Uncertainty Analysis of Carbon Ablation in the VKI Plasmatron

A. Turchi  
von Karman Institute

P. M. Congedo  
INRIA, Bordeaux

B. Helber  
von Karman Institute

T. E. Magin  
von Karman Institute

6th Ablation Workshop, April 10-14, Urbana-Champaign, Illinois
Motivations

- **To understand** the operational behavior of the TPS materials
- **To study** the gas/surface interaction physics occurring during reentry
- **To improve** the prediction capacity and **reduce** the design margins

---

**GALILEO MISSION**

Destination: Jupiter  
Date: 1989–2003

---

**THE BEST RACE CAR IS THE ONE THAT FALLS APART RIGHT AFTER THE FINISH LINE...**
LET’S INTRODUCE OUR PLAYERS

Player 1
"the oven"

Player 2
"the recipe"

Player 3
"the customer"

Test conditions
- \( T_e = 8000 \) K
- \( P_e = 1500 \) Pa
- \( v_e = 1500 \) m/s
Let’s introduce our players

Player 1

"the oven"

PLASMATRON

Player 2

"the recipe"

Test conditions
\[ T_e = 8000 \text{ K} \]
\[ P_e = 1500 \text{ Pa} \]
\[ v_e = 1500 \text{ m/s} \]

Rebuilding Code
(boundary layer)

Player 3

"the customer"

Stagnation-line code
(w/ ablative b.c.)

INTRODUCTION
**PLAYER #1: PLASMATRON FACILITY**

**Role:** performing reusable/ablative TPS tests

- **Gas:** Air, N₂, CO₂, Ar
- **Power:** 1.2 MW – most powerful ICP in the world –
- **Heat-flux:** up to 16 MW/m² (superorbital re-entry)
- **Pressure:** 10 – 800 mbar
PLAYER #1: PLASMATRON FACILITY

TPS MATERIAL OPERATIONAL TESTING IS ACHIEVABLE!!
**PLAYER #2: BOUNDARY-LAYER CODE**

*Role: rebuilding of enthalpy (calorimeter)*

### Description

- Solves the reacting boundary layer equations along the stagnation line.
- Assumes catalytic surface ($N + N \rightarrow N_2$ and $O + O \rightarrow O_2$).
- Rebuilds the boundary layer edge conditions to match the measured wall heat flux:

$$\dot{q}_{cw} = \dot{q}_{cw} \left( T_{cw}, \gamma_{ref}, h_e, p_e, \delta, \frac{\partial u_e}{\partial x}, v_e \frac{\partial}{\partial y} \left( \frac{\partial u_e}{\partial x} \right) \right)$$

### Pros & Cons

- **Thumbs up:** Limited computational cost
- **Thumbs down:** Ablative boundary condition not yet implemented

---

**PLAYER #3: STAGNATION-LINE CODE**

*Role: rebuilding of the ablation test (test sample)*

**Description**

- Solves a reduced form of the Navier–Stokes equations along the stagnation line
- Applicable to both sub- and supersonic flow over spheres and cylinders
- Chemistry solved via the Mutation++ Library. Up-to-date thermodynamic and transport properties dataset

**Pros & Cons**

- 🔄 Ablative boundary condition implemented
- 💿 Medium computational cost

---

**PLAYER #3: STAGNATION-LINE CODE**

### Surface Mass Balance

\[ \rho D_{im} \frac{\partial y_i}{\partial \eta} \bigg|_w + \dot{m}_{i,c} = (\rho v)_w y_{iw} \]

### Surface Energy Balance

\[ k \frac{\partial T}{\partial \eta} \bigg|_w + \sum_{i=1}^{N_c} h_{iw} \rho D_{im} \frac{\partial y_i}{\partial \eta} \bigg|_w + \dot{m}_c h_{cw} + \dot{Q}_{rad\text{net}} = (\rho v)_w h_w + \dot{Q}_{\text{cond}}^{ss} \]
**PLAYER #3: STAGNATION-LINE CODE**

The thermochemical ablation model considers the following reactions:

<table>
<thead>
<tr>
<th>Oxidation</th>
<th>Nitridation*</th>
<th>Sublimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_s + O \rightarrow CO$</td>
<td>$C_s + N \rightarrow CN$</td>
<td>$3C_s \rightarrow C_3$</td>
</tr>
<tr>
<td>$2C_s + O_2 \rightarrow 2CO$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Surface source terms are given in the form:

$$\dot{m}_i = \beta_{0i} \left( m_i n_i \sqrt{\frac{kT_w}{2\pi m_i}} \right)$$

Reaction probabilities evaluated experimentally

PUT THE PLAYERS TOGETHER

PLASMATRON

Experimental conditions
Geometry
Measurements

Rebuilding Code (boundary layer)

Stagnation-line code (w/ ablative b.c.)

