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A Mixed Methods Approach To Investigating Cognitive Load And Cognitive Presence In An Online And Face-To-Face College Algebra Course

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A MIXED METHODS APPROACH TO INVESTIGATING
COGNITIVE LOAD AND COGNITIVE PRESENCE
IN AN ONLINE AND FACE-TO-FACE COLLEGE ALGEBRA COURSE

DISSERTATION

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Education at the University of Kentucky

By
Jodi Jean Mills

Lexington, Kentucky

Director: Dr. Jennifer Wilhelm, Professor of Education

Lexington, Kentucky

2016

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A MIXED METHODS APPROACH TO INVESTIGATING
COGNITIVE LOAD AND COGNITIVE PRESENCE
IN AN ONLINE AND FACE-TO-FACE COLLEGE ALGEBRA COURSE

Most research on Cognitive Load Theory (Sweller, 1988) has uncovered many instructional design considerations for learning complex tasks. Additionally, the Community of Inquiry (Garrison, Anderson, & Archer, 2000) framework describes many of the learning experiences in online education. A gap existed in the literature for investigating cognitive load over the duration of a college algebra course and for investigating the relationship between cognitive load and cognitive presence. This research study has addressed this gap by investigating cognitive load and cognitive presence in an online and face-to-face college algebra course.

The results of this study revealed that face-to-face students earned statistically significant higher final course grades and homework grades than the online students. The face-to-face math course was slightly more efficient because it produced learners who exerted similar cognitive load as learners in the online course but the learners in the face-to-face earned higher performance score.

Online discussion prompts that ask student to apply their solution or defend their solution engaged students in cognitive presence differently. When students were prompted to apply their solution to a real world scenario, most students reached resolution in their initial posts, but they were often not cognitively present in their follow-up posts. When students were prompted to provide a defense of their solution, most of the posts demonstrated cognitive presence, but not as many individual students reached resolution.

Additionally, students progressed through the stages of cognitive presence when an instructor asked them a specific question about their math problem or real life scenario in a timely manner. When instructors post questions to their students that directly ask for an application of their hypothesis or an explanation how they arrived at their hypothesis, students can reach the highest stage of cognitive presence. When instructors post
messages that reach the highest stage of cognitive presence, students do not post messages that reach the highest stage of cognitive presence. Lastly, this study did not find a strong linear relationship between cognitive presence and cognitive load.

KEYWORDS: Cognitive Load, Cognitive Presence, Mixed Methods, Community of Inquiry
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CHAPTER 1. INTRODUCTION

Online education has surged into the mainstream of public and private education. Seventy-six percent of private postsecondary institutions and 98% of public postsecondary institutions offer online classes or programs (Allen & Seaman, 2014). Research has shown students in distance education classes perform as well as students in face-to-face classes (Bernard, Abrami, Lou, Borokhovski, Wozney, Wallet, Fiset, & Huang, 2004; Huh, Jin, Lee, Yoo, 2010; Shachar & Neumann, 2003; US Department of Education, 2010). Additionally, the percentage of academic leaders at postsecondary institutions who rate the learning outcomes for online courses as good as their face-to-face courses rose from 57% in 2003 to 74% in 2013 (Allen & Seaman, 2014). With online enrollment growing faster than overall higher education enrollment (Allen & Seaman, 2014), instructors of online courses need a deeper understanding of how students learn in an online classroom, what factors influence cognition, how students demonstrate cognitive presence, and how instructors can lead students to deeper conceptual understanding.

Online Education

In 1993, the first fully-accredited, fully-online institution, Jones International University, founded by Glenn R. Jones, began offering courses toward five bachelor degree programs and 24 master’s degree programs (Casey, 2008). By 2012, 62% of higher education institutions offered fully online programs, 86% offered online courses, and 33% (7.1 million) of students took at least one course online (Allen & Seaman, 2013; Allen & Seaman, 2014). Allen and Seaman (2003) define an online course “as one in which at least 80% of the course content is delivered online” (p.6). The remaining 20%
accounts for the possibility that a student may use computer software, a printed textbook, or other printed resource. Online courses do not meet face-to-face. Traditionally, a face-to-face course meets only in person, in a classroom. However, some face-to-face courses use printed textbooks and others use e-textbooks. The delivery mode of instruction usually defines whether a course is online or face-to-face, not the delivery mode of course material.

Online courses can be synchronous or asynchronous. Synchronous courses occur when the instructor and students meet online at the same time. Synchronous courses require students to log onto the internet, into the course management system (i.e. Blackboard, Moodle), or onto another type of communication tool (i.e. Skype, Adobe Connect) at the same time as the instructor and/or other classmates. Students can watch a real-time lecture by their instructor, ask him or her questions, and interact orally with their classmates. In most synchronous courses students are required to meet synchronously online once a week, with follow-up discussions and assignments delivered through the course management system. Course management systems also have the capability to incorporate various educational technologies.

Asynchronous courses occur when the instructor and students interact online at various, less confined times. The instructor and students are not synchronized to meet online at the same time. Students can learn independently from their instructor and classmates. Asynchronous courses require students log onto their course management system to interact with the course material, instructor, and classmates whenever it is convenient for them.
Purpose of Online Education

The purpose of distance education began as a mode of providing learning opportunities to students who could not complete coursework at an institution. At the beginning of the development of online courses Matthews (1999) stated that “distance education is first and foremost a movement that sought not so much to challenge or change the structure of higher learning, but to extend the traditional university and to overcome its inherent problems of scarcity and exclusivity” (p.54). Fifteen years later, equal access and equity has continued to be the primary purpose of online education. Jha (2014) stated “After food, education is the second greatest challenge for human community. Distance Education is a mode of education system that focuses on teaching methods and technology with the aim of delivering teaching, often on an individual basis, to the students who are unable to be present physically in a traditional educational setting such as classroom” (p.1).

Advantages of Online Education

Students perceive many advantages of online education. One advantage of online education is that it allows students opportunities to learn on their own time and at their own convenience (Jha, 2014). Anytime, anywhere is a common motto in the network of institutions offering distance education courses. It provides flexibility to many students, including part-time students (Jha, 2014; Song et al., 2004). Many part-time students and nontraditional, adult students juggle work and family responsibilities, along with school. Online educational programs allow these students to balance their lives on their own schedule, instead of conforming to university class schedules.
Online education also creates a valuable way for marginalized students to interact and communicate with their classmates and instructor (Jha, 2014). Many on these students do not feel comfortable asking questions in a face-to-face class. The platform of anonymity that distance education is built upon allows these students access to their classmates and instructor without having to worry about their perceived appearance or language barriers (US Department of Education, 2011).

Instructors perceive many advantages of online education, as well. Advantages include the ability to reach students in different geographical locations, the convenience and flexibility of posting discussions and course material at any time, the ease of communication and monitoring of students (Akdemir, 2010; Engelbrecht & Harding, 2005), and the wide range of available online resources (Engelbrecht & Harding, 2005). Kaifi, Mujtabe, and Williams (2009) note that faculty enjoy teaching online because it allows them time to meet the learning needs of their individual students. Other faculty report appreciating the ability to have a sense of control over the learning environment (Petricchi & Patchner, 2000).

In addition, institutions providing online education programs experience many tangible benefits. The enrollment increases, along with revenue, from students who otherwise would not have or could not have attended their campus (Jha, 2014). Furthermore, these students do not require classroom space or other campus amenities. Online education provides the institution with the opportunity to network with institutions and people around the globe. Allen and Seaman (2007) report that offering online courses increases student access, attracts students, is important for continued
growth in the institution, increases the rate of degree completion, enhances the value of their institution, and improves student retention.

**Disadvantages of Online Education**

Students perceive many disadvantages of online education, as well. Unreliable technology can keep students from accessing course materials when it is needed and required (Song et al., 2004; Kaifi, Mujtabe, & Williams, 2009). Although most online student have access to the universities’ online student support services and library, some students may feel uncomfortable accessing those services without speaking face-to-face with the support staff. Some students also lack the necessary advanced technology skills in order to succeed in an online education program (Distance Education, n.d.). Some students need consistent visible and audible contact with their instructor or their classmates, which is not always part of an online education course (Kaifi, Mujtabe, & Williams, 2009; Schrum & Ohler, 2005; Song et al., 2004). Schrum and Ohler (2005) also report that students feel overwhelming pressure of time requirements and workload.

Instructors experience disadvantages with online learning, too. Disadvantages include the increased time, experience, cost needed to design and develop course material, the increased time to actually communicate and teach the course compared to face-to-face classes (Akdemir, 2010; Engelbrecht & Harding, 2005; Kaifi, Mujtabe, & Williams, 2009). The absence of face-to-face contact with students and the lack of maturity of some students to become self-learners are clear disadvantages for online courses (Engelbrecht & Harding, 2005). Faculty also report feeling a lack of institutional support for teaching online courses (Schrum & Ohler, 2005).
One large weakness in the design of most online mathematics courses is the possibility of cheating or academic dishonesty. Technology to ensure academic honesty in writing based courses is prevalent in online and face-to-face classes. However, in mathematics and other fact based classes, the majority of tools that ensure academic honesty are in the form of proctored summative assessments. Although Trenholm (2007) advocates for proctored exams in online fact based classes, many institutions require students to sign an academic honesty statement.

For Institutions, creating online learning opportunities for their students can be complicated and problematic. The initial overhead cost of creating online learning courses is substantial. Technology infrastructure, technology support, electronic course materials, programs costs, instructor training, e-libraries, a system for identifying and addressing academic dishonesty, financial aid requirements, and legal issues related to copyright and intellectual property rights all need to be addressed before students even register and pay tuition for the distance education courses (Matthews, 1999).

**Cognition**

Reviewing these advantages and disadvantages of online education leads one to wonder about where learning and conceptual understanding fit. Does cognition hold an advantage or disadvantage in online education? Do students experience more or less cognitive load in an online course as compared to a face-to-face course? Research has shown mixed results when comparing learning outcomes or performance measures between distance education and face-to-face education. In Bernard et al.’s (2004) meta-analysis distance education students experienced higher achievement than face-to-face students. Distance education students also earned higher exams and course grades than
face-to-face students (Allen et al., 2004; Zhao et al., 2005). Additionally, online
discussions can exhibit higher-order thinking (Akyol and Garrison, 2011; Kanuka et al.,
2007; Meyer, 2003). However, face-to-face students perceived higher levels of learning
and metacognition that distance education students (Zhao et al., 2005). A void exists in
educational literature surrounding questions of cognition. This research study has
attempted to address this void and inform research into cognitive load and cognitive
presence in online and face-to-face courses.

**Background, Context, and Theoretical Framework**

Behaviorist theory, Cognitive theory, and Constructivist theory inform and aid in
the design and delivery of online education. The Behaviorist theory proposed that
behavior is changed through reinforcement (Skinner, 1953). Learning occurs by applying
verbal and situational stimuli, while knowledge is constructed through the repetition of
important behaviors (Skinner, 1953). The most well-known early Behaviorists include
Pavlov (1849-1936) and Skinner (1904-1990). Students taking a face-to-face math course
listen to their teacher lecture on the math concepts as they watch the teacher demonstrate
math problems (verbal stimuli). Students practice similar math problems (reinforcement)
until new knowledge is constructed. Students taking an online math course read the
textbook and follow the problems in the textbook (written stimuli), while they practice
similar math problems (reinforcement) until new knowledge is constructed. Drill and
practice could be the unofficial motto of education based on Behaviorism. Various types
of web-based educational software provide students the opportunity to practice skills and
factual knowledge through structured problems and instant feedback.
Cognitive Theory

Where the Behaviorist theory focused on the external control of behavior, the Cognitive theory focused on the internal mental processes. Cognitive theory purposed to explain how experiences and social influences affect learning. The earliest leader in this development was Piaget (1896-1980), who believed that not all behaviors can be explained through stimuli and reinforcement. Therefore, Piaget proposed stages of cognitive development primarily based on the accommodation and assimilation of experiences throughout a person’s life. The schema to be assimilated may be more or less complex (Huitt & Hummel, 2003). Cognitive theory views the nature of knowledge objectively, transferring knowledge effectively and efficiently (Bednar et al., 1995).

The effective and efficient assimilation of complex schema became an important task for cognitive researchers. Therefore, the Cognitive Load Theory (CLT) was developed to explain how some forms of problem solving interfere with learning (Sweller, 1988). CLT is used to measure the effectiveness and efficiency of various task characteristics (worked-examples versus conventional problems) by controlling the environmental characteristics (short, laboratory-type experiments) and accounting for the learner characteristics (levels of prior knowledge). CLT researchers uncovered many instructional design considerations for learning complex tasks, which will be addressed in the next chapter.

Constructivist Theory

Vygotsky (1896-1934), led the path toward the constructivist theory. Where behaviorism is teacher centered, constructivism is student centered. Where cognitive theory views learning as transferring knowledge, constructivist theory view learning as
interpreting one’s experiences to understand and learn. Constructivists contend that students learn by doing, through collaborating with other learners. Constructivists report that learning is constructed through experiences, negotiation of meaning, and collaborative learning (Merrill, 1991). Online learning environments which are learner-centered, knowledge-centered, assessment-centered, and community-centered (Swan, 2010) are important educational developments for constructivist theorists.

The Community of Inquiry (CoI) framework for online education is grounded in constructivist theory (Swan, 2010). The CoI framework (Garrison, Anderson, & Archer, 2000) presents the online learning experience as a relationship between cognitive presence, social presence, and teaching presence. Learning in the online environment occurs at the intersection of these three elements. Cognitive presence is “the extent to which the participants in any particular configuration of a community of inquiry are able to construct meaning through sustained communication” (p.89). Although cognitive presence may be the goal of education, Garrison argues that it does not occur in isolation. Social presence supports cognitive presence because it facilitates the process of critical thinking. Social presence is “the ability of participants in the Community of Inquiry to project their personal characteristics into the community” (p.89). Additionally, teaching presence supports both cognitive presence and social presence because teaching presence incorporates the design and facilitation of the educational experience (p.90).

In the Community of Inquiry framework instruction revolves around discussion, where discussions support exploration of multiple perspectives, complex understanding, and reflection (Parker & Germino, 2001, and Picciano, 2002). Additionally, Ice (2010) argues that “guided exploration and conceptual integration by a group of learners, in a
socially rich atmosphere, are catalysts and prerequisites for individual construction of meaningful knowledge” (p.141). Although the Community of Inquiry framework presents the online learning experience as a relationship between cognitive presence, social presence, and teaching presence, this research study will focus on cognitive presence in the online education experience of college algebra students.

The Cognitive Load Theory and the Community of Inquiry framework provide the theoretical framework for this research study. Cognitive Load Theory, which explores instructional effectiveness, instructional efficiency, and learner involvement, accounts for many of the complex learning activities students encounter. Additionally, the Community of Inquiry framework, which explores cognitive presence in an online learning environment, also describes many of the learning experiences in online education.

**Statement of the Problem**

Most research in Cognitive Load Theory has been designed to measure cognitive load in various task characteristics (worked-examples versus conventional problems) by controlling the environmental characteristics (short, laboratory-type experiments) and accounting for the learner characteristics (levels of prior knowledge). The CLT research uncovered many instructional design considerations for learning complex tasks. Using a different approach to cognitive load this study varied the environmental characteristics (online versus face-to-face) by controlling the task characteristics (same content and assignments) and accounting for the learner characteristics (levels of prior knowledge). A gap existed in the literature for investigating cognitive load over the duration of a college algebra course, in both online and face-to-face modalities. Additionally, a gap existed in
the literature for investigating the relationship between cognitive load and cognitive presence.

This research study proposed to uncover differences in cognitive load, efficiency, and cognitive presence for learning mathematics online and face-to-face. However, even if the task characteristics are controlled by covering the same mathematical content, the instructor-student and student-student interactions vary greatly in online courses (Walker, & Kelly, 2007; Wyatt, 2005) and these interactions impact learning (Zhao et al., 2005; Garrison & Cleveland-Innes, 2005; Hay et al., 2004). Therefore, investigating the relationship between cognitive load and cognitive presence attempted to uncover these relationships.

**Purpose of the Study**

The purpose of this study was to research whether there were differences in cognitive load, instructional efficiency, and learner involvement between online and face-to-face college algebra courses. Additionally, the purpose of this study was to determine if there was a relationship between cognitive load and cognitive presence in an online college algebra courses.

**Research Questions**

Research Question 1: How do instructional effectiveness and efficiency vary between online and face-to-face college mathematics courses?

Research Question 2: How do online college mathematics students demonstrate cognitive presence in a discussion forum?

Research Question 3: What is the relationship between cognitive load and cognitive presence in an online college mathematics course?
Definition of Terms

In order to clarify related terms, the following words are defined and used throughout this study with these intended meanings.

- **Face-to-face classes**: where the instructor and students meet in-person for all classes. However, the course content may be delivered online, via course management systems.

- **Online classes**: where the instructor and students do not meet in-person for any classes. Online classes are delivered through the use of the internet and course management systems. Online classes can be synchronous or asynchronous.

- **Hybrid classes**: where the instructor and students meet face-to-face for some classes as well as online for the rest of the classes.

- **Distance education**: the delivery of education through any type of non face-to-face method, including correspondence classes, video classes, and online classes.

Relevance of the Study

This research study offered the Cognitive Load Theory community valuable insight into the effectiveness and efficiency of online and face-to-face college algebra courses. This study is significant as it addressed the void in the literature for investigating cognitive load over the duration of a course. This study also addressed the gap in literature to explain how students experience cognitive presence in discussion forums which prompt for resolution. This research study can inform current research into cognitive learning processes. Instructors and instructional designers benefit from the
results of this study by gaining an understanding for why students post in each stage of
cognitive presence and how to encourage students to reach the integration and resolution
stage.

Assumptions of the Study

1. People can self-report the amount of cognitive load expended.

2. The subjective rating-scale technique is sensitive to small differences in cognitive
load.

Organization of the Remainder of the Study

This research study is organized into six chapters. Chapter 1 provided the
background, context, and Theoretical Framework for the study. It also identified the
statement of the problem, the purpose of the study, the Research Questions, the rational
and relevance of the study, and the assumptions and limitations of the study. Chapter 2
provided a literature review on Cognitive Load Theory as it pertains to instructional
effectiveness, instructional efficiency, and learner involvement. It also provided a
literature review on cognitive presence in the Community of Inquiry framework.
Additionally, Chapter 2 reviewed methodological issues in the research of these two
theories. Chapter 3 identified the research questions, hypotheses, and design. It described
the quantitative and qualitative data analysis procedures. It also described the dependent
variables, the independent variables, the data collection procedures, the data analysis
procedures, and the limitations of the research design. Chapter 4 provided the results of
the quantitative data analysis of the research questions. Chapter 5 provided the results of
the qualitative data analysis of the research questions. Chapter 6 provided a discussion of
the research results for all of the research questions. It also described the limitations of the study and recommendations for future research.
CHAPTER 2. LITERATURE REVIEW

Research has shown mixed results when comparing the effectiveness of distance education to face-to-face education. A review of three meta-analyses illustrates the variance across the effectiveness of the two modes of instruction. In Bernard et al.’s (2004) meta-analysis distance education was more effective than face-to-face education when measuring achievement, but face-to-face education was more effective than distance education when measuring student attitude and retention. In Allen et al.’s (2004) meta-analysis distance education was more effective than face-to-face education when measuring exams and course grades. In Zhao et al.’s (2005) meta-analysis distance education was more effective than face-to-face education when measuring student satisfaction, grades, student attitudes and beliefs, and student participation. However, face-to-face education was more effective than distance education when measuring student perception of learning and metacognition (Zhao et al., 2005). Therefore, in educational research the effectiveness of learning has many definitions. Effectiveness is defined differently and measured differently depending on a researchers’ area of interest.

Garrison (2003) defined the goal of an educational experience as “a deep and meaningful learning outcome” (p.50). He also stated that effective learning in asynchronous online education depends upon the integration of cognitive, social, and teaching presence. The combination of reflectivity and connectivity are the two unique properties of asynchronous online learning that enable and empower effective learning. Garrison (2003) linked effective learning to stages of higher-order thinking skills. Cognitive presence “concerns the process of both reflection and discourse in the initiation, construction, and confirmation of meaningful learning outcomes” (p.50).
However, measures of effectiveness alone cannot explain all of the cognitive learning processes students experience during learning experiences. Performance and cognitive presence do not always reveal effective learning. Although measures of effectiveness can be important to determine the power of different educational experiences, a meaningful interpretation of that effectiveness should be given in context of its associated cognitive load and efficiency.

**Cognitive Load Theory**

Sweller (1988) developed the Cognitive Load Theory (CLT) in the mid 1980’s with the intention to explain how some forms of problem solving interfere with learning. He started with three assumptions in order to explain the interference: “(a) the person has a fixed cognitive capacity; (b) both problem-solving and learning require some of that capacity; (c) the problem solving and learning mechanisms differ” (Sweller, 1988, p.275). Research by Miller (1956) showed that people can hold about seven items or elements in their working memory. If humans can only hold about seven items, then we can organize and interact with less than seven elements. However, working memory capacity can be increased when calling up processes from the large capacity of long-term memory. Our long-term memory can hold a seemingly unlimited amount of long and complex processes (Sweller, van Merrienboer, & Paas, 1998).

