be extracted. This technique can be useful for imaging and acquiring three dimensional information [3]. Traditional SL techniques use multi patterns for reliable and accurate reconstruction but this slows down the acquisition speed [4].
The main advantages of SLI are

a) Greater Precision  
b) Simple optical arrangement  
c) Cost effectiveness  
d) Simple computation  
e) Robust nature  
f) Speed

The geometric relationship between the camera, projector and the world coordinates is required for extracting the depth information. The parameters of the Structured Light System are obtained during the calibration process. Although uncalibrated cameras can also be used to obtain the 3D information, calibration is essential for accurate reconstruction\[5\]. The camera calibration involves determining the intrinsic parameters (attributes of the camera that affect the image like the focal length, width and height of the photo sensor\[6\]) or the extrinsic parameters (parameters related to its position and orientation in the real world). The linear model of Hall and Faugeras-Toscani use the least square technique but does not take distortions into account\[5\]. The non-linear model of Faugeras-Toscani and Tsai models the radial distortion and are more accurate. The perspective transformation is to be applied to obtain the 3D world coordinates from the 2D data. Using the Single value decomposition (SVD) or the Least Squares techniques the projection transformation matrices are found. The advantage of Least Squares is that it involves direct computation.

After the calibration is done one of the SLI techniques can be used for 3D data acquisition. The light pattern can be single stripe, multi stripe, binary, gray code, single spot etc. In this thesis we focus our research on composite pattern gray code SLI for obtaining the 3D data.

Section 1.2 gives the aim and organization of the thesis.
1.2 Thesis Organization

The aim of this thesis is to investigate the various data acquisition techniques and Structured Light Illumination scanning techniques. Then we discuss about the design of gray code composite pattern and using it for 3D reconstruction. This thesis also concentrates on the development of a simulation program that gives the camera image and the projector image.

The thesis is divided into 7 chapters. Chapter 1 gives an introduction to the Structured Light Illumination technique and its advantages and gives an overview of the thesis. Chapter 2 presents the various 3D data acquisition techniques, their advantages and disadvantages, Phase measuring profilometry, the concept of composite patterns and the issues related to SLI. Chapter 3 goes through the complete process, from design of composite pattern, projection and capture, demodulation, decoding to gray values till obtaining phase for 3D reconstruction. It also discusses the various problems encountered during each step and the rectification for the same. Chapter 4 gives the details about the mat5 format, calibration techniques based on SVD, Least Squares method and world coordinates reconstruction. The simulation program and its results are presented in chapter 5. The experimental results for the 3D reconstruction of various objects are presented in chapter 6. Chapter 7 gives the conclusions and the future work.
Chapter 2
BACKGROUND

In this chapter we discuss the various data acquisition techniques and their advantages and disadvantages. Details of Structured light illumination and performance comparison of the SLI scanning techniques is also made in this chapter. Sections 2.1-2.5 describe the various data acquisition techniques, section 2.6 explains the traditional phase measuring profilometry and section 2.7 gives the overview of the various 3D scanning algorithms.

Data acquisition means converting the real world data into a form that can be manipulated by a computer [7]. The various data acquisition methods are as follows and their details are discussed [8].

a) Laser ranging system
b) Structured light illumination
c) Moire Fringe method
d) Passive Stereoscopic methods
e) Active Stereoscopic methods

2.1 Laser ranging system

In a laser ranging system, a single spot or laser beam is scanned across the scene. The surface of the object reflects the laser pulses back towards the receiver. The time of flight i.e. time taken for transmission and reception of the laser pulse is measured to obtain the three dimensional data. The Laser rangefinders work at large distances but requires the scene to be static. An important advantage of laser ranging is that it does not require a triangulation angle. That is, it is a completely axial process. Typical Laser ranging system is given in figure 2.1.
2.2 Structured Light Illumination (SLI)

In the SLI method structured patterns are projected from projector onto an object and the camera captures the shape of the target object. Surface shape is reconstructed by measuring the distortions between the captured and the reflected images. Depth measurement by SLI is possible by Optical Triangulation. The major aspect of this system is that it is simple to use but require the target to remain static. The Structured Light system is shown in figure 2.2.
SL is used in depth measurement [3], accurate mounting of IC chips onto a circuit board (alignment) [2], machine vision, inspection in industry, surface defect detection [2], quality control [9], in drug packaging lines, edge detection, product development, biomedical topology [10], telecollaboration [11], and obstacle avoidance in robot navigation [12]. The uses of SLI are presented in figure 2.3. The use of SLI for obtaining the molar tooth and finger print scan is illustrated in figure 2.3(a). A 3D Scanner using SLI is given in figure 2.3(b).
Optical triangulation is a non-contact measurement technique used widely in the industry. The principle of Optical triangulation is illustrated in figure 2.4.
The 3D acquisition devices scan a single laser stripe progressively over the surface of an object. The problem with this is that there is a burden to capture all stripe images and also that the object should remain static during the scanning process. For reducing the technological burdens of scanning and processing each scan position of the laser stripe, the entire target surface is illuminated by projecting structured patterns like multi-stripe and sinusoidal fringe patterns. These multi-stripe patterns introduce ambiguities in the surface reconstruction around surface discontinuities; can be sensitive to surface reflectance (albedo) variations and suffer from low lateral resolution \[^2\]. The solution to this is to encode the surface repeatedly with multiple light striped patterns with variable spatial frequency.

For a real time system, either temporal multiplexed or color multiplexed image sequences can be used but these suffer from low SNR (Signal to Noise Ratio) and are sensitive to object motion and surface color variation. A method that is insensitive to albedo or surface color variations and has more accuracy that can be used to measure 3D objects is the Structured Light pattern.

A single pattern technique that is insensitive to albedo and uses binary coding to identify each line in a single frame \[^{15}\] was introduced by Maruyama and Abe but this is sensitive to highly textured surfaces. Therefore a method to combine multiple patterns into single Composite Pattern was developed by Dr. Hassebrook where the structured light systems are treated as wide bandwidth parallel communication channels \[^{16}\]. The individual patterns are spatially modulated along the orthogonal dimension, perpendicular to the phase dimension \[^{11}\]. This methodology can be applied to existing multi pattern techniques. Thus the existing procedure of Phase measuring profilometry (PMP) is used. The depth information is obtained by demodulating the patterns from the camera captured image. The formation of Composite pattern by modulating the PMP patterns is shown in figure 2.5. The detail of traditional PMP technique is given in section 2.6.
Figure 2.5 A composite pattern (CP) is formed by modulating traditional PMP patterns along the orthogonal direction \cite{17}.

### 2.3 Moire Fringe Method

This is an optical method used for non-contact surface evaluation. In this method moiré fringe pattern analysis is used to estimate the changes in depth and shape of an object. These patterns are formed by projecting a grating onto an object due to which an image is formed in the plane of some reference grating as in figure 2.6. This image interferes with the reference grating to form Moire patterns as shown in figure 2.7. Moire method produces accurate depth data but it is computationally expensive. Making the grating with enough accuracy can also be a limitation for the moiré technique \cite{18}.
2.4 Passive Stereoscopic method

The concept of passive stereoscopy is that a point on the scene will project a point on the two images taken by two different cameras at the same time but placed at different angles. By knowing the parameters of the camera such as focal lengths, orientations, positions, the equations of the projection lines through these image points is calculated and where they intersect is the position of the point in 3D \cite{17}. 

Figure 2.6 Moire System \cite{8}

Figure 2.7 Moire Patterns \cite{8}
Though Passive stereoscopy is a simple method, the major drawback is the “correspondence problem” i.e. locating every point in one image and finding the corresponding point in the other image and also the depth measurement is not very accurate, unless the correspondence point is clearly defined. Also the accuracy of depth obtained through Passive stereoscopy is of the order of millimeters \[^8\]. The setup for a passive stereoscopic system is given in figure 2.8.

