DETAILED MODELING OF MUFFLERS WITH PERFORATED TUBES USING SUBSTRUCTURE BOUNDARY ELEMENT METHOD

Balasubramanian Datchanamourty
University of Kentucky, d_balsu@yahoo.com

Recommended Citation
Datchanamourty, Balasubramanian, "DETAILED MODELING OF MUFFLERS WITH PERFORATED TUBES USING SUBSTRUCTURE BOUNDARY ELEMENT METHOD" (2004). University of Kentucky Master's Theses. 333.
https://uknowledge.uky.edu/gradschool_theses/333
ABSTRACT OF THESIS

DETAILED MODELING OF MUFFLERS WITH PERFORATED TUBES USING
SUBSTRUCTURE BOUNDARY ELEMENT METHOD

Perforated tubes in mufflers are generally modeled by the transfer impedance approach since modeling the actual geometry of the perforated tubes with holes is very expensive due to the enormity of the boundary elements required. With the development of the substructuring technique which greatly reduces the number of elements required detailed modeling of the perforated tubes has become possible. In this thesis mufflers with perforated tubes are analyzed by modeling the actual geometry and locations of holes on the perforated tubes. The Direct-mixed-body boundary element method with substructuring is used to model the mufflers. Mufflers of various geometry containing perforated tubes with holes of different sizes and porosity are tested. The results obtained from the analyses are compared with the empirical formula results and experimental results. A preliminary investigation on the detailed modeling of flow-through catalytic converters is also conducted.

Keywords: Boundary element method, Substructuring technique, Empirical formulas, Detailed modeling, perforated tubes.

Balasubramanian Datchanamourty

11/27/2004

Copyright © Balasubramanian Datchanamourty 2004
DETAILED MODELING OF MUFFLERS WITH PERFORATED TUBES USING SUBSTRUCTURE BOUNDARY ELEMENT METHOD

By

Balasubramanian Datchanamourty

Dr. Tingwen Wu

Director of Thesis

Dr. George Huang

Director of Graduate Studies

07/28/2004
RULES FOR THE USE OF THESIS

Unpublished thesis submitted for the Master’s degree and deposited in the University of Kentucky Library are as a rule open for inspection, but are to be used only with due regard to the rights of the authors. Bibliographical references may be noted, but quotations or summaries of parts may be published only with the permission of the author, and with the usual scholarly acknowledgements.

Extensive copying or publication of the thesis in whole or in part also requires the consent of the Dean of the Graduate School of the University of Kentucky.
DETAILED MODELING OF MUFFLERS WITH PERFORATED TUBES USING SUBSTRUCTURE BOUNDARY ELEMENT METHOD

THESIS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering in the College of Engineering at the University of Kentucky

By
Balasubramanian Datchanamourty

Lexington, Kentucky

Director: Dr. Tingwen Wu, Professor of Mechanical Engineering

Lexington, Kentucky

2004

Copyright © Balasubramanian Datchanamourty 2004
Dedication

To my uncle, for his support and encouragement all through my life
ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to Professor T.W. Wu, my academic advisor, for his invaluable guidance, support and encouragement throughout my research and graduate studies. I would also like to thank Dr. Andrew Seybert and Dr. Keith Rouch for serving on my thesis committee and for their encouragement and advice.

I am thankful to Nelson Industries for their financial support during part of my graduate studies. I would like to extend my gratitude to Dr. Raymond Cheng of Nelson Industries, Dr. Zeguang Tao and Dr. Andrew Seybert for providing the experimental data reported in this thesis.

Most of all, I would like to thank my family for their love and encouragement. I am indebted to my uncle Mr. Ramalingam E for being by my side throughout my life, without whose support I would not have achieved anything in my education.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Acknowledgements</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.1 General</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Empirical Formulas</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Literature Review</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Thesis Outline</td>
<td>5</td>
</tr>
<tr>
<td>2. DIRECT-MIXED-BODY BEM</td>
<td>7</td>
</tr>
<tr>
<td>2.1 General</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Direct-Mixed Body BEM for Reactive Mufflers and Packed Silencers</td>
<td>7</td>
</tr>
<tr>
<td>2.2.1 Two-Medium Problem</td>
<td>9</td>
</tr>
<tr>
<td>2.2.2 Complete Boundary Integral Equations</td>
<td>13</td>
</tr>
<tr>
<td>3. SUBSTRUCTURING</td>
<td>16</td>
</tr>
<tr>
<td>3.1 General</td>
<td>16</td>
</tr>
<tr>
<td>3.2 Impedance Matrix Synthesis</td>
<td>16</td>
</tr>
<tr>
<td>3.2.1 Two Substructures Connected by an Acoustic Filter Element</td>
<td>16</td>
</tr>
<tr>
<td>3.2.2 Two Substructures in Direct Contact</td>
<td>20</td>
</tr>
<tr>
<td>3.2.3. Multiple Substructures in Series Connection</td>
<td>21</td>
</tr>
<tr>
<td>4. BOUNDARY ELEMENT ANALYSIS OF MUFFLERS</td>
<td>23</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.1 General</td>
<td>23</td>
</tr>
<tr>
<td>4.2 Hole-Neck-Cavity Resonance Tube</td>
<td>23</td>
</tr>
<tr>
<td>4.3 Concentric-Tube Resonator</td>
<td>28</td>
</tr>
<tr>
<td>4.4 Concentric Tube Resonator with Flow Plug</td>
<td>54</td>
</tr>
<tr>
<td>4.5 Muffler with Two Parallel Perforated Tubes</td>
<td>62</td>
</tr>
<tr>
<td>5. BOUNDARY ELEMENT ANALYSIS OF CATALYTIC CONVERTERS</td>
<td>72</td>
</tr>
<tr>
<td>5.1 General</td>
<td>72</td>
</tr>
<tr>
<td>5.2 Flow - Through Catalytic Converter</td>
<td>74</td>
</tr>
<tr>
<td>6. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH</td>
<td>82</td>
</tr>
<tr>
<td>6.1 Summary and Conclusions</td>
<td>82</td>
</tr>
<tr>
<td>6.2 Future Work</td>
<td>82</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>84</td>
</tr>
<tr>
<td>VITA</td>
<td>89</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

2.1 Types of surfaces modeled in the direct mixed-body BEM .............................................. 8

2.2 A simplified two-medium problem with an I interface .................................................. 10

3.1 Two substructures connected by an acoustic filter element .......................................... 17

3.2 Two substructures in direct contact .................................................................................. 21

3.3 Silencer with multiple substructures in series connection ............................................... 22

4.2.1 Hole-Neck-Cavity Resonance tube ............................................................................. 24

4.2.2 Boundary element model of the Hole-Neck-Cavity Resonance tube ............................ 25

4.2.3 ANSYS mesh of a part of the resonance tube ............................................................... 26

4.2.4 Transmission loss of the Hole-Neck-Cavity Resonance tube ...................................... 27

4.3.1 Schematic diagram of the single perforated tube muffler ........................................... 29

4.3.2 Perforated tubes in the single perforated tube muffler ............................................... 30

4.3.3 ANSYS model of the perforated tube (tube1) in Concentric-tube resonator ............... 31

4.3.4 ANSYS model of the perforated tube (tube2) in Concentric-tube resonator ............... 31

4.3.5 ANSYS model of the perforated tube (tube3) in Concentric-tube resonator ............... 32

4.3.6 Boundary element model of substructure 1 of Concentric-tube resonator ................. 33

4.3.7 Boundary element model of the template of Concentric-tube resonator ..................... 33

4.3.8 Boundary element model of the last substructure of Concentric-tube resonator .......... 34

4.3.9 Boundary element empirical formula model of single perforated tube muffler (Tube 1: 276 holes) ........................................................................................................ 36

4.3.10 Boundary element hole model of single perforated tube muffler (Tube 1: 276 holes) ..................................................................................................................... 37

4.3.11 Comparison of the transfer impedances (Tube 1: 276 holes) .................................... 38
4.3.12 Transmission loss of single perforated tube muffler (Tube 1: 276 holes) - Comparison 1

