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FINITE ELEMENT SIMULATION OF INDENTATION OF POROUS MATERIALS

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ABSTRACT OF THESIS

FINITE ELEMENT SIMULATION OF INDENTATION OF POROUS MATERIALS

Finite element simulation of indentation is presented in this thesis. A rigid cylindrical indenter of flat end is used in all the cases, in which the simulation focuses on the effect of a hole on the indentation behavior of materials including elastic and elasto-plastic materials. In the simulation, the material is assumed to be a half-space. The relations between load and displacement are determined as a function of the hole size. Also indentation under cyclic loading is simulated for an elastic-perfect plastic half space. The influence of factors causing fatigue deformation like amplitude, median load and frequency is addressed. The propagation rate of plastic zone and Von Mises stress distribution at maximum and minimum loading are analyzed.

KEYWORDS: Indentation, Finite Element Analysis, Fatigue

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FINITE ELEMENT SIMULATION OF INDENTATION OF POROUS MATERIALS

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FINITE ELEMENT SIMULATION OF INDENTATION OF POROUS MATERIALS

THESIS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Materials Science and Engineering in the Chemical and Materials Engineering Department at the University of Kentucky

By

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Dedicated to my parents
ACKNOWLEDGEMENTS

I am very grateful to my parents, my younger brother and my relatives for their immense love, moral and financial support. I wouldn’t have been here without their help. With due respect and love I would like to dedicate this thesis to my parents.

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Chapter 1: INTRODUCTION

As technology is advancing towards the era of micro and nano sized materials tremendous research is going on in the field of nanomaterials. One such nano sized material which has set its place potentially as miniature sensors or as an effective platform being used as templates for producing nanotubes is the self ordered nanoporous material. Self ordered nanoporous material has displayed many advantages over conventional bulk material in use as sensor material due to the increased surface area and hence increased absorbent and adsorbent properties.

Mechanical characterization of nano materials is essential for their successful applications in the area of miniature sensors and actuators. Nanoindentation is one of the popular methods to characterize the properties of materials in the nano scale. It is a process in which an indenter is pushed into the surface of a material to be tested to a depth of few nanometers and the load-displacement data is monitored and recorded, from which the properties of the material can be determined. Nanoindentation has been successfully used to measure the properties of materials including Young’s modulus, hardness, yield strength, fracture toughness and fatigue.

Studying the mechanics of the nanoindentation process is essential for its effective use. Numerical method has become major approach to study the mechanics of the indentation process instead of conventional analytical methods due to the complexity of the problem such as geometric nonlinearities and nonlinear elasto-plastic behavior.

Finite element application program ABAQUS is used here to simulate the indentation of a half space. The effect of a hole underneath the indenter on the indentation deformation is analyzed in the third and the fourth chapters.
The third chapter discusses the indentation of an elastic half space with a hole. The effect of the hole size on the load-displacement relationship, the distribution of the Von Mises stress, and the surface morphology is analyzed.

In chapter 4, the indentation of an elastic-perfect plastic half space is studied instead of the elastic case. Using the loading-unloading curves, the dependence of the indentation stiffness, the energy dissipated during the indentation process on the hole size is discussed. The effect of Poisson’s ratio on the distribution of Von Mises stress is also addressed.

The chapter 5 analyzes the impression fatigue of an elasto-plastic material; the effects of frequency, loading amplitude and median load on the impression fatigue are discussed.
Chapter 2: LITERATURE REVIEW

2.1 Self ordered nanoporous materials

Tremendous progress has been made in the research and development of micro and nano sized materials in the past few years due to their numerous potential applications in different areas, such as biomedical sensors and actuators [1,2,3,4]. Among them self ordered nanoporous materials have been used extensively as sensors or as templates for the synthesis of nanomaterials. The advantage of using nanoporous materials is that, the surface area of solid increases with increasing porosity, which improves catalytic, absorbent and adsorbent properties.

Self ordered nanoporous materials are used as templates for ultra sensitive sensors with nanometer scale resolution and response time in the range of microsecond, e.g. ‘artificial nose’ containing a dense array of receptor sites could analyze harmful gases [1, 2, 3, 4]. They can also be used as chemical sensors in industry, food processing, meteorological studies, military combat situations, environmental monitoring [3, 5, 6, 7, 8] thermoelectric coolers for cooling computer chips [9, 10] and molecular devices.

The use of self ordered nanoporous alumina for sensing ammonia at room temperature was first reported by Oomman et al [5]. They used nanoporous alumina films of approximately 2cm × 2cm as resistive sensors and surface acoustic wave sensors. The films displayed a predominantly ordered pore structure with pore diameters 45, 38, and 13 nm respectively. The response and recovery times are largest for the smallest pore size as the time required for molecules to diffuse into and out of the pore increases with decreasing pore size. The ammonia measured by using resistive sensors showed increase in sensitivity with decreasing pore size. The
surface acoustic wave devices using alumina nanoporous films showed superior performance compared to less stable polymer and expensive metal counter parts. Their results suggest that nanoporous alumina based sensors do have practical application.

Porous silicon is also well known to be sensitive to environment due to its large surface vs. volume ratio [6]. Baratto et al [7] used porous silicon as multi parametric sensor to discriminate the presence of NO₂ (a dangerous pollutant) from interferant gases like humidity and ethanol. They enhanced the sensing capability of porous silicon by using an optical micro cavity made of porous silicon (PSM), which operates as a multi parametric pollutant sensor-sensing the electrical DC conduction, the photoluminescence intensity and the position of the peak of optical cavity all as a function of the presence of gases. The measurements showed that it is possible to distinguish between a pollutant like NO₂ and interfering gases like humidity and ethanol by coupling the measurement of optical and electrical quantities in porous silicon. Using the porous silicon, one can detect gas concentration of as low as 0.5 ppm of NO₂, 300 ppm of ethanol diluted in air and 40 % humidity.

The chemical and mechanical instability of the sensor films has limited porous silicon for biological and environmental applications. Hence the castings of polymer replicas of photonic porous silicon [11] are used instead, which possess the chemical and mechanical properties of the porous silicon and provides a substantial improvement in the design of experiments and devices made of nano-structured photonic materials. These polymers are used for vapor sensing and drug delivery applications [11, 12, 13, 14]. In addition thin films of porous silicon are used in reflective and fluorescent biosensors [13, 14].
The advantages of using nanoporous alumina over porous silicon as biomolecular sensor were discussed by Pan and Rothberg [15], who demonstrated the feasibility of sensing adsorption of untagged molecular analytes in an electrochemically fabricated porous aluminum oxide template with pores of approximately 40 nm diameter by monitoring reflectivity spectra. The large surface area in nanoporous materials allows for a substantial change in refractive index upon binding of analyte molecules, enabling relatively high sensitivity to be achieved to detect DNA oligonucleotides.

Sehayek et al [16] presented a new approach to fabricating gradient structures by electodepositing in insulating nanoporous alumina template. They showed the control of the electro deposition in the pores of nanoporous alumina membranes in both vertical and horizontal directions, thus producing a graded material. Graded materials are engineering materials wherein microstructures are spatially varied through non-uniform distribution of reinforcement phases. Functionally graded materials are ideal candidates for applications involving severe thermal gradients, ranging from thermal structures in advanced aircraft and aerospace engines to computer circuit boards.

2.2 Preparation of self organized nanoporous alumina

Self organized porous alumina is generally prepared by anodization technique. [17, 18, 19] This process has gained importance due to particular characteristics such as controllable pore diameter, periodicity and extremely narrow distribution of pore size, which offers a promising route to synthesize a large surface area and ordered nanostructure with high aspect ratio.

Anodization is considered to be the most successful technique to make a nanoporous surface. First a thin oxide layer is formed on the surface of an aluminum plate, which is used as
anode. Lead is used as cathode. The oxidation layer is characterized by numerous defects, originally caused by defects in the aluminum surface. Owing to the decreasing current with the increase of the thickness of the barrier layer, the formation of pores is initiated at defect positions. Supported by electric field, aluminum oxide and hydroxide at the pore walls are preferentially dissolved by the acidic electrolyte which may consist of diluted sulfuric acid, phosphoric acid or oxalic acid. When the electric field increases by increasing the voltage, the dissolution process is accelerated, i.e. larger pores are formed. To remove the membrane from the aluminum substrate, the oxidized plate can be dipped into dilute hydrochloric acid to dissolve part of the aluminum.

Wang et al [20] fabricated alumina templates by using a two step oxidization method at different temperatures and showed that pore diameter and spacing increase with increase in voltage. The sample prepared at lower voltage is more regular than the one prepared under higher voltage. There is an increase in the longitudinal growth of the pores with the enhancement of anodizing voltage. The pores can be adjusted by exposing them to phosphoric acid by pore widening process which results in monotonously increasing mean pore diameter and increasing the pore widening time.

Hideki et al [21] proposed a novel approach for precise control of the growth of the ordered channel array in the anodic porous alumina, by generating a textured pattern on the surface of alumina through the molding process. This film is then anodized in an oxalic acid solution. The predetermined pattern formed by mechanical deformation acts as initiation points and guides the growth of long channels in the alumina film. This results in the growth of an almost defect free channel array throughout the textured area when compared to the conventional lithographic techniques.
Because self ordered nanoporous materials have wide applications in today’s technological world, the study of their mechanical properties like Young’s modulus, yield strength etc. demands attention. Traditional techniques used to measure elastic modulus involves direct measurement of strain as a function of stress, plotting the data graphically and measuring the slope of the elastic portion of the curve which requires experiments being conducted under larger length scale like 3-point bending, 4-point bending or by uniaxial tensile test. They are not practically feasible for nano or micro sized structure. Hence nanoindentation testing of materials becomes an important approach to measure the properties of materials of smaller dimensions.
2.3 Nanoindentation

Nanoindentation is a method of measuring mechanical properties of materials like Young’s modulus, hardness, yield strength and fracture toughness by making indents on the surface of materials with indentation depth ranging from nm to µm. [22, 23, 24, 25, 26, 27]. Developed over the last decade it is now used routinely in mechanical characterization of small structural materials, thin films and thin surface layers [28, 29, 30, 31, 32]. In a typical nanoindentation test, the applied load and the displacement of the indenter are monitored as the indenter is driven onto and withdrawn from the surface of a material. Forces as small as few tenths of micro Newtons and displacements as small as nanometers can be detected.