![Figure 2.8 Passive Stereoscopic System](image)

2.5 Active Stereoscopic method

The problem with the passive stereoscopy system can be simplified by illuminating the whole scene with a source of light (e.g. Laser Scanner) which can be observed by both the cameras. The light source is swept across the whole scene and by knowing the optical properties of the camera and using active triangulation method depth data can be obtained. The schematic diagram of an Active stereo system is shown in figure 2.9.

![Figure 2.9 Active Stereoscopic System](image)
2.6 Traditional Phase Measuring Profilometry

One of the most commonly used and robust methods of structured light is the Phase measuring Profilometry. As stated before PMP is a multi pattern SLI technique. PMP projects phase shifted sine wave patterns onto a surface and the “Phase” is recovered for each pixel position by correlating across the shifted patterns. The depth measurement is more accurate with more pattern shifts and higher spatial frequency.

The projected light pattern is given as \[^{20}\]

\[
I_n(x^p, y^p) = A^p + B^p \cos[2\pi f y^p - 2\pi n/N]
\]  \hspace{1cm} \text{------ (2.1)}

Where \(A^p\) and \(B^p\) = constants of the projector

\((x^p, y^p) = \text{projector coordinates}\)

\(y^p\) is the phase dimension

\(x^p\) is the orthogonal dimension

\(f = \text{frequency of the sine wave}\)

\(n = \text{phase shift index}\)

\(N = \text{Total number of phase shifts}\)
The PMP patterns for base frequency projections for N = 4 are given in figure 2.10.

![Pattern 0](image1)
![Pattern 1](image2)
![Pattern 2](image3)
![Pattern 3](image4)

Figure 2.10 PMP base frequency patterns for N=4

Due to the topology of the target, the received image gets distorted. From camera view point the received image is expressed mathematically as

$$I_n(x^c, y^c) = A(x^c, y^c) + B(x^c, y^c) \cos[\phi(x^c, y^c) - 2\pi n/N]$$  \hspace{1cm} (2.2)

Where $\phi(x^c, y^c) = \text{phase of the sine wave}$

$\phi(x^c, y^c)$ can be calculated as
\[
\phi(x^e, y^e) = \arctan\left[\frac{\sum_{n=1}^{N} I_n(x^e, y^e) \sin(2\pi n / N)}{\sum_{n=1}^{N} I_n(x^e, y^e) \cos(2\pi n / N)}\right]
\]  

\begin{equation}
\text{------- (2.3)}
\end{equation}

It is clear from the above equations that

\[
y^p = \phi(x^e, y^e) / (2\pi f)
\]

\begin{equation}
\text{------- (2.4)}
\end{equation}

Thus by finding \(y^p\) and \(\phi(x^e, y^e)\) the 3D world coordinates can be calculated.

2.7 Comparison of various 3D Scanning Algorithms\[^{21}\]

As explained before, the SL patterns can be single spot, multi spot, single stripe, multi stripe, gray code, PMP etc. The 3D data acquisition devices using single spot and single stripe scan a spot or a stripe progressively on the surface of an object. The single spot technique is time consuming because of limited resolution. The multi spot scanning based on gray code is limited in resolution but has good depth range. PMP is another technique for 3D shape reconstruction by projecting a Composite Pattern of phase shifted sine waves. The advantage of PMP over single spot or gray code is that it uses fewer frames for a given precision. PMP technique gives very good resolution but major drawback with this is that it has limited depth range\[^{22}\]. Using single frequency PMP technique the reconstruction is quite noisy and therefore dual frequency PMP technique was used\[^{10}\]. In this technique the lower frequency is used for unwrapping the phase and a higher frequency is used for getting more accuracy. Multi frequency PMP technique is used for obtaining even better resolution. Dr. Veera Ganesh Yalla describes the procedure for finding the best choice of frequencies to obtain better resolution. Multi frequency PMP is better than single and dual frequency PMP technique for a given scan time\[^{2}\].
Chapter 3

DESIGN OF GRAY CODE COMPOSITE PATTERN

Using the principles of frequency modulation, multiple structured light patterns are combined to form a single pattern which is projected continuously on a 3D target object [4]. The target object should remain static during the scanning process. In this chapter the design of gray code composite pattern is being discussed where multiple gray patterns are combined to form single composite pattern. The composite pattern is then projected on an object and 3D depth is reconstructed. Because this technique is based on frequency modulation it is inherently insensitive to intensity variations [54]. The chapter also discusses the demodulation process.

This chapter is divided into following sections.

3.1 Composite pattern Synthesis
3.2 Acquisition and Analysis of projected pattern
   3.2.1 Gamma correction
   3.2.2 Optical roll-off correction
3.3 Carrier Peak detection and discrimination
3.4 Composite Pattern Demodulation
3.5 Binarising and decoding

The following sections describe about creating the composite pattern using gray code structured light patterns, projecting the composite pattern on a surface, demodulating the captured pattern to obtain the individual patterns, binarising and decoding. This decoded data is converted to phase and used to find the depth of objects which is easily obtained in a calibrated system [4].
3.1 Composite pattern Synthesis

Gray code is the code in which the numbers are represented as binary patterns and the consecutive numbers differ by only one bit position. Table 3.1 shows the binary and the gray code representation. The first step in creating a gray coded composite pattern is to create gray code structured light patterns. The gray coded structured light patterns are shown in figure 3.1. Each individual pattern is then modulated by a unique carrier frequency along the orthogonal direction \[32\] as shown in figure 3.2. The modulating frequencies are evenly distributed as \(f_1 = 32, f_2 = 64, f_3 = 96\) and \(f_4 = 128\).

<table>
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<tr>
<th>Decimal Value</th>
<th>Binary Code</th>
<th>Gray Code</th>
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<tr>
<td>0</td>
<td>000</td>
<td>000</td>
</tr>
<tr>
<td>1</td>
<td>001</td>
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<tr>
<td>2</td>
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<td>110</td>
<td>101</td>
</tr>
<tr>
<td>7</td>
<td>111</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 3.1 Gray Coded Structured Light Patterns
As shown in figure 3.3 the first column is the carrier patterns and the gray patterns are represented in second column. Each gray pattern is element wise multiplied with the carrier (cosine) pattern and summed together to form a composite pattern (CP) and is shown in figure 3.4. Thus the composite pattern to be projected is

\[
I^p(x^p, y^p) = A^p + B^p \cdot \sum_{n=1}^{N} I_{n^p}(x^p, y^p) \cdot \cos(2\pi f_n x^p)
\]  

where \(f_{n^p}\) are the carrier frequencies along the orthogonal direction

\(n\) is the shift index from 1 to \(N\)

\(A^p\) and \(B^p\) are the projection constants

\(I_{n^p}(x^p, y^p)\) are the gray coded patterns
Figure 3.3 Composite Pattern formed by modulating the Gray code patterns
3.2 Acquisition and analysis of the composite pattern

The composite pattern is then projected on a surface and the reflected image is captured as in figure 3.5. The reflected composite pattern image captured by the camera is

\[ I(x^c, y^c) = \alpha(x^c, y^c) \{ A^c + B^c \sum_{n=1}^{N} I_n(x^c, y^c) \cos(2\pi f_n x^c) \} + \beta \alpha(x^c, y^c) \]  \hspace{1cm} (3.2)

Where \( \alpha(x^c, y^c) \) is the albedo image

\[ \beta \alpha(x^c, y^c) \] represents the albedo image from ambient light with intensity \( \beta \).

The carrier frequencies in the captured image \( f_n^c \) may be different from the projected frequencies \( f_n^p \) due to the perspective distortion between the camera and the projector.
3.2.1 Gamma correction:

Gamma correction is important if accurate display of an image is desired. If the Projector is not gamma corrected then the images will be less satisfactory. If gamma correction is done properly, then the output should accurately reflect the image input. Gamma is simply defined as the non-linearity between the input voltage and output intensity and is given by the power law function where the exponent is the gamma value. Gamma correction is accomplished by raising the input value to the power of $1/\gamma$. For most of the projectors the gamma value is 2.2.