Knowledge is stored in long-term memory as schemas, where schemas are the thoughts which hold those processes. In CLT research Sweller (1988) defined schema as “a structure which allows problem solvers to recognize a problem state as belonging to a particular category of problem states that normally require particular moves” (p.259). Schemas reduce working memory load because these schemas can be called into working
memory to organize and process the elements. Schema automation is the development of schemas to process complex elements, and it occurs through extensive practice. Novice problem solvers, who do not fully understand mathematical operations and do not have a schema for a particular math problem, will require a large amount of cognitive processing capacity to understand that problem. Expert problem solvers will not need a large amount of cognitive processing capacity to understand the math problem because they have automated schemas to solve the math problem. In order to retrieve these schemas from long-term memory, the working memory capacity cannot be overwhelmed. Sweller, van Merrienboer, and Paas (1998) asserted that this process is the primary concern of CLT, under their designation that the primary goal of instruction is “the construction and the automation of schemas that are useful for solving problems of interest” (p.259).

**Factors of Cognitive Load**

Cognitive load is characterized by three factors: intrinsic cognitive load, extraneous cognitive load, and germane cognitive load. Figure 1 shows the factors and assessment of the Cognitive Load Theory. Intrinsic cognitive load represents the interaction between the natural complexity of a task and the expertise of the learner. Controlling for the expertise of the learner, simple tasks have a small intrinsic cognitive load (1+1), while complex tasks have a larger intrinsic cognitive load ($3^{8x-4} - 7 = 2$). However, as the expertise of the learner increases, the intrinsic cognitive load decreases. Although intrinsic cognitive load cannot be directly influenced by the design of a problem, educators need to consider the learners’ prior knowledge when designing a course (Sweller, van Merrienboer, & Paas, 1998).
Extraneous cognitive load represents the unnecessary mental energy that interferes with learning and with schema automation. Since Sweller (1988) argued that some forms of problem-solving interfere with learning, most of the CLT research has been conducted to explore how extraneous cognitive load can be reduced. Extraneous cognitive load is directly influenced by the design of a task or problem. If the design of an activity requires a learner to complete redundant or excessive tasks, then the extraneous cognitive load is high. If the design of an activity do not require a learner to complete redundant or excessive tasks, then the extraneous cognitive load is low (Paas, Renkl, & Sweller, 2003).

Figure 1: Factors and Assessment of the Cognitive Load Theory

Germane cognitive load represents the necessary mental energy relevant to learning. Germane cognitive load is also influenced by the design of the task. Any design
that directs learners’ attention toward cognitive processes that are relevant to learning and the construction of schemas increases germane cognitive load. Additionally, intrinsic, extraneous, and germane cognitive load are additive. For example, if a learner is asked to complete a complex task (high intrinsic cognitive load) where the instructions are confusing or the learner is unprepared for the task (high extraneous cognitive load), there only remains a low germane cognitive load for schema construction. However, if a learner is asked to complete a complex task (high intrinsic cognitive load) where the instructions are not confusing (low extraneous cognitive load), there remains a high germane cognitive load for schema construction.

**Performance and Mental Effort**

Sweller (1988) explained that a high cognitive load might influence aspects of performance. Novice problem solvers, who do not fully understand mathematical operations, will require a large amount of cognitive processing capacity to understand a math problem. Therefore, less cognitive processing capacity will be available to complete other aspects of the problem and may create more errors. Expert problem solvers will not need a large amount of cognitive processing capacity to understand the math problem. Therefore, more cognitive processing capacity will be available to complete the problem with fewer errors.

Cognitive load can be described as the amount of cognitive capacity a learner expends to gain deep understanding (Sweller, 1988). Although cognitive load is composed of the three factors of intrinsic, extraneous, and germane cognitive load, it can also be conceptualized as mental load and mental effort. Mental load reflects the intrinsic
aspects of a task, which are immune to instructional design. More complex tasks require more mental load.

Mental effort reflects the task characteristic, the individuals’ amount of cognitive capacity needed to complete the task, and the interaction between the task and the learner. When learners are asked to complete complex tasks, their mental load increases as well. They compensate for the increase in mental load by investing more mental effort in order to complete the task. However, their mental effort might not increase if they are experts in completing the task (Sweller, van Merrienboer, & Paas, 1998). Mental effort is the amount of cognitive resources allocated to learning. Therefore, “mental-effort measurements can reveal important information that is not necessarily reflected in performance and/or mental-load based measures. In combination with performance, measures of mental effort constitute the essence and the best estimator of cognitive load” (Paas & van Merrienboer, 1994a, p.357).

**Assessment of Cognitive Load**

Cognitive load is traditionally assessed through a subjective-rating Likert scale, through a performance measure, and through an efficiency model. The efficiency model is reviewed in the next section. The Likert measurement will be discussed here followed by the educational implications of CLT. Assessment of cognitive load began with a dual-task analysis. Researchers (Britton & Tesser, 1982) analyzed participants’ reaction times to an auditory click as the participants completed a secondary task. They found the participants’ reaction times depended upon the degree of complexity of the secondary task. Other researchers found that attending to a secondary task would interfere with
aspects of the primary task (Book & Garling, 1980; Lindberg & Garling, 1982; Fisk & Schneider, 1984).

Researchers have also used psychophysiological techniques to measure cognitive load. These techniques assumed that changes in cognitive load are reflected in changes in heart rate, brain activity, and eye activity. However, these measures were unreliable and they were not sensitive to small differences in cognitive load (Paas, van Merrienboer, & Adam, 1994; Paas & van Merrienboer, 1994b).

Subjective rating-scale techniques are based on the assumption that people can self-report the amount of cognitive load expended. Gopher and Braune (1984) have shown that subjects can introspect on their cognitive processes with no difficulty in assigning numerical values to the invested cognitive load. A modified perceived task difficulty scale, ranging from “1 – very, very low mental effort” to “9 – very, very high mental effort” has been shown to have high face validity (Paas, 1992; Paas et al., 2003) and internal consistency (deCroock & van Merrienboer, 2007; deCroock, van Merrienboer, & Paas, 1998; Paas & van Merrienboer, 1994a). The Cronbach’s coefficient alpha estimate ranged from .82 to .98 in these studies. Paas (1992) found a Spearman rank order correlation of 0.9 was obtained between objective and subjective task difficulty. Researchers comparing psychophysiological techniques and the subject rating-scale technique have also found the rating-scale to be valid, reliable, and nonintrusive (Paas, van Merrienboer, & Adam, 1994). Low cognitive load ranges from scores of 1 to 4 and high cognitive load ranges from scores of 5 to 9 (Paas & van Merrienboer, 1993).

Another rating-scale technique used by Cierniak, Scheiter, and Gerjets (2009) explored intrinsic, extraneous, and germane cognitive load separately. They asked
participants three six-point Likert-type questions. “How difficult was the learning content for you?” assessed participants’ intrinsic load because the question focused on the content. “How difficult was it for you to learn with the material?” assessed extraneous load because the question focused on the material. “How much did you concentrate during learning?” assessed germane load because concentration is shown to have a positive correlation on learning (Cierniak et al., 2009, p.319).

Decreasing Extraneous Cognitive Load

Most of the CLT research has been devoted to discovering instructional methods to decrease extraneous load when the intrinsic cognitive load is high. The majority of these instructional methods were examined in research studies conducted in short laboratory-type experiments, where the environment and learner characteristics are controlled. The task characteristic was the only varying piece of the experiment. Research has uncovered six task characteristics, which can decrease cognitive load. These six task characteristics include the worked example effect, the completion problem effect, the goal-free effect, the split-attention effect, the modality effect, and the redundancy effect.

The worked example effect occurs when some conventional problem-solving tasks are replaced with worked examples because it enables learners to focus on the problem states and solution states. Learners can induce generalized solutions or schemas. However, learners are not forced to study them (Cooper & Sweller, 1987; Paas & van Merrienboer, 1994b; Sweller et al., 1998; Sweller & Cooper, 1985; Zhu & Simon, 1987).

The completion problem effect happens when some conventional problem-solving tasks are replaced with partial solutions because it forces learners to focus on the problem states and the partial solution in order to complete the problem. Learners can induce

The goal-free effect occurs when learners solve problems where he or she can define the goal. For example, many story problems begin by telling students that a car has uniformly accelerated from rest for one minute and its final velocity is two km/min. However, instead of asking students to calculate the velocity of a car, ask them to calculate the value of as many variables as they can (Owen & Sweller, 1985; Sweller et al., 1998; Sweller, Mawer, and Ward, 1983).

The split-attention effect happens by replacing multiple sources of information with a single, integrated source of information. For example, instead of having a diagram with its descriptive statements separated from each other, integrate the statements inside of the diagram at the appropriate locations (Chandler & Sweller, 1996; Sweller et al., 1998; Tarmizi & Sweller, 1988).

The modality effect occurs by replacing a visual source of information and a written text with a visual source of information and a spoken text (Mousavi, Low, & Sweller, 1995; Tindall-Ford, Chandler, & Sweller, 1997; Sweller et al., 1998). Lastly, the redundancy effect happens when unnecessary, additional information or information presented in multiple forms is replaced with a single source of information (Bobis, Sweller, & Cooper, 1993; Chandler, & Sweller, 1991; Sweller et al., 1998).

**Decreasing Intrinsic Cognitive Load**

CLT researchers have investigated two instructional methods to decrease intrinsic cognitive load. The simple-to-complex method sequences problems or instruction from simple whole-tasks to more complex whole-tasks (van Merrienboer, Kirschner, Kester,
For example, begin with math problems which involve solving simple two-step equation and end with two-step equations involving fractions.

The progressive method sequences material with high element interactivity separately and then the learner can work on the interaction between them. (Pollock, Chandler, & Sweller, 2002). For example, begin with providing definitions and background information before providing procedural information.

**Increasing Germane Cognitive Load**

CLT research has also investigated one instructional method to increase germane cognitive load using longer experiments. The instructional method found to increase germane cognitive load is called the variability effect. The variability effect provides learners with a set of problems which vary on many levels. The variability effect increased cognitive load (van Merrienboer & Sweller, 2005). However, increasing cognitive load did not negate the positive effect this method had on learning. “High variability increased cognitive load during practice but yielded better schema construction and transfer of learning as indicated by a better ability to solve problems that were not solved for before” (van Merrienboer & Sweller, 2005, p.161). The variability effect increased germane cognitive load, not extraneous cognitive load (Paas & van Merrienboer, 1994b).

**Learner Characteristics**

Research has uncovered three learner characteristics which impact cognitive load. The expertise reversal effect is a combination of several other cognitive load effects and a learners’ level of expertise, when “instructional methods that work for novice learners have no effect or even adverse effects when learners acquire more expertise” (van
Merrienboer & Sweller, 2005, p.163). For example, novice learners experienced a high cognitive load when sources of information were not integrated (split-attention effect). When these sources were integrated the novice learners experienced a low cognitive load. However, expert learners did not experience a high cognitive load when sources of information were not integrated. In fact, when the sources were integrated, the expert learners experienced a high cognitive load due to the redundancy effect (Kalyung et al., 1998).

Research has also shown that age and motivation can influence cognitive load. Young adults performed better on cognitive tasks than older adults (Paas, Camp, & Rikers, 2001), and learners’ motivation can determine how much mental effort is expended (Paas et al., 2005).

CLT researchers have used these instructional techniques to show effective methods of instruction. Additionally, these methods can be accommodated in both face-to-face and online learning experiences. Although measures of effectiveness and cognitive load can be important to determine the power of different educational experiences, a meaningful integration of that effectiveness and cognitive load should be given in terms of instructional efficiency. The next section will review Paas and van Merrienboer’s (1993) model of efficiency and the research methodology using the efficiency model.

**Instructional Efficiency**

If two different instructional methods produced the same learning outcome (performance), how do researchers account for the differences in cognitive load? In order to address that issue Paas and van Merrienboer (1993) developed an efficiency measure,
which is a computational approach combining measures of mental effort with task performance. According to their model, if one instructional method produced learners with the same performance as learners in another instructional method but the learners in the first method exerted a lower cognitive load, the first instructional method is more efficient. Additionally, the first instructional method is more efficient if it produced learners with higher performances compared to learners in another instructional method when all of the learners exerted similar cognitive loads. The price of inefficiency is the “heavy use of limited cognitive-processing capacity” (Sweller, 1998, p.261).

Relative Efficiency

Paas and van Merrienboer (1993) defined relative efficiency as “the observed relation between mental effort and performance in a particular condition relative to a hypothetical baseline condition, in which each unit of invested mental effort equals one unit of performance” (p.739). These two measures are graphed on a coordinate system where the x-axis represents the mental effort z-scores and the y-axis represents the performance z-scores. The grand mean and standard deviation for mental effort and the grand mean and standard deviation for performance are calculated for each instructional method. Next, individual raw scores and group scores for mental effort and performance are converted to their z-scores using their grand mean and standard deviation. A mental effort z-score of zero indicates the average of all mental effort measures, and a performance z-score of zero indicates the average of all performances.

In Figure 2, the line $E=0$ indicates an efficiency of zero. Relative efficiency is calculated as the perpendicular distance from a point on the coordinate system to the line $E=0$, $E = (P - R) / \sqrt{2}$, where $P$ is the z-score of the mean of performance and $R$ is the z-
score of the mean of mental effort. Points above the line represent instructional efficiency, and points below the line represent instructional inefficiency.

Figure 2. Instructional Efficiency model

An Example of Relative Efficiency

Paas and van Merrienboer (1993) investigated three instructional methods on 45 business students who were learning basic statistics. Method CONV represents a
conventional set of problem solving tasks, where students had to solve each problem. Method WORK represents a worked-example set of problems, where students had to study example problems with their solutions before solving an entire problem. Method COMP represents a completion set of problems, where students had to study part of the problem then solve part of the problem before solving an entire problem. The participants’ performances on a test and their self-reported cognitive load using Paas’ (1992) 9-point Likert-scale were collected. Since Paas and van Merrienboer (1993) published their raw data, the author of this research study used their data to conduct multiple one-way ANOVAs. The findings are reported below.

Comparing the three methods on effectiveness (or students’ performance on a test), a one-way ANOVA revealed a statistically significant effect, F(2,42)=11.18, p<.001. Post-hoc multiple comparisons using Bonferroni’s procedure with p<.05 showed that the performance of the conventional method (P=53.5) was lower than both the worked-example method (P=77.7) and the completion method (P=67.2). Students using the worked-example method performed better than students using the completion method, but it was not statistically significant.

Comparing the three methods on cognitive load, a one-way ANOVA revealed a statistically significant effect, F(2,42)=16.41, p<.001. Post-hoc multiple comparisons using Bonferroni’s procedure with p<.05 showed that the cognitive load of the conventional method (R=4.84) was higher than both the worked-example method (R=3.79) and the completion method (R=3.15). Students using the completion method expended less mental effort than students using the worked-example method, but it was not statistically significant. Therefore, analyzing the data with these two techniques
revealed that the conventional instruction method required learners to carry a heavier cognitive load and they performed worse than students using the other two methods. Students using the worked-example instructional method performed better than students using the completion method, but the worked-example method required a higher cognitive load.

Therefore, comparing the efficiency of these three methods would provide a deeper picture of the experience of learning with these three instructional methods. Comparing the three methods on their efficiency, a one-way ANOVA revealed a statistically significant effect, F(2,42)=23.71, p<.001. Post-hoc multiple comparisons using Bonferroni’s procedure with p<.05 showed that the efficiency of the conventional method (E=-1.13) was lower than both the worked-example method (E=.57) and the completion method (E=.58). These results can be verified in Paas and van Merrienboer’s (1993) article. Conventional problem solving was an inefficient instructional method. However, the completion method and the worked-example method were equally efficient. Therefore, analyzing the data with the efficiency model revealed that students learning in the worked-example instruction method used more mental effort and outperformed learners using the completion method. Paas and van Merrienboer (1993) suggested the learners’ motivation could explain this result. After studying worked examples students were asked to solve a problem. The novelty of the new task could have increased their interest and motivation, thus influencing their willingness to use more mental effort to complete the problems.
Another Example of Relative Efficiency

Another interesting study investigating instructional efficiency compared the learning experiences of students using a discovery learning method to students using a worked-example method. Tuovinen and Sweller (1999) studied 32 college students, who were learning a new computer database system. Students in both instructional methods were introduced to the database program and shown basic procedures using it. The exploration group was provided with printed instructions, and they were told to create their own data-based situation to work through the lessons. The worked-example group was provided with examples of each lesson, and they were told to read through the example and then work out a problem similar to the example using a given database. The students’ test scores and their self-reported cognitive load using Paas’ (1992) 9-point mental effort Likert-scale were collected. To analyze the data, two-way ANOVA’s were conducted to compare the participants’ prior experience with database programs with their performance, cognitive load, and the instructional efficiency.

Comparing test performance, the worked-example group performed better compared to the exploration group when students had no prior experience with databases, but they scored slightly worse when they had some experience, $F(1,28)=4.18$, $p<.05$. The advantage of the worked-examples method was due to cognitive load. The worked-example group held a lower cognitive load compared with the exploration group, $F(1,28)=8.19$, $p<.05$. Additionally, students with no previous database experience reported higher levels of cognitive load for students in the exploration group, $F(1,28)=5.28$, $p<.05$, than in the worked examples group. The exploration group would have been a better instructional method if students in the exploration group held a high
cognitive load and also performed better than the worked-examples group. The high cognitive load for the exploration group is attributed to extraneous cognitive load because that load hindered learning instead of helping it. The worked-examples group was a more efficient instructional method for students with no prior experience with databases, F(1,28)=6.43, p<.05. Originally, the results of this research supported the expertise reversal effect, where instructional methods that work for novice learners have no effect or adverse effects for learners with expertise (van Merrienboer & Sweller, 2005, p.163). However, the results of this research can also be explained with Paas, Tuovinen, van Merrienboer, and Darabi’s (2005) learner involvement model. The next section will review the three dimensional instructional efficiency model and the research methodology using this model. The learner involvement model will be reviewed in the section following the three dimensional instructional efficiency model.

**Three Dimensional Instructional Efficiency**

Another model, the instructional efficiency model, was developed by Tuovinen and Paas (2004). The instructional efficiency model incorporates a learner’s mental effort during learning, a learner’s mental effort during a test, and a learner’s test performance. In reviewing how researchers have calculated instructional efficiency, Tuovinen and Paas (2004) determined that researchers have been calculating efficiency with mental effort scores during learning or during testing. When researchers use the mental effort during a learning phase to calculated efficiency, they are calculating learning efficiency. When researchers use the mental effort during the test phase to calculated efficiency, they are calculating test efficiency. Therefore, Tuovinen and Paas (2004) created a three
dimensional model which incorporated mental effort scores during learning and during testing.

The three dimensional relative efficiency is calculated as the perpendicular distance from a point on the coordinate system to the plane \(x+y-x=0\). Three dimensional efficiency, \(3D\ E = (P - R_L - R_T) / \sqrt{3}\), where \(P\) is the z-score of the mean of performance, \(R_L\) is the z-score of the mean of mental effort during learning, and \(R_T\) is the z-score of the mean of mental effort during the test. Using the data from the Tuovinen and Sweller (1999) study discussed earlier, Tuovinen and Paas (2004) calculated and interpreted the data in terms of the three dimensional efficiency model. Comparing the exploration group and the worked-examples group for students without prior knowledge, a statistically significant difference in learning efficiency \((t(15)=3.59, p<.05)\) and in three dimensional instructional efficiency \((t(16)=2.60, p<.05)\) was found in favor of the worked-examples group. These results provided supporting evidence of Tuovinen and Sweller’s (1999) study.

**Learner Involvement**

Most research about the effectiveness and efficiency of instructional methods using the Cognitive Load Theory occurred in short laboratory experiments. Because these experiments were short and because the participants were most likely rewarded for their participation, the participants did not experience low motivation (Paas et al., 2005). Therefore, the research on instructional effectiveness and efficiency has not addressed learner motivation (Paas and van Merrienboer, 1993). However, research has shown that complex learning tasks or environments impact motivation (Frankola, 2001; Spencer & Usher, 2007; Tsai, 2012) and motivation impacts learning (Bernard et al., 2004; Kanfer &
Ackerman, 1989; Tsai, 2012; Wadsworth et al., 2007; Zhao et al., 2005). Consequently, to appropriately research a face-to-face or online class using Cognitive Load Theory a learner’s motivation must be studied.

Since Paas and van Merrienboer (1994a) define mental effort as the amount of cognitive load allocated to learning, Paas et al. (2005) argued “the amount of mental effort invested in a certain learning task can be considered a reliable estimate of the learner’s motivation or involvement in that task” (p.28). Therefore, cognitive load can be a reflection of a learner’s involvement. Additionally, since a learner may appear to be involved in a task but may actually not be engaged the researchers developed a computational approach to learner involvement which combines measures of cognitive load with task performance. When learner involvement is high, more cognitive load effort is likely to also be used, which will most likely result in a higher performance (Paas et al., 2005).