Hardness of a material can be determined without imaging the size of impression in a nanoindentation test. The contact area is evaluated by using an empirically determined indenter shape function which relates cross sectional area of the indenter to the distance from its tip. The Young’s modulus is determined by calculating the slope of the unloading curve and using the projected contact area of the impression.

Some of the advantages of the technique are

1. It is a non-destructive method of evaluating basic properties of materials of small volumes.

2. Mechanical properties can be determined by analyzing the indentation load-displacement data alone, thereby avoiding the need to image the size of impression and facilitating property measurement at the submicron scale.

3. It simplifies specimen preparation and makes measurements possible in systems which would otherwise be difficult to test.
4. Measurements can be made without having to remove film or surface layer from its substrate.

5. It offers advantage such as the ability to obtain information about elastic and time dependent plastic properties of materials.

Based on the load-displacement curve in an indentation, the contact stiffness (S) can be obtained from the slope of the upper portion of the unloading curve [33], i.e.,

\[ S = \frac{dP}{d(\delta)} = \frac{2}{\sqrt{\pi}} E^* \sqrt{A} \]  \hspace{1cm} (2.1)

where \( P \) is the load, \( \delta \) is the indentation depth, \( A \) is the projected contact area, and \( E^* \) is the effective elastic modulus defined as

\[ \frac{1}{E^*} = \frac{1-v_s^2}{E_s} + \frac{1-v_i^2}{E_i} \]  \hspace{1cm} (2.2)

where \( E \) and \( v \) are Young’s Modulus and the Poisson ratio and the subscripts \( s \) and \( i \) refer to the specimen and indenter respectively.

A combination of nanoindenter and AFM was used by Kempf et al [34] to study the local mechanical properties of different phases and particles in metallic alloys since the indents can be positioned with AFM. This technique was used to characterize the nanohardness of the plastic zone, stress fields of cracks and grain boundaries. The hardness of the super alloy CMSX-6 was characterized and local changes of mechanical properties near crack tip in a NiAl single crystal were examined by using the nanoindenter and AFM [34].

Because the hardness of a material measured by the nanoindentation technique is affected by the properties of the material, the indenter shape, and partially by the experimental procedure and the surface condition of the specimen, many attempts have been made to obtain intrinsic properties from indentation tests such as yield or tensile strength and even flow properties.
2.4 Finite element simulation of nanoindentation

Conventional analytical methods weren’t successful to study the mechanics of the indentation process due to the following reasons;

1. It involves large geometric nonlinearity due to imposed large rotations and high strains at the tip of indenter.
2. It introduces a difficult contact problem at the interface of indenter and material.
3. It predicts stress concentrations of a complicated type close to the edges of indenter.
4. It involves scale problems since material is modeled as semi-infinite.
5. Non-linearities in the deformation of material exist due to plasticity, phase transformation, microcracking, friction, anisotropy and in the presence of residual stresses.

Considerable efforts were made in the past to improve the accuracy and the computational efficiency of finite element simulation of indentation tests. Hardy et al [35] used a finite element grid which captures the critical field quantities with sufficient resolution but, because of the computational effort, severely limits the extent to which the half space can be loaded. Lee et al [36] took the loading forward to higher levels experienced in practice, but sacrificed the resolution of a fine grid, thereby introducing some numerical errors into their results. Follansbee et al [37] did the finite element simulation of a quasi-static normal indentation of an elastic-plastic half space by a rigid sphere by using incremental elastic-plasticity. They simulated the analysis at higher loads with a finite element grid of sufficient resolution and accuracy by using Constant Strain Triangle elements.

Giannakopoulos et al [38] discussed the finite element analysis of the Vickers hardness test and contributed to the analytical background on the use of sharp indenters for the mechanical characterization of materials. Elastic and elastic-plastic FEM analyses were undertaken and the
results were compared to analytical and experimental results available. Universal relations between indentation load and indentation depth were found for elastic and elasto-plastic equation and the contact was found to be load independent. They concluded that other mechanical properties could be obtained from a Vickers test if the load-displacement relation and the degree of pile up are known. When the material is indented certain amount of material piles up on the surface of the material, this is called pile up of the material. Some indentation size effect, that is, higher average contact pressure than the steady state prediction, was found at low loads. The steady state mechanical stresses and deformation fields were computed for the elastic and elasto-plastic cases with no strain hardening and with low strain hardening. Strain hardening was found to suppress material pile up at the contact boundary.

Larsson et al [39] analyzed Berkovich indentation numerically by using FEM and experiments. Universal formulae for hardness and total indentation load were derived from the numerical results showing excellent agreement with experimental findings. Mechanical stress and strain fields were presented which were spherical in shape; size of plastic zone and possible locations for crack growth were also shown. The distribution of plastic strain did not form the commonly assumed concentric spheres. Also the contact area between material and indenter was computed. The nanoindentation results clearly exhibited size effects for the analyzed materials, indicating that higher applied loads were needed in order to obtain bulk macroscopic plastic properties.

Bolshakov et al [40] used finite element method to investigate the influences of pileup on the determination of hardness and elastic modulus. They showed that for elastic-perfect plastic materials with no work hardening; when pileup is significant, the areas obtained from the Oliver-Pharr analysis [26] can underestimate the true contact area by as much as 60% which leads to
huge errors in the measurement of hardness and elastic modulus. They also showed that it is not necessary to invoke indenter tip-rounding effects for explaining the power law exponents being less than 2 for conical indenters.

Laursen et al [41] used FEM to show how numerical approach may be used to characterize the indentation process and evaluate quantities that are difficult to evaluate experimentally like contact area, yielded zone etc. They studied alumina and silicon both in bulk form and in thin film-substrate system. They showed that negligence of pile-up and sink-in could be the reason for inaccurate determination of contact area from plastic depth, as the area of contact determined by using the plastic depth from straight line extrapolation of the unloading curve is significantly different from those attained from actual contact area.

Pharr et al [22] developed a conceptual framework to fit the unloading curves obtained by using sharp, geometrically self-similar indenters like pyramids and cones. They conducted finite element simulation for the indentation of a conical indenter into a half space. Two different constitutive behaviors were used to simulate the deformation of the half space. Their simulated results were in agreement with the experimental load-displacement data, conducted by a Berkovich pyramid, which implies that the edges of the pyramid do not significantly influence the load-displacement behavior of materials. Their results indicated that plastic deformation has a negligible influence on the load-displacement behavior after the first indentation. They tried to propose a mathematical form to fit the unloading curve by introducing effective indenter shape.

Because the surface impression, created during previous loading, adds certain complexity to the initial contact geometry, the evolution of contact area during subsequent unloading becomes more intricate. This problem was addressed by Li et al [42] who used FEM and AC indentation technique to evaluate the evolution of contact area during a repeated indentation
process. In the AC indentation technique, a microindentation system is upgraded to include an ac
capability by superposing a small displacement modulation on an otherwise linear indenter
motion. They proposed a methodology that could be employed to measure instantaneous contact
stiffness. They used FEM to show the dependence of contact area on the applied load for five
different materials - Si, Al, Al$_2$O$_3$, glass and amorphous carbon.

Knapp et al used FEM to develop a procedure [32] for determining mechanical properties
of MEMS materials like yield stress, Young’s modulus and intrinsic hardness for thin films.
Their model can also be applied to measure the properties of multi-layer materials whose
properties vary with depth. There are several limitations in their model because it assumed
perfect bonding between layers.

The measurement of micromechanical properties of nanoporous materials as well as the
interpretation of the experimental data poses a huge challenge to the society of mechanical
characterization. To better utilize the nanoindentation technique to quantify the mechanical
properties of nanoporous materials, it is necessary to explore their deformation behavior. It is the
purpose of this thesis to simulate the indentation behavior of porous materials. For simplicity, we
only consider the effect of one hole on the load-displacement curves in indentation tests. The
effect of multiple pores and the pattern of pores on indentation deformation will be addressed in
the future.
Chapter 3: ELASTIC INDENTATION

3.1 Introduction

Nanoindentation technique has been used extensively to study indentation deformation of materials and to determine mechanical properties of bulk materials [22, 23, 43, 44, 45, 46]. It is also used to study deformation behavior of thin films deposited on substrates [47, 48, 29, 30, 31, 32].

Indentation of elastic materials has been extensively studied [49, 50]. Sneddon [66] used the Hankel transforms to obtain the closed-form solution on the axisymmetric indentation of an elastic half space. Using his solution, one can easily obtain the load-displacement relation. His solution provides the basis for the nanoindentation technique. Olesiak [51] solved the problem of an annular flat end indenter pressed symmetrically on a semi-infinite space. The contact problem of a transversely isotropic elastic layer over a rigid foundation was solved by England [52]. Keer [53] analyzed a class of non-symmetric contact problems of an elastic half-space and an elastic layer over a rigid substrate. Li and his co-workers [54, 55, 56] studied the mechanics of indentation by a flat-ended indenter.

Pavliotis et al [57] obtained approximate analytical solutions to three dimensional axisymmetric indentation problem of the thin compressible linear elastic layer bonded to a rigid foundation. Their results were compared to the numerical results by using finite difference method. The singularity problems associated with the sharp end of the flat ended indenter were also incorporated in their problem by considering a flat end indenter without fillet at the sharp corner and the deformation gradient was calculated and the effectiveness to solve the problem was discussed. We use a sharp end flat end indenter in our case with a fillet.
Yang [58] deduced a load-displacement relationship for the indentation problem of an elastic layer by a rigid flat-ended cylindrical indenter. Frictionless boundary condition between the indenter and the layer and between the elastic layer and the substrate was used. A closed-form solution was derived for the normal stress underneath the indenter for contact radius being much larger than the film thickness. He found that the ratio of the indentation depth to the contact force is proportional to the indentation depth and the square of the contact radius and inversely proportional to the film thickness.

Solon et al [50] analyzed the axisymmetric contact problem of a rigid, bolt shaped punch pressed into an elastic half space or layer with a transverse cylindrical hole. They used Srivastav and Narain’s [59] results for axial displacement and axial stresses. The problem was reduced to the solution of the Fredholm integral equation.

Yang et al [60] used finite element method to analyze the effect of friction and cavity depth on the indentation of an elastic half space. They focused on the effect of different boundary conditions between the indenter and the elastic medium. They discussed the effects of boundary conditions and cavity on the distribution of the von Mises stress and the relationship between the punching stress and punch displacement which varied due to the presence of a cavity.