4.3.13 Transmission loss of single perforated tube muffler (Tube 1: 276 holes) - Comparison 2

4.3.14 Transmission loss of single perforated tube muffler (Tube 1: 276 holes) - Comparison 3

4.3.15 Boundary element empirical formula model of single perforated tube muffler (Tube 2: 162 holes)

4.3.16 Boundary element hole model of single perforated tube muffler (Tube 2: 162 holes)

4.3.17 Comparison of the transfer impedances (Tube 2: 162 holes)

4.3.18 Transmission loss of single perforated tube muffler (Tube 2: 162 holes) – Comparison 1

4.3.19 Transmission loss of single perforated tube muffler (Tube 2: 162 holes) - Comparison 2

4.3.20 Transmission loss of single perforated tube muffler (Tube 2: 162 holes) - Comparison 3

4.3.21 Boundary element empirical formula model of single perforated tube muffler (Tube 3: 72 holes)

4.3.22 Boundary element hole model of single perforated tube muffler (Tube 3: 72 holes)

4.3.23 Comparison of the transfer impedances (Tube 3: 72 holes)

4.3.24 Transmission loss of single perforated tube muffler (Tube 3: 72 holes) - Comparison 1

4.3.25 Transmission loss of single perforated tube muffler (Tube 3: 72 holes) - Comparison 2

4.3.26 Transmission loss of single perforated tube muffler (Tube 3: 72 holes) - Comparison 3
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4.1</td>
<td>Concentric-Tube muffler with a flow plug.</td>
</tr>
<tr>
<td>4.4.2</td>
<td>ANSYS mesh of tube ((L_2)) in concentric tube resonator with flow plug.</td>
</tr>
<tr>
<td>4.4.3</td>
<td>ANSYS mesh of tube ((L_3)) in concentric tube resonator with flow plug.</td>
</tr>
<tr>
<td>4.4.4</td>
<td>Boundary element empirical formula model of Concentric-Tube muffler with a flow plug.</td>
</tr>
<tr>
<td>4.4.5</td>
<td>Boundary element hole model of Concentric-Tube muffler with a flow plug.</td>
</tr>
<tr>
<td>4.4.6</td>
<td>Transmission loss of Concentric-Tube muffler with a flow plug – Comparison 1.</td>
</tr>
<tr>
<td>4.4.7</td>
<td>Transmission loss of Concentric-Tube muffler with a flow plug – Comparison 2.</td>
</tr>
<tr>
<td>4.5.1</td>
<td>Muffler with two parallel perforated tubes.</td>
</tr>
<tr>
<td>4.5.2</td>
<td>ANSYS mesh of the perforated tube.</td>
</tr>
<tr>
<td>4.5.3</td>
<td>Boundary element model of the first substructure.</td>
</tr>
<tr>
<td>4.5.4</td>
<td>ANSYS mesh of inlet and outlet to the substructures.</td>
</tr>
<tr>
<td>4.5.5</td>
<td>Boundary element mesh of the template.</td>
</tr>
<tr>
<td>4.5.6</td>
<td>Boundary element mesh of the last substructure.</td>
</tr>
<tr>
<td>4.5.7</td>
<td>Boundary element empirical formula model of muffler with two parallel perforated tubes.</td>
</tr>
<tr>
<td>4.5.8</td>
<td>Boundary element detailed hole model of muffler with two parallel perforated tubes.</td>
</tr>
<tr>
<td>4.5.9</td>
<td>Transmission loss of muffler with two parallel perforated tubes - Comparison 1.</td>
</tr>
<tr>
<td>4.5.10</td>
<td>Transmission loss of muffler with two parallel perforated tubes - Comparison 2.</td>
</tr>
<tr>
<td>5.1</td>
<td>Emission control device with (A) a through-flow catalyst converter (CC), (B) a wall-flow diesel particulate filter (DPF).</td>
</tr>
<tr>
<td>5.2</td>
<td>Muffler with a see-through catalytic converter.</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>5.3</td>
<td>ANSYS mesh of the curved surface of the catalytic converter.</td>
</tr>
<tr>
<td>5.4</td>
<td>ANSYS mesh of the partitions in the catalytic converter.</td>
</tr>
<tr>
<td>5.5</td>
<td>ANSYS mesh of the inlet and outlet to substructures</td>
</tr>
<tr>
<td>5.6</td>
<td>BEM mesh of the first substructure of muffler with a catalytic converter.</td>
</tr>
<tr>
<td>5.7</td>
<td>Boundary element model of template.</td>
</tr>
<tr>
<td>5.8</td>
<td>BEM mesh of the last substructure.</td>
</tr>
<tr>
<td>5.9</td>
<td>Boundary element bulk-reacting material model of muffler with a see-through catalytic converter.</td>
</tr>
<tr>
<td>5.10</td>
<td>Boundary element detailed model of muffler with a see-through catalytic converter.</td>
</tr>
<tr>
<td>5.11</td>
<td>Transmission loss of muffler with a flow-through catalytic converter.</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

1.1. General

Perforated tubes in mufflers and silencers have been modeled by the transfer impedance approach proposed by Sullivan and Crocker [1] since 1978. Since then, several alternative empirical formulas based on experiments have been proposed by other researchers to represent the acoustic impedance of perforates under different flow conditions [2, 3]. The impedance formulas are obtained based on the condition that the tube has a uniform hole-diameter and hole-pattern. In general the empirical formulas work well in most cases. However, there are some discrepancies in situations where the acoustic behavior of the perforated element can be influenced by the hole sizes and locations along the tube. The purpose of this study is to numerically model the actual geometry and location of the holes on the perforated tubes using the boundary element method (BEM) and to compare the results with empirical formulas.

To overcome the enormous task of modeling the details of each hole on the perforated tube, a newly developed substructuring BEM method [4] is used. A small section of the perforated tube which consists of identical number of holes and pattern is modeled and is used as a template for the repeating substructures. Since the geometry of repeating substructures remains the same, the impedance matrix does not need to be re-calculated for each substructure. The resultant impedance matrix of the entire muffler can be obtained by an impedance matrix synthesis procedure [4, 5]. Subsequently, the
transmission loss (TL) of the muffler can be calculated based on the resultant impedance matrix [6].

1.2. Empirical formulas

For simplicity, perforated tubes have been modeled by the equivalent transfer impedance approach originally proposed by Sullivan and Crocker [1]. In this approach, the pressure jump across the perforated tube wall is related to the equivalent normal velocity by

\[ p^- - p^+ = \rho c \xi v_n \]  

(1.1)

where \( p^+ \) is the sound pressure in the normal direction, \( p^- \) is the sound pressure on the other side of the perforated tube, \( \rho \) is the mean density of the fluid, \( c \) is the speed of sound, \( v_n \) is the equivalent normal velocity, and \( \xi \) is the dimensionless transfer impedance. The normal velocity on the rigid metal surface is zero, while acoustic waves can propagate through the holes. In the equivalent transfer impedance approach, the metal-and-holes combination is replaced by an equivalent porous surface with a known transfer impedance. Several empirical formulas for the transfer impedance have been proposed for use in Eq. (1). The simplest one, developed by Sullivan and Crocker [1], is

\[ \xi = \frac{1}{\rho c \sigma} (2.4 + 0.02 f) \]  

(1.2)

in SI units

where \( \sigma \) is the porosity (ratio between the open area and total area), \( f \) is the frequency, and \( i = \sqrt{-1} \). Sullivan and Crocker [1] also proposed a slightly different formula that takes the hole-diameter and wall thickness into consideration, which is
\[ \xi = \frac{1}{\sigma} \left[ 6 \times 10^{-3} + i k \left( t + 0.75 d_h \right) \right] \quad (1.3) \]

where \( k \) is the wavenumber, \( t \) is the wall thickness, and \( d_h \) is the hole diameter. The experiment was conducted by Sullivan and Crocker in the frequency range of 0 to 1.4 kHz. The sample porosity used in the experiment was 4.2%. Sullivan and Crocker recommended the use of equation (1.2) provided the porosity of the perforated tube considered is not too different from the sample porosity they used. But the equation (1.2) has been used in many BEM calculations with the porosity different from the one Sullivan and Crocker used and has been shown to produce fairly good results compared to the experimental results [7, 8].

Bento Coelho [2] developed an alternative empirical formula that includes the wall thickness and hole diameter. The formula is

\[ \xi = \frac{1}{\rho c \left[ R_0 + i X_0 \right]} \quad (1.4a) \]

with

\[ R_0 = \frac{1}{\sigma} \left[ \rho \left( \frac{d'}{d_h} \right) \sqrt{\frac{8 \nu \omega}{\omega}} + \left( \frac{\rho}{8 c} \right) (\omega d_h)^2 \right], \quad (1.4b) \]

\[ X_0 = \left( \frac{\omega \rho}{\sigma} \right) \left( d'' + \left( \frac{d'}{d_h} \right) \sqrt{\frac{8 \nu}{\omega}} \right) \text{ and} \quad (1.4c) \]

\[ d' = t + d_h, \quad d'' = t + \left( \frac{8}{3 \pi} \right) d_h \left( 1 - 0.7 \sqrt{\sigma} \right) \quad (1.4d, e) \]

where \( \nu \) is the kinematic viscosity of the gas and \( \omega = 2 \pi f \).

Equations (1.2) through (1.4e) are used for stationary acoustic media. Sullivan derived an empirical formula for perforated tubes with cross flow which is given by [30]
\[
\xi = \left( \frac{1}{\sigma} \right) \left[ 0.514d_1 \left( \frac{M}{l\sigma} \right) + i0.95k(t + 0.75d_h) \right] \quad (1.5)
\]

where \( d_1 \) is the diameter of the perforated tube, \( l \) is the length of the tube and \( M \) is the mean-flow Mach number in the tube. Rao and Munjal [3] developed the following empirical formula for perforated tubes with grazing flow

\[
\xi = \left( \frac{1}{\sigma} \right) \left[ 7.337 \times 10^{-3} (1 + 72.23M) + i2.2245 \times 10^{-5} f(1 + 5lt)(1 + 204d_h) \right] \quad (1.6)
\]

1.3. Literature review

The direct mixed body boundary element method for mufflers and packed silencers was proposed by Wu and his co workers [7-10]. This is different from the conventional multidomain BEM [13-16] in that it combines all the subdomains integral equations into single domain boundary integral equations by introducing the hypersingular integral equations [18-25]. Ji, et al. [5] in 1994 proposed the impedance matrix method for homogeneous medium to predict the acoustic performance of expansion chamber mufflers. Wu, et al. [8] proposed an improved method for deriving the four-pole parameters and tested different transfer impedance empirical formulas using the direct mixed body BEM.

