EXERGY METRIC FOR THE ASSESSMENT OF MATERIAL PROCESSING IN MANUFACTURING

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ABSTRACT OF THESIS

EXERGY METRIC FOR THE ASSESSMENT OF MATERIAL PROCESSING IN MANUFACTURING

Exergy utilization calculations have been in the past repeatedly used to quantify the quality and quantity of energy used in thermal energy processes. This thesis is an attempt to derive a common language – exergy utilization and compare for the first time two entirely different manufacturing processes, namely material processing by a mechanical method of straining of the material and simple heating of the same mass of the material using exergy utilization as a metric. The exergy utilization of material processing is determined by performed work and utilized heat transfer using 1) Ramberg-Osgood equation and 2) Lumped heat capacitance method. A comparison of these two methods is presented.

KEYWORDS: Manufacturing, Materials processing, Exergy utilization, Ramberg-Osgood equation, Lumped capacitance method

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EXERGY METRIC FOR THE ASSESSMENT OF MATERIAL PROCESSING IN MANUFACTURING

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September, 2006
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THESIS

Venkata-Sandeep Boddapati

The Graduate School
University of Kentucky
2006
EXERGY METRIC FOR THE ASSESSMENT OF MATERIAL PROCESSING IN MANUFACTURING

THESIS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Manufacturing Systems Engineering at the University of Kentucky
By
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Lexington, Kentucky
Director: Dr. Dusan P. Sekulic, Professor
Lexington, Kentucky
2006
DEDICATION

Dedicated to my beloved parents and my
dear brother
ACKNOWLEDGEMENTS

At the outset, I express my sincere and heartfelt gratitude to my advisor Dr. Dusan P. Sekulic, Professor, University of Kentucky, for his excellent guidance and profuse assistance in the course of completing my thesis. It is an outstanding experience to do such a thesis under his guidance. His support has been invaluable along with the timely suggestions have expanded my mental horizons to a new dawn. I would like to take this opportunity to express my earnest and profound thanks to Dr. I.S. Jawahir, Professor & Director of Graduate studies, CRMS, University of Kentucky for supporting me throughout the thesis. I would like to take this opportunity to thank Mr. Fadi Abu-Farha who has helped me in the conduction of experiments and also for guiding me at all time when I needed help. Also, I would like to thank Mr. Oscar Dillon for his help initially in my work and also for helping me find the right resources when I needed the most. I would also like to thank my dear friends, Mr. Chetan Babu Velkur, Mr. Anand Palki and Mr. Adithya Candadai for their help and support in programming. Also, I would like to take great pleasure in thanking my dear friend Ms. Rhonda G. Mckenzie for her continuous support and help in completion of my thesis. Finally, I would take immense pleasure in thanking my mother, Ms. Sunanda Vinod Boddapati, and my father, Mr. Vinod Kumar Boddapati, for their continuous support and encouragement and because of whom, I am the person, I am today.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS .................................................................................................................. IX

LIST OF TABLES .......................................................................................................................... XII

LIST OF FIGURES ...................................................................................................................... XIII

LIST OF FILES ............................................................................................................................ XV

1. INTRODUCTION ...................................................................................................................... 1
   1.1 Background ....................................................................................................................... 1
   1.2 Motivation ......................................................................................................................... 2
   1.3 Objective .......................................................................................................................... 3
   1.4 Thesis Layout .................................................................................................................... 4

2. LITERATURE REVIEW ............................................................................................................. 6
   2.1 Sustainability Overview ...................................................................................................... 6
   2.2 Concept of Exergy and Its Applications ............................................................................. 7

3. EXERGY UTILIZATION FOR MATERIALS PROCESSING OF AL & MG ALLOYS ...... 14
   3.1 Introduction ....................................................................................................................... 14
      3.1.1 Units Clarification ....................................................................................................... 17
      3.1.2 Initial Exergy of a Material ....................................................................................... 18
   3.2 Exergy Utilization Calculation (Method 1) ........................................................................ 21
      3.2.1 Experimental Determination of the True Stress-Strain Relationships ....................... 23
      3.2.2 Verification of the Validity of the Ramberg-Osgood Equation ..................................... 28
      3.2.3 Exergy Calculations (mechanical processing) .............................................................. 29
      3.2.3.2 Observations from the Tables and Graphs ............................................................... 34
   3.3. Analytical Method to Calculate Exergy Utilization (Method 2) ......................................... 35

4. EXERGY UTILIZATION FOR THERMAL PROCESSING FOR AL & MG ALLOYS ...... 38
List of Tables

1) Comparison of five sets of Al alloy data.........................................................30
2) Comparison of five sets of Mg alloy data.......................................................31
3) Total exergy change for Al alloy.................................................................35
4) Total exergy change for Mg alloy.................................................................35
List of Figures

1) Initial and final shape of the material before & after metal deformation………14
2) The stress-strain curve of a typical Al alloy........................................15
3) True stress-strain curve used in the analysis.......................................16
4) Universal tensile testing machine[UTM] MTS 810 material test system.........23
5) Representation of the work piece with dimensions used in the experiment……24
6) The work piece before (a) and after (b) the test..................................26
6a) Combined true stress-strain graphs for Al alloy.................................27
7) Combined true stress-strain graphs for Mg alloy.................................28
8) Graph comparing the theoretical and experimental data for Al alloy trail 1……29
9) Total and plastic exergy utilization for Al alloy....................................31
10) Total and plastic exergy utilization for Mg alloy..................................32
11) Comparison of total exergy utilization for the two alloys......................32
12) Comparison of elastic exergy utilization for the two alloys....................33
13) Comparison of plastic exergy utilization for the two alloys....................34
14) Difference in two methods for total exergy utilization for Al alloy............36
15) Difference in two methods for total exergy utilization for Mg alloy...........37
16) Figure representing a closed system heating.....................................40
17) Graph representing exergy utilization vs true strain for Al alloy..............49
18) Graph representing exergy utilization vs temperature raise for Al alloy........50
19) Graph representing exergy utilization vs temperature raise for Mg alloy........52
20) Graph representing exergy utilization vs true strain for Mg alloy............53
A.1) The area under the curve of interest ................................................................. 56
A.2) The area under the curve of interest ................................................................. 57
C.1) Al trail 1 true stress vs true strain ................................................................. 69
C.2) Al trail 2 true stress vs true strain ................................................................. 71
C.4) Al trail 4 true stress vs true strain ................................................................. 73
C.5) Al trail 5 true stress vs true strain ................................................................. 75
C.6) Mg trail 1 true stress vs true strain ................................................................. 77
C.7) Mg trail 2 true stress vs true strain ................................................................. 79
C.8) Mg trail 3 true stress vs true strain ................................................................. 81
C.9) Mg trail 4 true stress vs true strain ................................................................. 83
C.10) Mg trail 5 true stress vs true strain ............................................................... 85
H.1) Cooling of hot forging metal ........................................................................ 130
List of Files

1) Thesis.pdf (Acrobat Reader File) – 3.1 MB (File Size)
1. INTRODUCTION

1.1 Background

Production and consumption of goods is increasing more voluminous in manufacturing fields and will be even much faster in the future. What should be done to keep up with the increasing demand and customers wanting better quality products but with less pronounced impact on surroundings and with increased positive impact on society? There is a need to develop and maintain sustainable processes in every field and manufacturing is no different. The ever increasing demands and better quality creates a need to develop and continuously monitor and evaluate the appropriate metrics for assessment for manufacturing processes. One such metric for comparison is attempted to be developed in this thesis by the use of thermodynamics and it is based on the concept of exergy [1].

Exergy is the maximum extractable available energy from a system when it interacts with its surroundings. It carries the same units as work (energy), which is Joule (in SI system of units). However, exergy is not conserved like energy [2]. Every process in manufacturing is founded on the basic principles of conservation and transfer of energy between the involved systems, and thus deals also with transfer, use and destruction of exergy. So, the use of the exergy concept would be plausible as a metric to compare two processes. The concept of exergy is derived from the First and Second laws of Thermodynamics and with an emphasis on the Second Law of Thermodynamics [2].
The aim of this thesis work is to attempt a comparison between two entirely different materials processing activities relevant for manufacturing processes but using the same metric - exergy. The objective of the study is to see whether both can be evaluated and compared using a common metric. There has been not much work done in this particular field of study of systems involving materials processing for manufacturing. To the best of the author’s knowledge, there was not a common metric (exergy) that involves both quantity and quality of energy used to compare two entirely different processes. So, the main hypothesis of this work is that a common metric can be used to compare different materials processing with respect to resource (energy) utilization.

1.2 Motivation

There has been a large body of work about exergy utilization and system evaluation for energy systems such as large power plants, process industry and thermal engineering. However, in the field of sustainable engineering, in particular related to manufacturing such studies are very rare. One would like to explore a possibility of a use of common metric to evaluate diverse processes, such as purely thermal and dominantly mechanical.
1.3 Objective

The objective of this thesis is to provide a comparison of exergy utilization needed for performing a given task but for two entirely different processes of importance for manufacturing operations. The first being materials processing using mechanical deformation and the second being a thermal process of heating of the same unit of mass of the material. The main goals of this thesis are listed as follows.

- To define exergy potential of a given material
- To derive a theoretical expression for exergy utilization for straining of a material
- To calculate the theoretical exergy utilization for a given materials processing for metal forming
- To determine the exergy utilization based on experiments for a given metal forming materials processing
- Compare and discuss the results obtained using theoretical predictions and experimental measurements
- To determine the thermal exergy utilization for a heating of the work piece
- Compare and discuss the results obtained for selected two different processes (mechanical and thermal)
- Discuss a possible scope for future work in this field of study
1.4 Thesis Layout

The thesis starts with an introduction which sets the stage for the inquiry and explains the thesis content in brief, the background work done prior to this work, and provides the context review of the entire thesis. The second chapter contains the literature review. The review deals in detail with the work done prior to this particular effort, and summarizes work done to the related field of study. It provides a brief but in-depth coverage of all the sources used and materials consulted during the work on the thesis. This discussion is a basis for a well organized list of references and publications needed for future work in the field. Exergy has been selected as a common denominator, hence as a needed metric. It is hypothesized that this metric can also be used as an information carrier for subsequent economic calculations.

The third chapter narrows down the scope from a broad prospective and focuses on the primary area of interest—manufacturing. Two different types of materials are selected and the arguments are provided for this selection. The materials considered include Aluminum 3003 alloy and Magnesium AZ31 alloy. These are selected as representatives of most commonly used materials for lightweight constructions of interest in sustainable engineering. Furthermore, the theoretical derivation of the exergy utilization for a materials processing of straining a material of a certain mass and dimension is provided. The same quantities were determined experimentally by using a Universal Testing Machine [UTM]. A comparison of the deviations in theoretical and experimental studies is provided and the reasons for differences are discussed. The material is structured in such a way to extract only essential information and present it within the main body.
of the text of the chapter, with all the other relevant work, auxiliary discussions, and calculations given in a series of Appendices.

The fourth chapter contains an analysis of the exergy utilization of a thermal process of material heating. This analysis deals with the derivation and calculation of thermal exergy utilization for a material to be heated from a given state to a certain pre defined final temperature. This analysis provides the results for an assessment of the exergy utilization for thermal processing for same materials as used in the analysis of mechanical processes. The last chapter offers a comparison and discussion of the obtained results. This segment of work also gives an insight into the perceived future work. This text is accompanied with a number of appendices offering explicit derivations, calculations and also all the used computer programs.
2. LITERATURE REVIEW

2.1 Sustainability Overview

Over the recent years, the need to develop and use sustainable systems has been the primary concern of industries. Sustainable systems can be defined in several ways and according to Washington State Department of Ecology, “It is meeting the needs of the present without compromising the ability of future generations to meet their own needs” [3]. A similar definition is also presented by The Bruntland Commission [4] and this primary definition of sustainability couldn’t be properly understood as everyone has different ways of looking at it. However, researches applied the definition of sustainability to their systems in various ways to concentrate on their area of work. The 2002 World Summit on Sustainable Development on a global scale conducted in Johannesburg, South Africa [5] defines sustainability as “The design of human and industrial systems to ensure that humankind use of natural resources and cycles do not lead to diminished quality of life due rather to losses in future economic opportunities or adverse impact on social conditions, human health and the environment” [5, 6].

There has been a lot of work in this particular field. The development and improvements in this field has been renewed in detail by Mihelcic [4]. The topics discussed in this study are primarily the role of economic and social processes and sustainability indicators and illustrates the examples of the evolution of sustainability science and engineering. Schwarz [7] show how sustainability can be used to provide a framework for integrating environmental, social and economic interests into effective business strategies [7]. The basic metrics are defined which are called the basic indicators, and they are measured to provide a track of performance. These
metrics can be used by a manager to check, develop and improve a process in maintaining and developing a sustainable system.

### 2.2 Concept of Exergy and Its Applications

Exergy is a thermodynamic concept and is defined as “Optimal work that can be extracted from a system as it interacts with an environment” [2]. Exergy is additive but not conserved and the different types of exergy $B$ can be represented by the following equation [8].

$$B = B_k + B_p + B_{ph} + B_{ch}$$  \hspace{1cm} (2.1)

where $B_k$ represents kinetic exergy, $B_p$ represents potential exergy, $B_{ph}$ represents physical exergy and $B_{ch}$ represents chemical exergy (please refer to section 3.1.2 for better understanding of different types of exergy). There has been a lot of work done in the field of exergy analysis [12]. Exergy concepts have been applied to process industries, energy fields etc [30]. It is also mainly used in consideration of ecological concepts which can be used to assess the influence on the environment. The basic definition, the concept, the formulation and explanation of exergy use is clearly discussed by Rosen [1, 2]. Before the use of exergy analysis has been developed, the energy analysis has been used with a wide range of economic, environmental and efficiency studies. It is, however, very difficult to make energy quality accounting or calculations as energy follows a law of conservation, and efficiency studies not necessarily can give a complete representation of energy used, energy lost and energy not used [10], in particular by referring to quality. Brunhes [11] showed the drawbacks of energy assessments only.
The loss of exergy which is caused by the irreversibility of the process is directly linked to the entropy change. The relationship that defines this loss is called the Gouy-Stodola’s theorem represented by the following equation [12, 32].

\[ \Delta B = T_0 \sum (\Delta S) \]  

(2.2)

This particular equation is extensively used in this thesis work especially for the thermal exergy calculations. This provided the base to correlate the entropy generation and exergy utilization meaningfully. Berthiaume [10] provided a comprehensive analysis of the usage of exergy in environmental impact of paving material manufacture. Their work is a study of a macroscopic exergy indicator called the CNEx (The Concept of Net Exergy Consumption), proposed for calculation of the environmental impact of the transformation of raw materials into finished goods.

Another such exergy calculations related to the manufacturing industry is a study of Sodium Tripolyphosphate production by Atanasova [33]. Sodium Tripolyphosphate finds a wide range of applications and this study shows the exergy efficiency of the system in the related manufacturing process. It is found that the exergy efficiency coefficient is only 6.75 % [13]. This value is obviously very low; therefore the exergy analysis reveals clearly the possibilities for improvement. The study is done on material, energy and exergy balances of the system as a whole and of its separate elements. Masini [8], in their work presented an application of exergy concept on five basic metal industries i.e. Steel, Aluminum, Copper, Lead and Zinc industries. Their main concentration of interest was on resource and waste accounting purposes. In their study they have considered the manufacturing processes from the initial state to the final state of production and used exergy analysis to demonstrate how it can be used for resource and waste
accounting process. The data indicates that Aluminum industry is characterized by very large exergy consumption. The analysis illustrates that copper, lead and zinc represent an intermediate resources utilization between steel and aluminum industries [8].

An interesting study was conducted by Sorin [9] about the Exergy based approach for process synthesis. Process design guides transforming of raw materials into the finished products [9]. This study promotes two approaches for process design. One being hierarchic (called in the study) the other approach being mathematical approach. Using the exergy calculations, a new approach for chemical process synthesis has been developed. It shows that this approach may be used to define a more exergy efficient benzene synthesis process [9].

Exergy analysis has also been used extensively in the field of thermal engineering to calculate the energy utilization efficiencies. There has been an increasing application and recognition of the usefulness of exergy methods in industry, government and academia in recent years [13]. In Rosen, [13].work, the exergy analysis is described in detail and thermodynamic considerations in TES (Thermal Energy Storage) systems evaluation are discussed and then exergy analysis for a closed system is highlighted. The uses of exergy methods in optimization and design are illustrated by determining optimal discharge periods [13]. The authors have examined exergy analysis methodologies in detail and applied them to industrial systems for environmental impact assessments as well as TES.

