8-21-2014

The Meaning of WISE Colours – I. The Galaxy and its Satellites

Robert Nikutta  
*University of Kentucky*, robert@pa.uky.edu

Nicholas Hunt-Walker  
*University of Washington*

Maia M. Nenkova  
*Seneca College*

Željko Ivezić  
*University of Washington*

Moshe Elitzur  
*University of Kentucky*, moshe@pa.uky.edu

Right click to open a feedback form in a new tab to let us know how this document benefits you.

Follow this and additional works at: [https://uknowledge.uky.edu/physastron_facpub](https://uknowledge.uky.edu/physastron_facpub)

Part of the [Astrophysics and Astronomy Commons](https://uknowledge.uky.edu/astrophysics), and the [Physics Commons](https://uknowledge.uky.edu/physics)

Repository Citation  
https://uknowledge.uky.edu/physastron_facpub/185

This Article is brought to you for free and open access by the Physics and Astronomy at UKnowledge. It has been accepted for inclusion in Physics and Astronomy Faculty Publications by an authorized administrator of UKnowledge. For more information, please contact UKnowledge@lsv.uky.edu.
The Meaning of WISE Colours – I. The Galaxy and its Satellites

Notes/Citation Information

This article has been accepted for publication in Monthly Notices of the Royal Astronomical Society ©: 2014 The Authors Published by Oxford University Press on behalf of the Royal Astronomical Society. All rights reserved.

The copyright holder has granted the permission for posting the article here.

Digital Object Identifier (DOI)
http://dx.doi.org/10.1093/mnras/stu1087

This article is available at UKnowledge: https://uknowledge.uky.edu/physastron_facpub/185
The meaning of WISE colours – I. The Galaxy and its satellites

Robert Nikutta,1,2∗ Nicholas Hunt-Walker,3 Maia Nenkova,4 Željko Ivezić3 and Moshe Elitzur2

1Departamento de Ciencias Físicas, Universidad Andrés Bello, Av. República 252, Santiago, Chile
2Department of Physics & Astronomy, University of Kentucky, Lexington, KY 40506, USA
3Astronomy Department, University of Washington, Box 351580, Seattle, WA 98195-1580, USA
4School of English and Liberal Studies, Seneca College, Toronto, ON, M2J 2X5, Canada

Accepted 2014 May 29. Received 2014 May 27; in original form 2014 February 26

ABSTRACT

Through matches with the Sloan Digital Sky Survey (SDSS) catalogue we identify the location of various families of astronomical objects in WISE colour space. We identify reliable indicators that separate Galactic/local from extragalactic sources and concentrate here on the objects in our Galaxy and its closest satellites. We develop colour and magnitude criteria that are based only on WISE data to select asymptotic giant branch (AGB) stars with circumstellar dust shells, and separate them into O-rich and C-rich classes. With these criteria we produce an all-sky map for the count ratio of the two populations. The map reveals differences between the Galactic disc, the Magellanic Clouds and the Sgr Dwarf Spheroidal galaxy, as well as a radial gradient in the Large Magellanic Cloud (LMC) disc. We find that the C:O number ratio for dusty AGB stars increases with distance from the LMC centre about twice as fast as measured for near-IR selected samples of early AGB stars. Detailed radiative transfer models show that WISE colours are well explained by the emission of centrally heated dusty shells where the dust has standard properties of interstellar medium (ISM) grains. The segregation of different classes of objects in WISE colour space arises from differences in properties of the dust shells: those around young stellar objects have uniform density distributions while in evolved stars they have steep radial profiles.

Key words: radiative transfer – stars: AGB and post-AGB – Galaxy: structure – Magellanic Clouds – infrared: general – infrared: stars.

1 INTRODUCTION

Infrared emission from radiatively heated dust carries important information about the distribution of material around the heating source. This information can be deciphered either through detailed imaging and analysis of the spectral energy distribution (SED) of individual objects, or by studying the SED properties (such as colours) of statistically large populations of sources. The pioneering Infrared Astronomical Satellite (IRAS; launched in 1983) all-sky survey ushered in the era of modern highly successful astronomical surveys (Neugebauer et al. 1984; Olon et al. 1986). The IRAS survey provided unprecedented opportunity to classify the infrared properties of about 350 000 astronomical objects using a homogeneous data set obtained with a single facility (e.g. van der Veen & Habing 1988). Thanks to sensitivity improvements of up to three orders of magnitude compared to IRAS, the recent Wide-field Infrared Survey Explorer (WISE; launched in 2010) all-sky survey detected about 560 million objects (Wright et al. 2010). WISE represents the next major step in surveying and understanding of the infrared sky. For example, data from the IRAS point source catalogue (PSC) showed that certain Galactic objects tend to cluster in well-defined regions of IRAS colour–colour (CC) diagrams, and the same clustering is expected in the WISE photometric system but for a significantly larger sample that probes a much larger volume of the Galaxy. Furthermore, and perhaps more importantly, WISE data are deep enough for its catalogues to contain a very large number of extragalactic sources, and they too are clustered in well-defined regions of WISE CC and colour–magnitude (CM) diagrams. Although IRAS, too, detected some extragalactic objects (for a review see Soifer, Neugebauer & Houck 1987), WISE can be considered a true IR counterpart to modern optical surveys of extragalactic sources, such as the Sloan Digital Sky Survey (SDSS; York et al. 2000).

WISE mapped the sky at 3.4, 4.6, 12, and 22 μm with an angular resolution of 6.1, 6.4, 6.5 and 12.0 arcsec, respectively. Fig. 1 shows the WISE bandpasses and their comparison to other major infrared surveys. Throughout this paper we will refer to the filter names as ‘W1 filter’ (and W2, W3, and W4 filters). The source

∗E-mail: robert.nikutta@gmail.com

© 2014 The Authors
Published by Oxford University Press on behalf of the Royal Astronomical Society
These survey sensitivity limits apply in unconfused regions which correspond to Vega-based magnitudes 16.83, 15.6, 11.32 and 0.098, 0.86 and 5.4 mJy in the W1 through W4 bands, respectively, of an astronomical source, e.g. W1 ‘Colour’ is the magnitude difference between two measured fluxes such colour. bands 1 and 2. We will also use the short-hand notation W12 for Wi function.

SED \(\lambda F_\lambda\) is averaged in each band by the instrument’s transmission function \(W_i(\lambda)\), where \(i = 1, 2, 3, 4\). This yields the in-band fluxes \(F_i = \int W_i(\lambda) \cdot \lambda F_\lambda \, d\lambda\). The WISE Catalogue provides magnitudes calibrated in the Vega system using an empirical fit to the Vega spectrum (see equation 2 in Wright et al. 2010). Thus the calibrated in-band fluxes are

\[
f_i = \frac{F_i}{F_{\text{Vega}}} = \frac{\int W_i(\lambda) \cdot \lambda F_\lambda \, d\lambda}{\int W_i(\lambda) \cdot \lambda F_{\text{Vega}} \, d\lambda}.
\] (1)

‘Colour’ is the magnitude difference between two measured fluxes of an astronomical source, e.g. \(W1 - W2 = 2.5 \log(f_2/f_1)\) for bands 1 and 2. We will also use the short-hand notation W12 for such colour.

WISE attained 5\sigma point-source sensitivities better than 0.068, 0.098, 0.86 and 5.4 mJy in the W1 through W4 bands, respectively, which correspond to Vega-based magnitudes 16.83, 15.6, 11.32 and 8.0.\(^1\) These survey sensitivity limits apply in unconfused regions on the Ecliptic; they improve towards the ecliptic poles due to denser coverage and a lower zodiacal background. Photometric measurements of sources brighter than approximately 8.1, 6.7, 3.8 and −0.4 mag (Vega) are affected by saturation of the detectors. The astrometric precision for sources brighter than SNR = 40 is about 0.2 arcsec.

The WISE All-Sky Release comprises all data taken during the full cryogenic mission phase (2010 January 7 to 2010 August 6). It includes an Atlas of 18 240 images, a Source Catalogue with positional and photometric data for over 563 million objects, and also an Explanatory Supplement. Unlike IRAS catalogues which reported fluxes in Jansky, WISE measurements are reported on the Vega magnitude scale. Although this difference makes a direct comparison with some IRAS-based studies more difficult, we follow already published work based on WISE data and use unaltered catalogued Vega-based magnitudes. A more detailed discussion can be found in Wright et al. (2010) and at the WISE data release website.\(^2\)

Wright et al. (2010) have demonstrated that WISE detected all the main families of extragalactic sources, including star-forming (SF) galaxies, galaxies with active nuclei (AGN) and quasars (QSO). Their positions in a representative WISE CC diagram are shown schematically in the left panel of Fig. 2, and the challenge to theory is to come up with an explanation as to why different types of objects segregate the way they do. Similar structuring has been observed in IRAS-based CC diagrams; its details were explained by Ivezić & Elitzur (2000). The colours of a ‘naked’ radiation source correspond to a point in colour space; since WISE colours are normalized to Vega, colours of blackbody emission are close to 0. Embedding the source in a dust shell, the radiation is reprocessed to longer wavelengths and the colours are shifted to the red. The larger is the dust optical thickness, the redder are the colours. Therefore, increasing the shell optical depth while keeping all its other properties constant produces a track in CC diagrams with distance from the origin along the track increasing with optical depth. The right panel of Fig. 2 shows for illustration three tracks produced by spherical interstellar medium (ISM)-dust shells with power-law radial density distributions of varying steepness. The tracks for uniform density and for \(1/r\) density falloff nicely outline the boundaries of the region occupied by stars and extragalactic sources in the WISE CC diagram. Similarly, the WISE colours produced by clumpy torus models (Nenkova et al. 2008a,b), shown as grey-scale number density distribution, match the region of QSO and Seyferts. Our aim is to present in this paper a more detailed analysis of WISE colours of sources in the Galaxy and its satellites, and in a companion paper for AGN and quasars (Nikutta et al., in preparation).

