VALIDATION OF FINITE ELEMENT PROGRAM FOR JOURNAL BEARINGS -- STATIC AND DYNAMIC PROPERTIES

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ABSTRACT OF THESIS

VALIDATION OF FINITE ELEMENT PROGRAM FOR JOURNAL BEARINGS – STATIC AND DYNAMIC PROPERTIES

The analysis of bearing systems involves the prediction of their static and dynamic characteristics. The capability to compute the dynamic characteristics for hydrodynamic bearings has been added to Bearing Design System (BRGDS), a finite element program developed by Dr. R.W. Stephenson, and the results obtained were validated. In this software, a standard finite element implementation of the Reynolds equation is used to model the land region of the bearing with pressure degrees of freedom. The assumptions of incompressible flow, constant viscosity, and no fluid inertia terms are made. The pressure solution is integrated to give the bearing load, and the stiffness and damping characteristics were calculated by a perturbation method.

The static and dynamic characteristics of 60°, 120° and 180° partial bearings were verified and compared for a length to diameter (L/D) ratio of 0.5. A comparison has also been obtained for the 120° bearing with L/D ratios of 0.5, 0.75 and 1.0. A 360°-journal bearing was verified for an L/D ratio of 0.5 and also compared to an L/D ratio of 1.0. The results are in good agreement with other verified results. The effect of providing lubricant to the recesses has been shown for a 120° hybrid hydrostatic bearing with a single and double recess.

Keywords: Reynolds equation, Journal bearings, Stiffness, Damping, Sommerfeld Number.

Raja Shekar Balupari
07/28/2004

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VALIDATION OF FINITE ELEMENT PROGRAM FOR JOURNAL BEARINGS – STATIC AND DYNAMIC PROPERTIES

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07/28/2004
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THESIS

Raja Shekar Balupari

The Graduate School
University of Kentucky
2004
VALIDATION OF FINITE ELEMENT PROGRAM FOR JOURNAL BEARINGS – STATIC AND DYNAMIC PROPERTIES

THESIS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering in the College of Engineering at the University of Kentucky

By

Raja Shekar Balupari

Lexington, Kentucky

Director: Dr. Keith Rouch, Professor of Mechanical Engineering

Lexington, Kentucky
2004

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Dedication

To my family, for their everlasting love, support and encouragement
ACKNOWLEDGEMENTS

I would like to express deep gratitude to Dr. Keith Rouch for having given me this opportunity to work with him. The present work would not have been possible without the consistent guidance and patience shown by him. I would like to thank Dr. R. W. Stephenson from Mechcon, Inc., for his guidance and most importantly for providing me with important data. I would also like to thank Dr. Raymond LeBeau and Dr. David Herrin for serving on my thesis committee.

I would like to thank my family for their tremendous support and encouragement to pursue higher studies. I would particularly like to thank my brother Ravindra Balupari for his support and guidance throughout my graduate study. I would also like to thank all my friends for their assistance during my stay here at the University of Kentucky.
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CHAPTER ONE
INTRODUCTION

1.1 Introduction
A fluid film bearing may be defined as a bearing in which the opposing or mating surfaces are completely separated by a layer of fluid lubricant (Wilcock, 1979). A widely used bearing type is the plain journal bearing with application in compressors, turbines, pumps, electric motors and electric generators.

Journal Bearings consists of two cylinders rotating relative to each other. The outer cylinder (bearing) is stationary and the inner ring (shaft) rotating with an angular velocity is called the Journal as shown in figure 1.1. The main purpose of the journal bearing is to support the rotating machinery by providing sufficient lubrication to separate the moving parts and to minimize the friction due to rotation (Allaire, 1987). The high-pressure fluid film in the clearance between the journal and the bearing due to rotation of the journal provides the hydrodynamic film lubrication, and the load capacity to the bearing. The displacement of the shaft center relative to the bearing center is known as eccentricity.

Nearly all heavy industrial turbomachines use fluid film bearings of some type to support the shaft weight and control motions caused by unbalance forces. The two primary advantages of fluid film bearings over rolling element bearings are their superior ability to absorb energy to dampen vibrations, and their longevity due to the absence of rolling contact stresses. The damping is very important in many types of rotating machines where the fluid film bearings are often the primary source of the energy absorption needed to control vibrations. Fluid film journal bearings also play a major role in determining rotordynamic stability, making their careful selection and application is a crucial step in the development of superior rotor-bearing systems. Journal bearings are incredibly long-lived provided the lubricant is contaminant free and sufficiently supplied. Thus dynamic analysis of hydrodynamic bearings is important due to the forces imposed on the shaft from machine unbalance forces, aerodynamic forces, and external excitations from seals and couplings (Allaire, 1979).
1.2 Types of Journal Bearings
Journal bearings may be grouped into two main classes - full bearing in which the bearing arc completely surrounds the journal and partial bearing in which the arc is less than 360° (Raimondi and Boyd, 1958). Bearings are further classified as plain journal bearings, axial groove bearings, pressure dam bearings, multi-lobe journal bearings, tilting pad journal bearings, and hybrid hydrostatic bearings.

1.2.1 Partial Arc Bearings
Partial bearings as shown in figure 1.2 may be further classified as centrally or eccentrically loaded depending upon whether the load line bisects the bearing pad arc or divides it eccentrically. Typically the radius of the bearing is greater than the radius of the journal by an amount equal to the radial clearance. When the load applied is primarily unidirectional there is no need of full journal bearings and instead a single partial arc bearing may be used (Allaire, 1987). Partial arc bearings are used in relatively low speed applications. The partial arc pad may also be designed with a hydrostatic recess thus enabling fluid film lubrication due to both hydrostatic and hydrodynamic effects. Partial arc bearings are important because they form the building blocks for axial groove, multi-lobe and tilting pad bearings. The present work deals with the computation of dynamic characteristics of partial arc bearings.

1.2.2 Plain Journal Bearings (360°)
Plain bearings are composed of a cylindrical shaft of radius $R$, called a journal, rotating with an angular velocity, $\omega$, about its axis in a cylindrical bushing of radius $R+c$ and length $L$. The center of the bearing is labeled as $O_b$ and the center of journal as $O_j$ as shown in figure 1.1. Under steady state conditions the journal center remains at a constant eccentricity $e$ and attitude angle $\Phi$ for a given load $W$ acting on the shaft (Allaire et al., 1975, Allaire et al., 1980). The plain journal bearing is the simplest and most common radial bearing design, where a plain cylindrical shell encircles the shaft. Plain sleeve bearings have the highest cross coupling of all bearings and are suitable for highly loaded or low-speed shafts. Advantages are low cost and ease of manufacture. Examples include automotive crankshaft bearings and low-speed or highly loaded turbo machinery applications.
1.2.3 Axial Groove Bearings
The axial groove bearing is similar to the plain journal bearing, but with two or more axial grooves added for oil supply. As in the plain journal bearing, there is no preload and has a high tendency for instability (Allaire et. al., 1975). Advantages are low cost and ease of manufacture. Axial groove bearings are very common in many types of commercial machinery including turbines, generators, motors, pumps, and compressors, and have slightly better dynamic properties than do plain journal bearings.

1.2.4 Pressure Dam Bearings
The pressure dam bearing is a fixed-geometry bearing improves rotor dynamic stability than plain journal bearings. A pocket is milled in the upper (unloaded) half of the bearing that ends in a "dam". A sharp pressure peak is created at the dam due to fluid inertia effects. The pressure peak imposes a downward load on the journal, forcing the shaft down to a greater eccentricity that inherently improves stability because of the stiffness and damping asymmetry induced. Pressure dam bearings have relatively high power losses because of the built-in load. They are not suitable for applications where the load direction changes because the top half is pocketed for the dam. The dam also restricts them to unidirectional operation and the oil must be kept clean to prevent sludge accumulation in the pocket. Manufacturing of pressure dam bearings is more difficult and expensive than plain bearings, as the dam geometry is milled separately from the bore and must be precise. Pressure dam bearings are used primarily in higher speed applications to increase stability over other types of fixed-geometry bearings.

1.2.5 Multi-lobe Bearings
Multi-lobe bearings are essentially bearings with more than one bearing pad that enable a combination of number of pads, rotation of bearing, clearance, preload, and offset. This produces a stabilizing effect on the shaft and can increase load capacity. The center of curvature of the pad radius \((R+c)\) may be offset from the bushing/bearing center (preload). The preload factor in this case is given by the ratio of distance of center of curvature from bushing center to the clearance in the bearings (Allaire et al., 1975, Allaire et al., 1980). However, it can also consume more power due to the preload. Multi-lobe bearings can be either bidirectional or unidirectional, depending on whether the lobes have symmetric or asymmetric lobes. Multi-lobe bearings are
difficult and expensive to manufacture because of the precise machining operations required. They are commonly used in smaller, high-speed machines requiring high load capacity or high stability.

### 1.2.6 Tilting Pad bearings

The tilting pad bearing differs from the multi-lobed bearing in that each pad rotates about a pivot, enabling each pad to have higher degrees of freedom corresponding to movement about the pivot point. The pad tilts such that its center of curvature moves to create a converging pad film thickness. These pads are able to follow the shaft motion resulting in little cross-coupled stiffness and damping (Allaire et. al., 1975). They are widely used to stabilize machines that have sub synchronous vibrations. These bearings have higher power losses and higher cost to manufacture.

### 1.2.7 Hybrid Bearings

In hydrostatic bearings, high-pressure lubricant is fed to a recess in the pad. They have improved high-speed load carrying capacity compared to the hydrodynamic operation. Their design depends on the hydrodynamic effect in addition to the hydrostatic effect to achieve necessary load support. Hybrid bearings are advantageous over pure hydrodynamic bearings in that wear can be avoided at starting or stopping, they tolerate substantial loads above normal design load, withstand heavy dynamic loadings which vary widely in direction of rotation, and may allow for design with a smaller journal diameter, reducing initial cost and operating power consumption.

All the bearings apart from the tilting pad bearings discussed above are composed of fixed pad plain bearing geometry. Thus study of the plain bearing is significant, as the majority of the bearings possess similar characteristics as the plain bearing in geometry as well as operation. Full bearings and centrally loaded partial arc bearings of the clearance type are common, and it is to these bearings that the analysis presented in this dissertation is applicable. Also the dynamic analyses have been performed on the fixed partial arc bearings.
1.3 Literature Review

As already stated the dynamic analysis of hydrodynamic bearings is essential as they provide stability and control mechanical vibrations that occur in rotating machinery. Several numerical techniques have been proposed to provide the solution of the fluid film lubrication problem. Raimondi and Boyd (1958) provided the numerical solution of finite journal bearings for application in analysis and design using finite difference method. Li (1977) and Hays (1959) provided the analytical dynamics of partial journal bearings and finite journal bearings respectively using variational methods to solve the Reynolds equation. The finite element method has been used prominently for some years to continuum and field problems (Zienkiewicz, 1970). Reddi (1969) presented the finite element solution for incompressible lubrication problems of complex geometries without the loss of accuracy as the finite difference method. Wada, et al. (1971) has successfully used the finite element method for finite width and infinite width lubrication problems. They also showed that finite elements might produce more accurate solutions for journal bearings than finite difference methods. Huebner and Booker (1972) applied the finite element method to the general lubrication problem with a systematic description of procedures. Huebner (1975) provided the most recent and extensive treatment of fluid film lubrication. Allaire, et al. (1977) developed a systematic matrix approach for finite elements which automatically produce minimum bandwidth of algebraic equations. By organizing the labeling in matrix form throughout the analysis, the solution could be easily obtained using Gaussian elimination methods or other methods. Later, Allaire, et al. (1984) provided a pressure parameter method for the finite element solution of Reynolds equation to improve the accuracy significantly. Rao (1982) provided the mathematical formulation of the finite element method to the hydrodynamic lubrication problem governed by the Reynolds equation.

Allaire, et al. (1975) developed a finite element program FINBRG1 to compute the bearing characteristics of plain, partial arc, axial groove, multi-lobe and tilting pad bearings. Allaire, et al. (1980) also performed the dynamic analysis of incompressible fluid film bearings. Stephenson (1997) developed Bearing Design System, a program to perform bearing analysis using the finite element method.
1.4 Scope of Thesis

'BRGDS' is an acronym for Bearing Design System. The initial program developed by Dr. Stephenson has the capability to compute the bearing pressure distribution, film thickness, bearing forces and moments for a single partial arc bearing with multiple recesses in the pad. The program includes turbulence as an option. Either eccentricity or load can be input to the bearings. If load is input, the eccentricity and attitude angle are solved iteratively.

