EQUIVALENT BARGE AND FLOTILLA IMPACT FORCES ON BRIDGE PIERS
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Equivalent Barge And Flotilla Impact Forces On Bridge Piers

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# Equivalent Barge And Flotilla Impact Forces On Bridge Piers

**Abstract**

Bridge piers located in navigable inland waterways are designed to resist impact forces from barges and flotillas in addition to other design considerations (e.g., scour, dead and live loads, etc.). The primary design tool for estimating these forces is the AASHTO Guide Specification which provides a simple hand calculation method for determining an “equivalent impact force”. The simplicity comes at a cost of excluding the effect of the pier shape, impact duration, and interaction between barges in a flotilla. The objective of this report is to present a hand calculation method for determining barge or flotilla equivalent static impact forces on bridge piers. The primary advantage of this approach lies in its incorporation of pier geometry, interaction between barges, and impact duration. The proposed method is derived from the conduct of hundreds of finite element dynamic simulations of barges and various flotilla configurations impacting rigid and flexible rectangular and circular (or rounded end) bridge piers at different velocities. Results are presented and compared with ones derived from the AASHTO method and detailed finite element modeling. The results generated by the proposed method compare very well with ones derived from the FE modeling, while the AASHTO results are up to twice as large as one from the proposed method for the examples presented in this report.

**Key Words**

Equivalent impact load, Finite element method, Simulation, Barge, Flotilla, Bridge, Pier, Time history, Hand calculations

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**Supplementary Notes**

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NOTE: This report is the first (1st) in a series of two (2) reports for Project SPR 261: “Multi-Barge Flotillas Impact Forces on Bridges”. The two (2) reports are:

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1 INTRODUCTION

Barge and flotilla impact forces are important design considerations for bridges spanning navigable inland waterways. Approximately 26000 dry cargo barges, 3000 tanker barges, and 1200 towboats operate today on 40234 km (25000 miles) of inland waterways in the United States (CARIA 2005). A typical barge tow or flotilla navigating the Ohio River in Kentucky, consisting of one tow boat and fifteen attached barges, has a 20400 metric tons (45 x 10^6 lb) or 800000 bushel capacity, the equivalent of 225 train cars or 870 semi trucks. A single 366 m (1200 ft) long 15-barge tow carries as much coal or grain as 4.4 km (2.73 miles) of trains or 55.5 km (34.5 miles) of semi-trucks. It is much more energy efficient to move cargo through water than over land. On average, a gallon of fuel allows one ton of cargo to be shipped 95 km (59 miles) by truck, 325 km (202 miles) by rail, and 827 km (514 miles) by barge (USDOT 2005). Due to these advantages, barge traffic is expected to increase by 150 percent in the next 50 years (Carter 1999). The increased frequency and the serious consequences of a barge or flotilla colliding with a bridge pier necessitate the development of new methods for accurately determining the impact forces.

In order to evaluate or design bridges for barge or flotilla impact forces, three categories of information need to be taken into account: (1) bridge structure, (2) barge/flotilla, and (3) river. In respect to the bridge structure, the relevant information includes the pier geometry, rigidity, mass, and connections with the superstructure and footing. As to the barge or flotilla, its profile such as barge type, tonnage, layout, and traveling speed, should be considered based on navigation statistics and design criteria of the bridge. Regarding the river, the water level and velocity are among the most important.
The highest and lowest historical water levels suggest a most adverse impact position on the pier, and water velocity increases or decreases the barge impact energy.

The inland-water cargo movement is by means of flotillas, in which a number of barges are tied together and moved as one unit (Fig.1). The standardized Jumbo Hopper, 10.668 m (35 ft) wide and 61 m (200 ft) long, is the most widely used barge type in the U.S. for inland waterway barge operations (Whitney and Harik 1997). The number of barges in a flotilla is limited by the navigable channel width and the dimensions of the lock chambers along the flotilla’s route. The U.S. Army Corps of Engineers operates 275 lock chambers, which are generally 33.5 m (110 ft) wide, and either 182.9 m (600 ft) or 365.8 m (1200 ft) long (CARIA 2005). The most typical tow size through the locks on the Ohio River is three barges wide and five barges long. The smaller tributaries, such as the Alabama River, contain locks that are 25.6 m (84 ft) wide and 182.9 m (600 ft) long, which can support tows of two-barge width and length.

For inland waterways, including the Mississippi River, Ohio River, and those within the state of Kentucky, maximum legal velocities are not enforced for barge and flotilla traffic. Maximum velocity enforcement is not an issue since, in addition to maneuverability, maximum flotilla speed is determined relative to optimal fuel usage, which occurs at approximately 5 mi/h (2.24 m/s, 8.05 km/h, 4.34 knots) upstream and 5.5 mi/h (2.46 m/s, 8.85 km/h, 4.78 knots) downstream for common flotilla configurations. These speeds are uniform for all barge flotillas traversing Kentucky waterways. In extremely rare instances, flotilla speeds will increase between 7 mi/h (3.13 m/s, 11.27 km/h, 6.08 knots) upstream and 10 mi/h (4.47 m/s, 16.09 km/h, 8.69 knots) downstream. However, it should be noted that these speeds are the absolute upper limits of flotilla...
velocities and are rarely reached, if at all. In general, the bigger and heavier the flotilla is, the slower its speed.