We begin this first paper by identifying the WISE colours of families of astronomical sources through matching with the SDSS catalogue. The matching criteria and their results are described in Section 2. In particular, almost 40 million WISE sources are matched with Galactic stars, and in Section 3 we concentrate on the nature of these stellar sources. We proceed to calculate model colour tracks for the WISE photometric system which can be used to interpret the observed clustering of different stellar populations in the CC diagrams. The models and their comparison to WISE data are described in detail in Section 4. Our results are summarized and discussed in Section 5.

### 2 SDSS MATCHES FOR WISE OBJECTS

To enable a quantitative identification of source populations and subsequent comparison with our models, we use a sub-sample of

---

\(^1\) See the WISE Explanatory Supplement: http://wise2.ipac.caltech.edu/docs/release/allsky/expsup/

\(^2\) See http://wise2.ipac.caltech.edu/docs/release/allsky/
Figure 2. Qualitative comparison of WISE colours with models. Left: schematic distribution of WISE colours for various families of astronomical sources. This is fig. 12 from Wright et al. (2010), reproduced with permission of the authors and AAS. Right: sample model predictions for standard ISM dust. Solid lines show colour tracks for IR emission from dusty spherical shells that are centrally heated by a 5000 K blackbody point source to a dust temperature of 1000 K on the shell inner boundary. The shell radial density profile is $r^{-p}$, with $p = 0, 1$ and $2$ shown in red, black and blue tracks, respectively. Positions along each track correspond to the shell optical depth at visual, as marked on the red track. Results of AGN clumpy torus models (see Nikutta et al., in preparation) are shown as a grey-scale number density distribution normalized to its peak value.

Figure 3. Distribution of matching radii (in arcsec) between WISE and SDSS for our sample of stars. Based on this figure we adopt a matching radius of 1 arcsec in this paper.

WISE sources with SDSS-based identifications for five types of object: quasars, galaxies classified as AGN, SF and luminous red galaxies (LRG; essentially elliptical galaxies), and stars (optically unresolved objects not in the quasar sample). We first employ positional matching within a 1.0 arcsec radius of the WISE All-sky Data Release Catalogue, using procedures described in Obrči et al. (2006) and Covey et al. (2007): we match with SDSS Data Release 7 Catalogue for samples with spectroscopy, and Data Release 8 for unresolved sources in imaging data. The 1 arcsec cutoff matching radius is well justified. Fig. 3 shows the distribution of matching radii between WISE and SDSS for our sample of stars: the median and standard deviation are $0.26 \pm 0.17$ arcsec. For a sample of LRGs and star forming galaxies (not shown) it is $0.52 \pm 0.26$ arcsec. After positional matching, we enforce quality cuts for WISE photometry, as described further below. Fig. 4 shows three representative WISE CM diagrams for the five sub-samples selected as follows.

(i) Quasars: starting with 98 047 sources from the SDSS catalogue of quasars (Schneider et al. 2010) that have a WISE source closer than 1 arcsec, we further require that all four WISE magnitudes are brighter than $5\sigma$ limits and have a reliably measured uncertainty, yielding a sample of 14 795 quasars.

(ii) Star-forming and AGN galaxies: Galaxies with SDSS spectra are selected from the NYU VAGC Catalogue (Blanton et al. 2005). Galaxies with spectral emission lines are further classified as SF and AGN galaxies using the Baldwin–Phillips–Terlevich diagram (Baldwin, Phillips & Terlevich 1981). There are 12 379 AGN and 180 329 SF galaxies within 1 arcsec of a WISE source. The cuts for their WISE photometry are the same as for quasars, yielding samples of 4509 AGN and 38 092 SF galaxies.

(iii) LRGs: luminous red galaxies are adopted from a catalogue constructed for cosmological studies by Kazin et al. (2010). There are 66 093 galaxies from that catalogue within 1 arcsec of a WISE source. Requiring at least a $5\sigma$ detection in each of the four WISE bands leaves only 70 sources because of faint W3 and W4 magnitudes. As a result, for LRGs we enforce the $5\sigma$ cuts only in the W1 and W2 bands. This yields a sample of 47 989 galaxies, whose W2 − W3 and W3 − W4 colours are, of course, not reliable. To emphasize this point, the 10σ limits for the W3 and W4 bands are shown as solid lines in the two corresponding panels in Fig. 4.

(iv) Stars: SDSS Data Release 8 contains 119 million unresolved objects within 1 arcsec of a WISE source. To ensure high reliability of SDSS star/galaxy separation we limit the sample to $r < 20$. The majority of these stars are on the main sequence, and sample both disc and halo populations (see Ivezić, Beers & Jurić 2012, for detailed discussion). Similarly to the case of LRGs, we apply cuts sequentially. We first require $W1 < 15.8$ and $W2 < 14.8$ (10σ limits instead of $5\sigma$ because the sample is already fairly large), and $W12 < 0.8$ (to avoid contamination by faint unrecognized quasars; see below), yielding 39 million sources (bottom left panel in Fig. 4). Applying the 10σ cuts also to W3 and W4 bands reduces drastically

\[ \text{Available from http://sdss.physics.nyu.edu/vagc/} \]
Figure 4. Representative WISE CM diagrams for the five major families of objects (rows of panels) classified using SDSS data. The source distribution is rendered using a logarithmic grey-scale map and contours showing 5, 30, 60 and 90 per cent of peak density for each non-stellar sample. For the stars (lower panels), the left sample shows contours at 35, 50, 65, 80 and 95 per cent, the middle one at 55, 75 and 95 per cent, and the right one at 45, 70 and 95 per cent. The number of plotted objects is listed in the top-right corner of each left panel, and it is the same for the entire row unless printed otherwise. The solid lines in the two right-most panels for LRGs show the 10σ limit for the W3 band: the colours of objects with fainter magnitudes are not reliable. Vertical dashed lines in all other panels show median values μ for each distribution, which is indicated together with the corresponding interquartile range σ.

the sample size, yielding three million sources after the W3 < 10.5 cut (bottom centre panel), and ~80 000 sources when cuts in all four bands (W4 < 7.2) are enforced (bottom right panel).

The final sample sizes for all object classes and their mean WISE colours are listed in Table 1. The main conclusion derived from this analysis of SDSS-WISE matches is that subsamples of WISE sources with W1 < 11 must be dominated by Galactic objects, with the increasing fraction of extragalactic objects at fainter magnitudes.

2.1 Quasar contamination of the faint stellar sample

For faint ‘stars’, we required W12 < 0.8 because WISE sources with W1 ≳ 14 and W12 > 0.8 seem to be dominated by quasars that were too faint to be listed in the SDSS quasar catalogue. Strong support for this hypothesis is provided by their distribution in the SDSS g − r versus u − g CC diagram shown in Fig. 5. Objects that are unresolved in SDSS imaging data and which have WISE W1 − W2 > 0.8 and W1 ≤ 14 are shown as dots. All other selection criteria for stars as defined in Fig. 4 were also applied. The distributions of true SDSS stars, white dwarfs, and quasars are shown as blue, green, and red contours, respectively. We separated the WDs from the SDSS stellar catalogue (940 objects out of a total of > 430k) according to fig. 23 in Ivezić et al. (2007), requiring that u − g < 0.5 and g − r < 0.

Most of our faint WISE objects (218 in total, although only 190 dots are visible within the limits of the figure) are found in the region occupied by quasars (see e.g. fig. 1 in Smolčič et al. 2004). A smaller fraction is further consistent with anomalously red quasars, found along and just above the stellar locus (for details, see Richards et al. 2003). Finally, only very few of the faint objects could be misclassified WDs. White dwarfs, although of similar SDSS u − g colour as QSOs, tend to be significantly bluer in SDSS g − r.

3 COLOUR DISTRIBUTION OF BRIGHT OBJECTS

The analysis of SDSS-WISE matches shows that WISE sources with W1 < 11 must be dominated by Galactic objects. In this section we analyse this bright subset in more detail. We start by excluding objects close to the galactic plane (|b| < 10°) due to unknown levels
of extinction. We then require that the sources have magnitudes between the 5σ sensitivities and the saturation limits in all four bands. We also ask that the signal-to-noise ratio be greater than 5 in bands W1 and W2, and greater than 10 in bands W3 and W4. Finally, by imposing $W1 < 11$ and $W2 < 10$ we end up mostly with stars. We produced an all-sky map of the selected $\sim 16$ 000 bright sources by subdividing the sky into small patches of constant area on the sphere, about 13.43 deg$^2$, using the HEALPix tessellation scheme (Górski et al. 2005). The number of objects in each HEALPix patch was then counted to construct the logarithmic grey-scale map shown in the leftmost panel (0) of Fig. 6. This map, shown in Mollweide projection with Galactic coordinates, reveals the Galactic bulge and the Magellanic Clouds.

From the four WISE fluxes one can produce 12 different colours, but only three of them are independent. We choose $W12$, $W23$ and $W34$ as our three independent WISE colours. Their 3D distribution for the bright sources is shown in blue in the middle panel (1) of Fig. 6, with the 2D orthogonal projections shown as black scatter plots on the sides of the cube; these are the corresponding CC planes. These three projections are repeated in Panels 2–4, which show the WISE CC diagrams for Galactic sources; if Panels 2 and 4 are folded upwards along their common axes with Panel 3, the cube in Panel 1 is reproduced. We proceed now to discuss the structure evident in the WISE colour space.

### 3.1 Naked stars

The WISE magnitudes are normalized to the stellar spectrum of Vega, an A-type star. The colours of most stars should thus cluster around zero in the CC diagrams. Indeed, the highest concentration of objects in Figs 2 and 6 is found around zero colours. These are ‘naked’ stars, i.e. stars without significant circumstellar IR excess. In the case of IRAS, all filters sampled the Rayleigh–Jeans tail of every stellar blackbody (see Fig. 1) so that naked stars corresponded to a single point in all CC diagrams. In contrast, WISE bands 1 and 2 cover wavelengths short enough to be close to the peaks of some cool stellar spectra, therefore deviations from the zero-position in the CC diagrams can be expected, primarily for stars significantly cooler than Vega.