The motivation for the present work has been to include the capability to compute the damping characteristics for plain, partial arc and hybrid hydrodynamic bearings and to further validate the results. The finite element solution for the partial arc and full plain journal bearing including the bearing damping calculations are the highlight of this thesis. The bearing program is based on Fortran 77. The function STIFFCALC to compute stiffness and DAMPCALC that has been introduced to compute the damping characteristics of the bearings are shown in Appendix A2 and A3. The results obtained were compared and verified with other results. Most of the results used for comparison were obtained from Dynamic Rotor Bearing System program, ‘DyRoBes’. DyRoBeS™ is a powerful and sophisticated software tool authored by Wen Jeng Chen for rotor dynamics including comprehensive bearing analysis, with results courtesy of Stephenson R. W. from Mechcon, Inc.

The next chapter of this work deals with the solution of fluid film bearings, viscosity, assumptions that are used in the analysis and their validity. The Reynolds equation for fluid bearings has been shown and revised for finite width bearings. Also the finite element solution of the Reynolds equation that is the backbone of the present work has been detailed. Chapter three deals with the bearing design program (BRGDS). The general systems of equation, bearing coordinate system, solution procedure and convergence conditions used in the program are documented. However, the main focus is on the computation of stiffness and damping characteristics for fluid film bearings. The results that are obtained by the analysis are shown in chapter four. Comparisons have been made for the stiffness and damping characteristics to succeed the present program for further analysis. Plots showing the analysis of various other bearing types are also shown and comments on the results have been provided. Finally, the scope for future work has been outlined based on the present work.
Figure 1.1: Schematic diagram of a full plain journal bearing (Allaire et al., 1975)
Figure 1.2: Schematic diagram of partial arc bearing (Allaire et. al., 1975).
CHAPTER TWO
SOLUTION FOR FLUID FILM BEARINGS

2.1 Introduction
Fluid film bearings have been defined as bearings in which the opposing or mating surfaces are completely separated from each other by a layer of fluid lubricant. Thus it is essential to understand the fundamental process by which the fluid is maintained while supporting the load. This chapter deals with the basic principles of fluid film bearings, viscosity of fluids, the Reynolds equation and the finite element interpretation of the Reynolds equation.

The basic principle can be understood by considering high-speed fluid flow between two plane surfaces with a converging wedge and lubricant between the surfaces. The convergence coupled with high-speed fluid flow and fluid viscosity generates a high-pressure fluid film that supports the load. In the case of a plain journal bearing, the converging wedge is formed at the bottom of the bearing due to the weight of the shaft and any other external applied load. The fluid is pulled into the region under the shaft due to the shear forces generated by the shaft rotation. The fluid is forced into the converging film thickness at the bottom of the shaft producing a high-pressure film (Allaire, 1987). This film supports the weight of the rotating machine components and prevents the shaft from touching the bushing surface. For a given eccentricity the fluid film has converging and diverging geometry, such that cavitations may occur in the diverging portion. It is thus very important to be able to predict the pressure distribution and load capacity of the bearing.

2.2 Viscosity
The viscosity of the fluid lubricant is its most significant physical characteristic as far as fluid film bearings are concerned. In this analysis of hydrodynamic bearings the fluid is incompressible, and the viscosity remains constant throughout the flow. The amount of pressure generated depends on the viscosity and density of the fluid. Viscosity is the property that defines the resistance of the fluid to motion. The viscosity is due to the molecular attraction between the adjacent layers of fluid. Any applied shear force will cause the fluid to move, resulting in a
resistance to flow that depends on the viscosity of the fluid. When a fluid is at rest, there is no resistance to an applied shear force. In a positive sense, viscosity controls the fluid flow out of the bearing but it is the cause of power consumption of the bearing due to lubricating shear (Wilcock, 1979).

A mathematical equation to quantify viscosity can be obtained from figure 2.1. A fluid film of thickness \( H \) separates a lower plate that is stationary from an upper plate. The upper plate moves with velocity \( U \) due to application of force \( F \). The fluid particle \( O \) in contact with the lower plate has zero velocity, and the fluid particle \( Q \) attached to the upper plate moves with the velocity \( U \). For a Newtonian fluid the fluid velocity increases linearly from the stationary plate to the moving plate as shown in the figure 2.1. The viscosity \( \mu \) of any intermediate particle \( P \) located at a distance \( y \) from the lower plate moving with velocity \( v \) can be obtained from equation 2.1. Rearranging the terms we obtain equation 2.2, where \( \tau \) is the shear stress developed in the fluid, \( A \) is the area of moving plate in contact with the fluid, and \( U/H \) is the velocity gradient between the plates. The flow caused by viscosity is known as Couette flow.

\[
F = \mu \left( \frac{UA}{H} \right) \tag{2.1}
\]
\[
\mu = \frac{\tau}{v/y} = \frac{\tau}{U/H} \tag{2.2}
\]

Thus dynamic or absolute viscosity has been defined as the force required for moving a flat unit plate located at a unit distance from a stationary plate by a unit velocity where the space between the plates is filled by the fluid. The units of absolute viscosity are \( \text{lb.s/ft}^2 \). Fluids whose viscosity remains constant with change in velocity gradient are called Newtonian fluids, while fluids that do not have a linear relationship are called non-Newtonian fluids. The ratio of absolute viscosity to the density of the fluid is called kinematic viscosity (Szeri, 1999) as in equation 2.3. The units of kinematic viscosity are \( \text{ft}^2/\text{sec} \).

\[
\nu = \frac{\mu}{\rho} \tag{2.3}
\]
2.3 Solution of fluid film bearings

The wear free transfer of force in hydrodynamic lubricated bearings is based on the application of pressure in the lubricating film that balances the external forces on the bearing. Osborne Reynolds and others (Peeken, et al, 1983) derived from the common Navier-Stokes equations (2.4, 2.5 and 2.6) and the continuity equation (2.9), a differential equation for the calculation of the pressure distribution (Szeri, 1999). The Laplacian and the divergence operators are shown in equations 2.7 and 2.8.

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho f_x - \frac{\partial p}{\partial x} + \mu \left( \nabla^2 u \right) + \frac{\mu}{3} \frac{\partial}{\partial x} \left( \nabla \cdot u \right) \quad \text{X-direction (2.4)}
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \rho f_y - \frac{\partial p}{\partial y} + \mu \left( \nabla^2 v \right) + \frac{\mu}{3} \frac{\partial}{\partial y} \left( \nabla \cdot u \right) \quad \text{Y-direction (2.5)}
\]

\[
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \rho f_z - \frac{\partial p}{\partial z} + \mu \left( \nabla^2 w \right) + \frac{\mu}{3} \frac{\partial}{\partial z} \left( \nabla \cdot u \right) \quad \text{Z-direction (2.6)}
\]

\[
\nabla^2 (\ast) = \frac{\partial^2 (\ast)}{\partial x^2} + \frac{\partial^2 (\ast)}{\partial y^2} + \frac{\partial^2 (\ast)}{\partial z^2} \quad (2.7)
\]

\[
\nabla \cdot u = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \quad (2.8)
\]

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0 \quad (2.9)
\]

The Navier-Stokes equations in which the fluid inertia, body, pressure and viscous forces are included are too complicated to yield an analytical solution for most problems. The Reynolds equation is a simplified version of Navier-Stokes equation. The analytical solution of the journal bearing is obtained by using the most popular form of the Reynolds equation. The following assumptions have to be considered in order to obtain the solution.
2.3.1 Assumptions

The exact representation of the fluid flow in mathematical terms using Navier-Stokes equation is complicated for fluid film bearings. Certain assumptions may be made to obtain a standard equation known as Reynolds equation (2.10). The following assumptions are typical in the analysis of fluid film bearings (Cameron, 1917).

1. Body forces are neglected. This means that there are no inertial or other distributed forces acting on the fluid. The effect of gravity of the fluid is neglected.
2. The pressure of the fluid is assumed constant across the thickness of the film.
3. The curvature of the surfaces is large compared to the film thickness. Surface velocities are considered unidirectional.
4. There is no slip at the boundaries. The velocity of the oil layer adjacent to the boundary is the same as that of the boundary. The fluid attached to the bearing surface is stationary while the fluid near the rotor or shaft has the same angular velocity as the shaft itself.

Apart from these assumptions there are certain assumptions that are considered to enable simplification of the mathematical equations. These assumptions are not essential but make the equations simpler (Allaire, 1987).

5. The fluid lubricant used is Newtonian. This means the fluid obeys Newton’s law of viscosity and the fluid stress is proportional to the rate of shear.
6. Fluid film flow in the bearings is laminar.
7. The viscosity of the fluid does not change.
8. The density of the fluid remains constant.

2.3.2 Reynolds Equation

Reynolds equation with appropriate boundary conditions describes the pressure between two surfaces separated by a thin fluid film. Consider two surfaces moving relative to each other separated by a constant fluid film of thickness as shown in figure 2.2. The most generalized representation of Reynolds equation for fluid film lubrication, developed by Dowson (1962), can be represented as shown in equation 2.10.
\[
\frac{\partial}{\partial x} \left( \rho h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho h^3 \frac{\partial p}{\partial y} \right) = h U_2 \frac{\partial \rho}{\partial x} + h V_2 \frac{\partial \rho}{\partial y} + h \frac{\partial \rho}{\partial t} \\
- \frac{\partial}{\partial x} \left( U_2 - U_1 \right) \frac{\partial}{\partial x} (\rho h) \\
- \frac{\partial}{\partial y} \left( V_2 - V_1 \right) \frac{\partial}{\partial y} (\rho h) \\
+ \rho (W_2 - W_1)
\] 

(2.10)

Where,

\( U_2 \) = velocity of the upper plate
\( U_1 \) = velocity of the lower plate
\( i, j \) and \( k \) are unit velocity vectors in the \( x, y, z \) directions respectively.
\( U, V \) and \( W \) are velocity components in the \( x, y, z \) directions respectively.
\( \rho \) = fluid film density
\( h \) = fluid film thickness
\( \mu \) = fluid film viscosity
\( p \) = fluid film pressure
\( v \) = diffusion velocity
\( t \) = time
\( k_x \) = turbulent constant
\( k_x = 12 \), for laminar flow
\( k_x = 12 + 0.0136 \cdot Re^{0.9} \), for turbulent flow

The left hand side of equation 2.10 can be rewritten as shown in equation 2.11.

\[ LHS = \nabla \cdot \left( \frac{\rho h^3}{12 \mu} \nabla p \right) \] 

(2.11)

Here, \( \nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j \) and for incompressible fluid flow \( \frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial t} = 0 \)

Thus the right hand side can be rewritten as shown in equation 2.12; for slider bearings we consider equation 2.13.
\[ \text{RHS} = \frac{1}{2} \left\{ \rho (U_1 - U_2) \frac{\partial h}{\partial x} + \rho (V_1 - V_2) \frac{\partial h}{\partial y} \right\} + \frac{\partial}{\partial t} (\rho h) \]  
\[ \frac{\partial h}{\partial t} = W_2 - W_1 \]  

Neglecting stretch effects the right hand side can be represented as equation 2.14.

\[ \text{RHS} = \frac{1}{2} \left\{ \frac{\partial}{\partial x} (\rho h U_1) + \frac{\partial}{\partial y} (\rho h V_1) - \frac{\partial}{\partial x} (\rho h U_2) - \frac{\partial}{\partial y} (\rho h V_2) \right\} + \frac{\partial}{\partial t} (\rho h) \]  

Thus from equations 2.12 and 2.14, the full Reynolds equation can be written as equation 2.15.

\[ \nabla \cdot \left( \frac{\rho h^3}{k_x \mu} \nabla p \right) = \frac{1}{2} \frac{\partial}{\partial x} [\rho h (U_1 - U_2)] + \frac{1}{2} \frac{\partial}{\partial y} [\rho h (V_1 - V_2)] + \frac{\partial}{\partial t} (\rho h) \]  

With \( U = \frac{1}{2} [(U_1 - U_2)] \), the average velocity of upper and lower surfaces and neglecting the body forces \( (F) \) with diffusion effects, the Reynolds equation can be rewritten as in equation 2.16 (Reddi, 1969).

\[ \nabla \cdot \left( \frac{\rho h^3}{k_x \mu} \nabla p \right) = \nabla \cdot (\rho h U) + \frac{\partial}{\partial t} (\rho h) + \rho v \]  

Thus, equation 2.16 describes the Reynolds equation for incompressible and isoviscous fluid films. Body forces are included in the equation. Once the Reynolds equation is solved and the pressure is known, the average fluid velocity, \( u \) is given by equation 2.17 and the mass flow rate \( q \) is given by the equation 2.18.

\[ u = U - \frac{h^2}{k_x \mu} \nabla P \]  
\[ q = \rho h u \]  