Upto now most work in the literature only focused on the indentation of an elastic half space and multilayer structure. The study on the indentation of porous material is still at an early stage. It is the purpose of this chapter to consider the effect of a single hole on the indentation of an elastic material. In the simulation, a flat ended cylindrical indenter is chosen because it forms a conforming contact [61] between the indenter and the surface of elastic materials during loading and unloading. Thus the applied contact stress is linearly related to the applied load [62]
unlike other indenters like Berkovich, Vickers, conical, pyramidal indenters, where the contact area between the specimen and the indenter depends on the load and changes during the loading and unloading process of indentation [63, 64, 56, 60].

### 3.2 Mathematical formulation of the problem

Consider an isotropic, homogeneous elastic half space, bounded by the plane $z = 0$, with a transverse cylindrical hole of radius $R$. A rigid flat ended cylindrical indenter of radius $a$ ($a > R$) is pushed onto the surface of the elastic half space as shown in fig 3.1. A cylindrical coordinate system $(r, 0, z)$ is used. Because of the axial symmetry of the problem (no $\theta$ dependence), the displacement field has nonzero components $(u_r, u_z)$ only in the radial and vertical directions characterized by unit vectors $\hat{r}$ and $\hat{z}$.

$$\vec{u}(r, z) = u_r(r, z) \hat{r} + u_z(r, z) \hat{z}$$  \hspace{1cm} (3.1)

Assume that the contact between the indenter and the elastic half space is frictionless. The boundary conditions over the surface of the elastic half space ($z = 0$) are

$$\sigma_{rz}(r, 0) = 0 \text{ for } R \leq r < \infty$$  \hspace{1cm} (3.2)

$$u_z(r, 0) = D \text{ for } R \leq r \leq a$$  \hspace{1cm} (3.3)

$$\sigma_{zz}(r, 0) = 0 \text{ for } a < r < \infty$$  \hspace{1cm} (3.4)

$$2\pi \left[ \int_{R}^{a} \sigma_{zz}(r, z) r \, dr \right] = F$$  \hspace{1cm} (3.5)

where $\sigma_{rz}$ and $\sigma_{zz}$ are the shear stress and the normal stress respectively, $F$ is the force applied on the indenter, and $D$ is the indentation depth. In the following we only consider two different boundary conditions on the surface of the hole.

Case I: Constrained along the surface of the hole,
\[ u_r (R, z) = 0, \sigma_{rz} (R, z) = 0 \] \hspace{1cm} (3.6)

Case II: Traction free along the surface of the hole,

\[ \sigma_{rr} (R, z) = 0, \sigma_{rz} (R, z) = 0 \] \hspace{1cm} (3.7)

Using the displacement field, the equilibrium equation can be written as [60]

\[ \Delta^2 \ddot{u} + \frac{1}{1-2\nu} \Delta (\Delta \dot{u}) = 0 \] \hspace{1cm} (3.8)

where \( \nu \) is the Poisson ratio. In the theory of linear elasticity the relation between the components of the strain tensor and the components of the displacement vector is

\[ \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \] \hspace{1cm} (3.9)

### 3.3 Finite element analysis

#### 3.3.1 Finite element mesh

The finite element program ABAQUS [65] is used for the simulation of the indentation of the elastic half space. The elastic half space has a hole of radius \( R \), which goes through the material. The indenter has a radius of \( a \). Table 3.1 lists the material properties used for the simulation.

Figure 3.2 shows the finite element mesh, which consists of 6512 nodes, 2075 CAX8R eight node axisymmetric quadrilateral elements and 50 CINAX5R five node axisymmetric infinite elements. The mesh is refined around the contact region between the indenter and the elastic material for accuracy. The mesh gets coarser as it moves away from the region of contact as shown in fig 3.2.

To avoid the singularity in finite element analysis, due to the sharp edge of the indenter a fillet radius of 0.005 \( a \) is used. To simulate the contact between the indenter and the elastic half
space, the surface of the indenter is treated as the master surface and the surface of the elastic half space as the slave surface.

For case I, displacement constraint in the \( r \)-direction is applied to the surface \( S_r \). For case II, the surface \( S_r \) is traction free.

### 3.3.2 Verification of the mesh

To simulate the indentation, a pre-determined indentation depth is applied to the indenter; the indentation load is monitored during the indentation. The finite element mesh is first verified by comparing the simulation results with the analytical results given by Sneddon [66]

\[
F = \frac{4\mu aD}{1-\nu} \tag{3.10}
\]

where \( \mu \) is the shear modulus of the elastic half space.

Use \( a=0.4 \ \mu m \) and the material properties as listed in Table 1 in the simulation. Fig. 3.3 shows the dependence of the indentation load on the indentation displacement for the indentation of an elastic material with \( R=0 \). For comparison, the corresponding analytical solution of equation (3.10) is also depicted in fig. 3.3. The difference between the numerical results and equation (3.10) is less than \( 5.0 \times 10^{-2} \), suggesting that the finite element mesh is good enough to simulate the indentation of an elastic half space.

### 3.3.3 Load-displacement relationship

The effect of the hole size on the load-displacement relation is shown in fig. 3.4. The force required is greater for the constrained case when compared to the unconstrained case. With the increase in the hole size, the contact area between the indenter and the elastic half space decreases. Thus the indenter load required to displace the indenter into the elastic half space decreases as the hole size increases as expected.
Figure 3.5 illustrates the relationship between the ratio of the force to the displacement and Young’s modulus of the material. The force is proportional to Young’s modulus as expected in the theory of linear elasticity. From the fig 3.6 the relationship between the ratio of the force to displacement and the Poisson’s ratio at r/2a = 0.4 can be expressed as

\[ F = 2EaD(0.2916\nu^3 - 0.0667\nu^2 + 0.0088\nu + 0.0787) \]  

(3.11)

for the case I,

\[ F = 2EaD(0.1474\nu^3 + 0.0003\nu^2 - 0.00005\nu + 0.0753) \]  

(3.12)

for the case II.

### 3.3.4 von Mises stress distribution

Let \( \sigma_1 \), \( \sigma_2 \), \( \sigma_3 \) denote the principal values of the stresses in a solid, the von Mises stress can be expressed as

\[ \sigma_{\text{Mises}} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}} \]  

(3.13)

The distribution of the von Mises stress (\( \sigma_{\text{Mises}} \)) along the z-axis for an elastic impression of a flat half space by a flat end cylindrical punch [67] is given by

\[ \frac{\sigma_{\text{Mises}}}{\sigma_{\text{imp}}} = \frac{4(7-2\nu)z^2 + (1-2\nu)a^2}{4(z^2 + a^2)^2} \]  

(3.14)

where \( \sigma_{\text{imp}} \) is the punching stress.

The distribution of the von Mises stress in the elastic nanoindentation is shown in fig. 3.7. It shows that stress concentration exists near the edge of the indenter.

The von Mises stress distribution shows that maximum value of the von Mises stress is present near the surface of the hole and it gradually decreases as it moves away from the surface as shown in fig 3.8. The numerical and closed form solution of von Mises stress distribution
along the z-axis for $r/2a=0$ is shown in fig 3.8. Fig 3.9 shows the von Mises stress distribution along the depth of the hole for $r/2a=0.4$. The location of the maximum value of the von Mises stress along the depth of the hole for different hole radii is shown in fig. 3.10. As the size of the hole increases, the location of the maximum value of the von Mises stress moves towards the surface of the hole. The effect of hole radius on the maximum value of the von Mises stress is shown in fig 3.11. The plot shows that the maximum value of the von Mises stress increases with the increase in the hole radius.

The effect of Young’s Modulus and Poisson’s ratio on the maximum value of the von Mises stress is shown in fig 3.12 and fig 3.13 respectively. The plot illustrates the increase in the value of the von Mises stress with increasing Young’s modulus. Both the plots show a higher value of the von Mises stress for the constrained case when compared to the unconstrained case.

3.3.5 Surface profile along the depth of the hole

The surface profile is plotted along the depth of the hole and along the surface as shown in fig 3.14. The figure shows buckling at a point closer to the surface of the material.

3.3.6 Surface profile along the surface of the elastic half space

The axial displacement of the plane boundary seems to be similar to the axial displacement of an elastic half space without a hole. The axial displacement of the elastic half space is maximum for the maximum hole radius. With the increase in the radius of the hole the deformation seems to be more spread out. Thus the deformation is more spread out for hole radius of 0.32 µm when compared to that of hole radius 0.16 µm. The plot of the displacements in the z-direction along the surface of the plate is shown in fig 3.15.
3.4 Conclusions and results

Numerical analysis of elastic nanoindentation is done by using finite element method. The results can be summarized as below.

1. The displacement of the indenter is proportional to the applied load.

2. The load required to create the same indentation depth decreases as the radius of the hole increases.

3. The von Mises stress is proportional to the Young’s modulus of the elastic half space, and the maximum value of the von Mises stress increases and moves towards the surface of the elastic half space with the increase in the radius of the hole.

4. The surface morphology along the depth of the hole shows that the elastic half space buckles in compression. The surface morphology of the elastic half space along the surface of the elastic half space shows that sink in of the elastic half space takes place.
Table 3.1 Material properties of an elastic half space

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus E (GPa)</th>
<th>Poisson’s Ratio ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Aluminum</td>
<td>72</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Figure 3.1 Indentation of an elastic half space with a hole

Figure 3.2 Finite element mesh for the contact between an elastic half space and a rigid cylindrical indenter for \( r/2a = 0.4 \)
Figure 3. 3 Force vs. displacement relationship for numerical and analytical values

Figure 3. 4 Effect of hole radius on the Force applied on the indenter
Figure 3.5 Effect of Young's Modulus on force applied on the indenter for r/2a=0.4

Figure 3.6 Effect of Poisson's ratio on the force applied on the indenter for hole r/2a=0.4
Figure 3. 7 von Mises stress distribution $\sigma^{\text{mises}} / E$ for elastic half space for $r/2a=0.4$, for unconstrained and constrained cases respectively.
Figure 3.8 von Mises stress distribution along the z axis for hole r/2a=0

Figure 3.9 von Mises stress along the depth of the hole for r/2a=0.4
Figure 3. 10 The effect of hole radius on the position of the maximum value of the von Mises stress along the depth of the hole

Figure 3. 11 Effect of hole radius on maximum von Mises stress
Figure 3.12 Effect of Young's Modulus on the maximum von Mises stress for r/2a=0.4

Figure 3.13 Effect of Poisson's ratio on the maximum von Mises Stress for r/2a=0.4
Figure 3.14 Surface morphology along the depth of the hole with $r/2a=0.4$

Figure 3.15 Surface morphology along the surface of the elastic half space for $r/2a=0.4$
Chapter 4: INDENTATION OF ELASTO-PERFECT PLASTIC MATERIAL

4.1 Introduction

As mechanical systems continue to decrease in size and begin to approach atomic length scales, it is becoming important to develop experimental and corresponding theoretical tools to characterize material properties at these scales. Plastic deformation of materials has become a major concern in controlling the design and performance of engineering components at micro and nano scales. Extensive research has been done to study the plasticity at the micron scale by using nanoindentation [68, 69, 70, 71, 72, 73, 74].