Ji et al. [5] proposed the impedance matrix method for a homogeneous acoustic domain. Lou et al [6] used the impedance matrix synthesis approach to evaluate the transmission loss characteristics of multiply connected exhaust systems using the direct mixed body BEM. Based on the impedance matrix method, Lou, et al. [4] developed the substructuring technique for inhomogeneous medium and domains with
boundaries which need not be well defined. They also extended the impedance matrix method to analyze catalytic converters. More work on the direct mixed body boundary element method and substructuring technique can be found in [6-9].

Wave propagation in catalytic converters has been studied before and the different analytical and experimental methods can be found in references [33-39]. Cheng, et al. [27] used the direct mixed body BEM to study the transmission loss characteristics of catalytic converters and diesel particulate filters. They considered two different approaches to model the monolith substrate. In the first approach the catalytic converter is treated as a sound absorptive bulk-reacting material whose acoustic properties are characterized by a complex speed of sound and complex mean density and in the second method a four-pole transfer matrix is used to represent the monolith [33, 34].

1.4. Thesis outline

A review of the direct mixed body boundary element method is given in chapter 2. The content of this chapter include the direct mixed body BEM for reactive mufflers and packed silencers. The boundary integral equations involving the hypersingular equations are first expressed for a two medium problem. The equations are then extended to a generic problem involving different types of surfaces and the complete boundary integral equations are expressed.

In chapter 3 the substructuring technique used in the direct mixed body BEM is discussed. The impedance matrix synthesis approach is described. The impedance matrix is first derived for a muffler with two substructures connected by a filter element and then
the equations are reduced to the case of a muffler with two substructures in direct contact. Finally, the method is extended to multiple substructures in series connection.

Chapter 4 deals with the boundary element analysis of mufflers. The detailed hole model of the mufflers has been described. Four different muffler test cases including a Helmholtz resonator have been demonstrated. The results obtained from the detailed hole model are compared with the different empirical formula model results and also with results obtained from a commercial software (SYSNOISE), wherever available.

Chapter 5 is devoted to the boundary element analysis catalytic converters. A preliminary analysis of mufflers with flow through catalytic converters with a detailed model of the catalytic converter is conducted. The result obtained from the analysis is compared with that of the bulk-reacting-material model of the catalytic converter and the experimental result.

In chapter 6 the work done in this thesis is summarized and concluded. Recommendations for future research on the analysis of perforated tube mufflers and catalytic converters are suggested.
2.1. General

Mufflers and packed silencers used as acoustic filters in industries usually contain complex internal components such as perforated tubes, baffles, branched cavities, inlet/outlet tubes and sound absorbing bulk-reacting materials [7, 9]. The advantage of boundary element method (BEM) over other numerical techniques is that BEM meshes only the boundary surfaces. Traditionally, multi-domain BEM has been used to model mufflers and packed silencers in which the acoustic domain is divided into several sub-domains. Each sub-domain contains only one material [11]. Since mufflers and silencers contain components of complicated geometry, very often a lot of sub-domains may need to be created. It becomes extremely difficult to match so many sub-domains and define continuity conditions at each interface. The direct mixed-body BEM developed by Wu and his co-workers [7-10] overcomes this modeling difficulty. The advantage of the direct-mixed body BEM is that it reduces a multi-domain problem to a single domain problem thereby eliminating the need to define several sub-domains and artificial interfaces [4]. In this chapter, the theory of direct mixed-body BEM [9] as applied to mufflers and packed silencers is briefly reviewed.

2.2 Direct mixed-body BEM for reactive mufflers and packed silencers

The boundary element modeling for packed silencers is more complicated than perforated tube mufflers because they involve at least two different acoustic media, air
and the bulk-reacting sound absorbing material [11]. Bulk-reacting sound absorbing materials are characterized by complex speed of sound and complex mean density [12]. These two material properties can be measured experimentally by the two-cavity method [12] or calculated by using the empirical formulas, given the flow resistivity of the material.

![Diagram of Bulk-reacting material](image)

**Figure 2.1 Types of surfaces modeled in the direct mixed-body BEM [9]**

The following assumptions are made for analysis purposes [6]:

1. All the solid boundary surfaces are rigid.
2. Steady state linear acoustics with $e^{i\omega t}$ convention is followed, where $\omega$ is the angular frequency, $i = \sqrt{-1}$.

A typical packed silencer is shown in Figure 2.1 [9]. In Figure 2.2 $\Omega$ represents the interior acoustic domain of the silencer. The notations R, T, P, B, I, IP, IPC, ATB and BTB denote ‘Regular surface’, ‘Thin body’, ‘Perforated surface’, ‘Bulk-reacting’ material, ‘Interface’, ‘Interface with Perforated surface’, ‘Interface with Perforated surface and Cloth’, ‘Air-Thin-Bulk-reacting surface’ respectively [9]. The ‘Regular (R)’
surfaces include the exterior surfaces (with no bulk-reacting material) of the silencer, the inlet and outlet tubes and the inlet and outlet ends. The ‘Thin (T)’ surfaces include the thin components inside the silencer, like the extended inlet and outlet tubes, baffles, flow plugs and internal connecting tubes. ‘Thin’ surfaces have air on both sides. ‘Perforated (P)’ surfaces represent thin perforated tubes or plates with air on both sides. The ‘Bulk-reacting (B)’ surfaces include the exterior boundary of the bulk reacting medium. The ‘Interface (I)’ surfaces represent the interface between air and the bulk-reacting medium. The IP surfaces represent the perforated interfaces between air and the bulk-reacting medium. The IPC surface also has a perforated interface between air and bulk-reacting material. In addition to that a layer of protective cloth is placed between the bulk-reacting material and perforated plate so as to prevent the bulk-reacting material from oozing out through the perforates [4]. The ATB surfaces are the thin plates between air and the bulk-reacting material. The BTB surfaces are the thin plates which have bulk-reacting material on both sides.

2.2.1 Two Medium Problem

Figure 2.2 shows a simplified two-medium problem which can be solved by the direct mixed-body BEM. $\Omega_A$ and $\Omega_B$ represent the interior acoustic domain of air and the bulk-reacting material, respectively. $\mathbf{n}$ represents the unit normal vector pointing into the acoustic domain [9]. $R$ represents the ‘regular’ surface. $\rho_A$ and $\rho_B$ denote the mean densities of air and bulk-reacting material and $c_A$ and $c_B$ represent the speed of sound in air and bulk-reacting material, respectively. The relationships between the mean density,
The speed of sound, propagation constant and the characteristic impedance in the bulk reacting material are given by [9]

\[ \rho_B = \frac{Z_B}{c_B} \]

(2.1)

\[ c_B = \frac{i \omega}{\Gamma_B} \]

(2.2)

where \( i = \sqrt{-1} \), \( \omega \) is the angular frequency, \( \Gamma_B \) is the propagation constant and \( Z_B \) is the characteristic impedance.

The governing differential equations for a two-medium problem are given by [9]

\[ \nabla^2 p + k_A^2 p = 0 \quad \text{in air} \]

(2.3)

\[ \nabla^2 p + k_B^2 p = 0 \quad \text{in bulk-reacting material} \]

(2.4)

where \( p \) is the sound pressure and \( k_A \) and \( k_B \) denote the wavenumbers in air and the bulk-reacting medium, respectively. The boundary integral equations are given by [9]
where \( P \) is the collocation point and \( \psi_A \) and \( \psi_B \) are the free-space Green’s function in air and the bulk-reacting material, respectively. Since the normal at the interface between air and the bulk-reacting material is pointing into air, a negative sign is used in the second term in equation (2.6). The expressions for the free-space Green’s functions are given by [9]

\[
\psi_A = \frac{e^{-ik_A r}}{r} \\
\psi_B = \frac{e^{-ik_B r}}{r}
\]  

where \( i = \sqrt{-1} \) and \( r \) is the distance between the collocation point \( P \) and any point on the surface \( Q \), \( r = |P - Q| \). Equations (2.5) and (2.6) represent the boundary integral
equations of two sub-domains. The boundary integral equations for the direct mixed-body BEM are obtained by summing up these two equations.