If $C$ is the projected image then for gamma correction $C = C.^{(1/\gamma)}$

Finding gamma for the projector:

To find the gamma value for the projector a sine wave image is projected and captured. The projected and captured images are shown in figure 3.6 and figure 3.7 respectively. A row is extracted from the Fourier Transform of the captured image and is compared with
an ideal sine wave. By varying the value of gamma and multiplying the captured image with power of \(\frac{1}{\text{gamma}}\), the response is made closer to the ideal sine wave.

Table 3.2 shows the comparison of different gamma values against the mean squared error and the figure 3.8 shows the sine curve fitting for different values of gamma. From the table it is clear that mean squared error is minimum for gamma equal to 2.4. So the gamma of the projector is taken as 2.4.

![Figure 3.6 Projected sine Image](image1)

![Figure 3.7 Captured sine Image](image2)
Table 3.2 Comparing gamma values and mean squared error

<table>
<thead>
<tr>
<th>Gamma Value</th>
<th>Mean Squared Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.0799</td>
</tr>
<tr>
<td>2.4</td>
<td>0.0689</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0694</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0837</td>
</tr>
<tr>
<td>3.5</td>
<td>0.1054</td>
</tr>
</tbody>
</table>

Figure 3.8.1 Gamma =2.0
Figure 3.8.2 Gamma = 2.4

Figure 3.8.3 Gamma = 2.5

Figure 3.8.4 Gamma = 3.0
Figure 3.8.5 Gamma = 3.5

Figure 3.8 Gamma values

### 3.2.2 Roll-off correction

The optics of the camera and projector tend to attenuate higher spatial frequencies. The problem is that this will attenuate and corrupt the reconstruction of the patterns. The solution is to know the attenuation and multiply by a factor that will compensate. Therefore a carrier only composite pattern is created as in figure 3.9 and is projected. The captured pattern is given in figure 3.10. By performing 2D Fourier transform on the captured image in figure 3.10 and extracting first row, the peak locations are located as shown in figure 3.11. These peak locations correspond to the maximum likelihood estimate of the frequency modulation. The peak locations are calculated from peak1 to peak4. Then alphas (attenuation factors) are calculated as

\[
\alpha = \left[ \frac{\text{peak}_1}{\text{peak}_1}, \frac{\text{peak}_2}{\text{peak}_1}, \frac{\text{peak}_3}{\text{peak}_1}, \frac{\text{peak}_4}{\text{peak}_1} \right]
\]

These weights are multiplied to the individual carriers in the original gray code composite pattern that is to be projected. The concept of spatial deconvolution is explained in the appendix section.
Figure 3.9 Projected Carrier only Composite Pattern

Figure 3.10 Captured Carrier only Composite Pattern
The projected gray code composite pattern is now gamma corrected with gamma equal to 2.4 and also roll-off corrected and is shown in figure 3.13. This is projected on a surface and the reflected image is captured as in figure 3.14.
Figure 3.13 Projected Composite Pattern after gamma correction

Figure 3.14 Captured Composite Pattern after gamma correction
By performing gamma correction the lowest carrier was distorted and added to the dc value as observed in figure 3.15 and thereby the reconstructed gray code pattern was highly distorted. Therefore gamma correction for the projector was set to 1. The reason for the lowest carrier getting corrupted due to gamma correction is a part of future study. Now the projected pattern by setting gamma equal to 1 and including roll-off correction is shown in figure 3.16 and captured composite pattern is given in figure 3.17.
3.3 Carrier Peak detection and discrimination

We now process the captured image of figure 3.17 to reconstruct the patterns and thus the depth. The captured pattern is band pass filtered are centered at $f_{nc}$ to separate each channel. The Fourier spectrum of the four channel composite pattern is shown in figure 3.18. I, II, III and IV represents the carriers respectively.

The selection of Butterworth filter is made so that the cross talking between the channels is reduced [4]. By taking the 2D Fourier transform the average carrier energy is obtained. Then by selecting the first row the average energy distribution between the carriers is observed which is shown in figure 3.19. This gives an estimate for the band pass filter center position and cutoffs.
Figure 3.18 Fourier Spectrum of the four channel composite pattern

Figure 3.19 First row of 2D Fourier Transform
3.4 Composite Pattern Demodulation

After estimating the center positions and approximate spacing of the carriers, band pass filters are designed for each pattern. The Butterworth Band pass filter is shown in figure 3.21. Only one carrier is processed at a time by notching the other three.

After band pass filtering and notching we have \[^{[4]}\]

\[
I_{n}^{BP}(x^c, y^c) = I^c(x^c, y^c) * h_{N1}^{N}(x^c) * h_{N2}^{N}(x^c) * h_{N3}^{N}(x^c) \approx I^c(x^c, y^c) \cdot \cos(2\pi f_n x) \quad (3.3)
\]

Where \(h_{BP}^{n}(x)\) is the band pass filter centered at \(f_n\), \(h_{N1}(x^c)\), \(h_{N2}(x^c)\), \(h_{N3}(x^c)\) are one dimensional notch filters and \(*\) represents convolution operator. Hilbert transform is applied to the band pass filters i.e., we band pass filter as before but suppress one side of the band pass filter as given in figure 3.22.
\[ I_n^e(x^e, y^e) = \left| I_n^{BP}(x^e, y^e) + \hat{I}_n^{BP}(x^e, y^e) \right| \text{ given } y^e = \text{constant} \quad ---- \ (3.4) \]

Where \( \hat{I}_n^{BP} \) is the Hilbert transform of \( I_n^{BP} \).

The details of Hilbert transform and Band pass filter is given in appendix. After filtering, demodulation is done to obtain the individual patterns. The inverse Fourier transforms results in the individual demodulated patterns as shown in figure 3.23 and is used to obtain depth of the measured object.
3.5 Binarising and Decoding

The demodulated patterns are then binarized and decoded. The demodulated patterns are converted to binary images using thresholding method. That is if the pixel value for each pixel in the demodulated pattern is greater than a threshold value then that pixel value is set equal to 1 else it is set equal to 0. The demodulated patterns converted to binary and added together looks like in figure 3.24.
The binary patterns are then decoded to gray values using the look up table 3.3. The table consists of the three columns.

(a) Binary sequence
(b) Decoded value
(c) Look up value.

“P1 P2 P3 P4” gives the binary sequence, the decoded value is the decimal equivalent of the binary sequence and the Look up value is the gray code value. For example if the decimal equivalent of the binarised pattern 0101 is 5 then the gray code value from the look up table is 6. The image obtained by decoding the binary image in figure 3.24 is shown in figure 3.25.
Table 3.3 Look up Table

<table>
<thead>
<tr>
<th>Binary Sequence (P1P2P3P4)</th>
<th>Decoded Value (Decimal Value)</th>
<th>Look Up Value (Gray Value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0001</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0011</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>0010</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>0110</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>0111</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>0101</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>0100</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>1100</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>1101</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>1111</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>1110</td>
<td>14</td>
<td>11</td>
</tr>
<tr>
<td>1010</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>1011</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>1001</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>1000</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>
Now the composite pattern is projected on a target object (sphere) of radius 4.825 inches and captured. The captured view is given in figure 3.26.
Similar procedure of demodulation is applied to the captured image. The decoded integer values of the gray code that are obtained from the captured demodulated patterns are scaled between $[0, 2\pi]$ \cite{21}. This forms the phase information as shown in figure 3.27. The projector coordinates $y^p$ corresponding to each location of the camera pixel can be obtained as $y^p = \phi(x^c, y^c) \times (2\pi/16)$. Hence with the knowledge of $y^p$, $\phi(x^c, y^c)$ and using the transformation equations explained in chapter 4, the 3D world coordinates can be calculated. The 3D reconstruction using gray code composite pattern is discussed in detail in chapter 5. Figure 3.28 shows the phase obtained through multi-frequency PMP technique.