The computational approach begins with collecting measures of cognitive load and performance precisely as Paas and van Merrienboer’s (1993) instructional efficiency model. However, the instructional involvement line, I=0, runs perpendicular to the instructional efficiency line. For example, the instructional involvement is calculated as the perpendicular distance from a point on the coordinate system to the line I=0, I = (P + R)/√2, where P is the z-score of the mean of performance and R is the z-score of the mean of mental effort. Points above the line represent high instructional involvement, and points below the line represent low instructional involvement, see Figure 3.
Figure 3. Instructional Involvement model

**An Example of Learner Involvement**

Paas et al. (2005) used the data from the Tuovinen and Sweller (1999) study discussed earlier, calculated and interpreted the data in terms of instructional involvement. ENPK represents the exploration group with no prior knowledge (instructional involvement, I = 0.14). ESPK represents the exploration group with some
prior knowledge (instructional involvement, I=0.29). WSPK represents the worked-examples group with some prior knowledge (instructional involvement, I=0.23). WNPK represents the worked-examples group with no prior knowledge (instructional involvement, I=-0.18). Paas et al. (2005) did not report conducting any statistical analysis. However, it is interesting to note that both exploration groups showed higher instructional involvement than the worked-examples group. This view is consistent with the belief that active learning promotes student motivation and involvement (Boaler, 2002).

Additionally, the effect of using the discovery method was strongest for learners with some prior knowledge. The expertise reversal effect supports this conclusion, where worked-examples are more effective for novice learners. Darabi, Nelson, and Paas (2007) also examined instructional involvement and found learners who used worked-examples instructional method showed greater involvement than the students who used conventional problem solving methods. This result supports many research studies which have shown that worked-examples are more effective and efficient than conventional methods.

**Cognitive Presence**

In 1956 Dr. Benjamin Bloom developed a set of three categories (knowledge, skills, and attitudes) that organize learning behaviors aimed at promoting critical thinking in education. The knowledge category became known as the cognitive domain or the mental processes needed to learn. The cognitive domain is divided into a taxonomy of six subcategories which identify its components: knowledge, comprehension, application, analysis, synthesis, and evaluation (Bloom, 1956). Since that time, other cognitive
theories have evolved and taken precedence in research. Since Bloom’s (1956) taxonomy on higher-order thinking, many other researchers have developed theories on cognition. The Community of Inquiry (CoI) Theory (Garrison, Anderson, & Archer, 2000) is currently the most common theory among researchers interested in cognition.

The Community of Inquiry Theory

Garrison et al. (2000) created the CoI theory as a framework for lead successful learning experiences. Developed especially for online learning CoI identifies three core elements: social presence, cognitive presence, and teaching presence. These elements overlap and interact with each other in order to form a community of learning experiences. Cognitive presence is “the extent to which the participants in any particular configuration of a community of inquiry are able to construct meaning through sustained communication” (p.89). The cognitive presence element is important to a successful educational learning experience and is the basis of critical thinking. In addition to cognitive presence being the focus of the CoI articles in this literature review, cognitive presence is the focus of two of the research questions in this research study.

The four stages of cognitive presence include triggering event, exploration, integration, and resolution. The triggering event is the initial stage where the issue, problem, or task is presented to the student. The discourse surrounding the triggering event focuses on the background information and students asking for clarification. Exploration is the second stage where students brainstorm ideas and suggestions. They explore various opinions but it does not lead to further understanding. Integration is the third stage where students connect ideas and integrate information. They offer a tentative
solution or hypothesis. The last stage, resolution, is where student defend or test the hypothesis. The students may also apply the solution to other scenarios.

**Data Sources for Cognitive Presence**

Using the CoI theory as the theoretical framework enabled researchers to generate data through the CoI survey (Akyol & Garrison, 2011; Hosler & Arend, 2012), interviews (Akyol & Garrison, 2011), artifacts (Gregori et al., 2012), and online discussion boards (Akyol & Garrison, 2011; Arnold & Ducate, 2006; Hosler & Arend, 2012; Gregori et al., 2012; Kanuka et al., 2007; Meyer, 2003). The CoI survey is a 5-point Likert-type scale used for quantitative analysis. Nevertheless, Hosler and Arend (2012) added four open-ended questions to the end of the CoI survey. These data allowed the researchers to understand student perceptions of teaching presence and cognitive presence. The student and instructor interviews allowed Akyol and Garrison (2011) to find themes regarding student perceptions of their stage of cognitive presence. Gregori et al. (2012) analyzed learning activities in order to explore performance of online students through cognitive presence. However, they did not compare online students to face-to-face students.

Cozza and Oreshkina (2013) were the only authors who analyzed classroom discussions, as opposed to online discussions, for cognitive presence. They studied cognitive and metacognitive process during math problem solving think-aloud sessions in elementary classrooms in Hungary, Spain, Russia, and the United States. The authors were participant observers, where they observed and videotaped all four problem solving sessions. They generated data from the classroom discourse, student journals, and field notes and coded the data using Artzt and Armour-Thomas’s (1992) framework. They
found that students in all four countries showed similar combinations of cognitive and metacognitive processes.

While online discussion boards were the preferred method of gathering data, many of the researchers used content analysis to code and find themes in accordance with the four stages of cognitive presence (Akyol & Garrison, 2011; Arnold & Ducate, 2006; Kanuka et al., 2007; Meyer, 2003). However, Gregori et al. (2012) argued that the four stages do not serve as “decisive indicators of the quality of learning” (p.470). They were interested in cognitive attainment, as opposed to cognitive presence. Therefore, they created four categories to serve their purpose: student individual attainment grade, functional content use, socio-cognitive level, and epistemological referent nature (p.471).

**Data Analysis of Cognitive Presence**

For CoI researchers discussion boards were the most popular data source for understanding cognitive presence. Researchers compiled and organized the students and instructors discussion board posts. Data generated through discussion boards allowed for content analysis according to the CoI model (Akyol & Garrison, 2011; Arnold & Ducate, 2006; Darabi et al., 2011; Kanuka et al., 2007; Meyer, 2003). Content analysis is concerned with identifying what is occurring and how often it is happening. Garrison et al. (2000) provided a process for coding the four stages of the CoI theory. They suggested coding each message (or each discussion post) as one unit. The message will vary in length according to each student. They advised to code each message for the stage that is most clear. Higher levels of critical thinking involve using lower levels of cognitive presence, so it is not unusual for a message to have more than one stage presented
(Garrison et al., 2000). Content analysis allowed the researchers to draw conclusions about the patterns and themes (Williams, 2007).

After creating a frequency table for each category, the researchers conducted a statistical analysis in order to report the findings in a quantitative format. Chi-square tests allowed the researchers to statistically determine which category occurs most often or which intervention reached the highest cognitive presence (Akyol & Garrison, 2011; Darabi et al., 2011). T-tests and ANOVA tests allowed the researchers to compare cognitive presence across different variables (Bradley et al., 2008; Darabi & Jim, 2013; Mevarech & Amrany, 2008). Linear regression allowed Hosler and Arend (2012) to examine the relationship between cognitive presence and teaching presence. Factor analysis allowed Chilton and Gurung (2008) to validate their survey on student satisfaction and engagement. Correlational analysis allowed Gregori et al. (2012) to determine which learning strategies were similar in technological presence and cognitive attainment.

Most of the studies in this literature review involved analyzing discussion boards according to levels of cognitive presence. Exploration and integration occurred most often, while the resolution stage occurred least often (Akyol & Garrison, 2011; Arnold & Ducate, 2006; Gregori et al., 2012; Meyer, 2003). Although the resolution category contained the fewest elements, the discourse resulting in resolution occurred because the discussion board prompted students to find a solution to a problem. However, not every student reached resolution with this discussion board (Arnold & Ducate, 2006).
Educational Implications for Cognitive Presence

High cognitive thinking was discovered in online discussions which involved a real world scenario (Bradley et al., 2008; Weeks et al., 2013), brainstorming ideas or solutions to an issue (Bradley et al., 2008), debates (Kanuka et al., 2007), collaboration (Stahl, 2006, 2010; Zhou et al., 2008), WebQuests (Kanuka et al., 2007), and discussion questions involving metacognitive strategies (Mevarech & Amrany, 2008). How discussion boards were presented also influenced cognition. Providing an example post to the students, and limiting the number of posts displayed on each page encouraged student engagement and schema construction (Darabi & Jin, 2013). Students preferred high instructor involvement (Darabi et al., 2011; Hosler & Arend, 2012), and they preferred to collaborate with their peers and work in interactive environments (Rowan-Kenyon et al., 2012). Students also described online discussions as taking a lot of time, but were more thoughtful and fully participated in (Meyer, 2003). Additionally, learners from many different countries showed similar combinations of cognitive and metacognitive processes (Cozza & Oreshkina, 2013).

Course designers should vary their approach to discussion board questions, and stay thoughtful about which level of cognition is being achieved. Instructors should be involved in the discussions and provide timely and encouraging feedback. Instructors should also provide different cognitive tools to enable students to seek, select, organize, integrate, and generate knowledge. The reasons why learners use cognitive tools could bring clarity to a progression of cognitive understanding. Understanding how students seek, select, organize, integrate, and generate knowledge using various cognitive tools could be a starting point for future research into cognition.
Review of Methodological Issues

As opposed to the CoI model which describes the cognitive stages a learner passes through to gain deep understanding, CLT describes the mental energy, or cognitive load, a learner expends to gain deep understanding. Cognitive load is traditionally assessed through a mental effort scale, through a performance measure, and through an efficiency model. Most of the literature reviewed for this study pertained to quantitatively measuring cognitive load or to qualitatively recognizing the role of cognitive presence in online or face-to-face learning experiences. None of the literature generated qualitative data on cognitive load according to the tenets of the Cognitive Load Theory. However, Chilton and Gurung (2008) came relatively close. Quantitative data were collected from surveys on student engagement and satisfaction. They felt that if student satisfaction was high, this would indicate that the students were not overloaded with extraneous information. They also felt that if student engagement was high, this would indicate that the students were using germane cognitive load. Even though open-ended surveys could be used to generate data on student engagement and satisfaction, a students’ level of engagement and satisfaction may reflect conditions other than cognitive load.

This literature review also revealed that no research has been conducted to analyze face-to-face discussions for cognitive presence. Meyer (2003) gathered data about the face-to-face discussions from the end of course evaluations. However, she did not analyze any in class discussions for cognitive presences. Additionally, Akyol and Garrison (2011) asked their students to self-report their stage of cognitive presence. The students’ self-reported stage seemed to reflect the actual stage of cognitive presence.
which Akyol and Garrison discovered by analyzing online discussion boards. Therefore, analyzing in class discussions may not have added to their understanding of cognitive presences. Students could accurately self-report their stage of cognitive presence, whether the course is face-to-face or online. Most of the research in this literature review analyzed the online discussions according to the CoI model (Garrison et al., 2000). However, as Chilton and Gurung (2008) discovered, student satisfaction and engagement seem to be an appropriate measures of cognitive load. Their findings may contribute to possible future research.
CHAPTER 3. METHODOLOGY

Purpose of the Study

Cognitive Load Theory researchers have mostly investigated cognitive load in various task characteristics (worked-examples versus conventional problems) by controlling the environmental characteristics (short, laboratory-type experiments) and accounting for the learner characteristics (levels of prior knowledge). Their research has uncovered many instructional design considerations for learning complex tasks. However, a void existed on understanding cognitive load over the duration of an entire course and on understanding cognitive load in relation to cognitive presence. Using a different approach to CLT research this research study investigated various environmental characteristics (online versus face-to-face) by controlling the task characteristics (same content and assignments) and accounting for the learner characteristics (levels of prior knowledge). Therefore, the purpose of this study was to determine whether there were differences in cognitive load, instructional efficiency, and learner involvement between online and face-to-face college algebra courses. Additionally, the purpose of this study was to determine how online students demonstrate cognitive presence in online discussions, and whether there was a relationship between cognitive load and cognitive presence in the online college algebra courses.

A mixed methods approach was employed with quantitative and qualitative data sources and data analysis methods. This mixed methods study sought to understand cognitive load and cognitive presence in online and face-to-face college algebra courses. A convergent parallel mixed methods design was used. This is a type of design in which different but complementary data was collected at the same time and then integrated to
interpret the overall results (Creswell, 2013). In this study, the mental effort measure (Paas, 1992) was used to test the Cognitive Load Theory that predicts that task characteristics influence the cognitive load for students with similar learner characteristics. Concurrent with this data collection, qualitative analysis discussion forums was used to explore cognitive presence for students in online college algebra courses. The reason for collecting both quantitative and qualitative data was to bring together the strengths of both forms of research to understand cognitive load and cognitive presence. This approach is informed by a pragmatic worldview, in which the focus of research is on the consequences of actions and solutions to problems. Mixed methods research is consistent with pragmatism, since pragmatism focuses on the research problem and is not committed to any one system of philosophy (Creswell, 2013).

**Research Questions**

Research Question 1: How do instructional effectiveness and efficiency vary between online and face-to-face college mathematics courses?

Research Question 2: How do online college mathematics students demonstrate cognitive presence in a discussion forum?

Research Question 3: What is the relationship between cognitive load and cognitive presence in an online college mathematics course?

**Research Hypotheses**

The research hypotheses employed quantitative and qualitative analyses. This section explains the quantitative research methodology.
Prior Knowledge

The first set of hypothesis tests will determine whether students enter the college algebra course with similar prior knowledge of the math content. These hypotheses require a t-test with one fixed-factor independent variable, course type, which compares the online and face-to-face college algebra courses. If Hypothesis 1a is not statistically significant, then the other hypothesis tests can be conducted. If Hypothesis 1a is statistically significant, then the students’ pretest scores are statistically different. This would show that the two samples start this course with differences in their prior knowledge. However, students complete the pretest in MyMathLab where they can retake the pretest. This could skew the results. Therefore, time on task could also control for learner characteristics, along with the pretest. If Hypothesis 1a is statistically significant, then Hypothesis 1b can be conducted.

H1a: There are no statistically significant differences in mean pretest scores between the online and face-to-face students.

H1b: There are no statistically significant differences in mean time spent on homework between the online and face-to-face students.

Effectiveness – Performance

The effectiveness between each group of math classes according to their performance will be computed. The homework performance score will be calculated from the average of the students’ homework scores. The test performance score will be calculated from the average of the students’ quiz and exam scores. These hypotheses require a t-test with one fixed-factor independent variable, course type, which compares the online and face-to-face college algebra courses.
H2a: There are no statistically significant differences in mean homework performance scores between the online and face-to-face students.

H2b: There are no statistically significant differences in mean test performance scores between the online and face-to-face students.

**Effectiveness – Cognitive Load**

The effectiveness of each type math class will be computed according to the cognitive load. The following hypothesis test describe methods comparing mean mental effort scores. Since Paas and van Merrienboer (1994a) define mental effort as the amount of cognitive load allocated to learning, cognitive load is calculated from the average of the students’ mental effort scores on their homework. This hypothesis requires a t-test with one fixed-factor independent variable, course type, which compares the online and face-to-face college algebra courses.

H3: There are no statistically significant differences in mean cognitive load between the online and face-to-face students.

**Efficiency**

The efficiency of each type math class will be computed according to the cognitive load scores on homework, cognitive load scores tests, and performance scores on tests. The relative efficiency score is calculated by subtracting each student’s mean homework cognitive load z-score from their mean test performance z-score, then dividing by the square root of two (Paas, F. & van Merrienboer, J., 1993). The three dimensional relative efficiency score is calculated by subtracting each student’s mean homework cognitive load z-score and their mean test cognitive load z-score from their mean test performance z-score, then dividing by the square root of three (Tuovinen, J. & Paas, F.,
The learner involvement score is calculated by adding each student’s mean homework cognitive load z-score and their mean performance z-score, then dividing by the square root of two (Paas et al., 2005). Each hypothesis requires a t-test with one fixed-factor independent variable, course type, which compares the online and face-to-face college algebra courses.

H4a: There are no statistically significant differences in mean relative efficiency scores between the online and face-to-face courses.

H4b: There are no statistically significant differences in mean three dimensional relative efficiency scores between the online and face-to-face courses.

H4c: There are no statistically significant differences in mean learner involvement scores between the online and face-to-face courses.

Final Grades

The following hypothesis test will compare final grades of the students in the online and face-to-face college algebra course. The students’ final grade is categorized as numerical, continuous data. This hypothesis requires a t-test with one fixed-factor independent variable, course type, which compares the online and face-to-face college algebra courses.

H5: There are no statistically significant differences in mean final grades between the online and face-to-face students.

Cognitive Presence

Because the university policy states that instructors cannot require students in face-to-face classes to participate in online discussion forums, only the discussion forums from the online college algebra classes will be used in this research hypothesis.
Additionally, only four of the online discussion forums were used in this analysis. These discussion forums include Week 3: Comparing Metric Systems, Week 4: Linear Equations, Week 5: Managing Polynomials, and Week 6: The FOIL Method. These four discussion forums were chosen for analysis because they prompted the students to reach the resolution stage of cognitive presence. The other discussion forums did not prompt students to reach resolution. Some of them prompted students to reach integration, and some prompted students toward self-reflection.

Cognitive presence will be coded according to four stages of the CoI theory: triggering event, exploration, integration, and resolution (Garrison, Anderson, & Archer, 2000). This research hypothesis followed Garrison et al. (2000)’s process for coding the different stages of cognitive presence for the Community of Inquiry theory. Each message (or each discussion post) was coded as one unit. Although each message varied in length according to each student, Garrison et al. (2000) advised to code each message for the stage that is most clear. After creating a frequency table for each category, a statistical analysis was conducted in order to report the findings in a quantitative format. Chi-square tests have allowed researchers to statistically determine which category occurred most often or which intervention reached the highest cognitive presence (Akyol & Garrison, 2011; Darabi et al., 2011).

The second research question employed six Chi-Square hypothesis tests to investigate the five categories of cognitive presence within the four online college algebra discussion forums that prompted for resolution. The first three hypothesis tests will be performed in the Nonparametric, One Sample Chi-Square function and the last three Chi-Square tests will be performed in the Descriptive Statistics, Crosstabs Chi-Square function.
function in SPSS. These Chi-Square tests were performed to determine whether the five categories of cognitive presence met expected frequency counts. In order to prevent SPSS from treating the summaries as raw data, the Weight Cases function was performed on the frequencies.

H6a: The five categories of cognitive presence within the four discussion forums are consistent with expected probabilities.

H6b: The five categories of cognitive presence within the Apply discussion forums are consistent with expected probabilities.

H6c: The five categories of cognitive presence within the Defend discussion forums are consistent with expected probabilities.

H6d: Resolution and discussion type are independent of each other.

H6e: Cognitive presence and discussion type are independent of each other.

H6f: Cognitive presence and instructor participation are independent of each other.

**Cognitive Load and Cognitive Presence**

Because data on cognitive presence has been generated from the discussion forums in the online college algebra class, cognitive load data from the online college algebra classes will also only be used in this research hypothesis. Data from face-to-face college algebra classes will not be used in this analysis.

A void exists in the research literature on understanding the relationship between cognitive load and cognitive presence. Therefore, in order to understand the relationship a first-order multiple regression model relating cognitive load to the five categories of cognitive presence (triggering event, exploration, integration, resolution, and no cognitive presence) should be analyzed. Within the multiple regression analysis the first step is to
test model significance, that none of the five predictors is effective in explaining variation in cognitive load. Additionally, each of the five predictors should be tested whether it adds information about cognitive load to that provided by the other predictors.

\[
\text{Cognitive Load} = \alpha + \beta_1(\text{Triggering Event}) + \beta_2(\text{Exploration}) + \beta_3(\text{Integration}) + \\
\beta_4(\text{Resolution}) + \beta_5(\text{None}) + \varepsilon
\]

H7: There is no relationship between cognitive load and cognitive presence.

Research Design

A quasi-experimental, between-subject design was chosen for this project. Students were not randomly assigned to different groups. Students self-selected into the online or face-to-face modality of the college algebra course. The population in the study was the non-residential students taking a course during January to May 2015 from a private mid-western university. These students typically range in age from 20 – 60, males and females, of various ethnic and religious backgrounds. The size of the population was estimated to be over two thousand. With a convenience sampling method, students who are taking the college algebra course during January to May 2015 were invited to participate in this study. Data from the online students were pooled together, and data from the face-to-face students were pooled together.

The size of this sample was two hundred eighty-five, if every student gave consent to participate in the research project. Sixteen online sections of college algebra were offered during January to May 2015, with a sample size of one hundred sixty-six students. Ten face-to-face sections of college algebra were offered during January to May 2015, with a sample size of one hundred nineteen students. Table 1 outlines the college algebra section names, start dates, end dates, student counts, and locations.
The college algebra course is organized similarly, whether it is taken by online or face-to-face students. Each week of class covers one or two major topics. Since the majority of our courses are taught by adjunct instructors, the courses are prescribed. The discussion forums and assignments are housed in the course management system. Each
instructor is asked and expected to follow the predetermined course syllabus. This structure helps to ensure course expectations and student learning outcomes.

The college algebra course has two discussion forums each week. These discussion forums provide each student an opportunity to explore a math concept in a real life scenario, to explain their understanding of a math concept, or to reflect on their learning processes. The university does not allow instructors to require their face-to-face students to participate in any discussion forum. The points associated with the discussion forums become participation points for the face-to-face class. Therefore, the discussion forums will be used in the qualitative analysis of the online math course only. Data generated from the online discussion forums will be used for quantitative analysis in research hypotheses, H6 and H7.