Rosen and Dincer [14], in their study on an industrial steam process have used exergy analysis. They have found the technical factors that influence the feasibility of substituting steam supplied
heating for other energy sources for industrial heating. They concluded that substituting steam for some or all industrial purposes is beneficial to other energy resources using technical factors, by using of exergy as a metric (when a lower value energy currency such as steam is used in place of higher value currency such as electricity). Also, they have proposed alternative configurations for the steam supply for broadening the range of applicability in industries [14].

Another interesting concept introduced is the so called cumulative exergy analysis. This concept was presented by Morris and Szargut [31]. The cumulative exergy analysis involves raw materials, energy, equipment and products in calculation. It can evaluate the thermodynamic perfection (defined in the analysis) with different raw materials and production routes for the same product. Also, it concluded that the process parameters that have greater effect on energy performance cannot produce an optimal value of thermodynamic perfection [15]. The work of Feng [15] is using the same concept of cumulative exergy analysis applied to a heat exchanger manufacturing. The consumption of both energy and non-energy materials as well as the equipment used in the process has to be included in the analysis. The cumulative exergy of raw materials is the major component of the total value of exergy utilization. In the same work a method to optimize the heat exchange processes was presented [15].

Ayres [16] analyzed the U.S. exergy (energy) usage and linked it with the thermodynamic efficiency improvements. The basic concept called ‘growth engine’ is introduced. It provides a positive feedback loop involving declining costs of inputs and increasing demand for lower priced outputs. The purpose of the work is to reformulate the idea of this ‘growth engine’ in terms of the service provided by energy inputs. It is defined as the product of exergy inputs
multiplied by conversion efficiency. The attempt was to reconstruct the useful work performed in
the U.S. economy during the twentieth century. Two important conclusions were offered. The
first conclusion is that for the last century the locus of technical progress has moved from energy
(exergy) conversion efficiency to end use efficiency (service output per unit of work). Also,
purely thermodynamic efficiency improvements at the converter (equipment) level were largely
exhausted by the 1960’s [16]. The second important observation was that the growth of exergy
consumption has had an enormous impact on the past economy. A hypothesis emerged that the
improvement in the production of primary work may account for that portion of the economic
growth attributable to ‘technical progress’ [16].

Another economic study on energy efficiency is conducted by Stepanov [17]. This study is
similar to the case of Ayres [16]; but it is conducted for a different nation - former U.S.S.R. The
work consists of a determination of the thermodynamic efficiency of energy use in large scale
productions and calculates the theoretically minimum exergy consumption for the process
effectiveness. The processes include chemical transformations, deformation in rolled-stock
production, space heating and hot water supplies. The conclusion made was that the substitution
of useful energy by the value of minimal exergy consumption when studying chemical or
metallurgical processes and manufacturing a product makes it possible to apply such universal
indices as energy and exergy efficiencies [17].

Hammond [18] discusses how energy and exergy analysis may be used to evaluate appropriate
measures for sustainability. The area of focus of sustainability is clearly presented in the work
with a ‘Venn Diagram’ which was adopted from Parkin (2000) [18]. In this diagram, the society,
ecology & thermodynamics and economics & technology featured an interaction with one another, and the common region of interest for improvement of all these is called as sustainability [18]. In this paper, there is a discussion provided certain parameters. These parameters were used in this work provide a method of improving them in order to build a sustainable system. The exact interpretation of what sustainability is and what are the regions of focus for improvement were discussed.

A different study was conducted by Bakshi [19] on thermodynamic framework for ecologically conscious process systems engineering. The approach used by the author is to use thermodynamics to exploit the synergy between different methods from process systems engineering, systems engineering, systems ecology and life cycle assessment to overcome the shortcomings of individual methods from each field [19]. The proposed framework in this research is applicable to assist decision making in chemical and other engineering tasks. The conclusions that were drawn from this work were that a new approach for ecologically conscious process systems engineering can be formulated. The proposed framework is expected to be a step towards a systematic and rigorous approach for evaluating and satisfying the so called “triple bottom line” (triple bottom line consists of economic, environmental and social factors) in all engineering decisions [19].

As observed in all the above cases, exergy based metrics were used and practical conclusions were drawn for a variety of processes. The exergy calculations accounted were for particular processes and large systems – most often energy related and less prominent for manufacturing. As far as the knowledge of the author goes, there hasn’t been much work devoted to a
comparison of different processes using a simple exergy metric. This thesis is an attempt to compare two entirely different material processing i.e. mechanical deformation and a heating treatment, but using the exergy utilization as a metric.
3. EXERGY UTILIZATION FOR MATERIALS PROCESSING OF Al & Mg ALLOYS

3.1 Introduction

The analysis in this chapter offers an approach to both theoretical and experimental evaluation of mechanical (elastic and plastic) exergy utilization needed to perform certain mechanical deformation. This exergy amount constitutes a component of the overall exergy utilization within a material’s processing during manufacturing [34].

Assume that the material used to manufacture, say the heat exchanger tubes of a compact heat exchanger is initially in the form of a flat plate (without considering elastic/plastic exergy for due to a prior process of metal forming. The plate has a thickness \( t \), length \( l \), and width \( p \) as indicated in Fig 1. The plate material is characterized with a stress-strain diagram similar to the one shown in Fig 2. A tube manufactured from this sheet has a form as shown in Fig 1(b).

Fig 1. Initial (a) and final (b) shape of the material before and after metal forming
The material undergoes a significant strain during metal forming and there must be significant exergy utilization during this process. The amount of exergy used must be equal to the work done on the material. Hence, in a mechanical process (a process involving a mechanical work only); the exergy use is trivial to determine, if one has all the information needed for determination of the needed work. A need to express this work in terms of exergy, however, is not a semantical issue. The importance of the introduction of exergy is justified by the fact that exergy equivalent of a non-mechanical materials processing would not be possible to determine using the energy balancing only. Such exergy quantity would involve the quality of thermal interactions. In such a manner, a common metric (exergy) for both mechanical and non-

Fig 2. The stress-strain curves of a typical Aluminum alloy (Plot drawn to the trail 1 for Al alloy stress-strain values, see Fig. 7)
mechanical processing will be possible to evaluate. The work done in a mechanical process can be obtained by calculating the integral involving the stress-strain dependence (a theoretical amount with no losses). The general shape of the stress-strain curve is presented in Fig 2. The elastic and plastic exergy uses are presented in the Fig 3. Therefore, calculating the area under the stress-strain curve would give us the work which can be represented as an exergy use involved with material forming [20].

Fig 3. The true stress-strain curve used in this analysis
To calculate the required exergy demand, two equivalent but not identical procedures may be followed.

1) Integrate analytically the equation $\sigma = f(\varepsilon)$ up to the given strain to be accomplished (Method 1)

2) Determine the work needed for the given strain to be accomplished using experimental data use will follow both procedures directly (Method 2)

### 3.1.1 Units Clarification

There is a need to clarify the units used in the analysis for better understanding. All the values are represented in SI system.

The basic equation of work is represented as below.

$$w = \int \sigma d\varepsilon \ [\text{Pa}]$$

which also means

$$w = \int \sigma d\varepsilon \ [\text{N/m}^2]$$

Multiplying and dividing by the same unit will not change its representation, therefore using this principle, when we multiply and divide the previous equation by m gives,

$$w = \int \sigma d\varepsilon \ [\text{N/m}^2][\text{m/m}]$$

$$w = \int \sigma d\varepsilon \ [\text{N-m/m}^3]$$

And N-m is a Joule, J, therefore

$$w = \int \sigma d\varepsilon \ [\text{J/m}^3]$$
The work can also be represented by Joule per unit volume, and hence exergy can also be represented by the same units. These units are consistently used in the entire thesis. Some graphs and plots represent Joule per square meter for better understanding.

### 3.1.2. Initial Exergy of a Material

There is a need to know the initial exergy of a material in a given state. The majority of technical processes are flow processes. Hence, from a practical point of view, the exergy $B$ of a stream of substance crossing the immovable system boundary [29] is of importance. Kotas [30] has termed the exergy $B_{s}$ of a substance contained inside the system boundary as “stagnation exergy” [30]. The general form of total exergy excluding nuclear, electrical and interfacial exergy is given by the following equation. [31]

$$B = B_k + B_p + B_{ph} + B_{ch}$$

where $B =$ Total exergy in the material

$B_k =$ Kinetic exergy (Kinetic exergy is equal to the kinetic energy, when the velocity is considered relative to the surface of the earth) [31]

$B_p =$ Potential exergy (Potential exergy is equal to the potential energy when it is evaluated with respect to the average level of the surface of the earth in the locality of the process under consideration ) [31]

$B_{ph} =$ Physical exergy

Physical exergy is the work obtainable by taking the substance through reversible physical processes from its initial state temperature $T$ and pressure $p$, to the state determined by the temperature $T_0$, and the pressure $p_0$, of the environment [31].
And the physical exergy is also represented by the following equation

\[ B_p = -g \int_{X=0}^{X_{\text{max}}} \int_{\chi_b}^{\chi_e} A(\rho - \rho_0) dX d\chi \]

where \( A = \) Horizontal cross section of the body under consideration
\( \rho, \rho_0 = \) Density of the body and of surrounding matter
\( g = \) Gravitational acceleration (assumed to be constant)
\( \chi = \) Internal coordinate of the height, measured from the lowest point of the body
\( \chi = \) External coordinate of the height, measured from the reference level
\( \chi_b, \chi_e = \) The coordinate of the cross section under consideration at the actual moment an in the state of stable equilibrium.

\[ B_{ch} = \text{Chemical exergy} \]

Chemical exergy is the work that can be obtained by taking a substance having the parameters \( T_0, p_0, \) given composition and to the state of thermodynamic equilibrium with the datum level of the components of the environment [31]. There must be a selection of reference level to calculate the chemical exergy of the material. The selection of reference level was proposed by Szargut [31]. The exact chemical exergy of a material is impossible because of the lack of sufficiently exact thermodynamic data, the calculation should be made with currently available data and the result should be accepted as a conventional standard value of the chemical exergy of the element under consideration.[31].

In some cases the physical and chemical exergy are represented as thermal exergy given by \( B_{th}. \)
The value of the initial exergy of Al alloy and Mg alloy used can be evaluated using the above formulae and/or definitions. This value may be evaluated at the given state. The physical value
of exergy maybe equal to or close to zero if a material is in thermal equilibrium with surroundings, but the value of chemical exergy may not be. The procedure for calculating these exergy values is explicitly explained by Szargut, J et al [31] and the standard values of the chemical exergy of the element in kJ/mol for pure Al is 888.4 kJ/mol and for pure Mg is 633.8 kJ/mol. These are the solid reference species and their conventional average concentration in the external layer of the earth crust. The standard chemical exergy of the solid reference species for these elements can be estimated by a proposed idealized model of the solid environment, based on the assumption that the solid reference species can be treated as the components of an ideal solution [31]. Their standard chemical exergy is determined by the following equation which is based on conventional mole fractions $\chi_i$.

$$b_{ch}^0 = -RT\ln\chi_i$$

where $b_{ch}^0$ = the standard chemical exergy, $R$ = Universal gas constant, $T$ = Room temperature, $\chi_i$ = Conventional mole fraction

The author understands that this thesis deals with Al alloy and Mg alloy rather than the pure Al and pure Mg and there are bound to be changes in the values of the initial exergy of the material (presented above). As the initial exergy of the material is not of the primary region interest for the calculations, they do not affect the results given in this thesis as long as the chemical exergy changes are not involved with materials processing. If, however, during processing certain chemical processes take place, the correct representation of chemical exergy in any given state is necessary.
3.2 Exergy Utilization Calculation (Method 1)

To find the exergy utilization using an analytical procedure, the equations derived in solid mechanics theory for a given material should be considered. A general equation of stress-strain curve is given by the “Ramberg-Osgood equation” [21]. This equation is dimensionless and expressed explicitly in terms of $\varepsilon$, not $\sigma$, i.e.

$$\varepsilon = \frac{\sigma}{E} + \beta \left( \frac{\sigma}{E} \right)^n$$  \hspace{1cm} (1)

where

$$\beta = \frac{3}{7} \left[ \frac{E}{\sigma_{0.7}} \right]^{-1}$$  \hspace{1cm} (2)

where $E$ is the Young’s modulus of elasticity, $n$ is the characteristic constant depending on the shape of the curve, $\sigma_{0.7}$ is the secant modulus at 0.7E. The determination of the secant modulus is detailed in Appendix D.

By substituting equation (2) in equation (1), the following equation is obtained.

$$\varepsilon = \frac{\sigma}{E} + \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n - 1} \left( \frac{\sigma}{E} \right)^n$$  \hspace{1cm} (3)

The value of “$n$” can be obtained by the following equation [21].

$$n = 1 + \frac{\ln \frac{17}{7}}{\ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)}$$  \hspace{1cm} (4)

where $\sigma_{0.85}$ represents the secant modulus at 0.85E

As emphasized earlier, to calculate the elastic and plastic exergy utilization due to metal forming, there is a need to calculate the mechanical work needed to produce a certain strain [deformation].
The analytical procedure of this calculation is shown in Appendix A. The final results are given the following equations.

Total exergy change per unit area (pt)

\[
\frac{B_{m, ph}}{A_{\text{area}}} = \left[ \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right)^2 - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right) \left( \frac{\sigma^*}{n + 1} \right) \right] l \text{ J/m}^2 \tag{5}
\]

Plastic exergy change per unit area (pt)

\[
\frac{B_{m, ph, \text{plastic}}}{A_{\text{area}}} = \left[ \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right)^2 - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right) \left( \frac{\sigma^*}{n + 1} \right) - \frac{\sigma^2}{2E} \right] l \text{ J/m}^2 \tag{6}
\]

where \( B_{m, ph} \) is the total exergy used for mechanical deformation, \( B_{m, ph, \text{plastic}} \) is the plastic exergy use, \( \sigma^* \) is the strain at the final point of interest and \( \varepsilon^* \) is its corresponding strain. \( A_{\text{area}} \) represents the cross-section exposed to strain (refer to the Fig 5.).

In conclusion one can state that, to obtain the values of the total, elastic or plastic exergy demands for a process analytically, one needs to have information on accurate stress-strain curves. This will lead to a precise determination of the parameters involved in the correlation \( \sigma = f(\varepsilon) \). The most reliable would be to determine these values for a given material by performing a set of experiments (tensile testing). Therefore, the tensile testing is performed on materials by using a Universal Testing Machine to get the true stress-strain curves. The analyzed alloys are Al 3003 and Mg AZ31. The experimental setup and procedure is discussed next.
3.2.1 Experimental Determination of the True Stress-Strain Relationships

3.2.1.1. Experimental Setup and Materials

To find the values for the correlation $\sigma = f(\varepsilon)$ for the selected AA 3003 and Mg AZ31 alloys, a series of experiments were conducted. Tensile testing machine is used to find such results. The testing facility is illustrated in Fig 4.

Fig. 4. Universal testing machine [UTM] MTS 810 material test system

All experiments were performed at room temperature conditions. The work piece of the following dimensions is used in the experiments.

<table>
<thead>
<tr>
<th>Work Piece</th>
<th>Rectangular Cross-section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width: $p$</td>
<td>$0.00635 \text{ m (0.25 in)}$</td>
</tr>
<tr>
<td>Thickness: $t$</td>
<td>$0.003175 \text{ m (0.125 in)}$</td>
</tr>
<tr>
<td>Length: $l$</td>
<td>$0.0508 \text{ m (2 in)}$</td>
</tr>
</tbody>
</table>
The representation of the work piece is as shown in Fig 5.

![Diagram of work piece dimensions]

**All Dimensions are in Meters**

Fig 5. Representation of the work piece with dimensions used in the experiment

The experiment is conducted following a standard ASM procedures [28]. Of the materials used in the experiment are complied in the following table (Al 3003 and Mg AZ31 alloys).

<table>
<thead>
<tr>
<th>Al 3003 Alloy [24]</th>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Zn</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.60%</td>
<td>0.70%</td>
<td>0.05-0.2%</td>
<td>1-1.5%</td>
<td>0.10%</td>
<td>Remainder</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mg AZ31 Alloy[25]</th>
<th>Al</th>
<th>Zn</th>
<th>Mg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3%</td>
<td>1%</td>
<td>Remainder</td>
</tr>
</tbody>
</table>

A sheet of the thickness of the work piece was cut according to the dimensions shown in Fig 5 using a CNC machine. A pre defined program was used by the CNC machine to cut the work piece to the exact dimensions.
3.2.1.2 Experimental Procedure

The experimental procedure has been executed as described in the following steps. The work piece shown in Fig 5 is used in this experiment.

