2013

MODELING OF AN AIR-BASED DENSITY SEPARATOR

Tathagata Ghosh

University of Kentucky, tathagata.ghosh@uky.edu

Click here to let us know how access to this document benefits you.

Recommended Citation

Ghosh, Tathagata, "MODELING OF AN AIR-BASED DENSITY SEPARATOR" (2013). Theses and Dissertations--Mining Engineering. 7.

https://uknowledge.uky.edu/mng_etds/7

This Doctoral Dissertation is brought to you for free and open access by the Mining Engineering at UKnowledge. It has been accepted for inclusion in Theses and Dissertations--Mining Engineering by an authorized administrator of UKnowledge. For more information, please contact UKnowledge@lsv.uky.edu.
STUDENT AGREEMENT:

I represent that my thesis or dissertation and abstract are my original work. Proper attribution has been given to all outside sources. I understand that I am solely responsible for obtaining any needed copyright permissions. I have obtained and attached hereto needed written permission statements(s) from the owner(s) of each third-party copyrighted matter to be included in my work, allowing electronic distribution (if such use is not permitted by the fair use doctrine).

I hereby grant to The University of Kentucky and its agents the non-exclusive license to archive and make accessible my work in whole or in part in all forms of media, now or hereafter known. I agree that the document mentioned above may be made available immediately for worldwide access unless a preapproved embargo applies.

I retain all other ownership rights to the copyright of my work. I also retain the right to use in future works (such as articles or books) all or part of my work. I understand that I am free to register the copyright to my work.

REVIEW, APPROVAL AND ACCEPTANCE

The document mentioned above has been reviewed and accepted by the student’s advisor, on behalf of the advisory committee, and by the Director of Graduate Studies (DGS), on behalf of the program; we verify that this is the final, approved version of the student’s dissertation including all changes required by the advisory committee. The undersigned agree to abide by the statements above.

Tathagata Ghosh, Student
Dr. Rick Q. Honaker, Major Professor
Dr. Thomas Novak, Director of Graduate Studies
MODELING OF AN AIR-BASED DENSITY SEPARATOR

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Engineering at the University of Kentucky

By
Tathagata Ghosh
Lexington, Kentucky

Director: Dr. R.Q. Honaker, Professor of Mining Engineering
Lexington, Kentucky
2013
Copyright © Tathagata Ghosh 2013
ABSTRACT OF DISSERTATION

MODELING OF AN AIR-BASED DENSITY SEPARATOR

There is a lack of fundamental studies by means of state of the art numerical and scale modeling techniques scrutinizing the theoretical and technical aspect of air table separators as well as means to comprehend and improve the efficiency of the process. The dissertation details the development of a workable empirical model, a numerical model and a scale model to demonstrate the use of a laboratory air table unit.

The modern air-based density separator achieves effective density-based separation for particle sizes greater than 6 mm. Parametric studies with the laboratory scale unit using low rank coal have demonstrated the applicability with regards to finer size fractions of the range 6 mm to 1 mm. The statistically significant empirical models showed that all the four parameters, i.e, blower and table frequency, longitudinal and transverse angle were significant in determining the separation performance. Furthermore, the tests show that an increase in the transverse angle increased the flow rate of solids to the product end and the introduction of feed results in the dampening of airflow at the feed end. The higher table frequency and feed rate had a detrimental effect on the product yield due to low residence time of particle settlement.

The research further evaluated fine particle upgrading using various modeling techniques. The numerical model was evaluated using K-Epsilon and RSM turbulence formulations and validated using experimental dataset. The results prove that the effect of fine coal vortices forming around the riffles act as a transport mechanism for higher density particle movement across the table deck resulting in 43% displacement of the midlings and 29% displacement of the heavies to the product side. The velocity and vector plots show high local variance of air speeds and
pressure near the feed end and an increase in feed rate results in a drop in deshaling capability of the table.

The table was further evaluated using modern scale-modeling concepts and the scaling laws indicated that the vibration velocity has an integral effect on the separation performance. The difference between the full-scale model and the scaled prototype was 3.83% thus validating the scaling laws.

KEYWORDS: Dry Processing, Air-based density separator, Empirical model, CFD modeling, Scale Modeling.
MODELING OF AN AIR-BASED DENSITY SEPARATOR

By
Tathagata Ghosh

Dr. Rick Q. Honaker
Director of Dissertation

Dr. Thomas Novak
Director of Graduate Studies

Date
DEDICATION

I dedicate this work to my mother Madhabi Ghosh and my late father Alokesh Ghosh who have supported and inspired me all the way since the day I was born. Their dedication, sacrifice and hard work has been an inspiration for me and I cherish their love and belief in me. Without my parents this work would not have seen the light of the day.
ACKNOWLEDGEMENT

I would like to express my sincerest gratitude to Dr. Rick Honaker for his valuable guidance and motivation throughout the course of this study. Gratitude is also extended to Dr. Daniel Tao, Dr. Kozo Saito, Dr. B. K. Parekh and Dr. D. P. Patil of University of Kentucky for their time, guidance, assistance and suggestions for this research.

I would like to extend my thanks to CAST (Center for Advanced Separation Technologies) for the financial and technical support which made this work possible. Special appreciation is given to Dr. Abraham Salazar from Giffin, Inc. for his invaluable contribution towards developing the CFD model.

Appreciation is also extended to Ed Thompson as his guidance and technical support played a crucial role in this project. Many thanks are due to Dr. Tom Novak, Dr. Andrew Wala, Dr. Braden Lusk, Dr. Joe Sottile, Dr. Kyle Perry, Dr. Rick Sweigard, Dr. Kot Unrug for their assistance and support. I would also like to acknowledge Kathy Kotora, Megan Doyle, Christie Oliver, Joy McDonald and other department staff for their assistance.

Special thanks are given to all my friends, Dr. Deviprasad Udgata, Dr. Michael Vassalos, Fotis Moraitis, Dr. Rebecca Peyyala, Dr. Haralambos Symeonidis, Theodoros Kyriopoulos, Ken Hemmer, Cary Tsamas, Thomas Papanicolaou, Raghav Dube, Sampurna Arya, Dharmendra Kumar, Todor Petrov, Chad Wedding, Dr. Jhon Silva, Qing Huang, Ahmet Sohby, Mehmet Saracoglu, Mohd. Rezae, Hassan Amini and fellow graduate students and colleagues for their support and friendship. Last but not the least, I would like to acknowledge my wife, Sudeshna, for her constant inspiration and support.
# TABLE OF CONTENTS

**ACKNOWLEDGEMENT** .................................................................................................................. III

**LIST OF TABLES** ........................................................................................................................ VI

**LIST OF FIGURES** ...................................................................................................................... VII

**CHAPTER 1: INTRODUCTION** ...................................................................................................... 1

1.1 Background ..................................................................................................................................... 1

1.1.1 Air-based density separator upgrading concept ................................................................. 2

1.2 Goals and Objectives .................................................................................................................. 4

1.3 Dissertation Organization .......................................................................................................... 6

**CHAPTER 2: BACKGROUND** ....................................................................................................... 7

2.1 Introduction .................................................................................................................................... 7

2.2 Brief History of Dry Density Based Separator Technology Development....................... 9

2.2.1 Pneumatic Dry Separators ................................................................................................. 10

2.2.1.1 Air-based density separation fundamentals .......................................................... 17

2.2.1.2 Dual-Density Fluidized Bed Separator ...................................................................... 25

2.3 Problems with modern air-based density separators ......................................................... 27

2.4 Previous Modeling Attempts ................................................................................................... 28

2.5 Scale Modeling Concept .......................................................................................................... 32

**CHAPTER 3: PARAMETRIC EVALUATION AND EMPIRICAL MODELING** .................. 35

3.1 Introduction .................................................................................................................................... 35

3.2 Process Description .................................................................................................................... 36

3.3 Experimental ................................................................................................................................ 37

3.3.1 Laboratory Air Table .......................................................................................................... 37

3.3.2 Experimental Procedure ...................................................................................................... 39

3.3.3 Coal Sample Characterization ............................................................................................ 41

3.4 Results and Discussion .............................................................................................................. 43

**CHAPTER 4: NUMERICAL MODELING OF AN AIR-BASED DENSITY SEPARATOR** ............... 52

4.1 Introduction .................................................................................................................................... 52

4.2 Development of the Numerical Model ...................................................................................... 53
LIST OF TABLES

Table 2.1 Mineral properties used in different dry beneficiation techniques (Dwari and Rao, 2007) ................................................................................................................................. 10
Table 2.2 Fundamental aspects of scale modeling technique (Saito, 2008) .................. 33
Table 3.1 Air table parameters and their levels used in the test program which followed a 3-level Box-Behnken design ......................................................................................................................... 39
Table 3.2 Size-by-size analysis of the PRB coal sample * ........................................... 42
Table 3.3 Float-Sink data of the PRB coal sample * .................................................. 42
Table 3.4 Analysis of Variance of Table for product Ash .............................................. 45
Table 3.5 Analysis of Variance of Table for product yield ......................................... 45
Table 3.6 Results and comparison between the optimized tests .................................... 49
Table 4.1 Numerical Model development and CFD analysis process ................................. 54
Table 4.2 Average air-flow and pressure-drop measurement data .................................... 59
Table 4.3 Pre-processing model: Details of the mesh structure ........................................ 76
Table 4.4 Initial boundary condition parameters ............................................................... 77
Table 4.5 Hydraulic diameters of Air Inlet and the different outlet faces ...................... 78
Table 4.6 SUPERMICRO AMD server configuration specifications ............................... 79
Table 4.7 Comparative efficiency of increasing core-count relative to a single core desktop ............................................................................................................................................... 80
Table 4.8 Mass fraction distribution based on particle size and density in the injected feed ........................................................................................................................................... 83
Table 4.9 Air velocity comparison between k – ε and RSM models at 0.2 second .............. 89
Table 4.10 Static pressure distribution values above and below the screen deck .............. 90
Table 4.11 RSM vs k – ε turbulence intensity distribution (0.2 second) ....................... 91
Table 4.12 Correlating model predictions of pressure drop across the porous zone with measured experimental data .............................................................. 93
Table 4.13 Correlating model predictions of air velocity across different sections of the table deck with measured experimental data (0.2 second k – ε model) .............. 94
Table 4.14 Mass transfer summary of particle discharge (100 time steps) in 0.002 seconds ........................................................................................................................................... 97
Table 4.15 Comparison of yield and cut density between experimental data and model predictions for different feed rates ................................................................. 100
Table 5.1 Boundary Parameter values (Full-Scale Model vs Prototype) ......................... 108
Table 5.2 Average air velocity data measured at each section on top of the table deck (Full-scale model vs Prototype vs Experimental data) .................................................. 111
Table 5.3 Turbulent intensity plot (0.7 sec prototype vs 1 sec full-scale model) .......... 112
LIST OF FIGURES

Figure 1.1 Illustration of the FGX unit showing (a) the separation chamber and (b) helical particle motion and separation mechanism (Honaker et. al., 2008) ................................................... 3
Figure 1.2 Laboratory scale air-based density separator .......................................................... 3
Figure 2.1 Annual Capacity increase of Dry separators till 1965 (McCulloch et. al., 1968). ............................................................................................................................................. 8
Figure 2.2 Saxon Cleaner (Horsfall, 1980) .............................................................................. 13
Figure 2.3 Schematic of a typical air table separator (Alderman and Snoby, 2001) ............... 14
Figure 2.4 Roberts and Schaefer airflow jig (Frankland, 1995) ................................................... 15
Figure 2.5 (a) FX Dry Cleaner unit and (b) FGX Compound Dry Cleaner ....................... 16
Figure 2.6 Various forms of Fluidization (Kunii and Levenspiel, 1991) ............................ 19
Figure 2.7 Standard patterns of solid Fluidization (Chase, 2004) ....................................... 20
Figure 2.8 Partition curves for different size fractions (Luo et. al., 2002) ........................ 24
Figure 2.9 Experiments with coal mixtures (50% coal and 50% shale) for 3 air velocities: (a) 6.5 m/s, (b) 6.9 m/s and (c) 7.0 m/s; Size of coal and shale: 20-30 mm (Houwenlingen and Jong, 2004) ....................................................................................................................... 24
Figure 2.10 Experiments with moist and dirty coal mixtures (50% coal and 50% shale), for two air velocities: (a) 6.4 m/s and (b) 7.0 m/s; Size of mixture: 20-30 mm; Moisture content is 4.1% average and the adhering dirt 5% average (Houwenlingen and Jong, 2004) .......................................................... 25
Figure 2.11 Separation performance vs Particle size (Honaker, 2007) ..................................... 28
Figure 2.12 (a) Modified circuit flowsheet and (b) Predicted performance improvement. ........................................................................................................................................... 30
Figure 2.13 DEM simulating particle settlement on a vibrating bed (Gupta, 2011) ............... 31
Figure 3.1 Laboratory scale air-based density separator initial setup ........................................... 38
Figure 3.2 Modified air table setup.......................................................................................... 38
Figure 3.3 Figure depicting sample collection points A, B, C, D and E ............................... 40
Figure 3.4 Cleanability of the PRB coal sample, (a) - Cum. Ash (%) vs Cum. Wt. (%) and (b) Ash (%) vs Heating Value (BTU) ......................................................................................................................... 42
Figure 3.5 Plot showing typical data sets collected for each experiment ............................... 43
Figure 3.6 Effect of Fan Blower Frequency (Air Flow Rate) and Longitudinal Angle on Combustible Recovery .............................................................................................................. 47
Figure 3.7 Combustible Recovery of the test runs vs washability data .................................. 48
Figure 3.8 Plot showing partition numbers and separation performance (optimal tests) ........ 49
Figure 3.9 Photograph of the clean coal and reject generated from cleaning the 6 x 1 mm sub-bituminous coal using an air table .................................................................................. 50
Figure 4.1 Simple schematic of the particle paths on top of the deck and the sample collection points A, B, C, D and E................................................................. 56
Figure 4.2 Airflow and pressure drop measurements using a digital vane anemometer and a digital manometer, respectively. ................................................................. 57
Figure 4.3 Table deck divided into 6 sections to simplify calibration........................................ 57
Figure 4.4 Variance of air-velocity plotted against corresponding pressure drop measurements for different sections of the table. ................................................................. 58
Figure 4.5 Average air velocity vs pressure drop across the deck............................................ 59
Figure 4.6 Model CAD Geometry (developmental stage) isometric view. ............................ 74
Figure 4.7 Independent mesh structure for porous media......................................................... 75
Figure 4.8 Tet/hybrid mesh displaying the table deck ............................................................ 76
Figure 4.9 Velocity vector profile of airflow at various planes simulated for a static run. .................................................................................................................. 77
Figure 4.10 Plot signifying Relative Efficiency of increasing the number of cores with respect to a single core desktop. ................................................................. 80
Figure 4.11 Table deck motion profile (actual inclination angle $\theta$ vs the motion plane at an angle of 44° angle with the X,Y horizontal plane). ................................................. 81
Figure 4.12 Particle size Rosin Rammler distribution for individual density injections. . 82
Figure 4.13 DDPM Particle Phase properties setup in Ansys Fluent © .................................. 84
Figure 4.14 Transient airflow velocity profile plot (realizable $k - \varepsilon$ model). .................. 86
Figure 4.15 Transient airflow velocity profile streamline plot (realizable $k - \varepsilon$ model). 87
Figure 4.16 Air velocity vector plot displaying high variance near the feed end................. 88
Figure 4.17 Airflow profile plot at 0.2 seconds ($k - \varepsilon$ vs RSM). ....................................... 89
Figure 4.18 Pressure plot of $k - \varepsilon$ and RSM models at 0.2 second (scale is in Pascals). 90
Figure 4.19 Turbulent intensity plot of $k - \varepsilon$ and RSM models at 0.2 second. .......... 91
Figure 4.20 Pressure drop measurements recorded using point probing method................. 94
Figure 4.21 Air velocity contour plot on top of the table deck ($k - \varepsilon$ model). ............... 95
Figure 4.22 Particle injection turbulent intensity $%, k - \varepsilon$ model (6.0195 s) .................... 96
Figure 4.23 Plot of $k - \varepsilon$ model displaying vortices forming near the riffles................. 98
Figure 4.24 Top view schematic of the particle tracks (100 time steps, $k - \varepsilon$ model). .......... 100
Figure 4.25 Partition curve comparison between model prediction and experimental data for a feed rate of 200 kg/hr. ......................................................... 101
Figure 4.26 Partition curve comparison between model predictions for feed rates of 200 kg/hr and 400 kg/hr................................................................. 101
Figure 5.1 Schematic of the full-scale air-based density separator table geometry and simplified operating principle............................................................ 107
Figure 5.2 Volume mesh setup for the air-based density separator CFD model. ............. 108
Figure 5.3 Top view of the Table deck displaying the dividing sections (A, B, C, D, E, F). ............................................................................................................. 110
Figure 5.4 Comparison of the predicted air velocity values occurring directly above the table surface in defined cross-sectional areas using the full-scale and the prototype models with the average values obtained from direct measurements at several points in each area................................................................. 111
Figure 5.5 Air velocity contour plot (0.7 sec prototype vs 1 sec full-scale model)......... 112
Figure 5.6 Velocity vector plot (0.7 sec prototype vs 1 sec full-scale model)........... 113
Figure 5.7 Turbulent intensity (%) plot (0.7 sec prototype vs 1 sec full-scale model)... 113
CHAPTER 1: INTRODUCTION

1.1 Background

Modern dry processing technologies such as the air-based table separator offer significant benefits such as elimination of process water requirements, expensive dewatering technologies and waste slurry disposal systems, improvement in pit-waste mineral recovery, reduction of trace elements, sulfur and mercury and significant increase in thermal efficiency. The beneficiation process is augmented by the need to decrease transportation costs, improve operational and consumption properties, and diminish environmental impacts. At present, dry beneficiation techniques has provoked prospective interest not only due to the scarcity of fresh water in major mineral-producing countries, but also due to procedural benefits in downstream utilization. The economic advantages of dry beneficiation are obvious as no further energy is exhausted in drying the processed ore. Reduction in process equipment investment further supplements the relative ease of handling dry material.

Keeping the above argument in view, the following study focuses on one such viable technology, which is a air-based vibrating table separator. To improve the understanding of the fundamental separation mechanisms and separation performance a parametric evaluation and empirical modeling was utilized along with modern computational fluid dynamics (CFD) tools and state of the art scale modeling techniques. Research in the past
has relied heavily on empirical models as it was difficult to study the complex fundamental process variables without intricate turbulent fluid dynamic calculations. However, recent developments in computational and scale modeling techniques have made it possible to carry out elaborate simulations utilizing CFD concepts to better understand the procedure.

1.1.1 Air-based density separator upgrading concept

The dry cleaning system employs the separation principles of an autogenous medium and a table concentrator. The feed to the system is introduced into a feed hopper from which the flow is controlled using a vibrating feeder. The separation process generates multiple products of varying grades. Industrial and pilot scale units are equipped with dust collection systems to clean the recycled air and to remove about 90% of the recirculating dust from the system. Laboratory scale units are known to be notorious generators of fine coal dust; hence, the feed size fractions need to be closely monitored.

The separating compartment consists of a deck, vibrator, air chamber and vibratory feeder. A centrifugal fan provides air that passes through holes on the deck surface at a rate sufficient to transport and actually fluidize the small high density particles that the low density particles float on. Figure 1.1 shows the popular FGX industrial scale unit, while Figure 1.2 demonstrates the laboratory unit.
Figure 1.1 Illustration of the FGX unit showing (a) the separation chamber and (b) helical particle motion and separation mechanism (Honaker et. al., 2008).

Figure 1.2 Laboratory scale air-based density separator.

Introduction of the feed coal into the separation chamber is followed by the formation of a particle laden bed of certain depth. The vertical and horizontal angles of inclination of the bed can be altered by the help of mechanical or electronic control systems. The particles near the bottom of the bed directly contact the vibrating deck and move along...
the deck under the effect of the vibration-induced inertia force. Light particles create the upper layer of particles that are collected along the length of the table. Particles of sufficient density are able to settle through the autogenous medium formed due to the fluidized bed of fine, high density particles and report back to the deck surface. The particle movement pattern of the laboratory unit is slightly different from the FGX industrial unit as the lighter particles move towards the side of the table while the heavier particles move towards the front of the deck.

The air based density separator have been found to operate efficiently and provide high density separation at around 1.8 to 2.2 relative density range and for 75 mm to 6 mm particle size. However, for finer size fractions (6 mm x 1 mm range), the cleaning efficiency is found to be less than desirable. The operating parameters such as air flow rate, table vibration frequency, table amplitude and inclination angles can be adjusted to modify the fluidized air table separator to be applicable for finer fractions (6mm x 1 mm). The exposure of high moisture content fine particles can prove to be a major constraint. This limitation can be rectified to certain extent by sizing the feed at around 1 mm using screens or air-based classification systems, but, in most cases, efficient dry classification at 1 mm is found to be commercially unviable.