Specifying either the pressure or the average normal fluid velocity at each point along the thin fluid film boundary completes the formulation of the fluid film boundary value problem. Let \( C_P \) denote the portion of boundary \( C \) where the pressure is considered known and \( C_q \) represent the remaining portion of boundary where the normal fluid velocity or normal mass flow rate is known as shown in figure 2.3. This implies \( P \) is known on \( C_P \) and \( q.n \) is known on \( C_q \), where \( n \) is
the outward normal to $C_q$. It can be shown that the pressure must be specified at a minimum of 
one point along the boundary for the solution of $P$ to the boundary value problem to be unique. It 
has been shown that the pressure $P$ that minimizes this functional while satisfying the boundary 
conditions on $C_P$ is also a solution of the Reynolds equation. The variational principle for 
incompressible, isoviscous films of any thin film geometry is to find a pressure $P(x,y)$ which 
minimizes the function shown in equation 2.19 and satisfies the boundary condition on $C_P$ 
(Allaire et. al., 1977). The pressure thus obtained is the same as that which is the solution to the 
Reynolds equation 2.16. With these equations, the thin fluid film problem is completely 
formulated except for inertia and energy effects.

$$J(P) = \int_A \left( \frac{\rho h^3}{2k_x \mu} \nabla P - \rho h U \right) \cdot \nabla P + \left[ \frac{\partial}{\partial t} \left( \frac{\rho h}{\mu} + \rho h \right) \right] P \right) dA + \int_{C_q} (q \cdot n) PdC \quad (2.19)$$

For a fluid film journal bearing the lower surface is fixed, fluid film viscosity and density are 
constant, and body forces and diffusion forces are neglected. Thus, the Reynolds equation in 
cylindrical coordinate system for laminar flow of a circular bearing can be represented as in 
equation 2.20 and the corresponding functional as equation 2.21.

$$\frac{1}{r^2} \frac{\partial}{\partial \theta} \left[ \frac{h^3}{\mu} \frac{\partial p}{\partial \Theta} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right] = 6(\sigma_j \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t} \quad (2.20)$$

$$J(P) = \int_A \left\{ \frac{\rho h^3}{2k_x \mu} \left[ \left( \frac{1}{R} \frac{\partial P}{\partial \theta} \right)^2 + \left( \frac{\partial P}{\partial y} \right)^2 \right] - \frac{\rho}{2} \left( \sigma_b + \sigma_j \right) h \frac{\partial P}{\partial \theta} + \frac{\partial h}{\partial t} \right\} Rd\theta dy \quad (2.21)$$

2.4 Finite Element Method

Finite element procedures are very widely used in engineering analysis. The procedures are 
employed extensively in the analysis of solids and structures and of heat transfer and fluids. 
Finite element methods are useful in virtually every field of engineering analysis.

The finite element method is used to solve physical problems in engineering analysis and design. 
The physical problem typically involves the structure or structural component subjected to
certain loads. The idealization of the physical problem to the mathematical model requires certain assumptions that together lead to differential equations governing the mathematical model. The finite element analysis provides an approximate solution to the mathematical model. Since the finite element procedure is a numerical procedure it is necessary to assess the solution accuracy. If the accuracy criteria are not met, the numerical solution has to be repeated with refined solution parameters (such as a finer mesh) until a sufficient accuracy is reached. Also the choice of an approximate mathematical model is crucial and completely determines the insight into the actual physical problem that we can obtain by the analysis. A flow chart representation of the general procedure for finite element analysis is shown in figure 2.5 (Bathe, 1996).

In the fluid film bearing application the finite element method reduces the field problem in which the pressure is a continuous function of space to a problem in which the pressure is evaluated only at certain specific nodal points. Between the nodes the pressure is assumed to vary according to a simple function called an interpolation function. The problem then becomes one of evaluating a system of algebraic equations for unknown nodal pressures. The coefficients of algebraic equations depend upon the nodal coordinates, the interpolation polynomials chosen and other factors. The finite element method is conveniently derived from a variational principle. In the case of fluid film bearings the variational principle has been derived from the Reynolds equation. The variational principle consists of a functional in the form of an integral over the bearing area.

The bearing area to be analyzed is divided up into a finite number of small sub regions, called elements, of simple geometric shape. The elements must be chosen such that there are no gaps or overlapping occurs between the elements. On the boundary of each element, a number of nodes are chosen where pressure is to be evaluated. Interior nodes can be chosen but they are not usually employed. An interpolation function is chosen to approximate the pressure within each element. The function for a given element is not valid outside the element. It is found in terms of nodal pressure and nodal coordinates. Let us consider a polynomial of the form shown in equation 2.22 (Huebner, 1975 and Zienkiewicz, 1970). Here it is necessary that the element has six nodes placed, such that each node corresponds to a constant in the polynomial. In order to
insure accuracy of the solution, the interpolation function is usually chosen such that the pressure is continuous across the inter element boundaries (compatible formulation).

\[ a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 \quad (2.22) \]

Each element will make a contribution to the integral in the functional. These contributions must be evaluated for each element and added together. The solution to the problem is obtained when the nodal pressures are chosen such that the functional is a minimum. The algebraic system of equations resulting from a minimization process yields the nodal pressure. In summary the steps involved are

1. Express the problem as a functional.
2. Divide the area into elements.
3. Choose interpolation functions and nodes.
4. Evaluate element contributions to functional.

### 2.4.1 Finite element solution of Reynolds Equation

This section develops the basic theory of the finite element method for two-dimensional thin fluid films. The theory is generalized to a finite slider bearing with relative surface velocities as well as a squeeze velocity. The analysis applies to any bearing that can be unrolled onto a horizontal surface such as a journal bearing, partial arc bearing and tilt pad bearings.

Consider a general finite element slider shown in figure 2.2. The pressure must minimize the function in equation 2.23 neglecting the body forces (Allaire et. al., 1977).

\[
J(P) = \iint_A \left\{ \frac{\rho h^3}{2k_x \mu} \nabla P - \rho h U \right\} \cdot \nabla P + \left[ \frac{\partial}{\partial t} \left( \rho h \right) \right] P dA + \int_{C_q} (q \cdot n) PdC \quad (2.23)
\]

Here the subscripts \( x \) and \( y \) denote the vector components in the \( X \) and \( Y \) directions respectively, the functional may be written as show in equation 2.24. The boundary conditions along part of the boundary, \( C_p \), the pressure is specified as in equation 2.25 and along the remainder of the boundary, \( C_q \), the mass rate of flow is specified as shown in equations 2.26 and 2.27. The values
of \( p_a \) and \( q_a \) may vary around the boundary. If the pressure is specified around the entire boundary, \( C_p = C \), the last integral in equation 2.24 is simply ignored (Nicholas, 1977).

\[
J(P) = \iiint dA \left\{ \left( \frac{\rho h^3}{2k_x \mu} \left[ \left( \frac{\partial P}{\partial x} \right)^2 + \left( \frac{\partial P}{\partial y} \right)^2 \right] \right) - \rho h \left[ U_x \frac{\partial P}{\partial x} + U_y \frac{\partial P}{\partial y} \right] + \left[ \frac{\partial}{\partial t} (\rho h) \right] P \right\} dA
+ \int_{C_q} \left( q_x n_x + q_y n_y \right) P dC
\]  

(2.24)

\[
[P]_{on \ C_p} = [P_a]_{on \ C_p} \quad (2.25)
\]

\[
[q_x]_{on \ C_q} = [q_{ax}]_{on \ C_q} \quad (2.26)
\]

\[
[q_y]_{on \ C_q} = [q_{ay}]_{on \ C_q} \quad (2.27)
\]

The fluid is divided into a global system of elements in a manner such as that shown in figures 2.4 and 2.6. Let the region of interest in the \( X-Y \) plane be divided into \( E \) finite elements. First the bearing surface is divided into \( M \) quadrilaterals in the \( y \) direction and \( N \) quadrilaterals in the \( x \) direction. This permits the nodes at the intersections to be labeled in a rectangular matrix of order \( M \times N \). By expressing the pressure and the distribution of the various forcing functions in terms of their respective nodal values through interpolation function \( L_i(x,y) \), we obtain equations 2.28 and 2.29. Here, \( \vec{P}^{(e)} \) and \( \vec{U}^{(e)} \) are the vectors of nodal pressures and \( x \)-component velocities respectively, of element \( e \).

\[
p(x,y) = \sum L_i(x,y) P_i^{(e)} = [L(x,y)]\vec{P}^{(e)}
\]  

(2.28)

\[
U_x(x,y) = \sum L_i U_i^{(e)} = [L(x,y)]\vec{U}^{(e)}
\]  

(2.29)

Each of the nodal pressures \( P_i, P_j, P_k \) are labeled correspondingly and the coordinates of the nodes are denoted by \( (x_i,y_i), (x_j,y_j), (x_k,y_k) \). Within each element a linear interpolation function is represented as in equation 2.30.

\[
P^{(e)}(x,y) = L_i^{(e)}(x,y) P_i + L_j^{(e)}(x,y) P_j + L_k^{(e)}(x,y) P_k
\]  

(2.30)

Where, \( L_i^{(e)}(x,y) = \frac{a_n + b_n x + c_n y}{2A^{(e)}} \)

\( a_n, b_n, c_n \) are constants

\( A^{(e)} \) = area of the element

Since the coefficients \( a_n, b_n, c_n \) and the area of the elements are known, only the nodal pressures are unknown in the element. The function is broken up into integrals over the individual elements as shown in equations 2.31 and 2.24.
\[ J(P) = \sum_{e=1}^{E} J^{(e)}(P) \]  

(2.31)

The second integral is evaluated only for the portion of the element boundary that is located along \( C_q \). If no portion of boundary falls along \( C_q \), then the last integral is simply ignored. The pressure, shear in x-direction, shear in y-direction, squeeze and boundary flow constants are shown in equations 2.32, 2.33, 2.34, 2.35 and 2.36 (Rao, 1999 and Nicholas, 1977). Thus, the element functional in compact form and the total functional can thus be represented as in equations 2.37 and 2.38 respectively (Booker and Huebner, 1972).

\[
K_{pnn}^{(e)} = -\int_{\Omega^{(e)}} \left[ \frac{\rho h^3}{k_x \mu} \left( \frac{\partial L_n^{(e)}}{\partial x} \frac{\partial L_n^{(e)}}{\partial x} + \frac{\partial L_m^{(e)}}{\partial y} \frac{\partial L_m^{(e)}}{\partial y} \right) \right] dA^{(e)}
\]

(2.32)

\[
K_{uxn}^{(e)} = \int_{\Omega^{(e)}} \left[ \rho h U_x \frac{\partial L_n^{(e)}}{\partial x} \right] dA^{(e)}
\]

(2.33)

\[
K_{uyn}^{(e)} = \int_{\Omega^{(e)}} \left[ \rho h U_y \frac{\partial L_n^{(e)}}{\partial y} \right] dA^{(e)}
\]

(2.34)

\[
K_{hn}^{(e)} = -\int_{\Omega^{(e)}} \left[ \rho \frac{\partial h}{\partial t} L_n^{(e)} \right] dA^{(e)}
\]

(2.35)

\[
q_n^{(e)} = \int_{C_t^{(e)}} \left[ \left( q_x n_x + q_y n_y \right) \right] dC^{(e)}
\]

(2.36)

\[
J^{(e)}(P) = -\frac{1}{2} \{P\}^{(e)T} \left[ K_P \right]^{(e)} \{P\}^{(e)} - \frac{1}{2} \{P\}^{(e)T} \left[ K_{ux} \right]^{(e)} \{P\}^{(e)} + \{P\}^{(e)T} \left[ K_{uy} \right]^{(e)} \{P\}^{(e)} - \{P\}^{(e)T} \left[ K_{hn} \right]^{(e)} + \{P\}^{(e)T} \{g\}^{(e)}
\]

(2.37)

\[
J(P) = -\frac{1}{2} \sum_{e=1}^{E} \{P\}^{(e)T} \left[ K_P \right]^{(e)} \{P\}^{(e)} - \sum_{e=1}^{E} \{P\}^{(e)T} \left[ K_{ux} \right]^{(e)} \{P\}^{(e)} + \sum_{e=1}^{E} \{P\}^{(e)T} \left[ K_{uy} \right]^{(e)} \{P\}^{(e)} - \sum_{e=1}^{E} \{P\}^{(e)T} \left[ K_{hn} \right]^{(e)} + \sum_{e=1}^{E} \{P\}^{(e)T} \{g\}^{(e)}
\]

(2.38)