Since, the bulk-reacting material and air are in direct contact with each other, at interface I, the sound pressure $p$ and the normal velocity $v_n$ are continuous. Summing equations (2.5) and (2.6) gives [9]

\[
\int_{R} \left( p \frac{\partial \psi_A}{\partial n} + i \rho_A \omega v_n \psi_A \right) \, dS + \int_{B} \left( p \frac{\partial \psi_B}{\partial n} + i \rho_B \omega v_n \psi_B \right) \, dS \\
+ \int_{I} \left[ p \left( \frac{\partial \psi_A}{\partial n} - \frac{\partial \psi_B}{\partial n} \right) + i \omega v_n \left( \rho_A \psi_A - \rho_B \psi_B \right) \right] \, dS
\]

\[
\begin{cases}
4\pi p(P), & P \in \Omega_A + \Omega_B \\
2\pi p(P), & P \in R + B \\
4\pi p(P), & P \in I
\end{cases}
\]  
(2.9a)

(2.9b)

(2.9c)

The boundary condition specified on the surfaces $R$ and $B$ can be $p$, $v_n$ or the local impedance. This means that equation (2.9b) has only one unknown variable and it can be solved. But, at the interface both $p$ and $v_n$ are unknowns. Therefore, besides equation (2.9c), another equation is required to solve the two unknowns. The additional equation is derived by first taking the normal derivatives of equations (2.5b) and (2.6b) to obtain the hypersingular integral equations [9]

\[
\int_{R} \left( p \frac{\partial^2 \psi_A}{\partial n \partial n} + i \rho_A \omega v_n \frac{\partial \psi_A}{\partial n} \right) \, dS + \int_{I} \left( p \frac{\partial^2 \psi_A}{\partial n \partial n} + i \rho_A \omega v_n \frac{\partial \psi_A}{\partial n} \right) \, dS \\
= 2\pi \frac{\partial p}{\partial n}(P), & P \in I \text{ from the air side}
\]

(2.10)
The relationship between the normal derivative of pressure and the normal velocity is [9]

\[
\frac{\partial p}{\partial n} = -i\omega \nu_n \tag{2.12}
\]

Using equation (2.12) in equations (2.10) and (2.11) and summing the two equations yields [9]

\[
\int_R \left( p \frac{\partial^2 \psi_A}{\partial n^2} + i\rho_A \omega \nu_n \frac{\partial \psi_A}{\partial n} \right) dS + \int_B \left( p \frac{\partial^2 \psi_B}{\partial n^2} + i\rho_B \omega \nu_n \frac{\partial \psi_B}{\partial n} \right) dS
\]

\[
+ \int_I \left[ p \left( \frac{\partial^2 \psi_A}{\partial n^2} - \frac{\partial^2 \psi_B}{\partial n^2} \right) + i\omega \nu_n \left( \rho_A \frac{\partial \psi_A}{\partial n} - \rho_B \frac{\partial \psi_B}{\partial n} \right) \right] dS
\]

\[
= -2\pi i\omega (\rho_A + \rho_B) \nu_n (P), \quad P \in I \tag{2.13}
\]

Equation (2.13) supplements equation (2.9c) to solve the two unknowns. Thus with equations (2.9b), (2.14c) and (2.13) all the unknowns on the boundary can be solved for a two two-medium problem.

### 2.2.2 Complete Boundary integral equations

The interface considered in the two-medium problem can be of three types: I, IP, ATB. In the previous section, the boundary integral equations are derived for the I
interface. The boundary integral equations for the other types of interface and surfaces can be derived in a similar fashion. The complete boundary integral equations for a typical reactive muffler or packed silencer are given by [9]

\[
\begin{align*}
&\int_{R} \left( p \frac{\partial \psi_\text{A}}{\partial n} + i \rho_\text{A} \omega v_n \psi_\text{A} \right) dS + \int_{T+P} \left( \frac{\partial \psi_\text{A}}{\partial n} \left( p^+ - p^- \right) \right) dS + \int_{\text{BTB}} \left( \frac{\partial \psi_\text{B}}{\partial n} \left( p^+ - p^- \right) \right) dS \\
+&\int_{B} \left( p \frac{\partial \psi_\text{B}}{\partial n} + i \rho_\text{B} \omega v_n \psi_\text{B} \right) dS + \int_{I} \left[ p \left( \frac{\partial \psi_\text{A}}{\partial n} - \frac{\partial \psi_\text{B}}{\partial n} \right) + i \omega v_n \left( \rho_\text{A} \psi_\text{A} - \rho_\text{B} \psi_\text{B} \right) \right] dS \\
+&\int_{\text{IP} + \text{IPC}} \left[ p_\text{A} \left( \frac{\partial \psi_\text{A}}{\partial n} - \frac{\partial \psi_\text{B}}{\partial n} \right) - \rho_\text{A} c_A \xi^* v_n \frac{\partial \psi_\text{B}}{\partial n} + i \omega v_n \left( \rho_\text{A} \psi_\text{A} - \rho_\text{B} \psi_\text{B} \right) \right] dS \\
+&\int_{\text{ATB}} \left( p_\text{A} \frac{\partial \psi_\text{A}}{\partial n} - p_\text{B} \frac{\partial \psi_\text{B}}{\partial n} \right) dS
\end{align*}
\]

\[
\begin{cases}
4\pi p(P), & P \in \Omega \\
2\pi p(P), & P \in R + B \\
2\pi \left[ p^+(P) + p^-(P) \right], & P \in T + P + \text{BTB} \\
4\pi p(P), & P \in I \\
4\pi p_\text{A}(P) + 2\pi \rho_\text{A} c_A \xi^* v_n(P), & P \in \text{IP} + \text{IPC} \\
2\pi \left[ p_\text{A}(P) + p_\text{B}(P) \right], & P \in \text{ATB}
\end{cases}
\]

and

\[14\]
\[
\begin{align*}
&\int_R \left( \frac{\partial^2 \psi_A}{\partial n^2} + i\rho_A \omega v_n \frac{\partial \psi_A}{\partial n} \right) dS + \int_{T+P} \left( \frac{\partial^2 \psi_A}{\partial n^2} (p^+ - p^-) \right) dS \\
&+ \int_{BTB} \left( \frac{\partial^2 \psi_B}{\partial n^2} (p^+ - p^-) \right) dS + \int_B \left( \frac{\partial^2 \psi_B}{\partial n^2} + i\rho_B \omega v_n \frac{\partial \psi_B}{\partial n} \right) dS \\
&+ \int_I \left[ p \left( \frac{\partial^2 \psi_A}{\partial n^2} - \frac{\partial^2 \psi_B}{\partial n^2} \right) + i\omega v_n \left( \rho_A \frac{\partial \psi_A}{\partial n} - \rho_B \frac{\partial \psi_B}{\partial n} \right) \right] dS \\
&+ \int_{IP+IPC} \left[ p_A \left( \frac{\partial^2 \psi_A}{\partial n^2} - \frac{\partial^2 \psi_B}{\partial n^2} \right) - \rho_A c_A \xi^* v_n \frac{\partial^2 \psi_B}{\partial n^2} + i\omega v_n \left( \rho_A \frac{\partial \psi_A}{\partial n} - \rho_B \frac{\partial \psi_B}{\partial n} \right) \right] dS \\
&+ \int_{ATB} \left( p_A \frac{\partial^2 \psi_A}{\partial n^2} - p_B \frac{\partial^2 \psi_B}{\partial n^2} \right) dS \\
\end{align*}
\]

\[
\begin{cases}
0 & P \in T + BTB + ATB \\
4\pi \frac{ik_A}{\xi} \left[ p^+(P) - p^-(P) \right] & P \in P \\
-2\pi i\omega (\rho_A + \rho_B) v_n(P) & P \in I + IP + IPC
\end{cases}
\]

(2.15)

where \( \xi \) is the dimensionless transfer impedance for perforated surfaces [1], and \( \xi^* \) denotes transfer impedance for IP or IPC interfaces and it is dimensionless too. \( \xi^* \) is same as \( \xi \) on an IP interface. On an IPC interface, \( \xi^* \) is the sum of \( \xi \) and the dimensionless transfer impedance of the protective cloth [10].
CHAPTER 3
SUBSTRUCTURING

3.1. General

For large silencers and mufflers the number of boundary elements required to produce accurate results can be enormous. This makes the BEM computations difficult due to the limitations of computer memory and also it takes a long processing time to complete the computation. In the substructuring technique a large structure is broken into smaller substructures. Each substructure requires smaller computer memory and at any given time, only the substructure under consideration needs to be stored. Thus substructuring technique can greatly reduce matrix size and computation time. In this chapter, the substructuring technique using the impedance matrix synthesis developed by Lou, Wu and Cheng [4] is discussed.

3.2. Impedance matrix synthesis

The substructuring technique proposed by Lou, Wu and Cheng [4] is based on the impedance matrix synthesis approach [5]. In this method, the silencer is divided into a number of substructures and the impedance matrices of all substructures are combined together, two at a time, and the resultant impedance matrix is obtained. The following examples illustrate the impedance matrix synthesis method.