1.1 AASHTO Guide Specification

At present, design specifications used both nationally and internationally employ empirical equations as part of a codified procedure for determining the equivalent static design loads due to vessel impacts. Current bridge design practices in the U.S. follow the American Association of State Highway and Transportation Officials (AASHTO) Guide Specification and Commentary on Vessel Collision Design of Highway bridges (1991). The guide presents the following formulas to determine the barge damage depth, $a_B$, and impact force, $P_B$:

$$a_B = 3.1 \left( \sqrt{1 + 0.13E_k} - 1 \right) \text{ (m)} \quad (1)$$

$$P_B = \begin{cases} 
60a_s, & a_s < 0.1 \text{ m} \\
6 + 1.6a_s, & 0.1 \text{ m} \leq a_s
\end{cases} \text{ (MN)} \quad (2)$$

in which

$$E_k = \frac{1}{2} m_B V_i^2 \text{ (MJ)} \quad (3)$$

$E_k$ is the barge/flotilla initial kinetic energy at impact (Note: 1 MJ = 0.738x10⁶ ft-lbf), $V_i$ is the barge/flotilla initial velocity, including the river flow velocity, at impact in m/sec, and $m_B$ is the barge/flotilla mass in Mkg (1 Mkg = 2.205 10⁶ lbf), including the mass of the tow boat.
1.2 Recent Work on Flotilla Impact on Bridges

The AASHTO method is based on the equivalent static load method proposed by Meir-Dornberg (1983), and does not account for factors that affect the magnitude of impact forces and the bridge dynamic response. These factors include impact duration, pier geometry, barge-barge and barge-pier interactions, and structural characteristics of the bridge. Recent research efforts dealing with barge and flotilla impact forces have been carried out (Whitney and Harik 1997; Consolazio et al 2003, Modjeski and Masters 1985, Yuan et al 2005 and 2008) and are a first step towards better understanding of the problem. Many questions remain to be answered about barge/flotilla-bridge collisions (e.g., effect of pier shape and stiffness, connectivity between barges in a flotilla, etc.). Since inland-waterway cargo movement is primarily by means of flotillas, the study of impact forces generated by the flotillas is very significant. However, very little work has been conducted on flotilla-bridge collision problems. The analysis of barge/flotilla-bridge collisions can be carried out using the finite element (FE) method. However, FE simulations are very expensive regarding both model generation and computation time.

1.3 Objective of This Study

The objective of this study is to derive methods that are both rational with respect to the mechanics of the impact problem analysis and simple with respect to the prediction of the loads generated by barges or flotillas impacting bridge piers. In this report, two analysis methods, the time-history and equivalent static impact force methods are discussed, with emphasis on the later. These methods account for pier geometry and interaction between barges, and between the flotilla and bridge structure. The primary
advantage of this approach lies in its simplicity and familiarity to design engineers. The proposed equivalent impact force method transforms the complex dynamics of a barge or flotilla pier impact phenomena into a simple problem that can be solved through hand calculations.

2 SINGLE BARGE IMPACT SIMULATION

The study of a single barge collision with a bridge pier provides valuable insight leading to a better understanding of the more general problem of multi-barge flotilla and bridge collisions. During a barge-bridge collision event, a major part of the kinetic impact energy is dissipated through the deformation of the barge bow in contact with the pier. The impact force is tantamount to the crushing resistance of the bow structure. In general, the collision problem brought about by multi-barge flotillas is merely an extension of the single-barge collision.

In this study, the development of the Jumbo Hopper (JH) model is based on the blueprints and specifications provided by Jeffboat LLC, a barge manufacturer. The material and element descriptions for the generation of the finite element (FE) model of a JH and a pier (Fig. 2a) are presented by Yuan et al (2008). The model is applicable to a variety of FE simulation scenarios for single and multi-barge flotillas. The time dependant impact loads are generated by conducting dynamic simulations using the program LS-DYNA (2003). Figs. 2b and 2c show the crushed barge bow impacting a rectangular and circular or rounded end (hereinafter circular) pier, respectively. Fig. 3 presents the time history of the impact force and damage depth for a barge impacting rectangular and circular piers, of different widths or diameters, at a velocity of 4 knots (2.06 m/s, 7.41
km/h, 4.60 mi/h). Fig. 3a clearly shows that the rectangular pier shape greatly affects the barge-pier impact process. A wider rectangular pier produces a larger impact force, shorter time duration, and smaller barge damage distance. This is due to the fact that the contact force between the barge and pier is roughly proportional to the rectangular pier width, $b_c$, to barge width, $B$, ratio, $\alpha = b_c/B$, and the deformation in the barge absorbs energy that is closely related to the volume of the deformed steel in the crushed bow region. For a JH, $B = 10.668$ m (35 ft). Fig. 3b shows that the circular pier diameter, $D$, to barge width ratio ($\alpha = D/B$) does affect the impact force and barge crushing distance of circular piers. This is in contrast to rectangular piers where, for the same $\alpha$, the maximum impact force on a circular pier is much less than that on a rectangular pier. This is due to the gradually increasing contact area between the lead barge and circular pier during the impact process. It should be noted that the barge-impact forces derived from the AASHTO equations are independent of pier geometry. However, it should also be noted that, bridge piers in navigable inland waterways generally have rounded ends.