1.2 Goals and Objectives

The goal of this project was to characterize an air-based density separator and understand the fundamental aspects of the complex multi-phase process using empirical models, numerical analysis and scale modeling techniques with the emphasis on
improving the separation performance achieved on 6 x 1 mm material. If successful, the process information and the numerical model can be used in a proof-of-concept study to implement design changes and improve the performance of the separator.

To achieve the project goals, the following specific objectives were targeted:

- Develop empirical models describing the parametric effects on the response variables of the laboratory scale air-based density separator using data obtained from statistical parametric test program treating 6 mm x 1 mm material.
- Quantify the various process parameters such as airflow velocity, pressure drop across the screen deck, angles of inclination and vibration amplitude using experimental techniques.
- Develop suitable mathematical models describing the multi-phase phenomenon.
- Develop a comprehensive CFD model of the air-based density separator and validate the model using experimental data.
- Develop appropriate scaling laws for the process and compare the numerical model of the scaled prototype with the full-scale model and identify the divergence parameters.
- Analyze the results and identify process and design factors affecting the selectivity of the separation performance and ways to improve the process.
1.3 Dissertation Organization

This dissertation is divided into 6 (six) main chapters. Chapter 1 presents information, which provides the background leading to the research. It also presents the goals and objectives, which this study was designed to accomplish. Chapter 2 contains a literature review of the history and fundamentals of dry air based density separation and previous modeling attempts. Chapter 3 describes the parametric evaluation performed on the air-based density separator using a statistical test program. It also details the development of statistically significant empirical models for product ash and yield in terms of the process and operating parameters. Chapter 4 describes the theory and the development of a full-scale three dimensional CFD model of the laboratory scale air-based density separator. A discussion of the model results under different turbulence formulations and validation of the model are also contained in chapter 4. Chapter 5 details the development of a suitable scale model prototype of the air-table using numerical methods and subsequent comparison with the full-scale model, while Chapter 6 presents the major conclusions and recommendations for future work.
CHAPTER 2: BACKGROUND

2.1 Introduction

Dry cleaning technologies were prevalent in the mineral industry throughout the early to middle twentieth century. Major innovations and development of dry, density-based separators occurred during the 20 year period of 1910 and 1930. The U.S. coal industry processed a significant amount of coal using dry separators until about 1968 when production peaked at 25.4 million tons annually (Mcculloch et. al., 1968; Figure 2.1). Pennsylvania had the distinction of operating the largest dry cleaning plant with 14 units processing about 1400 tph of coal. By 1990, the production from dry cleaning processes fell to less than 3.6 million tons annually in the United States as most of the dry cleaning plants were either closed or their capacities severely stunted due to federal dust exposure regulations which required the use of a significant amount of water prior to the processing plant to suppress dust (Arnold et. al., 1991). Furthermore, many of the earliest dry cleaning units were manually operated and rendered inefficient by competing wet cleaning processes.

The effective top particle size for most of the dry mineral separators is around 75 mm and the effective size ratio for which a good separation is achieved is between 2:1 and 4:1. The reported probable error (Ep) values vary in the range of 0.15 up to 0.3 in treating a particle size range of 75 mm x 6 mm. These values indicate that the dry separation is inferior in separation efficiency in comparison with the wet coarse cleaning units. On the
other hand, the dry cleaning processes typically have lower capital and operating costs, required no waste water treatment or fine waste impoundment, provides lower product moistures and need less stringent permitting requirements. However, scarcity of water, downstream process water de-contamination issues and stricter environmental regulations concerning effluent discharge augmented by increasing capital expenditure on treating slurry impoundments has resulted in a significant increase in industry interest to look for effective dry cleaning solutions (Rezaee et. al., 2013; Wotruba et. al., 2010; Honaker et. al., 2008; Donnelly, 1999, Lockhart, 1984).

Figure 2.1 Annual Capacity increase of Dry separators till 1965 (Mcculloch et. al., 1968).
2.2 Brief History of Dry Density Based Separator Technology Development

The dry beneficiation processes exploit the inherent differences in physical properties between the mineral (or coal) and the waste rock such as the shape and size, density, luster, magnetic susceptibility, electrical and frictional properties. The difference in the above mentioned properties help decide the suitability of different types of dry cleaning equipment such as pneumatic jigs and tables, optical sorting, tribo-electrostatic separator, air-dense medium fluidized bed separator, etc., which are pertinent to various particle sizes (Dwari and Rao, 2007; Table 2.1). Besides the obvious advantages in handling of dry minerals, the reduction in capital expenditure and probability of ground water contamination, space and water requirements make dry cleaning an attractive proposition. Several modifications of processing technologies used during the peak years of dry beneficiation have been successfully developed and commercialized. Such technologies are: Accelerator (Honaker, 2007), Rotary Breaker (Bhattacharya, 2006), Allair Jig (Kelley and Snoby, 2002; Weinstein and Snoby, 2007), FGX unit (Li and Yang, 2006; Lu et. al., 2003), Air Dense Medium Fluidized Bed Separator (Luo and Chen, 2001), AKAFLOW (Rubarth et. al., 2009), Tribo-Electrostatic Separator (Dwari and Rao, 2009; Tao et. al., 2008; Tao et. al., 2011), MagMill (Oder et. al., 2008), optical and X-Ray Electronic Sorting Technology (Kiser et. al., 2013).
Table 2.1 Mineral properties used in different dry beneficiation techniques (Dwari and Rao, 2007)

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Characteristics</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Appearance/Color</td>
<td>Sorting</td>
</tr>
<tr>
<td>2</td>
<td>Shape</td>
<td>Screening</td>
</tr>
<tr>
<td>3</td>
<td>Friability/Elasticity</td>
<td>Rotary Breaker, Differential crushing</td>
</tr>
<tr>
<td>4</td>
<td>Density</td>
<td>Pneumatic, Fluid bed separators</td>
</tr>
<tr>
<td>5</td>
<td>Magnetic susceptibility</td>
<td>Magnetic separation</td>
</tr>
<tr>
<td>6</td>
<td>Electrical resistivity</td>
<td>Electrostatic separation</td>
</tr>
<tr>
<td>7</td>
<td>Radioactivity</td>
<td>Sorting</td>
</tr>
</tbody>
</table>

2.2.1 Pneumatic Dry Separators

Numerous patents of a variety of pneumatic separation techniques have been found dating back as far as 1850, which cover early attempts to separate materials of varying relative density ranges or of different shape and size by means of air based fluidized medium (Payne, 1913; Sutton, 1919; Delamater, 1927). The time period between 1910 and 1930 spanning two eventful decades saw the development of several dry, density-based separators used during much of the twentieth century (Osborne, 1988). These basic operational principles of pneumatic beneficiation technologies marked a striking resemblance to the mechanisms of more commonly utilized wet separators such as (i) dense medium separations, (ii) pulsated air jigging, (iii) riffled table concentration and (iv) air fluidized coal launders. Until 1930, several hundred patent applications covering these techniques have been filed and many of them were granted a national or
international patent. The different species of dry separators may be classified into four general groups, which are as follows:

1. Stationary devices with pulsating air currents. The separating deck is typically riffled with riders and air is supplied by fans or compressors. This group also includes air jigs, which have been used rather extensively.

2. Stationary devices with continuous air currents. These machines submit the material to a continuous current of air, either horizontal or vertical, e.g., chaff is blown from wheat by such a device.

3. Reciprocating or vibrating devices with pulsating air. A small group in which the pulsating air is supplied by a fan and some motion provided in the separating surface to move the stratified material to various discharge points, e.g., pneumatic conveyors.

4. Reciprocating or vibrating devices with a continuous air supply, e.g., the American Pneumatic Separator (Chapman et. al., 1928).

The last group was commissioned in 1924 and has turned out to be the most popular till present day. Modified clones of such devices have been developed ever since the development of the initial process. Examples of this type of separator can be found in agriculture, recycling businesses and minerals sector. All of the above groups involve the stratification of material by air and include often a combination of other principles such as friction, shape, resiliency and vibration. A similar air separator known as the Berry table was in use until 2003 in the coal preparation plant of Dr. Arnold Schäfer GmbH in
Saarwellingen, Germany. The particle size range of the feed to the machine is required to be within a 2:1 ratio. Size ranges can be handled from 2-4 mm up to 5-10 cm with a capacity ranging from 12 – 60 tons per hour.

Pneumatic or air tables separate the components in the feed based on particle density and shape by passing the material over a porous vibrating bed. The higher density material reports to the bottom of the bed and is removed by the action of the vibrating table. The lighter material floats on a suspended bed of high density particles and moves by mass action in the direction of the downward slope of the table. The two density fractions therefore move in different directions. The particle settling rates are much higher in air than those in water or any other medium. Hence, theoretically, air separation should be possible in a shorter time-span but this has been more difficult to achieve in practice.

A very simple air table referred to as the Saxon Cleaner (Horsfall, 1980; Figure 2.2) has been used to remove stones from agricultural products. Raw material is fed to the middle portion of the counter-current machine and a constant stream of air is blown through a wire mesh deck. The air helps lift the low-density minerals enough to allow the material to flow with gravity and against the direction of the flow of high density particles imparted by an eccentric drive to the deck. These devices have a relatively low capacity and are inefficient.
One of the earliest air tables, the Sutton, Sutton, and Steele (or Triple–S) table was installed at coal mines in Oklahoma and New Mexico prior to 1924 (Arms, 1924; Mitchell, 1942). The air table shown in Figure 2.3 is similar in appearance and function to a Deister Table. While both units share a riffled, sloping deck, the Triple S table employs air instead of water as the fluidizing agent. As with the Saxon Cleaner, low density particles flow with gravity while high density material is conveyed uphill in a counter-current flow by the eccentric motion of the deck. The riffles, installed over the wire mesh deck, channel the refuse toward the heavy product discharge. The lighter material assisted by air, can pass up and over the riffles, and discharges at the lower
corner of the deck, as indicated in Figure 2.3. Air tables must maintain a consistent feed quality and thin material bed to be effective. For these reasons, feed must be closely sized and unit capacity is low.

![Figure 2.3 Schematic of a typical air table separator (Alderman and Snoby, 2001).](image)

The Stump AirFlow Jig (Figure 2.4) also known as Roberts and Schaefer airflow jig was developed by Earl Stump in 1932 (U.S. Patent No. 1902917, issued on March 28, 1933). The machine is an inclined, vibrating perforated deck. Material is separated by passing the feed over a vibrating bed in the similar manner to the air tables, but in this operation both the high and low density particles move in the same direction and separation is achieved by positioning a splitter on the vertically stratified material. For 50 mm to 5 mm size fraction, a new air jig was developed at RWTH Aachen University (Aachen, Germany) and successfully commissioned into the industry (Weitkämper and Wotruba, 2004). This jig operates on the division of the air flow in a constant and pulsed
air flow, which differs from all of the existing jigs. A clean coal of below 20% ash content can be achieved from 52% ash feed coal with this jigging process. In the United States, recent developments with the modified Allair jig have been promising (Alderman and Snoby, 2001).

Figure 2.4 Roberts and Schaefer airflow jig (Frankland, 1995).

Tangshan Shenzhou Manufacturing Ltd., developed the FGX compound dry cleaner (Li and Yang, 2006; Figure 2.5) and later introduced a modified high capacity version known as the FX dry cleaner unit (Figure 2.5). The design is such that a helical motion is generated with air stratification and produces multiple products of light, middling, and heavy particles. The middling stream can be redirected to the product stream to facilitate better recovery. It operates acceptably on minus 80 mm coal size fraction, but the
efficiency reduces for particles having a size smaller than 6 mm. Generally less efficient than wet processing, a raw coal ash content of 35% is reduced to 25% ash with more than 75% ash content removal. Limited studies using a larger pilot scale unit on Wyoming coal achieved an ash reduction of over 60% and produced coal meeting end user specifications (Honaker et. al., 2008). Beneficiation of thermal coals in India with the help of a pilot scale FGX unit demonstrated the capability to effectively treat coarse particles rejecting almost 80% ash while recovering almost 90% of the combustible material (Gupta et. al., 2012). There are over 800 units of this dry cleaning unit successfully operating in China, Indonesia, Mongolia, North Korea, South Africa, The Philippines, Ukraine, United States and Vietnam.

![Figure 2.5 (a) FX Dry Cleaner unit and (b) FGX Compound Dry Cleaner.](image)

Pneumatic separation technologies originally developed and patented in the 1920’s have been recently redesigned and equipped with modern technologies including automation systems that provide a means of efficient separation performance control.
Air-based separation units utilize an upward flow of air to create a fluidized bed of high-density particles. Coal particles are not able to penetrate through the bed and thus float on top whereas high-density, high-ash content particles move through the bed. A vibrating table provides a mechanism to separate the coal floating on top of the bed from the reject material that forms the lower part of the bed. Honaker et. al. (2008), Patil and Parekh (2011), Zhang et. al. (2011) and Gupta et al. (2012) reported on the application of air table concentrators for cleaning coal.

### 2.2.1.1 Air-based density separation fundamentals

The dry beneficiation method utilizes the inherent density differences between mineral (or coal) and the host rock and might also utilize dry dense medium mixture of air-magnetite or air-sand suspension to affect their separation. The separation principle of a pneumatic air-based density separator is dependent on the behavior of solids under a state of fluidization and the relative settling rates of particles falling through a medium. The theory of equal settling of particles (Rittinger, 1867) explains the settling of particles having various diameters and specific gravities, which would each have equal terminal velocities falling through a medium of specific gravity $\rho_m$. The settling ratio is equal to the limiting ratio of diameters of particles that may be separated by free settling as given by Eq. (2.1):

$$ Settling \ Ratio = \frac{d_L}{d_H} = \frac{\rho_H - \rho_m}{\rho_L - \rho_m} $$

(2.1)
where $d_L$ is the diameter of particles of specific gravity $\rho_L$ and $d_H$ is the diameter of particles of specific gravity $\rho_H$ and $\rho_L < \rho_H$.

Hence, a light particle of density 1.40 gm/cm$^3$ and heavier particle of density 2.65 gm/cm$^3$ settling in air (density $\approx$ 0 gm/cm$^3$) have the settling ratio equal to 1.89 or about 2. Specifically, spherical particles with densities of 1.40 gm/cm$^3$ and 2.65 gm/cm$^3$ may be separated while falling in air if they are within a 2:1 ratio. Nevertheless, if air as a medium is replaced by water, the settling ratio will be 4.125 or about 4. Therefore a simple water medium process separation is possible within a 4:1 particle size ratio, as compared to a 2:1 particle size ratio for an air medium separator.

The various fluidization conditions in which solid particles are suspended and achieve fluid-like state by the passage of gas or liquid in the vertical direction opposite gravity are explained in Figure 2.6 (Kunii and Levenspiel, 1991). Initial introduction of upward moving fluid (gas or liquid) causes particulate resistance to the incoming flow resulting in a phenomenon called the "Fixed bed". However, with the increase of the upward fluid flow rate the particulate bed becomes agitated and expanding with void fractions developing in between until a point is reached when majority of the particles are suspended in the upward moving fluid. The threshold velocity of the fluid generates sufficient drag force to counterbalance the gravitational force acting on the particles. This is known as the state of "Minimum Fluidization". A further increase in fluid velocity (gas or liquid) results in significantly differing behavior between solid-liquid and solid-gas
bed. However, the dissertation considers the gas-solid or more specifically the behavior of the air-solid bed.

![Various forms of Fluidization](image)

Figure 2.6 Various forms of Fluidization (Kunii and Levenspiel, 1991).

Chase (2004) summarized the fluidization characteristics of most fine particle systems. Higher gas/air flow rate results in channeling and agitation and even can lead to bubbling of the fluidized solid bed. Feed flow rate however has a negative effect on fluidization and the bed becomes sluggish. The most undesirable situation arises with the turbulent
fluidization condition where solid particles start behaving erratically with aggressive momentum.

![Figure 2.7 Standard patterns of solid Fluidization (Chase, 2004).](image)

The pressure drop versus velocity diagram (Figure 2.7) describes the standard pattern of fluidization behavior of the packed bed and pressure drop across a cylinder. For the initial low velocity rate (less than $V_{om}$), the pressure drop is approximately proportional to the gas velocity. Initially, when the bed is packed, the pressure drop is slightly higher than the actual static pressure drop due to inter-particle cohesion at the junction point of the packed and fluidized bed. After this point, as the bed expands and voidage increases, the pressure drop remains insensitive to the increase in velocity ($V_0$). When the velocity of the gas decreases, the fluidized particles settle down into a loosely packed state with some voidage in between them.
Increased air-flow rate expands the bed surface and the bed surface stays horizontal irrespective of the angular inclination of the container. The pressure drop between any two points is given by Eq. (2.2) where the difference in pressure drop is equal to the difference between the hydrostatic heads of the two points (Luo and Chen, 2001),

\[ \Delta P = P_1 - P_2 = (h_1 - h_2)\rho \]  

(2.2)

where \( \Delta P \) is the pressure drop, N/m\(^2\); \( P_1 \) and \( P_2 \) are the pressures at the respective points, N/m\(^2\); and \( \rho \) is the density in Kg/m\(^3\).

The particle laden bed behaves like a fluid and particles lighter than the average density of the bed float on top while the heavier ones sink to the bottom. Optimum separation conditions require a stable dispersion fluidization rate and generation of micro-bubbles in the air-dense medium fluidized bed. The bed density must be time independent and uniform in the X,Y and Z plane and the bed medium must have low viscosity and high fluidity and can be expressed by Eq. (2.3):

\[ \rho_b = (1 - \varepsilon)\rho_p + \varepsilon\rho_g \]  

(2.3)

where \( \rho_b \) is the average density of the fluidized bed, \( \rho_p \) is the density of the solid particles, \( \rho_g \) is the density of the air and \( \varepsilon \) is the bed porosity.

Wei et. al. (1996) investigated the buoyancy effect along with the viscosity and the movement displaced distribution effects. Viscosity decreases with an increase in airflow rate. However, the movement displacement effect tends to have a large impact with minimum or maximum air velocity ranges. The displacement distribution effects can be
effectively controlled by regulating the medium particle size distribution and airflow rate. Wei and Chen (2001) studied the rheological characteristics and calculated the viscosity of fluidized bed by measuring the terminal velocity of spheres falling through the fluidized bed under the assumption that the fluidized particles behave as a Newtonian fluid. However, the experimental results showed that the bed behaves as a Bingham fluid and the plastic viscosity and yield stress can be obtained using a statistical regression model of the experimental data and the terminal settling velocity measurements of the falling spheres. The yield stress and the plastic viscosity is dependent on the size of the fluidized particles with larger particles generating a higher value. The drag coefficient can be calculated by the following equations (Chen and Wei, 2003):

\[ C_D = \frac{24}{Re_m} \left( 1 + 0.15 Re_m^{0.687} \right) \]  
\[ Re_m = \frac{d_0 u_r \rho_b}{\mu_e} \]  
\[ \mu_e = \frac{\mu + \tau_0 d_0}{3 \mu_r} \]

where \( C_D \) is the drag coefficient, \( Re_m \) is the particle Reynolds number, \( d_0 \) is the diameter of the falling object, \( \mu_r \) is the relative velocity between the object and the fluidized particles, \( \mu_e \) is the effective viscosity, and \( \tau_0 \) is the yield stress.

Chen and Yang (2003) investigated the air-dense medium fluidized bed separator for the beneficiation of 50 mm x 6 mm lump coal using a bed depth of 400 mm. However, the stable bed formation by suppressing of bubble growth do not provide enough space
for effective separation of particles larger than 50 mm size range. Experiments performed with a 1 m$^2$ laboratory air-dense medium fluidized bed showed that the bed height requirement increases to about 1200 mm to be effective in separating particles greater than 50 mm in size (Chen and Yang, 2003). The test program was successful in achieving an $E_p$ value of 0.02 and proved that a stable fluidized bed can be achieved with a constant bed density in order to treat coarse particles.

Wide-ranging studies were carried out on different size fractions by Luo et. al. (2002) and Houwelingen and Jong (2004) to investigate the parametric effects on an air-dense medium density separator (Figure 2.8). Luo and Chen (2001) confirmed that the behavior of the fine coal particles depends on the bubbles and back mixes due to the motion of the medium solids. Houwelingen and Jong (2004) further investigated the influence of airflow rate and moisture content on a 1:1 mixture of coal and shale (Figure 2.9 and Figure 2.10). The experimental results confirmed that with an increase in airflow rate through a sand bed the displacement of the lighter coal particles to the sink increases and the sink fraction gains in coal grade.
Figure 2.8 Partition curves for different size fractions (Luo et al., 2002).