3.2.1. Two substructures connected by an acoustic filter element

Consider a silencer consisting of two substructures as shown in Figure 3.1. The dotted lines represent a built-in acoustic filter usually a catalytic converter which connects the
two substructures [4]. Let the sound pressure and particle velocity at the inlet of the first substructure be denoted by $p_i$ and $v_i$, respectively and the corresponding variables at the outlet of the first substructure be denoted by $p_1$ and $v_1$. Let $p_2$ and $v_2$ represent the sound pressure and particle velocity, respectively, at the inlet of the second substructure and $p_0$ and $v_0$ represent the corresponding variables at the outlet of the second substructure. Since the sound pressure and particle velocity vary on any cross-section, the variables $p$ and $v$ represent vectors. The lengths of the vectors are determined by the number of elements at each cross section.

![Diagram of two substructures connected by an acoustic filter element](image)

**Figure 3.1** Two substructures connected by an acoustic filter element [4].

The impedance matrix relates the sound pressure at the inlet and outlet of the any substructure to the corresponding particle velocities. For the first substructure, this relationship is [4]

$$
\begin{bmatrix}
\{p_1\} \\
\{p_1\}
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
\{v_1\} \\
\{v_1\}
\end{bmatrix}

(3.1a,b)

The linear relationship in the above set of equations is based on the assumption that all boundaries other than the inlet and outlet are rigid or an impedance boundary condition is imposed on them. The impedance matrix can be generated by running the BEM on the
first substructure with different velocity boundary conditions. For example, the first column of the impedance matrix can be obtained by assigning \( v = 1 \) to the first element at the inlet of the first substructure and \( v = 0 \), elsewhere. The solution obtained for sound pressure with these velocity boundary conditions will give the first column of the impedance matrix. Similarly, by assigning \( v = 1 \) on every element at the inlet and outlet and \( v = 0 \) on the other elements, the rest of the columns of the impedance matrix can be generated.

Likewise, the impedance matrix for the second substructure can be obtained. The equations in matrix form are [4]

\[
\begin{pmatrix} p_2 \\ p_0 \end{pmatrix} = \begin{bmatrix} Z_{31} & Z_{32} \\ Z_{41} & Z_{42} \end{bmatrix} \begin{pmatrix} v_2 \\ v_0 \end{pmatrix}
\]

(3.2 a,b)

The acoustic property of the filter element that connects the two substructures are described by a ‘four-pole-type’ transfer matrix, given as [4]

\[
\begin{pmatrix} p_1 \\ v_1 \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} p_2 \\ v_2 \end{pmatrix}
\]

(3.3 a,b)

For a see-through type catalytic converter, the sub-matrices \( A, B, C \) and \( D \) of the transfer matrix are diagonal. This is because, the walls of the capillary duct are assumed to be rigid and this prevents any interaction between adjacent ducts. Since, the ducts have a small cross-section, the acoustics inside each duct can be described by a plane wave which travels forward and reflects backward. The sub-matrices of the transfer matrix can be written as [4]

\[
A = \begin{bmatrix} A_1 \\ & \ddots \\ & & A_1 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ & \ddots \\ & & B_1 \end{bmatrix}
\]
where $A_1$, $B_1$, $C_1$ and $D_1$ are the four-pole parameters of a straight duct. These four-pole parameters are functions of speed of sound and mean density which are both complex numbers. The objective is to obtain a set of equations that relate the inlet variables $p_i$, $v_i$ and the outlet variables $p_o$, $v_o$. In order to eliminate the interface variables $p_1$, $v_1$, $p_2$ and $v_2$ from Equations (3.1) – (3.3), first substitute Equation (3.3) into Equation (3.1b) to obtain [4]

$$Ap_2 + Bv_2 = Z_{21}v_1 + Z_{22} (Cp_2 + Dv_2)$$

(3.5)

Substitute for $p_2$ in Equation (3.5) from Equation (3.2a) to get

$$[(A - Z_{22}C)Z_{31} + (B - Z_{22}D)]v_2 = Z_{21}v_i - (A - Z_{22}C)Z_{32}v_o$$

(3.6)

Equation (3.6) can be solved for $v_2$. Let

$$L = (A - Z_{22}C)Z_{31} + (B - Z_{22}D)$$

(3.7)

From Equation (3.6),

$$v_2 = (L^{-1}Z_{21})v_1 - [L^{-1}(A - Z_{22}C)Z_{32}]v_o$$

(3.8)

Substituting Equation (3.8) into Equation (3.2a) gives,

$$p_2 = (Z_{31}L^{-1}Z_{21})v_i + [Z_{32} - Z_{31}L^{-1}(A - Z_{22}C)Z_{32}]v_o$$

(3.9)

Substituting Equation (3.8) and Equation (3.9) into Equation (3.3b) gives,
Finally, by substituting Equation (3.10) into Equation (3.1a) and substituting Equation (3.8) into Equation (3.1b) the relationship between the sound pressure and particle velocity can be obtained as follows [4]

\[
\begin{bmatrix}
    p_1 \\
    p_o
\end{bmatrix} =
\begin{bmatrix}
    Z_{51} & Z_{52} \\
    Z_{61} & Z_{62}
\end{bmatrix}
\begin{bmatrix}
    v_i \\
    v_o
\end{bmatrix}
\]  

(3.11)

where

\[
Z_{51} = Z_{11} + Z_{12} \left[ C \left( Z_{31} L^{-1} Z_{21} \right) + DL^{-1} Z_{21} \right]
\]  

(3.12)

\[
Z_{52} = Z_{12} C \left[ Z_{32} - Z_{31} L^{-1} (A - Z_{22} C) Z_{32} \right] - Z_{12} D \left[ L^{-1} (A - Z_{22} C) Z_{32} \right]
\]  

(3.13)

\[
Z_{61} = Z_{41} L^{-1} Z_{21}
\]  

(3.14)

\[
Z_{62} = Z_{42} - Z_{41} L^{-1} (A - Z_{22} C) Z_{32}
\]  

(3.15)

Usually, the sound pressure and particle velocity are uniform across any cross section in the inlet and outlet tubes. In that case, the vectors \(p_i, v_i, p_o,\) and \(v_o\) are each lumped into a single variable. Therefore, the impedance matrix in Equation (3.11) reduces to a 4 x 4 matrix [4].

### 3.2.2. Two substructures in direct contact

Figure 3.2 shows a silencer with two substructures in direct contact. When two substructures are in direct contact with each other, the transfer matrix in Equation (3.3) is
an identity matrix. In Equation (3.3), $A = D = I$ and $B = C = 0$, where $I$ is the identity matrix and 0 is the zero matrix. Equation (3.7) reduces to [4]

$$L = Z_{31} - Z_{22}$$

(3.16)

and Equations (3.12) to (3.15) become [4]

$$Z_{51} = Z_{11} + Z_{12} (Z_{31} - Z_{22})^{-1} Z_{21}$$

(3.17)

$$Z_{52} = -Z_{12} (Z_{31} - Z_{22})^{-1} Z_{32}$$

(3.18)

$$Z_{61} = Z_{41} (Z_{31} - Z_{22})^{-1} Z_{21}$$

(3.19)

$$Z_{62} = Z_{42} - Z_{41} (Z_{31} - Z_{22})^{-1} Z_{32}$$

(3.20)

Figure 3.2. Two substructures in direct contact [4].

3.2.3. Multiple substructures in series connection

Figure 3.3 shows a silencer with multiple substructures connected in series [4]. For a silencer with multiple substructures, the impedance matrix for the whole structure can be generated by first applying the impedance matrix synthesis procedure explained in section 3.2.2 to the first two substructures. The procedure is applied again to combine the resulting impedance matrix with the impedance matrix of the third substructure. The impedance matrix synthesis procedure applied repeatedly until all the substructures are
completed. Usually packed silencers used in practice have identical cross-sections in a major portion of the structure. In such cases, a template can be used. A template is a small section which represents the identical geometry. The four-pole matrix of the template can be used repeatedly in the impedance matrix synthesis procedure.

Figure 3.3. Silencer with multiple substructures in series connection [4].
CHAPTER 4
BOUNDARY ELEMENT ANALYSIS OF MUFFLERS

4.1. General

In this chapter the boundary element analysis of a resonance tube and three kinds of perforated tube mufflers is discussed. The mufflers are modeled by two different methods – the detailed hole model and the empirical formula model, with an exception to the Helmholtz resonator to which the empirical formula method cannot be applied. In the BEM analysis using the empirical formula model, the transfer impedance formulas proposed by Sullivan and Crocker [1] are used. The first one is the equation (1.3) which includes the details pertaining to the tube wall thickness and hole size and the second one is the equation (1.3) which doesn’t include those details. The results obtained from the detailed hole model and the empirical formula models are compared to the experimental result.