2.1 Barge Bow Stiffness

The barge bow stiffness relates the crushing distance of the bow and impact forces. The dissipated energy during a barge-bridge collision can be obtained using the damage depth of the bow and the impact force. More than one hundred three dimensional dynamic FE simulations were conducted on various pier geometries, barge mass, and initial velocities, to derive the following regression formulas for the barge bow stiffness, $k_{cr}$, (Yuan et al 2008):
\[
k_{cr} = 4.6 + 646.5\alpha - 270.0\alpha^2 \text{ (MN/m)} \quad \text{for rectangular piers} \quad (4a)
\]
\[
\alpha = \frac{b_c}{B} \leq 1.0 \quad \text{for rectangular piers} \quad (5a)
\]
\[
k_{cr} = 47.3 + 45.3\alpha - 16.0\alpha^2 \text{ (MN/m)} \quad \text{for circular piers} \quad (4b)
\]
\[
\alpha = \frac{D}{B} \leq 1.0 \quad \text{for circular piers} \quad (5b)
\]

in which, \(b_c\) is the width of the impacted face of the rectangular pier, \(D\) is the diameter of the circular pier, and \(B\) is the width of the impacting barge (Fig. 2).

### 2.2 Multi-Barge Impact Simulation

A rake barge, built with one end sloped or raked at a sharp angle to form a bow (Fig. 2) moves easier through water as compared to square-ended or box hopper barges. Rake barges are used primarily as lead barges in a flotilla and are also placed in the back of flotillas to permit towboat pilots to slow and turn the tow more quickly. Barge tows (flotillas) often include a mixture of both kinds of barges. This configuration takes advantage of both the storage capacity of box hopper barges and the fuel efficiency of raked hopper barges. Although barge flotillas are not entirely composed of one barge size or type, the vast majority of barges in a given flotilla generally consist of mostly the same barge size and type.

The connection between the individual barges in a flotilla is comprised of steel wire ropes. In this study, the FE models of flotillas, an extension of the single barge Jumbo Hopper model, are comprised of single jumbo hopper and box hopper barge models that are tied together using cable elements (Yuan et al 2008). The program LS-DYNA (2003)
was used to conduct crash simulations of multi-barge flotillas impinging perpendicularly upon a series of rectangular and circular piers.

Fig. 4 compares the impact force time histories generated by a single column in a flotilla impacting a rectangular pier of having an $\alpha = b/c/B = 0.1$ at an initial velocity of 4 knots (2.06 m/s, 7.41 km/h, 4.60 mi/h). The number of barges in the column is varied from a single barge (FL1) to five barges (FL5). The maximum impact force, which occurs immediately following the initial impact ($t < 0.1$ sec), varies with the number of barges in a column but its duration is very short. Following the maximum force, the force time history for the different flotilla columns follow a similar path, with the exception of the duration of the impact which increase with the number of barges in the column. The lead barge in a column, which is constrained by the barges behind it in a multi-barge column, provides the primary resistance to the impact by crushing. Consequently, the impact forces on piers are mostly dependent on the strength of the barge bow structure. This is the primary reason that impact forces do not increase proportionally with an increasing number of barges in a flotilla column. Similar to a non-hardening plastic spring, the bow absorbs impact energy through deformation while its resisting force remains at a relatively constant level. Fig. 4 also presents the AASHTO equivalent static loads for the different flotillas. It can be seen that, in this case, the AASHTO method overestimates the impact force for multi-barge flotillas. The maximum impact force, which has a short duration, is equivalent to that in AASHTO but the average forces are smaller. This is primarily due to the fact that pier geometry and interaction between barges in a flotilla are disregarded in the AASHTO method.
3 METHODS OF ANALYSIS FOR BARGE AND FLOTILLA IMPACT

Similar to the tools available for engineers dealing with seismic analysis of bridges, two methods are presented herein for the analysis of barges or flotillas impacting bridge piers: (1) The time history and (2) the equivalent impact force methods. Detailed derivations of the equations in the following sections are presented by Yuan et al (2008). Yuan et al also presented a response spectrum method that permits the determination, through hand calculations, of the displacements at the top of the pier. The displacements permit the designer to provide proper seat width for the superstructure bearings in case of roller or free support, or the shear forces in case of fixed bearings.

3.1 Time History Method

Yuan et al (2008) introduced an elastoplastic spring-mass model for the analysis of multi-barge flotillas colliding head on with bridge piers. The model accounts for the essential factors pertaining to flotillas impacting bridge piers, such as pier geometry and stiffness, dynamic interaction between barges, and dynamic interaction between the lead barge and bridge pier. The proposed model generates impact force time-histories for a multitude of flotilla configurations in a matter of minutes as compared to 40+ hours for a 3-D FE model of a 5-barge column time history derived from ANSYS (2004) using a Pentium 4 personal computer (3.0 GHz, and 1022 MB of RAM). This is especially valuable in probabilistic analysis requiring many collision simulations. The results from this study are compatible with the respective impact time-histories produced by exhaustive finite element simulations. Once the impact force time history is derived, dynamic
analysis of any bridge can be conducted. Details of the model derivation along with application of the time history method are presented by Yuan et al (2008).

3.2 Method of Equivalent Impact Force, $P_B$

Numerous attempts were made to generate a compact expression for $P_B$, similar to the one in AASHTO (Eq. 1). However, they resulted in a set of lengthy equations that depend on the magnitude of the kinetic energy, $E_k$, barge damage depth, $a_B$, pier shape, and pier width or diameter to barge width ratio, $\alpha = b/c/B$ or $\alpha = D/B$. In order to simplify the process and render it amenable to hand calculations, two categories were defined: Impacts with small kinetic energies ($E_k < 10$ MJ) and impacts with large kinetic energies ($10$ MJ < $E_k$). Within each category, distinction is made between rectangular and circular (or rounded end) piers.