Fig. 7 shows the three orthogonal WISE colours $W12$, $W23$, and $W34$ for naked stars, described by blackbody emission with a large range of surface temperatures $T_s$; AGB stars have $T_s > 2500$ K (see Sections 3.3 and 4.4), while O-type stars reach upward of 50 000 K. Because Wright et al. (2010) used for Vega a slight modification of a 14 454 K blackbody (see their equation 2), to take care of instrumental/calibration effects, the colours produced by an unmodified Planck function that at temperature are not precisely 0. At higher temperatures, all WISE bands fall in the Rayleigh–Jeans domain and the three colours become constant with a slight ($\sim 0.05$ mag) deviation from zero. At the cool end the deviations are larger, reaching $\sim 4.0$ mag for AGB stars. This is the scatter observed in the stellar blob around the colour origin (0,0,0) in Fig. 6 (Panels 1–4). All deviations larger than this intrinsic scatter must be due to radiation reprocessing by a dusty environment.

### 3.2 Dusty envelopes

The several plumes that stand out in the WISE colour space away from the origin reflect reprocessing of stellar radiation by surrounding dusty envelopes. Such reprocessing redistributes the energy to longer wavelengths, leading to redder colours. The discernible structure is reminiscent of the clustering in IRAS-based CC diagrams (e.g. van der Veen & Habing 1988; Ivezić & Elitzur 2000). In the case of Galactic IRAS sources, these CC structures correspond to different classes of stars. Ivezić & Elitzur (2000) found that they were predominantly young stellar objects (YSO) and AGB stars, and the same is expected for the Galactic WISE sources. The narrow, elongated plume parallel to the W23 axis at $W12 \approx 0$ in Panel 3 of Fig. 6 is produced by warm dust emitting predominantly at short IR wavelengths. Such warm emission requires heating by a hot central star and can be expected to be YSO signature. Indeed, Ivezić & Elitzur (2000) found that the locus of IRAS colours of YSOs could be reproduced with emission from dust shells with uniform density. Such shells produce the WISE colours outlined in Fig. 2 with a red track ($p = 0$), which passes through this plume. Thus this plume is the WISE colour locus of YSO sources, which we discuss further in Section 4.3. For now we exclude this plume
Figure 6. Distribution of 16,000 bright sources with Galactic latitude $|b| > 10^\circ$ on the sky and in WISE colour space; see text for all selection criteria. Each panel is numbered in its upper right corner. (0): All-sky map, shown in a Mollweide projection with Galactic coordinates. The logarithmic grey-scale corresponds to the number of sources per unit area in the sky. (1): 3D distribution (shown in blue) of the three independent WISE colours W12, W23 and W34. The orthogonal projections on the corresponding CC planes are shown in black scatter diagrams. (2–4): The three WISE CC diagrams (sides of the cube in panel 1). Several blobs and plumes are clearly discernible.

Figure 7. WISE colours of ‘naked stars’, represented by blackbody emission with surface temperatures $T_s$. The grey circle pinpoints the modified-blackbody empirical fit for Vega, adopted by Wright et al. (2010). The grey vertical line at $T_s = 2500$ K marks a typical AGB stellar temperature (see Sections 3.3 and 4.4).

by retaining only sources with an IR excess in both ‘warm’ colours, W12 > 0.2 and W23 > 0, concentrating on the other plumes that comprise the AGB sources found by WISE.

3.3 AGB stars

The progenitors of asymptotic giant branch (AGB) stars are red giants; their progeny are planetary nebulae and white dwarfs. According to photospheric chemical composition, they can be divided into oxygen-rich (O-rich) and carbon-rich (C-rich) stars. More evolved AGB stars have dusty winds driven by radiation pressure on the dust grains in their circumstellar shells (see e.g.: Elitzur & Ivezić 2001; Ivezić & Elitzur 2010, and references therein). The winds reprocess the stellar radiation, shifting the spectral shape towards the infrared. These dusty AGB stars can be divided into two major sub-groups according to dust chemical composition: O-rich stars are associated with silicate-rich dust chemistry and C-rich stars with carbonaceous dust grains.

Dusty AGB stars are observable in our Galaxy and in its close satellites, mostly the Magellanic Clouds, e.g. by IRAS (van der Veen & Habing 1988), ISO (Trams et al. 1999), MSX (Egan, Van Dyk & Price 2001), Akari (Ito et al. 2008), and Spitzer (Bolatto et al. 2007; Vijh et al. 2009) (but see also e.g. Menzies et al. 2008; Whitelock et al. 2009; Boyer et al. 2013; Dalcanton et al. 2012; Javadi et al. 2013, for other Local Group galaxies). Studies of the Large Magellanic Cloud (LMC) revealed an overabundance of C-rich objects among its AGB stars, a trend found both at the near-IR wavelengths of the DENIS survey (Cioni & Habing 2003, hereafter CH03) and the longer wavelengths of IRAS (e.g. Zijlstra et al. 2006). Observations by IRAS and 2MASS of AGB stars in our own Galaxy were limited by interstellar dust extinction and by the sensitivities of these surveys (e.g. Jackson, Ivezić & Knapp 2002; Nikolaev & Weinberg 2000). Thanks to the much improved sensitivity and spatial resolution of WISE, the counts of catalogued dusty AGB stars should increase dramatically. This is especially expected for the Galaxy, where a model-based estimate of the total population of dusty AGB stars count is about 200,000 (Jackson et al. 2002).

3.3.1 WISE-based selection of dusty AGB stars

To remove the YSOs and still maintain a sizeable sample of AGB sources, first we slightly relaxed the constraint on the Galactic latitudes and omitted all sources with $|b| < 6^\circ$. This excludes most YSOs, which cluster around the Galactic plane. We arrived at this choice by testing how well the plumes in Fig. 6 still separate visually when allowing latitudes closer and closer to the Galactic plane. We then removed the narrow, elongated YSO plume as described above (Section 3.2). Finally, we required that the stars must also have colours within boundaries determined from known AGB stars from the SIMBAD data base; a detailed discussion is given in Appendix A. The final sample has 8835 members. Its all-sky distribution is shown in Panel 0 of Fig. 8, where the logarithmic grey-scale corresponds to the normalized number of selected stars per unit area of the sky. Black circles are drawn around the LMC and SMC, with radii of $\sim 8^\circ$ and $4^\circ$, respectively.
extragalactic sources, but relatively few AGB stars (see Section 3.3 for details).

Figure 8. Selection of AGB stars. Each panel is numbered in its lower right corner. (0) All-sky map of bright sources with IR excess in Mollweide projection. The Galactic plane with $|b| < 6^\circ$ has been removed. The grey-scale corresponds to the logarithm of the source count per unit area on the sky. The LMC and SMC are circled in black. (1) Map of C-rich stars selected in the CC diagrams 3–5. This sample is dominated by stars in the Magellanic Clouds. (2) Map of the O-rich stars selected by our criteria. Stars in the Galactic bulge dominate this sample. (3–5) Three WISE CC distributions of the sources in Panel 0. The logarithmic colour scale (common to all CC diagrams) shows the number of sources per mag$^2$ bin. The black contours contain the peak and 68 per cent of all sources within the black circles in Panel 0; these are dominated by the C-rich stars in the LMC. The regions highlighted with blue and red lines delineate our respective selection criteria for C-rich and O-rich stars. The cluster of objects around W2–W3 $\approx 2$ and W3–W4 $\approx 2$ contains a mix of Galactic and extragalactic sources, but relatively few AGB stars (see Section 3.3 for details).

The colours of AGB sources form a substructure of the 3D blue structure in the WISE colour space, shown in Fig. 6. For simplicity, in Fig. 8 we show only the 2D projections of this substructure on the three orthogonal colour planes. Panels 3–5 of this figure show how the selected stars distribute in WISE CC diagrams. The black contours contain 68 per cent of all the stars within the two black circles drawn around the LMC and SMC in Panel 0, as well as the peaks of their CC distributions. Since dusty AGB stars in the Magellanic Clouds are dominated by C-rich stars, the plumes outlined with these black contours mark the locus of C-rich stars in the WISE colour space. To quantitatively improve the selection of C-rich stars, we define two slab-shaped regions shown with blue lines in Panels 3 and 4; the corresponding blue lines in Panel 5 are a consequence of these definitions. In the 3D colour space, the intersections of these slabs define a parallelepiped within which the selected C-rich stars fall. The centre line of this elongated volume follows roughly the greatest density of the C-rich star plume (see Appendix A for the mathematical details).

Two structures remain. From cross-checks with SIMBAD, the plume outlined with red is dominated by O-rich AGB stars. The separation between the O-rich and C-rich plumes is clearest in the W23–W34 diagram (Panel 3), where the red triangle is the locus of O-rich AGB stars. The slanted edge of our selection aligns with the C-rich star selection boundary. The edge in W34 is tuned to exclude additional objects (dominated by quasars; see below). The W23 edge is positioned to maximize the completeness of our O-rich star selection without significant overlap with stars which have very little IR excess, i.e. which might not be AGB stars with circumstellar dust shells. In the two other panels (4 and 5) the separation is not as clean, thus we use for selection criteria the simple rectangular areas outlined in red.

The other structure, around W12 $\approx 0.5$, W23 $\approx 2.7$, and W34 $\approx 2.2$, is disconnected from the origin of the WISE colour space. Its objects are a mix of Galactic and extragalactic sources outside the selection boundaries for O-rich and C-rich stars. Cross-checks with SIMBAD show that the blue end of this plume in Panel 3 comprises purely Galactic sources, mostly YSO, variable stars, emission-line stars and only $<10$ per cent AGB candidates. The YSOs most likely contribute to the W3 band through their polycyclic aromatic hydrocarbon (PAH) emission. Thanks to its especially broad filter ($\sim 7$–$18\mu m$; see Fig. 1), WISE band 3 covers the prominent PAH features at 7.7, 8.6, and 11.3 $\mu m$, indicative of star formation (e.g. Peeters, Spoon & Tielens 2004). Such non-thermal, fluorescent emission is outside the scope of our study. Towards the red end of this plume we find increasingly more and more extragalactic sources (red QSOs, Seyferts, etc.) that have slipped through our selections. Thus this plume is a diverse mix of objects and will be ignored hereafter.