4.2. Hole-neck-cavity resonance tube (Helmholtz Resonator)

Helmholtz resonator is an example of side-branch filters which have elements connected in parallel with the main duct in the form of branched acoustical filters. Such mufflers are also called narrow band silencers as they are effective only over a very narrow range of frequencies. The Helmholtz resonator consists of a cylindrical branch chamber of volume $V$ with length $L_0$ and a neck of area $S_0$. The transmission loss of the Helmholtz resonator is [26]

$$TL = 10 \log \left[1 + \mu^2 \left( \frac{f}{f_r} - \frac{f}{f} \right)^2 \right] \quad (4.1)$$
where

\[ \mu = \frac{\sqrt{c_0 V}}{2S_1} , \]  

(4.1a)

\[ c_0 = \frac{S_0}{L'} \quad \text{(duct conductivity)}, \]  

(4.1b)

\[ L' = L_o + 1.6 a , \]  

(4.1c)

and ‘a’ is the radius of the neck and \( S_1 \) is the area of cross section of the main duct. The resonance frequency \( f_r \) is calculated from the dimensions of the branch tube and the cavity volume [26]

\[ f_r = \frac{c}{2\pi \sqrt{\frac{c_0}{V}}} \]  

(4.2)

Figure 4.2.1 shows the sketch of the Helmholtz resonator considered in this study. The diameter of the duct is 0.04 m. The length of the cylindrical chamber is 0.07 m and the diameter is 0.07 m. The length \( (L_o) \) and diameter \( (2a) \) of the neck are 0.01 m and 0.006 m respectively. Figure 4.2.2 shows the boundary element model of the Helmholtz...
The model consists of semicircular inlet and outlet, flange and cylindrical tube at the inlet and outlet ends. All these components are modeled with ‘regular’ elements. The outer cylindrical tube lies between the inlet and outlet flange forming the boundary of the cavity which is also modeled with ‘regular’ elements. The internal tube is divided into

Figure 4.2.2. Boundary element model of the Hole-Neck-Cavity Resonance tube
In this problem, substructuring technique would not make a significant difference in computation time as the geometry of the internal tube is not uniform. The BEM analysis is conducted between a frequency range of 5Hz and 300Hz at a frequency increment of 5Hz. Transmission loss curve obtained from the boundary element analysis is shown in Figure 4.2.4. The theoretical resonant frequency calculated using the equation (4.2) is 146.88 Hz. The comparison of the two transmission loss curves shows that the boundary element result almost exactly matches with the analytical solution.

Figure 4.2.3. ANSYS mesh of a part of the resonance tube

In this problem, substructuring technique would not make a significant difference in computation time as the geometry of the internal tube is not uniform. The BEM analysis is conducted between a frequency range of 5Hz and 300Hz at a frequency increment of 5Hz. Transmission loss curve obtained from the boundary element analysis is shown in Figure 4.2.4. The theoretical resonant frequency calculated using the equation (4.2) is 146.88 Hz. The comparison of the two transmission loss curves shows that the boundary element result almost exactly matches with the analytical solution.
Figure 4.2.4. Transmission loss of the Hole-Neck-Cavity Resonance tube
4.3. Concentric-tube resonator

The second example considered in this thesis is a concentric-tube resonator which is shown in Figure 4.3.1. A concentric-tube resonator consists of an external cylindrical tube and a perforated internal tube located co-axially. The two ends of the external cylinder are covered by circular flanges which are mounted on the internal tube. The external tube is 8in long and 6.035in in diameter and the internal tube has a diameter of 1.375 in. Three different perforated tubes are considered in this analysis and each tube differs in the perforate pattern as shown in Figure 4.3.2. Tube 1 has a porosity of 0.098 and 276 holes with a hole diameter of 3.175 mm. Tube 2 has a porosity of 0.058 and 162 holes with a hole diameter of 3.175 mm. Tube 3 has a porosity of 0.058 and 72 holes with a hole diameter of 4.7625 mm. Tube 1 and tube 2 share the same hole diameter whereas tube 2 and tube 3 have the same porosity.

The boundary element analysis of the concentric tube resonator is conducted by empirical formula method and the detailed hole model method. In the empirical formula method, the perforated tube is modeled as a cylindrical tube with only the porosity percentage or both the porosity percentage and the thickness of the tube and hole diameter given as input depending on the formula used. But no details pertaining to the location and distribution of the holes are included. Whereas, in the detailed hole model, the physical tube is modeled exactly as it appears. Since the concentric-tube resonator is axi-symmetric, rotational symmetry is used and a 60° block is modeled as shown in Figure 4.3.6. As the perforations on the internal tube are uniform, substructuring technique is used and the entire muffler is divided into a number of substructures. For the detailed hole model, the internal tubes with holes are meshed by ANSYS.
Figure 4.3.1. Schematic diagram of the single perforated tube muffler

<table>
<thead>
<tr>
<th>Name</th>
<th>Thickness of tube (m)</th>
<th>Tube diameter (m)</th>
<th>Tube Length (m)</th>
<th>Hole diameter(m)</th>
<th>No. of holes</th>
<th>porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube 1</td>
<td>0.0003</td>
<td>0.034925</td>
<td>0.2032</td>
<td>0.003175</td>
<td>276</td>
<td>0.098</td>
</tr>
<tr>
<td>Tube 2</td>
<td>0.0003</td>
<td>0.034925</td>
<td>0.2032</td>
<td>0.003175</td>
<td>162</td>
<td>0.058</td>
</tr>
<tr>
<td>Tube 3</td>
<td>0.0003</td>
<td>0.034925</td>
<td>0.2032</td>
<td>0.0047625</td>
<td>72</td>
<td>0.058</td>
</tr>
</tbody>
</table>
Figure 4.3.2. Perforated tubes in the single perforated tube muffler (from the left, Tube1, Tube 2 and Tube 3).
Figure 4.3.3. ANSYS model of the perforated tube (tube1) in Concentric-tube resonator

Figure 4.3.4. ANSYS model of the perforated tube (tube2) in Concentric-tube resonator
Figure 4.3.5. ANSYS model of the perforated tube (tube3) in Concentric-tube resonator

Figure 4.3.3 shows the 60° curved surface with holes of part of tube1. Figure 4.3.4 and Figure 4.3.5 show that of tube 2 and tube 3. This component is imported to MAP and the rest of the boundary element mesh of the resonator is constructed within this program. Figure 4.3.6, Figure 4.3.7 and Figure 4.3.8 show the first substructure, the template and the last substructure of the muffler, respectively. All the external components are modeled as ‘regular’ elements while the internal components are modeled as ‘thin’ elements. In the empirical formula model the ANSYS mesh of the perforated tube is replaced with a mesh generated by MAP. The porosity of the perforated tube, the hole diameter and the wall thickness are specified depending on whether the Equation (1.2) or Equation (1.3) is used.
Figure 4.3.6. Boundary element model of the first substructure.

Figure 4.3.7. Boundary element model of the template.
Figure 4.3.7 shows the template that represents the portion of the muffler with uniform geometry between the first and the last substructures. Since the distribution of the holes is different for the three tubes, the number of substructures is different for each tube. The number of substructures between the first and last substructure is 7, 7 and 4 for tube 1, tube 2 and tube 3 respectively. Figure 4.3.8 shows the boundary element mesh of the last substructure.

In this test case a total of six boundary element models are created for the analysis of the concentric tube resonator, two (empirical formula model and detailed hole model) per tube for three perforated tubes. Figure 4.3.9 and Figure 4.3.10 show the complete boundary element models of the concentric-tube resonator with tube 1. Figure 4.3.15 and Figure 4.3.16 show that of the concentric-tube resonator with tube 2. 4.3.21 and Figure 4.3.22 show that of the concentric-tube resonator with tube 3.
Transfer impedances (equations 1.2 and 1.3) are plotted against frequency and are shown in figures for the 3 tubes respectively. Since equation (1.3) is complex the real and imaginary parts are plotted separately. The two transfer impedances are almost the same for tubes 1 and 2 but there is a significant difference for tube 3. The difference in tube 3 is due to the hole diameter effect. The transmission loss curves show three different comparisons for each of the three tube cases. In the first one, the detailed hole model and the empirical formula model (Equation 1.2) are compared with the experimental result. In the second graph, the detailed hole model and the empirical formula model (Equation 1.3) are compared with the experimental result. The third one is the comparison of the detailed hole model and SYSNOISE (Equation 1.3) result with the experimental result. Figure 4.3.12 – Figure 4.3.14 show the transmission loss curves for tube 1, Figure 4.3.18 – Figure 4.3.20 show the transmission loss curve for tube 2, Figure 4.3.24 – Figure 4.3.26 show the transmission loss curve for tube 3. It is inferred from the graphs that at low frequencies the empirical formula models and the detailed hole models produce nearly same results and deviation from experimental result is not very significant. But at mid frequencies and higher frequencies the detailed hole model results are better than the empirical formula models. At low frequencies SYSNOISE results match the experimental results better than the detailed hole models even though the detailed hole model doesn’t show much deviation. But at mid frequencies and higher frequencies the detailed hole model results are better than SYSNOISE (Equation (1.3)) results. The two empirical formulas produced almost same results. The trend is same for all the three tube cases.
Figure 4.3.9. Boundary element empirical formula model of single perforated tube muffler (Tube 1: 276 holes)
Figure 4.3.10. Boundary element hole model of single perforated tube muffler (Tube 1: 276 holes)
Figure 4.3.11. Comparison of the transfer impedances (Tube 1: 276 holes)
Figure 4.3.12. Transmission loss of single perforated tube muffler (Tube 1: 276 holes) - Comparison 1
Figure 4.3.13. Transmission loss of single perforated tube muffler (Tube 1: 276 holes) - Comparison 2
Figure 4.3.14. Transmission loss of single perforated tube muffler (Tube 1: 276 holes) - Comparison 3
Figure 4.3.15. Boundary element empirical formula model of single perforated tube muffler (Tube 2: 162 holes)
Figure 4.3.16. Boundary element hole model of single perforated tube muffler (Tube 2: 162 holes)
Figure 4.3.17. Comparison of the transfer impedances (Tube 2: 162 holes)
Figure 4.3.18. Transmission loss of single perforated tube muffler (Tube 2: 162 holes) – Comparison 1
Figure 4.3.19. Transmission loss of single perforated tube muffler (Tube 2: 162 holes) - Comparison 2
Figure 4.3.20. Transmission loss of single perforated tube muffler (Tube 2: 162 holes) - Comparison 3
Figure 4.3.21. Boundary element empirical formula model of single perforated tube muffler (Tube 3: 72 holes)
Figure 4.3.22. Boundary element hole model of single perforated tube muffler (Tube 3: 72 holes)
Figure 4.3.23. Comparison of the transfer impedances (Tube 3: 72 holes)
Figure 4.3.24. Transmission loss of single perforated tube muffler (Tube 3: 72 holes) - Comparison 1
Figure 4.3.25. Transmission loss of single perforated tube muffler (Tube 3: 72 holes) - Comparison 2
Figure 4.3.26. Transmission loss of single perforated tube muffler (Tube 3: 72 holes) - Comparison 3
4.4. Concentric tube resonator with flow plug