3.2.1 Impacts Resulting in Small Kinetic Energies, $E_k \leq 10$ MJ

A set of regression formulas (Yuan et al 2008), derived from extensive FE simulations, were solved to generate the relationship between the equivalent impact force, $P_B$, and the barge damage depth, $a_B$, in terms of the kinetic energy, $E_k$, and pier width to barge width ratio, $\alpha$. The results are presented in Fig. 5 and are applicable to barges and flotillas having any initial velocity $V_i$ as long as the kinetic energy $E_k \leq 10$ MJ.

3.2.2 Impacts Resulting in Large Kinetic Energies, $10$ MJ < $E_k$ and Initial Velocities $V_i \leq 5$ knots

In order to incorporate the size and shape of piers into the derivation of the equivalent static impact force, the following set of regression equations for $P_B$, in terms of the barge damage depth, $a_B$, barge bow stiffness $k_{cr}$, and the pier to barge width ratio, $\alpha$, are derived

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from extensive FE simulations (Yuan et al 2008) for initial velocities $V_i \leq 5$ knots (2.57 m/s, 9.26 km/h, 5.75 mi/h):

$$a_s = 29\left(\frac{E_k}{k_{cr}}\right) - 35\left(\frac{E_k}{k_{cr}}\right)^2 \quad \text{for rectangular piers} \quad (6a)$$

$$P_B = 10^{-3} k_{cr} \left(22\sqrt{a_s} + \frac{110}{e^{1.68a_s}}\right) \quad \text{(MN)} \quad \text{for rectangular piers} \quad (7a)$$

$$a_s = 15\left(\frac{E_k}{k_{cr}}\right) - 11\left(\frac{E_k}{k_{cr}}\right)^2 \quad \text{for circular piers} \quad (6b)$$

$$P_B = 10^{-3} k_{cr} \left(19a_s + \frac{77}{e^{0.35a_s}}\right) \quad \text{(MN)} \quad \text{for circular piers} \quad (7b)$$

4 INFLUENCE OF PIER GEOMETRY ON THE EQUIVALENT IMPACT FORCE

The equivalent static impact force, $P_B$, versus the damage depth, $a_s$, is plotted in Fig. 6 for different pier width or diameter to barge width ratio, $\alpha$. Fig. 6 clearly shows the effect of the pier geometry and the width or diameter of the impacted pier face, on the impact force, especially for rectangular columns. The AASHTO method underestimates the magnitude of the impact force for the majority of rectangular piers (Fig. 6a) and overestimates the impact force for circular piers (Fig. 6b) when the damage depth is greater than 0.5 m (1.64 ft). Yuan et al (2008) has shown that the impact force is not always proportional to the initial kinetic energy. The intensity of a collision is dependent on both the magnitude of the impact force and the impact duration. Unlike the impact duration, the impact force in an elastic collision is often much larger than that in a plastic collision. When the kinetic
energy of the barge/flotilla exceeds the maximum elastic strain energy that can be absorbed by the lead barge bow structure, the maximum impact forces decrease quickly as the pier entry deepens. The pier geometry’s influence on the impact forces is more significant for rectangular piers than for circular piers.

5 IMPACT DURATION

According to the impulse-momentum law, the time duration of impact, \( t_d \), is approximated as follows (Yuan et al 2008):

\[
t_d = \frac{1 + e_B m_B V_i}{P_s} \quad \text{(sec)}
\]

(8)

where \( m_B V_i \) is the initial momentum of the barge/Flotilla in N.sec, and \( e_B \) is the coefficient of restitution.

The coefficient of restitution \( (e_B) \) provides an indication of the elasticity of the barge/flotilla and bridge collision. Elasticity is a measure of how much of the initial kinetic energy, of the colliding objects (vessel and bridge), remains after the collision. For a perfectly elastic collision, \( e_B = 1.0 \), and for a perfectly inelastic collision, \( e_B = 0 \). More than one hundred three dimensional dynamic FE simulations were conducted on various pier geometries, flotillas, and initial velocities, to derive the following regression formulas to represent the coefficient of restitution (Yuan et al 2008):

\[
e_B = 0.01 \times \left| 28 + 4\alpha - (8 + 4\alpha)\ell nE_k \right| \quad \text{for } 0.1 \text{ MJ} < E_k \quad \text{for rectangular piers} \quad (9a)
\]

\[
e_B = 0.01 \times \left| 27 + 1\alpha - 5\ell nE_k \right| \quad \text{for } 0.1 \text{ MJ} < E_k \quad \text{for circular piers} \quad (9b)
\]

Impacts with kinetics energies \( E_k \leq 0.1 \text{ MJ} \) are not significant and the resulting impact forces on bridge piers are negligible.
6 LIMITATIONS OF THE EQUIVALENT IMPACT FORCE METHOD

The derivation of the equivalent impact force method in this study is limited to following:

1- Pier width or diameter to barge width ratios in the following range: $0.05 < \alpha \left( \frac{b}{B} \text{ or } \frac{D}{B} \right) < 1.0$. Very small or very large ratios, $\alpha < 0.05$ or $\alpha > 1.0$, respectively, are rare for bridges in navigable waterways.