The players

Alessandro Turchi
PUT THE PLAYERS TOGETHER

Experimental conditions
Geometry
Measurements

PLASMATRON

Rebuilding Code (boundary layer)

ICP code

Stagnation-line code (w/ ablative b.c.)

\[ q_{cw}^{(n)} = q_{cw}^{(exp)} \]

Yes

No

\[ Y_{ref,Cu} \]
\[ T_{cw} \]

\[ m_e, T_e, y_{i,e} \]

\[ P_{sta} \]
\[ R_{sample} \]
\[ P_{dyn} \]

\[ \dot{q}_{cw} \]

\[ y_{k,e} \]
PUT THE PLAYERS TOGETHER

PLASMATRON

Experimental conditions

Geometry

Measurements

ICP code

Rebuilding Code (boundary layer)

Stagnation-line code (w/ ablative b.c.)

\( Y_{ref,Cu, \gamma_{cw}} \)

\( m_e, T_e, y_{i,e} \)

\( \dot{m}_c, T_w, q_w \)

\( q_{cw}^{(n)} = q_{cw}^{(exp)} \)

Yes

No

\( P_{sta}, R_{sample}, P_{dyn}, q_{cw} \)

\( y_{k,e} \)
**Uncertain inputs generate...uncertain outputs!!!**

**STEP 1: BOUNDARY-LAYER CODE**

- Dynamic Pressure
- Static Pressure
- Cold Wall Heat Flux
- Cold Wall Temperature
- Catalycity
- Nitrogen/Oxygen ratio

**STEP 2: STAGNATION-LINE CODE**

- $C_S + O \rightarrow CO$
- $2C_S + O_2 \rightarrow 2CO$
- $C_S + N \rightarrow CN$
- $3C_S \rightarrow C_3$
- $N + N \rightarrow N_2$
- TPS wall emissivity

---

**THE PLAYERS**

Alessandro Turchi

33%
Objectives

1. **Evaluate** the ablative model uncertainty impact on the final QOIs

2. **Quantify** the influence of the free-stream condition uncertainties on the final QOIs
**POLYNOMIAL CHAOS (PC) EXPANSIONS**

1. The QOI $u$ is expanded in a convergent series

$$u(\xi) \approx u^{\text{PC}}(\xi) = \sum_{\alpha=0}^{P} u_\alpha \Psi_\alpha(\xi),$$

- $P = (n_\xi + N_0)!/n_\xi!N_0!$, $N_0$: expansion degree
- $\{\Psi_\alpha\}_{\alpha=0,\ldots,P}$ polynomial functions orthogonal w.r.t $p_\xi$ (input PDF)
- correspondence between $p_\xi$ and $\{\Psi_\alpha\}$
- $\{u_\alpha\}_{\alpha=0,\ldots,P}$: deterministic spectral coefficients

2. A non-intrusive spectral method is used to determine $\{u_\alpha\}$

$$u_\alpha = \|\Psi_\alpha\|^{-2} \int u(\xi) \Psi_\alpha(\xi) \approx \|\Psi_\alpha\|^{-2} \sum_{i=1}^{n} u(x, t, \xi_i) \Psi_\alpha(\xi_i) \omega_i$$

- $(\xi_i, \omega_i)$ quadrature formulae points and weights $\rightarrow$ deterministic code used as a black box

* Wiener 38; Cameron & Martin 47; Ghanem & Spanos 91
Sensitivity Analysis

From PC expansions of QOIs

1. **Means and Variances are obtained**

   \[ E(u^{PC}) = u_0, \quad \text{Var}(u^{PC}) = \sum_{\alpha=1}^{P} u_{\alpha}^2(x) \langle \psi_i^2 \rangle \]

2. **Sensitivity Analysis by ANOVA Decomposition**

   - **Sobol first order indices** \( \{ S_i \}_{i=1, \ldots, n_{\xi}} \)
     