![Figure 3.27 Phase through gray code](image)

Figure 3.27 Phase through gray code
Figure 3.28 Phase through multi frequency PMP
3.6 Block Diagram for the demodulation process

Action

Read in the Composite Pattern that is gamma corrected and optical roll-offs corrected

Perform 2D fft

Extract the first row of 2d fft image

Notch the 3 carriers and dc and IFFT2

Perform 1D fft
Design Butterworth Band Pass filters

Perform Hilbert Transform

Ifft and take magnitude to obtain the demodulated patterns
Chapter 4

SLI CALIBRATION AND 3D RECONSTRUCTION

Measurement accuracy can be obtained through calibration [23]. The 3D calibration involves the transformation of the three coordinate systems i.e. world, camera and projector coordinates. Let the world coordinates be represented as a 3D Euclidean space $X^w$, $Y^w$, $Z^w$ measured in metric units, the camera coordinates represented as $x^c, y^c$ measured in pixels and the projector coordinates as $x^p, y^p$ measured in pixels or $y^p$ in radian units. Uncalibrated cameras can also be used for 3D reconstruction but for accurate reconstruction camera calibration is essential. The camera calibration can be performed using a calibration grid whose 3D geometry is already known. A 3D calibration grid is essential though the intrinsic parameters are of little interest [24]. This type of calibration is accurate and comes under the category of photogrammetric calibration. The calibration grid used is shown in figure 4.1.

![Figure 4.1 Calibration grid](image)

The grid consists of 18 circles in black whose centers correspond to the known world coordinates. The number of circles used may vary depending on the technique being
used. The software that is used to obtain the albedo image of the grid, the X or Y phase information is shown in figure 4.2. This software makes use of the multi frequency PMP technique. The frequency settings can be specified in the file control of the software as shown in figure 4.2. The snapshot of the Uscanner software is shown in figure 4.3 (a) and (b).

Figure 4.2 Snapshot of the file control for the Uscanner software
Figure 4.3 (a) Snapshot of the Uscanner software

Figure 4.3 (b) Snapshot of the Uscanner software
Now that the calibration data is obtained, the Custom calibration software is used for the calibration process. This software generates the world coordinates corresponding to the projector and camera coordinates. The snapshot of the file control for the custom calibration software is given in figure 4.4.

![Calibration Grid Editor]

**Figure 4.4** Snapshot of the file control for the calibration software

The calibration data (the albedo image of the grid, the XP.byt, the YP.byt and the G.byt) can be specified in the file control. The details about the mat5 format are discussed in section 4.1. Also the number of calibration points can be selected. The snapshot of the custom calibration software displaying the camera and the phase image is given in figure 4.5.

![Calibration Filenames and Grid Size Editing]
The various techniques for calibrating are Single value decomposition (SVD) given by Tsai \[24\] where the camera is assumed to be a pin-hole one. Wei Su \[24\] has proposed a technique involving polynomial functions for calibration. The SVD and the Least squares technique are given in detail in section 4.3 and 4.4.

### 4.1 Mat5 Format \[25\]

Mat5 consists of 5 matrices containing the 3d data of a scan. The mat5 data is required for reconstruction. Let us assume that the mat5 is set with a name “test”. The first matrix or file will be testC.bmp. This file represents the texture map in BMP format. The second file is the testI.bmp which is an Indicator matrix. If the pixel in the indicator matrix is 1 then it is valid data else it is invalid data. The matrices testX.byt, testY.byt and testZ.byt contain the X, Y and the Z coordinates respectively and uses floating values. The pre mat5 format consists of A.byt which is a 4x4 transformation matrix, G.byt, which contains the calibration grid data i.e. Xw, Yw, Zw, Xp, Yp, Xc and Yc which are
floating point numbers, XP.by and YP.by which contains the phase of the projected patterns.

4.2 Experimental Setup

Calibration of a structured light system consists of a pin-hole camera (Canon 5.0 Mega Pixel 1944x2592 resolution) and a LCD projector (Epson with 1024x768 pixel resolution) connected to a Pentium 4 Windows XP computer through a frame grabber. Based on the orientation of the structured light stripes, the projector is displaced vertically relative to the camera in space \cite{26}. The experimental setup is shown in the figure 4.6.

![Figure 4.6 Experimental setup for SLI 3D scanner](image)

Figure 4.6 Experimental setup for SLI 3D scanner
4.3 Single Value Decomposition Technique (SVD)

Singular-value decomposition (SVD) is most commonly used technique used for calibration.

Let \((x^c, y^c)\) be the camera coordinates, \((x^p, y^p)\) be the projector coordinates and \((X^w, Y^w, Z^w)\) be the world coordinates.

The equations governing the transformation between the camera and the world coordinates are given as [7]

\[
x^c = \frac{m_{11} x^w + m_{12} y^w + m_{13} Z^w + m_{14}}{m_{31} x^w + m_{32} y^w + m_{33} Z^w + m_{34}} \quad ------ (4.1)
\]

\[
y^c = \frac{m_{21} x^w + m_{22} y^w + m_{23} Z^w + m_{24}}{m_{31} x^w + m_{32} y^w + m_{33} Z^w + m_{34}} \quad ------ (4.2)
\]

The 3x4 camera Transformation matrix is given as

\[
M_{wc} = \begin{bmatrix}
  m_{11} & m_{12} & m_{13} & m_{14} \\
  m_{21} & m_{22} & m_{23} & m_{24} \\
  m_{31} & m_{32} & m_{33} & m_{34}
\end{bmatrix} \quad ------ (4.3)
\]

The equations governing the transformation between the projector and the world coordinates are given as

\[
x^p = \frac{m_{11} x^w + m_{12} y^w + m_{13} Z^w + m_{14}}{m_{31} x^w + m_{32} y^w + m_{33} Z^w + m_{34}} \quad ------ (4.4)
\]

\[
y^p = \frac{m_{21} x^w + m_{22} y^w + m_{23} Z^w + m_{24}}{m_{31} x^w + m_{32} y^w + m_{33} Z^w + m_{34}} \quad ------ (4.5)
\]
The 3x4 projector Transformation matrix is given as

\[ M_{wp} = \begin{bmatrix}
    m_{11} & m_{12} & m_{13} & m_{14} \\
    m_{21} & m_{22} & m_{23} & m_{24} \\
    m_{31} & m_{32} & m_{33} & m_{34}
\end{bmatrix} \]  \hspace{1cm} (4.6)

In vector notation, the equation (4.3) can be written as

\[ m_c = \begin{bmatrix}
m_{11} & m_{12} & m_{13} & \ldots & m_{34}
\end{bmatrix}^T \]  \hspace{1cm} (4.7)

Similarly equation (4.6) can be written in vector notation as

\[ m_p = \begin{bmatrix}
m_{11} & m_{12} & m_{13} & \ldots & m_{34}
\end{bmatrix}^T \]  \hspace{1cm} (4.8)

The solution to \( A_c m_c = 0 \) is given by the coefficient vector \( m_c \) where \( A_c \) is the camera transformation matrix given as

\[ A_c = \begin{bmatrix}
    X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & -x_1 & x_1 & -y_1 & y_1 & -z_1 & z_1 \\
    0 & 0 & 0 & 0 & 1 & X_1 & Y_1 & Z_1 & -y_1 & Y_1 & -y_1 & Y_1 & -y_1 \\
    X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & -x_2 & X_2 & -x_2 & X_2 & -x_2 & X_2 \\
    0 & 0 & 0 & 0 & 1 & X_2 & Y_2 & Z_2 & -y_2 & Y_2 & -y_2 & Y_2 & -y_2 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    X_M & Y_M & Z_M & 1 & 0 & 0 & 0 & -x_M & X_M & -x_M & Y_M & -x_M & Z_M \\
\end{bmatrix} \]  \hspace{1cm} (4.9)

Where \( M \) is the number of calibration points.

Using SVD technique the coefficient vector is computed as

\[ A_c = UDV^T \]  \hspace{1cm} (4.10)

Where \( U \) is the 2Mx2M matrix whose columns are orthogonal vectors, \( D \) is the positive diagonal matrix and \( V \) is the 12x12 matrix whose columns are orthogonal.
The solution of this gives the perspective matrix in equation (4.3). Similarly the projector perspective matrix \( m_{wp} \) is calculated using \( A_c \) in equation (4.10) and using \( A_p m_p = 0 \).