The functional must be minimized with respect to all of the unknown nodal pressures. We assume that all the nodal pressures are unknown even though there are a few known nodal pressures. Thus minimization of equation 2.38, as shown in equation 2.39 results in equation 2.40. This equation is known as the finite element formulation of Reynolds equation for fluid film journal bearings (Nicholas, 1977). This equation can be conveniently solved in a recognized
matrix form as in equation 2.41. The elements thus obtained can be assembled into an overall system as in 2.42.

\[
\frac{\partial J(P)}{\partial P_{I,J}} = 0, \begin{cases} 
I = 1,2,\ldots,M \\
J = 1,2,\ldots,N 
\end{cases}
\] (2.39)

\[
\sum_{e=1}^{E} [K_p]^{(e)} \{P\}^{(e)} = -\sum_{e=1}^{E} [K_{v_i}]^{(e)} - \sum_{e=1}^{E} [K_{u_i}]^{(e)} - \sum_{e=1}^{E} [K_h]^{(e)} + \sum_{e=1}^{E} [g]^{(e)}
\] (2.40)

\[
[K_p]^{(e)} \tilde{P}^{(e)} = \tilde{Q}^{(e)} - [K_h^{(e)}] \tilde{U}^{(e)} - [K_h^{(e)}] \tilde{H}^{(e)} = \tilde{F}^{(e)}
\] (2.41)

\[
[K_p] \tilde{P} = \tilde{F}
\] (2.42)

Where, the order of matrix $K_p$ is equal to $M$. Applying the pressure boundary conditions can solve the equations thus obtained. Once the nodal pressure $\tilde{P}$ and flow $\tilde{U}$ are found, the bearing load capacity $W$ can be computed from 2.43.

\[
W = \iint_{S} p(x, y) dS
\] (2.43)
Figure 2.1: Velocity profile between two parallel plates to quantify viscosity (Szeri, 1999)

Figure 2.2: Fluid film geometry (Allaire et al., 1977).
Figure 2.3: Boundary conditions on unrolled bearing surface (Allaire et al., 1975)

Boundary $C_q$ where $\frac{\partial P}{\partial x} = 0$

Boundary $C_p$ where $P=0$

Leading edge

Tailing edge

$P=0$

$L/2$

$p=0$

$X_b$ (RΘ direction)

$Y_b$ (Z direction)

Figure 2.4: Division of fluid film into elements (Nicholas, 1977)

M x N quadrilateral elements
Physical Problem \[ \rightarrow \] Change of physical problem

**Mathematical Model**
Governed by differential equations considering assumptions on
- Geometry
- Kinematics
- Material Law
- Loading
- Boundary conditions etc.

**Finite Element Solution**
Choice of
- Finite Elements
- Mesh Density
- Solution Parameters
Representation of
- Loading
- Boundary conditions

Assessment of accuracy of finite element solution of mathematical model

Interpretation of Results \[ \rightarrow \] Refine Analysis

Design Improvements
Structural optimization

Refine mesh, solution parameters etc.

Figure 2.5: The process of finite element analysis (Bathe, 1996).
Figure 2.6: The $e^{th}$ two-dimensional simplex triangular element (Nicholas, 1977).
CHAPTER THREE
BEARING DESIGN PROGRAM

3.1 Introduction
The program name BRGDS is an acronym for Bearing Design. It is a finite element based computer program for the analysis of hydrodynamic partial arc bearings and bearings with hydrostatic recesses. Dr. Robert W. Stephenson initially developed the program. Additional work has been done to include the capability to compute the dynamic characteristics for partial arc, full and hybrid bearings. In the present chapter a detailed overview of the BRGDS program, coordinate system, flow charts for finite element method and convergence conditions are presented. The main focus is on the computation of the dynamic characteristics.

3.2 Overview
A standard finite element implementation is used to model the land region of the bearing. The finite element model of the bearing is generated using four-node quadrilateral and/or three-node triangular elements. The finite element solution of the Reynolds equation is used to obtain the pressure over the finite bearing surface.

The assumptions used in the solution of Reynolds equation in this program are incompressible flow, constant lubricant viscosity, and no fluid inertia terms. The bearing pad is assumed to be unrolled or flat, so that a two-dimensional mesh representation in the axial-circumferential plane can be used. The bearing edge is subjected to the boundary conditions. The nodes have either pressure or flow as the only degree of freedom. The system matrices are thus developed from the bearing geometry and operating parameters such as journal rotational speed, lubricant viscosity and clearance.

Either a specified load or a specified eccentricity is used to apply loading to the bearing. For a specified load the program iterates on the eccentricity, \( e \), and the eccentricity angle, \( \Phi \), until load equilibrium is reached. However, for specified eccentricity the iterations are performed on eccentricity angle and the bearing load is calculated. A conventional Newton-Raphson method is
used for adjusting the journal displacement to find equilibrium position. Pressures lower than the cavitation pressure are set equal to the specified cavitation pressure. Turbulence is included as an option and is implemented as a correction factor to the laminar solution. For turbulent solution, iterations are performed until pressure convergence is achieved since the correction factors in the system equations are pressure dependent.

The hydrostatic capability of the bearing is included in the analysis by specifying either flow or pressure boundary conditions on the bearing land around the edge of the hydrostatic recess. Here, all nodes around the recess periphery are assumed to be at the recess pressure itself. For specified recess pressure, the recess nodes at the periphery are set to have the boundary condition pressure and solution is achieved. The flow into and out of the recess is calculated by back substitution of the pressure solution into the system equations. In the other case of recess flow being defined, an iterative solution is performed to adjust the recess pressure, until the flow calculated by back substitution of pressure solution agrees with the specified recess flow within a specified tolerance.

The pressure is then integrated over the bearing to give the load capacity and moment on the bearing about a specified axial location. A 2X2 Gauss quadrature routine is used to numerically integrate the elements. A three-point integration scheme is used to numerically integrate the triangular elements (Zienkiewicz and Bathe). Linear stiffness and damping coefficients are obtained from small perturbations of displacement and velocity about the equilibrium position respectively. In addition, the bearing film thickness, torque loss and fluid flow into the bearing from hydrodynamic action are calculated. The effects of the axial misalignment of the shaft of the bearing are not considered in the current implementation.

3.3 Bearing Coordinate System

For the purpose of the program a right handed coordinated system fixed in space is set up as shown in Figures 3.1 and 3.2. $X$ and $Y$ are the horizontal and vertical coordinates of the geometrical center of the shaft. The angular coordinate, $\Theta$, is measured positive in the counterclockwise direction, starting always from the positive x-axis in the direction of rotation of the shaft. The two dimensional nodal coordinates required for the bearing model are in terms of
$Z$ (axial) and $\Theta$ (circumferential degrees). The bearing radius is specified as input and it is used along with $\Theta$ to correctly define the circumferential distance.

### 3.4 Solution Procedure

A block diagram flow chart of the program solution procedure is shown in Figure 3.3. The sequence of steps performed during bearing analysis by $BRGDS$ can be outlined as follows (Stephenson, 1997).

1. Read the bearing information from the input file and generate the bearing finite element model.
2. Assign the pressure boundary conditions to the edge node and recess nodes if any.
3. Assume initial values of hydrostatic recess pressures, turbulent correction factors and journal position.
4. Solve the finite element equation for unknown pressure. Compute the unknown flows by back substitution of pressure solution into system equations.
5. Integrate the pressure solution over the elements to compute the bearing forces and bearing moments.
6. Check for force convergence. If convergence conditions are not satisfactory then modify journal position and perform solution for unknown pressure (step 4).
7. Reset any pressure below the cavitation pressure to the specified cavitation pressure and perform step 4.
8. Verify pressure convergence for turbulent solution if included. If not converged then update turbulent correction factors and perform step 4.
9. In the event of a hydrostatic recess with specified recess flow, check recess flow convergence. If convergence condition is not achieved update recess pressure and perform step 4.
10. Display the converged bearing solution. Perform step 3 for more load cases.

### 3.5 General System Equations

The basic system equations for the hydrodynamic problem in matrix form can be represented as,

$$\begin{align*}
[K_p] \{p\} &= \{q\} - [K_u] \{u\}
\end{align*}$$

(3.1)
Where,

\[ K_p \] = system pressure fluidity matrix
\{p\} = nodal pressure vector
\{q\} = nodal flow vector
\[ K_u \] = system velocity fluidity matrix
\{u\} = nodal velocity vector

The system matrices \[ K_p \] and \[ K_u \] are assembled from the individual element matrices, which are dependent on the film thickness, lubricant viscosity, and element geometry (Booker et. al., 1972). Each entry of the vector \{u\} is a known value dependent of the rotational speed of the journal and is equal to the rotational speed of the journal \((-R\omega/2)\). Each node contains only one unknown degree of freedom in (3.1), either \(p\) or \(q\). Note that the pressures and flows are scalar values. For nodes with unknown pressures, the net flow values are always zero. Degrees of freedom are removed from (3.1) at nodes with specified pressures. Therefore, the right hand side becomes a known vector and all unknowns in the system are in \{p\}. The unknown flow values at nodes with specified pressures can be back calculated from the system equations after the complete system pressure solution is obtained.

### 3.5.1 Eccentricity

The position of the journal in the bearing is directly related to amount of loading on the bearing. When the journal bearing is sufficiently supplied with oil and with no load acting, the journal rotates concentrically with the bearing. However, when the load is applied, the journal moves to the equilibrium position or an increasingly eccentric position, thus forming a wedge shaped oil film to support the load. As show in Figure 1.1, the distance between the bearing center and the shaft center is known as eccentricity. The ratio of eccentricity to the bearing clearance is defined as eccentricity ratio, \(\varepsilon\). If \(\varepsilon=0\), then there is no load on the pad and for \(\varepsilon=1\) the journal would touch the bearing under larger external loads.

### 3.5.2 Film Thickness Calculation

The elements in the matrices \[ K_p \] and \[ K_u \] are functions of the film thickness, \(h\), which in turn varies as a function of the angular location, \(\theta\), at each node and is calculated from equation 3.2.
\[ h(\theta) = c_r - x_j \cos(\theta) - y_j \sin(\theta) \]  
\[ h(\theta) = c(1 + \varepsilon \cos(\theta)) \]  

Where,
\[ c_r = \text{bearing radial clearance} \]
\[ x_j = e \ c_r \cos(\phi) = \text{horizontal journal displacement} \]
\[ y_j = e \ c_r \sin(\phi) = \text{vertical journal displacement} \]
\[ e = \text{eccentricity} \]
\[ \phi = \text{eccentricity angle} \]
\[ \varepsilon = \text{eccentricity ratio} \]

An iterative solution is required to adjust the eccentricity and/or the eccentricity angle to yield new values of \( x_j, y_j \), and hence \( h \), at each node until the integration of the pressure solution yields force convergence on the journal. In the event of negative film thickness an error message is displayed since this means interference between the journal and the bearing.

### 3.5.3 Bearing Forces and Moments

The forces on the bearing journal are calculated from the pressure solution by a summation of the integration of the pressure over each element as

\[ F_x = \sum \int p \cos(\theta) dA \]  
\[ F_y = \sum \int p \sin(\theta) dA \]  

Where \( F_x \) and \( F_y \) are the horizontal and vertical forces on the journal, respectively. The summation implies that the integral is carried out over the elements. Similarly, the bearing moments about a specified axial coordinate, \( z_o \), are computed from the pressure solution by

\[ M_x = \sum \int p \sin(\theta)(z - z_o) dA \]  
\[ M_y = \sum \int p \cos(\theta)(z - z_o) dA \]  

Where \( M_x \) and \( M_y \), are defined as the moments about the global horizontal (x) and vertical (y) axis. The bearing torque loss, \( T \), due to shearing of the lubricant is calculated by,

\[ T = \mu \omega R^2 \sum \int (1/h) dA \]  

29
3.5.4 Hydrostatic Solution
The hydrostatic solution follows the similar procedure as for the hydrodynamic case. The major
difference is that the pressures around the recess or the recess periphery nodes are specified as an
additional boundary condition. In the case for the recess pressure being specified, the solution is
straightforward and recess flows are calculated by back substitution. However, if the recess
flows are specified, then iterations are performed to adjust the recess pressure until convergence
to a specified flow value is obtained. Here, the pressure boundary conditions are still applied to
the recess periphery nodes. The recess pressure is assumed at the beginning and then updated for
succeeding iterations.

3.6 Convergence Conditions
The convergence conditions are defined as the tolerance limits that control the iterations
performed in the program to attain equilibrium. These condition used in the bearing program are
defined as follows.