To summarize, we have demonstrated in this section that it is possible to use WISE photometry to select fairly clean samples of O-rich and C-rich AGB stars. We note that our results for the distribution of C-rich stars differ significantly from those in a recent study by Tu & Wang (2013). We believe that their results were affected by saturation effects in WISE photometry, as discussed in detail in Appendix A.

3.3.2 Near-IR colours of WISE-selected AGB stars

The WISE-based selection of AGB stars described in the preceding section is sensitive to the presence of circumstellar dust emission. The success of this selection, as well as the ability of WISE colours
Jand high Galactic latitudes, and from the LMC region. The expected AGB stars. These subsamples are selected positionally from low with 2MASS photometry for three subsamples of WISE and early AGB stars without much dust, have bluer J and near-IR photometry from Cioni (2009) does not include any stars. For example, the well-studied sample selected using visual stars, while samples selected using near-IR photometry (and some-dominated by the larger one.

The coordinates of these four regions are taken from the Sagittarius Dwarf Spherical Galaxy (Sgr DSph) is also clearly visible. The dynamical range spans almost four orders of magnitude.

Several regions stand out and are marked with (projected) circles: the Galactic Centre, and the LMC and SMC. Quite surprisingly, the Sagittarius Dwarf Spherical Galaxy (Sgr DSph) is also clearly discernible. The coordinates of these four regions are taken from the NASA/IPAC Extragalactic Database (NED) and are listed in Table 2 (lines 1 and 2). For the outlined circles we adopted radii 1.5 times the reported optical semi-major axes (line 3). For the Galactic Centre region we chose a radius identical to the LMC.

The right panel of Fig. 10 shows histograms of the C:O ratio for each sub-region, measured among all non-empty patches in that region. The histograms were obtained using the ‘Bayesian Blocks’ technique devised by Scargle et al. (2013). This method maximizes Bayesian evidence for a model under the assumption that data can be described by a piece-wise step function with arbitrarily located knots (analogous to a histogram with variable bin width). We used the implementation provided in the plotting_hist_tools.bayesian_blocks function from the astroML package (Vanderplas et al. 2012; Ivezić et al. 2014). The C:O-rich star ratio for Sgr DSph is about 1.5 orders of magnitude higher than in the Galactic plane, though less than that of the Magellanic Clouds. Table 2 lists the means (line 8) and ranges (lines 9 and 10) of the C:O ratios for Sgr DSph, but the number of contributing patches might be too small for meaningful conclusions.

### 3.3.3 The spatial variation of the C:O star count ratio

With the colour-selection criteria for C-rich and O-rich AGB stars established, Panels 1 and 2 of Fig. 8 show the distributions of these two families in the sky. The C-rich star sample is dominated by stars from the Magellanic Clouds, while O-rich stars are found predominantly towards the Galactic bulge and plane. This behaviour of dusty AGB stars is also seen for samples selected using near-IR photometry (recall that the latter are biased in favour of early AGB stars without thick dusty envelopes): while the Galaxy has more O-rich stars, the LMC and SMC are dominated by C-rich stars (CH03). This difference is likely due to higher metallicity in the Galactic disc than in LMC/SMC (Battinelli & Demers 2011).

The all-sky coverage and large size of the WISE sample of AGB stars allow us to compute a projected map of the C-to-O-rich star number ratio as a function of sky coordinates. The left panel of Fig. 10 shows in a colour map the variation of this ratio in the Galaxy and its immediate vicinity. The sky was divided into small 13.4 deg$^2$ patches (see Section 3), and the ratio was determined per patch. For this map we have dropped the previous requirement to exclude objects close to the Galactic plane, and selected all stars based on the colour and magnitude criteria defined previously. A strong gradient in the ratio, from the lowest values in the Galactic plane to the highest C-rich star concentration in the Magellanic Clouds, is clearly visible. The dynamical range spans almost four orders of magnitude.

The right panel of Fig. 10 shows histograms of the C:O ratio for Sgr DSph, but the number of contributing patches might be too small for meaningful conclusions.
3.3.4 The C:O star count ratio in LMC

To investigate the spatial variation of the C:O ratio in LMC, we analyse it as a function of radial distance from the LMC centre. We apply the coordinate transformations laid out in a two-paper series on the structure of the Magellanic Clouds, van der Marel & Cioni (2001) and van der Marel (2001) (hereafter MC1 and MC2). These transformations de-project the disc of the LMC to a face-on view and thus enable the measurement of the C:O ratio in concentric rings around the LMC centre. See Appendix B for the exact procedure. Fig. 11 shows in its left top panel the stars in the LMC in equatorial coordinates. The samples after de-projecting and applying our WISE colour and magnitude selection criteria for C-rich and O-rich AGB stars are shown in the left bottom panel. C-rich stars are shown in blue (nC = 1114), O-rich in red (nO = 238), and other stars as black dots (n = 330). The two grey rings are at 5 and 10 kpc radial distance from the LMC centre, and in the LMC disc. The two distributions of radial distances are indeed different. A two-sided Kolmogorov–Smirnov test of the two samples yields a p-value ≈ 6.5 × 10^{-6}, allowing us to reject the hypothesis that both are drawn from the same underlying distribution at a highly significant level. In fact, if we knew that both samples were distributed normally, with their respective means and variances, this p-value would afford rejection with \( \sqrt{2} \text{erf}^{-1}(1 - p) = 4.5 \sigma \) confidence.

To test for the existence of a radial gradient in the C:O ratio, we construct empirical cumulative distribution functions (CDF) of the radial distances of the C-rich and O-rich stars from the LMC centre. The CDFs are shown as blue and red lines in the right top panel of Fig. 11, and are defined as \( \text{CDF}(d) = \sum_{i=1}^{n} I[d_i < d] \). This function increases by one at each observed distance \( d \) in the sorted list of distances, i.e. it counts the number of stars observed up to a distance \( d \). A proper CDF would need to be normalized by 1/\( n \), but we refrained from doing so in the right top panel, to show how the absolute member counts in the two samples differ. To obtain analytic representations of the CDFs we fitted them with polynomials of various degrees, and also with the CDFs of Normal distributions. The best fits could be obtained with fourth degree polynomials, which were significantly better than third degree polynomials and Gaussians. The fitted polynomial function is \( P_i \equiv \sum_{i=0}^{4} a_i x^i \), with coefficients \( a_i = (-76.37, 256.03, 6.38, -4.54, 0.26) \) for the CDF of C-rich stars, and \( a_i = (-30.77, 107.91, -15.00, 0.65, 0.01) \) for O-rich stars (lowest order first). The fits are plotted with dotted lines in the top right panel of Fig. 11.

The second panel from the top on the right shows the fractional residuals of our fits (their absolute values). They are generally well-controlled at a level < 5 per cent, except in the innermost region. The third panel from the top shows as red and blue lines the probability density functions (PDF) of the two samples, obtained as derivatives \( \delta \text{CDF}/\delta d \) of the fitted CDFs and normalized to unit area. Also shown are histograms of the distributions of radial distances, which were obtained using Bayesian Blocks. The striking similarity of the PDFs obtained by polynomial fits of the empirical CDFs, and the

<table>
<thead>
<tr>
<th>No.</th>
<th>Quantity</th>
<th>Full sky</th>
<th>GC</th>
<th>Sgr DSph</th>
<th>SMC</th>
<th>LMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Galactic longitude (deg)</td>
<td>n/a</td>
<td>0. 5.57</td>
<td>302.80</td>
<td>280.47</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Galactic latitude (deg)</td>
<td>n/a</td>
<td>0. -14.17</td>
<td>-44.30</td>
<td>-32.89</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Radius (deg)</td>
<td>n/a</td>
<td>8.06</td>
<td>5.62</td>
<td>4.00</td>
<td>8.06</td>
</tr>
<tr>
<td>4</td>
<td>Number of C-rich stars n(C)</td>
<td>1629</td>
<td>60</td>
<td>21</td>
<td>161</td>
<td>1079</td>
</tr>
<tr>
<td>5</td>
<td>Number of O-rich stars n(O)</td>
<td>14 969</td>
<td>4500</td>
<td>62</td>
<td>24</td>
<td>235</td>
</tr>
<tr>
<td>6</td>
<td>Number of valid HEALPix patches log10 n(C)/n(O)</td>
<td>196</td>
<td>21</td>
<td>7</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>For entire selection</td>
<td>-0.96</td>
<td>-1.88</td>
<td>-0.47</td>
<td>0.83</td>
<td>0.66</td>
</tr>
<tr>
<td>8</td>
<td>Mean of valid patches</td>
<td>-0.85</td>
<td>-1.71</td>
<td>-0.19</td>
<td>0.84</td>
<td>0.87</td>
</tr>
<tr>
<td>9</td>
<td>Min of valid patches</td>
<td>-2.61</td>
<td>-2.61</td>
<td>-0.90</td>
<td>0.48</td>
<td>-0.40</td>
</tr>
<tr>
<td>10</td>
<td>Max of valid patches</td>
<td>1.38</td>
<td>-0.70</td>
<td>0.48</td>
<td>1.32</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Figure 10. Left: all-sky map of the local C-to-O-rich star number ratio, in Mollweide projection. The Galactic Centre is at (0,0). This map is a ratio of Panels 1 and 2 in Fig. 8, but without the |b| > 6° criterion. The colour scale shows the logarithm of the ratio per 13.43 deg^2 coordinate bin (defined by the HEALPix tessellation). The locations of the Galactic Centre and three Milky Way satellites are indicated with (projected) circles. Right: distribution of log10 n(C)/n(O) in each circular area from the left panel. The histograms were obtained using Bayesian Blocks (see text). See Table 2 for statistics.
range from \(-4\) to 18, a larger dynamic range as well. As discussed above, the two determinations of the C:O ratio involve different properties. The Cioni sample used by Feast et al. was selected from the photospheric emission of AGB stars while our WISE-selected samples are based on the radiation from the dusty shells that typify late stages of evolution of these stars. That a much larger C:O ratio is determined from dust re-radiation than from photospheric emission implies a higher efficiency of dust formation for the C-rich stars. A detailed comparison of the two results or, equivalently, predicting the variation of the C:O ratio for dusty AGB stars from the variation observed for early AGB stars, would require detailed modelling of AGB evolution (e.g. Marigo et al. 2008; Girardi et al. 2010). In addition to prescriptions for AGB evolution of both O-rich and C-rich stars, their age and metallicity distributions and the dust formation efficiency also need to be known or assumed. While such an effort is beyond the scope of this study, it has the potential to shed new light on the formation and evolution of the LMC.