The college algebra course provides the students an opportunity to practice their mathematical skills and demonstrate their mathematical understanding through a weekly homework assignment in MyMathLab (MML). MML is an interactive platform where students can access multimedia learning aids, videos, animations, and tutorial help. Students in both the online and face-to-face modalities complete the same weekly homework assignment in MML. Students in both the online and face-to-face modalities also complete the same weekly quiz, midterm exam, and final exam in MML.

**Qualitative Analysis**

Qualitative data were generated from four of the discussion forums in the online college algebra course. These discussion forums include Week 3: Comparing Metric Systems, Week 4: Linear Equations, Week 5: Managing Polynomials, and Week 6: The FOIL Method. These four discussion forums were chosen for analysis because they
prompted the students to reach the resolution stage of cognitive presence. The resolution stage is where students defend or test their hypothesis. In the resolution stage students may also apply their solution to other scenarios. The other discussion forums in the college algebra course did not prompt students to reach resolution. Some of those discussion forums prompted students to reach integration, while others prompted students toward self-reflection. Table 2 outlines the discussion prompts used in this study.

Table 2. Discussion Prompts. Resolution prompt is italicized.

<table>
<thead>
<tr>
<th>Discussion Prompt</th>
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<tbody>
<tr>
<td><strong>Week 3: Comparing Metric Systems</strong></td>
</tr>
<tr>
<td>a. Based on your understanding of the text, select a food recipe of your choice and convert all U.S. measurements (where applicable) into metric units. If the recipe is written using metric measurements, convert them into U.S. units of measure. Post the recipe and all of the conversions.</td>
</tr>
<tr>
<td>b. In your opinion, what is the most important function of signed numbers?</td>
</tr>
<tr>
<td>c. <em>Identify three real-life scenarios that call for the use of negative integers.</em></td>
</tr>
<tr>
<td><strong>Week 4: Linear Equations</strong></td>
</tr>
<tr>
<td>a. In your own words, describe the concept of like terms. How would you identify a like-terms expression?</td>
</tr>
<tr>
<td>b. Identify the reason(s) that the expression (x+y) cannot be simplified any further.</td>
</tr>
<tr>
<td>c. Develop a word problem that incorporates a linear equation.</td>
</tr>
<tr>
<td>d. Solve your word problem, outlining each step in the process, as if you were teaching a room of students how to solve your word problem.</td>
</tr>
<tr>
<td>e. In your own words, identify the unique considerations that must be taken into account when solving linear equations.</td>
</tr>
<tr>
<td>f. <em>Post a real-life example where you might need to solve a linear equation.</em></td>
</tr>
<tr>
<td><strong>Week 5: Managing Polynomials</strong></td>
</tr>
<tr>
<td>a. Based on your understanding of the text, develop three algebraic equations incorporating any combination of variables, radicals, and exponents.</td>
</tr>
<tr>
<td>b. For your initial post, post your three equations <em>without</em> the solutions.</td>
</tr>
<tr>
<td>c. Choose a classmate’s posted equation (one that has not been solved already). Post your solution to the equation and be sure that your answer is stated in its simplest form.</td>
</tr>
<tr>
<td>d. <em>By the end of the workshop you should post the solutions to your three equations. Indicate whether or not a classmate’s...</em></td>
</tr>
</tbody>
</table>
solution to your equation is correct. Defend your solution versus their solution.

| Week 6: The FOIL Method | a. Based on your understanding of the media you watched, develop three problems that can be solved using FOIL.  
b. For your initial post, post your three problems without the solutions.  
c. Choose a classmate’s posted problem (one that has not been solved already). Post your solution to the problem and be sure that your answer is stated in its simplest form.  
d. By the end of the workshop you should post the solutions to your three problems. Indicate whether or not a classmate’s solution to your problem is correct. Defend your solution versus their solution. |

**Coding Process**

The data were coded according to the stages of cognitive presence in the Community of Inquiry theory (Garrison, Anderson, & Archer, 2000). Coding followed Garrison et al.’s (2000) method of coding each message (or each discussion post) as one unit. Each message varied in length according to each student, therefore each message was coded for the stage that is most clear. Higher levels of critical thinking involve using lower levels of cognitive presence, so it is not unusual for a message to have more than one stage presented (Garrison et al, 2000). After coding each message for the stage that is most clear, a frequency table was created for each category to identify what is occurring and how often it is happening with the goal of drawing conclusions about the patterns and themes.

Hypothesis test, H6, employed a Chi-square test to statistically determine which category occurred most often in these four discussion forums that prompted students to reach resolution. Hypothesis test, H7, employed regression analysis to understand the relationship between cognitive load and cognitive presence. These two quantitative data analysis techniques were useful in understanding some aspects of cognitive presence and
its relationship to cognitive load. However, a deeper understanding of how students demonstrated cognitive presence in an online discussion forum required a thematic qualitative data analysis. Qualitative research is not about finding generalizations; it is about exploring complexities, culture, and phenomenon (Glesne, 2011). There are complexities and culture in online classroom discussions (Garrison et al., 2000). Qualitative research allowed for deep analysis of these discussions. The thematic qualitative data analysis investigated the following two areas of cognitive presence as related to the second research question: How do online college mathematics students demonstrate cognitive presence in a discussion forum?

**Apply or Defend your Solution**

Using the Community of Inquiry theory, researchers have investigated various discussion forum strategies to identify which discussion forum strategy reaches the highest stage of cognitive presence (Akyol & Garrison, 2011; Darabi et al., 2011). High cognitive thinking was discovered in online discussions which involved a real world scenario (Bradley et al., 2008; Weeks et al., 2013), brainstorming ideas or solutions to an issue (Bradley et al., 2008), debates (Kanuka et al., 2007), collaboration (Stahl, 2006, 2010; Zhou et al., 2008), WebQuests (Kanuka et al., 2007), and discussion questions involving metacognitive strategies (Mevarech & Amrany, 2008). How discussion boards were presented also influenced cognition. Providing an example post to the students, and limiting the number of posts displayed on each page encouraged student engagement and schema construction (Darabi & Jin, 2013).

However, these researchers compared discussion forum strategies that may or may not have prompted for the resolution stage of cognitive presence. The four
discussion forums used in this research project were chosen for analysis because they prompted the students to reach the resolution stage of cognitive presence. The resolution stage is where students apply their solution to other scenarios or where students defend or test their hypothesis. The first two discussion forums prompted each student to reach resolution stage by exploring a math concept in a real life scenario. The last two discussion forums prompted each student to reach resolution stage by giving the student an opportunity to defend or test their hypothesis. Therefore, this research project investigated the discussion forum strategies (apply their solution or defend their hypothesis) as they engaged students across the stages of cognitive presence.

A series of sub-questions guided the analysis. (1) What aspects of the two discussion forum strategies engaged the students in reaching the stages of cognitive presence? (2) What aspects of the two discussion forum strategies increased cognitive presence within a discussion thread? (3) What were the expected and unexpected patterns of response cycles within a discussion thread and across the two discussion forum strategies?

**Instructor Engagement**

Along with cognitive presence, the Community of Inquiry theory recognized the importance of social presence and teacher presence. Although the CoI theory provided a set of procedures to code discussion forum responses in order to identify social presence and teacher presence, this research project hoped to gain insight into the relationship of how instructor responses engaged students in developing deeper levels of cognition.

The four discussion forums in the sixteen online classes prompted students to reach the resolution stage of cognitive presence. However, the students’ initial posts may
not have thoroughly addressed the discussion prompt. Their initial post may or may not have reached the resolution stage. Therefore, this study investigated the presence or absence of the instructor responses to the students, identified themes in what type of response the instructor provided, and whether the students advanced in their stage of cognitive presence.

A series of sub-questions guided my analysis. (1) How did the instructors participate in the discussion forums? (2) How did the instructors respond to the students’ posts? (3) How did the instructors engage the students in cognitive presence?

**Quantitative Analysis**

Quantitative data will be collected from five sources: (1) Pretest, (2) Six weekly homework assignments, (3) Six weekly quizzes, (4) Two exams (Midterm and Final), and the (5) Final grade. The data analysis has been conducted according to the seven research hypotheses described above. These hypotheses employ t-tests, chi-square tests, or regression analysis.

**Dependent Variables**

The pretest score is a dependent variable (DV). It is a continuous variable, assumed to have a normal distribution. The pretest score can range from zero to one hundred points. The sample size should equal the number of participants in the research study.

The students’ weekly homework problems are completed in MyMathLab. Each students’ mental effort score and the time it took for them to complete the homework assignment will be collected from the weekly homework assignment in MyMathLab. The last homework question will ask each student to self-report their mental effort. The
mental effort measure is a 9-point Likert-type scale, ranging from “1 – very, very low mental effort” to “9 – very, very high mental effort” (Paas, 1992). Since Paas and van Merrienboer (1994a) define mental effort as the amount of cognitive load allocated to learning, cognitive load is calculated from the average of the students’ mental effort scores on their homework. The cognitive load score will be calculated using the scores on the mental effort question from each student’s homework assignments over the course of the class. The college algebra course has nineteen homework assignments. The mean cognitive load score for each student will be calculated by adding the nineteen homework mental effort scores and dividing by nineteen.

The mean cognitive load score is a continuous variable, assumed to have a normal distribution, ranging from one to nine. The sample size of this variable will equal the product of the number of participants in the research study and the number of weeks in their math class. However, after the mean cognitive load score is calculated, the sample size will equal the number of participants. Next, each student’s mean cognitive load z-score will be calculated by subtracting the grand mean cognitive load score from each student’s mean cognitive load score and dividing by the grand standard deviation cognitive load score. The Transform → Compute Variable feature in SPSS will be used for these calculations. The mean cognitive load z-score is a continuous variable, assumed to have a normal distribution.

MyMathLab also documents the time students needed to complete the assignment. The time is a continuous variable, assumed to have a normal distribution. Even though time could have any value possible, most likely the times will range from
zero minutes to four hours. The sample size should equal the number of participants in the research study.

The students’ weekly quiz and two exams were also completed in MyMathLab. Each student’s test mental effort and performance score was calculated using the quizzes and exams. The last question on the quizzes and exams asked each student to self-report their mental effort. The mental effort measure is a 9-point Likert-type scale, ranging from “1 – very, very low mental effort” to “9 – very, very high mental effort” (Paas, 1992). The mean test cognitive load score was calculated using the mental effort question from each student’s quizzes and exams over the course of the class. The college algebra course had six quizzes and two exams. The mean test cognitive load score for each student was calculated by adding the six quiz cognitive load scores with two exam cognitive load scores and dividing by eight.

The mean test cognitive load score is a continuous variable, assumed to have a normal distribution. The sample size of this variable was equal to the product of the number of participants in the research study and the number of weeks in their math class. However, after the mean test cognitive load score was calculated, the sample size equaled the number of participants. Next, each student’s mean test cognitive load z-score was calculated by subtracting the test grand mean cognitive load score from each student’s mean test cognitive load score and dividing by the test grand standard deviation cognitive load score. The Transform → Compute Variable feature in SPSS was used for these calculations. The mean test cognitive load z-score is a continuous variable, assumed to have a normal distribution.
In addition to the mental effort measurement on the quizzes and exams, the student’s homework, quizzes, and exams performance was collected. The mean performance was calculated for each student’s homework scores, quiz scores, and exam scores over the course of the class. For example, the college algebra course had nineteen homework assignments, six quizzes, and two exams. The mean homework performance score for each student was calculated by adding the nineteen homework scores and dividing by nineteen. The mean test performance score for each student was calculated by adding the six quiz scores and two exam scores and dividing by eight. The mean performance scores are continuous variables, assumed to have a normal distribution, ranging from zero to one hundred. The sample size of this variable was equal to the product of the number of participants in the research study and the number of weeks in their math class. However, after the mean performance scores was calculated, the sample size equaled the number of participants. Next, each student’s mean performance z-scores was calculated by subtracting the test grand mean performance score from each student’s mean performance score and dividing by the test grand standard deviation performance score. The Transform \( \rightarrow \) Compute Variable feature in SPSS was used for these calculations. The mean performance z-scores are continuous variables, assumed to have a normal distribution.

After these measures are collected and calculated each student’s relative efficiency was calculated according to Paas and van Merrienboer (1993). The three dimensional relative efficiency was calculated according to Tuovinen and Paas (2004), and the learner involvement measurement was calculated according to Paas et al. (2005). The Transform \( \rightarrow \) Compute Variable feature in SPSS was used for these calculations.
The relative efficiency score was calculated by subtracting each student’s mean cognitive load z-score from their mean test performance z-score, then dividing by the square root of two (Paas, F. & van Merrienboer, J., 1993). The three dimensional relative efficiency score was calculated by subtracting each student’s mean cognitive load z-score and their mean test cognitive load z-score from their mean test performance z-score, then dividing by the square root of three (Tuovinen, J. & Paas, F., 2004). The learner involvement score was calculated by adding each student’s mean cognitive load z-score and their mean performance z-score, then dividing by the square root of two (Paas et al., 2005). All of these computed scores are continuous variables, assumed to have normal distributions. The sample size equaled the number of participants in the research study.

Cognitive presence was identified through a qualitative coding process. Cognitive presence has four stages: triggering event, exploration, integration, and resolution. Another category was created for coding cognitive presence: no category. Coding the discussion forums according to the five categories of cognitive presence established a frequency table for each course. Each frequency was divided by the total number of elements in each course. Each category is a discrete variable, because the number of elements in each factor is countable. The sample size was determined by how many discussion forums are used, how many students participate in the discussion forums, and how often they participate.

**Independent Variables**

Six independent variables (IV) was collected for this study, although not all of them will be used for statistical analysis. The four demographic variables were independent variables since they do not change depending upon any type of math class
someone is enrolled in. The sample size for the demographic variables equaled the number of participants in the research study. Age is a continuous variable, assumed to have a normal distribution. Even though age could have any value possible, the ages ranged from 18 – 70. Gender is a categorical variable, with a binomial distribution (1= male, 2= female). Race is a categorical variable, with a multinomial distribution (1= American Indian or Alaska Native, 2= Asian, 3= Black or African American, 4= Native Hawaiian or Pacific Islander, 5= White or Caucasian). Marital status is a categorical variable, with a multinomial distribution (1= married, 2= single, 3= widowed, 4= divorced, 5= other).

Each student’s final grade was also collected. The final grade is a numerical, continuous variable, with a normal distribution ranging from zero to one thousand points. The main independent variable was course type. Course type is a between-subject factor. There are not any within-subject factors in this study. Course type is a nominal variable (1= online college algebra, 2= face-to-face college algebra). The sample size for the course type variable equaled the number of participants in the research study.

**Instrumentation**

One instrument was used in this project. The instrument was Paas’ (1992) 9-point Likert-type self-reported mental effort measure: “1 – very, very low mental effort” to “9 – very, very high mental effort”. It has been shown to have high face validity (Paas, 1992; Paas et al., 2003) and internal consistency (deCroock & van Merrienboer, 2007; deCroock, van Merrienboer, & Paas, 1998; Paas & van Merrienboer, 1994). The Cronbach’s coefficient alpha estimate ranged from .82 to .98 in these studies. Even though this scale is technically a categorical variable, it can be analyzed as a continuous
variable since it has nine categories (Lubke & Muthen, 2004). This nine-point Likert-type self-reported mental effort measure was the last question for students to answer on their weekly homework, their weekly quiz, and their two exams.

**Data Analysis Techniques**

The data collected were analyzed using SPSS 21. Table 3 outlines the Research Questions, according to the Quantitative Research Hypotheses, dependent and independent variables, as well as data collection method and data analysis method.

Because this research study could have encountered the issue of a low response rate, the procedures of how to handle missing data needed to be identified. According to Paas and van Merrienboer (1993), missing values of mental effort should be replaced with the grand mean of mental effort. Once the cases of missing data were managed, the appropriate statistical techniques was conducted. When the appropriate statistical techniques was conducted, the listwise missing data procedure was utilized. Each statistical technique is described next.

**T-tests**

T-tests include an assumption of homogeneity of variance. A statistically significant Levene’s test will indicate a violation of homogeneity of variance. The Levene’s test identifies whether the differences between means are due to inherent differences (violation of homogeneity of variance) or due to the differences of levels in the IV. Reporting the results from the Levene’s test include (1) degrees of freedom, (2) the F-statistic, and (3) the p-value. If Levene’s test is significant the following would be reported: F(k–1, N–k) = f-statistic, p<.05, where N is the sample size of the dependent variable and k is the number of levels in the IV.
Table 3. Quantitative Research Hypotheses, Data Collection Methods, Variables, and Data Analysis Methods for each Research Questions

<table>
<thead>
<tr>
<th>Research Hypothesis</th>
<th>Data Collection Source</th>
<th>Independent Variable</th>
<th>Dependent Variable</th>
<th>Data Analysis Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQ1: How do instructional effectiveness and efficiency vary between online and face-to-face college mathematics courses?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1a</td>
<td>Pretest</td>
<td>Course type</td>
<td>Performance</td>
<td>T-test</td>
</tr>
<tr>
<td>H1b</td>
<td>Homework</td>
<td>Course type</td>
<td>Time</td>
<td>T-test</td>
</tr>
<tr>
<td>H2a</td>
<td>Homework</td>
<td>Course type</td>
<td>Effectiveness - Performance</td>
<td>T-test</td>
</tr>
<tr>
<td>H2b</td>
<td>Tests</td>
<td>Course type</td>
<td>Effectiveness - Performance</td>
<td>T-test</td>
</tr>
<tr>
<td>H3</td>
<td>Homework</td>
<td>Course type</td>
<td>Effectiveness - Cognitive Load</td>
<td>T-test</td>
</tr>
<tr>
<td>H4a</td>
<td>Homework, Tests</td>
<td>Course type</td>
<td>Relative Efficiency</td>
<td>T-test</td>
</tr>
<tr>
<td>H4b</td>
<td>Homework, Tests</td>
<td>Course type</td>
<td>Three Dimensional Relative Efficiency</td>
<td>T-test</td>
</tr>
<tr>
<td>H4c</td>
<td>Homework, Tests</td>
<td>Course type</td>
<td>Learner Involvement</td>
<td>T-test</td>
</tr>
<tr>
<td>H5</td>
<td>Course Final Grade</td>
<td>Course type</td>
<td>Final Grade</td>
<td>T-test</td>
</tr>
<tr>
<td>RQ2: How do online college mathematics students demonstrate cognitive presence?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H6a-f</td>
<td>Discussion Forums</td>
<td>Online course</td>
<td>Cognitive Presence</td>
<td>Chi-Square test</td>
</tr>
<tr>
<td>RQ3: What is the relationship between cognitive load and cognitive presence?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H7</td>
<td>Discussion Forums, Homework, Tests</td>
<td>Cognitive Presence</td>
<td>Cognitive Load</td>
<td>Multiple Regression</td>
</tr>
</tbody>
</table>
By conducting t-tests, differences between the means of two independent groups can be detected. A t-test uses the sample’s mean and standard deviation to estimate values of the population. The requirements for a t-test include (1) one IV with only two levels and (2) only one dependent variable. The independent variable can be ordinal or nominal. The assumptions of a t-test include (1) the dependent variable is normally distributed and (2) homogeneity of variance. A statistically significant Levene’s Test will indicate a violation of homogeneity of variance. If homogeneity of variance is not violated (Levene’s Test: p > .05), the t-test can be interpreted with equal variances assumed. If homogeneity of variance is violated (Levene’s Test: p < .05), the t-test can be interpreted with equal variances not assumed (Ho, 2006). Reporting results from a t-test include (1) degrees of freedom, (2) the t-statistic, and (3) the p-value. These results are located in the Independent Samples T-test table in SPSS. If the t-test is significant the following would be reported: t(N – k) = t-statistic, p < .05, where N is the sample size of the dependent variable and k is the number of levels in the IV. For a t-test the IV should only have two levels, so k = 2. The means and standard deviations of each group should also be reported.

Chi-square Tests

The chi-square test is a nonparametric test because we cannot assume a normal distribution. By conducting Chi-square tests, differences between expected and observed frequencies can be detected. The chi-square test determines if the observed frequencies are statistically different from the expected frequencies. The expected frequencies are the values for each group if each group had equal chances of having an element (equally probable). Meaning, if there was a sample size of 30 with three groups, each group would
be expected to have 10 elements. A chi-square test can also determine whether categorical variables (nominal data) are independent or related. The assumptions include (1) random sampling, (2) independence between observations, and (3) the expected frequency of each category should be at least 5 (Ho, 2006).

If a chi-square test is performed on a single variable, the Analyze → Nonparametric Tests → Chi-Square function in SPSS should be used. Reporting results from a chi-square test should include (1) degrees of freedom, (2) the chi-square statistic ($\chi^2$), and (3) the p-value. The degrees of freedom, df, is the number of categories minus one. The chi-square statistic is the sum of the square of the difference between each observed value and the expected value, divided by the expected value. Using the Chi-Square Output table the following would be reported if the chi-square test was statistically significant: $\chi^2$ (df) = $\chi^2$ statistic, p<.05. The frequencies and percentages of each group need to be reported, indicating their increasing or decreasing order.