1) The work piece is fixed in the bottom gripper of the tensile testing machine as shown in Fig 4.
2) The top gripper is adjusted till it holds the work piece handle.
3) The grippers are adjusted till the work piece is firmly set.
4) The extensometer (INSTRON GL50mm 2630-111 is used in the experiment, it has 50 mm gauge length and measures +/- 5 mm tolerance, which implies it measures +/- 10% elongation) is placed on the region of interest which is the 0.0508 m at the center from the by point A in Fig 5.
5) The force is gradually applied using the load cells.
6) The readings (the loads applied and corresponding strain data) are monitored by the computer.
7) The force is applied till it reaches the fracture limit.
8) The loads applied and corresponding strain data are registered.

The readings as indicated in the step 6 above are recorded by the computer and the experiment is repeated for total of ten samples (five for each alloy). Note that the rectangular cross-section work pieces were used as shown in the Fig 5 rather than a round cross section work piece. The uncertainty for the strain measurements for the Al alloy is found to be $\Theta = \pm 0.50 \times 10^{-6}$, both
and the uncertainty for strain for Mg alloy is found to be $\Theta = \pm 0.76 \times 10^{-6}$ at 95% confidence intervals. The procedure for determination the uncertainty analysis is presented in Appendix G. A number of repeated tasks are done to secure more accurate readings. The pictures of the work pieces before and after testing are presented in Fig 6.

![Work piece before and after test](image)

Fig 6. The work piece before (a) and after (b) the test

The images correspond to the Al alloy work piece and almost an identical appearance was observed for Mg alloy as well.

The following data is obtained from the experiment

1) Time in second
2) Load in Newtons
3) Extension in millimeters
4) Tensile Strain, dimensionless
5) True Strain, dimensionless
6) Tensile Stress in MPa
7) True Stress in MPa

The values taken from the computer are listed in Appendix B. The prime task at this step was to determine the relationships between the true stress and true strain. The graphs of the trials for each of the two materials are as plotted in Fig 6 & Fig 7.
The trial 3 of Al alloy is eliminated from the further study as there was an error in the way the experiment is conducted. It was found that there was a small notch in the work piece in this trial. According to the stress concentration theory, a large and uniform notch does not affect the experimental results; however, a small notch within the range of the extensometer readings would give an improper data. This is what happened in this particular case. There is an apparent consistency except for the eliminated test in all the trials and the graphs are plotted up to a value of strain of 0.01 as this is the primary region of interest for subsequent calculations. Similarly, the graph for Mg alloy is plotted below, see Fig 7.
As observed, the consistency in the graphs of Mg alloys $\sigma$ vs $\varepsilon$ relationships also exists.

### 3.2.2 Verification of the Validity of the Ramberg-Osgood Equation

The theoretical “Ramberg-Osgood” equation is used to calculate the exergy values as work performed during the tensile testing. Using this theoretical equation for calculation should be verified for validity for given materials and stress-strain value ranges. To check the applicability of this particular equation, a verification of Ramberg-Osgood equation is performed (See Appendix C). This verification has demonstrated that difference between the theoretical (Ramberg-Osgood equation) and experimental results is in average less than 10% in the value of
stress vs given true strain magnitude of 0.01. An example of the comparison is given in Fig 8. This and the remaining verification graphs are given in Appendix C. Within the ranger of true strain relevant to this study agreement between the experimentally determined values of true stress and predicted is very small for the elastic part and periodically increases with the increase in true strain for most Al cases. A much better agreement is established for Mg alloy tests, except for one case, see Appendix C.

![Graph comparing theoretical and experimental data for Al alloy trial 1](image)

Fig 8. Graph comparing the theoretical and experimental data for Al alloy trial 1

### 3.2.3. Exergy Calculations (mechanical processing)

Using the prediction based on Ramberg-Osgood equation, the calculations of exergy utilization needed to perform a certain pre-defined task is executed. The task is to read the true-strain of 0.01. Subsequently, the same calculations of exergy utilization for the given task were given task were performed, by direct experimental data.
3.2.3.1 Exergy Utilization for Materials Processing by Mechanical Forming (Al and Mg alloys)

The summary of the results from the analysis performed by using both methods are shown below. The exergy utilization calculation details are presented in Appendix D. The comparison of the four sets of data for Al alloy is provided in the Table 1, and plotted in the Fig 9. Therefore, the 4218.03 kJ/m$^2$ on average is required to produce a deformation of 0.01 strain in Al 3003 alloy and 5338.39 kJ/m$^2$ on average for Mg AZ3 alloy for the same strain value.

<table>
<thead>
<tr>
<th>TRIAL #</th>
<th>TOTAL EXERGY USE</th>
<th>ELASTIC EXERGY USE</th>
<th>PLASTIC EXERGY USE</th>
<th>RATIO PLASTIC EXERGY /ELASTIC EXERGY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4204.9</td>
<td>316.23</td>
<td>3888.67</td>
<td>12.3</td>
</tr>
<tr>
<td>2</td>
<td>4227.65</td>
<td>316.31</td>
<td>3911.34</td>
<td>12.37</td>
</tr>
<tr>
<td>4</td>
<td>4248.96</td>
<td>316.31</td>
<td>3932.64</td>
<td>12.43</td>
</tr>
<tr>
<td>5</td>
<td>4190.6</td>
<td>316.31</td>
<td>3874.28</td>
<td>12.25</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>4218.03</td>
<td>316.29</td>
<td>3901.74</td>
<td>12.34</td>
</tr>
</tbody>
</table>

| Table 1 Comparison of five sets of Al alloy data |

Please note that the plastic and elastic exergy utilization calculations in this thesis are based on the assumption that the calculation of elastic exergy is done at yield point of the material.
Fig 9. Total and plastic exergy utilization for Al alloy, Note: Trail 3 is eliminated.

The comparison of the four sets of data for Mg alloy is provided in the Table 2 and is graphically represented in the Fig 10.

<table>
<thead>
<tr>
<th>TRIAL #</th>
<th>TOTAL EXERGY CHANGE KJ/m²</th>
<th>ELASTIC EXERGY CHANGE KJ/m²</th>
<th>PLASTIC EXERGY CHANGE KJ/m²</th>
<th>RATIO PLASTIC EXERGY/ELASTIC EXERGY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5338.9</td>
<td>2347.22</td>
<td>2991.68</td>
<td>1.27</td>
</tr>
<tr>
<td>2</td>
<td>5355.9</td>
<td>2384.78</td>
<td>2971.12</td>
<td>1.25</td>
</tr>
<tr>
<td>3</td>
<td>5338.75</td>
<td>2347.23</td>
<td>2991.52</td>
<td>1.27</td>
</tr>
<tr>
<td>4</td>
<td>5328.1</td>
<td>2347.3</td>
<td>2980.8</td>
<td>1.27</td>
</tr>
<tr>
<td>5</td>
<td>5330.3</td>
<td>2347.2</td>
<td>2983.1</td>
<td>1.27</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>5338.39</td>
<td>2854.75</td>
<td>2483.64</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Table 2 Comparison of five sets of Mg alloy data
The values in the above table are represented in the form of a plot as shown below. This histogram represents the comparison between the total exergy use and the plastic exergy use for mechanical deformation of the considered Mg alloy.

Fig 10. Total and plastic exergy utilization for Mg alloy

The plot given in Fig 11 represents a comparison of the total exergy changes for two different materials used. As seen, the total exergy use needed to do the same task for Mg alloy is greater than that of Al alloy. The task in both cases was to produce a material deformation up to a strain of 0.01.

Fig 11. Comparison of total exergy utilization for the two alloys
The plot in Fig 12 represents a comparison of elastic exergy utilization needed for presented mechanical task for the two different materials used. The elastic exergy requirement for Mg alloy is significantly greater than that of an Al alloy. This is due the fact that Al alloy has the yield point lower than the one for the Mg alloy. Therefore, it takes more exergy to deform Mg alloy than an Al alloy.

![Comparison for Elastic Exergy Utilization](image)

**Fig12. Comparison of elastic exergy utilization for the two alloys**

The plot Fig 13 shows a comparison between the plastic exergy between the two alloys used. In this case, the plastic exergy of the Al alloy is greater than that of Mg alloy. This is due to the fact that the yield point of an Al alloys starts very early when compared to that of Mg alloy.
3.2.3.2 Observations from the Tables and Graphs

1) Total exergy use needed to do the mechanical deformation for Mg alloy is greater than that of Al alloy for the same presented task, i.e., to achieve strain 0.01

2) There is much greater elastic exergy utilization in Mg alloy than that of Al alloy for the same strain 0.01

3) The plastic exergy utilization for Al alloy is much greater than that of Mg alloy for the same strain 0.01

4) Therefore, more exergy is required to do the same deformation for Mg alloy than for Al alloy

Fig 13. Comparison of plastic exergy utilization for the two alloys
3.3. Analytical Method to Calculate Exergy Utilization (Method 2)

To find the exergy needed for performing the same mechanical task using the experimental data directly, a simple program in MATLAB is defined. This approach calculates the work needed (i.e. exergy) by calculating the area under the experimental true stress-strain curve (i.e. without analytically interpretation of Ramberg-Osgood equation). The MATLAB program and the results are summarized in APPENDIX F. The results are as presented in the following Table 3 and Table 4 for Al and Mg alloys respectively.

<table>
<thead>
<tr>
<th>TRAIL #</th>
<th>TOTAL EXERGY USE IN kJ/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4175.13</td>
</tr>
<tr>
<td>2</td>
<td>4159.25</td>
</tr>
<tr>
<td>4</td>
<td>4127.50</td>
</tr>
<tr>
<td>5</td>
<td>4127.50</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>4147.35</td>
</tr>
</tbody>
</table>

Table 3 Total exergy utilization for Al alloy

<table>
<thead>
<tr>
<th>TRAIL #</th>
<th>TOTAL EXERGY USE IN kJ/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5492.75</td>
</tr>
<tr>
<td>2</td>
<td>5302.25</td>
</tr>
<tr>
<td>3</td>
<td>5302.25</td>
</tr>
<tr>
<td>4</td>
<td>5365.75</td>
</tr>
<tr>
<td>5</td>
<td>5175.25</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>5286.37</td>
</tr>
</tbody>
</table>

Table 4 Total exergy utilization for Mg alloy
As can be seen from Tables 3 and 4, the comparison of the theoretical values obtained using ‘Ramberg-Osgood’ equation and experimental data; it becomes obvious that there is a very small difference in the values of exergy utilization following two equivalent methods. This confirms the same conclusion from Appendix C, that the use of ‘Ramberg-Osgood’ equation is acceptable with a tolerable deviation in the respective data.

The following plot represents the comparison of values between experimental and theoretical results for Al alloy. The absolute difference between the theoretical and experimental exergy utilization for Al alloy is 71 KJ/m² (on average) till a strain of 0.01. That is there is a percentage of 1.71 % increase in the theoretical value than the experimental value.
The plot at Fig 15 is a similar representation for aluminum. The absolute difference between the theoretical and experimental exergy utilization (on average) for Mg alloy is 52 KJ/m² till a strain of 0.01. That is there is a percentage of 0.97 % increase in the theoretical value than the experimental value.
4. EXERGY UTILIZATION FOR THERMAL PROCESSING FOR Al & Mg ALLOYS

4.1 Introduction

The previous chapter deals with the determination of exergy utilization for materials processing by mechanical straining for Al 3003 alloy and Mg AZ31 alloy. In this chapter, a calculation of both exergy loss caused by finite temperature difference and exergy utilization for a thermal processing (i.e. heating) for the same two alloys is performed. Subsequently, a comparison of exergy utilization for materials processing and thermal processing will be discussed in the next chapter.

As discussed in chapter 1, exergy in a given state represents a theoretical maximum work obtainable for a given system versus a reference state of the system when it is allowed to interact with the environment to attain the equilibrium [1]. That means exergy can be calculated for a distinct set of states along the material processing. The difference between the exergy values would, in ideal case of fully reversible process indicate the exergy utilization needed to change a state from one to the other. This type of calculation was performed in the previous chapter for purely mechanical deformation (with no exergy losses caused by any secondary process). In this chapter, an attempt to determine how much exergy in form of heat is needed to perform different task – a thermal processing. In addition the calculation of the exergy loss caused by heat transfer across the finite temperature difference during given heating task is presented. The derivation and explanation of the exergy loss is presented in detail in Appendix H (Part I). This particular
equation is derived using the Gouy-Stodola’s Theorem [12]. The loss of exergy, which caused by the irreversibility of the process due to finite temperature difference is

\[ B_D = T_0 S_{gen} = \frac{Q}{T_L} T_0 \left( 1 - \frac{T_L}{T_H} \right) \]

(7)

where \( B_D \) represents the exergy loss, \( T_0 \) represents the reference temperature, \( S_{gen} \) represents the entropy production due to heat transfer across finite temperature difference, \( Q \) is the heat transfer during this process transferred from the higher to the lower temperature level and \( T_L, T_H \) represents average/mean temperature of the heated object and mean temperature of the heater.

The relationship Eq. (7) assumes that product exchanges energy in form of heat while being (in a first approximation) at a mean (constant) temperature between its initial and final values. In this application, however, this is not the case. If one determines a proper mean \( T_L \) value, the calculated exergy loss would be (most likely) close enough to the exact value. This has to be established rigorously. For such analysis, one should assume that \( T_L(t) \) changes between an initial value in a limit equal to \( T_0 \) to a desired value of \( T_L \). The analysis that follows attempts to determine entropy generation and exergy loss for such a case but in an exact rigorous manner.

Consider a system as shown in Fig.15; this is a metal bar of mass ‘m’. Let it be at an initial temperature of \( T_0 \) (which is the reference temperature in this case) and it is to be heated to a temperature of \( T_L \) where the source is at a temperature of \( T_H \).
The total energy transfer $Q$ occurring up to sometime $t$ is given by the following equation. The derivation and explanation of the equation is provided in Appendix H (Part II).

$$Q = (\rho Vc) \theta \left[ 1 - \exp \left( \frac{-t}{\tau_\theta} \right) \right]$$

(8)

The assumption that is used in this heating process is that the body heating follows the lumped capacitance conditions [25]. That is, this implies that the temperature gradients within the solid during heating are negligible. For this condition to be valid, the Biot Number has to be less than 0.1. The validation of this condition is also provided in the Appendix H (part II). It should be noted that many processes, for example, any brazing process must be performed with a great deal of care with respect to this temperature uniformity anyway – because the brazing conditions require it. So, such idealization is plausible from this point of view as well.

Differentiating Eq. (8), one gets

$$dQ = \frac{\rho Vc \theta}{\tau_\theta} e^{\frac{-t}{\tau_\theta}} dt$$

(9)

The differential of the entropy generation is now given by (see Eq. (H.9) in Appendix H)
\[ \int dS = \int \frac{dQ}{T_L} \left( 1 - \frac{T_L}{T_H} \right) \] 

(10)

The temperature reached by a solid at an instant \( t \) is given by [26]

\[ \frac{\partial}{\partial t} = \frac{T_L - T_H}{T_0 - T_H} = e^{\frac{-t}{\tau}} \] 

(11)

Eq. (11) can be re-written in the following format

\[ T_L = (T_0 - T_H)e^{\frac{-t}{\tau}} + T_H \] 

(12)

Substituting Eq. (12) and Eq. (9) in Eq. (10), one gets

\[ \int dS = \int_0^t \frac{\rho V c_\theta e^{\frac{-t}{\tau}}}{(T_0 - T_H)e^{\frac{-t}{\tau}} + T_H} \left[ -(T_0 - T_H)e^{\frac{-t}{\tau}} \frac{1}{T_H} \right] dt \] 

(13)

Integrating the above equation & substituting the limits, the following result is obtained.

\[ S_{gen} = \frac{\rho V c_\theta}{T_H} \left[ \frac{e^{\frac{-t}{\tau}} - 1}{(T_0 - T_H)} \ln \left( \frac{T_0}{(T_0 - T_H)e^{\frac{-t}{\tau}}} \right) \right] \] 

(14)

The exergy loss needed to thermally treat the material is given by Eq. (15)

\[ B_D = T_0 S_{gen} \] 

(15)

where \( S_{gen} \) is given by the Eq. (14)

4.1.1 Exergy Transfer Rate Associated with Heat Transfer Rate

As the work piece is heated the exergy in the work piece increases. In a simple closed system where the work piece is heated from a reference surrounding temperature to a higher
temperature, the differential (exergy utilization corresponding to a differential \(dQ\)) is given by Eq. (16) [32].

\[
dB = \int (1 - \frac{T_0}{T_L})dQ
\]  

(16)

Substituting the values of \(T_L\) from Eq. (12) and \(Q\) from Eq. (9), the following is obtained

\[
\int dB = \int \left[ \frac{\rho V c \theta}{\tau_t} \right] \left[ \left( \frac{\tau_t}{T_0 - T_H} e^{\frac{-t}{\tau_t}} \right) + \left( T_0 - T_H \right) \frac{e^{\frac{-t}{\tau_t}}}{T_0} \right] \frac{e^{\frac{-t}{\tau_t}}}{T_0} \left( T_0 - T_H \right) e^{\frac{-t}{\tau_t}} + T_H \right] dt
\]  

(17)

Upon integrating Eq (16) & substituting limits the following generalized equation is obtained

\[
B = -\rho V c \theta \left[ \left( \frac{T_0}{T_H - T_0} \right) \ln \left( \frac{T_0 - T_H}{T_0} \right) \right]
\]  

(18)

Subsequently, the calculations for the exergy loss and the exergy utilization are performed in next and finally a comparison of these values is presented in the chapter 5.