4 MODELLING WISE COLOURS

Why do objects segregate in colour space and what causes the particular segregation patterns of different families of WISE sources? These questions were already addressed by Ivezić & Elitzur (2000) in the context of IRAS sources. The dust radiative transfer problem possesses general scaling properties, as discussed in detail by Ivezić & Elitzur (1997). There are only two input quantities whose magnitudes matter: the dust optical depth at some fiducial wavelength and its temperature at some position in the source. All other input is defined by dimensionless, normalized profiles that describe (1) the spectral shapes of the dust absorption and scattering coefficients, (2) the spectral shape of the external heating radiation, and (3) the dust spatial distribution. Physical dimensions, e.g. luminosity and linear sizes, are irrelevant. Scaling applies to arbitrary geometries and implies that different dusty objects are expected to segregate in CC diagrams primarily because their dust shells are different, not because they have different central sources. This behaviour is displayed by the objects in the IRAS PSC and is similarly expected for WISE colours.

We now describe our basic modelling framework for dusty stellar environments and then discuss comparisons of models with data in more detail. All modelling was performed with our public code DUSTY (Ivezić, Nenkova & Elitzur 1999).

4.1 Model properties

We employ two families of dust grains. Our ‘standard ISM’ dust contains a standard Galactic mix of 53 per cent silicates, with optical constants from the Ossenkopf, Henning & Mathis (1992) ‘cold dust’ for silicate, and 47 per cent graphite, with optical properties from Draine (2003). The other family represents pure-composition dust typically found in AGB stars. Dust around C-rich stars comprises amorphous carbon, whose properties we take from Hanner (1988), while for O-rich stars we take the warm silicates from Ossenkopf et al. (1992). We assumed spherical grains and considered both single-size dust grains and models with MRN grain-size distribution (Mathis, Rumpl & Nordsieck 1977). DUSTY handles dust distributions through the composite grain approximation by averaging over the chemical composition and size distribution. This approach was shown to reproduce reasonably accurately the results from exact treatment of grain mixtures (see e.g. Efstathiou &

histograms computed independently via Bayesian Blocks, demonstrates that the results are not strongly affected by methodology. Encouraged by this agreement, we plot in the right bottom panel of Fig. 11 the ratio of the (unnormalized) PDFs of the C-rich and O-rich AGB samples, i.e. the radial C:O number count ratio in the LMC. The value increases from \(-3\) in the innermost regions to \(\lesssim 19\) at about 6 kpc, before turning down sharply. This downturn is probably not reliable because the number counts of both C-rich and O-rich stars are quite small at these distances. In fact, Feast, Abedigamba & Whitelock (2010), who re-analysed the DENIS data of Cioni (2009) by removing unpopulated areas in the LMC sky, find a similar trend in the C:O star count ratio (C:M star ratio, to be more precise), and furthermore derive large uncertainties for the data points beyond \(-7\) kpc (see their fig. 6).

It is reassuring that both the Feast et al. (2010) results and our analysis show the same trend of increasing C:O ratio with distance from the LMC centre. Nevertheless, there are significant differences. Between 2 and 6 kpc from the centre, the region where the results of both studies can be considered reliable, Feast et al. find a variation from approximately 0.25 to 0.5 while we find much larger ratios that...
Rowan-Robinson 1994). The extinction profiles used in the calculations are shown in Fig. 12.

With DUSTY we produced models of spherical dust shells around central sources; the code calculates exact solutions for the dust temperature profiles, emerging flux and other properties of interest. As noted above, the only input parameters whose magnitudes matter are the dust temperature on the shell inner boundary and the shell overall optical depth at a given wavelength; all other model input describes dimensionless, normalized profiles. We computed a range of dusty shell models with overall optical depth at visual from $\tau V = 0.1$ to 200 and dust temperature on the inner boundary from $T d = 300$ K to 1200 K. The spectral shape of the central source was taken as a blackbody with temperature $T s = 2500$ K, 5000 K and 10 000 K. For the radial density distribution of the spherical dusty shell we used a power-law $r^{-p}$ with $p = 0$, 1 and 2. In addition, for the AGB shells we used the radial density profiles determined from a solution of the full equation of motion of radiatively driven dusty winds, obtained with the RDW option of DUSTY (see Elitzur & Ivezić 2001). The shell relative thickness, ratio of outer to inner radius $Y \equiv R_{out}/R_{in}$, was taken as 100. Resulting SEDs for some representative cases are shown in Fig. 13.

### 4.2 Colour tracks

As noted above (Section 3.1), a naked star corresponds to a point near the origin of the WISE colour space. Surrounding the star with a dust shell will shift the colours to the red, moving the source away from the origin. Given the shell properties and varying only its overall optical depth, the dust temperature profile remains unchanged as long as the shell remains optically thin. Therefore the colours remain the same; all optically thin dusty shells with the same properties correspond to a single point in the colour space. Increasing the optical depth further, the shell temperature profile starts to change when it enters the optically thick regime, leading to deviation of the source colours away from the optically thin point towards redder colours. This results in a track in the 3D WISE colour space, with position along the track determined by the shell overall optical depth $\tau V$. Projections on the 2D colour planes produce tracks in CC diagrams, like those shown in the right panel of Fig. 2. This figure shows that the tracks for $p = 0$ and 1 shells of ISM dust trace the boundaries of the region occupied by extragalactic sources in the W12-W23 CC diagram. This behaviour is in agreement with the findings by Levenson et al. (2007) and Sirocky et al. (2008) that the deep silicate absorption features observed in ULIRG spectra require burial of luminous central sources in very optically thick ($\tau V \gtrsim 100$) dusty shells with flat ($p = 0$–1) radial density distributions. The figure also shows that stars occupy a region much closer to the colour origin so that the optical thicknesses of their dust shells are generally much smaller.

We now discuss in detail the WISE colours of Galactic sources. Tabulations of the colour tracks used in figures displayed below are provided in Table C1, where we also list the model in-band fluxes, useful for studies which depend on source luminosity.

### 4.3 YSO tracks

Ivezić & Elitzur (2000) found that IRAS colours of YSOs could be explained by the colour tracks of dust shells with flat ($p = 0$) density distributions. Fig. 14 explores in detail the WISE colours expected from $p = 0$ and 1 radial profiles for spherical shells of ISM dust with MRN size distribution and optical depths up to $\tau V = 30$, heated by a central 10 000 K blackbody. Models were produced for dust temperatures on the shell inner boundary varying from $T d = 300$ K to $T d = 1200$ K. This figure provides a zoom-in on the smaller-$\tau V$ region of the tracks for shallow density distributions that were shown in Fig. 2. Its various plots show the range of possibilities for colour tracks covering the likely region of parameter space for dust shells around YSOs. A characteristic feature of flat
density distributions is that even a small increase in optical depth can significantly enhance long wavelength emission because it places more dust at large radial distances with low temperatures. For this reason, the \( \tau_V \lesssim 1 \) portions of the tracks are parallel to the colour axis with longer wavelengths while maintaining a value of roughly 0 for the other axis with the 'warmer' colour. Deviations from this behaviour increase with \( T_b \) and are mostly noticeable in the W23–W34 diagram, especially for the \( p = 1 \) tracks.

Fig. 15 shows the comparison with observations. Its three panels reproduce the projections on to 2D colour planes of the 3D colour distribution of bright \textit{WISE} sources that were shown previously in Fig. 6, with model tracks plotted over the source distribution. The models producing these tracks are the same as the \( p = 0 \) models shown in Fig. 14 except for the spectral shape of the heating source: instead of a pure blackbody they use the empirical Vega fit employed for normalization by Wright et al. (2010) (see also Section 3.1). These tracks follow rather well the long, narrow plume of optically thick \( \approx 10^5 \) K sources that was shown in Fig. 14 and along the W34 axis at W23 \( \approx 0 \) in Panels 1 and 2 of Fig. 15 and along the W34 axis at W23 \( \approx 0 \) in Panel 0. This plume has been previously identified as the locus of YSOs among bright \textit{WISE} sources (Section 3.2). In most colour-colour combinations, little differentiates the tracks for different values of \( T_b \), the dust temperature on the inner boundary. The tracks diverge significantly only in the panels involving the W34 colour combination, where only the \( T_b = 1200 \) K track passes through the blobs roughly clustered around (2.5,2) in Panel 0 and (2.2,0.7) in Panel 2. It is not clear, though, that these blobs give a reliable indication of thermal dust emission because these sources could be dominated by PAH emission covered by the broad W3 filter (see Section 3.3). A reliable determination of the nature of these sources would require in-depth, detailed modelling of individual objects.