     Quantifies the contribution to the QOI variance of the \( i^{th} \) random parameter

   - **Sobol total order indices** \( \{ S_{T,i} \}_{i=1, \ldots, n_{\xi}} \)
     
     Quantifies the contribution to the QOI variance of the \( i^{th} \) random parameter including interactions with other parameter \( j \in \{1, \ldots, n_{\xi}\}, j \neq i \)

* Crestaux, Le Maitre & Martinez 09
Let’s start... from the end

**STEP 1: BOUNDARY-LAYER CODE**

- Dynamic Pressure
- Static Pressure
- Cold Wall Heat Flux
- Cold Wall Temperature
- Catalycity
- Nitrogen/Oxygen ratio

**STEP 2: STAGNATION-LINE CODE**

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DISTRIBUTION</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_S + O \rightarrow CO$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$2C_S + O_2 \rightarrow 2CO$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$C_S + N \rightarrow CN$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$3C_S \rightarrow C_3$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$N + N \rightarrow N_2$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>TPS wall emissivity</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

VARIABLE:
- Dynamic Pressure
- Static Pressure
- Cold Wall Heat Flux
- Cold Wall Temperature
- Catalycity
- Nitrogen/Oxygen ratio

RANGE:

**VARIABLE DISTRIBUTION MEAN ERROR**

- Dynamic Pressure: Normal 48 Pa 8.0%
- Static Pressure: Normal 20000 Pa 0.3%
- Cold Wall Heat Flux: Normal 2962 kW/m² 10.0%
- Cold Wall Temperature: Normal 350 K 10.0%

**CATALYST UNIFORMITY**

- Nitrogen/Oxygen ratio: Uniform (79/21) ±2%

**STEP 2: STAGNATION-LINE CODE**

- $m_{ef}$, $T_{ef}$, $Y_{ref,w}$
- $Y_{reac,w}$
- $\epsilon_w$
- $m_c$, $T_w$, $q_w$
Atomic oxygen

\[ C_s + O \rightarrow CO \]

Molecular oxygen

\[ 2C_s + O_2 \rightarrow 2CO \]
REACTION PROBABILITY UNCERTAINTIES ASSESSMENT

**Atomic oxygen**

\[ \text{Cs} + \text{O} \rightarrow \text{CO} \]

**Molecular oxygen**

\[ 2 \text{Cs} + \text{O}_2 \rightarrow 2 \text{CO} \]
**Define the Input Uncertainties**

### STEPM 1: BOUNDARY-LAYER CODE

- **Dynamic Pressure**
- **Static Pressure**
- **Cold Wall Heat Flux**
- **Cold Wall Temperature**
- **Catalycity**
- **Nitrogen/Oxygen ratio**

### STEPM 2: STAGNATION-LINE CODE

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DISTRIBUTION</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_S + O \rightarrow CO$</td>
<td>Uniform</td>
<td>0.37–1</td>
</tr>
<tr>
<td>$2C_S + O_2 \rightarrow 2CO$</td>
<td>LogUniform</td>
<td>0.00001–0.1</td>
</tr>
<tr>
<td>$C_S + N \rightarrow CN$</td>
<td>Uniform</td>
<td>0–0.3</td>
</tr>
<tr>
<td>$3C_S \rightarrow C_3$</td>
<td>LogUniform</td>
<td>0.01–1</td>
</tr>
<tr>
<td>$N + N \rightarrow N_2$</td>
<td>Uniform</td>
<td>0–0.5</td>
</tr>
<tr>
<td>TPS wall emissivity</td>
<td>Uniform</td>
<td>0.8–0.95</td>
</tr>
</tbody>
</table>
STAGNATION-LINE CODE NOMINAL OUTPUTS

**ABLATION QOI**

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass blowing rate</td>
<td>0.041[kg / m²s]</td>
</tr>
<tr>
<td>temperature</td>
<td>2534 [K]</td>
</tr>
<tr>
<td>mass blowing rate</td>
<td>0.021[kg / m²s]</td>
</tr>
<tr>
<td>temperature</td>
<td>2840 [K]</td>
</tr>
</tbody>
</table>