Plastic deformation of the material at the micron scale is discussed in materials point of view and in mechanics point of view. In materials point of view plasticity is discussed using dislocation mechanisms by performing nanoindentation on single crystals [75, 76, 77, 78, 79] and bulk materials. Plastic deformation in single crystals is used to study the initial points of yielding and the beginning of the plastic deformation under the action of the indenter. Dislocation nucleation and discrete movement of the dislocations is used to explain the phenomenon of plastic deformation in this case. In mechanics point of view, the plastic deformation of micro sized materials is explained using strain gradient plasticity theory [80, 81] which is independent of the grain boundary effects and the grain size.

To overcome the laborious numerical technique and the complex form of analytical equations finite element method was used to calculate the mechanical properties of materials in nanoindentation [82, 83, 84]. Finite element simulation allows better understanding of
deformation phenomenon at small scales and is also useful to examine the elastic-plastic transition regime. Finite element simulation of nanoindentation is also successfully used in measuring the elastic-plastic properties of thin films on substrates like work hardening coefficient, yield strength and residual stress and to extract the stress-strain curve of the coating [85]. The mechanical characteristics of surface coatings and thin films can be successfully extracted by combining finite element simulation and indentation tests [86, 87, 88].

Nanoindentation is a useful tool for studying the onset of plastic flow in small volumes, which can play a significant role in macroscopic deformation processes such as adhesion, friction and fracture. From nanoindentation experiments it was observed that initial plasticity is usually observed as a sudden deviation from elastic behavior. The sudden deviation from the elastic loading, termed yield point has been attributed to the nucleation, multiplication, motion, and pileup of dislocations.

As the indenter is driven into the material both elastic and plastic deformations take place. During the indenter withdrawal, only the elastic portion of the displacement is recovered, which facilitates the use of elastic solutions in modeling contact process. The Oliver-Pharr [89] data analysis begins by fitting the unloading curve to the power law relation

\[ P = B(h - h_f)^m \]  \hspace{1cm} (4.1)

where \( P \) is the indentation load, \( h \) is the displacement, \( B \) and \( m \) are empirically determined fitting parameters, and \( h_f \) is the final displacement after complete unloading. The slope of the unloading curve is a measure of the material stiffness called the contact stiffness, \( S \); it is calculated by differentiating equation (4.1) at the maximum depth of penetration \( h = h_{\text{max}} \), i.e.

\[ S = \frac{dP}{dh} (h = h_{\text{max}}) = mB(h_{\text{max}} - h_f)^{m-1} \]  \hspace{1cm} (4.2)
4.2 Mathematical formulation of the problem

Consider an isotropic, homogeneous elastic-perfect plastic half space, bounded by the plane $z = 0$, with a transverse cylindrical hole of radius $R$. A rigid flat ended cylindrical indenter of radius $a \ (a > R)$ is pushed onto the surface of the elasto-plastic half space as shown in Fig 3.1. A cylindrical coordinate system $(r, \theta, z)$ is used. Because of the axial symmetry of the problem (no $\theta$ dependence), the displacement field has nonzero components $(u_r, u_z)$ only in the radial and vertical directions characterized by unit vectors $\hat{r}$ and $\hat{z}$.

$$\vec{u} (r, z) = u_r (r, z) \hat{r} + u_z (r, z) \hat{z} \quad (4.3)$$

Assume that the contact between the indenter and the elasto-plastic half space is frictionless. The boundary conditions over the surface of the elasto-plastic half space ($z = 0$) are

$$\sigma_{rz} (r, 0) = 0 \quad \text{for} \quad R \leq r < \infty \quad (4.4)$$

$$u_z (r, 0) = D \quad \text{for} \quad R \leq r \leq a \quad (4.5)$$

$$\sigma_{zz} (r, 0) = 0 \quad \text{for} \quad a < r < \infty \quad (4.6)$$

$$2\pi \left[ \int_{R}^{a} \sigma_{zz} (r, z) r \, dr \right] = F \quad (4.7)$$

where $\sigma_{rz}$ and $\sigma_{zz}$ are the shear stress and the normal stress respectively, $F$ is the force applied on the indenter, and $D$ is the indentation depth. In the following we only consider two different boundary conditions on the surface of the hole.

Case I: Constrained along the surface of the hole,

$$u_r (R, z) = 0, \sigma_{rz} (R, z) = 0 \quad (4.8)$$

Case II: Traction free along the surface of the hole,

$$\sigma_{rr} (R, z) = 0, \sigma_{rz} (R, z) = 0 \quad (4.9)$$
4.3 Finite element analysis

Similar to the elastic analysis, the half space is assumed to be homogeneous, isotropic and the material is elastic perfect plastic. The elastic-plastic half space is meshed by using CAX4R four node axisymmetric reduced integration elements. The finite element mesh of the elastic-plastic half space is similar to that used for the elastic case as shown in fig 4.1. The plastic response of the material is governed by the von Mises yield criterion. The constitutive relationship in one dimension is given as

\[
\sigma = \begin{cases} 
E \varepsilon & \text{for } \sigma < \sigma_y \\
\sigma_y & \text{for } \sigma \geq \sigma_y
\end{cases}
\]

(4.10)

where \( E \) is the Young’s Modulus and \( \sigma_y \) is the yield strength of the half space respectively. The material properties of the material used are given in table 4.1.

The indentation procedure is simulated by two subsequent steps: loading and unloading. During loading the indenter moves along the z- direction penetrates into the specimen up to the maximum depth; during unloading, the tip returns to the initial position. At each depth increment during the static analysis the program makes many iterations according to a specified convergence rate.

4.3.1 Load-displacement relationship

When an indenter penetrates into the surface, a compressively strained zone is created and surrounded by an elastically constrained area. The related load-displacement curve of indentation for a cycle of loading-unloading can describe both elastic and plastic properties of samples in the vicinity of the indenter.
Typical load-displacement curve for the elastic-plastic half space is shown in fig.4.2. The plot shows that the load required to penetrate the indenter into the half space is almost double for the constrained case when compared to the unconstrained case for hole radius = 0.24 µm. Figure 4.3 shows the effect of Young’s Modulus on force, which illustrates the increase in the force with the increase of Young’s Modulus. There doesn’t seem to be any effect of Poisson’s ratio on the force. The effect of force on maximum depth of the indenter for case 1 and case 2 is shown in fig 4.4 and fig 4.5 respectively. The effect of force on the residual depth of the indenter for case 1 and case 2 is shown in fig 4.6 and fig 4.7 respectively.

The contact stiffness of the unloading and the loading curves is calculated. Both contact stiffnesses decrease with the size of the hole. The loading and unloading stiffness for varying hole radii is shown in fig 4.8. The stiffness of both the loading and the unloading curve are a function of the hole radius which can be expressed as

\[
\frac{dP}{dh} = f(a R^4 + b R^3 + c R^2 + d R + e) \tag{4.11}
\]

where \(\frac{dP}{dh}\) is the slope of the loading and the unloading curves, R is the radius of the hole, and a, b, c, d and e are constants. The values of the constants are given in table 4.2.

The energy dissipated in indentation is obtained by calculating the area of the load-displacement curve and is depicted in fig 4.9. The plastic energy dissipated in indentation decreases with the increase in the radius of the hole. The energy dissipation is lower for unconstrained case when compared to the constrained case which implies that the presence of the hole decreases the energy dissipation during the nanoindentation process. The effect of hole radius on the energy dissipated is shown in fig 4.9.
4.3.2 Von Mises stress distribution

Let $\sigma_1$, $\sigma_2$, $\sigma_3$ denote the principal values of stress, the Von Mises stress is defined as

$$\sigma^\text{Mises} = \frac{\sqrt{3}}{2} \left[ \left( \sigma_1 - \sigma_2 \right)^2 + \left( \sigma_2 - \sigma_3 \right)^2 + \left( \sigma_3 - \sigma_1 \right)^2 \right]^{1/2}$$  \hspace{1cm} (4.12)

The initiation of plastic behavior takes place when the von Mises stress developed is equal to the yield stress ($\sigma_y$) of the material. The material deforms plastically when the von Mises stress is greater than $\sigma_y$. Figure 4.16 and 4.17 show the distribution of the von Mises stress along the surface of the hole in the loading phase and in the unloading phase respectively. Due to the constraint, larger plastic zone is formed for case I.

Figures 4.12 and 4.13 show the dependence of the plastic zone on the indentation depth and the hole size respectively. Figures 4.14 and 4.15 shows the von Mises stress distribution along the surface of the half space in the loading and the unloading phase respectively. The plastic zone decreases with increasing hole size. Figures 4.16 and 4.17 show the contour of von Mises stress. Larger plastic zone is induced by the indentation. The plastic zone size after loading doesn’t seem to be affected a lot with varying Young’s modulus as shown in table 4.3.

4.2.3 Surface profile along the depth of the hole

The plot of surface profile along the depth of the hole in fig 4.18 and fig 4.19 after loading and unloading, respectively, illustrates sudden buckling at a certain point along the depth of the hole. This buckling is sharper than in the elastic case. As the applied load increases, the surface buckles along the depth of the hole in compression. If the load applied on the hole is increased, due to the effect of buckling, the surface would bend so much towards the hole that it almost covers the hole.
4.2.4 Surface profile along the surface of the elastic-plastic half space

The surface profile of the elastic-plastic half space along the surface of the hole at loading and unloading as shown in fig 4.20 and 4.21 respectively displays pileup of material on the surface. The size of the pileup is much more for the constrained case when compared to the unconstrained case. Also the pileup of the material is spread out for the constrained case when compared to the unconstrained case.