These perspective matrices are used to reconstruct the 3D world coordinates for a calibrated system. During a 3D scan the camera coordinates are obtained from the captured images and the coordinates of the projector are already known. (Because of DLP = Digital Light projection)

Thus we get

\[
C = \begin{bmatrix} m_{11}^w & -m_{31}^w & x^c & m_{12}^w & -m_{32}^w & x^c & m_{13}^w & -m_{33}^w & x^c & m_{14}^w \\
m_{21}^w & -m_{31}^w & y^c & m_{22}^w & -m_{32}^w & y^c & m_{23}^w & -m_{33}^w & y^c & m_{24}^w \\
m_{11}^p & -m_{31}^p & y^p & m_{12}^p & -m_{32}^p & x^p & m_{13}^p & -m_{33}^p & x^p & m_{14}^p \\
m_{21}^p & -m_{31}^p & y^p & m_{22}^p & -m_{32}^p & x^p & m_{23}^p & -m_{33}^p & x^p & m_{24}^p \\
\end{bmatrix} \tag{4.11}
\]

\[
D = \begin{bmatrix} m_{34}^w & x^c \\
m_{34}^w & y^c \\
m_{34}^w & x^p \\
m_{34}^w & y^p \end{bmatrix} \tag{4.12}
\]

Using equations 4.11 and 4.12 the 3d world coordinates are given as

\[
P^w = \begin{bmatrix} X^w \\
Y^w \\
Z^w \\
1 \end{bmatrix}^T = C^{-1} D \tag{4.13}
\]

For most of the applications the vertical phase of the projector i.e. \( y^p \) coordinate is calculated along \( x^c, y^c \) and thus the 3-D world coordinates are rewritten as

\[
C = \begin{bmatrix} m_{11}^w & -m_{31}^w & x^c & m_{22}^w & -m_{32}^w & x^c & m_{23}^w & -m_{33}^w & y^c \\
m_{21}^w & -m_{31}^w & y^c & m_{22}^w & -m_{32}^w & y^c & m_{23}^w & -m_{33}^w & y^c \\
m_{11}^p & -m_{31}^p & y^p & m_{22}^p & -m_{32}^p & y^p & m_{23}^p & -m_{33}^p & y^p \end{bmatrix} \tag{4.14}
\]
\[ D = \begin{bmatrix} m_{34}^{wc}X - m_{14}^{wc} \\ m_{34}^{wc}Y - m_{24}^{wc} \\ m_{34}^{wc}Z - m_{24}^{wp} \end{bmatrix} \quad \text{(4.15)} \]

\[ P = \begin{bmatrix} X^w & Y^w & Z^w \end{bmatrix}^T = C^{-1}D \quad \text{(4.16)} \]

### 4.4 Least Squares Technique

The SVD method involves the computation of the Eigen values and hence requires an iteration process where as the least square technique involves direct calculation.

The same set of equations from 4.1 – 4.8 can be used for least squares method. The coefficients \( m_{34}^{wc} \) and \( m_{34}^{wp} \) are assumed to be 1. This assumption can be made because the transformation matrices are defined to a scale factor \(^2\).

Using the least squares technique we get a linear equation of the form

\[ A m_c = B \]

Where \( A \) is given by

\[ A_{2i-1} = \begin{bmatrix} X_i^w \\ Y_i^w \\ Z_i^w \\ 1 \\ 0 \\ 0 \end{bmatrix}^T \]

\[ A_{2i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ X_i^w \\ Y_i^w \\ Z_i^w \\ 1 \end{bmatrix}^T \quad \text{(4.17)} \]
B is given by \[ B_{2i-1} = \begin{bmatrix} x_i^c \\ y_i^c \end{bmatrix} \quad B_{2i} = \begin{bmatrix} y_i^c \end{bmatrix} \] \quad ------ (4.18)

and \[ m^c = \begin{bmatrix} m_{11}^{wc} & m_{12}^{wc} & m_{13}^{wc} & \ldots & m_{33}^{wc} & m_{34}^{wc} \end{bmatrix} \] \quad ------ (4.19)

Thus the vector \( m_c \) obtained through the pseudo-inverse solution is given as \(^2\)

\[ m_c = (A^T A)^{-1} A^T B \]

Similarly solving \( A m_p = B \) we get the projector transformation matrix \( M_{wp} \). Once the perspective matrices \( M_{wc}, M_{wp} \) are known the 3-D world coordinates can be calculated. Using the vertical phase of the projector i.e. \( y^p \) coordinate along \( x^c, y^c \) the 3-D world coordinates are obtained as

\[
C = \begin{bmatrix}
    m_{11}^{wc} - m_{31}^{wc} x^c & m_{12}^{wc} - m_{32}^{wc} x^c & m_{13}^{wc} - m_{33}^{wc} x^c \\
    m_{21}^{wc} - m_{31}^{wc} y^c & m_{22}^{wc} - m_{32}^{wc} y^c & m_{23}^{wc} - m_{33}^{wc} y^c \\
    m_{21}^{wp} - m_{31}^{wp} y^p & m_{22}^{wp} - m_{32}^{wp} y^p & m_{23}^{wp} - m_{33}^{wp} y^c
\end{bmatrix} \quad ------ (4.20)
\]

\[
D = \begin{bmatrix}
    m_{34}^{wc} x^c - m_{14}^{wc} \\
    m_{34}^{wc} y^c - m_{24}^{wc} \\
    m_{34}^{wp} y^p - m_{24}^{wp}
\end{bmatrix} \quad ------ (4.21)
\]

\[ P = \begin{bmatrix} X^w \\ Y^w \\ Z^w \end{bmatrix}^p = C^{-1} D \quad ------ (4.22)\]

Dr. Veera Ganesh Yalla \(^{21}\) has tested the robustness and accuracy in reconstruction based on SVD and least squares and found that the least squares technique is the best choice for calibration. Also Least squares technique requires less computation.
As explained in chapter 3, the captured patterns are decoded to gray code integer values and converted to phase information and thus projector coordinates ($y''$) and equations (4.20 - 4.22) are used to obtain the 3D reconstruction. The resulting world coordinates are saved in the mat5 format as explained in section 4.1.
Chapter 5

SIMULATION PROGRAM

In this chapter the simulation program design and its results are discussed. The simulation program gives the camera and the projector view when a composite pattern is being projected onto a target without actually the pattern being projected. The output of the simulation program can then be compared with the actual image captured by a camera by projecting a pattern on the target. The simulated captured image can be used for post processing.

5.1 Simulation Program

The program requires the mat5 data of the target, the calibration data and the composite pattern as inputs and the outputs the simulated projector and camera images. The simulation program can be divided into three parts. In the first part the calibration data is read and the projector and the camera transformation matrices are calculated. In the second part the mat5 data of the target is mapped to projector space to create another set of mat5. The output texture image at this stage is the image which the projector views. In the third part the mat5 data obtained in step 2 is mapped to the camera space. The output at this stage is the image which the camera views.

Let X0, Y0, Z0, C0, I0 be the input mat5 data, let G file be the calibration data and Ip be the composite pattern.

Step 1: Read G file and calculate camera and projector transformation matrices.