Figure 2.9 Experiments with coal mixtures (50% coal and 50% shale) for 3 air velocities: (a) 6.5 m/s, (b) 6.9 m/s and (c) 7.0 m/s; Size of coal and shale: 20-30 mm (Houwenlingen and Jong, 2004).
2.2.1.2 Dual-Density Fluidized Bed Separator

Wei et al. (2003) investigated a three product dry-coal processing technology with dual-density air-dense medium fluidized bed by regulating the operational parametric effects as well as the physical properties of the medium solids and the particle size distribution effects. The simplified theory describing the physical principles governing the particle mixing or disintegration in a dual-density fluidized bed was proposed by Wei (1998),

\[
\frac{\rho_{sa} - \rho_f}{\rho_s - \rho_f} \left( 1 - \varepsilon_m + \frac{d_{sa}}{d_s} \varepsilon_m \right) - 1 = \frac{\rho_{sa}}{(\rho_s - \rho_f)g} \tag{2.7}
\]

\[
\rho_{sa} = \frac{\rho_{bm} - \rho_f \varepsilon_m}{1 - \varepsilon_m} \tag{2.8}
\]
where, $\rho_s$ is the actual magnetite powder (medium) density in gm/cm$^3$; $\rho_f$ is the density of air in gm/cm$^3$; $\rho_{sa}$ is the average density of the mixture in gm/cm$^3$; $\rho_{bm}$ is the average density of the fluidized bed in gm/cm$^3$; $d_s$ is the size of the magnetite powder in m; $d_{sa}$ is the size of mixture in m; $C_d$ is the coefficient of drag force; $U_r$ is the relative superficial velocity of gas-solid in m/s; $\varepsilon_m$ is the porosity of the mixture in %; $a$ is the acceleration of the particle in m/s$^2$; and $g$ is the acceleration due to gravity in m/s$^2$.

However, the density and size ($\rho_s$ and $d_s$ respectively) of the powder magnetite medium is greater than the average density and size of the feed particle-medium mixture the ratio of the total drag-force to buoyancy component can be expressed as:

$$
\frac{\rho_{sa} - \rho_f}{\rho_s - \rho_f} \left(1 - \varepsilon_m + \frac{d_{sa}}{d_s} \varepsilon_m\right) < 1
$$

(2.10)

The dual-density air-dense medium fluidized bed separator produces 3 products namely, clean coal, middling and reject and the optimum conditions demonstrate the formation of two stable beneficiation layers of varying densities in the axial direction (Wei, 1998). The stable stratification resulted in an upper layer with a density range of 1.5 to 1.54 gm/cm$^3$ and $E_p$ values of 0.06 - 0.09. The lower layer achieved a density range of 1.84 to 1.9 gm/cm$^3$ and $E_p$ values of 0.09 - 0.11 signifying acceptable separation efficiency (Wei, 1998).
Biswal et. al. (2002) and Sahu et. al. (2011) further investigated the applicability of an air-dense media fluidized bed separator for high ash low grade coal in Talcher coalfield, Orissa, India at a range of 25 mm x 6 mm feed particle size. The non-bubbling characteristics was confirmed by studying the dynamic stability of a 45 micron magnetite powder medium using varying parameters such as the bed expansion, bed density, superficial air velocity and bed voidage. The bed was reported to be stable under homogeneous conditions.

2.3 Problems with modern air-based density separators

The modern air-based density separators, such as the FGX compound separator, are effective in treating run-of-mine material in the size range of 75 mm to 6 mm and good separation is achieved between an effective size ratio of 2:1 and 4:1. However, separation performance declines considerably for particles smaller than 6 mm (Figure 2.11). It is noted that this effective particle size range is much smaller than most wet density-based separators due to the difference between the density of water and air (Patil and Parekh, 2011). Particles smaller than 6 mm do not segregate well and tend to report to the product side irrespective of their respective density ranges (Figure 2.11). Therefore, there is a demand to modify a dry density based separator for treating particles having size less than 6 mm. A study on a modified Bratney Company air table showed that dry separation of 6 mm x 1 mm fraction is feasible (Patil and Parekh, 2011; Gupta et. al., 2012). The dissertation focuses on the better understanding of the mechanism of the air-based density
separator and modifications that are needed to be applied to its design to effectively treat finer fraction of feed with less than 6 mm size using various modeling techniques.

2.4 Previous Modeling Attempts

Previous attempts to model the flow and separation performance of the air-based density separator have been mostly empirical in nature. Jianing (2006) and Luo et. al. (2006) attempted to develop a mathematical model to describe the phenomenological concepts of the complex physical phenomenon occurring in the air-based dense medium separator. Stress analysis, theoretical density models and movement equations over
various sections of the cross-section were derived lacking experimental validation. Honaker et. al. (2008) attempted a modified analysis using parametric process flow model to estimate the effect of recycling the middle density particle stream to the feed stream. A linear analysis was performed based on the flow diagram in Figure 2.12. If $F$, $P$, $M$, $L$ and $T$ are the mass flow rates of the feed, product, midlings, recirculating load and tailings respectively, the overall recovery $R_o$ can be determined according to the following expressions:

\[ P = L R_1 \quad (2.11) \]

\[ T = L(1 - R_1)(1 - R_2) \quad (2.12) \]

\[ R_0 = \frac{P}{F} = \frac{P}{P + T} = \frac{R_1}{R_1 + (1 - R_1)(1 - R_2)} \quad (2.13) \]

where, $R_1$ and $R_2$ are the partition numbers associated with the probability of a particle in a density fraction to report to the product and midlings stream respectively.

Using the washability data from each process stream and the resulting partition numbers, the overall circuit recovery values for particles in each density fraction were determined by Eqs 2.11, 2.12 and 2.13. The modified circuit with recycling middle density particle stream was predicted to provide a noteworthy improvement in separation efficiency as indicated to be around 40%. Recycling the middle density particle stream allows the rejection of about 95% of the high density rock while recovering nearly 100% of the 1.6 sp. gr. float material.
Similar applications based on the fluidized bed multi-phase flow concept have been modeled using CFD tools in the past (Gao et. al., 2009; Xu and Guan, 2003). Industrial vibrating screen decks have been modeled using the sliding mesh technique, though extensive literature is lacking on the subject. Several closure models have been derived to define suitable constitutive equations for multi-phase turbulent streams based on the kinetic theory of granular flow. Li et. al. (2009) studied the various drag models for simulating gas-solid flow in a turbulent fluidized bed. Classical drag models have been successful in many cases (Benyahia et. al., 2000), although dense phase fluidization models have been found to be less accurate. Syamlal and O’ Brien (1989) developed a suitable drag model in an attempt to simulate bubbles in a fluidized bed.

Akbari et. al. (2012) attempted to characterize the coal cleaning performance of the FGX separator using artificial neural network (ANN) model. The developed ANN model
was compared with a statistical regression model. However, neural network models require a large number of datasets to train properly (Boonkiatpong and Sinthupinyo, 2011). The total number of tests performed to generate the training dataset for the ANN model were 72 and hence would be insufficient to validate and train the model for performance improvement. Gupta (2011) attempted a simplified discrete element modeling (DEM) approach as shown in Figure 2.13. The DEM model was developed by simulating the settling of different density particles on top of a vibrating bed. The major drawback of the simplified DEM approach is the lack of air-flow modeling capabilities and hence would not be an ideal approach to model a vibrating air-table. However, the development of a full-scale CFD model of the laboratory scale air-based density separator have never been attempted before.

Figure 2.13 DEM simulating particle settlement on a vibrating bed (Gupta, 2011).
2.5 Scale Modeling Concept

The scale modeling concept involves the study of a physical system with the help a representative copy of the system which retains the relative magnitude of the material dimensions of the original system. Development and subsequent comparison of a scaled numerical prototype with the full-scale model helps in significant cost savings as the number of physical pilot scale plants is drastically reduced. The scaling theory relates to the fundamental aspects of the scientific method, which can be listed as, (a) experimentation, (b) theory and (c) computation (Wilson, 1990). The scale modeling technique belong to the experimental group (Emori et. al., 2000; Saito, 2008).

The primary objective of the scale modeling technique is to obtain a mathematical description of the dynamic behavior of the system and develop appropriate ratios which can subsequently be utilized to simulate a prototype model. The significant aspects of scale modeling method are listed in Table 2.2. Development of the scaling laws can be categorized into three different methods: (a) the Parameter approach, (b) the Eq. approach and (c) the Law approach (Saito, 2008). The Law approach (Emori, 1968) utilizes the dimensional analysis of Buckingham's $\pi$ theorem (Buckingham, 1914) and can satisfy each of the four aspects of the scale modeling technique described in Table 2.2. The fundamental aspect of the law approach assumes that all models are wrong but some are useful and consistent revision of the developed model with new physical and/or numerical results. The flowsheet in Figure 2.14 explains the concept of development of the scale model using numerical methods (Emori, et. al., 2000).
Table 2.2 Fundamental aspects of scale modeling technique (Saito, 2008)

<table>
<thead>
<tr>
<th>Statement of the problem</th>
<th>Study of the full-scale system and phenomenon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developing appropriate scaling laws</td>
<td>Understanding the governing mechanisms controlling the full-scale system</td>
</tr>
<tr>
<td>Model development</td>
<td>Validating the model (including CFD model) predictions</td>
</tr>
<tr>
<td>Optimization</td>
<td>Developing new design protocols and/or improving the performance of the existing system</td>
</tr>
</tbody>
</table>

Figure 2.14 Flowsheet describing the Scale model development and validation process (Emori et. al, 2000); SM - Scale Model, SL - Scaling Laws and NM - Numerical Model.

The scaling law approach with the help of numerical modeling have been successfully implemented to improve an over-spray paint capturing device (Tanigawa et. al, 2008), and a steel teeming process (Kuwana et. al., 2008). Scaling laws for combustion research (Emori and Saito, 1985) in pool and crib fires (Emori and Saito, 1983) and hazardous
forest fire whirl (Emori and Saito, 1982) were successfully developed and validated. Furthermore, scale modeling technique has also been utilized to investigate a fire accident in an automobile plant (Omar et. al., 2007).
CHAPTER 3: PARAMETRIC EVALUATION AND EMPIRICAL MODELING

3.1 Introduction

The laboratory scale air-based density separator was subjected to parametric evaluation using low rank Powder River Basin (PRB) coal. The PRB low rank coal was simulated as a model system to characterize the air table due to its wide range of size and density fractions. The Wyoming PRB coal obtained for the purpose of the test work was crushed and screened to prepare a 6 x 1 mm feed. A representative sample of the prepared material was characterized for proximate analysis and float sink analyses. The prepared material was processed through the modified laboratory air table separator. Currently, no laboratory or commercial air table equipment is available to economically dry process 6 x 1 mm size fraction. The goal of the test program was to investigate the possible development of an efficient dry processing method to process 6x1 mm finer size material and study the fundamental aspects of the separation process. The main components of a Bratney company air table manufactured by CIMBRIA Heid (Austria) developed for grain processing was used for the laboratory tests.

Operating parameters such as longitudinal and transverse angles, table frequency and air-flow rate were systematically varied in an effort to identify the important process parameters providing a significant impact on separation performance using a 3-level Box-Behnken design experimental program in which the central values were considered as well as the interactive effects of the significant parameters. The test results were used to
develop empirical models describing the performance response variables as a function of the operating parameter values and their associated interactions. Timed samples were collected for each test which was then weighed to determine mass yield and analyzed for ash and heating value content to quantify the amount of upgrading and energy recovery. The operating parameters were analyzed using t-statistics and tests were run at optimized conditions to maximize the product yield at a given product ash.

### 3.2 Process Description

The dry cleaning system employs the separation principles of a fluidized autogenous medium and a shaking table concentrator. The feed to the system is introduced into a hopper from which the underflow is controlled using vibrating feeder. The separation process generates multiple products of varying grades. Industrial and pilot scale units are equipped with dust collection systems to clean the recycled air and to remove the dust from air being emitted into the atmosphere. Major components of the table include a deck, vibrator, air-blower fan and table deck angle adjustment mechanism. A blower provides air that passes through holes on the deck surface at a rate sufficient to transport and fluidize the fine, high-density particles. An upward movement of air through the table deck suspends the light particles while the heavy particles remain on the deck and are driven by table vibration in a direction that is opposite of the light suspended particles. Upon introduction of the feed into the separation chamber, a particle bed of certain thickness is formed on the deck. The particles near the bottom of the bed directly contact the vibrating deck and move from the feed end towards the front discharge end.
under the effect of the vibration-induced inertia force. Upon striking an artificial lip existing along the edge of the deck, the heavier particles lose momentum and accumulate to build the bed while the lighter particles travel along the width of the table towards the opposite end of the table. Light particles are lifted up and travel along the deck mesh surface at a higher elevation than the higher density particles before discharging over the lip along the product side. As such, light particles create the upper layer of the bed and are collected along the side of the table. Particles of sufficient density are able to settle through the autogenous medium formed due to the fluidized bed of particles and report to the deck surface. These heavy particles are forced by both vibration and the continuous influx of new feed material to transport toward the front end of the table where the final refuse is collected.

3.3 Experimental

3.3.1 Laboratory Air Table

The laboratory scale air-based density separator used for the project designated as LAB-GA is manufactured by CIMBRIA HEID. The required setup was completed at University of Kentucky laboratories (Figure 3.1) and it was equipped with a fume hood and a dust collection system to prevent fire and deal with any other resultant hazards.
The table was subject to major modifications and the initial 6 mm screen deck was replaced with a 1 mm mesh screen. The flap gates controlling the outflow of material were disbanded and an artificial lip put in its place to facilitate a continuous flow of material and aid the formation of the bed. These modifications transformed the unit from a batch operation to a continuous process (Figure 3.2).
3.3.2 Experimental Procedure

Experiments were conducted based on the Box-Behnken design using the various air table parameters such as vibration frequency, air speed controlled by the fan, the cross-flow angles, and their levels as shown in Table 3.1. The location of the fan blower is closer to the low density particle discharge away from the feed end and thus has an augmenting effect on the mass flow rate of air at the light particle discharge end. A feed rate of about 204 kg/hr was determined to provide desirable table performance and thus was kept constant throughout the test program. The feed which had an ash content of 26% was prepared by screening the crushed sample (6x1 mm).

At the onset of a test, the air table angles and shaking frequency were adjusted before starting the fan which provided required airflow to the table. After achieving the required fan frequency, the feed to the table was initiated. A stable bed formed very quickly and samples were collected after achieving a steady state operation. A feed sample was also collected for each test.

Table 3.1 Air table parameters and their levels used in the test program which followed a 3-level Box-Behnken design

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan Frequency (Hz)</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Table Frequency (Hz)</td>
<td>30</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>Longitudinal Angle (°)</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>Transverse Angle (°)</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Figure 3.3 shows the sample collection points A, B, C, D and E for each test. Each sample was weighed and analyzed for ash content. This data provided an indication of the
direction of movement of rock and coal particles under various operating conditions. However, for final analysis, sample A and B were considered as reject, while samples C, D and E were treated as product.

![Figure 3.3](image)

**Figure 3.3** Figure depicting sample collection points A, B, C, D and E.

The 3-level Box-Behnken experimental design required a total of 26 tests to be conducted and six samples were collected for each test including a feed sample to quantify the inherent variability of the feed. A total of 156 samples were then subject to ash analysis. A representative sample collected from each of the test sample was crushed in a grinding machine to produce ultrafine samples for ash analysis. The data was analyzed using Design Expert software and empirical models were developed to describe product ash and yield in terms of operating parameter values. The sensitivity of each parameter and the developed empirical models were analyzed using method of analysis of variance. Additional tests were conducted to validate the optimum conditions predicted by the empirical models.
3.3.3 Coal Sample Characterization

Characterization performed on the delivered sub-bituminous coal sample revealed that the majority of the sample (58.36%) was over 2 mm in size with only a small amount (1.01%) being ultrafine with a particle size smaller than 0.60 mm. The sample had a very high inherent moisture content of around 21% with a dry ash % of around 26%. The heating value of the sample was found to be mid-range around 9305 BTU (Table 3.2). The calculated overall feed ash and heating value obtained from the washability data shown in table 3.3 is relatively close to the values from the particle size analysis. The difference can be attributed to propagated error. The washability data clearly shows significant feed weight having a density below 1.2 which reflects the higher inherent moisture content. The ash content of this fraction is 5.58 % which represents the lowest ash content achievable at a 1.2 density cut-point.

By plotting a cumulative float weight versus cumulative product ash content, the ultimate cleanability of the coal can be assessed as shown in Figure 3.4 (a). Based on this curve, the optimum separation would provide a decrease in ash content from 24.83% to around 8% while recovering 75% of the feed weight. The separation density required would be around 1.7 which is a theoretical projection and near the cut-point achievable by air table separators (Honaker et. al., 2008). Figure 3.4(b) shows a direct linear correlation between the dry heating value of the coal and the ash content. It clearly indicates the positive impact of coal cleaning on the heating value.
Table 3.2 Size-by-size analysis of the PRB coal sample *

<table>
<thead>
<tr>
<th>Size Ranges (mm)</th>
<th>Dry Wt. (%)</th>
<th>Moisture (%)</th>
<th>Dry Ash (%)</th>
<th>BTU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plus 2.00</td>
<td>58.36</td>
<td>19.17</td>
<td>32.43</td>
<td>8518</td>
</tr>
<tr>
<td>2.00</td>
<td>19.80</td>
<td>22.32</td>
<td>19.53</td>
<td>10358</td>
</tr>
<tr>
<td>1.68</td>
<td>20.83</td>
<td>22.70</td>
<td>18.24</td>
<td>10461</td>
</tr>
<tr>
<td>0.60</td>
<td>1.01</td>
<td>20.94</td>
<td>20.22</td>
<td>10300</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td>20.55</td>
<td>26.79</td>
<td>9305</td>
</tr>
</tbody>
</table>

Table 3.3 Float-Sink data of the PRB coal sample *

<table>
<thead>
<tr>
<th>Density Ranges (gm/cm³)</th>
<th>Dry Wt. (%)</th>
<th>Dry Ash (%)</th>
<th>BTU</th>
<th>Cum. Wt (%)</th>
<th>Cum. Ash (%)</th>
<th>Cum. BTU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Float 1.2</td>
<td>23.59</td>
<td>5.58</td>
<td>12230</td>
<td>23.59</td>
<td>5.58</td>
<td>12230</td>
</tr>
<tr>
<td>1.2</td>
<td>14.79</td>
<td>6.12</td>
<td>12103</td>
<td>38.37</td>
<td>5.79</td>
<td>12181</td>
</tr>
<tr>
<td>1.3</td>
<td>25.82</td>
<td>8.85</td>
<td>11727</td>
<td>64.20</td>
<td>7.02</td>
<td>11998</td>
</tr>
<tr>
<td>1.4</td>
<td>7.33</td>
<td>10.59</td>
<td>11368</td>
<td>71.52</td>
<td>7.39</td>
<td>11934</td>
</tr>
<tr>
<td>1.6</td>
<td>12.41</td>
<td>42.06</td>
<td>7172</td>
<td>83.93</td>
<td>12.51</td>
<td>11230</td>
</tr>
<tr>
<td>1.8</td>
<td>16.07</td>
<td>89.20</td>
<td>452</td>
<td>100.00</td>
<td>24.83</td>
<td>9498</td>
</tr>
<tr>
<td>Sink</td>
<td>1.8</td>
<td>1.3</td>
<td>1</td>
<td>42</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td>24.83</td>
<td>9498</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* where Wt. is "Weight", Cum. is "Cumulative"

Figure 3.4 Cleanability of the PRB coal sample, (a) - Cum. Ash (%) vs Cum. Wt. (%) and (b) Ash (%) vs Heating Value (BTU).
3.4 Results and Discussion

Figure 3.5 depicts typical sets of performance data collected for each experiment plotted in the form of cumulative yield versus cumulative product ash. The analysis of experimental results show that it is possible to get a product containing as low as 7% ash with 55% product yield. By comparing the best of the seven test performances at a 7% product ash with the theoretical performance (float-sink data), the lost yield resulting from the inefficiencies of the air table is 10% which results in an organic efficiency of 84.62%. Further increase in product yield up to 75% slightly increases the product ash to 8.5%.