4.3 Conclusions and results

Plastic nanoindentation of an elastic-perfect plastic half space is simulated by using finite element method. The results are summarized below.

1. The force required to push the indenter into the material increases with the increase in the ratio of Young’s Modulus to the yield stress.

2. The load displacement curve for Case 1 and Case 2 show almost similar slopes for the loading and the unloading curves. The stiffness of the curves decreases with the increase in the size of the hole. The energy dissipated also decreases with the increase in the size of the hole.

3. Larger plastic zone is formed for constrained case. The surface profile of the elastic-plastic half space shows pile up of the material, and buckling of the elastic-plastic half space in compression along the depth of the hole is observed.
Table 4.1 Material properties of an elastic-plastic half space

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus E (GPa)</th>
<th>Poisson’s Ratio $\nu$</th>
<th>Yield Strength (GPa)</th>
<th>Strain Hardening Coefficient $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>72</td>
<td>0.3</td>
<td>0.056</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.2 Stiffness constants calculation from the loading and unloading curves for hole radius of 0.32 $\mu$m

<table>
<thead>
<tr>
<th>Constants</th>
<th>Loading</th>
<th>Unloading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>A</td>
<td>2674.4</td>
<td>-19160</td>
</tr>
<tr>
<td>B</td>
<td>1969.6</td>
<td>11589</td>
</tr>
<tr>
<td>C</td>
<td>205.31</td>
<td>-2358</td>
</tr>
<tr>
<td>D</td>
<td>15.101</td>
<td>107.64</td>
</tr>
<tr>
<td>E</td>
<td>60.499</td>
<td>60.483</td>
</tr>
</tbody>
</table>

Table 4.3 Zone size ($\mu$m) after loading of case 1 and case 2 respectively for varying Young’s modulus for hole radius = 0.32 $\mu$m

<table>
<thead>
<tr>
<th>Young’s Modulus (E GPa)</th>
<th>Zone size (Case 1)</th>
<th>Zone size (Case 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0787257</td>
<td>0.0400449</td>
</tr>
<tr>
<td>20</td>
<td>0.157805</td>
<td>0.0778625</td>
</tr>
<tr>
<td>40</td>
<td>0.197413</td>
<td>0.0776262</td>
</tr>
<tr>
<td>65</td>
<td>0.197283</td>
<td>0.0775856</td>
</tr>
<tr>
<td>72</td>
<td>0.197238</td>
<td>0.077581</td>
</tr>
</tbody>
</table>
Figure 4.1 Finite element mesh of elastic-perfect plastic half space with $r/2a=0$

Figure 4.2 Load displacement curve for $r/2a=0.3$
Figure 4.3 Effect of Young's Modulus on force for hole radius = 0.32 \mu m

Figure 4.4 Effect of force on maximum depth of the indenter for case 1
Figure 4.5 Effect of force on maximum depth of the indenter for case 2

Figure 4.6 Effect of force on the residual depth after unloading for case 1
Figure 4.7 Effect of force on the residual depth after unloading for case 2

Figure 4.8 Stiffness vs. radius of the hole for loading & unloading curve
Figure 4.9 Effect of hole radius on the energy dissipated

Figure 4.10 Von Mises stress distribution along the depth of the hole in loading for $r/2a=0.4$
Figure 4.11 Von Mises stress distribution along the depth of the hole in unloading for
\( r/2a = 0.4 \)

Figure 4.12 Effect of indentation depth on von Mises stress zone size for hole radius = 0.32 µm
Figure 4.13 Effect of hole radius on the von Mises stress zone size after loading

Figure 4.14 Von Mises stress distribution along the surface of the elastic-plastic half space after loading for $r/2a = 0.4$
Figure 4.15 Von Mises stress distribution along the surface of the elastic-plastic half space at unloading for \( r/2a = 0.4 \)
Figure 4. Von Mises stress distribution $\sigma^{\text{Mises}} / 0.056$ for case 1 and case 2 at $D/a = 7.5E-3$ respectively after loading for $r/2a = 0.4$
Figure 4.17 Von Mises stress distribution ($\sigma^{Mises}/0.056$) for case 1 and case 2 at D/a = 5E-3 respectively after unloading for r/2a = 0.4

Figure 4.18 Surface profile along the depth of the hole after loading for r/2a=0.4
Case 2

Figure 4.19 Surface profile along the depth of the hole after unloading for $r/2a=0.4$

Figure 4.20 Surface profile along the surface of the elastic-plastic half space after loading for $r/2a=0.4$
Figure 4.21 Surface profile along the surface of the elastic-plastic half space after unloading for 
\(r/2a=0.4\)

\(a=0.4 \, \mu\text{m}, \, E=72 \, \text{GPa}, \, \sigma_y=0.056 \, \text{GPa}\)
CHAPTER 5: IMPRESSION FATIGUE

5.1 Introduction

Fatigue tests of micro/nano sized materials are essential due to the use of such materials in MEMS devices and other devices when subjected to dynamic loading. Fatigue has become progressively more prevalent as technology has developed a greater amount of equipment, in varying sizes from automobiles, aircraft, compressors, pumps, turbines, etc., to MEMS devices, and other micro, nano sized devices which are subject to repeated loading and vibration. It is often stated that fatigue accounts for at least 90 percent of all service failures due to mechanical causes [90]. Fatigue tests for materials of small volumes can be successfully done by using a nanoindenter by applying cyclic loading to the material. When the loading is applied repeatedly, the indenter can sink further into the specimen on a per cycle basis. This is called indentation fatigue test. If the indenter is a cylinder with a flat end, it is called the Impression fatigue test.

Impression fatigue is a localized fatigue test which can provide information on plastic zone propagation, cyclic hardening and softening, overloading and underloading effects, crack nucleation and growth, and other damage mechanisms. The impression fatigue can show a steady state depth increase per cycle after a transient period under a constant maximum load, similar to the traditional fatigue test [91]. The reason why the indenter sinks further than static loading may be explained by using the analogy of crack propagation.

Indentation fatigue test was carried out on stainless steel [92] by using a cyclic loading of sinusoidal wave form, brittle materials like soda lime glass [93, 94, 95, 96], silicon carbide and silicon nitride [97]. Impression fatigue test was done on a high purity \( \beta \)-tin crystal [98] which shows that fatigue loading enhances the propagation of plastic zone compared to static loading.
The indenter produced a plastic zone which propagated into the material without a crack. Even though there is no crack, impression fatigue has many of the characteristics of fatigue crack propagation, such as overloading and underloading effects as well as delayed retardation. Thus there exists the similarity between crack propagation and indentation fatigue.

5.2 Mathematical formulation of the problem

Consider an isotropic, homogeneous elastic-plastic half space, bounded by the plane \( z=0 \). A rigid flat end cylindrical indenter of radius \( a \) is used to apply dynamic loading with loading cycle as shown in fig 5.1. The displacement \( D \) of the indenter into the elastic-plastic half space varies with the applied load as shown in fig 5.2. A cylindrical coordinate system \((r, \theta, z)\) is considered. Because of the axial symmetry of the problem (no \( \theta \) dependence), the displacement field

\[
\hat{u}(r, z) = u_r(r, z) \hat{r} + u_z(r, z) \hat{z}
\]

has components that are nonzero only in the radial and vertical directions characterized by unit vectors \( \hat{r} \) and \( \hat{z} \).

It is assumed that the surfaces of contact between the indenter and the elastic-plastic half space are frictionless. The boundary conditions at the contact and free surfaces are

\[
\sigma_{rz}(r, 0) = 0 \text{ for } 0 \leq r < \infty \quad (5.2)
\]

\[
u_z(r, 0) = D \text{ for } 0 \leq r \leq a \quad (5.3)
\]

\[
\sigma_{zz}(r, 0) = 0 \text{ for } a < r < \infty \quad (5.4)
\]

\[
u_r(R, z) = 0, \quad \sigma_{rz}(R, z) = 0 \quad (5.5)
\]

\[
2\pi \left[ \int_0^a \sigma_{zz}(r, z) r dr + a \int_0^a \sigma_{rz}(a, z) dz \right] = F \quad (5.6)
\]
If there is no body force acting on the elastic-plastic half space, then the equations of equilibrium at each point of the half space is given by

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} = \rho \ddot{u}
\]  
(5.7)

\[
\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} = \rho \ddot{v}
\]  
(5.8)

where \( \rho \) is the density of the material.

The stress-strain relations for the elastic-plastic material is given as follows

\[
d\varepsilon_{ij} = \frac{1 + \nu}{E} d\sigma'_ij + \frac{1 + 2\nu}{E} \frac{d\sigma_{kk}}{2} \delta_{ij} + \frac{3}{2} \frac{dE}{\sigma} \sigma'_{ij}
\]  
(5.9)

where \( \sigma'_{ij} \) is the deviatoric stress.

### 5.3 Finite element analysis

Finite element simulation of impression fatigue test is presented in this chapter to study the effects of various factors on the propagation of the plastic zone, including the amplitude, number of cycles and the median load. The finite element mesh is similar to the one used for the simulation of plastic nanoindentation as shown in fig 5.2. Initially a static loading is applied followed by a dynamic analysis of sinusoidal wave load using a periodic loading command. The load vs. time curve is shown in fig 5.1.

Three basic factors are necessary to cause fatigue failure. These are (1) maximum tensile stress of sufficiently high value, (2) a large enough variation or fluctuation in the applied stress, and (3) a sufficiently large number of cycles of applied stress. In addition, there are many other variables, such as stress concentration, corrosion, temperature, overload, metallurgical structure, residual stresses, and combined stresses which tend to alter the conditions for fatigue. Here we
consider only the effect of the three parameters mentioned above on the growth of fatigue crack which is important in engineering design consideration.

We here simulate the impression fatigue test by varying the parameters like number of cycles, maximum tensile stress and median load. A sample sinusoidal loading curve is shown in fig 5.1.

5.3.1 Load amplitude changes

Considering constant number of cycles, and the median load, the amplitude of the loading curve is changed. Figure 5.3 shows the indentation depth as a function of time. The indenter continuously penetrates into the material. The slope of the indentation depth vs. time curve increases with the increasing amplitude. Thus the penetration rate increases with the increase in the amplitude of the loading. The effect of the amplitude of the load on the penetration rate is shown in fig 5.4.