The equations that are used to compute the Transformation matrices are

\[
x^{c} = \frac{m_{11} x + m_{12} y + m_{13} z + m_{14}}{m_{31} x + m_{32} y + m_{33} z + m_{34}}
\]

------ (5.1)
\[ y^c = \frac{m_{21} wc X^w + m_{22} wc Y^w + m_{23} wc Z^w + m_{24} wc}{m_{31} wc X^w + m_{32} wc Y^w + m_{33} wc Z^w + m_{34} wc} \]  \[ y^p = \frac{m_{11} wp X^w + m_{12} wp Y^w + m_{13} wp Z^w + m_{14} wp}{m_{31} wp X^w + m_{32} wp Y^w + m_{33} wp Z^w + m_{34} wp} \]  \[ y^p = \frac{m_{21} wp X^w + m_{22} wp Y^w + m_{23} wp Z^w + m_{24} wp}{m_{31} wp X^w + m_{32} wp Y^w + m_{33} wp Z^w + m_{34} wp} \]

Where \( M_{wc} = \begin{bmatrix} m_{11} wc & m_{12} wc & m_{13} wc & m_{14} wc \\ m_{21} wc & m_{22} wc & m_{23} wc & m_{24} wc \\ m_{31} wc & m_{32} wc & m_{33} wc & m_{34} wc \end{bmatrix} \)  \[ M_{wp} = \begin{bmatrix} m_{11} wp & m_{12} wp & m_{13} wp & m_{14} wp \\ m_{21} wp & m_{22} wp & m_{23} wp & m_{24} wp \\ m_{31} wp & m_{32} wp & m_{33} wp & m_{34} wp \end{bmatrix} \]

is the camera transformation matrix

is the projector transformation matrix

The transformation matrices are calculated using the Least Squares technique as explained in the section 4.4.

Step 2: Mapping to projector space

The mat5 in the projector space share the same x, y, z coordinates, same intensity image as the original mat5 but the texture image is different. This is the image which the projector sees. Thus the projector image C1 is obtained as \( C1 = \text{projected pattern times } C0 \).
Using the projector transformation matrix that transforms from world coordinates to the projector coordinates, the projector coordinates $X_p$, $Y_p$ are computed using equations 5.3 and 5.4. Using these coordinates, the composite pattern image and the initial $C_0$ image, image $C_1$ is obtained.

Step 3: Mapping to camera space

By using the mat5 obtained in step 2, the mat5 in the camera space is obtained. Using the camera transformation matrix that transforms from world coordinates to the camera coordinates $X_c$, $Y_c$ are computed as given in equations 5.1 and 5.2. The mat5 in the camera space again share the same $x$, $y$, $z$ and intensity image as the original mat5 data but has different texture image $C_2$. By using the projector image the simulated camera image is obtained.

5.2 Simulation outputs

Figure 5.1 Simulated projector view of sphere
Figure 5.2 Simulated camera view of sphere
Chapter 6

EXPERIMENTAL RESULTS

The main aim of this thesis is to form a gray code composite pattern and obtain 3D reconstruction based on this composite pattern. Experiments are conducted on various objects, world coordinates are computed for each case and the reconstruction results are presented in this chapter. The reason for adding a modified frequency to the existing composite pattern, its effects and results are also discussed.

The 3D reconstruction can be summarized as follows.

- Project and capture the gray code composite pattern on a target object
- Demodulate and decode the pattern
- Calculate the phase from the decoded image
- Obtain the projector coordinates $y''$ corresponding to each pixel location of the camera
- Use the transformation equations 4.20 - 4.22 to obtain the 3D world coordinates
- Save the world coordinates in the mat5 format

6.1 Experimental results

The 3D reconstruction through gray code composite pattern for various objects is compared in figure 6.1. The error measurement for free form shapes is difficult therefore it is presented in terms of surface reconstruction. Figure 6.1(a) is surface reconstruction for alice and 6.1 (b) is for sphere.
Figure 6.1 3D surfaces through gray code CP
(a) Alice    (b) Sphere

Figure 6.2 compares the 3D reconstruction of sphere using gray code and multi frequency PMP. The calibration inaccuracies also cause errors in the 3D reconstruction. The close view of the reconstructed sphere using gray code technique is given in figure 6.3.
Figure 6.2 3D reconstruction of Sphere
(a) Multi frequency PMP (b) Gray code
Figure 6.3 Close view 3D reconstruction of sphere using gray code technique

It is observed that the reconstruction errors are dominant with “blinds effect” caused due to the gray steps. The close view of the blinds or stair case like structure in the background is presented in figure 6.4 as observed in open GL 3D viewer. These steps can be minimized by performing an iterative search and constructing new data points based on known data. That is performing interpolation.
Figure 6.4 Stair case structure observed in the background of the reconstructed image as observed in open GL 3D viewer

### 6.2 Modified Composite Pattern

The composite pattern has small variation along the vertical dimension. Finding about where the errors (intensity variations) are along this dimension gives little information. In order to generate detectible intensity changes along the vertical lines for better intensity comparison when error occurs, the composite pattern is modified by adding a sine wave along the phase or vertical direction. By doing so the projected image has distinct gray level variations from its neighborhood along each vertical line. The modified composite pattern obtained by adding a sine wave of frequency 15 along the phase dimension is given in figure 6.5 (a) and 6.5 (b) gives the 2D Fourier spectrum for the modified frequency.
Figure 6.5 (a) Composite pattern with modified frequency (b) 2D Fourier spectrum of modified composite pattern
The addition of modified frequency though is useful for finding intensity variations the unwanted effect of this is that it decreases the SNR. The image of sphere projected with a modified composite pattern is given in figure 6.6. The image obtained after notching out the carriers from the composite pattern is given in figure 6.7. The 3D reconstruction of sphere with modified composite pattern is given in figure 6.8.

![Figure 6.6 Sphere projected with modified composite pattern](image1)

![Figure 6.7 Image after notching the carriers in the modified CP](image2)
Based on the figures illustrated above, it is clear that the reconstruction is improved on solid objects like sphere as compared to objects with discontinuities. The marked feature is the “Stair case structure or blinds” in the background. It can also be observed from the reconstruction results that the bulging area of the sphere is not exactly round or spherical but instead pointed which can be due to the scaling factor and calibration inaccuracies.
Chapter 7
CONCLUSIONS AND FUTURE WORK

This thesis emphasizes on the design of gray code composite pattern and 3D reconstruction of objects using it. It also discusses the development of simulation program that gives the projector and camera view. More importance is given to the design of composite pattern and demodulation of the captured pattern to obtain the phase for 3D reconstruction. The frequencies for modulating the gray code are selected such that they are evenly distributed to get better demodulation results.

7.1 Conclusions from Gray code Composite Pattern

The concept of Gray code composite pattern where multiple gray coded patterns are combined to form a single pattern based on the concept of modulation is introduced. Demodulation is carried out to the captured composite pattern to obtain phase. The attenuation in higher frequencies caused due to the optics of the camera is taken care of by weighing the carriers correctly in the projected pattern. Also the gamma value for the projector is calculated and gamma correction performed to the projected pattern.

The phase information obtained by demodulating the captured pattern and decoding the gray integers is used for obtaining 3D world coordinates. Based on the experiments conducted on different objects it was found that the 3D reconstruction is better for solid objects like a sphere when compared to alice which has marked discontinuities.
7.2 Future work

This thesis was confined to proposing and experimenting the concept of gray code composite pattern structured light illumination. The future work would be to obtain high resolution and non-ambiguous phase by using the concept of modified composite pattern. Interpolation can performed to minimize the stair case effect in the background of the 3D reconstructed image. Also the reason for distortion of the first carrier while modulating caused due to gamma correction of projector is to be known. Statistical analysis can be performed for the proposed system.
Appendix:

1. Matlab Code used in thesis:

1.1 Code for simulation program

%% Pratibha Gupta
%% Matlab Simulation %%
%% the matlab simulation program takes the mat5,G file
%% and the projected pattern as inputs
%% and outputs the camera view and the projector view
%% November 2006

clear all;

%%% Inputs
C0 = 'D:\2006cprog\UScanner\Calibrate\0C.bmp';
I0 = 'D:\2006cprog\UScanner\Calibrate\0I.bmp';
X0 = 'D:\2006cprog\UScanner\Calibrate\0X.by';
Y0 = 'D:\2006cprog\UScanner\Calibrate\0Y.by';
Z0 = 'D:\2006cprog\UScanner\Calibrate\0Z.by';
G = 'D:\2006cprog\UScanner\Calibrate\CalgridG.by';
Ip = 'D:\2006cprog\UScanner\Calibrate\composite pattern.bmp';

%%% Outputs
[Projview] = sim_prog_proj(x0,y0,z0,c0,i0,G,Ip);
[Camview] = sim_prog_cam(x0,y0,z0,c0,i0,G,Ip);
figure(1); imagesc(abs(Projview)); title('Projector View');colormap gray;
figure(2); imagesc(abs(Camview)); title('Camera View');colormap gray;
1.2) Part of code of simulation program that gives projector view