Figure 3.5 Plot showing typical data sets collected for each experiment.
The data shows that the clean coal ash content varies between about 7 % to 17 % within the test program matrix. This clearly indicates the flexibility of the equipment providing required product ash content while optimizing table operating parameters. The data thus obtained were analyzed using the Design Expert software. Empirical models describing the product yield and ash as a function of the operating parameter values can be derived in the form of:

Product Ash (%) = -25.24 + 0.15 × Fan Frequency + 0.72 × Table Frequency - 1.81 × Longitudinal Angle + 7.68 × Transverse Angle - 0.19 × Table Frequency x Transverse Angle

Product Yield (%) = 97.13 + 1.39 × Fan Frequency - 2.57 × Table Frequency - 16.33 × Longitudinal Angle + 4.67 × Transverse Angle

The analysis of variance of Eqs. (3.1) and (3.2) are provided in Tables 3.4 and 3.5. The associated p-values ("Prob > |F|") are interpreted as the probability of realizing a coefficient as large as that observed, when the true coefficient equals zero. In other words, small values of $p$ (less than 0.05) indicate significant coefficients in the model. The model Eqs. suggest that the effects of all four parameters are highly significant as indicated by their $F$-values (Tables 3.4 and 3.5).
### Table 3.4 Analysis of Variance of Table for product Ash

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F Value</th>
<th>p-value Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>156.47</td>
<td>5</td>
<td>31.29</td>
<td>12.27</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>A-Fan Blower Frequency</td>
<td>28.23</td>
<td>1</td>
<td>28.23</td>
<td>11.07</td>
<td>0.0034</td>
</tr>
<tr>
<td>B-Table Frequency</td>
<td>52.33</td>
<td>1</td>
<td>52.33</td>
<td>20.52</td>
<td>0.0002</td>
</tr>
<tr>
<td>C-Longitudinal Angle</td>
<td>9.84</td>
<td>1</td>
<td>9.84</td>
<td>3.86</td>
<td>0.0635</td>
</tr>
<tr>
<td>D-Transverse Angle</td>
<td>51.63</td>
<td>1</td>
<td>51.63</td>
<td>20.25</td>
<td>0.0002</td>
</tr>
<tr>
<td>BxD</td>
<td>14.43</td>
<td>1</td>
<td>14.43</td>
<td>5.66</td>
<td>0.0274</td>
</tr>
<tr>
<td>Residual</td>
<td>50.99</td>
<td>20</td>
<td>2.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lack of Fit</td>
<td>50.26</td>
<td>19</td>
<td>2.65</td>
<td>3.62</td>
<td>0.3947</td>
</tr>
<tr>
<td>Pure Error</td>
<td>0.73</td>
<td>1</td>
<td>0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>207.46</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3.5 Analysis of Variance of Table for product yield

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F Value</th>
<th>p-value Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>6151.88</td>
<td>4</td>
<td>1537.97</td>
<td>15.63</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>A-Fan Blower Frequency</td>
<td>2325.23</td>
<td>1</td>
<td>2325.23</td>
<td>23.63</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>B-Table Frequency</td>
<td>1980.89</td>
<td>1</td>
<td>1980.89</td>
<td>20.13</td>
<td>0.0002</td>
</tr>
<tr>
<td>C-Longitudinal Angle</td>
<td>800.25</td>
<td>1</td>
<td>800.25</td>
<td>8.13</td>
<td>0.0095</td>
</tr>
<tr>
<td>D-Transverse Angle</td>
<td>1045.51</td>
<td>1</td>
<td>1045.51</td>
<td>10.63</td>
<td>0.0037</td>
</tr>
<tr>
<td>Residual</td>
<td>2066.07</td>
<td>21</td>
<td>98.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lack of Fit</td>
<td>2065.55</td>
<td>20</td>
<td>103.28</td>
<td>196.04</td>
<td>0.0562</td>
</tr>
<tr>
<td>Pure Error</td>
<td>0.53</td>
<td>1</td>
<td>0.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>8217.95</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The product yield, combustible recovery and product ash values increase with decrease in longitudinal angle due to the augmented slope towards the product discharge end. During the testing, it was observed that with an increase in fan frequency the low density coal particles were easily lifted by upward air medium and move quite freely towards product end. The higher flow of air coupled with the increase in the longitudinal angle helps in maintaining a constant bed depth over the entire cross-section of the table thus maximizing combustible recovery and yield (Figure 3.6). Moreover, the table is designed in such a way that the section of the table where light particles travel to the low density discharge end comes closer to air blower outlet which may increase the airflow rate thus regulating the effect of slope on coal particles. Only particles light enough to float in the air travel to the discharge end, hence reducing the product ash and yield as the longitudinal slope is increased. The introduction of particle feed at a constant rate results in the dampening of airflow at the feed end, which directly affects the flow rate at the low density discharge end.

An increase in the transverse angle increased the product yield while increasing the product ash. This could be attributed to the higher flow rate of solids to the product end, as it becomes increasingly difficult for particles to overcome the effect of gravity to report to the reject end. As the higher fraction of stratified material move towards product end the separation density increases hence reducing the quality of clean coal.
The higher table frequency decreased the product yield and ash thus indicating larger movement of material towards reject end. During testing at higher table frequency, it was observed that the entire particle bed moved towards reject end. This behavior may be due to the fact that at higher table frequency, the particles residence time is very low and it starts discharging prior to the formation of the bed due to effect of mass action with increase in energy imparted on the table due to the vibration.

The results of the Box-Benkhen design tests are plotted in Figure 3.7. These results show that it is possible to obtain a clean coal having approximately 7.7 % ash with about 60 % yield from a feed coal having 26% ash with a combustible recovery of 65 %. The
ash rejection is about 80%. Many of the test runs achieved recovery values close to the washability analysis optimum.

![Graph of Combustible Recovery vs Product Ash Content](image)

**Figure 3.7** Combustible Recovery of the test runs vs washability data.

Further tests were conducted to verify the optimum test conditions identified using empirical relationships generated from data collected from the parametric experimental program. The optimal conditions and results are listed below:

<table>
<thead>
<tr>
<th>Fan Blower Freq</th>
<th>Table Freq</th>
<th>Long. Angle</th>
<th>Trans. Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 hz</td>
<td>35 hz</td>
<td>1.5°</td>
<td>5°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Feed Ash</th>
<th>Prod. Ash</th>
<th>Yield</th>
<th>Combustible Recovery (%)</th>
<th>Separation Efficiency (%)</th>
<th>Ash Rejection (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.62 %</td>
<td>7.04 %</td>
<td>66.60 %</td>
<td>84.37 %</td>
<td>66.74 %</td>
<td>82.4 %</td>
</tr>
</tbody>
</table>
Partition curve data from float sink analysis of the optimized test run product and the reject samples reveal that it is possible to obtain low Ep values of 0.12 and 0.22 and separation densities between 1.58 gm/cm$^3$ and 1.88 gm/cm$^3$ as shown in Figure 3.8 and provided in tabulated form in Table 3.6. Here Test 1 and Test 2 correspond to the optimized test runs.

Table 3.6 Results and comparison between the optimized tests

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimized Test 1</th>
<th>Optimized Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed Ash (%)</td>
<td>26.62</td>
<td>26.62</td>
</tr>
<tr>
<td>Product Ash (%)</td>
<td>9.21</td>
<td>7.04</td>
</tr>
<tr>
<td>Tailings Ash (%)</td>
<td>77.19</td>
<td>65.66</td>
</tr>
<tr>
<td>Mass Yield (%)</td>
<td>74.39</td>
<td>66.60</td>
</tr>
<tr>
<td>Combustible Recovery (%)</td>
<td>92.04</td>
<td>84.37</td>
</tr>
<tr>
<td>Separation Density (gm/cm$^3$)</td>
<td>1.88</td>
<td>1.58</td>
</tr>
<tr>
<td>Ep</td>
<td>0.22</td>
<td>0.12</td>
</tr>
<tr>
<td>Percentage (%) by-pass</td>
<td>7.86</td>
<td>2.77</td>
</tr>
<tr>
<td>Organic Efficiency (%)</td>
<td>92.14</td>
<td>97.23</td>
</tr>
</tbody>
</table>
The data from the sample analysis and optimized tests performed during the test program were statistically examined using commercially available Design Expert software and the empirical models were found to be statistically significant, accurately describing product ash and mass yield as a function of their operating variables. The analysis of variance revealed that all four parameters, i.e., fan frequency, table frequency, longitudinal angle and transverse angle were significant factors. Therefore, the four parameters must be considered for optimizing the process. Simulation results show that a sharp separation can be achieved for the 6x1 mm PRB coal fraction with minimal coal loss. Ash analysis of optimized test data shows that it is possible to obtain a clean coal
having approximately 7% ash with about 67% yield from a feed coal having 26% ash with a combustible recovery of 85% with an ash rejection of 82%.
CHAPTER 4: NUMERICAL MODELING OF AN AIR-BASED DENSITY SEPARATOR

4.1 Introduction

The importance of numerical modeling as a maturing tool for the mining and mineral industry is well documented. It has wide ranging applications in simulating mine ventilation networks (Heerden and Sullivan, 1993; Srinivasa et. al., 1993, Wala et. al, 2003), rock mechanics (Horii et. al, 1999; Sitharam et. al, 2001), predicting performance characteristics of various process technologies such as a classifying and dense medium cyclones (Hsieh, 1988; Suasnabar and Fletcher, 1999; Delgadilo and Rajamani, 2009; Brennan, 2006; Gupta et. al., 2008), fluidized bed systems (Bell et. al., 1997; Witt and Perry, 1996), spirals (Wang and Andrews, 1994; Mathews et. al, 1996), pneumatic conveying (Ratnayake et. al., 2004) and thickeners (Kahane et. al., 1997; Nguyen et. al., 2006; Nguyen et. al., 2012; Heath et. al., 2012). However, development of a comprehensive CFD model of an air-based density separator has not been previously undertaken.

Gupta (2011) developed a simplistic discrete element (DEM) model of a vibrating, fluidized air-table by simulating particle settlement according to density on top of a vibrating bed. Inherent limitations with the DEM model prevented the study of the effects of the air-flow patterns and how it affects the fluidizing of the bed. Similarly, theorized effects such as vortices being formed around riffles, which in turn might impact the high density particle movement on top of the bed, could not be investigated. This Dissertation
details the first attempt to develop a comprehensive CFD model of an air-based vibrating riffle table which was pursued to better understand the complex process. Upon model validation using experimental data, it can be used to better understand the separation mechanisms and the impact of the geometric parameters. A successful model will help in identifying the optimum design and assist in the development of full-scale industrial units.

4.2 Development of the Numerical Model

The exponential growth in modern computational capabilities have made it possible to solve a wide range of incompressible, compressible, laminar and turbulent fluid flow problems. Modern numerical and CFD modeling techniques have the capability to model complex geometries and an extensive array of transport phenomenon. Commercially available CFD softwares such as Ansys Fluent®, Cradle® and Ansys CFX® have the ability to simulate multiphase flows. These can be used to analyze gas-liquid, gas-solid, liquid-solid and gas-liquid-solid flows under various laminar and turbulent formulations. Numerical simulations do not attempt to replace the experimental measurements completely but the amount of experimentation and the overall costs involved in each pilot scale plant development and the testing phase can be significantly reduced. The commercial software used for the simulation in this study was Ansys Fluent®.

The operation of the laboratory air-based density separator can be defined as an incompressible multi-phase (constant density air phase and the particle phase) flow field with the added complexity of a vibrating deck oscillating with time and the screen mesh
acting as a porous media regulating the air-flow on top the deck. The steps in the development of a suitable CFD model is described in Table 4.1.

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Statement of the problem</td>
<td>Information about the flow</td>
</tr>
<tr>
<td>2. Mathematical model</td>
<td>Deriving the transport Eqs.</td>
</tr>
<tr>
<td>3. Pre-processing</td>
<td>Geometry and mesh generation</td>
</tr>
<tr>
<td>4. CFD software</td>
<td>Boundary conditions, implementation, debugging</td>
</tr>
<tr>
<td>5. Simulation run</td>
<td>Parameters, time-step</td>
</tr>
<tr>
<td>6. Post-processing</td>
<td>Data analysis and visualization</td>
</tr>
<tr>
<td>7. Verification</td>
<td>Model validation and adjustment</td>
</tr>
</tbody>
</table>

### 4.2.1 Experimental Data Collection

The laboratory air-table (Figure 3.1 and Figure 3.2, Chapter 3) is equipped with dual digital controls for the table vibration frequency and the air blower rate. The material is fed to the screen deck through a vibratory feeder. The table is equipped with a separate digital control to regulate the incoming feed. The original design consisted of flap gates closing all sides of the table deck and hence was a batch process. However, the table was modified and the flap gates were replaced with an artificial lip thus transforming the operation from batch to continuous process. The artificial lip helps in bed formation before discharge. The angle of inclination of the deck can also be controlled by two different angle control wheels, one for the transverse angle (range 0° to 8°) and the other
for the horizontal angle (range $0^\circ$ to $8^\circ$). Further modifications include the replacement of the original screen having 6 mm aperture with one containing 1 mm aperture to allow the treatment of finer particles. The parametric evaluation and the statistical test program is detailed in Chapter 3.

As established in Chapter 3, the major operating parameters are the feed rate, deck inclination angle, air-flow rate, vibratory motion frequency, density and size of the particles. The blower positioned below the deck provides the required airflow. It is noted that the blower position is not in the center of the table but located closer to the low density discharge end which contributes to the unequal airflow across the table. The blower frequency range is between 20 Hz and 55 Hz which controls the mass-flow of air passing through the deck. Air comes from below the deck and passes through the mesh and forms a fluidized medium of particles. The table is fed particles from the side and mixes with the fluidized air and forms the particle bed on top of the screen mesh. The fluidized particle bed with a defined bed density creates particle stratification based mainly on density and shape. heavy particles are fluidized or remain on the table deck where table vibration energy through frequency modulation and amplitude moves the high density particles across the table along the riffles and into streams A and B exiting the table front. There exist a stream comprised of both low and high density particles that remain unseparated that report in zones B and C (Figure 4.1) which may need to be retreated to recover the valuable material while meeting grade requirements. The bed formation and segregation is heavily modulated by the riffles on top of the deck.
Airflow velocity measurements (in m/s) and pressure drop (in Pascals) across the table deck were recorded with the help of a digital vane anemometer and a digital manometer respectively, at 6 different locations on the table deck (Figure 4.2). The deck has a honeycomb structure directly below it as shown in Figure 4.2, which regulates the uniformity of the air-flow. The table geometry was divided into 6 parts (Figure 4.3) to characterize air-flow in each section of the deck. The air velocity measurements were subsequently multiplied with the density of air (1.225 kg/m$^3$) in order to determine the mass flux of airflow entering the porous zone per second. The following plot shows the variance of air-velocity plotted against corresponding pressure drop measurements for different sections of the table (Figure 4.4). The table vibration frequency ranges from 20 Hz to 50 Hz and the amplitude measurement recorded a 7 mm oscillation range.
Figure 4.2 Airflow and pressure drop measurements using a digital vane anemometer and a digital manometer, respectively.

Figure 4.3 Table deck divided into 6 sections to simplify calibration.
Pressure and flow measurements recorded across the deck showed that the honeycomb structure directly below the mesh uniformly regulates the air-flow corresponding to the fan blower frequency. To calibrate the air-flow resistance factors and define the porous media characteristics of the mesh zone, an average dataset was prepared combining the measurements recorded in the 6 different sections (Table 4.2, Figure 4.5).
Table 4.2 Average air-flow and pressure-drop measurement data

<table>
<thead>
<tr>
<th>Fan Freq (Hz)</th>
<th>Air Velocity (m/s)</th>
<th>Air Flow Volumetric (m$^3$/s)</th>
<th>Mass Flow Air (Kg/s)</th>
<th>Pressure Drop (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.000000</td>
<td>0.00</td>
</tr>
<tr>
<td>20.00</td>
<td>0.93</td>
<td>0.25</td>
<td>0.308000</td>
<td>3.62</td>
</tr>
<tr>
<td>25.00</td>
<td>1.18</td>
<td>0.32</td>
<td>0.389674</td>
<td>5.08</td>
</tr>
<tr>
<td>30.00</td>
<td>1.42</td>
<td>0.38</td>
<td>0.468396</td>
<td>8.13</td>
</tr>
<tr>
<td>35.00</td>
<td>1.66</td>
<td>0.45</td>
<td>0.547118</td>
<td>10.80</td>
</tr>
<tr>
<td>40.00</td>
<td>1.92</td>
<td>0.52</td>
<td>0.633712</td>
<td>14.75</td>
</tr>
<tr>
<td>45.00</td>
<td>2.17</td>
<td>0.58</td>
<td>0.715387</td>
<td>18.92</td>
</tr>
<tr>
<td>50.00</td>
<td>2.38</td>
<td>0.64</td>
<td>0.784268</td>
<td>24.55</td>
</tr>
<tr>
<td>55.00</td>
<td>2.60</td>
<td>0.70</td>
<td>0.860038</td>
<td>29.17</td>
</tr>
</tbody>
</table>

Figure 4.5 Average air velocity vs pressure drop across the deck.

4.2.1.1 Porous Media Resistance Coefficients

The flow of a fluid through a porous medium and the resistance characteristics of the porous media were derived by Henry Darcy (1856). The flow is defined by a simple relationship between the instantaneous flux discharge rate through a porous medium, the
pressure drop over a given distance and the viscosity of the fluid. The screen mesh on the table deck acts as a porous zone and the porous resistance coefficients based on the recorded pressure drop and air velocity data were calculated based on a theoretical porous media thickness of 5 mm. The assumption of 5 mm thickness for the porous media is based on the fact that the table deck has a secondary screen of 6 mm aperture 5 mm below the original deck. Experimental data that is available in the form of pressure drop against velocity through the porous component can be extrapolated to determine the coefficients for the porous media. To affect a pressure drop across a porous medium of thickness, $\Delta n$, the coefficients of the porous media are determined in the manner described below.

Porous media are modeled by the addition of a momentum source term to the standard fluid flow Eqs.. The source term $S_i$ is composed of two parts: i) a viscous loss term (the first term on the right hand side of Eq. 4.1) and ii) an inertial loss term (the second term on the right hand side of Eq. 4.1) (Darcy, 1856; Ansys Fluent©, 2011).

$$S_i = -(\Sigma_{j=1}^{3} D_{ij} \mu v_j + \Sigma_{j=1}^{3} C_{ij} \frac{1}{2} \rho |v| v_j)$$ (4.1)

where $S_i$ is the source term for the $ith$ (x, y or z) momentum Eq., $|v|$ is the magnitude of the velocity, $\mu$ is the molecular viscosity, $\rho$ is the density of air while $D$ and $C$ are prescribed matrices. The momentum loss contributes to the pressure gradient in the porous zone, creating a pressure drop proportional to the velocity squared in the porous zone.
Considering the case of the simple homogeneous porous media:

\[
S_i = -\left( \frac{\mu}{\alpha} v_i + C_2 \frac{1}{2} \rho |v| v_j \right)
\]  \hspace{1cm} (4.2)

where \(\alpha\) is the permeability and \(C_2\) is the inertial resistance factor.

From the dataset recorded in Table 4.2, an \(x-y\) curve can be plotted to create a trend-line through these points yielding the following Eq. (Figure 4.5):

\[
\Delta P = 4.7343 \nu^2 - 1.187 \nu
\]  \hspace{1cm} (4.3)

where \(\Delta P\) is the pressure drop and \(\nu\) is the air velocity.

Note that a simplified version of the momentum Eq., relating the pressure drop to the source term, can be expressed as:

\[
\nabla P = S_i
\]  \hspace{1cm} (4.4)

or,

\[
\Delta P = -S_i \Delta n
\]  \hspace{1cm} (4.5)

Hence, comparing Eq (4.3) to the momentum Eq (4.2) yields the following curve coefficients:

\[
4.7343 = C_2 \frac{1}{2} \rho \Delta n
\]  \hspace{1cm} (4.6)
with default air density, $\rho = 1.225 \text{ kg/m}^3$ (Ansys Fluent®, 2011), and a porous media thickness, $\Delta n$, assumed to be 5 mm (0.005 m) in this example, the inertial resistance factor, $C_2$ has a value of 1545.8939.

Likewise,

$$-1.187 = \frac{\mu}{\alpha} \Delta n \quad (4.7)$$

with air viscosity, $\mu = 1.7894 \times 10^{-5}$ (Ansys Fluent®, 2011), the viscous inertial resistance factor, $\frac{1}{\alpha} = -13267016.88$.

The resultant factors were calculated to account for the porosity of the mesh. Though the best fit curve yielded negative coefficients only the positive magnitude is considered (Ansys Fluent®, 2011).

### 4.2.2 Numerical Model Theory

Development of a suitable mathematical model to characterize the multi-phase flow problem significant to the air-based density separator requires solving conservation Eqs. for mass and momentum. Since, the air table does not involve heat transfer or compressibility, no additional energy Eqs. need to be considered. The general form of the continuity Eq. (Pedlosky, 1987) valid for both compressible and incompressible flow is given by (Ansys Fluent®, 2011):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_m \quad (4.8)$$
where, $\rho$ is the density of the fluid, $t$ is time and $\mathbf{v}$ is the flow velocity vector field. The source term $S_m$ is the mass added to the continuous phase from a secondary dispersed phase. If $S_m = 0$, then Eq. 4.8 becomes one of Euler Eqs. (Euler, 1757).

The conservation of momentum in an inertial reference frame is described by (Batchelor, 1967):

$$\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \nabla \cdot (\bar{\mathbf{t}}) + \rho \mathbf{g} + \mathbf{F}, \quad (4.9)$$

where $p$ is the static pressure, $\bar{\mathbf{t}}$ is the stress tensor, and $\rho \mathbf{g}$ and $\mathbf{F}$ are the gravitational body force and external body forces respectively. $\mathbf{F}$ also contains other model-dependent source terms such as porous-media formulations. The stress tensor $\bar{\mathbf{t}}$ is given by:

$$\bar{\mathbf{t}} = \mu[(\nabla \mathbf{v} + \nabla \mathbf{v}^T) - \frac{2}{3} \nabla \cdot \mathbf{v} I] \quad (4.10)$$

where $\mu$ is the molecular viscosity, $I$ the unit tensor, and the second term on the right hand side the effect of volume dilation.