5.3.2 Median load changes

Considering constant number of cycles with $\omega = 60$ radians/sec, the median load is changed from 0.025, 0.035, 0.045, 0.5, 0.055 mN. This is also called a fluctuating stress cycle which is considered to be made up of two components, a mean, or a steady, stress $\sigma_m$, and an alternating or variable, stress. The mean stress is the algebraic mean of the maximum and the minimum stress in the cycle.

$$\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}$$  \hspace{1cm} (5.10)

The slope increases gradually with increase in the median load, thus the plastic zone propagation increases with the increase in the median load as shown in fig 5.5. This means that the penetration rate becomes greater with increase in the mean stress.
5.3.3 Change in frequency

Considering constant amplitude, and median load, the frequency of the loading cycles is increased. The different frequencies used for the loading curves are 60, 100, 150, 200 and 250 radians per second. The plastic zone propagation rate decreases as the frequency of the loading cycle increases as shown in fig.5.6

5.3.4 Von Mises stress distribution

Let $\sigma_1$, $\sigma_2$, $\sigma_3$ denote the principal values of stress, the Von Mises stress is defined as

$$\frac{\sqrt{2}}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2}$$

(5.11)

The variation of von Mises stress at a depth of 0.04 $\mu$m in the $z$ direction is shown in the fig 5.7; it varies according to the application of the load, in a sinusoidal form, increasing to a maximum value of the yield stress of the material at maximum load and decreases to a minimum value at the minimum load. The von Mises stress along the $z$-axis and contours of the von Mises stress are shown in fig 5.8 and 5.9 respectively. The von Mises stress seems to reach the yield stress value of the material at maximum load. At minimum load, the maximum stress value which is equivalent to the yield stress of the material is present near the edge of the indenter which is a characteristic of fatigue that a fatigue usually occurs at a point of stress concentration such as sharp corner. The stress regains the minimum value during unloading due to the elastic recovery of the material.

5.3.5 Surface morphology of the elastic plastic half space

The surface morphology of the material at maximum and minimum load is shown in fig. 5.10 and 5.11 respectively. Both the plots show pile up of the material along the surface of the elastic
plastic half space. The displacement vector on the elastic-plastic half space is illustrated in fig 5.12 and 5.13 at maximum and minimum loads respectively.

5.3.6 Effect of Young’s modulus

With the increase in the Young’s Modulus of the material the slope decreases that is the plastic zone propagation rate decreases as shown in fig 5.14

5.4 Conclusions and Results

Finite element simulation of indentation under a dynamic loading is studied on an elastic plastic half space by using a flat end cylindrical indenter.

1. The plastic zone propagation rate increases with the increase in the amplitude of the and with the increase in the mean load.

2. With the increase in the frequency, the plastic zone propagation decreases.

3. The von Mises stress varies according to the load applied; it reaches the yield stress value of the material at maximum load. Larger plastic zone is created at maximum load.

4. The surface morphology of the elastic-plastic half space shows pile up of the material at the maximum and the minimum loads.
### Table 5.1 Material properties of an elastic-plastic half space

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus E (GPa)</th>
<th>Poisson’s Ratio ( \nu )</th>
<th>Yield Strength (GPa)</th>
<th>Strain Hardening Coefficient ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aluminum</td>
<td>72</td>
<td>0.3</td>
<td>0.056</td>
</tr>
</tbody>
</table>
Figure 5.1 Sample Loading Curve
Figure 5.2 Finite Element Mesh for Impression fatigue

Figure 5.3 Displacement vs. time curve

ω = 60 rad/s, amplitude = 0.035, median load = 0.04

Load

Displacement (10\(^{-3}\)) μm

Time sec
\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{amplitude_slope_graph.png}
\caption{Slope at varying amplitude}
\end{figure}
\end{center}

\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{mean_load_slope_graph.png}
\caption{Slope at varying Mean Load}
\end{figure}
\end{center}

$a=0.4 \, \mu$m, $E=72$ GPa, $\omega=60$ rad/s, mean load = 0.045 mN
Figure 5.6 Slope at varying frequency

Figure 5.7 Von Mises stress at z=0.04 micro meters
Figure 5.8 Von Mises stress distribution along the z axis

\[
\frac{\pi a \sigma (1-\nu^2)E}{2D} = 0.035, \quad \text{mean load} = 0.03
\]

\[
\begin{align*}
a &= 0.4 \ \mu m, \ E &= 72 \ \text{GPa}, \ \nu &= 0.3 \\
\omega &= 60 \ \text{rad/s}, \ \text{amplitude} = 0.035, \\
D &= 0.00127 \ \mu m, \ t &= 0.0714 \ s \\
D &= 0.00237 \ \mu m, \ t &= 0.0309 \ s
\end{align*}
\]
Figure 5. 9 Von Mises stress distribution ($\sigma^{Mises}/\sigma_y$) at maximum load at step time = 1.131 and minimum load at step time = 1.183 respectively.
Figure 5. 10 Surface morphology of the surface of the half space at maximum load at step time=1.655 and load =0.00237

Figure 5. 11 Surface morphology of the surface of the half space at minimum load at step time =1.602 and force=0.00127
Figure 5. 12 Displacement vector at maximum load at step time = 1.655

Figure 5. 13 Displacement vector at minimum load at step time = 1.602
Figure 5. Effect of Young’s Modulus on Slope

$E/\sigma_y = 0.056$ GPa

Slope $= 10^{-4}$ µm s$^{-1}$
Chapter 6: CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

Finite Element simulation of indentation on a half space with a hole is studied for elastic and elastic-perfect plastic half space in this thesis. Finite element simulation of indentation under cyclic loading is also considered to study the propagation of plastic zone.

(1) Elastic indentation

Finite element simulation of indentation using a flat ended cylindrical indenter on an elastic half space with a hole is done to study the stress distribution and identify the effect of the hole on the load-displacement relationship. The simulation results are as follows:

- The force required to penetrate the indenter is proportional to the displacement of the indenter into the elastic half space and it decreases as the size of the hole increases.
- Force applied decreases initially and then increases as the Poisson’s ratio increases.
- Stress concentration is observed at the edge of the indenter.
- The surface morphology along the depth of the hole shows surface buckling of the elastic half space in compression.
- The surface morphology along the surface of the elastic half space shows sink in of the elastic half space when indented with a flat edge cylindrical indenter.

(2) Plastic indentation

Finite Element simulation of indentation on an elastic-perfect plastic half space with a hole is done to study the distribution of the von Mises stress. The following is the summary of the simulation.
• Higher force is required for the penetration of the indenter into the half space for the constrained case when compared to the unconstrained case.

• The stiffness of the loading and the unloading curves and the energy dissipated during the loading-unloading process of nanoindentation decreases with the increase in the size of the hole. The decrease is more rapid for the unconstrained case when compared to the constrained case.

• The force required increases with the increase in the ratio of Young’s Modulus to the yield stress.

• The von Mises plastic zone is larger for the constrained case than the unconstrained case

• The surface morphology along the depth of the hole shows buckling of the half space in compression.

• The surface morphology along the surface of the half space shows significant pile up of the material of the elastic-perfect plastic half space during the indentation process.

(3) Indentation under cyclic Loading

Finite element simulation of impression fatigue is done to study the propagation of plastic zone under sinusoidal loading by varying different parameters.

• The plastic zone propagation rate increases with the increase in the amplitude of the load and with the increase in the mean load of the dynamic load.

• With the increase in the frequency of the dynamic load, the propagation rate of the plastic zone decreases.

• The von Mises stress varies according to the load applied; it reaches the yield stress value of the material at maximum load
• The surface morphology of the elastic-plastic half space shows pile up of the material at the maximum and the minimum loads.

6.2 Future Work

In light of the simulation results more work could be done to extend and support the current study.

1. Finite element simulation for elastic-plastic material model with work hardening could be done for the half space with a hole in it to investigate the effect of work hardening on the indentation behavior.

2. Indentation experiments could be done on a material with a hole in it, to provide experimental verification for the elastic-plastic model

3. Impression fatigue could be done which can be used to support the numerical simulation.
APPENDIX

Finite Element Simulation of Equal Channel Angular Extrusion

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ABSTRACT

The mechanical response of elastoplastic billet in equal channel angular extrusion was simulated by using nonlinear finite element method. The plastic deformation of the billet was found to be a function of the process parameters, including the bent angle and channel width. Unstable extrusion was observed if the width of the outlet channel is larger than the entrance one. The simulation of multi-bent extrusion was highlighted, which shows the tremendous increase of plastic deformation.

Keywords: plastic deformation, ECAE, simulation, instability.

November 24, 2003
1. Introduction

Equal Channel Angular Extrusion (ECAE) was first developed by Segal et al [99, 100] as an approach to introduce severe plastic deformation in bulk materials. In ECAE, a rod-shape sample is extruded repetitively through a die with equal channels to produce intense plastic strains by simple shear, without changing the geometry of the sample. This procedure is capable of forming large fully-dense materials of ultra fine grain size in the submicrometer range [101,102,103]. Currently, the ECAE process has been used to process different materials including Al, Cu, Mg and Ni alloys.

Analytical approaches have been used to explain the deformation behavior of materials in the ECAE process [104,105]. However, due to the complexity of the ECAE process, numerical modeling becomes one of important approaches to understand the fundamental mechanisms for the formation of submicron grain structures and the deformation occurring in the ECAP process. Many FEM-based (finite element method) analyses have been performed to determine the deformation behavior of materials and estimate the developed strain in the ECAE process [106,107, 108, 109, 110, 111,112]. Prangnell et al [106] analyzed the effect of friction on material flow and found that the inhomogeneous deformation at billet ends increases with friction. Semiatin et al [107] has shown the influence of strain and strain rate hardening on the breadth of the shear zone and the uniformity of flow under isothermal conditions. Srinivasan [108] studied the effect of the bent angle between the channels on the deformation of billets for each pass. Using finite element simulation, Bowen et al [109] found that strain achieved in the billet is sensitive to the die angle, friction conditions, and the application of a back pressure. Rosochowski and Olejnik [113] simulated 2-bent ECAE process by using finite element method. Recently, Alkorta and Sevillano [114] addressed the effect of strain-hardening on the ECAE
process and found that the FEM solutions are in agreement with the upper-bound analysis. Baik et al [115], based on a dislocation model used the finite element method to simulate the behavior of the material under ECAE, from which the development of dislocation density and cell size was obtained. However, most of the work only focused on the development of plastic deformation in single pass. There has no report on the deformation of materials subjected to multi-bent ECAE process and the effect of die size, which play important role in determining the evolution of microstructure and plastic strain in ECAE processed materials.