4.2 Al 3003 Alloy Calculations for Exergy Loss

The exergy loss is defined by Eq. (15) and can be determined if the entropy generation is determined. Consider the process in which the Al 3003 alloy is heated. The same dimensions (effective dimension of the work piece – 0.0508x0.00635x0.003175 m\(^3\)) and mass of the material as in mechanical process are assumed. Let the work piece temperature be at an initial temperature of \(T_0=300\) K and the source temperature is at \(T_H = 900\) K. Let the work piece be
heated to a temperature of 600 K. So, the body is heated from 300 K to 600 K \((T_L = 600 \text{ K})\). The following input data are used.

\[
\rho = 2700 \text{ kg/m}^3, \quad V = 1.025 \times 10^{-6} \text{ m}^3, \quad A_s = 0.32 \times 10^{-3} \text{ m}^2 \quad \text{and} \quad c = 893 \text{ J/kg K}
\]

The next step in the analysis is to calculate the value of \(h\), heat transfer coefficient for the considered process of heating assuming a free convection condition.

Reference surface temperature is given by the following equation,

\[
T_S = \frac{T_0 + T_L}{2} = \frac{300+600}{2} = 450 \text{ K}
\]

The “film temperature” estimation at which the properties are to be evaluated is given by

\[
T_F = \frac{T_S + T_H}{2} = \frac{450+900}{2} = 675 \text{ K}
\]

From table A.4 from [27] the properties for material (air at \(T_F = 675 \text{ K}\)) is as follows

\[
\rho = 0.5165 \text{ kg/m}^3, \quad c = 1.069 \text{ kJ/kg K}, \quad \mu = 330.6 \times 10^{-7} \text{ N.s/m}^2, \quad \nu = 64.15 \times 10^{-6} \text{ m}^2/\text{s}, \quad k = 53.65 \times 10^{-3} \text{ W/m K}, \quad \alpha = 92.65 \times 10^{-6} \text{ m}^2/\text{s} \quad \text{and} \quad Pr = 0.693
\]

One has to calculate the Rayleigh number [27], first, to be able to determine Nusselt number.

The conditions, for this calculation are as listed below

1) External free convection flows
2) Horizontal surface up or cold surface down
3) Ambient air quiescent
4) Surfaces radiation effects are negligible

\[
R_{al} = \frac{g \beta (T_S - T_H) L^3}{\nu \alpha} = \frac{9.8 \times (1/675) (900-450) (0.0503^3)/ (64.15 \times 10^{-6} \times 92.65 \times 10^{-6})}{14.0 \times 10^4}
\]
Using the equation \( Nu_L = 0.54 R_{al}^{1/4} \) [29] gives the value of \( N_{UL} = 10.44 \)

Therefore, the value of \( h \), can be obtained by the following equation

\[
h = \frac{Nu_k}{L} = 11.14 \text{ W/m}^2\text{ K}
\]

Now, substituting all the values in Eq. (14), with the upper and lower limits to be time at \( t = 0 \) and \( t = 480.5 \text{ s} \) respectively. The following value of entropy generation is obtained.

\[
S_{gen} = 0.8893 \text{ J/K}
\]

Therefore, exergy loss involved with the raise of the Al 3003 alloy temperature in a heating process by one degree Kelvin Eq. (15), is as follows.

\[
B_D = 300 \times 0.8893 = 266.62 \text{ J}
\]

### 4.3 Mg AZ31 Alloy Calculations

Consider the exergy loss for performing the same task as for Al alloy, but for the magnesium material. Consider the same process in which the Mg AZ31 alloy is heated. The same dimensions and mass as in the case of Aluminum alloy are assumed. The exergy loss is defined by Eq. (15) and can be determined if the entropy generation is determined. Let the work piece temperature be at an initial temperature of \( T_0 = 300 \text{ K} \) and the source temperature is at \( T_H = 900 \text{ K} \). Let the work piece be heated to a temperature of 600 K. So, the body is heated from 300 K to 600 K (\( T_L = 600 \text{ K} \)).
The heat transfer coefficient is same as calculated in case of Al alloy as the conditions are the same. Therefore, 
\[ h = \frac{Nu_L k}{L} = 11.14 \text{ W/ m}^2 \text{ K} \]

Substituting all the relevant values in Eq. (14), with the upper and lower limits to be time at \( t=0 \) and \( t=310 \text{ s} \) respectively. The following value of entropy generation would be.

\[ S_{gen} = 0.667 \text{ J/K} \]

Therefore, exergy loss involved with the raise of the temperature of the Magnesium alloy by one degree Kelvin is given by \( B_D = T_0 S_{gen} \)

\[ B_D = 300 \times 0.667 = 200 \text{ J} \]

As seen from both the results, the exergy loss for Aluminum alloy is 67 J (i.e. 25\% greater) greater that the Magnesium alloy.

### 4.4 Al 3003 Alloy Calculations for Exergy Utilization

The value of exergy utilization for Al alloy 3003 by a process of heating by 300 degree Kelvin is calculated by the Eq. (18).

\[ B = -\rho V c_\theta \left[ \left( \frac{-\tau}{e^{\tau} - 1} \right) + \left( \frac{T_0}{T_H - T_0} \right) \ln \left( \frac{T_0 - T_H e^{\tau}}{T_0} \right) \right] \]

The values are same as used in the previous calculations, therefore

\[ B = -\rho V c_\theta \left[ \left( \frac{-\tau}{e^{\tau} - 1} \right) + \left( \frac{T_0}{T_H - T_0} \right) \ln \left( \frac{T_0 - T_H e^{\tau}}{T_0} \right) \right] = 973 \text{ J} \]
4.5 Mg AZ31 Alloy Calculations for Exergy Utilization

The value of exergy utilization for Mg AZ31 alloy by a process of heating by 300 degrees Kelvin is calculated by the Eq. (18).

\[
B = -\rho V c \theta_t \left[ \left( \frac{\tau}{e^{\tau} - 1} \right) + \left( \frac{T_0}{T_H - T_0} \right) \ln \left( \frac{\left( T_0 - T_H \right) e^{\tau} + T_H}{T_0} \right) \right]
\]

The values are same as used in the previous calculations, therefore

\[
B = -\rho V c \theta_t \left[ \left( \frac{\tau}{e^{\tau} - 1} \right) + \left( \frac{T_0}{T_H - T_0} \right) \ln \left( \frac{\left( T_0 - T_H \right) e^{\tau} + T_H}{T_0} \right) \right] = 730 \text{ J}
\]

If one would like to represent the loss of exergy due to heating of a material during processing, either an average (mean) temperature if the product must be determined, or the determination of exergy loss (Eq.(H.9) ) should be formulated starting with a differential exergy loss due to a delivery of \( dQ \) across the temperature difference \( \Delta T = T_H - T_L(t) \) that changes in time and which follows \( \int dS_{gen} = \int \left( \frac{1}{T_L(t)} - \frac{1}{T_H} \right) dQ \). The use of the above equation would give an accurate result of the exergy loss.
5. RESULTS FOR MATERIAL PROCESSING AND THERMAL PROCESSING OF 
Al ALLOY & Mg ALLOY

This chapter offers a discussion involving obtained results for both (i) mechanical and (ii) thermal processing. A comparison of these two entirely different processes is made in this chapter by using a unique metric of exergy utilization.

5.1 Results and Discussion for AA 3003

From chapter 3, the results obtained for the exergy utilization per unit area for Al alloy based either on analytical prediction or experimental work indicates a small variation. Therefore, it is concluded that the exergy utilization for both Aluminum and Magnesium materials can be performed using ‘Ramberg-Osgood’ calculations if the empirical values for exponent “n” is calculated. In that case, analytical integration can be performed without a need for experimental data handling. The average value of exergy utilization for mechanical process per unit area obtained from the theory using Ramberg-Osgood equation is $(\varepsilon_{m,ph}/A_{area}) = 4218.03 \text{ kJ/m}^2$.

And the value obtained from the experiment is $(\varepsilon_{m,ph}/A_{area}) = 4147.35 \text{ kJ/m}^2$. For better accuracy of the calculations, the value obtained from the experiments should be considered for comparison. The main point is to deduce that the total exergy utilization to achieve a strain of the body of 0.01 can be used as a metric for exergy utilization to be compared with corresponding values for other processes. The total exergy utilization for volume $(\text{plt} = 1.024 \times 10^{-6} \text{ m}^3 \text{ segment of the work piece considered in the experiment})$ is equal to the product of exergy utilization per
unit square meter times the area of the work piece (pt). That is equal to $(\varepsilon_{m,ph}) = 1338 \text{ J}$ of exergy utilization for a given mechanical deformation of 0.01 strain.

The results obtained for the exergy utilization for a heating of the same mass of material as considered for mechanical processing for a temperature raise of 300K is 973 J. So, for one degree Kelvin raise in temperature, it takes 3.24 J of exergy utilization. Therefore by comparison, with 1338 J of exergy, the temperature of the same mass of material can be raised by 413 K. And comparing the other way around, the same exergy of 3.24 J can be used to strain the same mass of material to $0.02 \times 10^{-3}$. For better understanding of the concept, the following graphs are presented.
The point A in the graph represents the strain that is achieved by the use of 1338 J of exergy and the point B represents the amount of strain that can be produced by the use of 3.24 J of exergy. Point B in the graph is the representation of true strain that can be produced in a mechanical deformation process by using the same amount of exergy as that to raise the same mass of material by 1K.
Fig 18. Graph representing exergy loss vs temperature raise for Al alloy

The point C in the graph represents the temperature raise that can be achieved by the use of 1338 J of exergy and the point D represents the temperature raise that is achieved by the use of 3.24 J of exergy. Point C in the graph is the representation of temperature raise that can be produced in a thermal process by using the same amount of exergy as that to produce a strain of 0.01 for the same mass of material.
5.2 Results and Discussion for Mg Alloy 3003

From chapter 3, the results obtained for the exergy utilization per unit area for Mg alloy based either on analytical prediction or experimental work features a small variation. The average value of exergy utilization per unit area obtained from the theory using Ramberg-Osgood equation is \( (\varepsilon_{m,ph}/A_{area}) = 5338.39 \text{ kJ/m}^2 \). And the value obtained from the experiment is \( (\varepsilon_{m,ph}/A_{area}) = 5286.37 \text{ kJ/m}^2 \). For better accuracy of the calculations, the value from the experiments is considered for comparison. However, the point is to deduce that the total exergy utilization per unit area to achieve strain of the body of 0.01. Therefore, the total exergy utilization for a volume of the work piece (plt=1.024x10^{-6}) is equal to the product of exergy utilization per unit times the area of the body (pt). That is equal to \( (\varepsilon_{m,ph}) = 1705 \text{ J of exergy utilization} \).

The results obtained for the exergy utilization for a heating of the same mass of material as considered for mechanical processing for a temperature raise of 300K is 730 J. So, for one degree Kelvin raise in temperature, it takes 2.43 J of exergy utilization. Therefore by comparison, with 1705 J of exergy, the temperature of the same mass of material can be raised by 701 K. And comparing the other way around, the same exergy of 2.43 J can be used to strain the same mass of material to 0.014 x 10^{-3}. For better understanding of the concept, the following graphs are presented.
Fig 19. Graph represents exergy utilization vs temperature raise for Mg alloy

The point E in the graph represents the temperature raise that can be achieved by the use of 1705 J of exergy and the point F represents the temperature raise that is achieved by the use of 2.43 J of exergy. Point E in the graph is the representation of temperature raise that can be produced in a thermal process by using the same amount of exergy as that to produce a strain of 0.01 for the same mass of material.
The point G in the graph represents the amount of strain that is achieved by the use of 1705 J of exergy and the point H represents the amount of strain that can be produced by the use of 2.43 J of exergy. Point H in the graph is the representation of true strain that can be produced in a mechanical deformation process by using the same amount of exergy as that to raise the same mass of material by 1K.
6. CONCLUSION AND FUTURE SCOPE

6.1 Conclusion

The first auxiliary conclusion that can be drawn is that, the use of “Ramberg-Osgood” equation is justified and can be used in similar cases of material straining for these particular material types (Al and Mg alloys). However, the use of the actual “Tensile testing” (experimental data) gives the best results to use in the analysis.

The main conclusion is drawn from the analysis from the previous chapter. Exergy utilization can be used as a common language (as a metric) to assess entirely different processes. If a material undergoes a series of different operations to attain its final shape, one can calculate the total exergy utilization involved in the whole process. And if there is a second series of operations to attain the same product, a comparison of these two methods can be made using exergy utilization as a metric to assess the better of the two methods.

One can also infer that the work done in a mechanical processing of straining can be better efficient than the thermal processing of heating for the same exergy utilization. The plots (Fig.17-20) actually support the above statement.

To explicitly explain the above conclusion, let one consider an example. Let 20 J of exergy utilization be used to produce an effect in mechanical straining in Al alloy and the same 20 J be used to produce an effect in thermal heating process of the same alloy. 20 J can produce a strain...
of 0.00149 (approx) in mechanical straining. In a thermal process 20 J of exergy utilization will be reduced to 14.5 J (See chapter 4 for calculations) due to the losses involved in a thermal process. The temperature of the same body (as used in mechanical straining) can be raised by a temperature of 4.4 K (approx). So, if one has 20 J of exergy utilization to spend, one can achieve a strain of 0.00149 in mechanical processing or raise the same body to a temperature of 4.4 K in thermal processing.

6.2 Scope of the Future Work

The comparison of the two entirely different material processing is apparently like comparing apples and oranges if no common yardstick is used. In this analysis we attempted to define such yardstick. There are some assumptions considered in this thesis which might lead to certain variations in the results. For a better analysis and results the following parameters might be changed to achieve a closer comparison. The material in the thermal processing may not follow lumped capacitance method. So, an exact heat transfer study maybe required. The heating may not be a free convective heat transfer over a rectangular surface, in which case the coefficient of heat transfer changes and the same happens also with the calculated exergy utilization results. One might consider a different mechanical processing (like bending, drawing or extrusion) and compare these to yet another heating process. A more rigorous analysis of this concept will still yield interesting results and will draw useful conclusions to manufacturing field. The purpose of such analysis may be driven by sustainability considerations (energy resources study).
APPENDIX A

CALCULATION OF THE EXERGY UTILIZATION DURING STRAINING

The Eq. (A.1) gives a relationship between the stress and the corresponding strain values, or vise-versa [21].

\[ \varepsilon = \frac{\sigma}{E} + \frac{3}{7} \left( \frac{E}{\sigma} \right)^{n-1} \left( \frac{\sigma}{E} \right)^n \]  \hspace{1cm} (A.1)

![Graph of stress-strain relationship](image)

**Fig A1. The Area to Be Calculated**

The exergy of straining during mechanical processing can be calculated as work needed for the process, i.e.

\[ w = \int \sigma \, d\varepsilon \]  \hspace{1cm} (A.2)

Note that \( B = \int \sigma \, d\varepsilon \), which is exergy of mechanical processing.
So, the area marked in Fig A.1 beneath the stress-strain curve needs to be determined. Calculating the area under curve can easily performed. The required portion of the area is obtained by subtracting the area above that curve from the area of the rectangle ($\sigma^* \epsilon^*$). Therefore, the area above the curve [shown in Fig A1] should be calculated first.