**S-L CODE RESULTS**

Alessandro Turchi

63%
STAGNATION-LINE CODE W/ NITRIDATION

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
<th>VARIANCE</th>
<th>( \Delta_{\text{stoch} - \text{nom}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass blowing rate</td>
<td>0.031 [kg / m(^2)s]</td>
<td>2.69e-05</td>
<td>-24.4%</td>
</tr>
<tr>
<td>temperature</td>
<td>2722 [K]</td>
<td>1.54e+04</td>
<td>+7.4%</td>
</tr>
</tbody>
</table>

NITRIDATION AND RECOMBINATION ARE STRONGLY RELATED!
STAGNATION-LINE CODE W/O NITRIDATION

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
<th>VARIANCE</th>
<th>$\Delta_{stoch-nom}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass blowing rate</td>
<td>0.021 [kg/m²s]</td>
<td>2.63e-10</td>
<td>-0.6%</td>
</tr>
<tr>
<td>temperature</td>
<td>2903 [K]</td>
<td>2.74e+03</td>
<td>2.2%</td>
</tr>
</tbody>
</table>

Wall mass blowing rate

Wall temperature

OXYGEN DIFFUSION LIMITS THE ABLATION RATE!
## Define the Input Uncertainties

### Step 1: Boundary-Layer Code

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DISTRIBUTION</th>
<th>MEAN</th>
<th>ERROR (±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Pressure</td>
<td>Normal</td>
<td>48 Pa</td>
<td>8.0%</td>
</tr>
<tr>
<td>Static Pressure</td>
<td>Normal</td>
<td>20000 Pa</td>
<td>0.3%</td>
</tr>
<tr>
<td>Cold Wall Heat Flux</td>
<td>Normal</td>
<td>2962 kW/m²</td>
<td>10.0%</td>
</tr>
<tr>
<td>Cold Wall Temperature</td>
<td>Normal</td>
<td>350 K</td>
<td>10.0%</td>
</tr>
<tr>
<td>Catalycity</td>
<td>Uniform</td>
<td>0.001–1</td>
<td>0.001–1</td>
</tr>
<tr>
<td>Nitrogen/Oxygen ratio</td>
<td>Uniform</td>
<td>(79/21) ± 2%</td>
<td>(79/21) ± 2%</td>
</tr>
</tbody>
</table>

### Step 2: Stagnation-Line Code

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DISTRIBUTION</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_S + O \rightarrow CO$</td>
<td>Uniform</td>
<td>0.37–1</td>
</tr>
<tr>
<td>$2C_S + O_2 \rightarrow 2CO$</td>
<td>LogUniform</td>
<td>0.00001–0.1</td>
</tr>
<tr>
<td>$C_S + N \rightarrow CN$</td>
<td>Uniform</td>
<td>0–0.3</td>
</tr>
<tr>
<td>$3C_S \rightarrow C_3$</td>
<td>LogUniform</td>
<td>0.01–1</td>
</tr>
<tr>
<td>$N + N \rightarrow N_2$</td>
<td>Uniform</td>
<td>0–0.5</td>
</tr>
<tr>
<td>TPS wall emissivity</td>
<td>Uniform</td>
<td>0.8–0.95</td>
</tr>
</tbody>
</table>
BOUNDARY-LAYER CODE ANALYSIS

**Edge mass flow rate**
error: ±8.00%

**Edge temperature**
error: ±9.41%

**Edge velocity gradient**
error: ±7.64%

EXPERIMENTAL UNCERTAINTIES ARE AFFECTING THE QOI THE MOST!
**Boundary-Layer Code Analysis**

**Mean Edge Mass Fractions**

<table>
<thead>
<tr>
<th>Species</th>
<th>Mass Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>O(_2)</td>
<td>1.24e-05</td>
</tr>
<tr>
<td>N(_2)</td>
<td>1.79e-01</td>
</tr>
<tr>
<td>NO</td>
<td>1.29e-03</td>
</tr>
<tr>
<td>O(^+)</td>
<td>2.42e-04</td>
</tr>
<tr>
<td>N(^+)</td>
<td>6.95e-04</td>
</tr>
<tr>
<td>O</td>
<td>2.32e-01</td>
</tr>
<tr>
<td>N</td>
<td>5.87e-01</td>
</tr>
<tr>
<td>e(^-)</td>
<td>3.55e-08</td>
</tr>
</tbody>
</table>