4.2.2.1 Approach to Multiphase Modeling

Modern computational techniques offer two distinct approaches for numerical calculation of multi-phase flows, (a) the Euler-Lagrange approach (Williams, 1985) and (b) the Euler-Euler approach (Gidaspow, 1994). The Euler-Lagrange approach treats the fluid phase as a continuum by solving the Navier-Stokes Eqs., while the dispersed discrete phase is solved by tracking a large number of particles, bubbles, or droplets.
through the calculated velocity field (Gauvin et. al., 1975). However, the basic assumption made in the Lagrangian model is that the dispersed secondary phase occupies a low volume fraction relative to the primary Eulerian fluid phase. Hence, the model is inappropriate for modeling fluidized bed systems like the air-based density separator where the volume fraction of the secondary particle phase is non-trivial and cannot be ignored. The theoretical model selected for the purpose of simulating the incompressible multi-phase flow field of the air-based density separator is the Eulerian multi-phase Dense Discrete Phase model (DDPM) available in Ansys Fluent©.

The Euler-Euler approach treats the different phases as interacting continuum. The concept of phase specific volume fraction is introduced as the specified volume of a particular phase cannot be occupied by another phase. This approach is frequently used for dense particulate flows as it is suitable to model the inter-particle stresses using space-time gradient of the volume fraction (Batchelor 1988, Gidaspow 1994). The distinct volume fractions are treated as continuous space-time functions with their summation equal to unity. The derived conservation Eqs. for each phase have uniform structure and constitutive relations are obtained by the application of kinetic theory for the granular particle flow is applied for closure. The Eulerian model is the most complex multi-phase model available in Ansys Fluent© which solves a set of \( n \) momentum and continuity Eqs. for each phase. Granular phase (air-solid) coupling is achieved through interacting phase exchange coefficients. However, the Eulerian model requires defining separate classes to handle different particle size distributions. The Dense Discrete Phase (DDPM) model operating under the Eulerian framework overcomes the limitations imposed by the
general Lagrangian model by extending the continuity and momentum Eqs. to account for the low volume fraction assumption of the particle phase thus eliminating the need to define separate classes to handle particle size distributions (Popoff and Braun, 2007).

4.2.2.2 Conservation Eqs. (Eulerian model)

Eulerian multiphase approach treats each phase as a separate continuum phase and the continuity (conservation of mass) and conservation of momentum Eqs. for any phase \( q \) in general form are described by Eqs. (4.11) and (4.12), respectively (Roco, 1993; Ansys Fluent©, 2011):

\[
\frac{\partial}{\partial t} (\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q) = \sum_{p=1}^{n} (\dot{m}_{pq} - \dot{m}_{qp}) + S_q, \tag{4.11}
\]

where \( \alpha_q \) is the \( q^{th} \) phase volume fraction, \( \vec{v}_q \) denotes the velocity of the phase \( q \), \( \dot{m}_{pq} \) and \( \dot{m}_{qp} \) represents the mass transfers from the \( p^{th} \) to the \( q^{th} \) phase and from the \( q^{th} \) to the \( p^{th} \) phase respectively. The mass source term \( S_q \) is zero by default; however, if there exists a different mass source, it must be defined as such.

\[
\frac{\partial}{\partial t} (\alpha_q \rho_q \vec{v}_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q \vec{v}_q) = -\alpha_q \nabla p + \nabla \cdot \vec{f}_q + \alpha_q \rho_q \vec{g} + \sum_{p=1}^{n} (\vec{R}_{pq} + \dot{m}_{pq} \vec{v}_{pq} - \dot{m}_{qp} \vec{v}_{qp}) + (\vec{F}_q + \vec{F}_{lift,q} + \vec{F}_{vm,q}), \tag{4.12}
\]

where \( \vec{F}_q \) is an external body force, \( \vec{F}_{lift,q} \) is a lift force, \( \vec{F}_{vm,q} \) is a virtual mass force and \( p \) is the pressure shared by all phases. The inter-phase velocity term \( \vec{v}_{pq} \) is defined by the modal mass transfer phenomenon, i.e., if mass is being transferred from the \( p^{th} \) phase
to the \( q^{th} \) phase \((\dot{m}_{pq} > 0)\) then \( \ddot{v}_{pq} = \ddot{v}_p \); if mass is being transferred from the \( q^{th} \) phase to the \( p^{th} \) phase \((\dot{m}_{pq} < 0)\) then \( \ddot{v}_{pq} = \ddot{v}_q \). Similarly, if \( \dot{m}_{qp} > 0 \) then \( \ddot{v}_{qp} = \ddot{v}_q \), if \( \dot{m}_{qp} < 0 \) then \( \ddot{v}_{qp} = \ddot{v}_p \) and the \( q^{th} \) phase stress-strain tensor is given by \( \ddot{\tau}_q \) (Eq. 4.13).

\( \ddot{R}_{pq} \) (Eq. 4.14) is the inter-phase force, dependent on several physical effects such as friction, cohesion force, pressure etc., defined in Eq. (4.12) and must be appropriately constituted to achieve closure. The interaction force, \( \ddot{R}_{pq} \) is also subject to conditions that \( \ddot{R}_{pq} = -\ddot{R}_{qp} \) and \( \ddot{R}_{qq} = 0 \).

The stress-strain tensor \( \ddot{\tau}_q \) Eq. (4.13) defines the shear \((\mu_q)\) and bulk \((\lambda_q)\) viscosity terms for the \( q^{th} \) phase:

\[
\ddot{\tau}_q = \alpha_q \mu_q (\nabla \ddot{v}_q + \nabla \ddot{v}_q^T) + \alpha_q (\lambda_q - \frac{2}{3} \mu_q) \nabla \cdot \ddot{v}_q I \quad (4.13)
\]

The simplified inter-phase interaction force \( \ddot{R}_{pq} \) defined by Ansys Fluent© takes the following form:

\[
\sum_{p=1}^{n} \ddot{R}_{pq} = \sum_{p=1}^{n} K_{pq} (\ddot{v}_p - \ddot{v}_q) \quad (4.14)
\]

where \( K_{pq} \) \((= K_{qp})\) is the interactive phasic momentum exchange coefficient.

**4.2.2.3 Dense Discrete Phase (DDPM) Model**

The Eulerian granular discrete phase modeling approach (Alder and Wainwright, 1960; Lebowitz, 1964; Ogawa et. al., 1980; Lun et. al., 1984; Chapman and Cowling, 1990; Ding and Gidaspow, 1990; Gidaspow et. al., 1992; Syamlal et. al., 1993) has been
traditionally based on the kinetic theory of granular flows (KTGF) which compares the random motion of particles due to the particle-particle interaction/collision forces to the thermal motion of gaseous molecules. No actual particles are modeled in this approach and each phase is treated as a separate continuum with a granular temperature term introduced to calculate the particulate energy exchange and conservation. The granular temperature is subsequently used to estimate fluid properties such as pressure and viscosity for the granular phase. The Eulerian granular model is also known as the two fluid model or TFM.

The Dense Discrete Phase (DDPM) model approach combines Lagrangian characteristics under an Eulerian framework. DDPM model tracks particle trajectories in a Lagrangian fashion but does not estimate the particle-particle interactions. The particle-particle interaction is simulated following the Eulerian KTGF approach thus allowing tracking of a much larger group of particles (Popoff and Braun, 2007; Cloete et. al., 2010). DDPM has some fundamental advantages over the TFM model, i.e,

- DDPM allows tracking schemes for a large number of particle trajectories and a significantly wider range of particle size distributions;
- Wall interactions cause significant uncertainty and computational error in the TFM model. In the DDPM approach, wall interactions can be better addressed;
- DDPM can avoid the phenomenon of "Delta shocks" observed by Passalacqua and Fox (2009) while simulating crossing dilute jets with the TFM approach.
The DDPM model is still in an early stage of development and have some significant limitations (Ansys Fluent©, 2011), i.e.,

a) The approach uses the Eulerian multiphase model framework and hence all its limitations are incorporated;

b) Large Eddy Simulation (LES) and Detached Eddy Simulation (DES) turbulence models are not available;

c) Parallel DPM with shared memory option is disabled;

d) Radiation, solidification, melting, wet steam and real gas models are not available;

e) The combustion models: PDF transport model, premixed, non-premixed and partially pre-mixed combustion models are not available.

The Lagrangian DPM (discrete phase model) is applicable under the assumption that the volume fraction of the discrete phase is significantly lower compared to the Eulerian continuum phase Eqs. (4.15) and (4.16). The DDPM model of Popoff and Braun (2007) overcomes this limitation by extending the Lagrangian conservation (continuity and momentum) equations to a new set of equations for multiple phases Eqs. (4.17) and (4.18) with the generalized form written for the $p^{th}$ phase.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_{DPM} + S_{other}$$  \hspace{1cm} (4.15)

$$\frac{\partial}{\partial t} (\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\vec{T}) + \rho \vec{g} + \vec{F}_{DPM} + \vec{F}_{other}$$ \hspace{1cm} (4.16)
The DDPM approach does not solve the conservation Eqs. for the discrete phase and the momentum exchange terms are considered only for the primary Eulerian continuum air phase. However, the appropriate volume fraction and the domain flow field velocity values are directly recorded from the discrete particulate phase analogous to the Lagrangian particle tracking solution. The equations of motion for the discrete phase (Cloete et. al., 2010) is predicted by integrating the force balance on each particle and can be written as:

\[
\frac{\partial}{\partial t} (\alpha_p \rho_p \vec{v}_p) + \nabla \cdot (\alpha_p \rho_p \vec{v}_p \vec{v}_p) = -\alpha_p \nabla p + \nabla \cdot \left[ \alpha_p \mu_p (\nabla \vec{v}_p + \nabla \vec{v}_p^T) \right] + \alpha_p \rho_p \vec{g} + F_{\text{vm, lift, user}} + \sum_{q=1}^{n_{\text{phases}}} \left[ \vec{K}_{qp} (\vec{v}_q - \vec{v}_p) + \dot{\vec{m}}_{qp} \vec{v}_q - \dot{\vec{m}}_{pq} \vec{v}_p \right] + K_{\text{DPM}} (\vec{v}_{\text{DPM}} - \vec{v}_p) + S_{\text{DPM, explicit}} \tag{4.17}
\]

\[
\frac{\partial}{\partial t} (\dot{\vec{v}}_p) = \frac{-1}{\rho_p} \nabla p + F_D (\vec{v} - \vec{v}_p) + \frac{\vec{g} (\rho_p - \rho)}{\rho_p} + \vec{F} + \vec{F}_{\text{interaction}}; \tag{4.18}
\]

where \( \vec{v} \) is the phase fluid (air) velocity, \( \vec{v}_p \) is the particle velocity, \( \rho \) is the density of the fluid (air), \( \rho_p \) is the density of the particle, \( F_D (\vec{v} - \vec{v}_p) \) is the drag force per unit of particle mass and the rest of the terms in the right hand side represent the pressure force, gravitational force, any supplementary force and the particle-particle interaction force respectively.
Eq. (4.20) gives the drag force $F_D$,

$$F_D = \frac{18\mu C_D Re}{2\rho_p d_p^2} \quad (4.20)$$

where $\mu$ is the molecular viscosity of the fluid, $d_p$ is the diameter of the particle and $Re$ is the relative Reynolds number where $\tilde{v}_p$ and $\tilde{v}_l$ are the particulate velocity and the fluid phase (air) velocity respectively (Eq. 4.21).

$$Re_p = \frac{\rho_g d_p |\tilde{v}_p - \tilde{v}_l|}{\mu_g} \quad (4.21)$$

The drag co-efficient $C_D$ selected to account for the non-spherical shape of coal and mineral particles (Haider and Levenspiel, 1989) is given by Eq. 4.22 with a shape factor of 0.536.

$$C_D = \frac{24}{Re_{p,\text{spherical}}} \left(1 + b_1 Re_{p,\text{spherical}}^{b_2} \right) + \frac{b_3 Re_{p,\text{spherical}}}{b_4 + Re_{p,\text{spherical}}} \quad (4.22)$$

where,

$$b_1 = \exp (2.3288 - 6.4581\phi + 2.4486\phi^2)$$

$$b_2 = 0.0964 + 0.5565\phi$$

$$b_3 = \exp (4.905 - 13.8944\phi + 18.4222\phi^2 - 10.2599\phi^3)$$

$$b_4 = \exp (1.4681 - 12.2584\phi + 20.7322\phi^2 - 15.8855\phi^3)$$
and, the shape factor $\phi$, is defined as:

$$\phi = \frac{s}{S} \quad (4.22a)$$

where $s$ is the surface area of a sphere having the same volume as the particle, and $S$ is the actual surface area of the particle. The shape factor is always a fraction and can attain a highest value of unity. The particulate Reynolds number $Re_{p, spherical}$ is calculated assuming the diameter of a sphere having the same volume as the particle.

The solids pressure provides a good estimation of the interaction force (Eq. 4.23),

$$F_{interaction} = \frac{-1}{\rho_p} \nabla P_p \quad (4.23)$$

and we derive the solids (particulate phase) pressure $\nabla P_p$ following Lun et al. (1984),

$$P_p = \alpha_p \rho_p \theta + 2 \rho_p (1 + e_{pp}) \alpha_p g_0 \theta \quad (4.24)$$

where, $e_{pp}$ is the coefficient of restitution for particle collisions, $g_0$ is the radial distribution function and $\theta$ is the granular temperature used in the KTGF model (Eq. 4.25; Ding and Gidaspow, 1990).

$$\frac{3}{2} \left[ \frac{\partial}{\partial t} (\alpha_p \rho_p \theta) + \nabla \cdot (\rho_p \alpha_p v_p \theta) \right] = \bar{\tau}_p : \nabla \tilde{v}_p + \nabla \cdot (k_\theta \nabla \theta) - \gamma_\theta + \phi_{pq} \quad (4.25)$$

where,

$$\bar{\tau}_p : \nabla \tilde{v}_p = \text{energy generation by the particulate stress tensor}$$

$$\nabla \cdot (k_\theta \nabla \theta) = \text{diffusion flux of granular energy (}k_\theta\text{ is diffusion coefficient})$$
\[ \gamma_\theta = \text{the collision dissipation energy} \]

\[ \phi_{pq} = \text{the energy exchange between the } p^{th} \text{ air phase and the } q^{th} \text{ solid phase.} \]

### 4.2.2.4 Turbulence Models

Turbulence describes the variance in the domain velocity fields characterized by chaotic changes in flow properties. The variance in transported quantities such as rapid changes of pressure and velocity, momentum, energy and species concentration in space and time. Ansys Fluent's Eulerian Dense Discrete phase model (DDPM) offers the following choices of Turbulence Models:

- **a) \( k - \epsilon \) models (Two Eq. model)**
  - i. Standard \( k - \epsilon \) model (Launder and Spalding, 1972)
  - ii. Renormalization-group (RNG) \( k - \epsilon \) model (Yakhot and Orszag, 1986)
  - iii. Realizable \( k - \epsilon \) model (Shih et. al., 1995)

- **b) Reynolds stress models (RSM) (Seven Eq. Model) (Launder et. al., 1975; Gibson and Launder, 1978; Launder, 1989)**
  - i. Linear pressure-strain RSM model
  - ii. Quadratic pressure-strain RSM model

The models used for simulating the turbulent flow field of the air-based density separator are the Realizable \( k - \epsilon \) model and the Linear pressure-strain RSM turbulence model respectively. The transport Eqs. and detailed theory is provided by Shih et. al.
(1995), Launder (1989) and Ansys Fluent© (2011). Independent simulations are necessary to characterize the flow behavior under the Realizable $k - \varepsilon$ model and the RSM model. Moreover, the RSM model accounts for the effects of swirl, rotation, curvature of streamlines and rapid strain rate variations and thus can give better predictions for multi-phase complex flows.

4.2.3 Pre-processing and Mesh Generation

Pre-processing involved developing a full-scale 3-D geometric model of the table deck utilizing the commercially available CAD software Gambit. The asymmetric nature of the geometry posed serious challenges to the adoption of the final model for meshing purposes; hence several design changes were needed to simplify the volumes. Ansys Fluent treats the entire model as a global volume which, if programmed, will read the basic model as empty through which the air-flow occurs without any substantial boundaries acting as riffle barriers. The engineering solution is to generate the volume and subsequently subtract the complex table geometry from the initial volume. The remaining volume would effectively simulate the table performance.

A total of 3 volumes were generated (Figure 4.6). One signifying the air-flow at the top of the table (Top Volume), the second one is the table deck which has been modeled as a 5 mm porous media (Screen Volume) and the third volume is the lower structure below the deck (Bottom Volume). All 3 volumes share common bases but were modeled as independent meshes with a very fine mesh being applied to the table deck porous
media. Figure 4.7 shows the independent mesh of the middle porous volume irrespective of the upper and lower superstructures. It demonstrates the possibility of having a connected geometry but distinct mesh structures.

Figure 4.6 Model CAD Geometry (developmental stage) isometric view.
The porosity of the screen volume is calculated by recording the pressure drop across the new mesh utilizing a modified Darcy formulation as described in section 4.2.1.1, Chapter 4 of the dissertation. The in-built functions in Ansys Fluent can define the porosity which in effect simulates the volume as porous media. The mesh was then exported to Fluent to check for any geometric or intrinsic anomalies. The volumes have been meshed using a coarse grid of spacing 10 using the Tet/Hybrid elements of type T-grid. The above mesh has a total of 794700 elements. The mesh was then exported as a “.msh” or mesh file to be imported by Ansys Fluent solver for analysis. The simulated geometrical model includes the stationary feeding tube, the vibrating-pan with the air inlet at the bottom and a portion of the surrounding area. The complex geometry of the
table separator deck offer no symmetries or periodicity to be exploited as simplification (Figure 4.8). Details of the mesh structure is provided in Table 4.3.

![Figure 4.8 Tet/hybrid mesh displaying the table deck.](image)

Table 4.3 Pre-processing model: Details of the mesh structure

<table>
<thead>
<tr>
<th>Domain Extents</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>X co-ordinate</td>
<td>Min (m) = -6.440528 $e^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Max (m) = 1.973480 $e^{-2}$</td>
</tr>
<tr>
<td>Y co-ordinate</td>
<td>Min (m) = -1.738639 $e^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Max (m) = 6.370839 $e^{-1}$</td>
</tr>
<tr>
<td>Z co-ordinate</td>
<td>Min (m) = -4.993122 $e^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Max (m) = 4.451743 $e^{-1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum volume (m³)</td>
<td>$1.039527 e^{-8}$</td>
</tr>
<tr>
<td>Maximum volume (m³)</td>
<td>$5.694778 e^{-7}$</td>
</tr>
<tr>
<td>Total volume (m³)</td>
<td>$1.418401 e^{-1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Face area Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum area (m²)</td>
<td>$6.450115 e^{-6}$</td>
</tr>
<tr>
<td>Maximum area (m²)</td>
<td>$1.582174 e^{-4}$</td>
</tr>
</tbody>
</table>

4.2.4 Solution set-up in Ansys Fluent

The mesh file generated in Gambit was imported in Ansys Fluent and the mesh was subjected to grid check and zone re-construction in Fluent and an exercise called “Case and Data” was written for the same. The model was calibrated for simulation using
rudimentary boundary conditions in a static mode with an initial simulation run for 100
time-steps, each time step being $2 \times 10^{-5}$ second solving 10 to 30 iterations per step. The
initial boundary conditions are provided in table 4.4. Convergence in the initial case took
a lot of computational time but was streamlined once pseudo steady state was achieved.
The total computation time was close to 12 hrs. The realizable $k - \varepsilon$ model was utilized
to account for the turbulent intensity in the static simulation. The following Figures
(Figure 4.9) shows the velocity vectors at two different planes above the table deck, one
at a scaled distance of 10 and the other at 20 along the vertical z-axis. It is evident from
the air-flow profiles shown in Figure 4.9 that the grid and the mesh are fully developed.