2- For barges or flotillas with impacts resulting in large kinetic energies ($10 \text{ MJ} < E_k$), the maximum initial velocities are limited to $V_i \leq 5$ knots (2.57 m/s, 9.26 km/h, 5.75 mi/h).

3- Flotillas made up of a maximum of 3-columns by 5-rows, and a total of 15 barges excluding the tow boat.

4- Relatively stiff piers where $k_p > 0.1 k_{cr}$, in which $k_p$ is the pier stiffness expressed in N/m and $k_{cr}$ is defined in Eq. (4). $k_p$ can be determined by considering the span(s) of the superstructure that join at the pier cap, applying a force $F$ of any reasonable magnitude to the structural model at the point of impact on the pier, and determining the corresponding displacement, $\Delta$, at the same point (Fig. 7). The stiffness $k_p = F/\Delta$. The case when $k_p < 0.1 k_{cr}$ is rare for bridges in navigable waterways.

5- Flotilla impacting bridge piers at zero angle of attack (i.e. head on or normal to the axis of the pier parallel the longitudinal direction of the bridge).

During the initial design stage, it can be assumed that all these limitations are satisfied. Validation can be carried out in the final design stage.
7 APPLICATION

The solution process for determining the equivalent impact force of a barge or flotilla is illustrated in the following three examples: (1) a single barge flotilla, (2) single column 3-barge flotilla, and (3) 3-columns by 5 rows, 15-barge flotilla.

7.1 Single Barge Flotilla

A fully loaded jumbo hopper barge with a mass $m_B = 1.724 \text{ Mkg (3.8 x 10}^6 \text{ lb)}$, including the mass of the tow boat, and an initial velocity $V_i = 3.09 \text{ m/s (6 knots, 11.12 km/h, 6.91 mi/h)}$, including the river flow velocity, collides head-on with a bridge pier having an impacted face width $b_c = 2.134 \text{ m (7.0 ft)}$ for a rectangular pier or a diameter $D = 2.134 \text{ m}$ for a circular or rounded end pier. The pier height is $L = 24.55 \text{ m (80.55 ft)}$, and the barge collides with the pier at a distance of $11.34 \text{ m (37.20 ft)}$ from the base of the pier. The pier stiffness, $k_p$, was determined to be $43.5 \text{ MN/m (2.98 x 10}^6 \text{ lbf/ft)}$. Refer to comments in the previous section for the determination of $k_p$.

Table 1 shows the process for the derivation of the equivalent impact force $P_B = 5.5 \text{ MN (1.24 x 10}^6 \text{ lbf)}$ and impact duration $t_d = 1.07 \text{ sec}$ for the rectangular pier, and $P_B = 4.3 \text{ MN (0.97 x 10}^6 \text{ lbf)}$ and $t_d = 1.45 \text{ sec}$ for the circular pier.

Fig. 8 compares the results from the proposed equivalent load method, the AASHTO method, and the impact force time-history derived from the FE simulation (LS-DYNA) for the rectangular pier. The proposed method compares very well with the average force derived from the finite element analysis (Fig. 8). The equivalent impact force derived from the AASHTO method [$P_B = 8.18 \text{ MN (1.84 x 10}^6 \text{ lbf)}$] is 48.7% larger than the force derived from the proposed method for the rectangular pier.
Although the LS-DYNA simulation provides detailed information that will better assist the bridge designer, it comes at a cost of time required for the solution. For this example, the proposed hand calculation method required 10-minutes to generate the results after the data for the bridge, river, and flotilla was defined. The FE simulation required 36 hours after the data was entered in the program on a Pentium 4 personal computer (3.0 GHz, 800 MHz, and 1022 MB of RAM).

7.2 Single Column 3-Barge Flotilla

A fully loaded single column 3-barge flotilla with a mass $m_B = 5.17$ Mkg ($11.4 \times 10^6$ lb), including the mass of the tow towboat, and an initial velocity $V_i = 2.57$ m/s (5 knots, 9.3 km/h, 5.7 mi/h), including the river flow speed, collides head-on with a circular bridge pier having a diameter of 3.2 m (10.5 ft). The bridge pier has an impacted face width $b_c = 3.2$ m (10.5 ft) for a rectangular pier or a diameter $D = 3.2$ m for a circular or rounded end pier. The pier stiffness $k_p$ was determined to be 75.5 MN/m ($5.17 \times 10^6$ lbf/ft).

Table 2 follows the same steps outlined in Table 1 for the derivation of the equivalent impact force $P_B = 6.36$ MN ($1.43 \times 10^6$ lbf) and impact duration $t_d = 2.15$ sec for a rectangular pier, and $P_B = 5.23$ MN ($1.18 \times 10^6$ lbf) and $t_d = 2.87$ sec for a circular pier.

Table 2 follows the same steps outlined in Table 1 for the derivation of the equivalent impact force $P_B = 6.36$ MN ($1.43 \times 10^6$ lbf) and impact duration $t_d = 2.15$ sec for a rectangular pier, and $P_B = 5.23$ MN ($1.18 \times 10^6$ lbf) and $t_d = 2.87$ sec for a circular pier.

Table 2 follows the same steps outlined in Table 1 for the derivation of the equivalent impact force $P_B = 6.36$ MN ($1.43 \times 10^6$ lbf) and impact duration $t_d = 2.15$ sec for a rectangular pier, and $P_B = 5.23$ MN ($1.18 \times 10^6$ lbf) and $t_d = 2.87$ sec for a circular pier.