Therefore, for ease of calculation, redraw the stress-strain curve as Fig A2. The plot in Fig A2 is a mirror image of the plot in Fig A1.

Let $A = (\epsilon^*, \sigma^*)$ be the points of our interest where the stress-strain curve terminates.

$I_1 = \text{Integral value of the area under the curve (shown in Fig A2); shares the same units as work done [J/m}^3]]$

$$I_1 = \int_0^{\sigma^*} \epsilon^* (\sigma) d\sigma \quad [\text{J/m}^3] \quad (A.3)$$
On substitution of the value of $\varepsilon(\sigma)$ from Eq. (A.2) in Eq. (A.3)

$$I_1 = \int_0^\sigma \left[ \frac{\sigma}{E} + \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{\sigma}{E} \right)^n \right] d\sigma \quad [J/m^3] \quad (A.4)$$

After integration, we have the following equation

$$I_1 = \left( \frac{\sigma^2}{2E} \right) \sigma^* + \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma_{n+1}}{n+1} \right) \left| \sigma^* \right| \quad [J/m^3] \quad (A.5)$$

After substituting the limits in the equation

$$I_1 = \left( \frac{\sigma^2}{2E} \right) + \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma_{n+1}}{n+1} \right) \quad [J/m^3] \quad (A.6)$$

We know that the area of the rectangle $= \sigma^* \varepsilon^*$

Therefore, the area under the stress-strain curve $= \text{Area of the rectangle} - I_1$

$$I_{\text{Total}} = \sigma^* \varepsilon^* - \left( \frac{\sigma^2}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma_{n+1}}{n+1} \right) \quad [J/m^3] \quad (A.7)$$

where $I_{\text{Total}} = \int \sigma d\varepsilon = W = \sigma^* \varepsilon^* - \left[ \varepsilon d\sigma \right]_0^\sigma \quad [J/m^3]$]

As we know, the area under the straight portion of the curve, $I_2 = \frac{1}{2} \sigma_y \varepsilon_y$

where $\sigma_y$ represents the stress at the yield point and $\varepsilon_y$ represents the corresponding strain.

This quantity represents as elastic work per unit volume of the strained material,

$$I_2 = \frac{1}{2} \sigma_y \varepsilon_y$$

The total exergy change of the material is given by the equation as follows
\[ B_{m, ph} = \int_V \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma_{n+1}}{n+1} \right) \right) dV \quad (A.8) \]

where \( B_{m, ph} = W \) represents work done during the mechanical deformation [J].

Upon integration, we have the following equation

\[ B_{m, ph} = \int_V \sigma^* \varepsilon^* dV - \int_V \left( \frac{\sigma^*}{2E} \right) dV - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma_{n+1}}{n+1} \right) dV \quad [J] \quad (A.9) \]

The volume \( V \) can be replaced by the plt in Eq. (A.9), where \( p \) represents the work piece width, \( t \) represents the work piece thickness and \( l \) represents the work piece length

\[ B_{m, ph} = \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma_{n+1}}{n+1} \right) \right) plt \quad [J] \quad (A.10) \]

\[ \frac{B_{m, ph}}{A_{area}} = \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma_{n+1}}{n+1} \right) \right) l \quad [J/m^2] \quad (A.11) \]

Eq. (A.11) represents the total exergy change per unit area. So, the final equation is given in Eq. (A.12).

\[ \frac{B_{m, ph\text{plastic}}}{A_{area}} = \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma_{n+1}}{n+1} \right) - \frac{(\sigma_y)^2}{2E} \right) l \quad [J/m^2] \quad (A.12) \]

Eq. (A.12) represents the plastic exergy change per unit area.

59
APPENDIX B

TRUE STRESS & TRUE STRAIN VALUES FROM EXPERIMENTS

To find the values of the true strain for the selected AA 3003 and Mg AZ31 alloys, the experimental procedure was specified as given in Chapter 3 (section 3.2.1.2). Tensile testing machine used was (Universal Tensile Testing Machine [UTM] see Fig 4.

The experiments were conducted on AA 3003 and Mg AZ31 alloys. Five trails for each of these alloys were conducted and the results obtained are as follows. Detailed trail 1 results for Al alloy and Mg alloy are recorded and a compilation data set is stored on a CD that constitutes the integral part of this thesis for illustration purposes. In the appendix data that follow, only the limited sets of data for each test are given for the illustration purposes (test date, specimen description, and geometry) and a sample of measured data presented for Al-1 and for Mg AZ31-1 is listed.
Trail 1 Experimental Results: Al alloy

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### Trail 3 Experimental Results: Al alloy

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<tr>
<td></td>
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</tr>
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### Trail 4 Experimental Results: Al alloy

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<td></td>
<td>Thickness: 0.003175 m (0.125 in)</td>
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<tr>
<td></td>
<td>Length: 0.00508 m (2 in)</td>
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Trail 5 Experimental Results: Al alloy

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Trail 1 Experimental Results: Mg alloy

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<th>Load (N)</th>
<th>Tensile strain (%)</th>
<th>Tensile stress (MPa)</th>
<th>True strain (%)</th>
<th>True stress (MPa)</th>
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<td>78.033966</td>
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<td>0.0033922</td>
<td>147.33737</td>
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<td>153.08428</td>
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<td>Specimen: Mg-2</td>
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<td>Geometry: Rectangular</td>
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### Trail 3 Experimental Results: Mg alloy

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<td>Geometry</td>
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</tr>
<tr>
<td>Thickness</td>
<td>0.003175 m (0.125 in)</td>
</tr>
<tr>
<td>Length</td>
<td>0.00508 m (2 in)</td>
</tr>
<tr>
<td>Area</td>
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### Trail 4 Experimental Results: Mg alloy

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<td>Thickness</td>
<td>0.003175 m (0.125 in)</td>
</tr>
<tr>
<td>Length</td>
<td>0.00508 m (2 in)</td>
</tr>
<tr>
<td>Area</td>
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### Trail 5 Experimental Results: Mg alloy

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<tr>
<td>Width: 0.00635 m (0.25 in)</td>
<td></td>
</tr>
<tr>
<td>Thickness: 0.003175 m (0.125 in)</td>
<td></td>
</tr>
<tr>
<td>Length: 0.00508 m (2 in)</td>
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</tr>
<tr>
<td>Area: 20.96 E-6 m² (0.0325 in²)</td>
<td></td>
</tr>
</tbody>
</table>

If one needs a complete set of data, please email to sekulicd@engr.uky.edu.
APPENDIX C

VERIFICATION OF THE VALIDITY OF THE RAMBERG-OSGOOD EQUATION

Before the Ramberg-Osgood equation is used, one needs to verify if application of this equation to this particular Al and Mg alloys is appropriate. So, a comparison of the values from the experiment to the Ramberg-Osgood equation must be performed. The percentage error between experiment and theory would subsequently be found. It was proposed that, if the average error between these is less than 10 %, then the use of Ramberg-Osgood equation is justified. Alternatively, one has to stick to the results obtained from experiment only.

This appendix provides a comparison of the true stress-strain values obtained from the experiment - tensile testing and the values obtained from the Ramberg-Osgood Equation. The series of values of each trail run are tabulated and the difference between the values and the percentage difference is calculated. A computer FORTRAN program is used to calculate these values. The FORTRAN program used and the values obtained are presented in Appendix E. Verification of this program is done in order to check if the program coded gives the true values and the error between the hand calculated and FORTRAN program is less than 1%, the difference being only due to the random error.
For Al Alloy

**ALUMINUM ALLOY TRAIL 1**

<table>
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<tr>
<th>TRUE STRAIN</th>
<th>TRUE STRESS[Experimental]</th>
<th>TRUE STRESS[Theory]</th>
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<th>% DIFFERENCE</th>
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<td>166.43</td>
<td>13.26</td>
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**AVG % DIFF** 3.97
Fig. C1. Aluminum trial 1 true stress vs true strain
## ALUMINUM ALLOY TRAIL 2

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<th>TRUE STRESS[Theory]</th>
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<th>% DIFFERENCE</th>
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Fig. C2. Aluminum trial 2 true stress vs true strain
### ALUMINUM ALLOY TRAIL 4

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<th>TRUE STRESS [Experimental]</th>
<th>TRUE STRESS [Theory]</th>
<th>DIFFERENCE</th>
<th>% DIFFERENCE</th>
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<td>0.80</td>
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**AVG % DIFF** 8.64
Fig. C4. Aluminum trial 4 true stress vs true strain
### ALUMINUM ALLOY TRIAL 5

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</table>

**AVG % DIFF** 4.00
Fig. C5. Aluminum trial 5 true stress vs true strain

OBSERVATIONS FROM THE TABLES & GRAPHS:

1) The percentage of error increase as we move towards the plastic region.

2) The area under the experimental curve is less than the area under the theoretical curve.

3) There is less plastic exergy involved in experimental curve than in the theoretically obtained one.
The percentage error between the theory and experiment is less than 10 %, in average. The Ramberg-Osgood equation is justified. Note, however, that the discrepancy significantly increases for larger strain values.

**For Mg Alloy**

The theoretical value is calculated using the Ramberg-Osgood equation and the following table presents the comparison between the theoretical. The experimental values and the percentage error between the two is also included.

**Mg ALLOY TRAIL 1**

<table>
<thead>
<tr>
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<th>TRUE STRESS[Theory]</th>
<th>DIFFERENCE</th>
<th>% DIFFERENCE</th>
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<td>8.10E-03</td>
<td>237.43</td>
<td>237.43</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>9.13E-03</td>
<td>249.27</td>
<td>249.27</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

AVG % DIFF  -4.71
Fig. C6. Magnesium trial 1 true stress vs true strain
### Mg ALLOY TRAIL 2

<table>
<thead>
<tr>
<th>TRUE STRAIN</th>
<th>TRUE STRESS[Experimental]</th>
<th>TRUE STRESS[Theory]</th>
<th>DIFFERENCE</th>
<th>% DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05E-03</td>
<td>48.97</td>
<td>41.36</td>
<td>-7.61</td>
<td>-18.39</td>
</tr>
<tr>
<td>2.13E-03</td>
<td>95.06</td>
<td>87.17</td>
<td>-7.89</td>
<td>-9.05</td>
</tr>
<tr>
<td>3.05E-03</td>
<td>131.10</td>
<td>124.09</td>
<td>-7.01</td>
<td>-5.65</td>
</tr>
<tr>
<td>4.15E-03</td>
<td>164.99</td>
<td>160.14</td>
<td>-4.85</td>
<td>-3.03</td>
</tr>
<tr>
<td>5.10E-03</td>
<td>188.40</td>
<td>184.31</td>
<td>-4.10</td>
<td>-2.22</td>
</tr>
<tr>
<td>6.09E-03</td>
<td>207.02</td>
<td>207.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7.19E-03</td>
<td>222.84</td>
<td>226.07</td>
<td>3.22</td>
<td>1.43</td>
</tr>
<tr>
<td>9.00E-03</td>
<td>247.15</td>
<td>252.83</td>
<td>5.68</td>
<td>2.25</td>
</tr>
</tbody>
</table>

**AVG % DIFF**  
-4.33
Fig. C7. Magnesium trial 2 true stress vs true strain
# Mg ALLOY TRAIL 3

<table>
<thead>
<tr>
<th>TRUE STRAIN</th>
<th>TRUE STRESS[Experimental]</th>
<th>TRUE STRESS[Theory]</th>
<th>DIFFERENCE</th>
<th>% DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.06E-03</td>
<td>44.90</td>
<td>44.90</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2.12E-03</td>
<td>90.70</td>
<td>90.70</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3.02E-03</td>
<td>126.69</td>
<td>120.58</td>
<td>-6.11</td>
<td>-5.06</td>
</tr>
<tr>
<td>4.12E-03</td>
<td>161.73</td>
<td>161.73</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5.07E-03</td>
<td>185.78</td>
<td>185.78</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>6.00E-03</td>
<td>204.44</td>
<td>204.44</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7.00E-03</td>
<td>220.44</td>
<td>223.66</td>
<td>3.22</td>
<td>1.44</td>
</tr>
<tr>
<td>9.07E-03</td>
<td>247.63</td>
<td>249.34</td>
<td>1.72</td>
<td>0.69</td>
</tr>
</tbody>
</table>

**AVG % DIFF** | -0.37
Fig. C8. Magnesium trial 3 true stress vs true strain
## Mg ALLOY TRAIL 4

<table>
<thead>
<tr>
<th>TRUE STRAIN</th>
<th>TRUE STRESS[Experimental]</th>
<th>TRUE STRESS[Theory]</th>
<th>DIFFERENCE</th>
<th>% DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.19E-03</td>
<td>54.16</td>
<td>54.16</td>
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<td>0.00</td>
</tr>
<tr>
<td>2.09E-03</td>
<td>92.50</td>
<td>84.86</td>
<td>-7.64</td>
<td>-9.01</td>
</tr>
<tr>
<td>3.18E-03</td>
<td>134.78</td>
<td>128.17</td>
<td>-6.61</td>
<td>-5.16</td>
</tr>
<tr>
<td>4.09E-03</td>
<td>162.84</td>
<td>157.46</td>
<td>-5.38</td>
<td>-3.42</td>
</tr>
<tr>
<td>5.21E-03</td>
<td>190.50</td>
<td>190.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.006028</td>
<td>205.60</td>
<td>205.60</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00711</td>
<td>221.47</td>
<td>221.47</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>9.13E-03</td>
<td>248.06</td>
<td>248.06</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**AVG % DIFF** = -2.20
Fig. C9. Magnesium trial 4 true stress vs true strain
Mg ALLOY TRAIL 5

<table>
<thead>
<tr>
<th>TRUE STRAIN</th>
<th>TRUE STRESS[Experimental]</th>
<th>TRUE STRESS[Theory]</th>
<th>DIFFERENCE</th>
<th>% DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.04E-03</td>
<td>43.44</td>
<td>43.44</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2.12E-03</td>
<td>89.93</td>
<td>89.93</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3.00E-03</td>
<td>125.22</td>
<td>125.22</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4.16E-03</td>
<td>162.02</td>
<td>162.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5.11E-03</td>
<td>185.85</td>
<td>185.85</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.006049</td>
<td>204.86</td>
<td>207.92</td>
<td>3.05</td>
<td>1.47</td>
</tr>
<tr>
<td>0.008165</td>
<td>234.90</td>
<td>237.67</td>
<td>2.77</td>
<td>1.17</td>
</tr>
<tr>
<td>9.18E-03</td>
<td>246.58</td>
<td>249.04</td>
<td>2.47</td>
<td>0.99</td>
</tr>
</tbody>
</table>

AVG % DIFF 0.45
The percentage difference in each trial is significantly less than 10 %, on average. Therefore the application of Ramberg-Osgood equation to calculate the exergy change is validated. Note that this equation much better represents the $\sigma$ vs $\varepsilon$ relationship for magnesium than it was the case for aluminum.
APPENDIX D

ELASTIC AND PLASTIC EXERGY CALCULATIONS

The equations for total and plastic exergy changes have been derived. In these calculations, consider the graphs obtained from the experiments. And calculate the value of “n” which is a characteristic constant which determines the shape of the curve and the values of $\sigma^*$, $\sigma_y$, $\varepsilon^*$ and $\varepsilon_y$ are taken from the graphs (which are drawn using the values obtained from the experiment).

The value of Young’s Modulus of Elasticity for Al alloy is taken as 68.95 GPa (given from the material supplier; McMaster) and for Mg alloy is taken as 45 MPa. In these particular calculations, the effect of the exergies between two different materials is analyzed. The calculations done are based on the true stress vs. true strain curves. First calculate the exergies of the four (3rd trial eliminated) sets for Al alloy followed by the Mg alloy. And the calculation of elastic and plastic exergies is performed at a true strain value of 0.01.

TRIAL 1: Aluminum alloy

To calculate the value of “n”, from Eq. (4)

$$n = 1 + \frac{17}{7} \ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)$$

Draw the true stress versus true strain graph for trail 1 for Al alloy using the experimental results. The stress-strain is limited to strain at 0.01 and graph obtained is as shown in he below figure.
Calculation of value of $\sigma_{0.7}$

It is known that the slope of stress strain curve gives the value of $E$. $\sigma_{0.7}$ is called the secant modulus at stress of 0.7E.

$$\frac{\sigma}{\varepsilon} = E = \text{Slope of Stress-Strain Curve [Hooke’s Law]}$$

Now for the same value of strain and for 0.7E; find the value of new stress.

i.e.

$$\frac{X}{\varepsilon} = 0.7E \Rightarrow X = \varepsilon \times 0.7E$$

Let one consider determining the secant modulus at 0.7 for trial 1 Al alloy for better illustration.