Table 4.4 Initial boundary condition parameters

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Airflow Rate</th>
<th>Longitudinal Angle</th>
<th>Transverse Angle</th>
<th>Table Frequency</th>
<th>Vibration Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Full-Scale</td>
<td>0.78 Kg/s</td>
<td>$3^0$</td>
<td>$8^0$</td>
<td>35 Hz</td>
<td>7 mm</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.9 Velocity vector profile of airflow at various planes simulated for a static run.
In order to account for the turbulence in the airflow and calculate the backflow at each wall and boundary in accordance to the geometric asymmetry of the table deck it was necessary to define the Hydraulic diameters of the different inlet and outlet opening faces. The relevant hydraulic diameters of the inlet and outlet faces are listed in Table 4.5 according to their randomly generated Ansys Fluent identifiers. Hydraulic diameter of any face and opening is calculated by the following relationship (Eq. 4.26):

\[
\text{Hydraulic dia (m) of a face} = \frac{4 \times \text{Face Area (m}^2\text{)}}{\text{Perimeter of Face (m)}} \quad (4.26)
\]

<table>
<thead>
<tr>
<th>Name of Face</th>
<th>Area (m(^2))</th>
<th>Perimeter (m)</th>
<th>Hyd. Dia (4*Area/Perimeter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Inlet</td>
<td>0.314516</td>
<td>2.260600</td>
<td>0.556517</td>
</tr>
<tr>
<td>Opening</td>
<td>0.142937</td>
<td>1.570800</td>
<td>0.363986</td>
</tr>
<tr>
<td>Opening:022</td>
<td>0.378901</td>
<td>2.600162</td>
<td>0.582888</td>
</tr>
<tr>
<td>Opening:023</td>
<td>0.044694</td>
<td>0.869737</td>
<td>0.205552</td>
</tr>
<tr>
<td>Opening:024</td>
<td>0.041225</td>
<td>0.879236</td>
<td>0.187548</td>
</tr>
<tr>
<td>Opening:025</td>
<td>0.114762</td>
<td>1.367200</td>
<td>0.335759</td>
</tr>
<tr>
<td>Opening:026</td>
<td>0.038724</td>
<td>0.974480</td>
<td>0.158952</td>
</tr>
<tr>
<td>Opening:027</td>
<td>0.046310</td>
<td>0.914677</td>
<td>0.202518</td>
</tr>
<tr>
<td>Opening:027:030</td>
<td>0.058774</td>
<td>1.011434</td>
<td>0.232437</td>
</tr>
<tr>
<td>Opening:027:031</td>
<td>0.034390</td>
<td>0.742000</td>
<td>0.185391</td>
</tr>
<tr>
<td>Opening:028</td>
<td>0.133308</td>
<td>1.835020</td>
<td>0.290587</td>
</tr>
<tr>
<td>Opening:028:032</td>
<td>0.010471</td>
<td>0.471046</td>
<td>0.088913</td>
</tr>
<tr>
<td>Opening:028:033</td>
<td>0.004671</td>
<td>0.367479</td>
<td>0.050847</td>
</tr>
<tr>
<td>Opening:029</td>
<td>0.049530</td>
<td>1.190600</td>
<td>0.166403</td>
</tr>
</tbody>
</table>

4.2.4.1 Server Calibration

The simulation was run on a state-of-the-art AMD server configured with RAID 5 (up-to 3 hard disks can fail) for superior data protection and four 2 Terabyte Seagate
Constellation drives. The server also has 128 GB of random access memory which complements parallel processing with its four 12 core AMD 6238 processors. The specifications of the server is provided in Table 4.6.

Table 4.6 SUPERMICRO AMD server configuration specifications

<table>
<thead>
<tr>
<th>Server Name:</th>
<th>SUPERMICRO, 2042G-6RF Black 2U Rack Server Platform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor:</td>
<td>AMD 4 x Opteron 6238 Twelve-Core 2.6 GHz, LGA1944 G34, HT 6400MHz, 16MB L3 cache, 115W (total of 48 cores)</td>
</tr>
<tr>
<td>RAM:</td>
<td>CRUCIAL, 128GB (16 x 8 GB) Dual-Rank PC3-10600 DDR3 1333MHz CL9 SDRAM DIMM, ECC Registered</td>
</tr>
<tr>
<td>Hard Disk:</td>
<td>4 x SEAGATE, 2TB Constellation ES, SAS 6 Gb/s, 7200 RPM, 16MB cache</td>
</tr>
<tr>
<td>RAID Controller:</td>
<td>LSI, MegaRAID SAS 9265-8i SAS/SATA RAID Controller, 8-ports, 6 Gb/s, Levels 0/1/5/6/10/50/60, 1GB DDR3 cache, PCIe x8</td>
</tr>
<tr>
<td>RAID Formation:</td>
<td>RAID, RAID 5 (fault tolerance), 3 hard drives to fail</td>
</tr>
<tr>
<td>Optical Drive:</td>
<td>TEAC, Slim Black 8x DVD-ROM Optical Drive</td>
</tr>
<tr>
<td>Serverboard:</td>
<td>LGA1944 /4, AMD SR5690/SR5670</td>
</tr>
<tr>
<td>Power Supply:</td>
<td>1400W Rdt PFC</td>
</tr>
<tr>
<td>OS:</td>
<td>CENTOS Linux 64-bit Latest Available Enterprise Edition</td>
</tr>
</tbody>
</table>

The AMD server was calibrated for maximum efficiency using iteration time analysis. Relative efficiency in terms of a single-core desktop was calculated by increasing the number of cores running the same number of time-steps. It is evident from table 4.7 and Figure 4.10 that as we increase the number of cores running the simulation to solve 50 time steps for a total of 268 iterations, the solution registers faster and faster convergence times in terms of a single-core desktop. A significant pattern emerges as the server has 4 processors, 12 cores each, the time-scale jump for each multiple of 6 cores is optimal instead of a multiple of 2. Hence, when the simulation was run at 30 cores it is almost 8.906 times faster than a single core desktop but any subsequent increase in core count
displays a sharp deterioration of simulation performance as observed in Figure 4.10. Therefore, 30 cores proved to be optimal in running a multi-core nodal analysis.

Table 4.7 Comparative efficiency of increasing core-count relative to a single core desktop

<table>
<thead>
<tr>
<th>No. of Cores</th>
<th>Time (s) 50 iterations</th>
<th>No. of total iterations</th>
<th>Days to finish 30s</th>
<th>Relative Efficiency (DT/Server)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28.955</td>
<td>268</td>
<td>2694.42</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>17.968</td>
<td>268</td>
<td>1672.02</td>
<td>1.611</td>
</tr>
<tr>
<td>4</td>
<td>9.643</td>
<td>268</td>
<td>897.33</td>
<td>3.003</td>
</tr>
<tr>
<td>6</td>
<td>6.307</td>
<td>268</td>
<td>586.90</td>
<td>4.591</td>
</tr>
<tr>
<td>8</td>
<td>5.013</td>
<td>268</td>
<td>466.49</td>
<td>5.776</td>
</tr>
<tr>
<td>12</td>
<td>4.884</td>
<td>268</td>
<td>454.48</td>
<td>5.929</td>
</tr>
<tr>
<td>16</td>
<td>4.711</td>
<td>268</td>
<td>438.38</td>
<td>6.146</td>
</tr>
<tr>
<td>24</td>
<td>3.365</td>
<td>268</td>
<td>313.13</td>
<td>8.605</td>
</tr>
<tr>
<td>30</td>
<td>3.251</td>
<td>268</td>
<td>302.52</td>
<td>8.906</td>
</tr>
<tr>
<td>32</td>
<td>7.105</td>
<td>268</td>
<td>661.16</td>
<td>4.075</td>
</tr>
</tbody>
</table>

Figure 4.10 Plot signifying Relative Efficiency of increasing the number of cores with respect to a single core desktop.
4.2.4.2 User Defined Function (UDF) for deck vibration motion

The motion of the vibrating deck was accounted for by using dynamically moving grids using an user defined function (UDF). Figure 4.11 displays the deck motion profile relative to the horizontal axis. The problem setup required the mesh to be located at a plane which is inclined at $3^0$ with the horizontal axis and $8^0$ with the transverse axis. However, the movement of the deck occurs in a different plane, which is at $44^0$ angle with the X,Y horizontal plane, with respect to its primary inclination angle $\theta$. The UDF was developed as a trigonometric function taking into account the horizontal and vertical components of the deck motion vectors. The developed UDF code is provided in Table A-2 (Appendices). Consequently, due to the vibrating pan mechanism of the air-table, the simulation has to be transitory with the time step restricted by some fraction of the vibration cycle time.

Figure 4.11 Table deck motion profile (actual inclination angle $\theta$ vs the motion plane at an angle of $44^0$ angle with the X,Y horizontal plane).
4.2.4.3 Particle Feed injection DDPM setup

The Dense Discrete Phase Model (DDPM) particle injection method was utilized to insert particles into the turbulent flow-field. Four different specific gravity particles are being injected into the system at a constant feed rate of 200 kg/hr. The size distribution of each injected particle stream follows the Rosin-Rammler distribution (Figure 4.12) with a top size of 6 mm (6000 microns). The Dense Discrete phase simulation with 4 different density injections are designated as Cd1 (1300 kg/m$^3$), Cd2 (1450 kg/m$^3$), Cd3 (1800 kg/m$^3$) and Cd4 (2650 kg/m$^3$) with 20 particles being injected for each stream totaling 80 particles per time step. The mass fraction distribution of the particle feed based on the size and density is given in Table 4.8. The particle injection was initiated after 3s of the initial model run and 30 iterations were performed for each time-step.

![Figure 4.12 Particle size Rosin Rammler distribution for individual density injections.](image)
Table 4.8 Mass fraction distribution based on particle size and density in the injected feed

<table>
<thead>
<tr>
<th>Density</th>
<th>% in Feed</th>
<th>gms/s</th>
<th>Kg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1300 kg/m³</td>
<td>68.60</td>
<td>38.11</td>
<td>0.038110</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size Range (microns)</th>
<th>in 100%</th>
<th>Mass Fraction</th>
<th>Cumulative Mass Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000.00 3350.00</td>
<td>22.31</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>3350.00 1000.00</td>
<td>65.27</td>
<td>0.65</td>
<td>0.88</td>
</tr>
<tr>
<td>1000.00 0.00</td>
<td>12.42</td>
<td>0.12</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>1.00</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density</th>
<th>% in Feed</th>
<th>gms/s</th>
<th>Kg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1450 kg/m³</td>
<td>4.28</td>
<td>2.38</td>
<td>0.002376</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size Range (microns)</th>
<th>in 100%</th>
<th>Mass Fraction</th>
<th>Cumulative Mass Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000.00 3350.00</td>
<td>16.24</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>3350.00 1000.00</td>
<td>61.57</td>
<td>0.62</td>
<td>0.78</td>
</tr>
<tr>
<td>1000.00 0.00</td>
<td>22.19</td>
<td>0.22</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>1.00</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density</th>
<th>% in Feed</th>
<th>gms/s</th>
<th>Kg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800 kg/m³</td>
<td>3.48</td>
<td>1.93</td>
<td>0.001931</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size Range (microns)</th>
<th>in 100%</th>
<th>Mass Fraction</th>
<th>Cumulative Mass Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000.00 3350.00</td>
<td>40.19</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>3350.00 1000.00</td>
<td>50.94</td>
<td>0.51</td>
<td>0.91</td>
</tr>
<tr>
<td>1000.00 0.00</td>
<td>8.87</td>
<td>0.09</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>1.00</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density</th>
<th>% in Feed</th>
<th>gms/s</th>
<th>Kg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2650 kg/m³</td>
<td>23.65</td>
<td>13.14</td>
<td>0.013138</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size Range (microns)</th>
<th>in 100%</th>
<th>Mass Fraction</th>
<th>Cumulative Mass Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000.00 3350.00</td>
<td>48.72</td>
<td>0.49</td>
<td>0.49</td>
</tr>
<tr>
<td>3350.00 1000.00</td>
<td>47.43</td>
<td>0.47</td>
<td>0.96</td>
</tr>
<tr>
<td>1000.00 0.00</td>
<td>3.85</td>
<td>0.04</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>1.00</strong></td>
<td></td>
</tr>
</tbody>
</table>
The particle injections were characterized by the Gidaspow (1994) granular viscosity model and Lun et. al. (1984) granular bulk viscosity model (Figure 4.13). The solids bulk viscosity describes the resistance of the granular phase particles to elastic deformation. The Lun et. al. and the Gidaspow model assumes that the volume fraction is close to the packing limit and hence the radial function tends to be infinity, which can then be used to describe the frictional viscosity directly. Transient particle tracking was done in real time using X,Y and Z positional tracking scheme of Ansys Fluent© and the escaping mass flow calculated for each designated mesh zone.

Figure 4.13 DDPM Particle Phase properties setup in Ansys Fluent©.
4.3 Results and Discussion

The transient realizable $k - \epsilon$ simulation was run for a time period of 6 seconds with particle injection starting at the 3 second mark. The time-step size was $2 \times 10^{-5}$ seconds with 10 to 30 iterations required for convergence every time step. The total number of iterations performed each second was close to 5,00,000. The results for the airflow profile have been found to be quite interesting. Figure 4.14 shows the velocity profile of the airflow and it displays high local air-speeds near the feed end of the table. The streamlines plot (Figure 4.15) proves that the blocked end at the back of the table has a controlling effect on the airflow movement and direction of dissipation of momentum.

The air velocity vector plot (Figure 4.16) displays rapid variance during the time of the deck motion. With each motion cycle of the deck the velocity vectors undergo a sudden modification in momentum with the airflow pattern near the walls and the blocked off area at the back is affected the most. The dissipation of momentum of the airflow and the instantaneous shift in kinetic energy is evident from the high local flow speeds almost close to 7.0 m/s at the feed end. However, the rest of the grid show modest variance in speed and momentum of the airflow. The air velocity contours on top of the table deck show much less change in air speeds as shown in Figure 4.14 when compared to the wall effect at the feed end. The RSM turbulent formulation displays similar airflow patterns, however, the pressure drop and turbulent intensity differs significantly (section 4.3.1).
Figure 4.14 Transient airflow velocity profile plot (realizable $k - \epsilon$ model).
Figure 4.15 Transient airflow velocity profile streamline plot (realizable $k - \epsilon$ model).
4.3.1 Comparison between Realizable $k - \epsilon$ model and RSM model

The air flow profile after 1.0 second of the realizable $k - \epsilon$ model shows high peaks and variance. However, subsequent RSM formulation simulations prove that the difference between the two models are non-trivial. The RSM model displays similar airflow patterns but the domain maxima and minima are significantly different. The RSM turbulent model at 0.2 second (10,000 time steps) is compared with the realizable $k - \epsilon$ model at 0.2 second (10,000 time steps) real time simulation. The airflow profile data (Figure 4.17) shows high degree of variance in air velocities between the two models.
with the absolute percentage difference between the minima and maxima being close to 94.96% and 48.35% respectively. The domain average values show much less variance (Table 4.9).

![K- Epsilon airflow 0.2 s vs RSM airflow 0.2 s](image)

Figure 4.17 Airflow profile plot at 0.2 seconds ($k - \epsilon$ vs RSM).

<table>
<thead>
<tr>
<th>Air Velocity</th>
<th>RSM</th>
<th>K-Epsilon</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time - 0.2 sec</td>
<td>Time - 0.2 sec</td>
<td>% age</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>0.000325</td>
<td>0.006455</td>
<td>94.96</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>5.97</td>
<td>11.55</td>
<td>48.35</td>
</tr>
</tbody>
</table>

### Average Velocity for different Volumes

<table>
<thead>
<tr>
<th>Volume</th>
<th>RSM</th>
<th>K-Epsilon</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Top Volume</strong></td>
<td>2.08</td>
<td>2.12</td>
<td>1.89</td>
</tr>
<tr>
<td><strong>Bottom Volume</strong></td>
<td>2.92</td>
<td>2.85</td>
<td>-2.46</td>
</tr>
<tr>
<td><strong>Screen Volume</strong></td>
<td>3.12</td>
<td>3.06</td>
<td>-1.96</td>
</tr>
<tr>
<td><strong>Net Domain</strong></td>
<td>2.30</td>
<td>2.31</td>
<td>0.65</td>
</tr>
</tbody>
</table>
The pressure plot results and the turbulent intensity values display the maximum variance between the two models (Table 4.10 and Table 4.11). The RSM model showed a remarkable characteristic, whereby, the average pressure on top of the screen deck is -6.47 Pascal which is negative, hence it is indicative that a portion of the air is getting sucked back into the system when the deck is in motion. The $k – \varepsilon$ model showed a positive pressure value of 11.66 Pascal. The scenario described by the RSM simulation is more plausible as the deck is in constant motion thus forming a negative pressure situation on top of the deck augmented by the incoming feed. The pressure and turbulence intensity plots are shown in Figure 4.18 and Figure 4.19 respectively.

Table 4.10 Static pressure distribution values above and below the screen deck

<table>
<thead>
<tr>
<th>Static Pressure</th>
<th>RSM</th>
<th>K-Epsilon</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time - 0.2 sec</td>
<td>Time - 0.2 sec</td>
<td>% age</td>
</tr>
<tr>
<td>Average Screen Top</td>
<td>-6.47</td>
<td>11.66</td>
<td>155.49</td>
</tr>
<tr>
<td>Average Screen Bottom</td>
<td>95.949</td>
<td>120.178</td>
<td>20.16</td>
</tr>
</tbody>
</table>

Figure 4.18 Pressure plot of $k – \varepsilon$ and RSM models at 0.2 second (scale is in Pascals).
Figure 4.19 Turbulent intensity plot of $k - \varepsilon$ and RSM models at 0.2 second.

Table 4.11 RSM vs $k - \varepsilon$ turbulence intensity distribution (0.2 second)

<table>
<thead>
<tr>
<th>Turbulence Intensity</th>
<th>RSM 0.2 sec</th>
<th>K-Epsilon 0.2 sec</th>
<th>Difference % age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.0201</td>
<td>0.0487</td>
<td>58.73</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.9896</td>
<td>5.8299</td>
<td>65.87</td>
</tr>
</tbody>
</table>

Average Turbulence Intensity for different Volumes

<table>
<thead>
<tr>
<th>Volume</th>
<th>RSM</th>
<th>K-Epsilon</th>
<th>Difference % age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Volume</td>
<td>0.2539</td>
<td>0.5174</td>
<td>50.93</td>
</tr>
<tr>
<td>Bottom Volume</td>
<td>0.1040</td>
<td>0.1543</td>
<td>32.60</td>
</tr>
<tr>
<td>Screen Volume</td>
<td>0.2131</td>
<td>0.4748</td>
<td>55.12</td>
</tr>
<tr>
<td>Net Domain</td>
<td>0.2159</td>
<td>0.4258</td>
<td>49.30</td>
</tr>
</tbody>
</table>

Turbulence intensity (Pope, 2000) or the turbulence level is described as the velocity fluctuations created by eddies in the flow field and is defined as:

$$ I \equiv \frac{u'}{\bar{u}} $$  \hspace{1cm} (4.27)
where $u'$ is the root mean square of the turbulent velocity fluctuations and $U$ is the Reynolds averaged mean velocity. Ansys Fluent© calculates the turbulent energy, $k$, for the flow-field and subsequently, $u'$, can be calculated by averaging the axial components of the velocity vectors as given by Eq. (4.28). Similarly, the Reynolds averaged mean velocity, $U$, can be calculated from the axial mean velocity components Eq. (4.29). $L$ and $\rho$ represent the reference values for length and density respectively Eq. (4.30).

$$u' \equiv \sqrt{\frac{1}{3}(u'^2_x + u'^2_y + u'^2_z)} = \sqrt{\frac{2}{3}k}$$  \hspace{1cm} (4.28)$$

$$U \equiv \sqrt{U^2_x + U^2_y + U^2_z}$$  \hspace{1cm} (4.29)$$

$$Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho UL}{\mu}$$  \hspace{1cm} (4.30)$$

The air-based density separator is characterized by low air-speed flow with a measured experimental maximum value of 3.04 m/s at 50 Hz blower frequency. This would correspond to a low Reynolds number Eq. (4.30) flow where a turbulence intensity value between 1% and 5% signifies a low to medium turbulence domain. The RSM turbulent intensity maximum of 1.9896% at 0.2 seconds proves that the flow can be effectively treated as laminar, however, the $k-\epsilon$ model displays a medium to high turbulence scale with a domain maximum of 5.8299%. The apparent difference between the RSM and the $k-\epsilon$ model turbulence intensity values at the 0.2 second mark signify the major variance between the two turbulence models.
4.3.2 Model Validation

Model validation was done using point probing method for pressure drop and velocity measurements. The experimental pressure drop measurements at specific points above and below the 5 mm porous zone signifying the screen deck was compared with the simulated results at similar points in the domain as shown in Figure 4.20 and Table 4.12. Air velocity measurements on top of the deck showed high variance near the riffle block or section "B" of the table and also near the high density discharge end, section "F" (Figure 4.21). At different sections the divergence was found to be as large as 10 % and as small as 1 % (Table 4.13) but the maximum experimental value of 3.04 m/s is lower than the maximum simulated value of 5.81 m/s near the blocked end. Nevertheless, the average values conform well with the measured experimental values thus validating the numerical model.