%% Pratibha Gupta
%% sim_prog_proj takes the mat5, the calibration data Gfile
%% the projected pattern as inputs
%% G file should contain both xp, yp info
%% the output is the image which the proj views
%% November 2006

function [P1] = sim_prog_proj(x0,y0,z0,c0,i0,G,ip)

%% Read Mat5 Data
[c0name] = double(imread(c0));
i0name = double(imread(i0));
[C0] = c0name(:,:,1);
[I0] = i0name(:,:,1);
[X0] = read_mat_data(x0);
[Y0] = read_mat_data(y0);
[Z0] = read_mat_data(z0);
[Iindex0]=find(I0==0);
Z0(Iindex0)=0;

clear x0, clear y0; clear z0;
clear c0name, clear i0name, clear i0; clear c0; clear Iindex0;

%% Read Composite Pattern
im1 = double(imread(ip));
im2 = imresize(im1,[1944,2592]);
Ip = im2(:,:,1);
[Myp,Nxp] = size(Ip);
clear im1;

%% Read the G file
[Gdata] = read_calib_data(G,18);
[N,M] = size(Gdata);

%% G data
xw = Gdata(:,1);
yw = Gdata(:,2);
zw = Gdata(:,3);
xct = Gdata(:,4);
yct = Gdata(:,5);
xpt = Gdata(:,6);
ypt = Gdata(:,7);
xc = xct;
yc = yct;
xp = xpt*Nxp/(2*pi);
yp = ypt*Myp/(2*pi);

%% Calculate the Camera and Projector Transformation Matrices %%
%mwp : m for world cood. to projector coord. %mwc : m for world cood. to camera coord.

[Ap,mwp] = calibrate(xp,yp,xw,yw,zw,N);
mwp = -mwp/(mwp(3,4));

clear xw;clear yw;clear zw;
clear xc;clear yc;
clear xp;clear yp;
clear xct;clear yct;
clear xpt;clear ypt;
clear Ac, clear Ap;
clear N; clear M; clear Gdata;

%% Map the mat5 to the projector space

X1 = X0;
Y1 = Y0;
Z1 = Z0;
I1 = I0;

%% Get Projector coordinates from world coords,mwp
[Xp,Yp] = getprojcoords(X0,Y0,Z0,mwp);

mp=floor(Yp+0.5);
np=floor(Xp+0.5);
[index] = find(mp<1);
mp(index)=1;
[index] = find(mp>Myp);
mp(index)=Myp;
[index] = find(np<1);
np(index)=1;
[index] = find(np>Nxp);
np(index)=Nxp;

P1 = zeros(Myp,Nxp);
for m = 1:Myp
    for n = 1:Nxp
        P1(mp(m,n),np(m,n)) = C0(m,n) .* Ip(mp(m,n),np(m,n));
        C1(m,n) = C0(m,n) .* Ip(mp(m,n),np(m,n));
    end
end

%% P1 is the projector view
1.3) Part of code of simulation program that gives camera view

%% Pratibha Gupta
%% sim_prog_cam takes the mat5, the calibration data Gfile
%% the projected pattern as inputs
%% G file should contain both xp, yp info
%% the output is the image which the camera views
%% November 2006

function [C2] = sim_prog_cam(x0,y0,z0,c0,i0,G,ip)

%% Mat5 Data
[c0name] = double(imread(c0));
i0name = double(imread(i0));
[C0] = c0name(:,:,1);
[I0] = i0name(:,:,1);
[X0] = read_mat_data(x0);
[Y0] = read_mat_data(y0);
[Z0] = read_mat_data(z0);
[Iindex0]=find(I0==0);
Z0(Iindex0)=0;
clear x0, clear y0; clear z0;
clear c0name, clear i0name, clear i0; clear c0; clear Iindex0;

%% Composite Pattern
im1 = double(imread(ip));
im2 = imresize(im1,[1944,2592]);
Ip = im2(:,:,1);
[Myp,Nxp] = size(Ip);
clear im1;
%% reading the G file %%
[Gdata] = read_calib_data(G,18);
[N,M] = size(Gdata);
%% G data %%
xw = Gdata(:,1);
yw = Gdata(:,2);
zw = Gdata(:,3);
xct = Gdata(:,4);
yct = Gdata(:,5);
xpt = Gdata(:,6);
ypt = Gdata(:,7);
xc = xct;
yc = yct;
xp = xpt*Nxp/(2*pi);
yp = ypt*Myp/(2*pi);
%% Calculate the Camera and Projector
%% Transformation Matrices %%
%mwp : m for world coord. to projector coord.
%mwc : m for world coord. to camera coord.

[Ap,mwp] = calibrate(xp,yp,xw,yw,zw,N);
[Ac,mwc] = calibrate(xc,yc,xw,yw,zw,N);
mwp = -mwp/(mwp(3,4));
mwc = -mwc/(mwc(3,4));

clear xw;clear yw;clear zw;
clear xc;clear yc;
clear xp;clear yp;
clear xct;clear yct;
clear xpt;clear ypt;
clear Ac, clear Ap;
clear N; clear M;
clear Gdata;

%% We want to create a camera image given the projector pattern

%% Map the mat5 to the projector space
X1 = X0;
Y1 = Y0;
Z1 = Z0;
I1 = I0;

%% Get Projector coordinates from world coords,mwp
[Xp,Yp] = getprojcoords(X0,Y0,Z0,mwp);

% figure(1); imagesc(abs(Xp)); colormap gray;
% figure(2); imagesc(abs(Yp)); colormap gray;

mp=floor(Yp+0.5);
np=floor(Xp+0.5);

[index] = find(mp<1);
mp(index)=1;
[index] = find(mp>Myp);
mp(index)=Myp;
[index] = find(np<1);
np(index)=1;
[index] = find(np>Nxp);
np(index)=Nxp;
P1 = zeros(Myp,Nxp);
for m = 1:Myp
    for n = 1:Nxp
        \( P1(mp(m,n),np(m,n)) = C0(m,n) \cdot Ip(mp(m,n),np(m,n)); \)
        \( C1(m,n) = C0(m,n) \cdot Ip(mp(m,n),np(m,n)); \)
    end
end

%%% Map the new mat5 to camera space
%%% Get Camera coordinates from world coords,mwc

[Xc,Yc] = getcameracoords(X1,Y1,Z1,mwc);
clear X1; clear Y1; clear Z1; clear I1;

mc = floor(Yc+0.5);
nc = floor(Xc+0.5);

[index] = find(mc<1);
mc(index)=1;
[index] = find(mc>Myp);
mc(index)=Myp;
[index] = find(nc<1);
nc(index)=1;
[index] = find(nc>Nxp);
nc(index)=Nxp;

C2=zeros(Myp,Nxp);
for m = 1:Myp
    for n = 1:Nxp
        C2(mc(m,n),nc(m,n)) = C1(m,n);
    end
end

%%% C2 is the camera view
1.4) Code for writing mat5 data

%% MATLAB LIBRARY (DR. HASSEBROOK)
% Veeraganesh Yalla
% template script to write the
% MAT5 files
% Date: Jan 30 2004

function [result] = mat5write(matfile,xw,yw,zw,imageI,imageC);
% open the world coordinate files
fnamex = strcat(matfile,'X.byt');fnamey = strcat(matfile,'Y.byt');
fnamez = strcat(matfile,'Z.byt');fnameC = strcat(matfile,'C.bmp');
fnameI = strcat(matfile,'I.bmp');
%get the dimensions
[my,nx,pz] = size(imageI);
%reshape the 1D world coordinate vectors
x = reshape(xw',1,my*nx)';y = reshape(yw',1,nx*my)';z = reshape(zw',1,nx*my)';
%f
fpx = fopen(fnamex,'wb');fwrite(fpx,x,'float32');fclose(fpx);
%f
fpy = fopen(fnamey,'wb');fwrite(fpy,y,'float32');fclose(fpy);
%f
fpz = fopen(fnamez,'wb');fwrite(fpz,z,'float32');fclose(fpz);
%f
imwrite(imageC,fnameC,'bmp');imwrite(imageI,fnameI,'bmp');
%result = 1;
1.5) Code for reading mat5 data