The purpose of the present paper is to examine in detail the deformation behavior of elastoplastic materials processed by the ECAE. The effect of the process variables on the development of the plastic strain and the force required for the extrusion in single pass is discussed. The finite element method is used because it can provide us direct information on the evolution of plastic deformation during the ECAE and enable us to simulate the deformation of materials subjected to multi-bent ECAE. In the simulation, the following assumptions are made; (1) the material is isotropic and homogeneous; (2) the materials is elasto-perfectly plastic; (3) the system is isothermal; (4) the von-Mises flow rule is used to construct the constitutive relation for the simulation; and (4) there is no friction between the surface of the material and the die wall.

2. Formulation of Equal Channel Angular Extrusion

It is known that the extruded billet experiences large plastic deformation in the ECAE process. The model based on continuum mechanics needs to allow for arbitrary finite strains. The elastoplastic behavior of the billet is described by using a generalization of $J_2$ flow theory to finite strain [116]. In the current configuration $x_j$, the displacement components and the metric tensors are denoted $u_i$ and $G_{ij}$, respectively. In the reference coordinate system $x'$, the
displacement components and metric tensors are denoted $u^i$ and $g_{ij}$ respectively. The determinants of $G_{ij}$ and $g_{ij}$ are $G$ and $g$ respectively. Then the Lagrangian strain tensor $\eta_{ij}$ is

$$\eta_{ij} = (u_{i,j} + u_{j,i})/2 + (u^k u_{k,j})/2$$  \hspace{1cm} (1)

where $( )_{,j}$ denotes the covariant derivatives in the reference frame. The contravariant components of the Kirchhoff stress $\tau^{ij}$ and the Cauchy stress tensor $\sigma^{ij}$ on the current base vectors are related by

$$\tau^{ij} = \sqrt{G/g} \sigma^{ij}$$  \hspace{1cm} (2)

Using a generalized $J_2$ flow theory [116], the tensor of instantaneous moduli $L^{ijkl}$ relating the stress and strain increments in the constitutive law, $\tau^{ij} = L^{ijkl} \dot{\eta}_{kl}$, is given by

$$L^{ijkl} = \frac{E}{1 + \nu} \left[ \frac{1}{2} (G^{ik} G^{jl} + G^{il} G^{jk}) + \frac{\nu}{1 - 2\nu} G^{ij} G^{kl} - \alpha \frac{3}{2} \frac{E / E_t - 1}{E / E_t - (1 - 2\nu) / 3} \frac{\sigma^2}{\sigma^2} \right]$$

$$- \frac{1}{2} [G^{ik} \tau^{jl} + G^{jk} \tau^{il} + G^{il} \tau^{jk} + G^{jl} \tau^{ik}]$$  \hspace{1cm} (3)

and

$$\alpha = 1 \quad \text{if } \dot{\sigma}_e \geq 0 \text{ and } \sigma_e = (\sigma_e)_{\text{max}}$$

$$\alpha = 0 \quad \text{if } \dot{\sigma}_e < 0 \text{ and } \sigma_e < (\sigma_e)_{\text{max}}$$

Here $E$ and $\nu$ are Young’s modulus and Poisson’s ratio respectively, $\sigma_e = (3s_i s^i / 2)^{1/2}$ is the effective Mises stress, and $s^j = \tau^{ij} - G^{ij} G_{kl} \tau^{kl} / 3$ is the stress deviator. The tangent modulus $E_t$ is the slope of the uniaxial true stress vs logarithmic strain curve at the stress level $(\sigma_e)_{\text{max}}$. The curve is represented by a piecewise power law.
\[
\varepsilon = \begin{cases} 
\frac{\sigma}{E} & \text{for } \sigma \leq \sigma_y \\
\sigma_y \left( \frac{\sigma}{\sigma_y} \right)^n & \text{for } \sigma > \sigma_y
\end{cases}
\]  

(4)

where \( n \) is the strain hardening exponent and \( \sigma_y \) is the initial yield stress. The equations of equilibrium are defined in terms of the principle of virtual work, which gives

\[
\int_V \left\{ \tau^j \delta \eta_{ij} + \tau^{ij} \delta \hat{u}_j (u_{k,j}) \right\} dV = \int_S \overline{T}^i \delta u_i dS - \left[ \int_V \tau^{ij} \delta \eta_{ij} dV - \int_S T^i \delta u_i dS \right]
\]

(5)

where \( S \) and \( V \) denote the surface and the volume, respectively, in the reference state, and \( T^i \) are the contravariant components of the nominal surface tractions. The bracketed terms are included to prevent drifting away from the true equilibrium path.

Boundary conditions need to be defined to completely determine the deformation behavior of the billet. Figure 1 shows the general principle of the ECAE process. The surface of the billet can be divided into 6 segments \( (S_{AB}, S_{BC}, S_{CD}, S_{DE}, S_{EF}, \text{ and } S_{FA}) \), depending on the extrusion process. In the analysis, we only consider plane-strain problem and assume the contact between the billet and the surface of die and punch is frictionless. For the segments \( S_{BC} \) and \( S_{EF} \), there are

\[
u_2 = u_0, \quad \sigma^{12} = 0, \quad \text{and} \quad F = \int_{S_{bc}} \sigma^{32} dS \quad \text{on the segment } S_{BC}
\]

(6)

\[
\sigma^{ij} n_i n_j = 0 \quad \text{on the segment } S_{EF}
\]

(7)

where \( n_i \) are the components of the unit normal vector of the segment \( S_{EF} \). The boundary conditions for the segments \( S_{AB}, S_{CD}, S_{DE}, \text{ and } S_{FA} \) involves the inequality equations, which can be expressed as

\[
u_i \geq 0 \quad \text{and} \quad \sigma^{21} = 0 \quad \text{on the segment } S_{AB}
\]

(8)
\[ u_i \leq 0 \text{ and } \sigma^{21} = 0 \quad \text{on the segment } S_{CD} \quad (9) \]
\[ u_i n_i' \leq 0 \text{ and } \sigma^d n_i n_j = 0 \quad (i \neq j) \quad \text{on the segment } S_{DE} \quad (10) \]
\[ u_i n_i' \geq 0 \text{ and } \sigma^d n_i n_j = 0 \quad (i \neq j) \quad \text{on the segment } S_{FA} \quad (11) \]

where \( n_i' = n_i \). The inequality indicates that the billet cannot penetrate onto the surface of the die, while separation of the billet from the die can occur as we observe in the following FEM simulation.

3. Finite Element Modeling and Computation

Our simulation of the equal channel angular extrusion used the large elastoplastic feature of the ABAQUS finite element code [65]. To avoid possible convergence problems arising from the sharp corner of the die structure, fillets of different radius were used at the intersection AD as shown in Fig. 1. In most of the simulation, we used 3 cm as the width of the channels (t). To simplify the analysis and increase the computational efficiency, a two-dimensional model was used instead of simulating 3-dimensional deformation. The finite element mesh, shown in Fig. 2, consisted of four-node quadrilateral elements CPE4R. Along the interface, the contact elements were used, which automatically monitor the changes of contact zone during the extrusion. To simulate the extrusion process, the billet was pushed down by giving a desired displacement to the nodes on the top surface, \( S_{BC} \). The force applied to each node was recorded as a function of the extrusion length, from which the total force required to push the billet was calculated. The simulation was performed under isothermal condition, and heat generation in the billet due to plastic deformation was not considered here.

The material used in the simulation is aluminum. Table 1 lists the mechanical properties of Al. As shown in the table, we only consider elasto-perfectly plastic deformation.
4. Results and Discussions

**Evolution of overall deformation behavior**

Before assessing the effect of the process variables on deformation of the billet in the ECAE, the overall deformation behavior of the billet during the extrusion is discussed here. The evolution of the deformation in the billet is depicted in Fig. 3, in which the values of the plastic strain ($\varepsilon_{p}^{\text{pl}}$) are displayed. As the billet is pushed through the ECAE die, it starts contacting with the die at the leading edge $EF$. This causes local deformation around $F$, as shown in Fig. 3a. Further pushing the billet leads to the growth of the deformation and the formation of a uniform deformation band connecting the surface $CDE$ and $FAB$. This is a critical step for the ECAE process, which starts introducing uniform shear deformation in the billet. The deformation band grows with the further push and reaches steady state as shown in Fig. 3b, which remained the same during the extrusion. Continuous extrusion causes the propagation of the uniform deformation band over the billet and creates the uniform shear deformation in the billet as observed in Fig. 3c. However, there exists dead zone around the corner $E$, where the material is at stress-free state after only one-pass. Non-uniform deformation zone is observed around the corner $F$, in which the billet experiences largest plastic deformation.

Figure 4 shows the force required to push the billet through the die, having the bent angle of 90° and fillet radius of 0.20 cm. The force first increases with the extrusion, reaches the maximum, then decreases with the extrusion, and finally remains constant during the ECAE process. The increase of the force is related to the growth of deformation zone, and the maximum force corresponds to the formation of the largest deformation band connecting the surface $CDE$ and $FAB$. Continuously extruding the billet, the size of the deformation band remains the same and it requires less force, suggesting the occurrence of the recovery of plastic deformation.
The variation of the plastic strain in the billet is shown in Fig. 5. Both the largest plastic strain in the billet and the uniform plastic strain in the deformation band increase with the extrusion and reach constant values of 1.28 and 0.91, respectively. The constant values of the largest and uniform plastic strain are defined as \( \varepsilon_{\text{max}}^0 \) and \( \varepsilon_{\text{uniform}}^0 \). It is interested in observing the increase of both the strains after the external force reaches the maximum load. The deformation in the billet involves the recovery of plastic deformation and the propagation of deformation zone caused by the interaction between the punch and the billet and the contact between the surface of the billet and that of the die. The difference between the uniform plastic strain and largest plastic strain indicates the inhomogeneous deformation around the leading edge, which is consistent with our experimental results for the indentation of the ECAE processed Al [117]. It should be pointed out that the development of such large plastic strain in the billet requires the change of microstructure, such as the formation of dislocation cells.