3.6.1 Load Convergence
In the event of an applied load being specified on the bearing, iterations are needed to adjust the
bearing eccentricity $e$ and the eccentricity angle $\Phi$. The two load convergence conditions 3.8
and 3.9 are to be satisfied. For specified eccentricity, iterations only on eccentricity angle are
required to satisfy the convergence condition 3.9.

\[ \text{ABS}(F_x - F_a) / F_a < \varepsilon_L \]  \hspace{1cm} (3.8) \\
\[ \text{ABS}(F_y) / F_{mag} < \varepsilon_L \]  \hspace{1cm} (3.9)

Where,

$\varepsilon_L$ = load convergence tolerance

$F_a$ = specified applied load

$F_x$ = radial load along load angle direction

$F_y$ = tangential load to load angle direction

$F_{mag}$ = magnitude of calculated bearing load
3.6.2 Pressure convergence

The calculated pressures are checked for pressure below cavitation pressure and are set equal to the specified cavitation pressure for all the solutions. The process is repeated until there are no more pressures below the cavitation pressure. In this case no pressure convergence is required. However, if turbulence is included as an option, a separate pressure iteration loop is needed since the turbulent correction factors are pressure dependent. The current pressure solutions are used to compute the correction factors for next iteration and new pressure are calculated until convergence is achieved.

\[
ABS(P_{i+1} - P_i)/P_i < \varepsilon_p
\]  

(3.10)

Where \( P_i \) and \( P_{i+1} \) are node pressures at \( i^{th} \) and \( i^{th}+1 \) iterations respectively. Here, \( \varepsilon_p \) is specified pressure convergence tolerance equal to 0.001.

3.6.3 Recess flow convergence

Similar to the pressure flow convergence, recess flow convergence (\( \varepsilon_Q \)) is defined as,

\[
ABS(Q_{i+1} - Q_i)/Q_i < \varepsilon_Q
\]  

(3.11)

Where \( Q_i \) and \( Q_{i+1} \) are recess flow values at \( i^{th} \) and \( i^{th}+1 \) iterations respectively. Here, \( \varepsilon_Q \) is specified recess flow convergence tolerance equal to 0.001.

3.7 Damping and Stiffness calculation

All machines especially turbo machines where journal bearings are extensively used are subject to horizontal and vertical vibrations due to the unbalance and other forces. The shaft or journal center always moves about an equilibrium position described by the eccentricity, \( e \) and the eccentricity angle, \( \Phi \). However, the orbit of the shaft motion is small compared to the bearing clearance. Thus, the stiffness and damping coefficients obtained by considering small changes in displacement and velocity respectively can be used to replace the dynamic fluid film forces.

The stiffness and damping properties of a journal bearing can be treated in a manner analogous to a spring and viscous damper from simple vibration theory. Consider the spring and mass system subjected to a steady loading as shown in Figure 3.4. Here, under the action of weight, \( W \),...
the spring stretches vertically by $e$, such that the equilibrium position is attained. In order to provide further displacement of $\Delta y$, an external force of $\Delta F$ is needed. Thus the positive spring constant or stiffness is defined as the amount of force that is required to obtain a unit displacement in a particular direction. Similarly, damping can be defined as the amount of force required providing a unit velocity in a particular direction to the spring mass system.

The fluid film forces are general functions of journal center displacements and velocities. It is assumed that the journal is subjected to small amplitude motion i.e., the dynamic journal displacements are less than the bearing clearance. Thus we can express the bearing forces as a Taylor series expansion around the journal position as in equations 3.12 and 3.13.

\[
F_x = F_{x_o} + \frac{\partial F_x}{\partial X} \Delta X + \frac{\partial F_x}{\partial Y} \Delta Y + \frac{\partial F_x}{\partial \dot{X}} \dot{X} + \frac{\partial F_x}{\partial \dot{Y}} \dot{Y} \tag{3.12}
\]

\[
F_y = F_{y_o} + \frac{\partial F_y}{\partial X} \Delta X + \frac{\partial F_y}{\partial Y} \Delta Y + \frac{\partial F_y}{\partial \dot{X}} \dot{X} + \frac{\partial F_y}{\partial \dot{Y}} \dot{Y} \tag{3.13}
\]

When the shaft is subject to a small displacement in either the $x$ ($\Delta x$) or $y$ ($\Delta y$) direction, a corresponding change in force occurs in both $x$ ($\Delta F_x$) and $y$ ($\Delta F_y$) directions. Thus, stiffness can be defined as in equations 3.14 to 3.17 (Allaire, 1987).

\[
K_{xx} = -\frac{\Delta F_x}{\Delta x_j} = -\frac{\partial F_x}{\partial x_j} \tag{3.14}
\]

\[
K_{yy} = -\frac{\Delta F_y}{\Delta y_j} = -\frac{\partial F_y}{\partial y_j} \tag{3.15}
\]

\[
K_{xy} = -\frac{\Delta F_y}{\Delta y_j} = -\frac{\partial F_x}{\partial y_j} \tag{3.16}
\]

\[
K_{yx} = -\frac{\Delta F_y}{\Delta x_j} = -\frac{\partial F_y}{\partial x_j} \tag{3.17}
\]

The coefficients $K_{xx}$ and $K_{yy}$ are the principal or direct stiffness parameters and $K_{xy}$ and $K_{yx}$ are the cross-coupled stiffness coefficients. These stiffness values are often converted into dimensionless numbers for the analysis using the following equation 3.18 (Allaire, 1987).

\[
K_{xx} = \frac{K_{xx} c}{W} \tag{3.18}
\]
Similarly the damping coefficients are obtained as in equations 3.19 to 3.22 when a change in force is obtained in both \( x (\Delta F_x) \) and \( y (\Delta F_y) \) directions due to a change in velocity in either \( x \) or \( y \) direction.

\[
C_{xx} = -\frac{\Delta F_x}{\Delta \dot{x}_j} = -\frac{\partial F_x}{\partial \dot{x}_j} \quad (3.19)
\]

\[
C_{yx} = -\frac{\Delta F_y}{\Delta \dot{x}_j} = -\frac{\partial F_y}{\partial \dot{x}_j} \quad (3.20)
\]

\[
C_{xx} = -\frac{\Delta F_x}{\Delta \dot{y}_j} = -\frac{\partial F_x}{\partial \dot{y}_j} \quad (3.21)
\]

\[
C_{yy} = -\frac{\Delta F_y}{\Delta \dot{y}_j} = -\frac{\partial F_y}{\partial \dot{y}_j} \quad (3.22)
\]

Similar to stiffness, \( C_{xx} \) and \( C_{yy} \) are the principal or direct damping parameters and \( C_{yx} \) and \( C_{xy} \) are the cross coupled damping parameters. The non-dimensional value of the damping coefficients is often obtained by the equation 3.23 (Allaire, 1987).

\[
\bar{C}_{yy} = \frac{C_{yy} c \varpi}{W} \quad (3.23)
\]

The perturbation constants that are selected are the fractional amounts of the shaft displacement or shaft motion of the equilibrium point used in calculating the stiffness and damping coefficients. The perturbation constants are chosen carefully. Too small values result in inaccurate coefficients due to round off errors. It can however be stated that over small changes on either the shaft position or motion, the dynamic coefficients will be approximately constant. Thus the following perturbation values are chosen as in 3.24 and 3.25. (Kocur and Allaire, 1990)

\[
\Delta x = \Delta y = 0.05 * c \quad (3.24)
\]

\[
\Delta \dot{x} = \Delta \dot{y} = 0.05 * c * \varpi \quad (3.25)
\]

The sample-bearing program input and output files are shown in appendix A4 and A5 respectively.
Figure 3.1: Bearing Coordinate system (Stephenson, 1997).

Figure 3.2: Side view and pressure profile of journal bearing (Allaire, 1987)
START

Generate finite element model of bearing geometry

Set Pressure boundary conditions

Assume initial values for journal position, turbulent correction factors and recess pressure.

Solve finite element solution and calculate back flow rates

Update e and $\Phi$

Reset Pressure

Update film thickness

Film thickness convergence

Yes

Yes

Structural Deflections

Yes

No

Flow convergence

Yes

No

Pressure below cavitations pressure

Turbulent solution

Pressure convergence

Yes

No

Load convergence

Update correction factors

Update recess pressures

No

Yes

STOP

Figure 3.3: Flow chart of BRGDS Bearing analysis program (Stephenson, 1997)
Figure 3.4: The physical representation of Dynamic force coefficients of a fluid film bearing
(San Andres, 2000)

\[ K_{ij} = -\frac{\Delta F_i}{\Delta Y_j} \]

\[ C_{ij} = -\frac{\Delta F_i}{\Delta X_j / \Delta t} \]
CHAPTER FOUR
RESULTS AND DISCUSSIONS

4.1 Introduction
The bearing design system (BRGDS) program has been developed to analyze the bearing performance characteristics for partial arc bearings, full plain journal bearings and partial arc bearings with recesses. The accuracy of the results that were obtained by BRGDS has been verified with other results from various sources. In the current dissertation the results have been shown and verified for 60°, 120° and 180° partial arc plain journal bearings with length over diameter ratio of 0.5. Also, the results obtained for the 360° bearing has been verified for an L/D ratio of 0.5. Various simulations have been performed for these bearings for eccentricity ratios ranging from 0.05 to 0.95. Most importantly, the stiffness and damping characteristics have been plotted with respect to the bearing characteristics number (Sommerfeld number) and were compared to other verified results. The dynamic properties of 60°, 120° and 180° partial arc plain journal bearings were compared for a length to diameter ratio of 0.5. Similarly, the properties for a 120° partial arc bearing were compared for length over diameter ratios of 0.5, 0.75 and 1.0. In addition to the above results, comparisons have been performed for a 360° plain journal bearing with L/D ratios of 0.5 and 1.0. The behavior of partial hybrid hydrodynamic bearings with hydrostatic recesses is also shown.

The stiffness and damping properties that are the main focus of this dissertation have been obtained by considering small values of perturbation for displacement and velocity of the shaft respectively. The perturbations have been considered in the positive X and Y directions and the bearing forces were computed. The ratio of the bearing forces and the perturbed displacements give the stiffness coefficients as shown in equations 3.14 to 3.17. Similarly the ratio of bearing forces with respect to the perturbed velocities generates the damping coefficients as in equations 3.19 to 3.22. The non-dimensional stiffness and damping coefficients are shown in equations 3.18 and 3.23 respectively. Most of the results in the present analysis were compared to DyRoBes. DyRoBeS™ is a powerful and sophisticated software tool authored by Wen Jeng Chen.
for rotor dynamics including comprehensive bearing analysis. Stephenson R.W., from Mechcon Inc, has supplied the results from DyRoBes.

4.2 Sommerfeld Number

The characteristics in steady running of a journal bearing of specified design are usually expressed non-dimensionally as functions of a single parameter called the Sommerfeld Number (Smith, 1969). It is often referred to as the bearing characteristic number. The Sommerfeld number, \( S \), has been conveniently used to compare the various non-dimensional characteristics of varied bearing arcs. The Sommerfeld Number can be mathematical represented as in equation 4.1.

\[
S = \left( \frac{R}{C} \right)^2 \frac{\mu NL R}{30*W}
\]

(4.1)

Where,
\( \mu \)=lubricant viscosity
\( N \)=journal RPM
\( L \)=bearing length
\( R \)=bearing radius
\( W \)=load capacity
\( C \)=bearing clearance

This number units into one dimensionless variable all the factors that influence the design of fluid film bearings except for the bearing arc (Raimondi and Boyd, pt.1, 1958). When the Sommerfeld number is established for a given condition of operation, all of the operating characteristics become fixed. As the eccentricity ratio of the bearing increases the load capacity increases. Since load capacity is inversely proportional to Sommerfeld number, as the eccentricity ratio increases the Sommerfeld Number decreases and vice versa.

4.3 Partial Arc Bearing

Analysis has been performed on 60\(^\circ\), 120\(^\circ\) and 180\(^\circ\) partial arc bearings. The results obtained using BRGDS were verified with DyRoBes. These results have been further explained in detail in the further sections.
4.3.1 60° Partial Bearing

A 60° plain partial arc bearing with the dimensions specified in Table 4.1 has been modeled and studied using BRGDS program. The bearing configuration with \( L/D \) equal to 0.5 was divided using 9 nodes along the axial direction and 61 nodes along the circumferential direction. The starting and the ending angle measured in the counterclockwise direction beginning at the positive \( X \)-axis were 240° and 300° respectively (Figure 3.1). The fluid film pressure distribution, bearings forces, and most importantly the bearing stiffness and damping properties were computed for input eccentricity ratios ranging from 0.05 to 0.90. These properties were plotted and compared to the results obtained from DyRoBes.