If a chi-square test on more than a single variable is performed, the Analyze → Descriptive Statistics → Crosstabs function in SPSS should be used. The degrees of freedom, df, is the number of rows minus one times the number of columns minus one. In the Chi-Square Output table the Expected count needs to be reviewed to make sure the value is greater than five. In the Chi-Square Tests table the Pearson chi-square statistic is used to determine whether there are differences (or a relationship) between the expected and observed frequencies. The Pearson chi-square statistic needs to be reported: $\chi^2$ (df) = $\chi^2$ statistic, p<.05. Results for each category needs to be reported according to its’ count and percentage within the category. The significant differences occur where the observed count is more or less than the expected count.
Regression Analysis

Lastly, regression analysis is a tool to understand and quantify relationships between variables. Since a theoretical model between the variables is unknown an empirical model based on probabilities is used to approximate the main features of the relationships between independent and dependent variables (Mendenhall & Sincich, 2012). A regression model is used to predict the value of a dependent variable and estimate its’ mean value for given values of the predictors. For multiple regression analysis the assumptions include the random errors component follows a normal probability distribution and the random errors are independent of each other, meaning the cognitive load of any one student is not dependent upon the cognitive load of any other student. A normal probability plot of the residuals will determine normality. The least squares method minimizes the sum of squared errors in order to fit a multiple regression model.

The regression model is: Cognitive Load = α + β1(Triggering Event) + β2(Exploration) + β3(Integration) + β4(Resolution) + β5(None) + ε. The independent variables (Triggering Event, Exploration, Integration, Resolution, and None) are first-order terms. The dependent variable and the independent variables are all identified as a Scale measure in SPSS. A scatterplot is constructed in SPSS to determine if the relationship is approximately linear between the dependent variable and each independent variable. In the Model Summary the R Square term identifies the percentage of the variability of the cognitive load that is explained by the fitted regression model. To test model significance the p-value for the F-statistic must be less than the level of significance, .05. This test shows whether at least one of the five predictors is
significantly different from zero, meaning at least one of the five predictors is effective in explaining variation in cognitive load. Additionally, each of the five predictors is tested for significance using the p-value associated with each Unstandardized Beta Coefficients. If any $\beta$ is shown to not be significant it can be removed from the model. Each estimated beta is interpreted as: for each one unit increase in resolution, the estimated mean increase in cognitive load is $\beta_1$ when all other independent variables are held constant. The intercept is interpreted as: the average cognitive load for zero levels of cognitive presence is $\alpha$. This interpretation of the intercept may not be interpretable. Therefore, the predictor data could be centered in the fixed model by subtracting the grand sample mean of that predictor from the original sample data of that predictor. The regression model could also be fitted using the generalized linear model.

**Data Collection Procedures**

Data were collected and generated in accordance with the University of Kentucky’s research guidelines. The following steps occurred once the research proposal was approved and IRB approval granted from the University of Kentucky.

- January to April 2015: About a month before each course section began the Principal Investigator (PI) contacted each instructor to ask for their blessing to use their class for this research study.

- Students in each face-to-face class were invited to participate in the research study. A video was posted inviting students to participate in the research study with a link to access the electronic consent form and the survey, which asked demographic questions.
• In each online class a video was posted inviting students to participate in the research study with a link to access the electronic consent form and the survey. The same survey was used for the face-to-face and online courses.

• The PI collected the consent form information and the survey information.

• Paas’ (1992) 9-point Likert-type self-reported measure was embedded in each homework assignment, quiz, and exam. Students earned one point each time they answered the mental effort question, regardless of how they answered it.

• Data from the online students were pooled together, and data from the face-to-face students were pooled together.

• February to August, 2015: The PI collected the data from each course via the course management system. Quantitative data were organized when each course section ends.

• August to November, 2015: Qualitative data were transcribed, coded, and recoded throughout the data analysis process. Quantitative data were analyzed.

• November to January, 2016: The PI synthesized the data and wrote the remaining chapters of the dissertation.

This chapter depicted the quantitative and qualitative research methodology intended to investigate cognitive load and cognitive presence. This chapter described the research design, population, sample, and instruments to be used in the research study. A description of each dependent and independent variable according to its level of measurement, distribution, and sample size has been identified. This chapter identified each hypothesis test with the appropriate statistical technique. Additionally, the procedure for handling missing data has been explained.
CHAPTER 4. QUANTITATIVE ANALYSIS

The purpose of this study was to determine whether there were differences in cognitive load, relative efficiency, and learner involvement between online and face-to-face college algebra courses. Additionally, the purpose of this study was to determine how students demonstrate cognitive presence in online discussions and whether there was a relationship between cognitive load and cognitive presence in online college algebra courses. This chapter presents descriptive and inferential analyses of the quantitative study data as related to the study’s research questions:

Research Question 1: How do instructional effectiveness and efficiency vary between online and face-to-face college mathematics courses?

Research Question 2: How do online college mathematics students demonstrate cognitive presence in a discussion forum?

Research Question 3: What is the relationship between cognitive load and cognitive presence in an online college mathematics course?

Description of the Sample

The population in the study was the non-residential students taking a course at a private mid-western university during the spring of 2015. Students self-selected into the online or face-to-face modality of their course. With a convenience sampling method, students who were taking college algebra were invited to participate in this study. Sixteen online sections were offered with a sample size of one hundred sixty-six students. Of those one hundred sixty-six online students, one hundred fifty-nine students (95.8%) consented to participate in the study. Ten face-to-face sections were offered with a sample size of one hundred nineteen students. Of those one hundred nineteen face-to-face
students, one hundred six students (89.1%) consented to participate in the study. Out of a total of two hundred eighty-five students, two hundred sixty-five students (93%) consented to participate in this study. Out of the total sample size of two hundred sixty-five students, 60% were online and 40% were face-to-face. The typical online student was a 35 year old, married, Caucasian female. The typical face-to-face student was a 37 year old, Caucasian female. The typical marital status of face-to-face students could not be determined since those classes consisted of an equal number of married and single students. Table 4 outlines the demographics of the sample.

Table 4. Demographics of the sample

<table>
<thead>
<tr>
<th>Gender, n (%)</th>
<th>Online (n = 159)</th>
<th>Face-to-Face (n = 106)</th>
<th>Total (n = 265)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>50 (32%)</td>
<td>50 (47%)</td>
<td>100 (38%)</td>
</tr>
<tr>
<td>Female</td>
<td>107 (68%)</td>
<td>56 (53%)</td>
<td>163 (62%)</td>
</tr>
<tr>
<td>Age, n (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18-22</td>
<td>5 (3%)</td>
<td>4 (4%)</td>
<td>9 (3%)</td>
</tr>
<tr>
<td>23-29</td>
<td>29 (18%)</td>
<td>33 (31%)</td>
<td>62 (24%)</td>
</tr>
<tr>
<td>30-39</td>
<td>62 (39%)</td>
<td>33 (31%)</td>
<td>95 (36%)</td>
</tr>
<tr>
<td>40-49</td>
<td>44 (28%)</td>
<td>25 (24%)</td>
<td>69 (26%)</td>
</tr>
<tr>
<td>50-59</td>
<td>17 (11%)</td>
<td>10 (9%)</td>
<td>27 (10%)</td>
</tr>
<tr>
<td>60+</td>
<td>0</td>
<td>1 (&lt;1%)</td>
<td>1 (&lt;1%)</td>
</tr>
<tr>
<td>Marital Status, n (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Married</td>
<td>83 (52%)</td>
<td>47 (44%)</td>
<td>130 (49%)</td>
</tr>
<tr>
<td>Single</td>
<td>53 (33%)</td>
<td>47 (44%)</td>
<td>100 (38%)</td>
</tr>
<tr>
<td>Widowed</td>
<td>3 (2%)</td>
<td>1 (&lt;1%)</td>
<td>4 (2%)</td>
</tr>
<tr>
<td>Divorced</td>
<td>18 (11%)</td>
<td>10 (9%)</td>
<td>28 (10%)</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
<td>1 (&lt;1%)</td>
<td>1 (&lt;1%)</td>
</tr>
<tr>
<td>Race, n (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>American Indian</td>
<td>1 (1%)</td>
<td>1 (&lt;1%)</td>
<td>2 (1%)</td>
</tr>
<tr>
<td>Black</td>
<td>39 (25%)</td>
<td>38 (36%)</td>
<td>77 (29%)</td>
</tr>
<tr>
<td>Caucasian</td>
<td>109 (69%)</td>
<td>61 (58%)</td>
<td>170 (64%)</td>
</tr>
<tr>
<td>Hispanic</td>
<td>1 (1%)</td>
<td>3 (3%)</td>
<td>4 (2%)</td>
</tr>
<tr>
<td>Other</td>
<td>6 (4%)</td>
<td>2 (2%)</td>
<td>8 (3%)</td>
</tr>
</tbody>
</table>
Summary of the Results

The data collected were analyzed using SPSS for Windows, Version 21. The research hypotheses employed t-tests, chi-square tests, and regression analysis. Tables 5-8 outline the summary of the results for each research question.

Table 5. Summary of the results for Research Question 1
RQ1: How do instructional effectiveness and efficiency vary between online and face-to-face college mathematics courses?

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Online Mean (SD)</th>
<th>Face-to-face Mean (SD)</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1: Pretest (0-100%)</td>
<td>4.05 (12.1)</td>
<td>3.46 (13.6)</td>
<td>(-4.258, 4.501)</td>
</tr>
<tr>
<td>H2a: Homework Score (0-100%)</td>
<td>92.49 (9.9)</td>
<td>96.25 (5.2)</td>
<td>(-5.942, -1.640)*</td>
</tr>
<tr>
<td>H2b: Test Score (0-100%)</td>
<td>82.25 (13.8)</td>
<td>85.45 (10.3)</td>
<td>(-6.742, .375)</td>
</tr>
<tr>
<td>H3: Cognitive Load (1-9)</td>
<td>6.42 (1.4)</td>
<td>6.59 (1.5)</td>
<td>(-.638, .306)</td>
</tr>
<tr>
<td>H4a: Relative Efficiency</td>
<td>-.02 (1.1)</td>
<td>.07 (0.9)</td>
<td>(-.395, .217)</td>
</tr>
<tr>
<td>H4b: Three Dimension Relative Efficiency</td>
<td>-.02 (1.4)</td>
<td>-.02 (1.3)</td>
<td>(-.427, .421)</td>
</tr>
<tr>
<td>H4c: Learner Involvement</td>
<td>.01 (0.8)</td>
<td>.25 (0.8)</td>
<td>(-.501, .019)</td>
</tr>
<tr>
<td>H5: Final Grade (0-100%)</td>
<td>88.24 (12.9)</td>
<td>93.33 (7.0)</td>
<td>(-7.99, -2.27)*</td>
</tr>
</tbody>
</table>

*p<.05
Table 6. *Summary of the results for Research Question 2*
RQ2: How do online college mathematics students demonstrate cognitive presence?

<table>
<thead>
<tr>
<th>Cognitive Presence</th>
<th>Triggering Event</th>
<th>Exploration</th>
<th>Integration</th>
<th>Resolution</th>
<th>None</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined Discussions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>19.2</td>
<td>10.0</td>
<td>25.7</td>
<td>22.9</td>
<td>22.3</td>
<td>183.55*</td>
</tr>
<tr>
<td>Expected</td>
<td>18.3</td>
<td>18.3</td>
<td>18.3</td>
<td>26.7</td>
<td>18.3</td>
<td></td>
</tr>
<tr>
<td>Apply Discussions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>10.6</td>
<td>18.0</td>
<td>11.4</td>
<td>20.1</td>
<td>39.9</td>
<td>367.39*</td>
</tr>
<tr>
<td>Expected</td>
<td>17.9</td>
<td>17.9</td>
<td>17.9</td>
<td>28.6</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>Defend Discussions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>26.8</td>
<td>2.8</td>
<td>28.5</td>
<td>25.4</td>
<td>6.5</td>
<td>537.68*</td>
</tr>
<tr>
<td>Expected</td>
<td>18.8</td>
<td>18.8</td>
<td>18.8</td>
<td>24.9</td>
<td>18.8</td>
<td></td>
</tr>
<tr>
<td>Students Who Reached Resolution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apply Discussions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Defend Discussions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage of Discussion Posts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apply Discussions</td>
<td>10.6</td>
<td>18.0</td>
<td>11.4</td>
<td>20.1</td>
<td>39.9</td>
<td>647.35*</td>
</tr>
<tr>
<td>Defend Discussions</td>
<td>26.8</td>
<td>2.8</td>
<td>38.5</td>
<td>25.4</td>
<td>6.5</td>
<td></td>
</tr>
<tr>
<td>Instructors Participation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>17.3</td>
<td>10.4</td>
<td>22.4</td>
<td>25.0</td>
<td>24.8</td>
<td>25.10*</td>
</tr>
<tr>
<td>No</td>
<td>21.0</td>
<td>9.5</td>
<td>29.1</td>
<td>20.8</td>
<td>19.7</td>
<td></td>
</tr>
</tbody>
</table>

*p<.05*
Table 7. *Summary statistics for the results for Research Question 3*

**RQ3: What is the relationship between cognitive load and cognitive presence?**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Pearson Correlation with Cognitive Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive Load</td>
<td>6.45</td>
<td>1.42</td>
<td></td>
</tr>
<tr>
<td>Triggering event</td>
<td>2.65</td>
<td>1.13</td>
<td>.113</td>
</tr>
<tr>
<td>Exploration</td>
<td>1.36</td>
<td>1.49</td>
<td>.017</td>
</tr>
<tr>
<td>Integration</td>
<td>3.54</td>
<td>2.26</td>
<td>.193*</td>
</tr>
<tr>
<td>Resolution</td>
<td>3.19</td>
<td>2.56</td>
<td>-.035</td>
</tr>
<tr>
<td>No category</td>
<td>3.09</td>
<td>2.15</td>
<td>-.003</td>
</tr>
</tbody>
</table>

*p<.05

Table 8. *Linear model of predictors of cognitive load.*

<table>
<thead>
<tr>
<th></th>
<th>( b )</th>
<th>SE ( B )</th>
<th>( \beta )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>6.022</td>
<td>.207</td>
<td>( .207 )</td>
<td>( p = .000 )</td>
</tr>
<tr>
<td></td>
<td>(5.613, 6.430)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integration</td>
<td>.121</td>
<td>.049</td>
<td>( .193 )</td>
<td>( p = .015 )</td>
</tr>
<tr>
<td></td>
<td>(.024, .219)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

95% confidence intervals reported in parentheses

**Detailed Data Analysis for Research Question 1**

Research Question 1: How do instructional effectiveness and efficiency vary between online and face-to-face college mathematics courses?

**Assumptions of T-tests**

**Normality.** The assumption of normality is the assumption that requires the dependent variable to be normally distributed. The numerical and graphical methods to assess normality include using the Shapiro-Wilk Test and reviewing Normal Q-Q plots, histograms, and boxplots. If the significance value of the Shapiro-Wilk Test is greater than 0.05, then the data is normal. Also, if the data points are close to the diagonal line on
the Normal Q-Q plots, then the data is normal. In testing for normality of the research data most of the data did not have a significance value of the Shapiro-Wilk Test greater than 0.05, nor were the data points close to the diagonal line on the Normal Q-Q plots. Also, the boxplots showed outliers for most of the data. However, histograms of the data show that the shape of the distributions were similar but the sample sizes of the two factors of the dependent variable are unequal (online, n=159, and face-to-face, n=106). Wilcox (2010) notes that the presence of outliers increases type II error because outliers influence the mean and standard error. Additionally, with unequal sample sizes and non-normal population distributions type I error is not maintained. Therefore, when the data violates the normality assumption a non-parametric bootstrapped procedure could be performed and reported.

**Homogeneity of variance.** The assumption of homogeneity of variance is the assumption that requires the dependent variables to have equal variances. The Levene’s Test assesses homogeneity of variance. A statistically significant Levene’s Test will indicate a violation of homogeneity of variance. If homogeneity of variance is not violated (Levene’s Test: p>.05), the t-test can be interpreted with equal variances assumed. If homogeneity of variance is violated (Levene’s Test: p<.05), the t-test can be interpreted with equal variances not assumed (Ho, 2006).

**Bootstrapping.** Bootstrapping is used as a non-parametric procedure when the normality assumption for using the t-test is not met. Wilcox (2010) explains that bootstrapping produces a random sampling with replacement of the sample data. The sample data essentially becomes the population data and samples are drawn from that data of equal sample size. Each sample provides an estimate of the parameter of interest.
Bootstrapping repeats this procedure of random sampling with replacement of the sample data multiple times and creates an empirical sampling distribution. Therefore, instead of assuming a normal sampling distribution, bootstrapping uses the sample data to create a sample distribution with a bootstrapped mean and standard deviation. Additionally, a bootstrapped confidence interval is created corresponding to the 2.5th and 97.5th percentiles (for $\alpha = .05$). To use with hypothesis testing, “reject $H_0: \mu_1 = \mu_2$, the hypothesis that two groups have equal means, if the [bootstrap confidence interval] does not contain 0” (Wilcox, 2010, p.98).

**Prior Knowledge**

The first hypothesis test determined whether students entered the college algebra course with similar prior knowledge of the math content.

**H1:** There are no statistically significant differences in mean pretest scores between the online and face-to-face students.

For the pretest scores, the Shapiro-Wilk test was significant ($p=.000$) and the boxplot showed evidence of outliers. Therefore, the data was not normal, so the significance of the differences in pretest scores were tested using bootstrapping procedures. The Levene’s test was not significant, ($F(1,198)=.137$, $p=.712$). Without a violation of homogeneity of variance equal variances were assumed.

Differences in pretest scores were computed for 10,000 bootstrapped samples for online students ($M=4.05$, $SD=12.1$) and face-to-face students ($M=3.46$, $SD=13.6$). The 95% confidence interval was computed at the 2.5th and 97.5th percentiles. The bootstrapped 95% confidence interval ranged from -4.258, 4.501. The bootstrapped confidence interval contained the zero, so the differences in pretest scores was not
statistically significant. Online and face-to-face students did not enter the course with statistically significantly different prior knowledge of the math content.

Effectiveness – Performance

The effectiveness between each group of math classes according to the students’ performance on the homework, quizzes and exam was compared.

H2a: There are no statistically significant differences in mean homework performance scores between the online and face-to-face students.

For the homework performance scores, the Shapiro-Wilk test was significant (p=.000) and the boxplot showed evidence of outliers. Therefore, the data was not normal, so the significance of the differences in homework performance scores were tested using bootstrapping procedures. The Levene’s test was not significant, (F(1,263)=.057, p=.812). Without a violation of homogeneity of variance equal variances were assumed.

Differences in homework performance scores were computed for 10,000 bootstrapped samples for online students (M=92.49, SD=9.9) and face-to-face students (M=96.25, SD=5.2). The 95% confidence interval was computed at the 2.5th and 97.5th percentiles. The bootstrapped 95% confidence interval ranged from -5.942, -1.640. The bootstrapped confidence interval did not contain the zero, so the differences in homework performance scores was statistically significant. The homework grades the online and face-to-face students earned were statistically significant, with face-to-face students earning higher homework grades than the online students.

H2b: There are no statistically significant differences in mean test performance scores between the online and face-to-face students.
For the test performance scores, the Shapiro-Wilk test was significant (p=.000) and the boxplot showed evidence of outliers. Therefore, the data were not normal, so the significance of the differences in test performance scores were tested using bootstrapping procedures. The Levene’s test was not significant, (F(1,263)=.313, p=.576). Without a violation of homogeneity of variance equal variances were assumed.

Differences in test performance scores were computed for 10,000 bootstrapped samples for online students (M=82.25, SD=13.8) and face-to-face students (M=85.45, SD=14.5). The 95% confidence interval was computed at the 2.5th and 97.5th percentiles. The bootstrapped 95% confidence interval ranged from -6.742, .375. The bootstrapped confidence interval contained the zero, so the differences in test performance scores was not statistically significant. Online and face-to-face students did not earn statistically significantly different test scores.

Effectiveness – Cognitive Load

The effectiveness between each group of math classes according to their cognitive load was compared. The mental effort measure is a 9-point Likert-type scale, ranging from “1 – very, very low mental effort” to “9 – very, very high mental effort” (Paas, 1992). Since Paas and van Merrienboer (1994a) define mental effort as the amount of cognitive load allocated to learning, cognitive load is calculated from the average of the students’ mental effort scores on their homework.

H3: There are no statistically significant differences in mean cognitive load between the online and face-to-face students.

For the cognitive load, the Shapiro-Wilk test was significant (p=.000) and the boxplot showed evidence of outliers. Therefore, the data were not normal, so the
significance of the differences in cognitive load was tested using bootstrapping procedures. The Levene’s test was not significant, (F(1,263)=2.183, p=.141). Without a violation of homogeneity of variance equal variances were assumed.

Differences in cognitive load were computed for 10,000 bootstrapped samples for online students (M=6.42, SD=1.4) and face-to-face students (M=6.59, SD=1.5). The 95% confidence interval was computed at the 2.5th and 97.5th percentiles. The bootstrapped 95% confidence interval ranged from -.638, .306. The bootstrapped confidence interval contained the zero, so the differences in cognitive load was not statistically significant. Online and face-to-face students did not experience statistically significantly different cognitive load in their math courses.