Table 4.12 Correlating model predictions of pressure drop across the porous zone with measured experimental data

<table>
<thead>
<tr>
<th>Model</th>
<th>Pressure drop (Pascals)</th>
<th>Difference with experimental data in % age</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k - \varepsilon$ turbulence model</td>
<td>23.64</td>
<td>7.29</td>
</tr>
<tr>
<td>RSM turbulence model</td>
<td>24.82</td>
<td>2.67</td>
</tr>
<tr>
<td>Experimental</td>
<td>25.50</td>
<td>-----</td>
</tr>
</tbody>
</table>
Table 4.13 Correlating model predictions of air velocity across different sections of the table deck with measured experimental data (0.2 second $k - \epsilon$ model)

<table>
<thead>
<tr>
<th>Section</th>
<th>k - $\epsilon$ model data (m/s)</th>
<th>Experimental data (m/s)</th>
<th>Difference % age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section A</td>
<td>2.34774</td>
<td>2.324608</td>
<td>1.00</td>
</tr>
<tr>
<td>Section B</td>
<td>2.77014</td>
<td>2.503424</td>
<td>10.65</td>
</tr>
<tr>
<td>Section C</td>
<td>2.61793</td>
<td>2.637536</td>
<td>-0.74</td>
</tr>
<tr>
<td>Section D</td>
<td>3.07661</td>
<td>3.039872</td>
<td>1.21</td>
</tr>
<tr>
<td>Section E</td>
<td>2.25986</td>
<td>2.190496</td>
<td>3.17</td>
</tr>
<tr>
<td>Section F</td>
<td>2.41201</td>
<td>2.190496</td>
<td>10.1125</td>
</tr>
<tr>
<td>Average</td>
<td>2.580715</td>
<td>2.481072</td>
<td>4.016127</td>
</tr>
</tbody>
</table>

Figure 4.20 Pressure drop measurements recorded using point probing method.
4.3.3 Particle Paths, effect of Vortices and Separation Performance

The \( k - \epsilon \) model was run for an additional 3 seconds to characterize the particle separation characteristics using the Dense discrete phase particle (DDPM) model. Four different particle injection streams of 1.3, 1.45, 1.8 and 2.65 gm/cm\(^3\) respectively were initiated after 3 seconds of dry run to bring the total simulation run time to 6 seconds of experimental time scale. The particle streams followed the Rosin Rammler distribution scheme as described in section 4.2.4.3 with 80 particles being injected each time step. The total number of particle paths to be tracked after 1 second of injection is 4,000,000. The "non-spherical" drag model was utilized with a shape factor of 0.536 signifying coal...
particles and 30 iterations were performed each time step. It was observed that pseudo-steady state was achieved after 2 seconds of particle injection, i.e., the number of particles entering the simulation domain was found to be equivalent to the number of particles exiting the domain with no change in mass flux. Transient particle tracking was done in real time using the axial positional tracking scheme of Ansys Fluent© and the escaping mass calculated for each Fluent designated grid zone. The mass transfer summary provided in Table 4.14 show the amount of variance in relocation and discharge of particles through various zones of the table mesh structure. Zone 7 denotes the discharge end on the product and reject side and the initial mass of material flow is balanced by the outflow with the maximum mass output passing through Zone 7. Turbulent intensity (Figure 4.19) of only 2% signifies that airflow tends to be mostly laminar in the absence of particles but once particles are injected into the system the turbulence dynamics changes drastically and the region becomes highly turbulent as shown in Figure 4.22.

![Figure 4.22 Particle injection turbulent intensity %, $k - \epsilon$ model (6.0195 s).](image)

96
Table 4.14 Mass transfer summary of particle discharge (100 time steps) in 0.002 seconds

<table>
<thead>
<tr>
<th>Fate</th>
<th>Number</th>
<th>Initial (kg)</th>
<th>Final (kg)</th>
<th>Change (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Escaped - Zone 6</td>
<td>2244</td>
<td>6.12E-05</td>
<td>6.12E-05</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 22</td>
<td>12448</td>
<td>8.49E-05</td>
<td>8.49E-05</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 23</td>
<td>102</td>
<td>3.79E-06</td>
<td>3.79E-06</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 24</td>
<td>63</td>
<td>2.62E-06</td>
<td>2.62E-06</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 25</td>
<td>42</td>
<td>4.44E-07</td>
<td>4.44E-07</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 26</td>
<td>57</td>
<td>5.18E-07</td>
<td>5.18E-07</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 27</td>
<td>3</td>
<td>1.07E-11</td>
<td>1.07E-11</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 28</td>
<td>424</td>
<td>5.44E-06</td>
<td>5.44E-06</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 29</td>
<td>1298</td>
<td>4.93E-05</td>
<td>4.93E-05</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 30</td>
<td>132</td>
<td>7.93E-07</td>
<td>7.93E-07</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 31</td>
<td>189</td>
<td>3.56E-08</td>
<td>3.56E-08</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 32</td>
<td>43</td>
<td>1.40E-06</td>
<td>1.40E-06</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 33</td>
<td>17</td>
<td>7.37E-07</td>
<td>7.37E-07</td>
<td>0E+00</td>
</tr>
<tr>
<td>Escaped - Zone 7</td>
<td>58949</td>
<td>8.64E-04</td>
<td>8.64E-04</td>
<td>0E+00</td>
</tr>
<tr>
<td>Total Escaped</td>
<td>76011</td>
<td>0.001075</td>
<td>0.001075</td>
<td>0E+00</td>
</tr>
</tbody>
</table>

In a turbulent flow field the pseudovector field governing the evolution of vortices (Batchelor, 1967; Chorin, 1994; Ohkitani, 2005) describing the curl of the momentum is given by Eq. 4.31. Formation of vortices are a result of the instantaneous change of the fluid velocity in axial direction defining a spinning motion of the flow field.

\[
\vec{\omega} = \vec{\nabla} \times \vec{v}
\]  

(4.31)

where \( \vec{\omega} \) is the pseudovector field, \( \vec{\nabla} \) is the del operator or nabla (Schey, 1997) and \( \vec{v} \) is the velocity field. The vorticity plot of the Plot of \( k - \epsilon \) model describing the formation of the vortices near the riffles on top of the deck per second is shown in Figure 4.23. The vortices play an important role as a transport mechanism for the high density particles.
reporting to the product end negating selectivity and increasing bypass. The existing riffle design is triangular in shape which facilitates the formation of local vortices.

![Vorticity plot](image)

Figure 4.23 Plot of \( k - \epsilon \) model displaying vortices forming near the riffles.

A schematic view of the particle paths for 100 time steps (0.002 second) is shown in Figure 4.24 and it depicts the Lagrangian tracking properties of the DDPM model. The particles are colored according to their respective density values; particles of density 1300 kg/m\(^3\) are depicted by red spheres, 1450 kg/m\(^3\) particles are depicted by green spheres, the blue spheres depict particles of density 1800 kg/m\(^3\) and the heaviest particle stream of 2650 kg/m\(^3\) is depicted by white spheres. The red and green particles which are lighter overwhelmingly report to the product side, but, there are streams which tend to travel down towards the midlings and reject but change direction to move towards the product side. The zone escape data records every light particle as escaped through the
midlings and the product zone. Some are even so small and light that they escape as dust particles through the top of the domain boundary. However, there is some displacement with regards to heavier particles reporting to the product as is evident from the white streak traveling along the length of the table and reporting to the midling side. The displacement of the heavier particles to the product is an example of smaller heavy density particles riding on the riffle vortices throughout considerable length of the table and subsequently losing momentum due to its higher gravity and tapers off towards the midlings. The displacement of the heavier particles as an effect of vorticity can be prevented by increasing the selectivity of the riffles by implementing a change in the riffle design. The partition curves generated from the particle segregation data obtained by analyzing a time-capsule of 100 time steps or 0.002 seconds of simulation time is provided in Figure 4.25. There is a difference of 11% between the yield values of the simulated data and the experimental results for a feed rate of 200 kg/hr. The simulation data also shows 43% displacement of midlings and 29% displacement of the heavies reporting to the product side. But the overall cut-density is closer to the experimental value though the difference of 5% can be considered to be minimal and can effectively validate the model results. Increasing the feed rate to 400 kg/hr results in a drop in the yield and the effective deshaling capability of the table is severely restricted as shown in Figure 4.26. The various density cut-points of the simulated particle feed rates of 200 kg/hr and 400 kg/hr are provided in Table 4.15.
Figure 4.24 Top view schematic of the particle tracks (100 time steps, $k - \epsilon$ model).

Table 4.15 Comparison of yield and cut density between experimental data and model predictions for different feed rates

<table>
<thead>
<tr>
<th>Feed Rate</th>
<th>400 kg/hr (Predicted)</th>
<th>200 kg/hr (Predicted)</th>
<th>200 kg/hr (Experimental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield (%)</td>
<td>66.07</td>
<td>80.74</td>
<td>88.04</td>
</tr>
<tr>
<td>Cut-density ($\rho_{50}$)</td>
<td>1.75</td>
<td>1.95</td>
<td>2.04</td>
</tr>
<tr>
<td>Difference in $\rho_{50}$ (%)</td>
<td>14.22</td>
<td>4.41</td>
<td>-----</td>
</tr>
</tbody>
</table>
Figure 4.25 Partition curve comparison between model prediction and experimental data for a feed rate of 200 kg/hr.

Figure 4.26 Partition curve comparison between model predictions for feed rates of 200 kg/hr and 400 kg/hr.
5.1 Introduction

The modern air-based density separators are effective in treating run-of-mine material in the size range of 75 mm down to 6 mm but separation performance declines considerably in the finer size range of 6 mm x 1 mm. The modified Bratney table has been successful in treating finer size fraction Wyoming coal and the CFD model developed showed that the riffles on top of the deck need major redesign to make the separation process more selective in nature. However, the laboratory scale table can only treat 200-300 kg/hr and hence an appropriate scaling law (Emori, 1968; Saito, 2008) needs to be developed to augment its capacity to treat industrial feed rates and to study the mechanism in greater detail. The air-table is modeled as a semi-dynamic system handling energy. Sets of generalized energy variables and force balances are developed. The energetic restrictions introduced by the parametric effects on the system are used to specify certain basic constraints and hence obtain mathematical models.

The fundamental physical forces involved in the air-based density separator are: Vibration force in the form of inertia \( F_i \), gravitational force acting on the particles \( F_g \), frictional force between the particles and the deck \( F_\mu \), force of air acting on the particles in the form of inertia \( F_i \), pressure \( F_P \) and the viscous force \( F_\nu \), and the particle-particle interaction in the form of collision or friction.
5.2 Deriving the Scaling Laws

Scale modeling is based on scaling laws, and if they are correct, describe the actual system, and the prototype model can be used for the design and optimization of the full-scale model; otherwise the assumptions made to derive scaling laws should be reconsidered.

To derive the scaling laws, the reference X, Y plane (horizontal and transverse coordinates) are located on the deck; the particle-particle interactions are ignored to simplify the model; the force of air acting on particles are assumed to be mostly viscous forces (due to low Reynolds number of air flow passing on to the deck prior to particle injection), and velocity of air is considered as a constant, which is equal to the value needed to lift up the coarsest particle to facilitate its movement along the deck.

**Z direction:**

Deck motion has an angle relative to the horizontal plane so the inertia force caused by the vibration of the deck acts on both X and Z direction. Therefore, the main forces in the vertical Z direction are: gravity force (Fg), inertia force caused by the vibration (Fi), and the viscous force caused by the air acting on the particles (Fv), as defined in Eqs. 5.1 to 5.3:

\[ F_i = \frac{\rho d^3 l}{t^2} \]  

\[ F_g = \rho d^3 g \]
The governing \( \pi \) numbers (Bertrand, 1878; Rayleigh, 1892; Buckingham, 1914) are Froude numbers (Froude, 1877; Belanger, 1828), the ratio of inertia force over gravity, and Reynolds number over Froude number, the ratio of gravity force over viscous force (Eqs. 5.4 and 5.7).

\[
F_v = \mu \frac{d^2 v_{air}}{d} \quad (5.3)
\]

\[
\pi_1 = \frac{F_i}{F_g} = \frac{\rho d^3 l}{\rho d^3 g} = \frac{l}{t^2 g} = \frac{v}{t g} = \frac{v^2}{l g} \quad (5.4)
\]

Therefore,

\[
\frac{v_2}{v_1} \approx \sqrt{\frac{l_2}{l_1}} \quad (5.5)
\]

\[
and \frac{t_2}{t_1} \approx \sqrt{\frac{l_2}{l_1}} \quad (5.6)
\]

The vertical velocity of the vibration and time are proportional to the root square of the length characteristic of the table.

\[
\pi_2 = \frac{F_g}{F_v} = \frac{\rho d^3 g}{\mu d^2 v_{air}} \quad (5.7)
\]

Therefore,

\[
v_{air} = \frac{\rho d^2 g}{\mu} = const. \quad (5.8)
\]
X direction:

In the horizontal plane, particle movement along the X direction is governed by the frictional force between the particles and the deck and the inertia force caused by vibration of the table. Therefore, the pi-number is (Eq. 5.9):

\[
\pi_1 = \frac{F_1}{F_{\mu}} = \frac{\rho d^3 l}{\mu' d^2} = \frac{\rho dl}{\mu' l^2} = \frac{\rho du^2}{\mu' l}
\] (5.9)

Therefore,

\[
\frac{u_2}{u_1} \approx \sqrt{\frac{l_2}{l_1}}
\] (5.10)

and

\[
\frac{t_2}{t_1} \approx \sqrt{\frac{l_2}{l_1}}
\] (5.11)

So the horizontal velocity of the vibration and time are proportional to the root square of the table length.

Y direction:

In the Y direction, particle movement is governed by the viscous force caused by the air acting on the particles (Fv) and the gravity force. Therefore, the \(\pi\) number can also be described by Eq. 5.7.
5.3 Numerical Simulation

A numerical simulation was performed to simulate the operation of the full-scale air-based density separator and the prototype model with size reduction ratio of ½ in all three axes and the results were then compared to evaluate the validity of the scaling laws.

The commercial CFD software ANSYS-Fluent was used for the simulation. A pre-processing model was developed using GAMBIT (Figure 5.1). The vibration of the separator pan was accounted for by using dynamically moving grids while the feeder tube, contained in a different sub-domain, was kept stationary. The airflow pattern of the blower was simulated using Fluent's Eulerian model. The simulated geometrical model includes the feeding tube, the vibrating pan with the air supply and a portion of the surrounding area. The geometry of the simulated air-based density separator offer no symmetries or periodicity to be exploited as simplification. As a result, the full 3-D simulation was mandatory which increased the size of the computational model and the inherent cost of the simulations. In addition, due to the pan vibration, the modeling approach has to be transient with a time step restricted by some fraction of the vibration period. To perform these demanding calculations in a timely manner, parallel calculation using a server with 30-cores running at 2.2 GHz each and 128 GB of random access memory was utilized.
The user defined function (UDF) utilized to account for the amplitude of the table vibration occurring in a plane which is positioned at 44° angle with the horizontal axis was modified for the prototype model to simulate a smaller amplitude of 3.5 mm. The mesh structure contained 151679 nodes and a total of 794319 elements. The mesh was divided into 3 separate volumes (Figure 5.2), the Top, Bottom and the Screen volume. The screen volume was setup as a porous media. The modified Darcy law formulations were utilized to account for the pressure drop and define the porous media resistance factors. The K-Epsilon model was used to define the turbulence regime around the screen volume. Table 5.1 describes the parameter values used to define the boundary conditions.
and simulations were performed to characterize the airflow patterns for 1 second simulation of full-scale model and 0.7 second simulation for the scaled prototype model.

Table 5.1 Boundary Parameter values (Full-Scale Model vs Prototype).

<table>
<thead>
<tr>
<th>Model</th>
<th>Airflow Rate</th>
<th>Longitudinal Angle</th>
<th>Transverse Angle</th>
<th>Table Frequency</th>
<th>Vibration Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-Scale Model</td>
<td>0.78 Kg/s</td>
<td>3°</td>
<td>8°</td>
<td>35 Hz</td>
<td>7 mm</td>
</tr>
<tr>
<td>Prototype Model</td>
<td>0.196 Kg/s</td>
<td>3°</td>
<td>8°</td>
<td>35 Hz</td>
<td>3.5 mm</td>
</tr>
</tbody>
</table>

The time-step size was $2 \times 10^{-5}$ s with 10 to 30 iterations per time step. 1 second flow-time simulation translates to approximately 500,000 iterations. Since the iterative solver scheme solves 2 Eqs. for K-Epsilon one second flow time simulation was completed in around 20 days. The scaled prototype was run for 0.7 seconds real time simulation.

Figure 5.2 Volume mesh setup for the air-based density separator CFD model.
5.4 Validation of the scaling laws

In order to validate the scaling laws and evaluate the modifications needed in the design of the full scale model of the dry air-based density separator to effectively treat particles having a size range of 1 to 6 mm, a prototype model scaled down by the factor of $\frac{1}{2}$ in the X, Y and Z directions was simulated.

Since the air velocity is kept constant and the area of the table decreases by $\frac{1}{4}$, the volumetric air flow and feed rate were reduced by a factor of $\frac{1}{4}$. Since the vertical (Z) and horizontal (X) velocity component of the table vibration are proportional with the root square of the characteristic length of the table, and the scaling ratio of the prototype model is $\frac{1}{2}$ of the full-scale model, these parameters were reduced by a factor of 0.7 (root square of $\frac{1}{2}$). Since the time is proportional with the root square of length characteristic, the time period for the prototype model should be 0.7 times that of the time period of the full-scale model. Hence, the full-scale model was run for 1 second while the prototype was simulated for 0.7 seconds. In order to better compare the results, the area of the table is divided to the six sections (Figure 5.3).
Figure 5.3 Top view of the Table deck displaying the dividing sections (A, B, C, D, E, F).

Results of the air velocity profile on top of the deck in 1 second of the full-scale model, 0.7 second in prototype model and in the experiment are presented in Table 5.2 and Figure 5.4. The average absolute percent difference of the air velocity on the top of the deck in two models is 3.83% which shows that variance between the model data is trivial and thus can validate the scaling laws. Slightly higher variance is recorded with respect to the experimental data. Nevertheless, in section D of the table the experimental data matches the simulated values. The experimental measurements have been recorded on various sections of the table and an average dataset was prepared while the simulated dataset is imported from a post processor using average sectional domain analysis. However, the difference between the average simulated data for each section and the experimental data is less than 15% hence the numerical results are validated.
Table 5.2 Average air velocity data measured at each section on top of the table deck (Full-scale model vs Prototype vs Experimental data).

<table>
<thead>
<tr>
<th>Sections</th>
<th>Full-Scale Model</th>
<th>Prototype Model</th>
<th>Absolute % Diff.</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.52</td>
<td>2.69</td>
<td>6.75</td>
<td>2.32</td>
</tr>
<tr>
<td>B</td>
<td>3.03</td>
<td>3.05</td>
<td>0.66</td>
<td>2.50</td>
</tr>
<tr>
<td>C</td>
<td>3.22</td>
<td>3.29</td>
<td>2.17</td>
<td>2.64</td>
</tr>
<tr>
<td>D</td>
<td>2.93</td>
<td>3.09</td>
<td>5.46</td>
<td>3.04</td>
</tr>
<tr>
<td>E</td>
<td>2.69</td>
<td>2.74</td>
<td>1.86</td>
<td>2.19</td>
</tr>
<tr>
<td>F</td>
<td>2.62</td>
<td>2.46</td>
<td>6.11</td>
<td>2.19</td>
</tr>
<tr>
<td>Average</td>
<td>2.84</td>
<td>2.89</td>
<td>3.83</td>
<td>2.48</td>
</tr>
</tbody>
</table>

Figure 5.4 Comparison of the predicted air velocity values occurring directly above the table surface in defined cross-sectional areas using the full-scale and the prototype models with the average values obtained from direct measurements at several points in each area.
The air velocity magnitude contours on the top of the deck of the full-scale model in 1 second and prototype model in 0.7 second are demonstrated in Figure 5.5. The velocity vectors for 1 second simulated time for the full-scale model and 0.7 second for the prototype model are depicted in Figure 5.6. The air-velocity contour plot and the vector plot show modest variation between the two models, validating the scaling laws.

The minimum, maximum, and area average of turbulent intensity on top of the table deck prior to particle injection for 1 second full-scale and 0.7 second prototype model are listed in Table 5.3, and turbulent intensity contours are shown in Figure 5.7.

Table 5.3 Turbulent intensity plot (0.7 sec prototype vs 1 sec full-scale model).

<table>
<thead>
<tr>
<th>Statistical Parameters</th>
<th>Values (%)</th>
<th>Full-Scale Model</th>
<th>Prototype Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td></td>
<td>0.0046</td>
<td>0.0059</td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td>3.64</td>
<td>1.72</td>
</tr>
<tr>
<td>Area Average</td>
<td></td>
<td>0.38</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Figure 5.5 Air velocity contour plot (0.7 sec prototype vs 1 sec full-scale model).
5.5 Discussion

The turbulent intensity data proves that the initial assumption of laminar flow was adequate. However it should be noted that the region would be highly turbulent with the
injection of particles in the system as the chaotic nature of the particle movement on the
deck and the formation of the bed would create a restrictive zone for the airflow passing
through the porous media, hence the intensity values might prove to be non-trivial with
the introduction of particles into the system. The variation between the two models was
found to be minimal and the absolute % difference of the air velocity between the full-
scale model and the prototype was 3.83%.