**Effect of fillet radius**

To examine the effect of the fillet on the plastic deformation of the billet processed in the ECAE process, several fillet radii \( r/t : 0.05 - 0.117, \ r \) (the radius of the fillet) are used for the die structure having the bent angle of 90°. Figure 6 shows the dependence of the maximum force \( F_{\text{max}} \) required in the extrusion of the billet on the fillet radius. The \( F_{\text{max}}/4Et \ (10^{-4}) \) decreases from 2.40 at \( r/t = 0.05 \) to 2.28 at \( r/t = 0.117 \). Smaller fillet radius requires larger force to form the uniform deformation band.

The evolution of the plastic deformation in the billet as a function of the fillet radius is depicted in Fig. 7. Both the largest and uniform plastic strains decrease with the increase of the fillet radius. Less deformation is introduced in the billet for larger fillet radius and less force is required to extrude materials, which is in agreement with the results as shown in Fig. 6.
$r/t \leq 0.08$, the fillet radius has little effect on the development of the plastic deformation in the billet. One interesting observation is that the size of the non-uniform deformation increases with the fillet radius, suggesting that more plastic recovery occurs due to less confinement on the deformation of the billet for die structure with larger fillet radius.

**Effect of bent angle**

It is known that the development of plastic deformation in the billet depends on the geometric configuration of the die. To examine the effect of the bent angle on the formation of the uniform deformation band, different die structure having the fillet radius of 0.2 cm and bent angle of $90^\circ$ to $130^\circ$ are used in the simulation. Figure 8 shows the dependence of the maximum force on the bent angle. The $F_{\text{max}} / 4Et \times 10^{-4}$ decreases from 2.37 at $90^\circ$ to 0.93 at $130^\circ$. Larger bent angle imposes less confinement on the extrusion of the billet, which requires less force to push the billet through the die.

Figure 9 shows the effect of the bent angle on both the maximum and uniform plastic strains developed in the billet. Both the largest and uniform plastic strain decreases with the increase of the bent angle. For larger bent angle, there is less confinement on the deformation of the billet and less plastic deformation is created in the billet. The billet experiences more uniform shearing, and large uniform deformation zone is formed. For comparison, the approximate expression obtained by Segal [99],

$$
\varepsilon_{\text{pl}} = \frac{2}{\sqrt{3}} \cot \phi
$$

(12)

is also plotted in the Fig. 9. The plastic strain as expressed in Eq. (12) is larger that the numerical results, suggesting that Eq. (2) overestimates the plastic deformation created in the billet. However,
the difference between the numerical results and Eq. (12) over 90° to 130° remains the same, which suggests that Eq. (12) might still be used to estimate the plastic deformation developed in the billet with the correction given in Fig. 9.

**Effect of die width**

To utilize the ECAE process for the fabrication of large severe deformed bulk materials, especially in industry, the effect of the die width on the deformation behavior of the billet was simulated. Different width of the die ranging from 2 cm to 7.5 cm is used. No significant change in the plastic strain was observed. Uniform deformation band was formed, similar to Fig. 3b. The plastic deformation developed in the billet is similar in all the cases. However, the size of the deformation zone increases with the width of the die, while the stress-free zone at the leading edge also increases.

**Effect of unequal channel width**

In analyzing the effect of the die width on the deformation behavior of the billet, the width of the entrance channel and that of the outlet channel was kept the same. One may vary the ratio of the entrance channel width to the outlet channel width in order to create large plastic deformation and the change of microstructure in the billet.

To examine the effect of the ratio of the entrance channel width to the outlet channel width on the plastic deformation in billet, the width of the entrance channel is kept to be 3 cm, while the different width of the outlet channel ranging from 2 cm to 3 cm is used. Figure 10 shows the dependence of the maximum force in the ECAE extrusion on the ratio (≤ 1). The force required to develop the uniform deformation band increases with the decrease of the ratio. Smaller width of the outlet channel confines the motion of the billet in the die, which creates more deformation in the billet along the direction of the outlet channel.
The effect of the ratio (≤1) on the largest plastic strain developed in the billet is depicted in Fig. 11. Consistent with the increase of the pushing force, larger plastic deformation is introduced for smaller width of the outlet channel. For same amount of material being extruded through the ECAE die, the distance over the uniform deformation band decreases with the decrease of the ratio. This leads to the increase of the plastic deformation in the billet and the shrinkage of the inhomogeneous deformation zone. More uniform plastic deformation is created.

In simulating the deformation behavior of the billet extruded through two unequal channels, surface distortion of the billet is observed for the larger outlet channel, as shown in Fig. 12. Different from the extrusion through the die with the same size of the entrance and outlet channels, there is less confinement for the deformation of the billet in the outlet channel. The internal stress created in the ECAE causes the elastic recovery in the billet and induces surface instability to reduce the system energy. As shown in Fig. 12, the width of the billet becomes a little larger and a worm-like shape is formed. The final extruded billet no longer has the same dimensions as that of the original metal piece. It should be pointed out that, under this condition the billet experiences non-uniform plastic deformation.

**Effect of multi-bent extrusion**

To simulate the multi-pass extrusion process, the multi-bent extrusion as shown in Fig. 13 is simulated. A long billet of length equal to the length of the die is forced into the die, which consists of 5-bents. The angle at each jointing is 90° and the radius of the fillet is 0.2 cm. As shown in Fig. 13, the billet experiences shear deformation after passing through each bent. Severe plastic deformation is created in the billet. The billet experiences shear deformation in one direction after passing through the first bent, and then in the other direction when passing through the next bent, regaining their original shape. Such multi-bent extrusion is similar to the
multi-pass ECAE process and can be used to illustrate the deformation behavior of the billet subjected to multi-pass extrusion.

Table 2 lists the constant value of the uniform plastic strain in the billet at each channel. The uniform plastic strain increases tremendously from a value of 0.93 after the first bent to 5.34 after the fifth bent. Such high plastic deformation requires the change of microstructure in the billet, which may involve the formation of dislocation cell structure and refinement of grain size.

5. Conclusions

The deformation behavior of elasto-perfectly plastic materials subjected to the ECAE process was simulated by using finite element method. The effect of the process variables on the evolution of plastic deformation in the billet is summarized as below.

(1) A deformation band in the billet was created, which was cross over the sharp corners at the joint. It is such a deformation band that leads to the formation of uniform deformation zone in the ECAE deformed material.

(2) Non-uniform deformation zone was formed around the leading edge, in which the billet experienced largest plastic deformation. Both the largest and uniform plastic strains reached constant values, respectively, after certain length of the billet was extruded.

(3) Both the largest and uniform plastic strains increased with the decrease of the radius of the fillet, and the force required to push the billet through the die decreased with the increase in the radius of the fillet.

(4) Both the largest and uniform plastic strains increased with the decrease of the bent angle at the joint of two channels. Larger uniform plastic deformation zone was created for smaller bent angle.
(5) Uniform plastic deformation can only be formed in the billet when the size of the outlet channel is equal to or less than that of the entrance channel. For the size of the outlet channel being larger than that of the entrance channel, worm-like surface distortion was created in the billet, leading to the surface instability.

(6) From the viewpoint of the continuum mechanics, the size of the ECAE die does not show any significant effect on the evolution of the uniform plastic strain as long as both the entrance channel and the outlet channel are same.

(7) Multi-bent extrusion introduces much severe plastic deformation in the billet, comparing to single-bent test. It is anticipated that such high plastic deformation require the change of microstructure in the billet, which may involve the formation of dislocation cell structure and refinement of grain size.

Acknowledgement

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Table 1. Material Properties of Aluminum

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<tr>
<th>Material</th>
<th>Young’s Modulus</th>
<th>Poisson’s Ratio</th>
<th>Yield Strength</th>
<th>Straining Hardening</th>
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<td>Al</td>
<td>72</td>
<td>0.3</td>
<td>0.056</td>
<td>0</td>
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</table>

Table 2. Dependence of the uniform plastic strain on the No. of bents

<table>
<thead>
<tr>
<th>No. of bents</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{uniform}$</td>
<td>0.93</td>
<td>2.67</td>
<td>3.23</td>
<td>4.80</td>
<td>5.34</td>
</tr>
</tbody>
</table>
Figure Captions:

1. Schematic diagram of the ECAE process
2. Finite element mesh for the extrusion of a billet
3. Evolution of the equivalent plastic strain in the billet; (a) formation of local deformation, (b) formation of the deformation band, and (c) propagation of the deformation zone.
4. Change of force with the extrusion length for the bent angle of 90° and fillet radius of 0.20 cm (δ: the extrusion length)
5. Variation of plastic strain with the extrusion length for the bent angle of 90° and fillet radius of 0.20 cm
6. Effect of the fillet radius on the extrusion force (bent angle: 90°)
7. Effect of the fillet radius on the development of the plastic strain (bent angle: 90°)
8. Effect of the bent angle on the extrusion force (fillet radius: 0.2 cm)
9. Effect of the bent angle on the development of the plastic strain (fillet radius: 0.2 cm)
10. Effect of the ratio of the width of the entrance channel to the width of the outlet channel on the pushing force (fillet radius: 0.2 cm)
11. Effect of the ratio of the width of the entrance channel to the width of the outlet channel on the maximum plastic strain in the billet (fillet radius: 0.2 cm)
12. Deformed shape the billet extruded though the die with larger width of the outlet channel
13. Multi-bent extrusion of a billet
Figure 1. Schematic diagram of the ECAE process

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Figure 5. Variation of plastic strain with the extrusion length for the bent angle of 90° and fillet radius of 0.20 cm.

Figure 6. Effect of the fillet radius on the extrusion force (bent angle: 90°).
Figure 7. Effect of the fillet radius on the development of the plastic strain (bent angle: 90°)

Figure 8. Effect of the bent angle on the extrusion force (fillet radius: 0.2 cm)
Figure 9. Effect of the bent angle on the development of the plastic strain (fillet radius: 0.2 cm)

Figure 10. Effect of the ratio of the width of the entrance channel to the width of the outlet channel on the pushing force (fillet radius: 0.2 cm)
Figure 11. Effect of the ratio of the width of the entrance channel to the width of the outlet channel on the maximum plastic strain in the billet (fillet radius: 0.2 cm)

Figure 12. Deformed shape the billet extruded though the die with larger width of the outlet channel
Figure 13. Multi-bent extrusion of a billet
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