The small range of optical depths spanned by the main plume of SF sources implies that \textit{WISE} has uncovered a large number of YSO with small amounts of dust around them while giving little indication of such objects covered by deep layers of dust. This is surprising because heavy dust obscuration is expected in many YSOs. Indeed, the SF regions identified by \textit{IRAS} were characterized by large optical depths (\( \tau_V \sim 1-10 \); see fig. 3 in Ivezić & Elitzur 2000). These differences reflect the starkly different sensitivity limits of the two facilities. \textit{IRAS} required for detection strong IR emission, which precluded sources with optically thin dusty shells. However, optically thick YSO shells, which were a prerequisite for \textit{IRAS} detection, produce \textit{WISE} colour differences that exceed 4. Such large differentials are eliminated by the cuts we place on saturation limits, which remove sources too bright at short wavelengths, and on signal-to-noise ratio, which remove sources too weak at the long wavelength bands. As is evident from Fig. 2, the W23 colour of optically thick \( p = 0 \) objects is larger than 4, entering the domain populated by extragalactic sources which we deliberately sought to avoid in this analysis. Relaxing our selection criteria should therefore reveal a heavily obscured YSO population mixed with the extragalactic sources.

To test this hypothesis we identified in our basic selections those sources that are moderately bright in W1 and are matched by high optical depths of our model colour tracks. These would most likely be YSOs with high shell optical depths. Such a population, compatible with the previous \textit{IRAS} detections, indeed exists in the \textit{WISE} data base. It can be found approximately within \( 10 \lesssim W1 \lesssim 11 \), \( 0.4 \lesssim W12 \lesssim 1.4 \), and \( 4 \lesssim W23 \lesssim 5.7 \). All sources with \( 10 \lesssim W1 \lesssim 11 \) are shown in the CC scatter diagram in Fig. 16 (left panel); all other selection criteria from Fig. 15 apply, except that the constraint on Galactic latitude was dropped. The CC diagram contains 29 892 sources while the blob outlined with a blue rectangle contains 6192 sources that were shown previously in Fig. 6; each panel is numbered at its top right corner. Overplotted are model colour tracks of spherical dusty shells with standard ISM dust grains and uniform density profiles \( p = 0 \). The shells surround central stars with Vega-like emission. The dust temperature on the shell inner boundary is either 600 K (red tracks) or 1200 K (green). Position along the track depends only on the dust optical depth; a few values of the \( V \)-band optical depth are marked. The tracks delineate the plume of YSOs.
objects, presumably YSOs with high dust optical depths. Indeed, the centre of the selection cube, $W1 = 10.5, W2 = 0.9, W3 = 4.85$, implies that $W3 = 4.75$ (Vega). With this magnitude we can use $F_\nu = F_\nu^{1200} \times 10^{-\mu_{W3}/2.5}$ to compute the in-band flux density $F_\nu$, where $F_\nu^{1200}$ is the zero-magnitude flux density in Jansky. It is given as 31.674 Jy for band 3 in Jarrett et al. (2011, see their table 1). Thus our $W3$ magnitude $m_{\text{Vega}} = 4.75$ corresponds to $\approx 0.4$ Jy at 12 $\mu$m (W3) while the faint limit of the IRAS Faint Source Catalogue at 12 $\mu$m is 0.2 Jy. Therefore, our findings are compatible with IRAS discovering only sources in the blue box but not those with lower 12 $\mu$m fluxes. Also plotted in the left panel of Fig. 16 are the same model colour tracks from Panel 1 of Fig. 15, but over an expanded colour range. The green track ($T_d = 1200$ K) shows a turn-over at $W2 \sim 5.5$, and all its $\tau_V$ values between 0.025 and 9.4 stay perfectly within the blue YSO rectangle. The right panel of Fig. 16 shows the all-sky distribution of the ~6200 sources from the YSO plume. The distribution is extremely heavily concentrated towards the Galactic plane, a preferred location to find local SF regions. This is further evidence that our conclusions are consistent. The choice of dusty shell models for YSOs can be debated because some young stars, e.g., T Tauri stars, are known to be surrounded by dusty disks. Vinkovic et al. (2003) showed that a flared disk produces an SED identical to that of an equivalent optically thin spherical dust shell. The latter is thus a good proxy for the more realistic disk model. There is no way to distinguish between them, or any combination of the two, with flux measurements alone; imaging is necessary for that. From equations 7 and 8 in section 2.1 of Vinkovic et al., a constant density shell (as applied in our YSO models) is equivalent to a disk with linear flaring. The very small optical depths we find in the WISE data of YSOs may be also interpreted as the difference in flaring angle across the disk (Equation 7), but it is difficult to decide without imaging.

### 4.4 AGB tracks and model parameter constraints

Radiatively driven winds (RDW) have been shown to well describe infrared emission from dusty envelopes around AGB stars (see e.g.: Elitzur & Ivezić 2001; Ivezić & Elitzur 2010, and references therein). Their circumstellar dust distribution approximately follows a power-law $1/r^2$. DUSTY can calculate RDW models exactly and in Fig. 17 we show RDW model colour tracks for a central source at $T_s = 2500$ K and a $Y = 100$ shell. Tracks are shown for two temperatures at the inner dust boundary, $T_d = 600$ K and $T_d = 1200$ K. AGB stars are classified as O-rich or C-rich stars (see Section 3.3) and models were produced with two corresponding types of dust: warm silicates from Ossenkopf et al. (1992) (Sil-Ow, left panels), and amorphous carbon (amC, right panels). Both single-size grains and an MRN size distribution were used, as indicated in the legend. As seen from Fig. 17, the effect of different grain sizes on the colour tracks is negligible compared to varying the dust temperature.

Amorphous carbon absorbs more in the W1 and W2 bands, while silicates absorb more in the W3 and W4 bands (see Fig. 12). Therefore, given two circumstellar shells with the same overall optical depth but different chemical composition, the silicate shell is expected to be redder in the W2 – W3 colour and bluer in the W1 – W2 colour. As illustrated in Fig. 17, the WISE colour diagrams can separate sources with steep density profile and predominantly silicate dust (e.g., typical AGB stars) from those with carbonaceous dust (e.g., carbon stars), for the same dust sublimation temperature.

How do the RDW model tracks compare to AGB colours measured by WISE? First, we can fix the stellar temperature $T_s$ at 2500 K and the ratio of inner-to-outer radius of the shells at 100, since both are quite typical values for AGB stars. We also investigated other $T_d$, but they did not agree as well with the data (the allowed temperature range is only a few hundred K wide). The two most important free model parameters are dust interface temperature, $T_s$, and optical depth, $\tau_V$. This two-dimensional space of model parameters maps to WISE colour space (and vice versa).

We have demonstrated in Section 3.3 that AGB stars can be selected using only WISE photometry. Equations (A3)–(A8) in Appendix A are the mathematical definitions of our selection criteria. The corresponding loci of AGB stars are visualized in Fig. 8 with red and blue lines. These equations also define lower and upper bounds of several WISE linear colour combinations. For instance, equation (A7) can be written as $x = 0.629 y \in [-0.198, 0.359]$ (where $x = W1 - W2, y = W2 - W3$). With given dust properties
Figure 18. Translation of our AGB colour selection criteria [defined in equations (A3)–(A8)] into model parameter constraints. Left for C-rich AGB stars, right for O-rich stars. Each equation translates into curved areas in the \((T_d, \tau_V)\) parameter space. The excluded areas are shaded (see legend for the colour/equation code). Only the white fraction of each panel contains permissible model parameter combinations. See text for details.

Since our C-rich star colour selection volume is confined quite tightly (cf. Fig. 8), the allowed parameter space for DUSTY models is also well-constrained. The dust shells must have interface temperatures not lower than \(\approx 600\) K and the visual optical depths are in the range \(\approx 0.2–20\). For O-rich star models the selection in CC space was not as tight; consequently the permissible parameter ranges are less well-constrained. Our results seem to indicate that C-rich AGB stars tend to have higher maximal optical depths than O-rich AGBs, and that they lack the very lowest \(\tau_V\) values. In contrast, O-rich AGB models extend to smaller optical depths, and values larger than a few are excluded. This conclusion is supported by samples analysed in Ivezić & Elitzur (2010); see their figs 5 and 6, with data from Young (1995), Richards & Yates (1998) and Olofsson et al. (1993) (we caution the reader that those data sets may be affected by unknown selection effects). We emphasize that our results should be viewed qualitatively and note that the parameter boundaries are ‘soft’ because they depend on the exact colour selection criteria.

The distribution of C-rich and O-rich AGB stars in the WISE colour space, as previously selected in Section 3.3 and Appendix A, is shown in Fig. 19. Overplotted are fiducial RDW model colour tracks, for which the temperature at the inner boundary is \(T_d = 800\) K (solid lines) or \(T_d = 500\) K (dotted), in accordance with the parameter ranges found above. All tracks are computed for a shell thickness of 100, and with a stellar temperature of \(T_S = 2500\) K, as explained before. Panels 00–02 (left side) show our C-rich star selection and colour tracks computed with amorphous carbon dust, while Panels 10–12 (right) show our O-rich star selection and tracks with ‘warm’ circumstellar silicates from Ossenkopf et al. (1992). The model tracks do not match all AGB plumes in all three orthogonal projections simultaneously, but the range spanned by models of these dust interface temperatures brackets well the observed WISE colours of AGB stars.

The \(T_d = 800\) K RDW model with amorphous carbon dust (blue solid line) follows the plume of C-rich stars reasonably well in all three two-dimensional projections of the WISE colour space (Panels 00–02). Although the agreement is not perfect, it is likely that these 0.2–0.3 mag large offsets could be rectified with minor modifications of the dust opacity. We have not attempted a detailed study because such small changes could also be explained by deviations from spherical symmetry, non-uniform dust distribution, variations in photospheric temperature as the star ascends the asymptotic
5 SUMMARY AND DISCUSSION

With detections for about 560 million objects, WISE represents a major step forward in the surveying and understanding of the infrared sky. Its combination of up to 1000 times higher sensitivity than previous IR surveys, and a relatively high angular resolution, enables studies of source samples with unprecedented statistical significance. We have computed colour tracks in the WISE photometric system for two families of models: smooth spherical shells and clumpy dust torus models, and decided to study local/Galactic populations in this paper, and the extragalactic sky in Nikutta et al., (in preparation).