The third example considered in this study is the concentric tube resonator with flow plug in the middle of the perforated tube. This example was analyzed by Wu, Zhang and Cheng [8] to compare Sullivan and Crocker’s formulas (Equation (1.2), (1.3)) and Bento Coelho’s formula (Equation (1.4)) and study if the details such as the perforate diameter and the tube thickness influence the transmission loss. In this thesis, the same example is considered to study the effect of the detailed hole model on the transmission loss. The BEM results obtained from the detailed hole model and the empirical formula models are compared with the experimental results.

Figure 4.4.1 shows the details of the model. The main duct (internal tube) contains both perforated and thin surfaces. The first region (length $L_2$ in Figure 4.4.1) has a porosity of 21.68 % and the second region (length $L_5$ in Figure 4.4.1) has a porosity of 3.54 %. The substructuring technique is not employed in this analysis as the geometry along the length of the muffler does not exhibit a repetitive pattern. Since the muffler exhibits rotational symmetry about its axis, a 30$^\circ$ sector is considered for the boundary element model. The analysis involves modeling the muffler in two methods- the detailed hole model and the empirical formula method. In the empirical formula method Sullivan and Crocker’s formulas (Equations (1.2) and (1.3)) are used. The analysis is performed for a total of three models. For the detailed hole model the perforated tubes (lengths $L_2$ and $L_5$ in Figure 4.4.1) are meshed in ANSYS as shown in Figure 4.4.2 and Figure 4.4.3, respectively.
$L_1 = 0.0317 \text{ m}, L_2 = 0.0952 \text{ m}, L_3 = 0.0255 \text{ m}, L_4 = 0.0063 \text{ m}, L_5 = 0.0952 \text{ m}, L_6 = 0.0953 \text{ m}, R = 0.0538 \text{ m}, r = 0.0254 \text{ m}$. For perforations in $L_2$: $\sigma = 0.2168$, $d_h = 0.00635 \text{ m}$, $t = 0.0011938 \text{ m}$; for perforations in $L_5$: $\sigma = 0.1354$, $d_h = 0.00635 \text{ m}$, $t = 0.0011938 \text{ m}$.

Figure 4.4.1. Concentric-tube muffler with a flow plug [8].
Figure 4.4.2. ANSYS mesh of tube $L_2$ in concentric tube resonator with flow plug.

Figure 4.4.3. ANSYS mesh of tube $L_3$ in concentric tube resonator with flow plug.
The tubes L₁, L₃, L₄, L₆ (Figure 4.4.1) are modeled as ‘thin’ elements while the external components are modeled as ‘regular’ elements. In the empirical formula method the tubes L₂ and L₅ (Figure 4.4.1) are modeled as perforated tubes whereas in the detailed hole model the meshes of the tubes are imported from ANSYS.

The BEM prediction of the transmission loss is compared to the experimental data and also compared to the BEM results obtained by Wu, Zhang and Cheng [8] using the empirical formulas for perforated tubes. Figure 4.4.6 shows the transmission loss of the BEM predictions (thin line) of the detailed hole model compared to the BEM prediction (dotted line) using the Sullivan and Crocker’s Equation (1.2) and the experimental data (thick line). Figure 4.4.7 shows the transmission loss of the BEM predictions (thin line) of the detailed hole model compared to the BEM prediction (dotted line) using the Sullivan and Crocker’s Equation (1.3) and the experimental data (thick line). Wu, Zhang and Cheng [8] showed that Equation (1.3) in which the hole diameter and the wall thickness are included, produced slightly better results than Equation (1.2) in which those details are not included. These results are shown in Figure 4.4.6 and Figure 4.4.7. The detailed hole model results almost match the empirical formula results and the experimental results at low frequencies. At mid and high frequencies, there are shifts in the peaks for both the empirical formula method and the detailed hole model method.
Figure 4.4.4. Boundary element empirical formula model of Concentric-Tube muffler with a flow plug
Figure 4.4.5. Boundary element hole model of Concentric-Tube muffler with a flow plug
Figure 4.4.6. Transmission loss of Concentric-Tube muffler with a flow plug – Comparison1
Figure 4.4.7. Transmission loss of Concentric-Tube muffler with a flow plug – Comparison 2
4.5. Muffler with two parallel perforated tubes

The fourth example considered in this study is the muffler with two parallel perforated tubes. This is another example analyzed by Wu, Zhang and Cheng [8] to compare the different empirical formulas and to study the effect of hole diameter and wall thickness on the transmission loss. Figure 4.5.1 shows the details of the model. The muffler consists of two inner tubes (Tube 1 and Tube 2 in Figure 4.5.1) parallel to each other. These two tubes are enclosed within an external tube (Tube 3 in Figure 4.5.1). Part of the internal tubes within the chamber is perforated (length $L_2$ in Figure 4.5.1). The right end of Tube 1 and the left end of Tube 2 are closed by flanges. Sound entering through the inlet of one of the tubes (Tube 1 in Figure 4.5.1) enters the chamber enclosed by the outer tube (Tube 3 in Figure 4.5.1) and passes into the other inner tube (Tube 2 in Figure 4.5.1) through the perforates in the inner tubes and emerges out at the outlet. The external tube has a radius of 0.1016m and a length of 0.508m. The internal tubes are each 0.0254m in radius and are offset by a distance of 0.0381m from the axis of the external tube. The perforations on the internal tubes start at 0.0254m from the left flange and are uniformly distributed over a length of 0.4572m on either tube. The diameter of the holes is 0.003175m. The wall thickness of the perforated tubes is 0.0011938m and the porosity of the tubes is 14.4 percent.

The boundary element analysis of the parallel tube muffler is basically conducted on the empirical formula models and the detailed hole model. In the empirical formula method Sullivan and Crocker’s formulas (Equations 1.2 and 1.3) are used. Figure 4.5.7 and Figure 4.5.8 show the boundary element mesh of the empirical formula model and the detailed hole model of the muffler with two parallel perforated tubes, respectively.
\[ L_1 = 0.0254 \text{ m}, \ L_2 = 0.4572 \text{ m}, \ L_3 = 0.0254 \text{ m}, \ L_4 = 0.0381 \text{ m}, \ L_5 = 0.0381 \text{ m}, \ R = 0.1016 \text{ m}, \ r = 0.0254 \text{ m}. \]  

For both tubes: \( \sigma = 0.144 \), \( d_h = 0.003175 \text{ m} \), \( t = 0.0011938 \text{ m} \).

Figure 4.5.1. Muffler with two parallel perforated tubes [8].
For the detailed hole model, part of the perforated tube mesh is generated in ANSYS and imported to MAP for the boundary element analysis. Figure 4.5.2 shows the $180^\circ$ model of the ANSYS generated mesh of the perforated tube. Figure 4.5.3 shows the boundary element model of the first substructure. Figure 4.5.5 shows the template that represents the uniform geometry portion of the muffler. There are 14 substructures between the first and last substructure and they are identical to the template. The impedance matrix for the portion of the muffler between the template and the last substructure is duplicated from that of the template. Figure 4.5.6 shows the boundary element mesh of the last substructure. All the internal components are modeled with ‘thin’ elements while the external components are modeled with ‘regular’ elements.

Figure 4.5.2. ANSYS mesh of the perforated tube.
Figure 4.5.3. Boundary element model of the first substructure.

Figure 4.5.4. ANSYS mesh of inlet and outlet to the substructures.
Figure 4.5.5. Boundary element mesh of the template.