Figure 4.1, 4.2, 4.3 and 4.4 shows comparison of the results obtained by using BRGDS and DyRoBes for the direct stiffness coefficients \( (K_{XX}, K_{YY}) \), direct damping \( (C_{XX}, C_{YY}) \), cross coupled stiffness \( (K_{XY}, K_{YX}) \) and cross coupled damping \( (C_{XY}, C_{YX}) \) coefficients respectively with respect to Sommerfeld number for eccentricity ratios ranging from 0.05 to 0.9. The coefficients \( C_{XY}, C_{YX} \) and \( K_{XY} \) are negative for the entire range and their absolute values were needed to obtain the plots. From these figures it can be clearly seen that the results obtained using BRGDS are in very good agreement with those the DyRoBes results.

From figure 4.1 it can be stated that the direct stiffness property \( (K_{XX} \) and \( K_{YY} \)) of the partial arc bearing increases as the journal eccentricity in the bearing increases, due to increased bearing forces at larger eccentricities. The relationship between Sommerfeld number and eccentricity ratio is shown in figure 4.9. On the contrary, the direct damping property decreases as the eccentricity increases as shown in figure 4.2, which can be attributed to the decrease in the fluid film as the eccentricity increases. In figure 4.3 the coefficient \( K_{XY} \) approaches zero close to the eccentricity ratio of 0.675, and becomes negative as the eccentricity further increases. In addition, the forces normal to the applied load angle are very much greater than tangential forces. Figure 4.4 shows that the cross-coupled damping coefficients \( C_{XY} \) and \( C_{YX} \) are equal, and are in good agreement with DyRoBes results.
4.3.2 120° Partial Bearing

Similar analysis was performed on a 120° partial arc bearing with L/D ratio of 1.0. The dimensions of the bearing used are shown in table 4.1. The bearing land was divided into 61 circumferential and 9 axial nodes. Figure 4.5 and 4.6 shows the comparisons for all the dimensionless stiffness and damping coefficients with DyRoBes results respectively. It can be clearly seen that the results obtained are good agreement for a higher L/D ratio also. However, the coefficients $K_{yx}$ and $C_{yy}$ show considerable difference at small eccentricity ratios.

4.3.3 180° Partial Arc Bearing

A 180° partial arc bearing with dimensions specified in table 4.1 has been modeled using 9 axial and 61 circumferential nodes for an L/D ratio of 0.5. The starting and ending angles were 180° and 360° respectively.

Figure 4.7 shows a very good correlation for the four stiffness coefficients ($K_{xx}$, $K_{xy}$, $K_{yx}$ and $K_{yy}$) with DyRoBes results. However, the stiffness values deviate for large values of Sommerfeld number (for small eccentricity ratio). The cross-coupled stiffness coefficient $K_{xy}$ has larger values than direct stiffness $K_{xx}$ and $K_{yy}$ and approaches zero for very small eccentricity ratios. The $K_{yx}$ coefficient is greater than the other stiffness coefficients for the range of eccentricities. The comparison for the four damping coefficients is shown in Figure 4.8. The direct damping coefficients have a larger values than the corresponding cross-coupled coefficients for a given eccentricity ratio. The cross-coupled coefficients are almost constant for the entire range and for large Sommerfeld number the damping coefficients deviate from DyRoBes results. However, a very good agreement has been found for the operating range of eccentricity ratios between 0.2 and 0.7.

4.4 Effect of Bearing Arc

Analysis was performed on the 60°, 120° and 180° partial arc bearings for a length to diameter ratio of 0.5. The effect of bearing arc on the stiffness and damping characteristics was studied.

Figure 4.9 shows that the Sommerfeld number decreases as the eccentricity ratio increases and as the angle of the bearing is increased from 60° to 180°. It is important to note that the value of
Sommerfeld number hardly drops when the bearing arc is reduced to 120° from 180°. Thus, it would be possible to reduce the bearing arc by 30°-40° with no considerable loss of load-carrying capacity (Figure 4.12), but with considerable reduction in friction (Cameron, 1976). The eccentricity angle increases as the Sommerfeld number increases as the bearing arc decreases for a given eccentricity (Figure 4.10). Figure 4.11 shows the pressure distribution along the circumferential nodes for a 60°, 120° and 180° bearings with L/D=0.5 for eccentricity ratios of 0.4 and 0.6 with respect to the bearing angle. The 180° bearing has the maximum pressure for all eccentricity ratios and the 60° bearing has the least pressure. Also, the position of the peak pressure changes with the bearing arc. Figure 4.12 shows the load capacity variation with respect to the eccentricity ratio. The 180° bearing has the maximum load capacity due to the large bearing surface area and high pressures generated. The fluid flow increases as the Sommerfeld number increases and remains constant for very small eccentricity ratios (Figure 4.13).

The comparisons for the dimensionless direct stiffness parameters for the 60°, 120° and 180° bearing with L/D=0.5 are shown the figure 4.14. It can be clearly stated that the stiffness coefficient $K_{XX}$ decreases and the coefficient $K_{YY}$ increases as the bearing arc is reduced from 180° to 60°. This behavior can be attributed to the increase in the bearing force in the $y$-direction. Figure 4.15 and 4.16 shows the comparisons of the direct damping and cross-coupled stiffness parameters the three different bearing arcs. The damping parameter $C_{XX}$ and stiffness parameter $K_{XY}$ decreases as the bearing arc is increased, and the parameters $C_{YY}$ and $K_{YX}$ remain almost constant for eccentricity ratio greater than 0.4 and increase for ratios less than 0.4. From figure 4.17 it can be stated that the cross-coupled damping parameter $C_{XY}$ increases with the angle of the bearing arc. However, the 120° bearing arc has greater values for eccentricity ratios less than 0.5.

### 4.5 Effect of Length to Diameter Ratio

A 120° bearing arc has been studied for various lengths over diameter ratios of 0.5, 0.75 and 1.0 for an eccentricity range of 0.05 to 0.85. The Sommerfeld number increases as the L/D ratio is decreased from 1.0 to 0.5 (figure 4.18). The eccentricity angle increases slightly as the L/D ratio decreases and is largest for small eccentricity ratios as shown in figure 4.19. Figure 4.20 indicates that the load-carrying capacity increases as the L/D ratio increases. This is due to the
higher pressure generated for larger \(L/D\) ratio and the larger bearing surface area. The bearing fluid flow decreases as the eccentricity ratio increases and largest for the bearing with \(L/D\) ratio of 1.0. (Figure 4.21)

The variations of the direct stiffness properties are shown in figure 4.22. The parameters \(K_{XX}\) and \(K_{YY}\) decrease as the \(L/D\) ratio is increased from 0.5 to 1.0. However, \(K_{XX}\) reaches a constant value for eccentricity ratios less than 0.3 and \(K_{YY}\) between 0.45 and 0.7. The direct damping property \(C_{YY}\) remains constant for eccentricity ratio greater than 0.65 and increases as the \(L/D\) ratio is increased from 0.5 to 1.0 as shown in figure 4.23. The parameter \(C_{XX}\) remains almost constant for all the bearing \(L/D\) ratios. The cross-coupled stiffness parameter \(K_{YX}\) remains constant for all the \(L/D\) ratios as shown in figure 4.24. The parameter \(K_{XY}\) remains constant for all the bearings with eccentricity ratios less than 0.5 and decreases as the \(L/D\) ratio is increased from 0.5 to 1.0. Figure 4.25 shows that the parameter \(C_{XY}\) increases as the \(L/D\) ratio decreases from 1.0 to 0.5.

### 4.6 Full Journal Bearing (360°)

A 360° plain journal bearing with the dimensions specified in Table 4.1 was modeled to study the performance using BRGDS program. The bearing configuration with \(L/D\) equal to 0.5 was divided using 9 axial and 61 circumferential nodes. The starting and ending bearing angles were 0° and 360° respectively. The bearing stiffness and damping properties were computed and compared to the results obtained from DyRoBes.

Figures 4.26, 4.27, 4.28 and 4.29 compare the results obtained by using BRGDS and DyRoBes for the direct stiffness coefficients \((K_{XX}, K_{YY})\), direct damping \((C_{XX}, C_{YY})\), cross-coupled stiffness \((K_{XY}, K_{YX})\) and cross coupled damping \((C_{XY}, C_{YX})\) coefficients respectively with respect to Sommerfeld number for eccentricity ratios ranging from 0.05 to 0.9. The results obtained by using BRGDS were in very good agreement with the DyRoBes results. Thus the Bearing Design System program can be effectively used to perform the analysis of full plain journal bearings.

From figure 4.26, the direct stiffness property \(K_{YY}\) of the bearing decreases as the journal eccentricity in the bearing decreases and the property \(K_{XX}\) increases slightly as the eccentricity ratio decreases. The direct damping properties increase as the eccentricity decreases as shown in
figure 4.27. In figure 4.28 the coefficient $K_{XY}$ approaches zero close to the eccentricity ratio of 0.725, and becomes negative as the eccentricity further increases. Figure 4.29 show that the cross-coupled damping coefficients $C_{YX}$ increase slightly as the eccentricity ratio decreases. Very good agreement has been found between the frequently operating eccentricity ratio ranges of 0.3 to 0.7.

4.6.1 Effect of Length to Diameter ratio ($360^\circ$)

The properties obtained from $360^\circ$ bearing with $L/D$ ratios of 0.5 and 1.0 have been compared. Figure 4.30 shows that the bearing with $L/D$ ratio of 0.5 has higher Sommerfeld number than $L/D=1.0$. The eccentricity angle for a bearing with $L/D$ ratio of 1.0 is higher than $L/D=0.5$ and increases as the eccentricity ratio decreases (Figure 4.31). Figure 4.32 shows the pressure profile along the circumferential nodes for eccentricity ratios of 0.4 and 0.6. The load capacity and the dimensionless flow are higher for bearing with $L/D$ ratio of 1.0 as shown in figure 4.33 and figure 4.34. The direct stiffness property $K_{YY}$ is less for bearing with $L/D=1.0$ than 0.5 and $K_{XX}$ remains constant for the entire range of eccentricities as shown in figure 4.35. The damping property $C_{XX}$ is larger for bearing with $L/D=1.0$ than 0.5 as in figure 4.36. The coefficient $C_{YY}$ remains constant between eccentricity ratio of 0.5 and 0.7. $C_{YY}$ increases of eccentricity ratio less than 0.5. $C_{YY}$ for $L/D=0.5$ is less than $C_{YY}$ for $L/D=1.0$ for eccentricity ratio less than 0.5 and greater for eccentricity ratio greater than 0.7. $C_{XX}$ increases as eccentricity ratio decreases and $C_{XX}$ for $L/D=1.0$ is always greater than $L/D=0.5$. The cross-coupled stiffness behavior is shown in figure 4.37. The coefficient $K_{XY}$ remains constant for eccentricity ratio between 0.5 and 0.7. $K_{XY}$ for $L/D=0.5$ increases for eccentricity ratio less than 0.5 but is always less than $L/D=1.0$.

4.7 Partial Arc Hybrid Bearing

The Bearing Design System program has been used to perform the analysis for partial arc bearings with hydrostatic recesses. Firstly, a $120^\circ$ partial arc bearing with a hydrostatic recess in the bearing midland has been analyzed. The bearing recess has a circumferential length of $20^\circ$ and axial length of 0.5 inches. The bearing with specifications mentioned in table 4.1 has been divided into 9 axial nodes and 48 circumferential nodes. For the hydrostatic option, all nodes around the periphery of the hydrostatic recess are assumed to be at the same pressure as the recess itself. Figure 4.38 shows the pressure profile of the bearing circumferential nodes at the
bearing midland for fluid flow (0.5in³/sec) through the recess for concentric position and eccentricity ratio of 0.45. It can be stated that as the fluid flow through the recess increases higher pressures are generated thus increasing the load capacity of the bearing. Also, it can be seen that the hydrostatic effect dominates the hydrodynamic effect. Figure 4.39 shows the effect of journal RPM on the bearing pressure profile. The dynamic characteristics are shown in table 4.3.

Secondly, a 120° bearing with two hydrostatic recesses has been analyzed. The geometry of the bearing is mentioned in table 4.1. Each recess is 20° circumferentially and 0.5 inch axially. The recesses are placed on the bearing axial midline with the midpoint of the recess at 35° and 85° respectively on the bearing circumference. Figure 4.40 and 4.41 variation in the pressure profile for various fluid flows into the recess and for various journal speeds respectively. The dynamic characteristics are shown in table 4.4.
Table 4.1: Bearing design data for 60°, 120° and 180° partial arc bearings

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, D (inches)</td>
<td>2.0</td>
</tr>
<tr>
<td>Length, L (inches)</td>
<td>1.0, 2.0</td>
</tr>
<tr>
<td>Speed of Rotation, N (rev/sec)</td>
<td>2000 rpm</td>
</tr>
<tr>
<td>Viscosity, μ (lbf-sec/in²)</td>
<td>1.8e-6</td>
</tr>
<tr>
<td>Density, ρ (lb/in³)</td>
<td>1.0e-6</td>
</tr>
<tr>
<td>Radial Clearance, c (inches)</td>
<td>0.005</td>
</tr>
<tr>
<td>Eccentricity Ratios</td>
<td>0.05 to 0.9</td>
</tr>
<tr>
<td>Nodes</td>
<td>9X61</td>
</tr>
</tbody>
</table>

Table 4.2: Stiffness (lbf/in) and damping (lbf-s/in) coefficients for a 120° bearing with no recess.