%% MATLAB LIBRARY (DR. HASSEBROOK)
% Veeraganesh Yalla
% template script to read the
% MAT5 files
% Date: Jan 30 2004

function [xw,yw,zw,imageI,imageC] = mat5read(matfile);
% open the world coordinate
% files
% the x,y,z are 1-D arrays
% need to be reshaped based on
% the dimensions of I and C images
fnamex = strcat(matfile,'X.byt');fnamey = strcat(matfile,'Y.byt');
fnamez = strcat(matfile,'Z.byt');fnameC = strcat(matfile,'C.bmp');
fnameI = strcat(matfile,'I.bmp');
%
fpx = fopen(fnamex,'rb');x = fread(fpx,'float');fclose(fpx);
%
fpy = fopen(fnamey,'rb');y = fread(fpy,'float');fclose(fpy);
%
fpz = fopen(fnamez,'rb');z = fread(fpz,'float');fclose(fpz);
%open the I and C images
imageC = imread(fnameC);imageI = imread(fnamex);
%get the dimensions
[my,nx,pz] = size(imageI);
%reshape the 1D world coordinate vectors
xw = reshape(x,nx,my)';
yw = reshape(y,nx,my)';
zw = reshape(z,nx,my)';
1.6) Code to perform SVD calibration

%% MATLAB LIBRARY (DR. HASSEBROOK)
%% Veeraganesh Yalla
%% Compute the Calibration Matrices
%% SVD
%% Date: 15 Feb 2006

clc;
close all;
clear all;
warning off;
%
infname = 'D:\2006cprog\Toyota_Scanners\Grab_Composite\Calibrate\troughgrid.byt';
[matdata] = read_calib_data(infname,28);
%
xw = matdata(:,1);
yw = matdata(:,2);
zw = matdata(:,3);
xct = matdata(:,4);
yct = matdata(:,5);
xpt = matdata(:,6);
ypt = matdata(:,7);
%
xc = xct;
yc = yct;
xp = xpt*1280/(2*pi);
yp = ypt*1024/(2*pi);
%
[N,M] = size(matdata)

% PARAMETER ESTIMATION
% mwp : m for world coord. to projector coord.
% mwc : m for world coord. to camera coord.

[Ap,mwp] = calibrate(xp,yp,xw,yw,zw,N);
[Ac,mwc] = calibrate(xc,yc,xw,yw,zw,N);

% normalize
mwp = mwp/abs(mwp(3,4))
mwc = mwc/abs(mwc(3,4))
% Reconstruction of World Coordinates
% % calculate the world coordinates using Xp and Yp
for i=1:N
    c(1,1) = mwc(1,1)-mwc(3,1)*xc(i);
    c(1,2) = mwc(1,2)-mwc(3,2)*xc(i);
    c(1,3) = mwc(1,3)-mwc(3,3)*xc(i);
    
    c(2,1) = mwc(2,1)-mwc(3,1)*yc(i);
    c(2,2) = mwc(2,2)-mwc(3,2)*yc(i);
    c(2,3) = mwc(2,3)-mwc(3,3)*yc(i);
    
    c(3,1) = mwp(2,1)-mwp(3,1)*yp(i);
    c(3,2) = mwp(2,2)-mwp(3,2)*yp(i);
    c(3,3) = mwp(2,3)-mwp(3,3)*yp(i);
    
    d(1,1) = mwc(3,4)*xc(i)-mwc(1,4);
    d(1,2) = mwc(3,4)*yc(i)-mwc(2,4);
    d(1,3) = mwp(3,4)*yp(i)-mwp(2,4);
    
    Pw(i,:)  = (inv(c)*d')';
end

% errval(:,1)= xw-Pw(:,1);errval(:,2)= yw-Pw(:,2);errval(:,3)= zw-Pw(:,3);
% erval = sqrt(errval(:,1).^2+errval(:,2).^2+errval(:,3).^2)
%write the projector parameters
fp = fopen('proj.txt','w');
fprintf(fp,'%f ',mwp(1,1));fprintf(fp,'%f ',mwp(1,2));
fprintf(fp,'%f ',mwp(1,3));fprintf(fp,'%f
',mwp(1,4));
fprintf(fp,'%f ',mwp(2,1));fprintf(fp,'%f ',mwp(2,2));
fprintf(fp,'%f ',mwp(2,3));fprintf(fp,'%f
',mwp(2,4));
fprintf(fp,'%f ',mwp(3,1));fprintf(fp,'%f ',mwp(3,2));
fprintf(fp,'%f ',mwp(3,3));fprintf(fp,'%f
',mwp(3,4));
fclose(fp);

%write the camera parameters
fp = fopen('cam.txt','w');
fprintf(fp,'%f ',mwc(1,1));fprintf(fp,'%f ',mwc(1,2));
fprintf(fp,'%f ',mwc(1,3));fprintf(fp,'%f
',mwc(1,4));
fprintf(fp,'%f ',mwc(2,1));fprintf(fp,'%f ',mwc(2,2));
fprintf(fp,'%f ',mwc(2,3));fprintf(fp,'%f
',mwc(2,4));
fprintf(fp,'%f ',mwc(3,1));fprintf(fp,'%f ',mwc(3,2));
fprintf(fp,'%f ',mwc(3,3));fprintf(fp,'%f
',mwc(3,4));
fclose(fp);
2. Band Pass filters and Hilbert transforms \[56\]

2.1 Band Pass filters

Let \( x(t) \) be a band pass filter with center frequency \( \omega_c \) specified by the impulse response, \( h(t) \) or transfer function \( H(\omega) \). There the filter output is

\[
Y(\omega) = X(\omega) H(\omega)
\]

The spectrum of the band pass filters is confined to a band not including 0 Hz in the frequency domain.

2.2 Hilbert transform

The Hilbert transform is a convenient tool to use in dealing with band pass signals. It is an ideal 90 degree phase shifter. The Hilbert transform of a signal \( x(t) \) is denoted by \( \hat{x}(t) \) and is obtained by passing \( x(t) \) through a filter with transfer function

\[
H(\omega) = -j \text{ sign } \omega = \begin{cases} 
-j & \text{ for } \omega > 0 \\
0 & \text{ for } \omega = 0 \\
j & \text{ for } \omega < 0
\end{cases}
\]

The system forming the Hilbert transform is shown in figure 2.2
Figure: 2.2 System for forming Hilbert transforms
3. Spatial deconvolution

Let $I_p(x,y)$ be the projected pattern and $I_{out}(x,y)$ be the captured pattern and $h(x,y)$ be the response.

$$I_{out}(x,y) = I_p(x,y) * h(x,y)$$

In the fourier domain, $I_{out}(u,v) = I_p(u,v)H(u,v)$

We want $I_{out}$ to be equal to $I_p$.

Therefore

Let $g(x,y) = h^{-1}(x,y)$

Then $I_{1}(x,y) = g(x,y)*I_p(x,y)$

$$I_{out}(x,y) = I_{1}(x,y)*h(x,y)$$

$$I_{out}(x,y) = g(x,y)*I_p(x,y)*h(x,y)$$

In fourier domain

$$I_p(u,v) = G(u,v).I_p(u,v).H(u,v)$$

$$I_{out}(u,v) = I_p(u,v) \text{ (Spatial Deconvolution)}$$

Thus the input pattern is multiplied with roll-off weights to make the captured pattern as close to the input pattern.
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Pratibha Gupta was born in Hyderabad, India on August 16, 1982. She received her Bachelor of Technology degree in Electrical and Electronics Engineering from VNRVJET College which is affiliated to Jawaharlal Nehru Technological University. She received a gold medal for her academic excellence during under graduation. She is currently working as a Research Assistant at Center for Visualization and Virtual Environment, Lexington.