**Efficiency**

Each student’s relative efficiency, three dimensional relative efficiency, and learner involvement measurement was calculated for each student. The relative efficiency score was calculated by subtracting each student’s mean cognitive load z-score from their mean test performance z-score, then dividing by the square root of two (Paas, F. & van Merrienboer, J., 1993). The three dimensional relative efficiency score was calculated by subtracting each student’s mean homework cognitive load z-score and their mean test cognitive load z-score from their mean test performance z-score, then dividing by the square root of three (Tuovinen, J. & Paas, F., 2004). The learner involvement score was calculated by adding each student’s mean homework cognitive load z-score and their mean performance z-score, then dividing by the square root of two (Paas et al., 2005). Each hypothesis requires a t-test with one fixed-factor independent variable, course type, which compares the online and face-to-face courses.
H4a: There are no statistically significant differences in mean relative efficiency scores between the online and face-to-face courses.

For the relative efficiency scores, the Shapiro-Wilk test was significant (p=.031) and the boxplot showed evidence of outliers. Therefore, the data were not normal, so the significance of the differences in relative efficiency scores was tested using bootstrapping procedures. The Levene’s test was not significant, (F(1,263)=1.251, p=.264). Without a violation of homogeneity of variance equal variances were assumed.

Differences in relative efficiency scores were computed for 10,000 bootstrapped samples for online students (M=-.02, SD=1.1) and face-to-face students (M=.07, SD=.9). The 95% confidence interval was computed at the 2.5th and 97.5th percentiles. The bootstrapped 95% confidence interval ranged from -.395, .217. The bootstrapped confidence interval contained the zero, so the differences in relative efficiency scores was not statistically significant. The online and face-to-face courses did not demonstrate statistically significantly differences in relative efficiency.

H4b: There are no statistically significant differences in mean three dimensional relative efficiency scores between the online and face-to-face courses.

For three dimensional relative efficiency, the Shapiro-Wilk test was not significant (p=.202). Since the data was normal, the significance of the differences in three dimensional relative efficiency was tested using a t-test. The Levene’s test was not significant, (F(1,263)=.015, p=.903). Without a violation of homogeneity of variance the t-test can be interpreted with equal variances assumed.

There was not a significant difference in three dimensional relative efficiency for online students (M=-.02, SD=1.4) and face-to-face students (M=-.02, SD=1.3); t(198)=-
The online and face-to-face courses did not demonstrate statistically significantly differences in three dimensional relative efficiency.

H4c: There are no statistically significant differences in mean learner involvement scores between the online and face-to-face courses.

For learner involvement, the Shapiro-Wilk test was significant (p=.000) and the boxplot showed evidence of outliers. Therefore, the data were not normal, so the significance of the differences in learner involvement was tested using bootstrapping procedures. The Levene’s test was significant, (F(1,263)=5.20, p<.05). With a violation of homogeneity of variance equal variances were not assumed.

Differences in learner involvement were computed for 10,000 bootstrapped samples for online students (M=.01, SD=.8) and face-to-face students (M=.25, SD=.8). The 95% confidence interval was computed at the 2.5th and 97.5th percentiles. The bootstrapped 95% confidence interval ranged from -.501, .019. The bootstrapped confidence interval contained the zero, so the differences in learning involvement scores was not statistically significant. The online and face-to-face courses did not demonstrate statistically significantly differences in learner involvement.

Final Grades

The following hypothesis test will compare final grades of the students in the online and face-to-face course. The students’ final grade is categorized as numerical, continuous data. This hypothesis requires a t-test with one fixed-factor independent variable, course type, which compares the online and face-to-face courses.

H5: There are no statistically significant differences in mean final grades between the online and face-to-face students.
For final course grades, the Shapiro-Wilk test was significant (p=.000) and the boxplot showed evidence of outliers. Therefore, the data were not normal, so the significance of the differences in final course grades was tested using bootstrapping procedures. The Levene’s test was significant, (F(1,263)=4.134, p<.05). With a violation of homogeneity of variance equal variances were not assumed.

Differences in final course grades were computed for 10,000 bootstrapped samples for online students (M=88.24, SD=12.9) and face-to-face students (M=93.33, SD=7.0). The 95% confidence interval was computed at the 2.5th and 97.5th percentiles. The bootstrapped 95% confidence interval ranged from -7.99, -2.27. The bootstrapped confidence interval did not contain the zero, so the differences in final course grades was statistically significant. The difference in final scores online and face-to-face students earned was statistically significant different, indicating the face-to-face students earned higher final course grades than the online students.

**Detailed Data Analysis for Research Question 2**

Research Question 2: How do online college mathematics students demonstrate cognitive presence in a discussion forum?

**Assumptions of Chi-Square tests**

**Random sampling.** Although this research study did not employ random sampling, each student participating in this study could choose whether modality of course selection. The students had an equal chance of taking the math class online or face-to-face. Therefore, the assumption for random sampling has been met.
Independence between observations. Each observation was independent of all of the other observations. There was only one observation per student. The sum of all of the frequency counts equals the total number of students.

No expected frequency under 5. None of the categories had expected frequencies below five.

Cognitive Presence

Students’ stage of cognitive presence was explored by analyzing four discussion forums in the sixteen online classes. In order to explore the students’ stage of cognitive presence, the posts by the course instructors were excluded from this analysis. The four discussion forums were chosen for analysis because they prompted the students to reach the resolution stage of cognitive presence. Two of the discussion forums prompted for resolution by asking students to apply their solution to a real world scenario. One hundred fifty two students and one hundred forty eight students participated in these two discussion forums, for a total of 1050 student posts. The other two discussion forums prompted for resolution by asking students to defend their solution. One hundred forty eight students and one hundred forty five students participated in these two discussion forums, for a total of 1178 student posts. The average number of postings per student was 3.5 in the discussion forums where students were asked to apply their solution and the average number of postings per student was 4.0 in the discussion forums where students were asked to defend their solution.

Cognitive presence was coded according to five categories of the CoI theory: triggering event, exploration, integration, resolution, and no category (Garrison, Anderson, & Archer, 2000). Each discussion post was coded as one unit, and although
each discussion post varied in length, Garrison et al. (2000) advised to code for the stage that is most clear. After creating a summary frequency table for each category, a chi-square statistical analysis was conducted in order to report the findings in a quantitative format.

The second research question employed six Chi-Square hypothesis tests to investigate the five categories of cognitive presence within the four online discussion forums that prompted for resolution. The first three hypotheses employed a one-way Chi-Square Goodness-of-fit test, performed in the Nonparametric, One Sample Chi-Square function in order to determine whether the five categories of cognitive presence met expected frequency counts. In order to prevent SPSS from treating the summaries as raw data, the Weight Cases function was performed on the frequencies. The last three hypotheses employed Chi-Square tests of Independence, performed in the Descriptive Statistics, Crosstabs Chi-Square function in SPSS.

H6a: The five categories of cognitive presence within the four discussion forums are consistent with expected probabilities.

For this one-way Chi-Square Goodness-of-fit hypothesis test the total frequencies of all five categories of cognitive presence across all four discussion forums were combined to determine whether the observed frequency counts of cognitive presence met expected frequency counts. In these four discussion forums, each student was expected to reach the resolution stage of cognitive presence because the discussion prompt asked them to apply or defend their solution. In these four discussion forums there were 2228 student discussion posts. If every student reached resolution in every discussion forum, then 26.7% of the student posts should have reached resolution. The remaining 73.3% of
the student posts would be split equally between the other four categories of cognitive presence. So, 73.3% divided by four equals 18.3% for the no category, triggering event, exploration, and integration categories. The Chi-Square Goodness-of-fit analysis was customized according to those expected probabilities (triggering event = .183, exploration = .183, integration = .183, resolution = .267, and no category = .183).

Because the Pearson $X^2$ was statistically significant, $X^2 (4, N = 2228) = 183.55, p < .05$, we could conclude that there was a statistically significant difference in observed and expected frequencies for the five categories of cognitive presence. The integration category contributed the most to the chi-square statistic, where a large discrepancy is evident with observed frequency is higher than the expected frequency. Additionally, the no cognitive presence category contributed to the chi-square statistic, where the observed frequency is higher than the expected frequency. Two other categories, exploration and resolution, also contributed to the chi-square statistic, where the observed frequency is lower than the expected frequency. The only exception is the triggering event category, where the observed and expected frequencies are consistent with each other. Figure 4 shows cognitive presence across the four discussion forums.
H6b: The five categories of cognitive presence within the Apply discussion forums are consistent with expected probabilities.

This one-way Chi-Square Goodness-of-fit test was performed to determine whether the five categories of cognitive presence met expected frequencies for two of the discussion forums that prompted for resolution, where students are asked to apply their solution to a real world scenario. In these two discussion forums, each student was expected to reach the resolution stage of cognitive presence because the discussion prompt asked them to apply their solution. In these two discussion forums there were 1050 student discussion posts. If every student reached resolution in every discussion forum, then 28.6% of the student posts should have reached resolution. The remaining 71.4% of the student posts would be split equally between the other four categories of cognitive presence. So, 71.4% divided by four equals 17.9% for the no category, triggering event, exploration, and integration categories. The Chi-Square Goodness-of-fit analysis was customized according to those expected probabilities (triggering event = 

Figure 4. Cognitive Presence within the Four Discussion Forums
exploration = .179, integration = .179, resolution = .286, and no category = .179).

Figure 5 shows cognitive presence within the Apply discussion forums.

![Cognitive Presence within the Apply Discussion Forums](image)

Figure 5. Cognitive Presence within the Apply Discussion Forums

Because the Pearson $X^2$ was statistically significant, $X^2(4, N = 1050) = 367.39, p < .05$, we can conclude that the five categories of cognitive presence have statistically significant differences within the discussion forums where students are asked to apply their solution to a real world scenario. The no cognitive presence category contributed the most to the chi-square statistic, where a large discrepancy is evident with observed frequency is higher than the expected frequency. Three other categories, triggering event, integration, and resolution, also contributed to the chi-square statistic, where the observed frequency is lower than the expected frequency. The only exception is the exploration category, where the observed and expected frequencies are consistent with each other.

**H6c:** The five categories of cognitive presence within the Defend discussion forums are consistent with expected probabilities.
This one-way Chi-Square Goodness-of-fit test was performed to determine whether the five categories of cognitive presence met expected frequencies for two of the discussion forums that prompted for resolution, where students had to defend their solution. In these two discussion forums, each student was expected to reach the resolution stage of cognitive presence because the discussion prompt asked them to defend their solution. In these four discussion forums there were 1178 student discussion posts. If every student reached resolution in every discussion forum, then 24.9% of the student posts should have reached resolution. The remaining 75.1% of the student posts would be split equally between the other four categories of cognitive presence. So, 75.1% divided by four equals 18.8% for the no category, triggering event, exploration, and integration categories. The Chi-Square Goodness-of-fit was customized according to expected probabilities (triggering event = .188, exploration = .188, integration = .188, resolution = .249, and no category = .188). Figure 6 shows cognitive presence within the Defend discussion forums.

![Defend Discussions Chart]

Figure 6. Cognitive Presence within the Defend Discussion Forums
Because the Pearson $X^2$ was statistically significant, $X^2 (4, N = 1178) = 537.68, p < .05$, we can conclude that the five categories of cognitive presence have statistically significant differences within the discussion forums where students had to defend their solution. The integration category contributed the most to the chi-square statistic, where a large discrepancy is evident with observed frequency is higher than the expected frequency. Additionally, the triggering event category contributed to the chi-square statistic, where the observed frequency is higher than the expected frequency. Two other categories, exploration and no category, also contributed to the chi-square statistic, where the observed frequency is lower than the expected frequency. The only exception is the resolution category, where the observed and expected frequencies are consistent with each other.

H6d: Resolution and discussion type are independent of each other.

This Chi-Square test of independence was performed to determine whether there was a difference in the proportion of students who reached resolution in the discussion forums where they were asked to apply their solution to a real world scenario compared to when they had to defend their solution. In the two discussion forums where student had to apply their solution to a real world scenario, out of the three hundred students who participated, one hundred ninety-three (64.33%) students reached resolution in their discussion thread. In the two discussion forums where students had to defend their solution, out of the two hundred ninety-three students who participated, eighty-five (29.01%) students reached resolution in their discussion thread. Figure 7 shows students who reached resolution in each discussion forum.
Figure 7. Students who Reached Resolution in each Discussion Forum

Because the Pearson $X^2$ was statistically significant, $X^2 (1, N = 593) = 74.27, p < .05$, we can conclude that there are statistically significant differences between the two types of discussion forums that prompt students to reach resolution. Individual students reached resolution in their discussion thread statistically more often when they are asked to apply their solution than when they are asked to defend their solution.

H6e: Cognitive presence and discussion type are independent of each other.

This Chi-Square test of independence was performed to determine whether there was a difference in the proportions of the five categories of cognitive presence in the discussion forums where they were asked to apply their solution to a real world scenario compared to when they had to defend their solution. In the two discussion forums where students had to apply their solution to a real world scenario, 300 students are represented in the 1050 student discussion posts. Those discussion posts coded according to cognitive presence resulted in the following proportions: triggering event = 10.6%, exploration = 18.0%, integration = 11.4%, resolution = 20.1%, and no category = 39.9%. In the two
discussion forums where students had to defend their solution 293 students are represented in the 1178 student discussion posts. Those discussion posts coded according to cognitive presence resulted in the following proportions: triggering event = 26.8%, exploration = 2.8%, integration = 38.5%, resolution = 25.4%, and no category = 6.5%. Figure 8 shows cognitive presence in each discussion forum.

Because the Pearson $X^2$ was statistically significant, $X^2 (4, N = 2228) = 647.35, p < .05$, we can conclude that there are statistically significant differences between the categories of cognitive presence in the two types of discussion forums that prompt students to reach resolution. The triggering event, integration, and resolution categories occurred statistically more often in the discussions when they are asked to defend their solution than when they are asked to apply their solution. The exploration category occurred statistically more often in the discussions when they are asked to apply their solution than when they are asked to defend their solution. Additionally, there are more instances of students posting without reaching any category of cognitive presence in the
discussions when they are asked to apply their solution than when they are asked to
defend their solution.

H6f: Student posts of cognitive presence and instructor participation are independent of
each other.

This Chi-Square test of independence was performed to determine whether there
was a difference in the proportions of the five categories of cognitive presence in the
discussion forums where instructors participated compared to the discussion forums
where instructors did not participate. In the discussion forums where the instructors
participated, 265 students are represented in the 1120 student discussion posts. The
student discussion posts coded according to cognitive presence resulted in the following
proportions: triggering event = 17.3%, exploration = 10.4%, integration = 22.4%,
resolution = 25.0%, and no category = 24.8%. In the discussion forums where instructors
did not participate, 328 students are represented in the 1108 student discussion posts. The
student discussion posts coded according to cognitive presence resulted in the following
proportions: triggering event = 21.0%, exploration = 9.5%, integration = 29.1%,
resolution = 20.8%, and no category = 19.7%. Figure 9 shows cognitive presence when
instructors participated and did not participate.
Figure 9. Cognitive Presence when Instructors Participated and Did Not Participate

Because the Pearson $X^2$ was statistically significant, $X^2 (4, N = 2228) = 25.104, p < .05$, we can conclude that there are statistically significant differences between the categories of cognitive presence in discussion forums where instructors participated and where instructors did not participate. Triggering event and integration posts occurred statistically more often in the discussions where instructors did not participate. Exploration and resolution posts occurred statistically more often in the discussions where instructors participated. However, there are more instances of students posting without reaching any category of cognitive presence in the discussions where instructors participated than in discussions where instructors did not participate.

**Detailed Data Analysis for Research Question 3**

Research Question 3: What is the relationship between cognitive load and cognitive presence in an online college mathematics course?
Cognitive Load and Cognitive Presence

Because data on cognitive presence had been generated from the discussion forums in the online classes, cognitive load data from the online classes will also only be used in this research hypothesis. Data from face-to-face classes will not be used in this analysis. A void existed in the research literature on understanding the relationship between cognitive load and cognitive presence. Therefore, in order to understand the relationship a first-order multiple regression model relating cognitive load to the five categories of cognitive presence (triggering event, exploration, integration, resolution, and no cognitive presence) was analyzed. Within the multiple regression analysis the first step was to test model significance, that none of the five predictors is effective in explaining variation in cognitive load. Additionally, each of the five predictors was tested whether it adds information about cognitive load to that provided by the other predictors. However, the assumptions of conducting a regression analysis were checked first.

Assumptions of Regression

Continuous variables. Multiple regression assumes that the dependent variable is continuous and two or more of the independent variables are continuous. For this research study the dependent variable, cognitive load, is continuous and the five independent variables are also continuous.

Linear relationship. Multiple regression assumes there is a linear relationship between the dependent variable and each independent variable. Visually inspecting scatterplots and partial regression plots determined that this assumption was met.
**Homoscedasticity.** Multiple regression assumes that the variance of the dependent variable is consistent for all values of the independent variables. Visually inspecting a plot of the residuals against the predicted values have determined that this assumption was met.
**Normality of random errors.** Multiple regression assumes that the residuals to be normally distributed. Visually inspecting the Normal Q-Q Plot of the standardized residuals have determined that this assumption was met.
Independence of random errors. Multiple regression assumes that the residual terms for any two observations should be uncorrelated or independent of each other. The cognitive load of any one student is not dependent upon the cognitive load of any other student. Therefore, the data did not violate the assumption of the independence of random errors. Additionally, this assumption was tested with the Durbin-Watson test (d=2.006), confirming this assumption.

Regression Analysis

Cognitive Load = α + β₁(Triggering Event) + β₂(Exploration) + β₃(Integration) +
β₄(Resolution) + β₅(None) + ε

H7: There is no relationship between cognitive load and cognitive presence.

Multiple regression analysis was used to develop a model for predicting cognitive load from the stages of cognitive presence (no category, triggering event, exploration, integration, and resolution). Scatterplots and Pearson correlations were constructed in SPSS to determine if the relationship is approximately linear between the dependent variable and each independent variable. The only independent variable that was statistically significantly correlated to cognitive load was the integration category. Since the VIF value equaled one, there was no multi-collinearity. Summary statistics are shown in Table 7 and the regression model is shown in Table 8.

The Model Summary table showed that model using integration as the independent variable had significant results. The Pearson correlation, R = .193, showed that there is a weak but positive correlation between the integration category of cognitive presence and cognitive load. The R Square value, .037, showed that integration accounted for 3.7% of the variation in cognitive load. The Adjusted R Square, .031, had a difference
of 0.6% (3.7-3.1) from the R Square. This showed that if the model was derived from the population rather than a sample it would account for approximately 0.6% less variance in cognitive load.

The ANOVA table tested the null hypothesis that the predictors in the model are effective in explaining variation in cognitive load.

\[ H_0: \beta_I = 0; \quad H_1: \beta_I \neq 0 \]

The null hypothesis was rejected, \( F(1, 157) = 6.060, p < 0.05 \). There was statistically significant evidence that the predictor in the model was useful and significant for the fitted model.

The Coefficients table tested the null hypothesis that the integration predictor added no information about Cognitive Load.

\[ H_0: \beta_I = 0; \quad H_1: \beta_I \neq 0 \]

Since \( p < 0.05 \), the null hypothesis is rejected. There is statistically significant evidence that \( \beta_I \neq 0 \), that integration adds information about cognitive load. Additionally, the Unstandardized Coefficient for integration had a positive \( b \) value, 0.121. This indicated that as integration increased, cognitive load also increased.

Therefore, the regression analysis indicated that there is a very weak positive linear relationship between cognitive load and the integration stage of cognitive presence.

\[ \text{Cognitive Load} = 6.022 + .121(\text{Integration}) + \varepsilon \]

Research has shown that instructor-student and student-student interactions vary greatly in online courses (Walker, & Kelly, 2007; Wyatt, 2005) and these interactions impact learning (Zhao et al., 2005; Garrison & Cleveland-Innes, 2005; Hay et al., 2004). However, the results of this study did not demonstrate that the interactions influenced
cognitive load. Although this result did not produce a strong linear relationship between cognitive presence and cognitive load, this study was the first attempt to find a relationship between those two theories. The cognitive load data was sourced from the homework, and the cognitive presence data was sourced from the online discussion forums. Even though the same group of students completed the homework and the discussion forums, the students’ amount of mental effort invested in learning the math content did not influence their stage of cognitive presence in the online discussions. Their cognitive load and cognitive presence may derive from different sources of mental processes. However, further studies are needed to investigate a possible relationship. A different perspective of quantifying cognitive presence may result in a relationship with cognitive load. Studies with larger samples may also produce stronger results. Additionally, reproducing this study using students with different demographics at a different school may influence the results.

Summary of the Findings

This study addressed the differences in cognitive load, instructional efficiency, and learner involvement between online and face-to-face college algebra courses. Additionally, this study investigated how students demonstrated cognitive presence in an online discussion forum and whether there was a relationship between cognitive load and cognitive presence in online and face-to-face college algebra courses. This study also identified the demographics of the face-to-face and online student. The typical online student was a 35 year old, married, Caucasian female. The typical face-to-face student was a 37 year old, Caucasian female. The typical marital status of face-to-face students
could not be determined since those classes consisted of an equal number of married and single students.

**Research Question 1:** How did instructional effectiveness and efficiency vary between online and face-to-face college mathematics courses?

- Online and face-to-face students earned statistically significantly different final course grades, with the face-to-face students (90.97) earning higher final course grades than the online students (88.08).
- Online and face-to-face students earned statistically significantly different homework grades, with the face-to-face students (96.25) earning higher homework grades than the online students (92.49).