<table>
<thead>
<tr>
<th>e/c</th>
<th>Kxx</th>
<th>Kxy</th>
<th>Kyx</th>
<th>Kyy</th>
<th>Cxx</th>
<th>Cxy</th>
<th>Cyx</th>
<th>Cyy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>2538</td>
<td>1584</td>
<td>13030</td>
<td>5505</td>
<td>16</td>
<td>12.74</td>
<td>11.6</td>
<td>125.5</td>
</tr>
<tr>
<td>0.45</td>
<td>7296</td>
<td>2755</td>
<td>24390</td>
<td>16310</td>
<td>33.65</td>
<td>42.05</td>
<td>37.95</td>
<td>241.2</td>
</tr>
<tr>
<td>0.65</td>
<td>17500</td>
<td>2030</td>
<td>53360</td>
<td>58790</td>
<td>58.96</td>
<td>98.38</td>
<td>86.25</td>
<td>532.6</td>
</tr>
</tbody>
</table>

Table 4.3: Stiffness (lbf/in) and damping (lbf-s/in) coefficients for a 120° bearing with single recess. (Flow=0.5in³/sec)

<table>
<thead>
<tr>
<th>e/c</th>
<th>Kxx</th>
<th>Kxy</th>
<th>Kyx</th>
<th>Kyy</th>
<th>Cxx</th>
<th>Cxy</th>
<th>Cyx</th>
<th>Cyy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>2934</td>
<td>2209</td>
<td>5387</td>
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Table 4.4: Stiffness (lbf/in) and damping (lbf-s/in) coefficients for a 120° bearing with two recesses. (Flow=0.1in³/sec).

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Figure 4.1: Dimensionless direct stiffness coefficients for a $60^\circ$ bearing with $L/D=0.5$.

Figure 4.2: Dimensionless direct damping coefficients for a $60^\circ$ bearing with $L/D=0.5$. 

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Figure 4.3: Dimensionless cross-coupled stiffness for a $60^\circ$ bearing with $L/D=0.5$.

Figure 4.4: Dimensionless cross-coupled damping for a $60^\circ$ bearing with $L/D=0.5$. 
Figure 4.5: Dimensionless stiffness coefficients vs. Sommerfeld number for a 120º bearing with $L/D=1.0$. 

[Graph showing dimensionless stiffness coefficients vs. Sommerfeld number for different cases labeled Kxx (BRGDS), Kyy (BRGDS), Kxy (BRGDS)].
Figure 4.6: Dimensionless damping coefficients vs. Sommerfeld number for a 120° bearing with $L/D=1.0$. 
Figure 4.7: Dimensionless stiffness coefficients vs. Sommerfeld number for a 180° bearing with $L/D=0.5$. 

Dimensionless Stiffness Coefficients vs. Sommerfeld Number for $180^\circ$ Bearing with $L/D=0.5$. 

- $K_{xx}$ (BRGDS)
- $K_{yy}$ (BRGDS)
- $K_{xy}$ (BRGDS)
- $K_{yx}$ (DyRoBeS)

180 Degree Bearing
Figure 4.8 Dimensionless damping coefficients vs. Sommerfeld number for a 180° bearing with $L/D=0.5$. 
Figure 4.9: Sommerfeld number vs. eccentricity ratio for 60°, 120° and 180° bearings.

Figure 4.10: Sommerfeld number vs. eccentricity angle for 60°, 120° and 180° bearings.
Figure 4.11: Pressure at circumference vs. bearing angle for $60^\circ$, $120^\circ$ and $180^\circ$ bearings.

Figure 4.12: Load capacity vs. eccentricity ratio for $60^\circ$, $120^\circ$ and $180^\circ$ bearings.
Figure 4.13: Bearing flow vs. Sommerfeld number for 60°, 120° and 180° bearings.

Figure 4.14: Dimensionless direct stiffness properties for 60°, 120° and 180° bearings.
Figure 4.15: Dimensionless direct damping properties for $60^\circ$, $120^\circ$ and $180^\circ$ bearings.

Figure 4.16: Cross-coupled stiffness properties for $60^\circ$, $120^\circ$ and $180^\circ$ bearings.
Figure 4.17: Cross-coupled damping properties for 60°, 120° and 180° bearings.

Figure 4.18: Sommerfeld number vs. eccentricity ratio for $L/D=0.5$, $0.75$ and $1.0$. 
Figure 4.19: Eccentricity angle vs. Sommerfeld number for $L/D=0.5$, 0.75 and 1.0.

Figure 4.20: Load capacity vs. eccentricity ratio for $L/D=0.5$, 0.75 and 1.0.
Figure 4.21: Total flow vs. eccentricity ratio for \( L/D = 0.5, 0.75 \) and \( 1.0 \).

Figure 4.22: Dimensionless direct stiffness characteristics for \( L/D = 0.5, 0.75 \) and \( 1.0 \).
Figure 4.23: Dimensionless direct damping characteristics for $L/D=0.5$, 0.75 and 1.0.

Figure 4.24: Dimensionless cross coupled stiffness for $L/D=0.5$, 0.75 and 1.0.
Figure 4.25: Dimensionless cross-coupled damping for $L/D=0.5$, 0.75 and 1.0.

Figure 4.26: Dimensionless direct stiffness for a $360^\circ$ bearing with $L/D=0.5$. 
Figure 4.27: Dimensionless direct damping for a $360^\circ$ bearing with $L/D=0.5$.

Figure 4.28: Dimensionless cross-coupled stiffness for a $360^\circ$ bearing with $L/D=0.5$. 
Figure 4.29: Dimensionless cross-coupled damping for a 360° bearing with $L/D=0.5$.

Figure 4.30: Sommerfeld number vs. eccentricity ratio for 360° bearing ($L/D=0.5$ and 1.0).
Figure 4.31: Eccentricity angle vs. Sommerfeld number for a 360° bearing ($L/D=0.5$ and 1.0).

Figure 4.32: Nodal pressures vs. angle of bearing for a 360° bearing ($L/D=0.5$ and 1.0).
Figure 4.33: Load capacity vs. eccentricity ratio for a 360° bearing ($L/D=0.5$ and 1.0).

Figure 4.34: Dimensionless flow vs. Sommerfeld number for a 360° bearing ($L/D=0.5$ and 1.0).
Figure 4.35: Dimensionless direct stiffness characteristics for a 360° bearing ($L/D=0.5$ and 1.0).

Figure 4.36: Dimensionless direct damping characteristics for a 360° bearing ($L/D=0.5$ and 1.0).
Figure 4.37: Dimensionless cross-coupled stiffness for a 360° bearing ($L/D=0.5$ and 1.0).

Figure 4.38: Pressure profile at various recess flows for a 120° bearing with single recess.
Figure 4.39: Pressure profile at various journal speeds for a 120° bearing with single recess.

Figure 4.40: Pressure profile at various recess flows for a 120° bearing with two recesses.
Figure 4.41: Pressure profile at various journal speeds for a 120° bearing with two recesses.
CHAPTER FIVE
CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

Bearing Design System (BRGDS) is a finite element based computer program for analysis of hydrodynamic bearings with hydrostatic recesses. The capability to compute the dynamic characteristics for plain partial arc, plain 360° and partial arc recessed hybrid hydrodynamic journal bearings has been successfully added to the Bearing Design System (BRGDS) program. The standard finite element implementation of the Reynolds equation was used to model the land region of the bearing. The usual assumptions of constant fluid viscosity, constant fluid density and no fluid inertia terms were used. For the hydrostatic option, all nodes around the periphery of the hydrostatic recess are assumed to be at the same pressure as the recess itself. The bearing pad or arc was assumed to be unrolled or flat and a two-dimensional mesh representation in an axial-circumferential plane was used. Boundary conditions are applied as required around the bearing edge. In the present work a specified eccentricity has been used to apply the load on the bearing. The pressure solution is integrated to give the bearing load. In addition, the bearing film thickness, fluid flow into the bearing from hydrodynamic action, the bearing stiffness, and damping characteristics were calculated.

Analysis was performed on the 60° partial arc bearing with an \( L/D \) ratio of 0.5, 120° bearing with \( L/D \) ratios of 0.5, 0.75 and 1.0, and 180° bearing with \( L/D \) ratio of 0.5 using BRDGS. The results obtained were successfully validated with verified results from DyRoBes. In addition, comparisons have been performed to study the effect of bearing arc on the dynamic properties using 60°, 120° and 180° partial arc bearings with an \( L/D \) ratio of 0.5. The effect of bearing length to diameter ratio was also studied for a 120° bearing with \( L/D \) ratios of 0.5, 0.75 and 1.0.

A 360° plain journal bearing has also been analyzed using BRGDS for \( L/D \) ratios of 0.5 and 1.0. The results for \( L/D \) ratio of 0.5 have been verified with DyRoBes results. The results are in good agreement with the verified results except for very small eccentricity ratios. However the results exhibit good agreement for the usual operating range of eccentricity ratios between 0.3 and 0.7.
Also the effect of length to diameter ratio on the dynamic characteristics was studied for $L/D$ ratios of 0.5 and 1.0.

Analysis was also performed on the partial arc hybrid hydrostatic journal bearing. Firstly, a $120^\circ$ partial arc bearing of $L/D$ ratio of 1.0 with a single recess in the bearing mid land has been analyzed for the performance characteristics. Secondly, a study was performed on a $120^\circ$ bearing with $L/D$ ratio of 1.0 having two recesses with recess midpoints located at $35^\circ$ and $85^\circ$ respectively on the bearing axial mid plane. Theses recesses have a circumferential length of 20° and axial length of 0.5 inches. The variations of the bearing pressure profile for various amounts of fluid flow into the bearing through the recess and for various journal speeds of rotation for both the bearings have been studied. Also the variation of the bearing dynamic properties with respect to a $120^\circ$ bearing with no recess has been studied.

Thus, it has been established that Bearing Design System can be used effectively to perform the analysis of plain partial arc bearings, partial arc journal bearings with hydrostatic recess, and full plain journal bearings in the operating range of eccentricities.

5.2 Future Work

It has previously been mentioned that the Bearing Design System can be effectively used to perform the analysis for partial arc bearings. However there are certain limitations in the present program based on few assumptions considered for the analysis. There has been a need to incorporate the fluid film inertia effects in order to obtain more accurate results. Also for the hydrostatic option the program does not have the capability to include the effect of flow restrictors and valves that control the flow of fluid into the recess. Also a skyline matrix approach to store the finite element matrices would reduce requirements and the process time for the analysis. Thus it is needed to include these capabilities in order for the program to be more effective. The long-term goal would be to include the capability to compute the dynamic characteristics for Multi-pad journal bearings.
APPENDIX

A.1 Nomenclature

\( R, r \) radius of the shaft/bearing, \textit{inches}
\( c \) clearance in the bearing, \textit{inches}
\( L \) length of the bearing, \textit{inches}
\( O_b \) center of the bearing
\( O_j \) center of the journal
\( e \) eccentricity, \textit{inches}
\( \varepsilon \) eccentricity ratio, \( e/c \)
\( \overline{W} \) load on the bearing, \textit{lbf}
\( H, h \) film thickness, \textit{inches}
\( h(\Theta) \) film thickness distribution
\( P, p \) fluid film pressure, \textit{psi}
\( X, x \) Cartesian coordinate along \textit{X}-axis
\( Y, y \) Cartesian coordinate along \textit{Y}-axis
\( Z, z \) Cartesian coordinate along \textit{Z}-axis
\( \omega \) angular velocity, \textit{rev/sec}
\( \mu \) lubricant absolute viscosity, \textit{lbf/in}^2\textit{sec}
\( \Phi \) eccentricity angle, \textit{degrees}
\( \Psi \) load angle, \textit{degrees}
\( \rho \) lubricant density, \textit{lb/in}^3
\( \Theta \) angular coordinate, \textit{degrees}
\( \nu \) kinematic viscosity, \textit{ft}^2\textit{sec}
\( \tau \) shear stress, \textit{lbf/in}^2
\( A \) area of moving plate, \textit{in}^2
\( P, Q, O \) fluid particles
\( U, u \) velocity of fluid in \textit{x} direction
\( V, v \) velocity of fluid in \textit{y} direction
\( W, w \) velocity of fluid in \textit{z} direction
\( k_x \) turbulent constant
\( q, Q \) flow rate, \textit{in}^3\textit{sec}
\( f_x \) fluid film force in \( x \)-direction, lbf
\( f_y \) fluid film force in \( y \)-direction, lbf
\( f_z \) fluid film force in \( z \)-direction, lbf
\( F_x \) bearing force in \( x \)-direction, lbf
\( F_y \) bearing force in \( x \)-direction, lbf
\( M_x \) bearing moment about \( x \)-axis
\( M_y \) bearing moment about \( y \)-axis
\( S \) Sommerfeld Number