The model tracks provide guidance for the observed distribution of sources in WISE CC diagrams, such as illustrated in the left panel in Fig. 2. The main results of our computations are shown in the right panel in Fig. 2. The model-based colour tracks outline reasonably well the distribution of sources expected to be associated with dust emission (i.e. all source types except T dwarfs and stars without dusty shell). The location of dust-free stars coincides very well with the optically thin point of all dust-shell models irrespective of the dust composition. Deviations from the zero-point are caused by WISE filters W1 and W2 being sensitive to the blackbody peaks of the coolest stars, thus producing colours other than those for the Rayleigh–Jeans slope. In the other extreme, at high optical depths, the DUSTY model tracks of ISM dust shells with shallow radial density distributions overlap with those CC areas populated by Ultra-Luminous Infrared Galaxies (ULIRGs), which are known to be deeply shrouded in dust (Sirocky et al. 2008).

We find that WISE photometry alone is sufficient to reliably classify C-rich and O-rich asymptotic giant branch stars with dusty shells (Section 3.3). With the large population of AGB stars in the WISE Catalogue we were able to define magnitude and colour selection criteria that distinguish between the two classes. It is necessary to regard the distribution of WISE colours as a three-dimensional arrangement in colour–colour–colour space. The traditional two-dimensional orthogonal projections are very helpful, but not sufficient. With the ability to cleanly separate the local populations of C-rich and O-rich AGB stars we computed the most detailed map of the C/O-rich star count ratio for the Milky Way and its satellites (see Section 3.3.3). We find a strong spatial gradient away from the Galactic plane, with the Magellanic Clouds showing a C/O ratio about 1000 times higher than the Galactic bulge. We also clearly uncover the Sagittarius Dwarf Spheroidal Galaxy in the map. We find that the C/O dusty AGB star ratio increases with distance from the LMC centre about twice as fast as measured for near-IR selected samples of early AGB stars. Detailed modelling of these measurements has the potential to shed new light on the formation and evolution of LMC.

DUSTY model tracks for shells with RDWs follow the identified AGB plumes well. By mapping the previously found colour selection criteria to model parameters, specifically inner dust shell temperature $T_d$ and shell optical depth $\tau_V$, we found observational constraints on these model parameters. They seem to indicate slight differences in the physical properties of C-rich and O-rich AGB shells (see Section 4.4).

Spherical dusty shell models with flat radial density distributions generate WISE colours compatible with YSOs. YSO samples selected using IRAS and WISE surveys have very little overlap due to vastly different imaging depths: while IRAS was sensitive to bright sources with relatively high visual optical depths, WISE detects much fainter YSOs with relatively small optical depths (Section 4.3).

To investigate further how well models describe the WISE colours of IR sources, sufficiently large samples need to be studied object-by-object, with SED fitting for each source over a large wavelength range. The Spitzer SAGE Survey (Meixner et al. 2006), for instance, has been previously used for such a task. Riebel et al. (2012) matched the IR Spitzer spectra with optical and near-IR data to produce broad-band SEDs for more than 30 000 AGB stars in the LMC, and fitted them with GRAMS models (Sargent, Srinivasan & Meixner 2011; Srinivasan, Sargent & Meixner 2011). Thanks to WISE, a similar study can be now undertaken for AGB stars in the Galaxy.

CLUMPY models of AGN dust torus emission, with their more complicated geometry and dust distributions, are much better than simple spherical shells in explaining the general location of QSOs and other AGNs in the WISE CC diagram. The location of the highest CLUMPY model density aligns almost perfectly with the centre of the circular region identified as ‘Seyferts’ by Wright et al. in the left panel of Fig. 2. A detailed study of the WISE colours of AGNs will be presented in the companion paper (Nikutta et al., in preparation).

ACKNOWLEDGEMENTS

We are grateful to Maria-Rosa Cioni for illuminating discussions and for making her data available to us. We reproduced fig. 12 from Wright et al. (2010) as part of our Fig. 2 by permission of the AAS and the authors. We thank Chao-Wei Tsai for providing us a high-quality version. We thank both referees for their helpful suggestions which improved the paper. We express our gratitude to Xavier Koenig for reminding us of the disk/shell dichotomy of YSO models (see Section 4.3), and to James Davenport for his insightful post-submission comments on the variability information in the WISE database. We are also thankful to Patricia Arévalo and Ron Wilhelm for helpful discussions. RN acknowledges support by the ALMA-CONICYT fund, project No. 31110001, and by Fondecyt grant No. 3140436. ŽI thanks the Hungarian Academy of Sciences for its support through the Distinguished Guest Professor grant No. E-1109/6/2012. ŽI and NHW acknowledge support by NSF grant AST-1008784 to the University of Washington. ME acknowledges NASA support and the award of an NPP Senior Fellowship from ORAU, which supported a Sabbatical leave at JPL/Caltech where much of this work was completed. This publication makes use of data products from the Wide-field Infrared Survey Explorer, which is a joint project of the University of California, Los Angeles, and the Jet Propulsion Laboratory/California Institute of Technology, funded by the National Aeronautics and Space Administration. Funding for SDSS-III has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, and the U.S. Department of Energy Office of Science. The SDSS-III web site is http://www.sdss3.org/. SDSS-III is managed by the Astrophysical Research Consortium for the Participating Institutions of the SDSS-III Collaboration including the University of Arizona, the Brazilian Participation Group, Brookhaven National Laboratory, Carnegie Mellon...
University, University of Florida, the French Participation Group, the German Participation Group, Harvard University, the Instituto de Astrofísica de Canarias, the Michigan State/Notre Dame/JINA Participation Group, Johns Hopkins University, Lawrence Berkeley National Laboratory, Max Planck Institute for Astrophysics, Max Planck Institute for Extraterrestrial Physics, New Mexico State University, New York University, Ohio State University, Pennsylvania State University, University of Portsmouth, Princeton University, the Spanish Participation Group, University of Tokyo, University of Utah, Vanderbilt University, University of Virginia, University of Washington, and Yale University. This research has made use of the NED which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Some of the results in this paper have been derived using the HEALPix package (Górski et al. 2005).

REFERENCES

Covey K. R. et al., 2007, AJ, 134, 2398
Ita Y. et al., 2008, PASJ, 60, 435
Ivezić Ž. et al., 2007, AJ, 134, 973
Schneider D. P. et al., 2010, AJ, 139, 2360
Vijh U. P. et al., 2009, AJ, 137, 3139
Wright E. L. et al., 2010, AJ, 140, 1868

APPENDIX A: SELECTION CRITERIA FOR AGB STARS

In order to develop selection criteria for AGB stars, we follow Tu & Wang (2013) and analyse the WISE colour distribution of known AGB stars listed in the SIMBAD data base. Aided by their results, our procedure is simpler. They first selected about 76 million sources from the full WISE All-Sky source catalogue with detections in W1, W2 and W3 bands (and free of contamination and confusion flags in all four bands) that have a unique 2MASS source within 3 arcsec. Then they positionally matched this sample to objects listed in the SIMBAD data base within 2 arcsec (except in the case of high proper-motion stars which were matched within 4 arcsec), and further restricted the sample to 20° from the Galactic plane, yielding a sample of 3.2 million sources. This sample includes AGB stars, as well as all other sources known to SIMBAD and detected by WISE, which satisfy the selection criteria.

We begin by obtaining a sample of stars from SIMBAD whose main identifier was consistent with AGB stars (AGB star, C star, S star, OH/IR star, or Mira-type variable), resulting in 32 556 objects. The positions of these stars were matched to the positions of objects in the WISE All-Sky source catalogue within 2 arcsec, rejecting sources with more than one association. The resulting sample of 24 942 stars is further reduced by rejecting all sources with contamination and confusion flags, as well as removing sources outside of saturation limits (bands W1 through W3) and 5σ faint limits (see Section 1), yielding the final sample of 6171 objects with...
presumably both reliable \textit{WISE} photometry and reliable SIMBAD classifications.

We did not enforce any cut on Galactic coordinates. The distribution of this sample in representative CC and CM diagrams constructed with 2MASS and \textit{WISE} photometry is shown in Fig. A1. As known from previous work, most AGB stars have 2MASS colour $J - K_s > 1$. Most of them also show clear \textit{WISE} colour excess, $W1 > 0$ and $W2 > 0$, as well as $K_s - W3 > 0$. The two magnitude limits discernible from the source distribution in the top right panel are due to adopted faint limit $W3 < 11.32$ and saturation limit $W1 > 8.1$.

The latter is responsible for a significant difference between the data distribution in our $W2 - W3$ versus $W1 - W2$ diagram (the bottom left panel) and the diagram shown in fig. 4 from Tu & Wang (2013). Their version displays C stars below the dashed line in our bottom left panel, and OH/IR stars above the dashed line. Based on this separation, they proposed the dashed line as a separator between OH/IR and C-rich AGB stars. However, in our version, all the stars below the line are absent, as well as all OH/IR stars, and the region above the line is dominated by C stars, and not by OH/IR stars! It turns out that these differences are caused by rejecting stars brighter than saturation limits — when we relax this criterion, we obtain a data distribution reminiscent of fig. 4 from Tu & Wang (2013). This is shown in Fig. A2, similar to Fig. A1, but now includes formally saturated stars. All saturated stars, with $W1$<8.1 and $W2$<6.7, are shown with red dots, while the black dots show unsaturated sources. Notice in the lower left panel that almost all saturated stars fall below the dashed line that Tu & Wang (2013) have devised to separate C-rich from O-rich AGB stars. They did not mention saturation limits in their work. We believe that sub-standard \textit{WISE} photometry for saturated AGB stars has induced unreliable source distribution in fig. 4 from Tu & Wang (2013). The $W2$ band is most likely to blame, as evidenced by fig. 8 (in section VI.3.c.i.4) of the \textit{WISE} Explanatory Supplement (see footnote 1), where the bias for saturated sources is most pronounced in $W2$.