Fig. 9 compares the results from the proposed equivalent load method, the AASHTO method, and the impact force time-history derived from the FE simulation (LS-DYNA 2003) for the circular or rounded end pier. The proposed method gives an equivalent impact force $P_B = 5.23$ MN ($1.18 \times 10^6$ lbf), which compares very well with the average force derived from the finite element analysis (Fig. 9). The equivalent impact
force derived from the AASHTO method \([P_B = 9.94 \text{ MN} (2.23 \times 10^6 \text{lbf})]\) is 90% larger than the force derived from the proposed method.

### 7.3 3-Columns by 5-Rows, 15-Barge Flotilla

A bridge pier, having an impacted face width \(b_c = 2.0 \text{ m} (6.56 \text{ ft})\) for a rectangular pier or a diameter \(D = 2.0 \text{ m}\) for a circular or rounded end pier, is impacted by a 15-barge flotilla traveling at an initial speed \(V_i = 1.54 \text{ m/s} (3.0 \text{ knots}, 5.54 \text{ km/h}, 3.44 \text{ mi/h})\). The flotilla has a total mass \(m_B = 25.8 \text{ Mkg} (56.9 \times 10^6 \text{ lb})\), including the mass of the tow boat. The pier stiffness \(k_p\) was determined to be 143 MN/m \((9.80 \times 10^6 \text{ lbf/ft})\).

The problem is solved by the proposed method and the results are presented in Table 3 and Fig. 10. The impact force time history presented in Fig. 10 is compared with the average impact force derived from the proposed method and the AASHTO method (1991). The proposed method gives an equivalent impact force \(P_B = 5.84 \text{ MN} (1.31 \times 10^6 \text{lbf})\) for a rectangular pier, which compares very well with the average force derived from the finite element analysis (Fig. 10). The equivalent impact force derived from the AASHTO method \([P_B = 12.11 \text{ MN} (2.72 \times 10^6 \text{lbf})]\) is 107% larger than the force derived from the proposed method. This is primarily due to the fact that the proposed method accounts for the interaction between the barges in the flotilla. Consequently, the energy dissipation is higher and the average impact force is smaller than the value derived using the AASHTO method (Yuan et al 2008).

It should be noted that the damage depth for the lead barge in the flotilla impacting the circular pier \([a_B = 4.93 \text{ m} (16.17 \text{ ft})]\) is smaller than the damage depth for the lead barge in the flotilla impacting the rectangular pier \([a_B = 5.21 \text{ m} (17.09 \text{ ft})]\). This is due to the fact
that, as the lead barge damage depth increases for flotillas impacting circular piers, the pier resistance increases leading to an $a_B$ that is smaller than that for impacts with rectangular piers.

### 8 CONCLUSIONS

A hand calculation method is presented in this paper for the determination of the equivalent static impact force resulting from a barge or flotilla impacting a bridge pier. Similar to the AASHTO method (1991), the proposed method renders a very complex dynamic problem to a simple static one that can be used for the evaluation or design of bridge piers in navigable waterways. Unlike the AASHTO method, the pier shape and stiffness, and the dynamic interaction between barges in a flotilla and bridge pier are accounted for in the proposed method.

The results from this study for the average impact force and impact duration compare very well with ones derived from detailed finite element dynamic analyses that take into account the thousands of elements in each barge and the cables connecting the barges in a flotilla. The results presented herein for the 15-barge flotilla have shown that the AASHTO impact force can be more than double the forces generated by the detailed finite element method or the proposed method.

In all the scenarios studied by Yuan et al (2005), using the finite element method, for flotillas impacting bridge piers, the stresses in the cables (or wire ropes) connecting the barges together were below the ultimate cables' stresses. Consequently, the separation between the columns in a flotilla during an impact was not observed.
REFERENCES


Table 1. Determination of the Equivalent Flotilla Impact Force for a Single Barge Flotilla Impacting a Rectangular and a Circular (or Rounded End) Pier