**Subscripts**

- \( b \) bearing
- \( j \) journal
- \( A \) bearing surface area
- \( e \) element
- \( n \) node

**A2. STIFFCALC subroutine**

SUBROUTINE STIFCALC(NDOF,JTURB,FX0,FY0)
PARAMETER (MAXN=400)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON /COMM1/
    SKP(MAXN,MAXN),SKU(MAXN,MAXN),SKHDOT(MAXN,MAXN)
COMMON /COMM2/ U(MAXN),UOLD(MAXN)
COMMON /BDATA/ RADIUS,RADC,VIS,DEN,RPM,BRGLEN,BETA,ZREF
COMMON /EASAVE/ EE,AA
DIMENSION NLIST(MAXN),DELX(2),DELY(2),DELXDOT(2),DELYDOT(2)
DATA PI,FACT /3.141592654, .01 /
C     CALCULATE BEARING STIFFNESS IN GLOBAL XY SYSTEM
C     1ST SOLUTION : SHIFT X
C     2ND SOLUTION : SHIFT Y
C     NOTE: NO CHANGES ARE MADE TO DEFLECTIONS OR PRESSURE BC'S
DELX(1) = FACT * RADC
DELX(2) = 0.
DELY(1) = 0.
DELY(2) = FACT * RADC
DELXDOT(1) = 0.
DELXDOT(2) = 0.
DELYDOT(1) = 0.
DELYDOT(2) = 0.
BLANG = 0.

C SAVE ORIGINAL PRESSURE SOLUTION (FOR TURB CORRECTION FACTORS)
DO 10 J = 1, NDOF
    UOLD(J) = U(J)
10    CONTINUE

XJDOT = 0.
YJDOT = 0.

DO 50 JTIM = 1, 2
    XJ = EE * RADC * COS(AA * PI / 180.) + DELX(JTIM)
    YJ = EE * RADC * SIN(AA * PI / 180.) + DELY(JTIM)
    XJDOT = XJDOT + DELXDOT(JTIM)
    YJDOT = YJDOT + DELYDOT(JTIM)
    CALL STOREH(RADC, XJ, YJ, BETA, ZREF, HMIN, NHMIN)
    CALL ASSEM(U, NDOF, NLIST, JTURB)
    CALL GAUSSJ(SKP, NDOF, MAXN, U, 1)
    CALL STOREPQ(NLIST, NDOF, PMAX, NPMX)
    CALL BEARFORC(RADIUS, FMAG, ALPHA, BLANG, RPM, FX, FY, VIS, FXP, FYP, TORQ, ZREF, BMOMX, BMOMY)

IF (JTIM .EQ. 1) THEN
    SKXX = (FX0 - FX) / DELX(1)
    SKYX = (FY0 - FY) / DELX(1)
    DSKXX = SKXX * (RADC / FMAG)
    DSKYX = SKYX * RADC / FMAG
ELSE
    SKXY=(FX0-FX)/DELY(2)
    SKYY=(FY0-FY)/DELY(2)
    DSKXY=SKXY*RADC/FMAG
    DSKYY=SKYY*RADC/FMAG
    DO 20 J=1,NDOF
        U(J)=UOLD(J)
    20    CONTINUE
END IF

50    CONTINUE
    WRITE(11,100) SKXX,SKXY,SKYX,SKYY
    WRITE(11,120) DSKXX,DSKXY,DSKYX,DSKYY
RETURN

100   FORMAT(/5X,'Bearing Stiffness Coefficients :'/10X,
      1       G12.4,3X,G12.4/10X,G12.4,3X,G12.4)
120   FORMAT(/5X,'Dimensionless Bearing Stiffness Coefficients :'/10X,
      1       G12.4,3X,G12.4/10X,G12.4,3X,G12.4)
END

A3. DAMPCALC subroutine

SUBROUTINE DAMPCALC(NDOF,JTURB,FX0,FY0)
PARAMETER (MAXN=400)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON /COMM1/
    SKP(MAXN,MAXN),SKU(MAXN,MAXN),SKHDOT(MAXN,MAXN)
COMMON /COMM2/ U(MAXN),UOLD(MAXN)
COMMON /BDATA/ RADIUS,RADC,VIS,DEN,RPM,BRGLEN,BETA,ZREF
COMMON /EASAVE/ EE,AA
DIMENSION NLIST(MAXN),DELX(2),DELY(2),DELXDOT(2),DELYDOT(2)
DATA PI,FACT /3.141592654, .01 /
C    CALCULATE BEARING DAMPING IN GLOBAL XY SYSTEM
C 1ST SOLUTION : SHIFT X
C 2ND SOLUTION : SHIFT Y
DEGX(1)=0.
DEGX(2)=0.
DEGY(1)=0.
DEGY(2)=0.
DEGXDOT(1)=FACT*RADC*3.141592654*RPM/30.
DEGXDOT(2)=0.
DEGYDOT(1)=0.
DEGYDOT(2)= FACT*RADC*3.141592654*RPM/30.
BLANG=0.
XJDOT=0.
YJDOT=0.
C SAVE ORIGINAL PRESSURE SOLUTION (FOR TURB CORRECTION FACTORS)
DO 10 J=1,NDOF
   UOLD(J)=U(J)
10 CONTINUE
DO 50 JTIM=1,2
   XJDOT=0
   YJDOT=0
   XJ=EE*RADC*COS(AA*PI/180.)+DEGX(JTIM)
   YJ=EE*RADC*SIN(AA*PI/180.)+DEGY(JTIM)
   XJDOT=XJDOT+DEGXDOT(JTIM)
   YJDOT=YJDOT+DEGYDOT(JTIM)
   CALL STOREH(RADC,XJ,YJ,BETA,ZREF,HMIN,NHMIN)
   CALL STOREHDOT(RADC,XJDOT,YJDOT,BETA,ZREF,HMIN,NHMIN)^M
   CALL ASSEM(U,NDOF,NLIST,JTURB)
   CALL GAUSSJ(SKP,NDOF,MAXN,U,1)
   CALL STOREPQ(NLIST,NDOF,PMAX,NPMX)
   CALL BEARFORC(RADIUS,FMAG,ALPHA,BLANG,RPM,FX,FY,
       VIS,FXP,FYP,TORQ,ZREF,BMOMX,BMOMY)
IF(JTIM.EQ.1) THEN
  DCXX=(FX-FX0)/DELXDOT(1)
  DCYX=(FY-FY0)/DELXDOT(1)
  DDCXX=(RADC*2*PI*RPM/60.)*(1/FMAG)*DCXX
  DDCYX=(RADC*2*PI*RPM/60.)*(1/FMAG)*DCYX
ELSE
  DCXY=(FX-FX0)/DELYDOT(2)
  DCYY=(FY-FY0)/DELYDOT(2)
  DDCXY=(RADC*2*PI*RPM/60.)*(1/FMAG)*DCXY
  DDCYY=(RADC*2*PI*RPM/60.)*(1/FMAG)*DCYY
DO 20 J=1,NDOF
  U(J)=UOLD(J)
20     CONTINUE
END IF
50 CONTINUE
WRITE(11,110) DCXX,DCXY,DCYX,DCYY
WRITE(11,120) DDCXX,DDCXY,DDCYX,DDCYY
RETURN
110 FORMAT(/5X,'Bearing Damping Coefficients :'/10X,
1     G12.4,3X,G12.4/10X,G12.4,3X,G12.4)
120 FORMAT(/5X,'Dimensionless Bearing Damping Coefficients :'/10X,
1     G12.4,3X,G12.4/10X,G12.4,3X,G12.4)
END

A4. Sample Input file

TestCase 8: 120 Deg Arc L/D=1.0 9X48 Nodes with recess
Include Turbulence, Solution at eccentricity ratio=0.45
  2 1 0 0 0
  1 1 1 0 0 0
  1.0 .005 2000. 1.8e-6 1.0e-6 .00 .00
  .001 .001 .001 .001 0.00
  25 25 25 0
  .45 .0 .0 270. 300.0 270.0
  1 48 1 0. 210. 0. 2.5
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A5. Sample Output file

BRGDS Bearing Analysis Program, Version 1.2, July 2004

TestCase 8: 120 Deg Arc L/D=1.0 9X48 Nodes with recess
Include Turbulence, Solution at eccentricity ratio=0.45

Solution for specified eccentricity:
Minimum eccentricity = 0.4500
Maximum eccentricity = 0.0000
Eccentricity increment = 0.0000
Load angle = 270.00
Starting angle = 300.00
Minimum angle = 270.00

Solution control:
Include turbulence = Yes
Include deflections = No
Max load iterations = 25
Max pressure iterations = 25
Max flow iterations = 25
Max deflection iterations = 25
Load convergence tolerance = 0.0010
Press convergence tolerance = 0.0010
Flow convergence tolerance = 0.0010
Deflection tolerance = 0.0010

Bearing definition:
Radius = 1.000
Radial clearance = 0.5000E-02
Journal RPM = 2000.00
Lubricant viscosity = 0.1800E-05
Lubricant density = 0.1000E-05
Cavitation pressure = 0.000
Axial misalignment = 0.000
Z reference = 0.000

Pressure boundary conditions:

BC set 1
Pressure = 0.000
Nodes:
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49
BC set 2
Pressure = 0.000
Nodes :
393 394 395 396 397 398 399 400 401 402
403 404 405 406 407 408 409 410 411 412
413 414 415 416 417 418 419 420 421 422
423 424 425 426 427 428 429 430 431 432
433 434 435 436 437 438 439 440 441

BC set 3
Pressure = 0.000
Nodes :
50 99 148 197 246 295 344

BC set 4
Pressure = 0.000
Nodes :
98 147 196 245 294 343 392

Bearing Recess Data :

Recess 1
Specified flow = 0.5000
Starting pressure = 300.0
TAU factor = 2.000
Edge nodes:
169 170 171 172 173 174 175 176 225 274
273 272 271 270 269 268 267 266 217 168
Elements:
1 2 3 4 5 6 7 8

Finite Element Model Summary :

Total number of nodes in model = 434
Number of bearing land nodes = 434
Number of internal recess nodes = 0
Total number of elements = 376
Number of bearing land elements = 368
Number of recess elements = 8
Number of Nodes with Pres BCs = 112
Number of Recesses = 1
Bearing length = 2.000
Solution for eccentricity = 0.4500

Total iterations = 18
Deflection iterations = 1
Flow iterations = 5
Pressure iterations = 2
Load iterations = 1

Force magnitude = 102.7  Moment magnitude = 102.7
Radial force = 102.7  X moment = 102.7
Tangential force = 0.1788E-01  Y moment = 0.1787E-01

Eccentricity angle = 299.69
Sommerfeld No = 0.9348E-01
Torque = 0.4616

BC Set 1: Flow = -0.1914
BC Set 2: Flow = -0.1914
BC Set 3: Flow = 0.4985
BC Set 4: Flow = -0.6154

Recess 1: Flow = 0.4996  Pressure = 89.34

Bearing Stiffness Coefficients:
  7142.  5546.
  -0.1056E+05  -3414.

Dimensionless Bearing Stiffness Coefficients:
  0.3459  0.2696
  -0.5115  -0.1659

Bearing Damping Coefficients:
  28.06  -26.39
  -18.07  110.8

Dimensionless Bearing Damping Coefficients:
  0.2867  -0.2662
  -0.1846  1.117

Calculated Pressure Solution:

Maximum pressure = 89.34  at node 168

Calculated Film Thickness Solution:

Minimum film thickness = 0.2750E-02 at node 37
REFERENCES


Peeken Heinz., Gunter Knoll., Klaus Jacoby., ‘Calculation of the characteristics of journal bearings with angular displacement and bending of shaft’, VDI Forschungsheft, verlag des vereins deutscher ingenieure (publishing house of the association of German engineers), Nr. 617, 1983.


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