Figure 4.5.6. Boundary element mesh of the last substructure.
Figure 4.5.9 shows the transmission loss curves of the detailed hole model and empirical formula model using Sullivan and Crocker’s Equation (1.2) compared with the experimental results. Figure 4.5.10 shows the transmission loss curves of the detailed hole model and empirical formula model using Sullivan and Crocker’s Equation (1.3) compared with the experimental results. At low frequencies both the detailed hole model and empirical formula model produce identical results. At mid frequencies the empirical formula models yield slightly better results compared to the hole model. Between the two empirical formula models Equation (1.3) which includes the wall thickness of the tube and hole diameter gives better results compared to Equation (1.2). At high frequencies for either model the peaks are shifted compared to the experimental results. But the detailed hole model peaks are more consistent with the experimental peaks and match slightly better compared to the empirical formula peaks.
Figure 4.5.7. Boundary element empirical formula model of muffler with two parallel perforated tubes.
Figure 4.5.8. Boundary element detailed hole model of muffler with two parallel perforated tubes.
Figure 4.5.9. Transmission loss of muffler with two parallel perforated tubes - Comparison 1
Figure 4.5.10. Transmission loss of muffler with two parallel perforated tubes – Comparison 2
5.1. General

Catalytic converters (CC) and diesel particulate filters (DPF) are required to be used as emission control devices in trucks and automobiles [28]. The primary purpose of using the CC and DPF are to reduce the gaseous exhaust pollutants. Besides reducing pollution, they have a significant effect on the overall design and acoustical performance of the exhaust system.

Figure 5.1. Emission control device with (A) a through-flow catalyst converter (CC), (B) a wall-flow diesel particulate filter (DPF) [28].
The commonly used substrates in catalytic converters and diesel particulate filters are the extruded ceramic monolithic honeycombs [28]. Monoliths used in catalytic converters are known as through-flow monoliths. In this type of monoliths, the substrate consists of a number of small parallel ducts which allow the exhaust gas to flow through the catalytic converter. The catalytic process, in which the harmful pollutant gases are converted into harmless gases, is enabled by the catalyst material coated on the surfaces of the substrate [28]. Wall – flow monoliths are similar to the flow-through monoliths except that they have only one end of the ducts open and the other ends are plugged. This type of monoliths is used in DPF. When gas flows through the wall-flow monoliths, they are forced to penetrate through the walls of the ducts as the other ends are closed. This process results in the filtration of the particulate matter by the wall pores.

In this study, the catalytic converter analyzed by Cheng, Wu and Lou [28] is considered for the BEM analysis using a detailed model. They considered two different approaches. In the first one, the filter substrate is treated as a sound absorptive bulk-reacting material which is characterized by complex speed of sound and complex mean density. The second method is the impedance matrix synthesis approach [6] in which the monolith is represented by a four-pole-type transfer matrix as discussed in chapter 2. In this thesis, a third approach is investigated in which the monolith with all its ducts is modeled in detail. Since modeling the details of the monolith results in a large number of elements, the substructuring technique is used to reduce the matrix size and save computation time. In the analysis performed in this study, the walls of the ducts are modeled in 2D while in the physical model the ducts have a finite wall thickness. Since the wall thickness affects the acoustic properties of the converter, besides modeling the
ducts, the bulk reacting properties of the converter is also included in the model. The boundary element model of the catalytic converter basically consists of two parts, the outer curved surface and the inner duct partition. Besides these two components, other geometric models required for running the substructure analysis such as the inlet and outlet and the interface surfaces are meshed using ANSYS and imported to MAP where the BEM analysis of the catalytic converter is conducted.

**Figure 5.2. Muffler with a see-through catalytic converter [28].**

### 5.2 Flow-through Catalytic converter

Figure 5.2 shows the physical model of the muffler with a flow-through catalytic converter. The muffler has an inlet tube with a diameter of 1.9 inches followed by a conical tube of length 7 inches and a larger end diameter of 3.25 inches. The conical tube is followed by a cylindrical tube of length 3 inches and diameter of 3.25 inches. The see-through catalytic converter is attached at the end of the cylindrical tube. The catalytic
The analysis consists of modeling the catalytic converter in two methods: (i) bulk reacting material (ii) bulk reacting material with the duct partition included. Figure 5.9 shows the boundary element model of the muffler with the catalytic converter treated as a bulk reacting material and Figure 5.10 shows the boundary element model of the muffler with the catalytic converter treated as a bulk reacting material and the partition included.

The boundary element model of the muffler consists of three substructures with ten substructures between the first and the last substructure. ANSYS is used to generate the meshes of the following components: curved surface of the converter (Figure 5.3), partitions of the converter (Figure 5.4), inlet and outlet faces between the substructures (Figure 5.5) and the interface mesh between air and bulk reacting material (Figure 5.5).

Figure 5.3. ANSYS mesh of the curved surface of the catalytic converter.

The analysis consists of modeling the catalytic converter in two methods: (i) bulk reacting material (ii) bulk reacting material with the duct partition included. Figure 5.9 shows the boundary element model of the muffler with the catalytic converter treated as a bulk reacting material and Figure 5.10 shows the boundary element model of the muffler with the catalytic converter treated as a bulk reacting material and the partition included.

The boundary element model of the muffler consists of three substructures with ten substructures between the first and the last substructure. ANSYS is used to generate the meshes of the following components: curved surface of the converter (Figure 5.3), partitions of the converter (Figure 5.4), inlet and outlet faces between the substructures (Figure 5.5) and the interface mesh between air and bulk reacting material (Figure 5.5).
Figure 5.4. ANSYS mesh of the partitions in the catalytic converter

Figure 5.5. ANSYS mesh of the inlet and outlet to substructures
Figure 5.6. BEM mesh of the first substructure of muffler with a catalytic converter

Figure 5.7. Boundary element model of template.
Figure 5.6 shows the boundary element mesh of the first substructure. The interface between air and bulk reacting material is modeled as ‘interface’ elements. The outer curved surface of the catalytic converter is modeled as ‘B’ (Bulk reacting material) elements, the partitions in the catalytic converter are modeled as ‘BTB’ (Bulk reacting – Thin – Bulk reacting) elements. In the bulk reacting material model the partition in the catalytic converter is not included. Figure 5.7 shows the boundary element model of the template. Figure 5.8 shows the boundary element mesh of the last substructure.

![BEM mesh of the last substructure.](image)

The analysis was conducted between a frequency range of 50 Hz to 3000 Hz. Figure 5.11 shows the transmission loss curves of the bulk reacting material model and partitioned catalytic converter model compared to the experimental results. The results of both the models are identical except for minor variations at high frequencies.
Figure 5.9. Boundary element bulk-reacting material model of muffler with a see-through catalytic converter
Figure 5.10. Boundary element detailed model of muffler with a see-through catalytic converter
Figure 5.11. Transmission loss of muffler with a flow-through catalytic converter
CHAPTER 6
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

6.1 Summary and Conclusions

In this paper, the holes on perforated tubes are modeled in detail using the direct mixed-body BEM. A substructuring technique is used to facilitate the modeling of the complex perforate pattern. The advantage of the substructuring technique is that when a large structure is broken into smaller substructures, at any given time only the substructure under consideration needs to be stored. Further for mufflers with uniform geometry substructuring technique considerably reduces the number of elements required to model the structure. In such cases it is sufficient to model only a portion of the uniform geometry of the muffler called the template. The impedance matrix generated for the template is duplicated to complete the rest of the unmodeled portion of the muffler thereby reducing computation time and storage.

Results show that the detailed perforate modeling can predict similar or sometimes slightly better results than the empirical formulas. Although the empirical formula approach is still the simplest way of modeling perforated surfaces, the detailed perforate modeling provides an alternative when the empirical formulas do not work well for perforated tubes having non-uniform hole-diameter or hole-pattern.

6.2 Recommendations for future work

The perforated tube mufflers considered in this study have a uniform hole pattern and hole diameter. The detailed model approach can be tested for perforated tubes with non-uniform hole
pattern and hole diameter. As empirical formula method may not be suitable for such cases, experiments need to be conducted to validate the results of the detailed model method.

The analysis conducted on the flow through catalyst converter in only a preliminary investigation. Although the partitions in the catalyst converter are modeled the wall thickness is not included due to modeling difficulties. The thickness of the partitions is an important factor in the determination of the impedance characteristics of the catalyst converter. Future work can be done on modeling the actual geometry of the catalyst converter and compared to experimental results. Also the analysis can be extended to wall-flow catalyst converters.
REFERENCES


VITA

Author’s Name – Balasubramanian Datchanamourty

Birthplace – Pondicherry, India

Birthdate – May 25, 1978

Education

Bachelor of Technology in Mechanical Engineering
Pondicherry Engineering College, India
May – 2001

Research Experience
Research Assistant, Department of Mechanical Engineering
University of Kentucky
Lexington, KY
1/02 – 8/02

Teaching Experience
Teaching Assistant, Department of Civil Engineering
University of Kentucky
Lexington, KY
6/03 – present

Work Experience
Engineer, Ancillary Development Department
Tractors and Farm Equipment Limited
Chennai, India
6/99 – 7/01
Publications

Awards
Best Undergraduate Project of the Year 1998 - 1999 in the Mechanical Engineering Department.