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bridge, Barge/Flotilla, and River/Waterway Information</strong></td>
<td></td>
</tr>
<tr>
<td>Bridge</td>
<td>( b_c = 2.134 \text{ m} ) ( k_p = 43.5 \text{ MN/m} ) ( D = 2.134 \text{ m} ) ( k_p = 43.5 \text{ MN/m} )</td>
</tr>
<tr>
<td>Barge</td>
<td>( B = 10.668 \text{ m} ) ( V_i = 3.09 \text{ m/s (6 knots, 11.1 km/h, 6.9 mi/h)} ) ( m_B = 1.724 \text{ Mkg (including tow boat)} )</td>
</tr>
<tr>
<td>River/Waterway</td>
<td>River flow speed included in ( V_i )</td>
</tr>
<tr>
<td><strong>Applicability of the Equivalent Impact Force Method</strong></td>
<td></td>
</tr>
<tr>
<td>( \alpha ) [Eq. (5)]</td>
<td>( \frac{2.134}{10.668} = 0.2 ) ( 0.2 )</td>
</tr>
<tr>
<td>( k_{cr} ) [Eq. (4)]</td>
<td>( 4.6 + 646.5 \times 0.2 - 270.0 \times 0.2^2 = 123.1 \text{ MN/m} ) ( 55.7 \text{ MN/m} )</td>
</tr>
<tr>
<td>( k_p &gt; 0.1 k_{cr} )</td>
<td>The Equivalent Force Method is applicable</td>
</tr>
<tr>
<td><strong>Barge/Flotilla Initial Kinetic Energy</strong></td>
<td></td>
</tr>
<tr>
<td>( E_k ) [Eq. (3)]</td>
<td>( \frac{1}{2} \times 1.724 \times 3.09^2 = 8.23 \text{ MJ} &lt; 10 \text{ MJ} )</td>
</tr>
<tr>
<td><strong>Barge/Flotilla Equivalent Impact Force</strong></td>
<td></td>
</tr>
<tr>
<td>( a_B ) (Fig. 5)</td>
<td>1.50 m ( 1.90 \text{ m} )</td>
</tr>
<tr>
<td>( P_B ) (Fig. 5)</td>
<td>5.50 MN ( 4.30 \text{ MN} )</td>
</tr>
<tr>
<td><strong>Impact Duration</strong></td>
<td></td>
</tr>
<tr>
<td>( \epsilon_B ) [Eq. (9)]</td>
<td>( 0.28 + 0.04 \times 0.2 - (0.08 + 0.04 \times 0.2) \ell_n 8.23 \right</td>
</tr>
<tr>
<td>( t_d ) [Eq. (8)]</td>
<td>( \frac{1+0.10}{5.50} \times 1.724 \times 3.09 = 1.07 \text{ sec} ) ( 1.45 \text{ sec} )</td>
</tr>
<tr>
<td><strong>AASHTO Method (1991)</strong></td>
<td></td>
</tr>
<tr>
<td>( a_B ) [Eq. (1)]</td>
<td>( 3.1(\sqrt{1 + 0.13 \times 8.23} - 1) = 1.36 \text{ m} )</td>
</tr>
<tr>
<td>( P_B ) [Eq. (2)]</td>
<td>( 6 + 1.6 \times 1.36 = 8.18 \text{ MN} )</td>
</tr>
</tbody>
</table>
Table 2. Determination of the Equivalent Flotilla Impact Force For a Single Column 3-Barge Flotilla Impacting a Rectangular and a Circular (or Rounded End) Pier

<table>
<thead>
<tr>
<th>Item</th>
<th>Rectangular Pier</th>
<th>Circular Pier</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bridge, Barge/Flotilla, and River/Waterway Information</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bridge</strong></td>
<td>$b_c = 3.2 \text{ m}$</td>
<td>$D = 3.2 \text{ m}$</td>
</tr>
<tr>
<td></td>
<td>$k_p = 75.5 \text{ MN/m}$</td>
<td>$k_p = 75.5 \text{ MN/m}$</td>
</tr>
<tr>
<td><strong>Barge</strong></td>
<td>$B = 10.668 \text{ m}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_i = 2.57 \text{ m/s (5 knots, 9.3 km/h, 5.8 mi/h)}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_B = 5.17 \text{ Mkg (including tow boat)}$</td>
<td></td>
</tr>
<tr>
<td><strong>River/Waterway</strong></td>
<td></td>
<td>River flow speed included in $V_i$</td>
</tr>
</tbody>
</table>

**Applicability of the Equivalent Impact Force Method**

| | $\alpha\text{[Eq. (5)]}$ | $k_{cr}\text{[Eq. (4)]}$ |
| | 0.3 | $\frac{3.2}{10.668} = 0.3$ |
| | | $174.25 \text{ MN/m}$ |
| | $k_p > 0.1k_{cr}$ | | The Equivalent Force Method is applicable |

**Barge/Flotilla Initial Kinetic Energy**

| | $E_k\text{[Eq. (3)]}$ |
| | $\frac{1}{2} \times 5.17 \times 2.57^2 = 17.07 \text{ MJ > 10 MJ}$ |

**Barge/Flotilla Equivalent Impact Force**

| | $a_B\text{[Eq. (6)]}$ | $P_B\text{[Eq. (7)]}$ |
| | 2.51 m | 6.36 MN |
| | $15 \left( \frac{17.07}{59.45} \right) - 11 \left( \frac{17.07}{59.45} \right)^2 = 3.40 \text{ m}$ | $10^{-3} \times 59.45 \left( 19 \times 3.40 + \frac{77}{e^{0.35 \times 3.40}} \right) = 5.23 \text{ MN}$ |

**Impact Duration**

| | $e_B\text{[Eq. (9)]}$ | $t_d\text{[Eq. (8)]}$ |
| | 0.03 | 2.15 sec |
| | $0.27 + 0.01 \times 0.3 - 0.05 \times \ell_n 17.07 = 0.13$ | $1 + 0.13 \times 5.17 \times 2.57 = 2.87 \text{ sec}$ |

**AASHTO Method (1991)**

| | $a_B\text{[Eq. (1)]}$ | $P_B\text{[Eq. (2)]}$ |
| | $3.1 \sqrt{1 + 0.13 \times 17.07 - 1} = 2.46 \text{ m}$ | $6 + 1.6 \times 2.46 = 9.94 \text{ MN}$ |
Table 3. Determination of the Equivalent Flotilla Impact Force for a 3-Columns by 5-Rows, 15-Barge Flotilla Impacting a Rectangular and a Circular (or Rounded End) Pier