The scaling laws indicate that the vibration velocity has an integral effect on the
separation performance, and its vertical and horizontal components need to be changed
by the square roots of the length characteristic ratios. The air velocity vector and contour
plots as well as the turbulent intensity contours in the two models displayed similar
behavior, all validating the scaling laws. Hence, to modify the scale of the laboratory air-
based density separator to treat industrial capacity feed rates, the air velocity should be
kept at a constant value as calculated by the scaling laws, while the volumetric airflow
rate needs to be changed based on the deck area ratio.
CHAPTER 6: CONCLUSION

6.1 Conclusions

The objective of the dissertation was to characterize an air-based density separator using empirical modeling from data generated from a statistical experimental program, CFD analysis and scale modeling techniques to find novel ways to improve the separation performance in the fine particle range and to implement design modifications necessary to increase selectivity.

The laboratory tests were conducted in two phases using the 6 x 1 mm low rank Powder River basin coal as a model system due to its wide range of particle size and density. The first phase of the test work involved evaluating the different operating parameters such as longitudinal and transverse angles, table vibration frequency and the rate of air-flow utilizing a 3-level Box-Behnken experimental design program. The process parameters were systematically varied to identify the important process factors having an impact on performance. The performance data was analyzed using a commercially available Design Expert software and statistically significant empirical models were developed for product ash and product yield in terms of the operating parameters. The statistical analysis showed that all four parameters, i.e., the fan blower frequency, the table vibration frequency, longitudinal angle and transverse angle, were highly significant in affecting the separation performance. The results prove that it is possible to obtain a clean product having approximately 7.7 % ash at a mass yield of
around 60% from a feed coal having around 26% ash. The combustible recovery was 65%
and ash rejection was 80%. Several test runs achieved recovery values close to the
optimum as predicted from the float-sink analysis data. The second phase of the test work
involved further tests to verify the empirically predicted optimum. The results validate
the possibility of achieving a sharp separation with low $E_p$ values of 0.12 and 0.22 in the
fine particle range of the low rank Powder River basin coal. During the test work it was
observed that the higher air flow rate and increase in the longitudinal angle helps in the
formation of a homogeneous fluidized bed. The feed particle stream act as a dampener of
the airflow at the feed end directly affecting the feed rate at the discharge end. The
transverse angle controls the reject discharge with an increase in the inclination resulting
in higher flow rates towards the product end limiting the discharge at the reject end. The
table frequency has a direct correlation with the residence time of the particles on the
table bed. Increase in table frequency decreases the residence time and discharge occurs
prior to the formation of a homogeneous bed.

The comprehensive numerical model of the air-based density separator was developed
using a full-scale 3D mesh generated in Gambit. Experimental measurements of airflow
and pressure drop was collected at different sections of the table using a digital
anemometer and a digital manometer, respectively. The porosity across the deck mesh
was calculated based on an average dataset using the modified Darcy's law formulation.
To account for the turbulence in the airflow in accordance to the geometric asymmetry of
the table deck, it was necessary to define the hydraulic diameters of the different inlet and
outlet faces. The vibration of the separator pan was simulated by applying a user defined
function (UDF) to emulate the dynamic movement of the grid while the feeder tube contained in a different sub-domain was kept stationary. The relatively new Eulerian granular phase dense discrete (DDPM) model of Ansys Fluent® was used to account for a large number of particles analogous to a Lagrangian tracking scheme. In addition, due to the pan vibration, the modeling approach was required to be transient with a time step restricted by some fraction of the vibration period. A high precision time step of $2 \times 10^{-5}$ second with 10 to 30 iterations per time step was utilized. The simulation was run on a 30 core x 2.2 Ghz server running at 128 GB Ram.

The DDPM approach was evaluated using two distinct turbulence schemes, i.e., realizable $k - \epsilon$ and RSM models. The time scale of one second simulation for RSM was 50 days (approx. 500,000 iterations) and 50,000 time steps. The iterative solver scheme solves seven Eqs. for RSM instead of two Eqs. for the $k - \epsilon$ model, hence the extended time scale. The realizable $k - \epsilon$ model by contrast took 20 days to complete one second of real time simulation which showed 2.5 time faster convergence time. The $k - \epsilon$ model was run for an additional 3 seconds to characterize the particle feed separation. The particulate feed to the model was comprised of particles with varying density and size. The four different particle density values were 1.3, 1.45, 1.8 and 2.65 gm/cm$^3$ respectively. The particle size distribution followed the Rosin Rammler model with 80 particles being injected at each time step. The "Non-Spherical" drag model was utilized with a shape factor of 0.536. Particle injection was initiated after 3 seconds of dry run and 30 iterations were performed each time step. Pseudo steady state was achieved after 2s of particle injection which was defined as the condition where the number of particles
entering the simulation domain was equivalent to the number of particles exiting the domain thereby following the law of conservation of mass. Transient particle tracking was realized in real time using x,y,z positional tracking scheme of fluent and escaping mass calculated for each zone. The "Gidaspow" granular viscosity model was used and the granular bulk viscosity was defined by the "Lun-et-al" formulation.

In general, it is believed that a stable and well distributed air-flow profile above the air-table surface is necessary to achieve a high level of separation performance, especially for fine particles. The air-flow profile showed high peaks and variances near the feed end. However, there were significant differences in the profiles obtained between the RSM and k – ε models. The absolute % difference between the k – ε and the RSM models air velocity data was 48.35 % for the domain maximum. RSM recorded a maximum airspeed of 5.97 m/s in the domain but k – ε model domain maximum was almost double at 11.55 m/s. Both the models were validated using point prediction method and comparison with the experimental values. The experimental pressure drop measurements at specific points above and below the screen deck was compared with the simulated results at similar points in the domain. The k – ε model shows a variance of about 7.29% with the measured data whereas the RSM values showed better correlation and the difference was 2.67%. Air velocity data was validated in similar way by point prediction on top of the screen deck. At different sections the divergence was found to be as large as 10% and as small as 1% but the maximum measured value of 3.04 m/s is lower than the maximum simulated value of 5.8 m/s which is due to the inherent
numerical error of the two Eq. $k - \epsilon$ model. However, the average values conform well with the measured values.

The pressure drop results and the turbulent intensity values display the maximum variance between the two models. The RSM model showed a remarkable characteristic whereby the average pressure on top of the screen deck is -6.47 Pascals which is negative, hence it is indicative that the portion of the air is getting sucked back into the system when the deck is in motion. The $k - \epsilon$ model showed a positive pressure value of 11.66 Pascal. The scenario described by the RSM simulation is more plausible as the deck is in constant motion thus forming a negative pressure situation on top of the deck augmented by the incoming feed. Velocity vectors showed a variance at the time of deck motion. High pressure fluctuations were observed near the feed end. Turbulent intensity of 1.98 % in RSM model and 5.83 % in $k - \epsilon$ at 0.2 second mark show major variance between turbulence models. The $k - \epsilon$ model showed higher peak values in the simulation domain than the RSM model but the time scale difference in the iterative scheme between the two models prevent any substantive investigation to gauge the separation performance with RSM.

The partition curves generated from the particle segregation data obtained by analyzing a time-capsule of 100 time steps or 0.002 seconds of simulation time shows that there is a difference of 11% between the mass yield values of the simulated data and the experimental results for a feed rate of 200 kg/hr. The simulation data also shows 43% displacement of middle density particles and 29% displacement of the high density
particles reporting to the low density particle stream. The overall density cut-point predicted by the CFD model is within 5% of the experimentally measured value which is sufficiently accurate for validation purposes. Increasing the feed rate to 400 kg/hr results in a drop in the yield and the effective deshaling capability of the table is severely restricted.

A very interesting phenomenon of vortices forming around the riffles on the table deck was observed which would have a negative effect on the separation performance. The existing riffle design is triangular in shape which facilitates the formation of the local vortices. The particle tracking data show that there is considerable displacement of high density particles preferentially occur for particles in the finest particle size fractions which are more easily carried by the vortices across the length of the table before losing momentum which further declines towards the middle part of the table. The displacement of the heavier particles by the action of vortices can be prevented by implementing a change in the riffle design.

The next phase of the project involved evaluating the air table using modern scale modeling techniques to augment the capacity of the laboratory table to treat industrial feed rates. To achieve this goal, scaling laws were derived and the full-scale model of a laboratory air-based density separator and a prototype model scaled down by the factor of \( \frac{1}{2} \) in X, Y and Z directions were simulated using numerical methods. The user defined function (UDF) was modified to simulate a smaller amplitude of 3.5 mm for the prototype model. The air velocity and turbulent intensity were used to validate the scaling
laws. The variation between the full-scale and the prototype model was found to be minimal and the absolute percent difference of air velocity between the two models was 3.83 %. The air velocity vector and contour plots as well as the turbulent intensity contours in two models displayed similar behavior, all validating the scaling laws. Therefore, to modify the scale of the laboratory air-based density separator to treat industrial feed rates, the air velocity should be kept at a constant value as calculated by the scaling laws, while the volumetric air flow rate needs to be changed based on the deck area ratio. Furthermore, the scaling laws show that the vibration velocity has an integral effect on the separation performance and its vertical and horizontal components needs to be changed by the square roots of the length characteristic ratios.

6.2 Recommendations for Future Work

Based on a review of the previous investigations and findings of the research described in this dissertation, the recommendations for future study are as follows:

- Investigating a staggered design with adjustable height of the riffle using CFD analysis and scale modeling techniques to reduce the effect of the momentum curl with formation of vortices and improve the selectivity of the separation performance.
- Exploring the potential benefits of momentum conservation using different materials for deck design. The goal would be to restrict the effect of dissipation of momentum of the particles after hitting the deck surface.
• Evaluate a Lagrangian tracking scheme with Large Eddy and Detached Eddy simulation turbulence models and correlation of the results with the DDPM Eulerian scheme and experimental data.

• Introduction of the particulate phase in the prototype scale model and evaluate the separation performance achieved by the prototype.
## APPENDICES

Table A-1 Statistical Test Program Data: Feed = 204.38 kg/hr

<table>
<thead>
<tr>
<th>Run</th>
<th>Fan Freq</th>
<th>Table Freq</th>
<th>Long. Angle</th>
<th>Trans. Angle</th>
<th>Feed Ash %</th>
<th>Prod. Ash %</th>
<th>Yield (%)</th>
<th>Comb. Rec</th>
<th>Ash Rej</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>35</td>
<td>1</td>
<td>6</td>
<td>26.62</td>
<td>8.67</td>
<td>76.84</td>
<td>93.09</td>
<td>74.98</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>30</td>
<td>1.5</td>
<td>8</td>
<td>29.89</td>
<td>17.04</td>
<td>87.31</td>
<td>99.02</td>
<td>43.66</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>35</td>
<td>1</td>
<td>4</td>
<td>26.72</td>
<td>8.40</td>
<td>71.48</td>
<td>86.26</td>
<td>77.52</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>40</td>
<td>1.5</td>
<td>4</td>
<td>28.12</td>
<td>7.68</td>
<td>58.35</td>
<td>71.23</td>
<td>84.06</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>30</td>
<td>1.5</td>
<td>4</td>
<td>26.81</td>
<td>8.09</td>
<td>69.80</td>
<td>92.14</td>
<td>79.22</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>35</td>
<td>2</td>
<td>4</td>
<td>23.78</td>
<td>7.68</td>
<td>60.35</td>
<td>64.62</td>
<td>80.51</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>35</td>
<td>1.5</td>
<td>6</td>
<td>26.11</td>
<td>9.43</td>
<td>77.62</td>
<td>87.96</td>
<td>72.68</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>35</td>
<td>1.5</td>
<td>8</td>
<td>29.03</td>
<td>16.90</td>
<td>95.58</td>
<td>100.00</td>
<td>19.87</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>35</td>
<td>1.5</td>
<td>4</td>
<td>25.56</td>
<td>8.93</td>
<td>73.71</td>
<td>86.73</td>
<td>74.24</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>35</td>
<td>1</td>
<td>8</td>
<td>27.03</td>
<td>13.86</td>
<td>86.60</td>
<td>100.00</td>
<td>49.88</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
<td>40</td>
<td>1</td>
<td>6</td>
<td>27.97</td>
<td>7.68</td>
<td>53.63</td>
<td>90.38</td>
<td>85.28</td>
</tr>
<tr>
<td>12</td>
<td>40</td>
<td>40</td>
<td>1.5</td>
<td>8</td>
<td>29.49</td>
<td>9.04</td>
<td>71.64</td>
<td>90.67</td>
<td>78.05</td>
</tr>
<tr>
<td>13</td>
<td>40</td>
<td>35</td>
<td>2</td>
<td>8</td>
<td>28.53</td>
<td>9.15</td>
<td>74.90</td>
<td>99.15</td>
<td>75.97</td>
</tr>
<tr>
<td>14</td>
<td>50</td>
<td>35</td>
<td>1</td>
<td>6</td>
<td>28.34</td>
<td>10.57</td>
<td>77.74</td>
<td>93.70</td>
<td>71.02</td>
</tr>
<tr>
<td>15</td>
<td>50</td>
<td>30</td>
<td>1.5</td>
<td>6</td>
<td>29.64</td>
<td>13.34</td>
<td>79.89</td>
<td>100.00</td>
<td>62.95</td>
</tr>
<tr>
<td>16</td>
<td>40</td>
<td>40</td>
<td>2</td>
<td>6</td>
<td>26.86</td>
<td>7.51</td>
<td>51.40</td>
<td>52.37</td>
<td>85.63</td>
</tr>
<tr>
<td>17</td>
<td>30</td>
<td>35</td>
<td>1.5</td>
<td>4</td>
<td>29.49</td>
<td>7.35</td>
<td>23.48</td>
<td>29.69</td>
<td>93.30</td>
</tr>
<tr>
<td>18</td>
<td>40</td>
<td>35</td>
<td>1.5</td>
<td>6</td>
<td>29.25</td>
<td>8.22</td>
<td>66.06</td>
<td>94.42</td>
<td>82.73</td>
</tr>
<tr>
<td>19</td>
<td>30</td>
<td>40</td>
<td>1.5</td>
<td>6</td>
<td>30.15</td>
<td>7.77</td>
<td>31.42</td>
<td>43.80</td>
<td>91.90</td>
</tr>
<tr>
<td>20</td>
<td>50</td>
<td>40</td>
<td>1.5</td>
<td>6</td>
<td>25.44</td>
<td>7.63</td>
<td>64.49</td>
<td>70.41</td>
<td>80.66</td>
</tr>
<tr>
<td>21</td>
<td>30</td>
<td>30</td>
<td>1.5</td>
<td>6</td>
<td>28.66</td>
<td>9.16</td>
<td>70.97</td>
<td>86.02</td>
<td>65.29</td>
</tr>
<tr>
<td>22</td>
<td>40</td>
<td>30</td>
<td>2</td>
<td>6</td>
<td>26.12</td>
<td>11.59</td>
<td>70.08</td>
<td>100.00</td>
<td>76.02</td>
</tr>
<tr>
<td>23</td>
<td>40</td>
<td>30</td>
<td>1</td>
<td>6</td>
<td>27.24</td>
<td>13.15</td>
<td>76.95</td>
<td>100.00</td>
<td>65.85</td>
</tr>
<tr>
<td>24</td>
<td>50</td>
<td>35</td>
<td>2</td>
<td>6</td>
<td>32.67</td>
<td>8.27</td>
<td>62.38</td>
<td>92.80</td>
<td>84.21</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td>35</td>
<td>1.5</td>
<td>8</td>
<td>27.87</td>
<td>7.03</td>
<td>35.11</td>
<td>61.46</td>
<td>91.14</td>
</tr>
<tr>
<td>26</td>
<td>30</td>
<td>35</td>
<td>2</td>
<td>6</td>
<td>28.85</td>
<td>7.25</td>
<td>30.41</td>
<td>46.13</td>
<td>92.36</td>
</tr>
</tbody>
</table>
Figure A-1 Optimized Test vs Washability data (Statistical Test Program)
Table A-2 User Defined Function (UDF) defining the table deck motion

/*****************************/
* Compiled UDF
* Time-dependent Screen Deck position
* Formula used for Screen deck position:
* $x[t] = x_0 - a * \cos(36 + 8) * \sin(2 \pi f \cdot t)$
* $y[t] = y_0$
* $z[t] = z_0 + a * \sin(36 + 8) * \sin(2 \pi f \cdot t)$
* Formula used for Screen Deck Velocity:
* $u[t] = -2\pi f \cdot a * \cos(44) \cdot \cos(2 \pi f \cdot t)$
* $v[t] = 0.0$
* $w[t] = 2\pi f \cdot a * \sin(44) \cdot \cos(2 \pi f \cdot t)$
* $(x_0, y_0, z_0)$ is the position of the Screen at the starting of simulation
* Total displacement is $2a$ in the direction of the movement
/*****************************/

#include "udf.h"

real freq = 35.0;
real ampl = 0.0035;
real pvelx;
pvely;
pvelz;

DEFINE_ZONE_MOTION(fmotion,omega,axis,origin,vel,time,dtime)
{
/* Compute Components of Screen Velocity */
pvelx = -2.0 * M_PI * freq * ampl * cos(44.0 * M_PI / 180.0);
pvelx = pvelx * cos(2.0 * M_PI * freq * time);
pvelz = 2.0 * M_PI * freq * ampl * sin(44.0 * M_PI / 180.0);
pvelz = pvelz * cos(2.0 * M_PI * freq * time);
/* Assign Velocities */
vel[0] = pvelx;
vel[1] = pvely;
vel[2] = pvelz;

return;
}
REFERENCES


Arms, R. W., 1924, Dry cleaning of coal, Transactions of the American Institution of Mining and Metallurgy, 70, 758-774.


International Conference on CFD in the Mineral & Metal Processing and Power Generation Industries, CSIRO, Melbourne, Australia.


Bertrand, J., 1878, Sur l'homogénété dans les formules de physique, Comptes rendus 86 (15), 916–920.


Brennan, M., 2006, CFD Simulations of hydrocyclones with an air core: Comparison Between Large Eddy Simulations and a Second Moment Closure, Chemical Engineering Research and Design, 84, 495-505.


Emori, R. I. and Saito, K., 1982, Model experiment of hazardous forest fire whirl, Fire Technology, 18, 319 - 327.


Emori, R. I., Saito, K. and Sekimoto, K., 2000, Scale Models in Engineering (in Japanese: Mokei Jikken no Riron to Ohyou), Gihodo, Tokyo, Japan.

Euler, L., 1757, General principles concerning the motion of fluids (Principes généraux du mouvement des fluides), Mémoires de l'académie des sciences de Berlin, 11, 274-315.


Gupta, R., Kaulaskar, M. D., Kumar, V., Sripriya, R., Meikap, B. C. and Chakraborty, S., 2008, Studies on the understanding mechanism of air core and vortex formation in a hydrocyclone, Chemical Engineering Journal, 144, 153-166.


Jianing, H., 2006, Market prospect of application of the compound dry coal cleaning technology in USA, China Coal, Technology Monograph of the Tangshan Shenzhou Machinery Co., Ltd., 29-33.


Luo, Z., Chen, Q. and Yaomin, Z., 2002, Dry beneficiation of coarse coal using an air dense medium fluidized bed (ADMFB), Coal Preparation, 22, 57-64.


Rayleigh, 1892, On the question of the stability of the flow of liquids, Philosophical magazine 34, 59–70.


Riabouchinsky, D., 1911, Méthode des variables de dimension zéro et son application en aérodynamique, L’Aérophile, 407–408.

Rittinger, P. R., 1867, Lehrbuch der Aufbereitungskunde (Textbook of the Processing Art), Berlin: Verlag von Ernst & Korn.


Saito, K. (editor), 2008, Progress in scale modeling: summary of the first international symposium on scale modeling (ISSM in 1988) and selected papers from subsequent symposia (ISSM II in 1997 through ISSM V in 2006), Springer.

Saito, K., 2012, Lecture notes ME 565: Scale modeling in engineering, Department of Mechanical Engineering, University of Kentucky, Lexington, Kentucky, USA.


VITA

Tathagata Ghosh was born in Kolkata, India. He obtained his undergraduate degree in Mining Engineering from Bengal Engineering and Science University, Howrah, India in 2004. He was then employed at Tega Industries Limited in Kolkata, India. In 2005, he went to Poland to pursue Master of Science in Mechanical Engineering from AGH University of Science and Technology in Krakow, Poland. He completed the post graduate M. Sc. and returned to Kolkata in March, 2007. He was then employed as a Mining Engineer Consultant at the International Mining Consultants (IMC-SRG) company. In August 2007, he came to the University of Kentucky to pursue a Doctor of Philosophy degree in Mining Engineering.