Referring to the values of true stress and true strain given by the experiment, any random value is
selected (within the linear region of the slope for Hooke’s law to be valid). Let one pick a stress value of 61 MPa and the corresponding strain is 0.00092.

The next step is calculating the value of $X$, substituting the value in the above equation, one gets the value of $X$ to be 44.2 MPa. A line is drawn from origin passing through this value of $X$ and the line is extended till it intersects the stress-strain curve. The point A in the curve is the point of intersection of the stress-strain curve and the line (passing thorough origin and point $X$). The corresponding value of stress at this point A is the secant modulus at 0.7 strain, which is found to be $\sigma_{0.7} = 132$ MPa. The point B in the above figure represents the secant modulus at 0.85 strain, i.e. $\sigma_{0.85} = 117$ MPa

**Calculation of value of n**

The equation for the value of n and given as below.

\[
n = 1 + \frac{\ln \frac{17}{7}}{\ln \left(\frac{\sigma_{0.7}}{\sigma_{0.85}}\right)}
\]

The value of $\sigma_{0.85}$ is calculated similar to that of $\sigma_{0.7}$. The two dotted lines seen in the above diagram represent this secant modulus.

The calculated values of $\sigma_{0.7}$, $\sigma_{0.85}$ and n are as follows.

$\sigma_{0.85} = 117$ MPa

$\sigma_{0.7} = 132$ MPa

n = 8.35
As the values of $\sigma_{0.7}$, $\sigma_{0.85}$ and n are known now, proceed further in the calculations. And length $l$ = 2 in = 0.0508 mm and $E = 68.95$ GPa

1 psi = 6894.76 Pa

$$\sigma_y = 17000 \text{ psi} = 117,210,869.00 \text{ Pa} = 117.21 \text{ Mpa} \text{ [from McMaster, supplier]}$$

$$\varepsilon_y = 0.0022$$

[True strain corresponding to the true stress, from appendix B]

$$E = 68.95 \text{ GPa} = 68947.59 \text{ MPa} = 68947.59 \text{ N/mm}^2$$

$$\sigma^* = 155.14 \text{ MPa}$$

$$\varepsilon^* = 0.01$$

Exergy of the material per unit area $= \frac{B_{m,ph}}{A_{Area}}$ i.e. Equation (4)

$$\frac{B_{m,ph}}{A_{Area}} = \left( \sigma^* \varepsilon^* - \frac{\sigma^*}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n+1} \right) l$$

On substitution of these values in the above equation,

$$\frac{B_{m,ph}}{A_{Area}} = [1.5514-0.17454-0.05246] l$$

$$\frac{B_{m,ph}}{A_{Area}} = 4.2049 \text{ MJ/m}^2 = 4204.9 \text{ KJ/m}^2 = \text{Exergy of the material per unit area}$$

Therefore, plastic exergy of the material per unit area can be calculated from Eq. (6)
\[ \frac{B_{m,ph \text{ Plastic}}}{A_{\text{Area}}} = \frac{B_{m,ph}}{A_{\text{Area}}} - l \left( \frac{\sigma^2}{2E} \right) \]

\[ \frac{B_{m,ph \text{ Plastic}}}{A_{\text{Area}}} = 4.2049 - 0.31623 \]

\[ \frac{B_{m,ph \text{ Plastic}}}{A_{\text{Area}}} = 3.8886 \text{ MJ/m}^2 = 3888.67 \text{ KJ/m}^2 = \text{Plastic Exergy of the material per unit area} \]

**TRIAL 2: Aluminum alloy**

To calculate the value of “n”, from Eq. (4)

\[ n = 1 + \frac{\ln 17}{\ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)} \]

The values of \( \sigma_{0.7} \), \( \sigma_{0.85} \) and \( n \) are already calculated, proceeding further in the calculations.

\( \sigma_{0.85} = 117 \text{ MPa} \)

\( \sigma_{0.7} = 135 \text{ MPa} \)

\( n = 7.20 \)

These values are calculated in the same procedure as shown in the previous case.

And length \( l = 2\text{in} = 0.0508 \text{ mm} \) and \( E = 68.95 \text{ GPa} \)

1 psi = 6894.76 Pa

\[ \sigma_y = 17000 \text{ psi} = 117,210,869.00 \text{ Pa} = 117.21 \text{ Mpa} \text{ [from McMaster, supplier]} \]

\[ \varepsilon_y = 0.0022 \]
[True strain corresponding to the true stress, from appendix B]

\[ E = 68.9475 \text{ GPa} = 68947.59 \text{ MPa} = 68947.59 \text{ N/mm}^2 \]

\[ \sigma^* = 154.75 \text{ MPa} \]

\[ \varepsilon^* = 0.01 \]

Exergy of the material per unit area = \( \frac{B_{m,ph}}{A_{Area}} \) i.e. Eq. (5)

\[
\frac{B_{m,ph}}{A_{Area}} = \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma^*} \right)^n \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n + 1} \right) \right) l
\]

On substitution of these values in the above equation,

\[
\frac{B_{m,ph}}{A_{Area}} = [1.54755-0.173676-0.04233] l
\]

\[
\frac{B_{m,ph}}{A_{Area}} = 4.22765 \text{ MJ/m}^2 = 42276.5 \text{ KJ/m}^2 = \text{Exergy of the material per unit area}
\]

Therefore, plastic exergy of the material per unit area can be calculated from Eq. (6)

\[
\frac{B_{m,ph Plastic}}{A_{Area}} = \frac{B_{m,ph}}{A_{Area}} - l \left( \frac{\sigma^2}{2E} \right)
\]

\[
\frac{B_{m,ph Plastic}}{A_{Area}} = 4.22765-0.31631
\]
\[ B_{m,ph\text{ Plastic}} \]
\[ \frac{3.91134 \text{ MJ/m}^2 = 3911.34 \text{ KJ/m}^2 = \text{ Plastic Exergy of the material per unit area}}{A_{\text{Area}} \]  

**TRIAL 4: Aluminum alloys**

To calculate the value of “n”, from Eq. (4)

\[
n = 1 + \frac{\ln \frac{17}{7}}{\ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)}
\]

The values of \( \sigma_{0.7}, \sigma_{0.85} \) and n are already calculated and proceeding further in the calculations.

\( \sigma_{0.85} = 120 \text{ MPa} \)

\( \sigma_{0.7} = 137 \text{ MPa} \)

n = 7.78

And length \( l = 2 \text{ in} = 0.0508 \text{ mm} \) and \( E = 68.95 \text{ GPa} \)

1 psi = 6894.76 Pa

\( \sigma_y = 17000 \text{ psi} = 117,210,869.00 \text{ Pa} = 117.21 \text{ Mpa} \) [from McMaster, supplier]

\( \varepsilon_y = 0.00223 \)

[True strain corresponding to the true stress, from appendix B]

\( E = 68.9475 \text{ GPa} = 68947.59 \text{ MPa} = 68947.59 \text{ N/mm}^2 \)

\( \sigma^* = 155.32 \text{ MPa} \)

\( \varepsilon^* = 0.01 \)
Exergy of the material per unit area $= \frac{B_{m,ph}}{A_{Area}}$ i.e. Equation (4)

$$\frac{B_{m,ph}}{A_{Area}} = \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{E} \right)^2 \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n + 1} \right) l$$

On substitution of these values in the above equation,

$$\frac{B_{m,ph}}{A_{Area}} = [1.553211-0.174949-0.04006] l$$

$$\frac{B_{m,ph}}{A_{Area}} = 4.24896 \text{ MJ/m}^2 = 4248.96 \text{ KJ/m}^2 = \text{Exergy of the material per unit area}$$

Therefore, plastic exergy of the material per unit area can be calculated from Eq. (6)

$$\frac{B_{m,ph\text{ Plastic}}}{A_{Area}} = \frac{B_{m,ph}}{A_{Area}} - l \left( \frac{\sigma^2}{2E} \right)$$

$$\frac{B_{m,ph\text{ Plastic}}}{A_{Area}} = 4.24896-0.31631$$

$$\frac{B_{m,ph\text{ Plastic}}}{A_{Area}} = 3.9326 \text{ MJ/m}^2 = 3932.64 \text{ KJ/m}^2 = \text{Plastic Exergy of the material per unit area}$$

**TRIAL 5: Aluminum alloy**

To calculate the value of “n”, from Eq. (4)
\[ n = 1 + \frac{\ln \frac{17}{7}}{\ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)} \]

The values of \( \sigma_{0.7} \), \( \sigma_{0.85} \) and \( n \) are already calculated and proceeding further in the calculations.

\( \sigma_{0.85} = 125 \text{ MPa} \)

\( \sigma_{0.7} = 135 \text{ MPa} \)

\( n = 12.53 \)

And length \( l = 2\text{in} = 0.0508 \text{ mm} \) and \( E = 68.9475 \text{ GPa} \)

1 psi = 6894.76 Pa

\[ \sigma_y = 17000 \text{ psi} = 117,210,869.00 \text{ Pa} = 117.21 \text{ Mpa} \text{ [from McMaster, supplier]} \]

\[ \varepsilon_y = 0.0022 \]

[True strain corresponding to the true stress, from appendix B]

\[ E = 68.9475 \text{ GPa} = 68947.59 \text{ MPa} = 68947.59 \text{ N/mm}^2 \]

\( \sigma^* = 154.4728 \text{ MPa} \)

\( \varepsilon^* = 0.01 \)

Exergy of the material per unit area = \( \frac{B_{m,ph}}{A_{Area}} \) i.e. Equation (4)

\[ \frac{B_{m,ph}}{A_{Area}} = \left[ \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right)^2 \right] - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n+1} \right) l \]

On substitution of these values in the above equation,
\[
\frac{B_{m,ph}}{A_{\text{Area}}} = [1.544728-0.17304-0.0518]l \\
\]
\[
\frac{B_{m,ph}}{A_{\text{Area}}} = 4.1906 \text{ MJ/m}^2 = 4190.644 \text{ KJ/m}^2 \text{ = Exergy of the material per unit area}
\]

Therefore, plastic exergy of the material per unit area can be calculated from Equation (5)

\[
\frac{B_{m,ph,\text{Plastic}}}{A_{\text{Area}}} = \frac{B_{m,ph}}{A_{\text{Area}}} - l \left( \frac{\sigma^2}{2E} \right)
\]

\[
\frac{B_{m,ph,\text{Plastic}}}{A_{\text{Area}}} = 4.1906-0.3163 = 3.87428 \text{ MJ/m}^2 = 3874.28 \text{ KJ/m}^2 \text{ = Plastic Exergy of the material per unit area}
\]

**TRAIL 1: Magnesium alloy**

To calculate the value of “n”, from Eq. (4)

\[n = 1 + \frac{\ln \frac{17}{7}}{\ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)} \]

The values of \(\sigma_{0.7}, \sigma_{0.85}\) and n are already calculated and proceeding further in the calculations.

\(\sigma_{0.85} = 200 \text{ MPa}\)

\(\sigma_{0.7} = 240 \text{ MPa}\)

n = 5.87

And thickness length \(l = 2\text{in} = 0.0508 \text{ mm}\) and \(E = 45 \text{ GPa}\)
\[ \sigma_y = 260 \text{ Mpa} \]

\[ E = 45 \text{ GPa} = 45,000 \text{ MPa} = 45,000 \text{ N/mm}^2 \]

\[ \sigma^* = 257.54 \text{ MPa} \]

\[ \varepsilon^* = 0.01 \]

Exergy of the material per unit area = \[ \frac{B_{m,ph}}{A_{\text{Area}}} \]
i.e. Eq. (5)

\[ \frac{B_{m,ph}}{A_{\text{Area}}} = \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right)^2 \right) - \frac{3}{7} \left( \frac{E}{\sigma^*} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n+1} \right) l \]

On substitution of these values in the above equation,

\[ \frac{B_{m,ph}}{A_{\text{Area}}} = [2.575-0.7369-0.129] l \]

\[ \frac{B_{m,ph}}{A_{\text{Area}}} = 5.3389 \text{ MJ/m}^2 = 5338.90 \text{ KJ/m}^2 \]

= Exergy of the material per unit area

Therefore, plastic exergy of the material per unit area can be calculated from Eq. (6)

\[ \frac{B_{m,ph \text{Plastic}}}{A_{\text{Area}}} = \frac{B_{m,ph}}{A_{\text{Area}}} - l \left( \frac{\sigma^*}{2E} \right) \]

\[ \frac{B_{m,ph \text{Plastic}}}{A_{\text{Area}}} = 5.3389-2.347 \]
\[ B_{\text{m,ph Plastic}} \]
\[ \frac{A_{\text{Area}}}{2.99 \text{ MJ/m}^2 = 2991.67 \text{ KJ/m}^2} \]

**TRIAL 2: Magnesium alloy**

To calculate the value of “n”, from Eq. (4)

\[ n = 1 + \frac{\ln \frac{17}{7}}{\ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)} \]

The values of \( \sigma_{0.7}, \sigma_{0.85} \) and n are already calculated and proceeding further in the calculations.

\( \sigma_{0.85} = 205 \text{ MPa} \)

\( \sigma_{0.7} = 245 \text{ MPa} \)

\( n = 5.37 \)

And length \( l = 2 \text{ in} = 0.0508 \text{ mm} \) and \( E = 45 \text{ GPa} \)

\( \sigma_y = 260 \text{ Mpa} \)

\( E = 45 \text{ GPa} = 45,000 \text{ MPa} = 45,000 \text{ N/mm}^2 \)

\( \sigma^* = 256.48 \text{ MPa} \)

\( \varepsilon^* = 0.01 \)

Exergy of the material per unit area = \( \frac{B_{\text{m,ph}}}{A_{\text{Area}}} \) i.e. Eq. (5)

\[ \frac{B_{\text{m,ph}}}{A_{\text{Area}}} = \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right) \right) \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n + 1} \right)^n \]

97
On substitution of these values in the above equation,

\[
\frac{B_{m,ph}}{A_{Area}} = [2.5648-0.7309-0.120] \ l
\]

\[
\frac{B_{m,ph}}{A_{Area}} = 5.36 \ \text{MJ/m}^2 = 5355.9 \ \text{KJ/m}^2 = \text{Exergy of the material per unit area}
\]

Therefore, plastic exergy of the material per unit area can be calculated from Eq. (6)

\[
\frac{B_{m,ph\ \text{Plastic}}}{A_{Area}} = \frac{B_{m,ph}}{A_{Area}} - l \left( \frac{\sigma^2}{2E} \right)
\]

\[
\frac{B_{m,ph\ \text{Plastic}}}{A_{Area}} = 5.3389-2.3847
\]

\[
\frac{B_{m,ph\ \text{Plastic}}}{A_{Area}} = 2.97 \ \text{MJ/m}^2 = 2971.12 \ \text{KJ/m}^2 = \text{Plastic Exergy of the material per unit area}
\]

**TRIAL 3: Magnesium alloy**

To calculate the value of “n”, from Eq. (4)

\[
n = 1 + \frac{\ln \frac{17}{7}}{\ln \left( \frac{\sigma_{0,7}}{\sigma_{0,85}} \right)}
\]

The values of \(\sigma_{0,7}\), \(\sigma_{0,85}\) and n are already calculated and proceeding further in the calculations.