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pier Shape</strong></td>
<td><strong>Rectangular Pier</strong></td>
</tr>
<tr>
<td>Bridge, Barge/Flotilla, and River/Waterway Information</td>
<td><strong>Circular Pier</strong></td>
</tr>
<tr>
<td>Bridge</td>
<td>( b_c = 2.0 \text{ m} ) ( k_p = 143 \text{ MN/m} )</td>
</tr>
<tr>
<td>Barge</td>
<td>( B = 10.668 \text{ m} ) ( V_i = 1.54 \text{ m/s (3 knots, 5.6 km/h, 3.5 mi/h)} ) ( m_B = 25.8 \text{ Mkg (including tow boat)} )</td>
</tr>
<tr>
<td>River/Waterway</td>
<td>River flow speed included in ( V_i )</td>
</tr>
<tr>
<td><strong>Applicability of the Equivalent Impact Force Method</strong></td>
<td></td>
</tr>
<tr>
<td>( \alpha ) [Eq. (5)]</td>
<td>( \frac{2.0}{10.668} = 0.1875 )</td>
</tr>
<tr>
<td>( k_{cr} ) [Eq. (4)]</td>
<td>( 4.6 + 646.5 \times 0.1875 - 270.0 \times 0.1875^2 = 116.3 \text{ MN/m} ) ( 55.2 \text{ MN/m} )</td>
</tr>
<tr>
<td>( k_p &gt; 0.1 k_{cr} )</td>
<td>The Equivalent Force Method is applicable</td>
</tr>
<tr>
<td><strong>Barge/Flotilla Initial Kinetic Energy</strong></td>
<td></td>
</tr>
<tr>
<td>( E_k ) [Eq. (3)]</td>
<td>( \frac{1}{2} \times 25.8 \times 1.54^2 = 30.59 \text{ MJ} &gt; 10 \text{ MJ} )</td>
</tr>
<tr>
<td><strong>Barge/Flotilla Equivalent Impact Force</strong></td>
<td></td>
</tr>
<tr>
<td>( a_B ) [Eq. (6)]</td>
<td>( 29 \left( \frac{30.59}{116.3} \right) - 35 \left( \frac{30.59}{116.3} \right)^2 = 5.21 \text{ m} ) ( 4.93 \text{ m} )</td>
</tr>
<tr>
<td>( P_B ) [Eq. (7)]</td>
<td>( 10^{-3} \times 116.3 \left( 22 \sqrt{5.21} + \frac{110}{e^{1.665.21}} \right) = 5.84 \text{ MN} ) ( 5.93 \text{ MN} )</td>
</tr>
<tr>
<td><strong>Impact Duration</strong></td>
<td></td>
</tr>
<tr>
<td>( e_B ) [Eq. (9)]</td>
<td>( 0.28 + 0.04 \times 0.1875 - (0.08 + 0.04 \times 0.1875) /</td>
</tr>
<tr>
<td>( t_d ) [Eq. (8)]</td>
<td>( 1 + 0.012 \times 25.8 \times 1.54 = 6.89 \text{ sec} ) ( 7.38 \text{ sec} )</td>
</tr>
<tr>
<td><strong>AASHTO Method (1991)</strong></td>
<td></td>
</tr>
<tr>
<td>( a_B ) [Eq. (1)]</td>
<td>( 3.1 \left( \sqrt{1 + 0.13 \times 30.59 - 1} \right) = 3.82 \text{ m} )</td>
</tr>
<tr>
<td>( P_B ) [Eq. (2)]</td>
<td>( 6 + 1.6 \times 3.82 = 12.11 \text{ MN} )</td>
</tr>
</tbody>
</table>
Fig. 1. Fifteen barge flotilla made up of 3-columns and 5-rows
Fig. 2. Finite element model: (a) jumbo hopper impacting a rectangular pier; and simulation of the crushing of barge bows impacting, (b) rectangular pier and (c) circular pier
Fig. 3. Barge bow crushing distance, $a_B(t)$, impact force, $P(t)$, and time-histories for a fully loaded single barge impacting bridge piers of different widths with an initial velocity, $V_i$, of 2.06 m/s (4 knots, 7.41 km/h, 4.60 mi/h): (a) rectangular piers, (b) circular piers.
Fig. 4. Impact force time history for a single and multi-barge column flotilla impacting a rectangular bridge pier (FL1 = single barge column,..., FL5 = 5-barge column)

\[ \alpha = \frac{b_c}{B} = 0.1 \]

\[ V_i = 2.06 \text{ m/s (4 knots)} \]
Fig. 5. Impact force $P_B$ and barge damage length $a_B$ in relation to the kinetic impact energy $E_k$: (a) rectangular piers; (b) circular piers
Fig. 6. Equivalent static impact force $P_B$ vs. damage depth $a_B$: (a) rectangular piers; (b) circular piers.
Fig. 7. Model of bridge pier and attributed portion of the superstructure for determining the pier stiffness $k_p$. 

$$k_p = \frac{F}{\Delta}$$
Finite Element Simulation

Equivalent Impact Force

AASHTO

AASHTO ($P_B = 8.18$ MN)

$P_B = 5.50$ MN

$t_d = 1.07$ sec

Fig. 8. Impact force time history for single barge flotilla impacting a rectangular bridge pier (Refer to Table 1 for additional information)
Fig. 9. Impact force time history for a single column 3-barge flotilla impacting a circular bridge pier (Refer to Table 2 for additional information)
Fig. 10. Impact force time history for a 3-columns by 5-rows, 15-barge flotilla impacting a rectangular bridge pier (Refer to Table 3 for additional information)