With saturation limits enforced, the distribution of C stars, by and large from LMC and SMC, clearly stands out in Fig. A1 (see the bottom right panel). We define the 3D colour space $x = W1 - W2$, $y = W2 - W3$, $z = W3 - W4$. The box that outlines a high concentration of C-rich AGB stars is defined by

\begin{align}
1.59x - 0.57 < y < 1.59x + 0.32 & \quad (A1) \\
-0.63x + 1.20 < y < -0.63x + 5.07 & \quad (A2)
\end{align}

In the $W2 - W3$ versus $W1 - W2$ diagram shown in the bottom left panel of Fig. A1, O-rich stars with silicate dust are seen as a low-density cloud towards the left from the C-rich star box. Further insight in the distribution of O-rich and C-rich AGB stars in \textit{WISE} CC diagrams is based on complete \textit{WISE} samples and is discussed in Section 3.3.
Based on this analysis our samples of AGB stars for Fig. 8 are selected as follows. The initial criteria are:

- \[ \frac{W_1}{W_2} < 16.83/15.60/11.32/8.0 \text{ } 5\sigma \text{-limits} \]
- \[ W_1 < 8.1, W_2 < 6.7 \text{ only bright sources} \]
- \[ \text{SNR1} \& 2 > 5, \text{SNR3} \& 4 > 10 \text{ signal-to-noise ratio}. \]

The regions with \(|b| < 6^\circ\) were also excluded because YSOs tend to cluster in the Galactic plane (Ivezic & Elitzur 2000), and also because of the uncontrolled/unknown levels of Galactic extinction at these latitudes. In the \(xyz\) colour space defined above we also require \( x > 0.2, 0 < y < 4 \), excluding the long narrow plume of stars with constant-density dust shells. This so-selected initial AGB sample consists of 8835 sources.

To select O-rich AGB stars from this initial sample we used the CC diagrams 3–5 in Fig. 8 as a guideline. We defined a wedge-shaped volume bounded by these criteria:

- \( 0.2 < x < 2 \)
- \( 0.4 < y < 2.2 \)
- \( 0 < z < 1.3 \)
- \( z > 0.722y - 0.289 \)

The selected O-rich star sample has 1608 members.

The C-rich stars, visible as a strongly elongated plume in Fig. 8, were selected by carving out a 3D elongated volume around the plume. It is the intersection of the following two slabs:

\[
0.629y - 0.198 < x < 0.629y + 0.359 \quad \text{(A7)}
\]

\[
0.722y - 0.911 < z < 0.722y - 0.289. \quad \text{(A8)}
\]

Note that (A7) is the inverse of (A1), and that we abandon (A2). The C-rich star sample contains 1383 sources.

For the C:O-rich star ratio map in Fig. 10 we have relaxed the \(|b| < 6^\circ\) requirement, and have selected C-rich and O-rich star candidates through the CC volumes and magnitudes defined above. The preselected sample contains 90 114 sources, of which 14 969 end up in the O-rich star group. The size of the C-rich star selection grows to 1629 objects.

**APPENDIX B: LMC COORDINATE TRANSFORMATIONS**

To obtain a de-projected map of the LMC disc, for our analysis of the radial gradient of the C:O-rich star ratio in Section 3.3.3, we follow the procedure outlined in two papers on the structure of the Magellanic Clouds, MC1 and MC2. We begin with the right ascension and declination angles \( \alpha \) and \( \delta \) of our colour and magnitude selected C-rich and O-rich AGB stars (given in degrees or radians).

We further assume as the origin of this equatorial coordinate system \((\alpha_0, \delta_0) = (82.25, -69.5)\), as defined in Section 4.1 in MC1. This origin corresponds to the centre of the density iso-contours of the DENIS survey, but is slightly offset w.r.t. the standard locus of the LMC centre \([\text{e.g. } (\alpha_0, \delta_0) = (80.9, -69.76)]\) given in the NED database.
data base). From equation (1) of MC1 we obtain the formula for transformed angular coordinate \( \rho \):

\[
\rho = \arccos(\cos \delta \cos \delta_0 \cos(\alpha - \alpha_0) + \sin \delta \sin \delta_0).
\]

Merging equations (2) and (3) from the same paper we get the other transformed coordinate \( \phi \):

\[
\phi = \arctan \left( \frac{\sin \delta \cos \delta_0 - \cos \delta \sin(\alpha - \alpha_0)}{-\cos \delta \sin(\alpha - \alpha_0)} \right).
\]

MC2 then defines in equation (2) the de-projected coordinates \( x' \) and \( y' \) as linear distances (in kpc) from the LMC centre, in the plane of its disc

\[
x' = \frac{D_0 \cos i \sin \rho \cos(\phi - \Theta_{\text{tar}})}{\cos i \cos \rho - \sin i \sin \rho \sin(\phi - \Theta_{\text{tar}})},
\]

\[
y' = \frac{D_0 \sin \rho \sin(\phi - \Theta_{\text{tar}})}{\cos i \cos \rho - \sin i \sin \rho \sin(\phi - \Theta_{\text{tar}})},
\]

where \( D_0 \) is the distance from Earth to the LMC, 51 kpc, and \( i = 34.7^\circ \) is the inclination of the LMC disc plane with respect to our line of sight, \( \Theta_{\text{tar}} = \Theta + 90^\circ \), where \( \Theta = 122.5^\circ \) is the position angle (PA) of the ‘line of nodes’, the intersection of the LMC disc plane and the plane of the sky. See the discussion in MC2 as to why \( \Theta \) differs from the PA of the LMC semi-major axis.

MC2 finally rotates the \((x',y')\) coordinates by the angle \( \Theta_{\text{tar}} \) to align them with another set of coordinates in their paper. For our work, this step is not necessary, but we execute it to obtain a map oriented identically to MC2

\[
x'' = x' \cos \Theta_{\text{tar}} - y' \sin \Theta_{\text{tar}},
\]

\[
y'' = x' \sin \Theta_{\text{tar}} + y' \cos \Theta_{\text{tar}}.
\]

These are the coordinates we plot in the lower left panel of our Fig. 11. We select all C-rich and O-rich stars that fall within a \( \sqrt{x''^2 + y''^2} \leq 10\text{kpc} \) circle. Their counts are, respectively, \( n_\text{C} = 1114 \) and \( n_\text{O} = 252 \).

**APPENDIX C: TABULATION OF MODEL COLOUR TRACKS AND FILTER FLUXES**

Equation (1) averages the observed SED of a source or model with the instrument’s total transmission curves, and calibrates the in-band fluxes to an empirical fit for Vega (Wright et al. 2010). Recall that the WISE colour between two bands 1 and 2 is defined as

\[
W1 - W2 = 2.5 \log \frac{F_2}{F_1} = 2.5 \left( \log \frac{F_2^{\text{IRC}}}{F_1^{\text{IRC}}} + \log \frac{F_2^{\text{Vega}}}{F_1^{\text{Vega}}} \right),
\]

\[
= (W1 - W2)^{\text{IRC}} - (W1 - W2)^{\text{Vega}}.
\]

To compute colours, the absolute scale of the SED is irrelevant, because all scales cancel out. However, to allow luminosity-dependent studies using our models, we list in Table C1, together with the Vega-calibrated WISE colours, the unnormalized in-band fluxes \( F_i^{\text{IRC}} \) of the DUSTY models used in Figs 2, 14–17, and 19. The pure Vega in-band fluxes, using the aforementioned empirical fit, are constants. Their values are: \( F_1^{\text{Vega}} = 5.576 \times 10^{-13}, 2.956 \times 10^{-13}, 7.289 \times 10^{-14}, 4.360 \times 10^{-15} \) for bands \( i = 1–4 \), in the same units as the models. The DUSTY model SEDs are normalized with the bolometric flux, thus for comparison with observational data the reported \( F_i^{\text{IRC}} \) values must be multiplied with the true bolometric flux of the source.

The entire tabulations are available as an electronic supplement file model_tables.dat to this paper. All physical properties of the models are listed in the header of each table, and are tabulated in the electronic file. They are: the shell radial optical depth at visual (0.55 \( \mu \text{m} \)), the dust chemistry, grain size distribution, the photospheric temperature \( T_c \) of the central star, the radial density distribution (either a power-law \( r^{-\beta} \), or RDW), and the dust temperature \( T_d \) at the inner radius of the shell. All models have radial shell thickness \( Y = R_{\text{out}}/R_{\text{in}} = 100 \).

**SUPPORTING INFORMATION**

Additional Supporting Information may be found in the online version of this article:

**Table C1.** Tabulation (excerpt) of WISE colours and filter-averaged fluxes \( F_i^{\text{IRC}} \) for all DUSTY shell model tracks shown in Figs 2, 14, 15, 16, 17, and 19, as a function of the shell radial optical depth at visual (left column). The entire tabulations are available as an electronic supplement to this paper (model_tables.dat).

<table>
<thead>
<tr>
<th>( \tau_V )</th>
<th>( \tau_{W1} )</th>
<th>( \tau_{W2} )</th>
<th>( \tau_{W3} )</th>
<th>( \tau_{W4} )</th>
<th>( F_{1}^{\text{IRC}} )</th>
<th>( F_{2}^{\text{IRC}} )</th>
<th>( F_{3}^{\text{IRC}} )</th>
<th>( F_{4}^{\text{IRC}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td># Fig. = 12, ( T_d = 14454 \text{ K, dust = ISM, grain_size = MRN, } T_d = 600 \text{ K, density_law = 1/r^{0.0}} )</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>0.000e+0</td>
<td>5.780e−3</td>
<td>1.541e−2</td>
<td>1.303e−2</td>
<td>1.571e−2</td>
<td>1.199e−2</td>
<td>2.102e−3</td>
<td>1.114e−3</td>
<td>2.640e−4</td>
</tr>
<tr>
<td>1.000e−4</td>
<td>6.098e−3</td>
<td>2.228e−1</td>
<td>1.769e+0</td>
<td>2.102e−3</td>
<td>1.114e−3</td>
<td>3.196e−4</td>
<td>1.085e−4</td>
<td></td>
</tr>
</tbody>
</table>

This paper has been typeset from a \TeX/X\LaTeX file prepared by the author.