**Research Question 2:** How did online college mathematics students demonstrate cognitive presence in a discussion forum?

- There are statistically significant differences in observed and expected frequencies for the five categories of cognitive presence.
  - The integration category contributed the most to the chi-square statistic, where a large discrepancy is evident with observed frequency is higher than the expected frequency.
  - The no cognitive presence category also contributed to the chi-square statistic, where the observed frequency is higher than the expected frequency.
  - Exploration and resolution contributed to the chi-square statistic, where the observed frequency is lower than the expected frequency.
The triggering event category had observed and expected frequencies consistent with each other.

- The five categories of cognitive presence have statistically significant differences within the discussion forums where students were asked to apply their solution to a real world scenario.
  - The no cognitive presence category contributed the most to the chi-square statistic, where a large discrepancy is evident with observed frequency is higher than the expected frequency.
  - Three other categories, triggering event, integration, and resolution, also contributed to the chi-square statistic, where the observed frequency is lower than the expected frequency.
  - The exploration category had the observed and expected frequencies consistent with each other.

- The five categories of cognitive presence have statistically significant differences within the discussion forums where students had to defend their solution.
  - The integration category contributed the most to the chi-square statistic, where a large discrepancy is evident with observed frequency is higher than the expected frequency.
  - The triggering event category also contributed to the chi-square statistic, where the observed frequency is higher than the expected frequency.
  - Exploration and no category contributed to the chi-square statistic, where the observed frequency is lower than the expected frequency.
• The resolution category had observed and expected frequencies consistent with each other.

• There are statistically significant differences between the two types of discussion forums that prompted students to reach resolution.
  
    • Individual students reached resolution in their discussion thread statistically more often when they are asked to apply their solution than when they are asked to defend their solution.
  
• There are statistically significant differences between the categories of cognitive presence in the two types of discussion forums that prompted students to reach resolution.
  
    • Collectively, the triggering event, integration, and resolution categories occurred statistically more often in the discussions when they were asked to defend their solution than when they were asked to apply their solution.

    • Collectively, the exploration category occurred statistically more often in the discussions when they were asked to apply their solution than when they are asked to defend their solution.

    • There are statistically more instances of students posting without reaching any category of cognitive presence in the discussions when they were asked to apply their solution than when they are asked to defend their solution.

• There are statistically significant differences between the student posts in the categories of cognitive presence in the discussion forums where instructors participated compared to discussion forums where instructors did not participate.
• Collectively, the triggering event and integration categories occurred statistically more often in the discussions where instructors did not participate.

• Collectively, the exploration and resolution category occurred statistically more often in the discussions where instructors did participate.

• There are statistically more instances of students posting without reaching any category of cognitive presence in the discussions where instructors participated than in the discussions where instructors did not participate.

**Research Question 3:** What was the relationship between cognitive load and cognitive presence in an online college mathematics course?

• Regression analysis indicated there was a very weak positive linear relationship between cognitive load and the integration stage of cognitive presence.

• Cognitive Load = 6.022 + .121(Integration) + ε
CHAPTER 5. QUALITATIVE ANALYSIS

Cognitive presence was coded according to five categories of the CoI theory: triggering event, exploration, integration, resolution, and no category (Garrison, Anderson, & Archer, 2000). The four stages of cognitive presence include triggering event, exploration, integration, and resolution. The triggering event is the initial stage where the issue, problem, or task is presented to the student. The discourse surrounding the triggering event focuses on the background information and students asking for clarification. “This phase initiates the inquiry process through a well-thought out activity to ensure full engagement and buy-in from the students. This has several positive outcomes in terms of involving students, assessing the state of knowledge and generating unintended but constructive ideas” (Akyol, Z. & Garrison, D. R., 2011, p.236).

Exploration is the second stage where students brainstorm ideas and suggestions. They explore various opinions but it does not lead to further understanding. “This phase focuses first on understanding the nature of the problem and then searching for relevant information and possible explanation” (Akyol, Z. & Garrison, D. R., 2011, p.236).

Integration is the third stage where students connect ideas and integrate information. They offer a tentative solution or hypothesis. “This phase moves into a more focused and structured phase of constructing meaning. Decisions are made about integration of ideas and how order can be created parsimoniously” (Akyol, Z. & Garrison, D. R., 2011, p.236).

The last stage, resolution, is where students defend or test the hypothesis. The students may also apply the solution to other scenarios. “This phase is the resolution of the dilemma or problem, whether that is reducing complexity by constructing a
meaningful framework or discovering a contextually specific solution. This confirmation or testing phase may be accomplished by direct or vicarious action” (Akyol, Z. & Garrison, D. R., 2011, p.236).

**Description of the Sample**

Students’ stage of cognitive presence was explored by analyzing four discussion forums in the sixteen online classes. In order to explore the relationship between the students’ stage of cognitive presence and instructor engagement, the posts by the course instructors were not excluded from this analysis. The four discussion forums were chosen for analysis because they prompted the students to reach the resolution stage of cognitive presence.

Two discussion forums prompted for resolution by asking students to apply their solution to a real world scenario. One hundred fifty-two students participated in the Week 3 discussion, totaling 558 student posts, while the sixteen instructors posted sixty-seven messages. One hundred forty-eight students participated in the Week 4 discussion, totaling 492 student posts, while the sixteen instructors posted fifty-six messages. The other two discussion forums prompted for resolution by asking students to defend their solution. One hundred forty-eight students participated in the Week 5 discussion, totaling 605 student posts, while the sixteen instructors posted eighty-three messages. One hundred forty-five students participated in the Week 6 discussion, totaling 573 student posts, while the sixteen instructors posted sixty-one messages. Across the four discussion forums in the sixteen online classes, this research project coded and analyzed sixty-four discussion forums with 2228 student posts and 267 instructor posts, totaling 2495 messages. Table 9 outlines the descriptors, instructor indicators, and student indicators of
cognitive presence in this research study. Indicators were provided by the Community of Inquiry theory (Garrison, Anderson, & Archer, 2000).

Table 9: Descriptors, Instructor Indicators, and Student Indicators of Cognitive Presence

<table>
<thead>
<tr>
<th>Stage (Descriptor)</th>
<th>Instructor Indicator</th>
<th>Student Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triggering Event</td>
<td>Discussion prompt</td>
<td>Recognize problem</td>
</tr>
<tr>
<td></td>
<td>Assessing their state of knowledge</td>
<td>Puzzlement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Assessing their state of knowledge</td>
</tr>
<tr>
<td>Exploration</td>
<td>Asking questions</td>
<td>Divergence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Information exchange</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Suggestions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Brainstorming</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Intuitive leaps</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Understanding the nature of the problem</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Searching for relevant information</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Searching for possible explanations</td>
</tr>
<tr>
<td>Integration</td>
<td>Answering questions</td>
<td>Convergence</td>
</tr>
<tr>
<td></td>
<td>Posting the answers</td>
<td>Agreement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Synthesis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Decision is made</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Posting the answer</td>
</tr>
<tr>
<td>Resolution</td>
<td>Explaining the solution</td>
<td>Applying the solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Defending or testing the solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constructing a meaningful framework</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discovering a contextually specific solution</td>
</tr>
<tr>
<td>No Category</td>
<td>Encouragement</td>
<td>Encouragement</td>
</tr>
<tr>
<td></td>
<td>Off topic responses</td>
<td>Off topic responses</td>
</tr>
</tbody>
</table>

**Coding Decisions**

During the coding phase of the qualitative analysis, the researcher made conscientious decisions regarding how to code the data. Each discussion post was coded
as one unit, and although each discussion post varied in length, Garrison et al. (2000) advised to code for the stage that is most clear. However, sometimes two stages were clear. For example, a student posted a response to a classmate,

   My friend uses Rice Chex instead of Rice Krispies. I’ve also used Fruit Loops before. They were okay if you like the fruity option. I didn’t even think of the stock market for signed numbers. People use signed numbers all the time to see if they want to buy or sell. Great job.

This message contained messages that did not show any stage of cognitive presence when she wrote about different kinds of cereal to use in a recipe. However, the message also showed the exploration stage because she searched for relevant information and possible explanations of using signed numbers. Although the message included off topic responses, the researcher coded this message as exploration. Whenever a post included messages that did not demonstrate a stage of cognitive presence and messages that did demonstrate a stage of cognitive presence, the message was coded for the cognitive presence stage.

   This research study did not employ a qualitative analysis software program nor a colleague to code the data generated from the discussion forums. However, after coding the first two discussions, the researcher reflected upon the descriptors and indicators of cognitive presence to form concrete examples of each stage of cognitive presence. Reviewing and recoding the data occurred intentionally and thoughtfully. The researcher decided upon the final code after reviewing each message at least three times.

   In order to explore the relationship between the students’ stage of cognitive presence and instructor engagement, the posts by the course instructors were not
excluded from this analysis. The relationship between instructor engagement and cognitive presence will be analyzed later in this chapter. Although the posts of non-consenting students were excluded from analysis, the follow-up posts of consenting students were not excluded. Because these consenting students replied to a non-consenting student, these messages could not be analyzed for themes regarding how students reach the resolution stage. However, these posts were coded to aid in understanding each stage of cognitive presence.

A few students attached a word document of their response to a discussion forum prompt, instead of writing in the discussion forum. However, the researcher lost access to the courses after a couple weeks. In the Apply discussions, it is impossible to know or assume what the students posted, so these messages were not coded and were excluded from analysis. In the Defend discussions, these documents either reached integration or resolution because the students said they were attaching their answers or solutions. Therefore, these messages were coded as reaching the resolution stage. If the messages had only reached the Integration stage (e.g., \( x = 3 \)), then the students could have easily typed in the answers into the discussion forum.

All of the discussion prompts asked students to answer multiple parts. If a student did not answer the part of the discussion prompt that prompted for resolution, the researcher had to assume (1) that the student either did not know or understand what to do, or (2) that the student did not want to answer that part, but (3) that the student still wanted to get some credit for posting. Therefore, those messages were coded as a triggering event. Additionally, if a student posted an answer to a few parts of the prompt
and then created another post answering the other parts of the prompt, the researcher counted those two posts as one message.

The researcher used Excel to organize the coding for each college algebra class. Table 10 provides an example of this organization. The first row, next to “Section-41” identifies the stage of cognitive presence each student reached in his or her initial post. Each response was coded according to the categories of cognitive presence. If that student posted another response with his or her own discussion thread, it was underlined. If an instructor posted a response, it was italicized. For example, the second student (column R) to post a response reached resolution. This student post elicited four responses. The first response reached the exploration stage of cognitive presence. The second response came from the instructor, and it also reached the exploration stage. The original student replied to the instructor in the triggering event stage, coded T. The third response was also the original student, reaching the integration stage. The fourth response reached the exploration stage, but it also equally conveyed messages that did not meet any stage of cognitive presence.

Table 10: Example of Coding for One Section
Examples of Student Posts

Student posts from four discussion forums in the sixteen online college algebra courses were used in this research project. These discussion forums were chosen for analysis because they prompted the students to reach the resolution stage of cognitive presence. The resolution stage is where students apply their solution to other scenarios or where students defend or test their hypothesis. The first two discussion forums prompted each student to reach resolution stage by applying a math concept to a real life scenario. These discussions were identified by the research as the Apply discussions. The last two discussion forums prompted each student to reach resolution stage by giving the student an opportunity to defend or test their hypothesis. These discussions were identified by the research as the Defend discussions. Students’ stage of cognitive presence was coded according to the descriptors and indicators described in Table 9 (Garrison, Anderson, & Archer, 2000).

Triggering Event

The triggering event is the initial stage where the issue, problem, or task is presented to the student. The discourse surrounding the triggering event focuses on the background information and students asking for clarification. Nineteen percent of the student posts reached the triggering event stage. Since the triggering event is the initial stage where the task is presented, the discussion prompts usually served as the triggering event. In the Apply discussions students were asked to apply their solution to a real world scenario. Therefore, the only posts reaching the triggering event stage would be focused on students assessing their state of knowledge. For example,
Bourbon chicken sounds delicious! I never thought about sky divers and calculations below/above the surface. I also did not think of measurement as a source of using negative integers when I did this assignment. Good job!

These triggering event posts could also be negative comments as they assess their knowledge. For example, “…I don’t understand the point of this discussion topic. I thought the point of the course was to teach us about math…”

In the Defend discussions students were asked to explain and defend their solution. In these discussions the discussion prompt was coded for analysis since the discussion prompt was not embedded into the existing curriculum in the learning management system. Students were asked to create and post three math problems for their classmates to answers. The initial student posts were supposed to reach the triggering event stage. Out of the two hundred ninety-three initial posts (two hundred ninety-three students), two hundred eighty-nine of the posts were triggering events. For example, the Week 6 discussion prompt read “Develop three problems that can be solved using FOIL. For your initial post, post your three problems without the solutions.” One example of a students’ initial post is “1. (8x+7)(2x-2) 2. (x+11)(5x+6) 3. (24x-34)(14x+10)”.

**Exploration**

Exploration is the second stage where students brainstorm ideas and suggestions. They explore various opinions but it does not lead to further understanding. Ten percent of the student posts reached the exploration stage. In the Apply discussions the exploration stage was demonstrated by students asking questions, providing suggestions, or commenting on the nature of the math problem. For example,
I guess I went overboard with my explanation I thought we needed a lot of information. Your example was great. How hard or complicated are variable to you? What math have you found that you like if any?

In the Defend discussions the exploration stage occurred in only three percent of the posts. In these messages students tried to understand and solve the problems posted by their classmates. Similar to the Apply discussions, the students reaching the exploration stage asked questions, provided suggestions, or commented on the nature of the math problem. For example, a student asked a classmate, “When you simplified did you take out numbers? Please let me know. Thanks!”

**Integration**

Integration is the third stage where students connect ideas and integrate information. They offer a tentative solution or hypothesis. “This phase moves into a more focused and structured phase of constructing meaning. Decisions are made about integration of ideas and how order can be created parsimoniously” (Akyol, Z. & Garrison, D. R., 2011, p.236). Twenty-six percent of the student posts reached the integration stage. In the Apply discussions, integration occurred when a student offered a real-life example, but did not explain the context or framework of the example. For example,

The real-life scenarios of negative integers would be weather, weight loss and in many jobs. For me I deal with negative numbers on a daily basis with my companies’ balance sheet.

In the Defend discussions, integration occurred when a student posted answers to a classmate’s math problem, but did not post the solution (show or explain the work). For
example, five days after a student posted her three math problems for her classmates to solve, she posted the answers, “the solutions 1) $9 + \sqrt{3}$, 2) $-7x^3 + 11y^2 + 3xy$, 3) $x^2 + 22x + 5y$”. This post was coded for integration.

**Resolution**

In the last stage, resolution, students defend or test their hypothesis. The students may also apply the solution to other scenarios. “This phase is the resolution of the dilemma or problem, whether that is reducing complexity by constructing a meaningful framework or discovering a contextually specific solution” (Akyol, Z. & Garrison, D. R., 2011, p.236). Twenty-three percent of the student posts reached the resolution stage. In the Apply discussions, resolution occurred when a student posted a real-life example and also explained the context or framework of the example. Resolution is supposed to occur in the student’s initial post since the discussion prompt asked them to “Identify three real-life scenarios that call for the use of negative integers” or “Post a real-life example where you might need to solve a linear equation”. For example,

Due to the recent cold weather I have experienced this winter there have been negative wind chill and negative temperatures. If a person’s bank overdraws on their bank account they will end up with a negative integer, meaning they have gone into the negative on their account. In the game of football if the offensive team has a first and ten and the quarterback hands the ball off the running back and the runner does not cross the line of scrimmage and losses yards instead of gaining yards. The running back has created a negative integer or negative yards. (Student J)
In the Defend discussion, resolution occurred when a student posted answers and solutions to a classmate’s math problem. Resolution is not supposed to occur in the student’s initial post. The discussion prompt says “By the end of the workshop you should post the solutions to your three equations. Indicate whether or not a classmate's solution to your equation is correct. Defend your solution versus their solution.” Therefore, resolution is supposed to occur in their second or third post. For example, two days after Student K posted her three math problems for her classmates to solve, she posted her solutions showing the resolution stage.

Answers are:
1. \( \sqrt{300} = \sqrt{100 \times 3} = \sqrt{100} \times \sqrt{3} = 10 \sqrt{3} \)
2. \( 5\sqrt{4} + \sqrt{16} = 5\sqrt{4} + \sqrt{4^2} = 5\sqrt{4} + 2\sqrt{4} = 7\sqrt{4} \)
3. \( \frac{4}{\sqrt{x^2}} = \frac{2}{x} = x^{\frac{1}{2}} \)

No Category

Twenty-two percent of all student posts did not reach any stage of cognitive presence. These messages either provided encouragement or acknowledgement to their classmates, or they were tangential conversations off of the discussion prompt. Here are examples of both types of messages: “This is very helpful. I always just guess about how much gas it will take. I am going to use this on my next trip. Great post” (Student W). “I will have to agree with you wholeheartedly I love pumpkin bread and I love chocolate. I did not even think of it till I read this recipe. It is definitely on my must try list now” (Student G).

Interestingly, forty percent of the Apply discussion student posts were coded for no category, but only seven percent of the Defend discussion student posts were coded
for no category. In the Defend discussions the student posts coded for no category were usually short messages of acknowledgement. For example, “Good job, Student T!”

**Examples of Instructor Posts**

**Triggering Event**

The discourse surrounding the triggering event focuses on the background information through “a well-thought out activity to ensure full engagement and buy-in from the students” (Akyol, Z. & Garrison, D. R., 2011, p.236). Only two percent of the instructor posts reached the triggering event stage. Since the triggering event is the initial stage where the issue, problem, or task is presented, the discussion prompts usually served as the triggering event. In the Apply discussions the discussion prompt was not coded for analysis since the discussion prompt was embedded into the existing curriculum in the learning management system. However, there were still instances of instructors posting in the triggering event stage because they were assessing their own state of knowledge. For example, “I didn’t think of negative yardage with football! Good job!” Another example of instructors posting in the triggering event stage occurred when they directed their students’ attention back to answering the discussion prompt: “Great job! Be sure to address the most important function of signed numbers. I like the way you neatly arranged your work.”

In the Defend discussions the discussion prompt was coded for analysis since the discussion prompt was not embedded into the existing curriculum in the learning management system. In these discussions students created and posted three math problems for their classmates to solve. Therefore, these posts were coded as students reaching the triggering event stage, not for instructors reaching the triggering event stage.
However, there were still instances of instructors posting in the triggering event stage because they corrected their students’ original three math problems: “Very good. In equation 1 I think you meant $x^2 - 4 = 0$”. Another example of instructors posting in the triggering event stage occurred when they directed their students’ attention back to answering the discussion prompt: “You guys are doing a good job of working through your peer’s problems. Do not forget to post the final answers to your 3 problems by Tuesday evening.”

**Exploration**

Exploration is the second stage where students brainstorm ideas and suggestions. Therefore, instructor posts in the exploration stage occurred when instructors asked their students questions or guided them to search for relevant information or possible explanations. Thirty-three percent of the instructor posts reached the exploration stage. Instructors posted in the exploration stage more often than the other three stages of cognitive presence. It was only eclipsed by instructor posts that did not reach any category of cognitive presence (39%).

Instructors posted in the exploration stage in order to provide guidance to their students. They wanted to help their students understand the nature of the problem. Instructors provided this guidance via the exploration stage in two different ways. First, they explained the nature of the problem to their students. For example,

You did a great job with your response. The recipe that you posted sounds just as good as the spaghetti recipe that your classmate posted. By the way, how did you find your conversions? You are correct in stating the signed numbers are used in many different ways. One useful function of negative numbers is to illustrate a
change in direction. Thanks for the examples that you provided and keep up the
good work.

For another example,

You did a good job with your responses, however, all problem #3 is an equation,
as the first two problems are expressions. An equation should include and equal
sign between two expressions. Don’t worry about changing the problems, let’s see
if your classmates can simplify them. Thanks and hang in there.

Secondly, instructors wanted to help their students understand the nature of the
problem by asking direct and specific questions.

It did work! Good explanations. In (f), using your $100 budget, can you write that
equation for us? I will give you a few more data points. Let’s say you need to buy
two items for the party: hotdogs/buns combination and pizza slices. Hotdog/Buns
are 87c each, pizza slices are 1.05. How would you organize this info into a linear
equation? Thank you.

**Integration**

Integration is the third stage where students connect ideas, integrate information,
and offer a tentative solution. Although only eighteen percent of the instructor posts
reached the integration stage, these posts are also integrate information and provide
solutions. In the Apply discussions most instructor posts that reflected a level of
integration offered an example post to guide students in answering the discussion prompt.

For example,

For the discussion this week, most of the equivalent measurements between US
system and metric system can be found of slide 18 of Measurements
presentation... Keep the recipe simple and only include the ingredients and measurement parts. Here is mine: French Éclair shell. 100g butter – 6 tbsp butter. 100g water – 1/3 cup + 4 tsp water. 100g flour – 2/3 cup + 2 tbsp flour. 2 eggs.

In the Defend discussion, most of the instructor posts that reached the integration stage either provided solutions to the students’ math problems or confirmed whether the posted answers were correct or incorrect. “Hi Student K, you did all three problems correctly. Thank you so much!”