\(\sigma_{0,85} = 200\ \text{MPa}\)

\(\sigma_{0,7} = 240 \ \text{MPa}\)
\[ n = 5.87 \]

And length \( l = 2 \text{ in} = 0.0508 \text{ mm} \) and \( E = 45 \text{ GPa} \)

\[ \sigma_y = 260 \text{ Mpa} \]

\[ E = 45 \text{ GPa} = 45,000 \text{ MPa} = 45,000 \text{ N/mm}^2 \]

\[ \sigma^* = 257 \text{ MPa} \]

\[ \varepsilon^* = 0.01 \]

Exergy of the material per unit area = \( \frac{B_{m,ph}}{A_{\text{Area}}} \) i.e. Eq. (5)

\[
\frac{B_{m,ph}}{A_{\text{Area}}} = \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n+1} \right) l
\]

On substitution of these values in the above equation,

\[
\frac{B_{m,ph}}{A_{\text{Area}}} = [2.57-0.734-0.127] l
\]

\[
\frac{B_{m,ph}}{A_{\text{Area}}} = 5.33 \text{ MJ/m}^2 = 5338.75 \text{ KJ/m}^2 = \text{Exergy of the material per unit area}
\]

Therefore, plastic exergy of the material per unit area can be calculated from Eq. (6)

\[
\frac{B_{m,ph \text{ Plastic}}}{A_{\text{Area}}} = \frac{\varepsilon_{m,ph}}{A_{\text{Area}}} - l \left( \frac{\sigma^2}{2E} \right)
\]

\[
\frac{B_{m,ph \text{ Plastic}}}{A_{\text{Area}}} = 5.33-2.34
\]
\[ B_{m,\text{ph Plastic}} \]
\[ \frac{A_{\text{Area}}}{2.99 \text{ MJ/m}^2 = 2991.52 \text{ KJ/m}^2} = \text{Plastic Exergy of the material per unit area} \]

TRIAL 4: Magnesium alloy

To calculate the value of “n”, from Eq. (4)

\[ n = 1 + \frac{\ln \frac{17}{7}}{\ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)} \]

The values of \( \sigma_{0.7}, \sigma_{0.85} \) and n are already calculated and proceeding further in the calculations.

\( \sigma_{0.85} = 200 \text{ MPa} \)

\( \sigma_{0.7} = 238 \text{ MPa} \)

n = 6.10

And length \( l = 2\text{in} = 0.0508 \text{ mm} \) and \( E = 45 \text{ GPa} \)

\( \sigma_y = 260 \text{ Mpa} \)

\( E = 45 \text{ GPa} = 45,000 \text{ MPa} = 45,000 \text{ N/mm}^2 \)

\( \sigma^* = 256.84 \text{ MPa} \)

\( \varepsilon^* = 0.01 \)

Exergy of the material per unit area = \( \frac{B_{m,\text{ph}}}{A_{\text{Area}}} \) i.e. Eq. (5)

\[ \frac{B_{m,\text{ph}}}{A_{\text{Area}}} = \left( \sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2 E} \right) \right) - \frac{3}{7} \left( \frac{E}{\sigma_{0.7}} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n + 1} \right) l \]

On substitution of these values in the above equation,
\[
\frac{B_{m,ph}}{A_{\text{Area}}} = [2.57-0.73-0.13] l
\]

\[
\frac{B_{m,ph}}{A_{\text{Area}}} = 5.32 \text{MJ/m}^2 = 5328.10 \text{KJ/m}^2 = \text{Exergy of the material per unit area}
\]

Therefore, plastic exergy of the material per unit area can be calculated from Eq. (6)

\[
\frac{B_{m,ph \text{ Plastic}}}{A_{\text{Area}}} = \frac{\varepsilon_{m,ph}}{A_{\text{Area}}} - l \left( \frac{\sigma^2}{2E} \right)
\]

\[
\frac{B_{m,ph \text{ Plastic}}}{A_{\text{Area}}} = 5.32 - 2.35
\]

\[
\frac{B_{m,ph \text{ Plastic}}}{A_{\text{Area}}} = 2.98 \text{ MJ/m}^2 = 2980.8 \text{KJ/m}^2 = \text{Plastic Exergy of the material per unit area}
\]

**TRIAL 5: Magnesium alloy**

To calculate the value of “n”, from Eq. (4)

\[
n = 1 + \frac{\ln \frac{17}{7}}{\ln \left( \frac{\sigma_{0.7}}{\sigma_{0.85}} \right)}
\]

The values of \(\sigma_{0.7}, \sigma_{0.85}\) and n are already calculated and proceeding further in the calculations.

\[\sigma_{0.85} = 200 \text{ MPa}\]

\[\sigma_{0.7} = 240 \text{ MPa}\]

\[n = 5.87\]

And length \(l = 2\text{in} = 0.0508 \text{ mm}\) and \(E = 45 \text{ GPa}\)
\( \sigma_y = 260 \text{ Mpa} \)

\[
E = 45 \text{ GPa} = 45,000 \text{ MPa} = 45,000 \text{ N/mm}^2
\]

\( \sigma^* = 254.35 \text{ MPa} \)

\( \varepsilon^* = 0.01 \)

Exergy of the material per unit area \( \frac{B_{m,ph}}{A_{\text{Area}}} \)

\[
B_{m,ph} = \frac{\sigma^* \varepsilon^* - \left( \frac{\sigma^*}{2E} \right) - \frac{3}{7} \left( \frac{E}{\sigma^*} \right)^{n-1} \left( \frac{1}{E} \right)^n \left( \frac{\sigma^*}{n+1} \right)}{E_{\text{Area}}}
\]

On substitution of these values in the above equation,

\[
B_{m,ph} = \frac{[2.54-0.72-0.12]}{l}
\]

\[
\frac{B_{m,ph}}{A_{\text{Area}}} = 5.33 \text{ MJ/m}^2 = 5330.30 \text{ KJ/m}^2 = \text{Exergy of the material per unit area}
\]

Therefore, plastic exergy of the material per unit area can be calculated from Eq. (6)

\[
\frac{B_{m,ph\text{Plastic}}}{A_{\text{Area}}} = \frac{\varepsilon_{m,ph}}{A_{\text{Area}}} - l \left( \frac{\sigma^*}{2E} \right)
\]

\[
\frac{B_{m,ph\text{Plastic}}}{A_{\text{Area}}} = 5.32-2.35
\]
\[ \frac{B_{m,ph\,Plastic}}{A_{Area}} = 2.98\text{MJ/m}^2 = 2983.1\text{KJ/m}^2 = \text{Plastic Exergy of the material per unit area} \]
APPENDIX E

FORTRAN PROGRAM AND RESULTS

Nomenclature used in the FORTRAN program

1) e = E = Young’s Modulus of Elasticity

2) sig07 = $\sigma_{0.7}$ = Secant Modulus at 0.7E

3) cnst = 3/7

4) rn = n = Characteristic Constant (describing the shape of the curve)

5) epsi = $\varepsilon$ = True Strain

6) i = Iteration Number (which equals to the number to stress values as input)

TRIAL 1 [Al alloy]

! Program stress strain

IMPLICIT NONE

! implicit real*8 (a-h,o-z) !i,j,k,l,m,n

!-- Declaring Arrays

real(8), Dimension(1:1000):: sig, epsi

INTEGER:: i

REAL(KIND=8):: e,cnst,sig07,rn

!-- Opening input data files

Open(18,file='sundy-Al_1.dat')

104
Open(19,file='Al_1-graph.dat')

-- Reading input file into an array. i counter must be varied depending
! on number of input data points.
Do i =1,145
read(18,*) sig(i)
End Do
close(18)

-- Echo above read
!Do i =1,999
!Write(*,*) sig(i), i
!End Do

-- Defining constants in the equation
  e = 68947.59d0
  cnst = 3.0d0/7.0d0
  sig07 = 132d0
  rn = 8.35569d0

-- Formula
  Do i =1, 145
    epsi(i) = (sig(i)/e)+(cnst*((e/sig07)**(rn-1))*((sig(i)/e)**rn))
  End Do

-- Writing to a file for output plots
Do i =1,997
Write(19,*) epsi(i), sig(i)

End Do

End program

As an example, some of the input and output values for this particular program are as listed in the following table.

<table>
<thead>
<tr>
<th>INPUT STRESS VALUES</th>
<th>OUTPUT STRAIN VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.522585347</td>
<td>2.21E-05</td>
</tr>
<tr>
<td>6.300360816</td>
<td>9.18E-05</td>
</tr>
<tr>
<td>11.70382139</td>
<td>0.00016975</td>
</tr>
<tr>
<td>17.44861219</td>
<td>0.000253071</td>
</tr>
<tr>
<td>130.3888405</td>
<td>0.002631609</td>
</tr>
<tr>
<td>131.9958439</td>
<td>0.002734721</td>
</tr>
<tr>
<td>133.076817</td>
<td>0.00280825</td>
</tr>
<tr>
<td>134.7925538</td>
<td>0.002932345</td>
</tr>
<tr>
<td>155.2528961</td>
<td>0.005434984</td>
</tr>
<tr>
<td>160.2449403</td>
<td>0.006470965</td>
</tr>
<tr>
<td>274.4282767</td>
<td>0.375494337</td>
</tr>
<tr>
<td>276.4282767</td>
<td>0.398762637</td>
</tr>
</tbody>
</table>

TRIAL 2 [Al alloy]

! Program stressstrain

! implicit real*8 (a-h,o-z) !i,j,k,l,m,n
!-- Declaring Arrays

real, Dimension(1:1000):: sig, epsi

INTEGER:: i

REAL(KIND=8):: e, cnst, sig07, rn

!-- Opening input data files

Open(18, file='sundy-Al_2.dat')

!-- Opening file for output

Open(19, file='Al_2-graph.dat')

!-- Reading input file into an array. i counter must be varied depending
! on number of input data points.

Do i = 1, 145

read(18, *) sig(i)

End Do

close(18)

!-- Echo above read

! Do i = 1, 1999

! Write(*, *) sig(i), i

! End Do

!-- Defining constants in the equation

e = 68947.59d0

cnst = 3.0d0/7.0d0

sig07 = 135d0

rn = 7.20084481d0
TRIAL 4 [Al alloy]

! Program stressstrain

! implicit real*8 (a-h,o-z) !i,j,k,l,m,n

!-- Declaring Arrays
real(8), Dimension(1:1000):: sig, epsi
INTEGER:: i
REAL(KIND=8)::e,cnst,sig07,rn

!-- Opening input data files
Open(18,file='sundy-Al_4.dat')

!-- Opening file for output
Open(19,file='Al_4-graph.dat')

!-- Reading input file into an array. i counter must be varied depending
! on number of input data points.
Do i =1,145
read(18,*) sig(i)
End Do
close(18)
!-- Echo above read
!Do i =1,999
!Write(*,*) sig(i), i
!End Do
!-- Defining constants in the equation
   e = 68947.59d0
   cnst = 3.0d0/7.0d0
   sig07 = 137d0
   rn = 7.69717d0
!-- Formula
   Do i =1, 145
      epsi(i) = (sig(i)/e)+ (cnst*((e/sig07)**(rn-1))*((sig(i)/e)**rn))
   End Do
!-- Writing to a file for output plots
   Do i =1,576
      Write(19,*) epsi(i), sig(i)
   End Do
End program
TRIAL 5 [Al alloy]

!Program stress strain

! implicit real*8 (a-h,o-z) !i,j,k,l,m,n

!-- Declaring Arrays

real(8), Dimension(1:1000):: sig, epsi

INTEGER:: i

REAL(KIND=8)::e,cnst,sig07,rn

!-- Opening input data files

Open(18,file='sundy-Al_5.dat')

!-- Opening file for output

Open(19,file='Al_5-graph.dat')

!-- Reading input file into an array. i counter must be varied depending
! on number of input data points.

Do i =1,145

read(18,*) sig(i)

End Do

close(18)

!-- Echo above read

!Do i =1,999

!Write(*,*) sig(i), i

!End Do
!-- Defining constants in the equation

\[ e = 68947.59d0 \]
\[ \text{cnst} = 3.0d0/7.0d0 \]
\[ \text{sig07} = 135.0d0 \]
\[ \text{rn} = 12.52925d0 \]

!-- Formula

\[
\text{Do } i = 1, 145 \\
\text{epsi}(i) = (\text{sig}(i)/e) + (\text{cnst}*((e/\text{sig07})**(\text{rn}-1))*((\text{sig}(i)/e)**\text{rn}))
\]
End Do

!-- Writing to a file for output plots

\[
\text{Do } i = 1, 997 \\
\text{Write}(19,*) \text{epsi}(i), \text{sig}(i)
\]
End Do

End program

TRIAL 1 [Mg alloy]

! Program stressstrain

! IMPLICIT NONE

! implicit real*8 (a-h,o-z) !i,j,k,l,m,n

!-- Declaring Arrays

real(8), Dimension(1:1000):: sig, epsi
INTEGER:: i
REAL(KIND=8):: e, cnst, sig07, rn
!-- Opening input data files
Open(18, file='Mg1_cp.dat')

!-- Opening file for output
Open(19, file='Mg_1-graph.dat')

!-- Reading input file into an array. i counter must be varied depending
! on number of input data points.
Do i = 1, 620
    read(18, *) sig(i)
End Do

close(18)

!-- Echo above read
!Do i = 1, 999
!Write(*, *) sig(i), i
!End Do

!-- Defining constants in the equation
    e = 41204.42d0
    cnst = 3.0d0/7.0d0
    sig07 = 240.d0
    rn = 5.8667d0
!
-- Formula
    Do i = 1, 620
        epsi(i) = (sig(i)/e) + (cnst* ((e/sig07)**(rn-1))*((sig(i)/e)**rn))
    End Do
TRIAL 2 [Mg alloy]

! Program stressstrain

! IMPLICIT NONE

! implicit real*8 (a-h,o-z) !i,j,k,l,m,n

!-- Declaring Arrays

real(8), Dimension(1:1000):: sig, epsi

INTEGER:: i

REAL(KIND=8)::e,cnst,sig07,rn

!-- Opening input data files

Open(18,file='Mg2_cp.dat')

!-- Opening file for output

Open(19,file='Mg_2-graph.dat')

!-- Reading input file into an array. i counter must be varied depending
! on number of input data points.

Do i =1,709

read(18,*) sig(i)

End Do
close(18)

!-- Echo above read

!Do i = 1, 999

!Write(*,*) sig(i), i

!End Do

!-- Defining constants in the equation

e = 41204.42d0

cnst = 3.0d0/7.0d0

sig07 = 245.d0

rn = 5.372d0

!-- Formula

Do i = 1, 709

\[ \varepsilon(i) = \frac{\sigma(i)}{e} + \left( cnst \times \left( \frac{e}{\sigma07} \right)^{rn-1} \times \left( \frac{\sigma(i)}{e} \right)^{rn} \right) \]

End Do

!-- Writing to a file for output plots

Do i = 1, 709

Write(19,*) epsi(i), sig(i)

End Do

End program

TRIAL 3[Mg alloy]

! Program stressstrain

! IMPLICIT NONE
! implicit real*8 (a-h,o-z) !i,j,k,l,m,n

!-- Declaring Arrays
real(8), Dimension(1:1000):: sig, epsi
INTEGER:: i
REAL(KIND=8):: e, cnst, sig07, rn

!-- Opening input data files
Open(18, file='Mg3_cp.dat')

!-- Opening file for output
Open(19, file='Mg_3-graph.dat')

!-- Reading input file into an array. i counter must be varied depending
! on number of input data points.
Do i = 1, 764
   read(18, *) sig(i)
End Do

Close(18)

!-- Echo above read
! Do i = 1, 999
! Write(*, *) sig(i), i
! End Do

!-- Defining constants in the equation
   e = 41204.42d0
   cnst = 3.0d0/7.0d0
   sig07 = 240.0d0
rn = 5.8667d0

!-- Formula
Do i =1, 764
   epsi(i) = (sig(i)/e) + (cnst*((e/sig07)**(rn-1))*((sig(i)/e)**rn))
End Do

!-- Writing to a file for output plots
Do i =1, 764
Write(19,*) epsi(i), sig(i)
End Do

End program

TRIAL 4[Mg alloy]

! Program stressstrain

! IMPLICIT NONE

! implicit real*8 (a-h,o-z) !i,j,k,l,m,n

!-- Declaring Arrays
real(8), Dimension(1:1000):: sig, epsi
INTEGER:: i
REAL(KIND=8):: e, cnst, sig07, rn

!-- Opening input data files
Open(18,file='Mg4_cp.dat')

!-- Opening file for output
Open(19,file='Mg_4-graph.dat')
!-- Reading input file into an array. i counter must be varied depending
! on number of input data points.
Do i =1,645
read(18,*) sig(i)
End Do
close(18)

!-- Echo above read
!Do i =1,999
!Write(*,*) sig(i), i
!End Do

!-- Defining constants in the equation
e = 41204.42d0
cnst = 3.0d0/7.0d0
sig07 = 238.d0
rn = 6.1008d0

!-- Formula
Do i =1, 645
  epsi(i) = (sig(i)/e)+ (cnst*((e/sig07)**(rn-1))*((sig(i)/e)**rn))
End Do

!-- Writing to a file for output plots
Do i =1,645
Write(19,*) epsi(i), sig(i)
End Do
END PROGRAM

TRIAL 5[Mg alloy]

! Program stressstrain

! IMPLICIT NONE

! implicit real*8 (a-h,o-z) !i,j,k,l,m,n

!-- Declaring Arrays

real(8), Dimension(1:1000):: sig, epsi

INTEGER:: i

REAL(KIND=8):: e, cnst, sig07, rn

!-- Opening input data files

Open(18, file='Mg5_cp.dat')

!-- Opening file for output

Open(19, file='Mg_5-graph.dat')

!-- Reading input file into an array. i counter must be varied depending
! on number of input data points.