**N\(_2\) Mass Fraction**

- Error: ±69.00%

**N Mass Fraction**

- Error: ±12.76%

**O Mass Fraction**

- Error: ±0.96%

Oxygen practically unaffected by the uncertainties!
COUPLED ANALYSIS: INPUT UNCERTAINTY DISTRIBUTIONS

Stagnation-line code (w/ ablative b.c.)
Rebuilding Code (boundary layer) $q_{cw}(n) = q_{cw}(exp)$

Yes
No

$P_{sta}$ $R_{sample}$
$P_{dyn}$ $q_{cw}$
$\gamma_{k,e}$

$T_{cw}$ $\gamma_{ref,Cu}$
$T_{cw}$ $\gamma_{reac,w}$
$\varepsilon_{w}$
$mc$, $Tw$, $qw$

$yk,e$

$\mu = 2.540 \times 10^{-1}$ $\sigma = 2.467 \times 10^{-2}$
10 bins
Scott's bin width
Kernel density

$\mu = 7.494 \times 10^{3}$ $\sigma = 8.522 \times 10^{2}$
10 bins
Scott's bin width
Kernel density

Frequency

Mean=2.540e-01 $\sigma =2.467e-02$
10 bins
Scott's bin width
Kernel density

Mean=7.494e+03 $\sigma =8.522e+02$
10 bins
Scott's bin width
Kernel density

Frequency

Samples

Value

Samples

Value

Alessandro Turchi
COUPLED ANALYSIS W/ NITRIDATION

### ABLATION QOI

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
<th>VARIANCE</th>
<th>$\Delta_{\text{stoch-nom}}$</th>
<th>$\varepsilon_{\text{old}}$ (±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass blowing rate</td>
<td>0.029 [kg /m$^2$s]</td>
<td>3.48e-5</td>
<td>-28.4%</td>
<td>16.72%</td>
</tr>
<tr>
<td>temperature</td>
<td>2661 [K]</td>
<td>2.17e+4</td>
<td>+5.0%</td>
<td>4.56%</td>
</tr>
</tbody>
</table>

CONSIDERING ALL THE UNCERTAINTIES SLIGHTLY AFFECT THE ERROR!
**COUPLED ANALYSIS W/O NITRIDATION**

### ABLATION QOI

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
<th>VARIANCE</th>
<th>$\Delta_{stoch - nom}$</th>
<th>$\varepsilon_{old(\pm)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass blowing rate</td>
<td>0.020 [kg/m²s]</td>
<td>1.94e-6</td>
<td>-2.9%</td>
<td>1.15%</td>
</tr>
<tr>
<td>temperature</td>
<td>2818 [K]</td>
<td>1.39e+4</td>
<td>+0.8%</td>
<td>1.80%</td>
</tr>
</tbody>
</table>

**Rebuilding uncertainties affect the mass blowing rate!**
CONCLUDING REMARKS

CONCLUSIONS

- **DECOUPLED ANALYSIS**
  - STRONG IMPACT ON THE QOIs OF A QUESTIONABLE PHENOMENON SUCH AS THE SURFACE NITRIDATION WHEN CONSIDERED
  - SMALL VARIATIONS OF THE QOIs UNCERTAINTIES WHEN NITRIDATION IS NEGLECTED: CONSEQUENCE OF THE ANALYZED ABLATION REGIME

- **COUPLED ANALYSIS**
  - THE INFLUENCE OF THE NITRIDATION UNCERTAINTIES REMAINS THE BIGGER
  - MEASUREMENT AND MODEL UNCERTAINTIES FROM THE REBUILDING PROCEDURE CAUSE THE ERROR TO GROW WHEN NITRIDATION IS NEGLECTED

PERSPECTIVES

- ASSESS MORE PLAUSIBLE RANGES FOR THE MOST INFLUENTIAL PARAMETERS
- ANALYZE DIFFERENT ABLATION REGIMES
- COMPARE THE OBTAINED RESULTS WITH THE EXPERIMENTAL MEASUREMENTS
Uncertainty Analysis of Carbon Ablation in the VKI Plasmatron

A. Turchi  
von Karman Institute

P. M. Congedo  
INRIA, Bordeaux

B. Helber  
von Karman Institute

T. E. Magin  
von Karman Institute

6th Ablation Workshop, April 10-14, Urbana-Champaign, Illinois