Do i = 1, 757

read(18, *) sig(i)

End Do

close(18)

!-- Echo above read

! Do i = 1, 999

!Write(*,*) sig(i), i
!End Do

!-- Defining constants in the equation

\[ e = 41204.42 \text{d}0 \]
\[ \text{cnst} = 3.0 \text{d}0 / 7.0 \text{d}0 \]
\[ \text{sig07} = 240.0 \text{d}0 \]
\[ \text{rn} = 5.8669 \text{d}0 \]

!-- Formula

\[
\text{Do } i = 1, 757
\]
\[
\text{epsi}(i) = (\text{sig}(i)/e) + (\text{cnst} * ((e/\text{sig07})^{(\text{rn}-1)} * ((\text{sig}(i)/e)^{\text{rn}}))
\]

End Do

!-- Writing to a file for output plots

\[
\text{Do } i = 1, 757
\]

Write(19,* epsi(i), sig(i))

End Do

End program
MATLAB Program for Al Alloy:
The input values for this program are the stress and strain values from taken from APPENDIX B [till value of strain 0.01]

* Reading the input values from the excel file
a=xlsread('Al_Input_Values','Aluminum 5');

* Assigning the matrix dimensions to x and h
x=a(:,1);
h=a(:,2);

[b m]=size(x);
[c n]=size(h);

* Initializing the value of area to be zero for the start
area=0;

* Repeated loop to calculate the area and store the value of calculated result in term area
for i=1:b-1
    area;
    area=area+((x(i+1)-x(i))*(0.5)*(h(i+1)+h(i)));
end

area
Results for the Al Program:

A1 = 1.315
A2 = 1.31
A4 = 1.30
A5 = 1.30

Converting these in terms of exergy we will have the following total exergy change

<table>
<thead>
<tr>
<th>TRAIL #</th>
<th>TOTAL EXERGY CHANGE IN KJ/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4175.13</td>
</tr>
<tr>
<td>2</td>
<td>4159.25</td>
</tr>
<tr>
<td>4</td>
<td>4127.5</td>
</tr>
<tr>
<td>5</td>
<td>4127.5</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>4147.345</td>
</tr>
</tbody>
</table>

MATLAB Program for Mg Alloy:

The input values for this program are the stress and strain values from taken from APPENDIX B [till value of strain 0.01]

* Reading the input values from the excel file
a=xlsread('Mg_Input_Values','Aluminum 5');

* Assigning the matrix dimensions to x and h
x=a(:,1);
h=a(:,2);
[b m]=size(x);
[c n]=size(h);

*Initializing the value of area to be zero for the start
area=0;

*Repeated loop to calculate the area and store the value of calculated result in term area
for i=1:b-1
    area;
    area=area+((x(i+1)-x(i))*(0.5)*(h(i+1)+h(i)));
end

area

Results for the Mg Program:

A1= 1.7255
A2= 1.6697
A3= 1.6691
A4= 1.6880
A5= 1.6291

Converting these in terms of exergy we will have the following total exergy change values

<table>
<thead>
<tr>
<th>TRAIL #</th>
<th>TOTAL EXERGY CHANGE IN KJ/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5492.75</td>
</tr>
<tr>
<td>2</td>
<td>5302.25</td>
</tr>
<tr>
<td>3</td>
<td>5302.25</td>
</tr>
<tr>
<td>4</td>
<td>5365.75</td>
</tr>
<tr>
<td>5</td>
<td>5175.25</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>5286.375</td>
</tr>
</tbody>
</table>
APPENDIX G

UNCERTAINTY ANALYSIS

The process of measurement is defined as the process of assigning a value to a physical variable. In every measurement there is an error as the true value is not exactly known. Therefore, the measured value provides an estimate of the true value. Therefore, the error cannot be calculated exactly but a probability of an error is estimated. This process of quantifying and identifying errors is called uncertainty analysis [24].

This particular measurement is performed on the Universal Testing Machine [UTM] experiment to find the error probability. This analysis is performed to make sure that the values obtained for true stress-true strain are within decent probability intervals. The relation between stress and strain is given by the following equation.

\[ \sigma = E \varepsilon \]  

\[(G.1)\]

The relation between the stress and force is given by the following equation.

\[ \sigma = \frac{F}{A} \]  

\[(G.2)\]

Comparing Eq. (G.1) and Eq. (G.2), the following equation is obtained.

\[ F = AE \varepsilon \]  

\[(G.3)\]

There are some assumptions to be made in performing the uncertainty analysis. They are as listed below

1) Area \( A \) assumed to be a constant value
2) Young’s Modulus $E$ has constant value

The uncertainty of the strain can be calculated from Eq. (G.3) after its rearrangement, i.e.,

$$\varepsilon = \frac{F}{AE}$$  \hspace{1cm} (G.4)

The product of area of cross section and Young’s modulus can be replaced by a constant value as each of them is a constant. Replacing $AE$ by some constant $1/C$, the Eq. (G.4) transforms to the following equation.

$$\varepsilon = FC$$  \hspace{1cm} (G.5)

Based on Eq.(5.8) & Eq.(5.9) from the “Theory and Design for Mechanical Measurements”[24], the following equations are written.

$$\bar{\varepsilon} = f(F) \text{ and } u_{\varepsilon} = f(u_F)$$  \hspace{1cm} (G.6)

From Eq.(5.11) [24], the uncertainty in the strain $\varepsilon = FC$ is given by the following equation.

$$u_{\varepsilon} = \pm[(\theta_F u_F)^2]^{1/2}$$  \hspace{1cm} (G.7)

Where the sensitivity indices are evaluated from Eq.(5.10) [24] as

$$\theta_F = \frac{\partial \varepsilon}{\partial F} = C = \frac{1}{AE}$$  \hspace{1cm} (G.8)

Therefore, the value is given by the following equation.

$$u_{\varepsilon} = \pm[(Cu_F)^2]^{1/2}$$  \hspace{1cm} (G.9)

Therefore, substituting the values in Eq. (G.9), the uncertainty of the value of strain is obtained.
For Al alloy:

\[ C = \frac{1}{A \cdot E} = \frac{1}{(20.16 \times 10^{-6} \text{ m}^2 \times 68.95 \times 10^9 \text{ N/m}^2)} = 0.71 \times 10^{-6} \text{ (1/N)} \]

Therefore, the error in approximating the strain for this particular experiment is given by

\[ u_e = \pm 0.71 \times 10^{-6} \frac{1}{\text{N}} \times 0.689 \text{ N} = \pm 0.50 \times 10^{-6} \]

For Mg Alloy:

\[ C = \frac{1}{A \cdot E} = \frac{1}{(20.16 \times 10^{-6} \text{ m}^2 \times 45 \times 10^9 \text{ N/m}^2)} = 1.1 \times 10^{-6} \text{ (1/N)} \]

Therefore, the error in approximating the strain for this particular experiment is given by

\[ u_e = \pm 1.1 \times 10^{-6} \frac{1}{\text{N}} \times 0.689 \text{ N} = \pm 0.76 \times 10^{-6} \]

Therefore, the uncertainty for strain for Al alloy is found to be \( \pm 0.50 \times 10^{-6} \) and the uncertainty for strain for Mg alloy is found to be \( \pm 0.76 \times 10^{-6} \) at 95% confidence interval.
APPENDIX H

PART I

IRREVERSIBILITY AND ENTROPY GENERATION DERIVATIONS

Exergy Change for a Closed System

Exergy is a measure of the departure of the state of a closed system from the state the same system would have in equilibrium with the environment [32]. The exergy change between two states of a closed system is given as in the following equation. [32]

\[ B_2 - B_1 = (U_2 - U_1) + p_0 (V_2 - V_1) - T_0 (S_2 - S_1) \] (H.1)

where \( B \) represents the exergy, \( U \) represents the energy, \( S \) represents entropy and \( V \) represents the volume at states 1 and 2 respectively. And \( p_0, T_0 \) are pressure and temperature determined by the state of the environment.

Development of the Exergy Balance

The forms of the energy and entropy balances used in the development are as follows.

\[ E_2 - E_1 = \int_1^2 \partial Q - W \] (H.2)

\[ S_2 - S_1 = \int_1^2 \left( \frac{\partial Q}{T} \right)_b + \sigma \] (H.3)
where \( Q \) and \( W \) represent the heat and work transfers for the system of interest and its surroundings respectively and \( \sigma = S_{gen} \) represents the entropy generation. In this equation, instead of using only internal exergy \( U \) as in Eq. (H.1), all exergy contribution is accounted for (kinetic, potential etc.).

Multiplying the entropy balance by the temperature \( T_0 \) and subtracting the results from the energy balance, the following equation is obtained.

\[
(E_2 - E_1) - T_0 (S_2 - S_1) = \frac{2}{1} \frac{\delta Q - T_0}{T} (\frac{\delta Q}{T})_b - W - T_0 S_{gen} \tag{H.4}
\]

Rearranging the terms and using the above Eq. (H.1), the following equation is obtained.

\[
(B_2 - B_1) - p_0 (V_2 - V_1) = \frac{2}{1} \left(1 - \frac{T_0}{T_b}\right) \delta Q - W - T_0 S_{gen} \tag{H.5}
\]

Again rearranging the terms in the above equation, the following equation is obtained.

\[
B_2 - B_1 = \frac{2}{1} \left(1 - \frac{T_0}{T_b}\right) \delta Q - [W - p_0 (V_2 - V_1)] - T_0 S_{gen} \tag{H.6}
\]

The terms on the left hand side of the equation represents the exergy change and the term \( T_0 S_{gen} \) represents the exergy destruction and the remaining terms represent the exergy transfer. The term \( T_0 S_{gen} \) refers to the \( B_D \) of exergy destruction due to irreversibilities within the system.

Therefore, the exergy loss can be represented as the following.

\[
B_D = T_0 S_{gen} \tag{H.7}
\]

Let a body of certain mass be heated from an average temperature of \( T_L \) by a source at an average higher temperature\( T_H \). The entropy generation across the temperature difference due to transfer of heat is represented by the following equation.
Calculating the exergy loss for this particular case using Eq. (H.7), the following equation is obtained.

\[ B_D = T_0 S_{\text{gen}} = \frac{Q}{T_L} T_0\left(1 - \frac{T_L}{T_H}\right) \]  \hspace{1cm} (H.9)

Note, in the considered case it is assumed that the both temperatures levels were kept at an average (constant) temperature. So, if one would like to represent the loss of exergy due to heating of a material during processing, either an average (mean) temperature of the product must be determined, or the determination of exergy loss Eq. (H.9) should be formulated starting with a differential exergy loss due to a delivery of \(dQ\) across the temperature difference \(\Delta T = T_H - T_L(t)\) that changes in time, represented as shown below.

\[ \int dS_{\text{gen}} = \int \left(\frac{1}{T_L(t)} - \frac{1}{T_H}\right)dQ \]  \hspace{1cm} (H.9 A)
PART II

LUMPED CAPACITANCE METHOD EQUATIONS AND VALIDATION

Let a body be quenched from a higher temperature \( T_H \) to lower temperature \( T_L \). And let the quenching liquid be at a temperature of \( T_0 \) as shown in the below figure.

Neglecting the temperature gradients (assumption validated by Biot Number) and balance equation. That is the rate of heat loss at the surface to the rate of change of the internal energy, the following equation is obtained.

\[ -\dot{E}_{\text{out}} = \dot{E}_{st} \]  \hspace{1cm} (H.10)

This can also be represented as the following.
\[-hA_s(T_L - T_0) = \rho Vc \frac{dT}{dt}\]  \hspace{1cm} (H.11)

Let \( \theta = T_L - T_0 \) and also realizing that \( \frac{dT}{dt} = \frac{d\theta}{dt} \), the above equation can be represented as the following

\[\frac{\rho Vc}{hA_s} \int_0^t \frac{d\theta}{\theta_i} dt = -\int_0^t \frac{d\theta}{\theta_i}\]  \hspace{1cm} (H.12)

where \( \theta_i = T_H - T_0 \).

Performing the integration for the above equation, the following equation is obtained.

\[\frac{\rho Vc}{hA_s} \ln \frac{\theta_i}{\theta} = t (\text{or}) \frac{T_L - T_0}{T_H - T_0} = \exp \left[ -\left(\frac{hA_s}{\rho Vc}\right)t \right]\]  \hspace{1cm} (H.13)

And to calculate the energy transfer \( Q \) at sometime \( t \), the following equation is written

\[Q = \int_0^t q \, dt = hA_s \int_0^t \theta \, dt\]  \hspace{1cm} (H.14)

Substituting \( \theta \) from Eq. (H.13) in the above equation, the following equation for heat transfer is obtained.

\[Q = (\rho Vc)\theta_i \left[ 1 - \exp\left(\frac{t}{\tau_t}\right) \right]\]  \hspace{1cm} (H.15)

where \( \tau_t = \frac{\rho Vc}{hA_s} \)

**Validation of Lumped Capacitance System to this particular case**

Lumped Capacitance System is valid when the Biot Number, which is a dimensionless parameter value, is less than 0.1[25]. The equation for this is given as follows.

130
where $h$ is the heat transfer coefficient, $L_c$ is the ratio of volume to the surface area of the material used and $k$ is the thermal conductivity of the material.

**Calculation of Biot Number for Al alloy**

\[
L_c = \frac{V}{A_s} = (1.025 \times 10^{-6}/0.00032) = 3.2 \times 10^{-3}
\]

\[
h = \frac{N_{UL}K}{L} = 11.14 \text{ W/m}^2 \text{ K}
\]

\[k = 233.6 \text{ W/m K}\]

Hence, the Biot Number is equal to $\text{Bi} = 0.00015 < 0.1$

Therefore, the assumption of lumped capacitance method is valid for Al alloy 3003 for that particular mass.

**Calculation of Biot Number for Mg alloy**

\[
L_c = \frac{V}{A_s} = (1.025 \times 10^{-6}/0.00032) = 3.2 \times 10^{-3}
\]

\[
h = \frac{N_{UL}K}{L} = 11.14 \text{ W/m}^2 \text{ K}
\]

\[k = 155 \text{ W/m K}\]

Hence, the Biot Number is equal to $\text{Bi} = 0.00022 < 0.1$

Therefore, the assumption of lumped capacitance method is valid for Mg alloy AZ31 for that particular mass.
NOMENCLATURE

\( \sigma \) = Stress

\( \varepsilon \) = Strain

E = Young’s Modulus of Elasticity

\( \sigma_{0.7} \) = Secant Stress corresponding Secant Modulus 0.7

\( \sigma_{0.85} \) = Secant Stress corresponding Secant Modulus 0.85

n = Characteristic constant which determines the shape of the curve

\( \beta \) = Material constant obtained from tests

(\( \sigma^*, \varepsilon^* \)) = The point of the end of stress-strain curve

(\( \sigma_y, \varepsilon_y \)) = The point of the end of elastic limit

t = Thickness of the flat plate

l= Length of the flat plate

A= Area (lt)

I_t= Area of the curve

\( B_{m,ph} \) = Total exergy change

\( B_{m,ph\,plastic} \) = Plastic exergy change

\( W_{Max} \) = Maximum obtainable work

\( \Delta E \) = Change in exergy utilization

\( T_L \) = Lower temperature of the work piece

\( T_o \) = Temperature of the environment
\[ T_{H} = \text{Higher temperature of the work piece} \]

\[ Q = \text{Heat transfers between the system under study and its surroundings} \]

\[ t = \text{Time} \]

\[ S_{\text{gen}} = \text{Entropy generation because of heating} \]

\[ W_{\text{max}} = \text{Maximum theoretical work obtainable} \]

\[ h = \text{Heat transfer coefficient} \]

\[ A_{S} = \text{Surface area of the work piece} \]

\[ V = \text{Volume of the work piece} \]

\[ \rho = \text{Density of the work piece} \]

\[ c = \text{Specific heat at constant pressure} \]

\[ K = \text{Thermal conductivity of the work piece} \]

\[ l = \text{Length of the work piece} \]

\[ \Theta = \text{Uncertainty} \]
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