**Resolution**

The last stage, resolution, is where students construct a meaningful framework or discover a contextually specific solution. Instructor posts that reached resolution also either constructed a meaningful framework or provided a solution. Only nine percent of the instructor posts reached the resolution stage. In the Apply discussions only one of the instructor posts reached resolution. “Hi, when we deposit money into savings account, it represents the positive number. When we withdrawal money from saving account, it represents the negative number.”

In the Defend discussions most of the instructor posts that reached resolution provided a solution (showing steps and work) for a student.

Great job! Sure: put all your “x” terms on the left side of equal sign and all your non-“x” terms on the right side to get: \( \frac{x}{4} + 2x = \frac{15}{2} - 2 \). Then: \( \frac{x}{4} + 8x/4 = \frac{15}{2} - \frac{4}{2} \) (having common denominators makes it easier to add/subtract the fraction). Then: \( 9x/4 = 11/2 \)… Continuing from above gives you: \( x = 22/9 \).
No Category

Thirty-nine percent of all instructor posts did not reach any stage of cognitive presence. These messages either provided encouragement to their students or they were tangential conversations to the discussion prompt. For example, an instructor responded to a student, “Hi, wow! Your steps are very detailed. Thank you for taking your time to explain this problem very thoroughly.” In a second example, another instructor replied to one of her students,

Those beignets sound very good right about now. I have not visited New Orleans in several years, so it has been a while since I had some, however, I may be able to make my own with that simple recipe that you provided. You also did a good job of describing situations where signed numbers are used. Thanks and keep up the good work!!

Research Question 2

Research Question 2: How do online college mathematics students demonstrate cognitive presence in a discussion forum?

Two quantitative data analysis techniques, hypothesis tests H6 and H7, were useful in understanding some aspects of cognitive presence. However, a deeper understanding of how students demonstrated cognitive presence in an online discussion forum required a thematic qualitative data analysis. Because of the complexities and culture in online classroom discussions, qualitative research allowed for deep analysis of these discussions. The thematic qualitative data analysis uncovered themes of how online college mathematics students demonstrated cognitive presence in discussion forums
where they were prompted to reach resolution and in discussion forums where the instructors were engaged.

**Apply or Defend your Solution**

Research has shown presence of high cognitive thinking in online discussions which involved a real world scenario (Bradley et al., 2008; Weeks et al., 2013), brainstorming ideas or solutions to an issue (Bradley et al., 2008), debates (Kanuka et al., 2007), collaboration (Stahl, 2006, 2010; Zhou et al., 2008), WebQuests (Kanuka et al., 2007), and discussion questions involving metacognitive strategies (Mevarech & Amrany, 2008). Additionally, providing an example post to the students, and limiting the number of posts displayed on each page encouraged student engagement and schema construction (Darabi & Jin, 2013).

However, these researchers compared discussion forum strategies that may or may not have prompted for the resolution stage of cognitive presence. The four discussion forums used in this research project were chosen for analysis because they prompted the students to reach the resolution stage of cognitive presence. The resolution stage is where students apply their solution to other scenarios or where students defend or test their hypothesis. Two discussion forums prompted each student to reach the resolution stage by asking them to apply a math concept to a real life scenario. The other two discussion forums prompted each student to reach resolution stage by asking them to defend or test the solutions to their math problems.

Throughout the following section each category of cognitive presence (triggering event, exploration, integration, resolution, and no category) was explored and analyzed within the two different types of discussions forums (apply your solution and defend your
solution). A series of sub-questions guided the analysis. (1) What aspects of the two discussion forum strategies engaged the students in reaching the stages of cognitive presence? (2) What aspects of the two discussion forum strategies increased cognitive presence within a discussion thread? (3) What were the expected and unexpected patterns of response cycles within a discussion thread and across the two discussion forum strategies?

**Triggering event: Apply your solution.** In the discussion prompt for the Apply discussions, students were prompted to reach resolution in their initial post. Although most initial posts reached resolution, eleven percent of initial posts only reached the triggering event stage. This occurred when a student skipped the part of the discussion prompt that asked them to apply the solution. For example, the week 3 discussion prompt asked students to (1) convert a recipe from U.S. units of measurement into metric units, (2) discuss the importance of signed numbers, and (3) identify three real-life scenarios that call for the use of negative integers. Student S posted in the triggering event stage because she converted the recipe for her grandma’s banana bread, but did not discuss the importance of signed numbers or identify three real-life scenarios that call for the use of negative integers. A classmate, thereby, did not post in any category of cognitive presence when she said “Banana bread is a great recipe to use! I always make sure to grab some at Lynds Fruit Farm when I visit the pumpkin patch yearly.”

In every instance where the initial post only reached the triggering event stage, the resulting follow-up posts did not extend beyond the exploration stage, with two exceptions to this pattern. In the first exception, Student S did not identify a real-life
example of solving a linear equation in her initial post. Her instructor responded in the integration stage by providing examples,

Hi Student S, There are many cases that we can use a linear equation in our life. For example, investing money (savings account), traveling distances, mixing two solutions, renting a rental car, comparing cell phone plans,…etc. are all required to solve the linear equation. It is very practical and useful in our real life. Thank you. (Instructor C)

Although her instructor provided some examples that she could explain, Student S only responded in the exploration stage by posting how she understood the nature of the problem. “I am just noticing that I didn’t answer that last question, oops! I agree there are many linear equations dealing with real life situation.” Her instructor could have asked her to provide a real-life scenario, but he only responded by saying “Hahaha! That’s what I thought. Sometimes I forgot some problems too” (Instructor C).

The second exception occurred when Student C vented his frustration in not being able to make a real-life application. He ended his post by saying “I am looking forward to the discussion post of the rest of the class to hopefully be able to have a better understanding of how this would apply in daily life outside of class.” Thankfully, one of his classmates responded to him in the integration stage.

Interesting post. I understand that sometimes it's hard to put something into real-world examples when we feel that we don't use it. We all use algebra and don't realize it. I use it a lot when I pay my bills or try and figure out a budget for the next few months. As a finance manager I'm sure that you're used to dealing with
budgets and what percentages of money go into which accounts. It may not seem like algebra, but you do it every day. Have a great day.

In both of these instances a classmate or instructor provided a real-life application. However, the students never followed-up by applying a real-life scenario, thereby reaching resolution or even integration. In the Apply discussions none of the students that started in the triggering event stage ever reached integration or resolution within their discussion thread.

Other than as the initial post, students posted in the triggering event stage as follow-up responses. In reality, the triggering event stage of cognitive presence was skipped in the Apply discussions unless a student posted an assessment of their state of knowledge. The follow-up responses meeting the triggering event stage resulted in responses reaching no category or exploration. For example, Student T’s initial post met the triggering stage because he did not respond to the discussion prompt asking for real life examples. Student C responded,

By putting yourself in a position to be able to teach how to solve a word problem involving that of a linear equation it shows that you understand how to solve them fully. It's like learning anything for that matter, you don't understand it fully until you can teach someone else how to do it. It's just a perspective discussion to see how much you understand about linear equations and your ability to explain it thoroughly, the objective of most discussions are to discuss what you have learned through the homework and resources available.

As a follow-up response to Student C, Student T confessed his lack of understanding.
Thanks Student C, I know I still have a lot to learn about linear equations and I still don’t understand them well enough to be able to teach anyone else about them. I can follow steps provided but I do not always get the explanation behind the steps, even after doing them several times.

Student C provided a suggestion, posting in the exploration stage, “Try to outsource for help, I use Khanacademy.com sometimes for explanations or YouTube as I find videos very helpful for me.” Student T did not respond to Student C. In the Apply discussions none of the follow-up posts in the triggering event stage ever resulted in students reaching integration or resolution within that discussion thread.

**Triggering event: Defend your solution.** In the Defend discussions, the discussion prompt intended students to reach the triggering event stage in their initial post because the discussion prompt asked students to post three unsolved math problems. The vast majority of initial posts followed the directions in the discussion prompt, thereby reaching the triggering event stage. The four initial posts that did not reach the triggering event stage either reached the integration or resolution stages because the students posted the answers or solutions to their three problems.

For example, in one instance an initial post reached integration because the student posted three problems with their answers. Student T posted,

1. \(7(4y - 6) + 3(4y + 5)\). Answer is = 40y - 27.

2. \(-8(n + 4) + 36 = -5n - 8\). Answer is = 4.

3. \(7^2 - [14 - (72 ÷ (6 x 2) + \sqrt{9}) + 6^2]\). Answer is = 8.

Even though Student T posted her answers, a classmate attempted to work out the solution to one of her problems. Student S responded in the resolution stage,
Hi, I will be doing question 1: $7(4y-6) + 3(4y+5)$. First, I will multiply $(7) (4)$ and $(7) (-6)$, which comes out to be $28y - 42 + (3) (4) + (3) (5)$. So now I have $28y - 42 + 12y + 15$. Next, I’m going to add $28y + 12y = 40y$. Then solve the other half by $-42 + 15 = -27$. My final answer = $40y - 27$.

Her instructor reached integration by commenting on the accuracy of Student S’ answers, “Hi Student S, thank you for showing all steps. You did the problem correctly. Good job!” Thereby, even though Student T posted her answers, it did not result in lost potential of students reaching resolution.

If the initial posts were removed from analysis, only three percent of the posts in the Defend discussions met the triggering stage. These posts were not ‘I don’t know how to solve or defend’ responses, as opposed to the majority of the triggering event posts in the Apply discussions were ‘I don’t know how to apply’. The triggering event posts in the Defend discussions focused on correcting mathematical mistakes in the initial triggering event stage post. For example, Student C had posted “1. $x^2 - 4 = 0$…” as his initial post, but then corrected an error in that initial post by posting a reply “The first problem should read: $x^2 - 4 = 0$”.

**Exploration: Apply your solution.** In the Apply discussions, the discussion prompt intended students to reach resolution in their initial post because it asked students to post a real life scenario of a math concept. There was two main observations about the explorations stage of cognitive presence in the Apply discussions. First, exploration was not the least occurring stage of cognitive presence in the Apply discussions. Even though eighteen percent of the total posts reached exploration, only twelve percent reached integration.
Secondly, when students were asked to apply a solution to a real life scenario, the data showed students to either skip that part of the prompt (triggering event), post a real life scenario but not explain it (integration), or post a real life scenario with an explanation (resolution). There was not room for exploration to occur with a discussion prompt that asked students to apply a solution. Therefore, none of the initial posts reached the exploration stage, and all of the posts reaching exploration stage occurred as follow-up posts.

The exploration stage was demonstrated by students in three different ways. Students demonstrated exploration by asking questions, providing suggestions, or commenting on the nature of the math problem. When students asked questions, they created an atmosphere where an answer was expected, even though only twelve percent of the exploration posts asked questions. For example, in Student S’s initial post his real life application was “I think that in the kitchen you can use a linear equation to increase or decrease a recipe for the amount you’d like to make.” Although his post met the requirements of resolution, a classmate replied, “Would you like to provide an example of the linear equation that you would use to increase or decrease a recipe?” Student S responded with clearer resolution,

You take your desired servings and divide it by the recommended servings, such as 4 desired servings divided by 10 recommended servings. This will give you the number to multiply all of the other ingredients against to get the decreases portion needed to complete the recipe. \( \frac{4}{10} = .4 \). 2 cups of milk for 10 servings: \( 2 \times .4 = .8 \) cups for a 4 serving portion. 1 tablespoon of vanilla for 10 servings: \( 1 \times .4 = .4 \) tablespoon for a 4 serving portion. So on and so on.
The classmate replied,

Great example, it really did make it clearer for me to understand, ask my wife, I don't cook much, and I have hardly ever used a recipe to make a meal. Now I have an idea what I would need to do to scale a recipe up or down. Thank you!

The exchange between these classmates showed the only instance where a classmates’ specific question helped a student reach resolution. In half of the instances where a student posed a question to his or her classmate, the question was not answered. For example, Student E posted her initial post without an example of a real life scenario. Her classmate, Student B, responded by asking questions about her post,

Banana Nut bread is a huge weakness for me. I am taking this recipe and will try & make soon. Yummy! Will you also be adding to your posting with examples you found for negative integers? I read Sunny's and had the same ones I noted so I tried to branch out and think of new ones. What did you come up with? Thanks for sharing the recipe.

Although Student B asked Student E a specific question about the absence of her real life application, Student E ignored the questions and responded without any cognitive presence, “This is a delicious recipe. I can get you the copy without the metric versions as well. The key is to make sure the bananas are really ripe.”

In only one instance exploration increased cognitive presence in the follow-up posts. Student A skipped the part of the discussion prompt that asked them to apply the solution. Her classmate, Student C replied in the exploration stage,
Do you think these linear equations are something you would use in real day math? My problem is I don't use this kind of math every day at my job, hard to make myself understand it again at the age of 32.

Student A responded in the exploration stage, “I don't know if I would use linear equations in my career or every day. I don't use them now at home or at work. I think this is why it is so hard to comprehend certain equations.” Although Student A’s initial post met the triggering event stage of cognitive presence, her response to Student C showed willingness to explore possible real life applications. Student A’s post met the exploration stage of cognitive presence, but she did not reach integration or resolution. The conversation ended and Student A never reached beyond exploration. These data showed how students did not capitalize on an exploration stage post, and it did not extend the progression of cognitive presence.

**Exploration: Defend your solution.** Similar to the Apply discussions, none of the initial posts in the Defend discussions reached the exploration stage. In the initial posts students either posted the three unsolved math problems (triggering event), posted the problems and the answers (integration), or posted the problems and the worked out solutions (resolution). There was not any reason or purpose for exploration to occur in the initial posts for the Defend discussions.

In fact, exploration was the least occurring stage of cognitive presence in the Defend discussions. Only three percent of the total student posts in the Defend discussions reached exploration. All of the posts reaching exploration stage occurred as follow-up posts. The exploration stage was demonstrated when students asked specific questions trying to understand the problems posted by their classmates, provided
suggestions to their classmates, or commented on the nature of the math problems.

Unexpectedly, only one post showed a student posting in the exploration stage by asking a question. Student J posted three math problems in her initial post,

1) \((4x^3 + 2x^2 + 1x - 15) - (10x^3 + 6x^2 - 4x + 15)\)

2) \((5x + 6) (5x^2 + 4x - 6)\)

3) \(3\sqrt{729}\)

A classmate, Student T, responded in the integration stage,

1. \((4x^3 +2x^2 + 1x - 15) - (10x^3 + 6x^2 - 4x + 15) = -6x + 8x^2 - 3x\)

2. \((5x + 6) (5x^2 + 4x - 6) = 9x + 5x^2\)

3. \(3\sqrt{729}\). I am still working on this one.

The next day Student T posted again, “\(3\sqrt{729} = 729^{1/3}\).” Student J provided resolution to Student T for the third problem, “In this case the answer is 9 because 9\(\times\)9\(\times\)9 = 729. Thanks!” However, Student J also followed-up with an exploration post, “Student T, when you simplified did you take out numbers? Please let me know. Thanks!” Student T did not respond, never answering Student J’s question. Student exploration stage posts were not an important stage of cognitive presence in the Defend discussions.

There were not any instances of exploration increasing cognitive presence in the follow-up posts of the Defend discussions. For example, Student R posted three problems that needed to be foiled or factored,

1. \((x + 5) (x + 3)\)

2. \(6x^2 + 4x - 16\)

3. \((x - 9) (x + 7)\)
A classmate posted his answer to the second problem, “2. \(6x^2 + 4x - 16 = (2x + 4)(3x - 4)\).” Student R replied, “Before you start factoring the problem, all numbers share a common factor that needs to be pulled out first. Then you would factor. Or if you look at the factored form one of them can be simplified further.” Although Student R used the exploration stage to provide a suggestion to his classmate about his post, the classmate did not respond and did not reach resolution. This exchange showed an exploration stage post, but it did not extend the discussion or extend the progression of cognitive presence. The data in the Apply or Defend discussions showed that student posts in the exploration stage did not play an important or consistent role in the progression of cognitive presence.

**Integration: Apply your solution.** In the Apply discussions, the discussion prompt intended students to reach resolution in their initial post because it asked students to post a real life scenario of a math concept. However, some students posted a real life scenario but did not explain how it was an example of a negative number or an example of a linear equation. The data showed that twenty-five percent of the initial posts only reached integration. In the Week 3 discussion students were asked to post a recipe converting U.S. measurements to metric measures, discussed the importance of signed numbers, and also posted three real-life scenarios that call for the use of negative numbers. For example, Student G reached integration because in her initial post she identified three real-life scenarios that call for the use of negative numbers, “Checkbook registers, Company bookkeeping, Stock market”. She also posted a conversion of her recipe for chocolate chip banana bread and discussed the importance of signed numbers, “Signed numbers help you to keep things in perspective so that you can make things in a
regular order the same way every time they help to keep you on track”. Student G did not reach resolution in her initial post because she did not explain how negative numbers were used in a checkbook registry, company bookkeeping, or the stock market. She needed a classmate or instructor to ask her how those examples used negative numbers. Unfortunately, only one classmate, Student T responded to her in the exploration stage.

That sounds like a good twist on pumpkin bread, love chocolate chips. Having an order to the way we do things is important to most of us, once we figure out how to do something then we work on ways to make it work better for us. Kinda like the recipe you provided, pumpkin bread is good, when you add chocolate chips that makes it better.

Student G responded to her classmate without any category of cognitive presence, “I will have to agree with you wholeheartedly I love pumpkin bread and I love chocolate. I did not even think of it till I read this recipe it is defiantly on my must try list now.” Although Student T could have discussed Student G’s examples of negative numbers, their exchange demonstrated a lack of resolution.

Individually, the same twenty-six percent of students who reached integration in their initial post were the same twenty-six percent of students who reached integration within their Apply discussion thread. The data does not show any instances of students reaching the integration stage who did not already reach it in their initial post. Even though eleven percent of the initial posts only reached the triggering stage, none of those students reached integration throughout the course of the discussion.

Additionally, there is only one instance of a student initially posting in the integration stage, but eventually meeting resolution. This occurred when the instructor
responded to her initial post, asking specific questions about the students’ real life scenario. Student D’s initial post met the integration stage because she posted that driving under the speed limit was a real life example of using negative numbers, but Student D did not explain how or why. Instructor M responded by asking a specific question, “Hi Student D. How would you use negative numbers for “Driving under the speed limit”?” Student D explained, “Say the speed limit is 45 mph and you are going 40 mph, then you are driving -5 under the speed limit.” Because Instructor M asked a specific question about her real life example, Student D answered by connecting the mathematical concept to her example. If Instructor M had not asked the question, Student D would not have made the mathematical connection or reached resolution.

Integration: Defend your solution. In the discussion prompt for the Defend discussions, students were prompted to reach resolution in their final posts for the week. They were asked to post three different unsolved math problems, let their classmates solve them, and then respond by explaining what was correct or incorrect. They were also supposed to post their solutions, showing their work and their answers, by the end of the week. Therefore, none of the initial posts were supposed to reach integration. They were all supposed to be triggering events. There was one instance of an initial post reaching integration, where the student posted three problems and also included the answers to those three problems. Student T posted,

1. 7(4y - 6) + 3(4y + 5). Answer is = 40y - 27
2. -8(n + 4) + 36= -5n - 8. Answer is = 4
3. 7^2 – {14 – [72 ÷ (6 x 2) + √9]} + 6^2. Answer is = 8
Even though she posted the answers to the three problems, it did not affect the cognitive presence stage in the follow-up responses of her classmates. A classmate, Student S, responded with her solution to the first problem.

Hi, I will be doing question 1. 7(4y-6) + 3(4y+5). First I will multiply (7) (4) and (7) (-6), which comes out to be 28y – 42 + (3) (4) + (3) (5). So now I have 28y – 42 + 12y + 15. Next I'm going to add 28y + 12y = 40y. Then solve the other half by -42 + 15 = -27. My final answer = 40y - 27.

Student T’s instructor responded to Student S and also responded to Student T’s initial post. However, Student T never reached resolution herself because she did not post the worked out solutions to her initial three problems.

Individually, 37.5% of students reached integration within their Defend discussion thread. In the context of the Defend discussion, the integration stage is where students post an answer or confirm an answer. For example, Student L posted,

I actually really like the FOIL method. Here’s a few…

1. (x + 5) (x^2 - 8x + 3)
2. (x + 6) (x - 5)
3. X (3y - 4) + x (8y + 2)

A classmate, Student J posted in the resolution stage,

Ok here is what I think #2 is?!?!

(x + 6)(x - 5) =

F = x^2
O = 5x
I = 6x
L= -30

\[ x^2 - 5x + 6x -30 \] Simplified = \[ x^2 + 1x - 30 = x^3 -30 \]. I think that one of the x’s needed to be an \( x^2 \) because you end up with -5x and 6x which can be simplified.

Student J replied to his own post with a correction in the integration stage, “Actually I simplified wrong at the end! I believe that the simplified answer would be: \[ x^2 + 1x - 30 \]”.

Student L responded with an integration stage post, “You are correct. That is the answer for number 2”. Another classmate, Student J, responded to Student L’s initial post with answers to the three problems. Student J posted in the integration stage as well,

I too like using the FOIL method. Here are my answers:

1. \( x^3 - 3x^2 - 37x + 15 \)
2. \( x^2 + 11x - 3 \)
3. \( 11xy - 2y \)

Student L responded in the integration stage, “Thank you for solving all of them. Those are the answers I came up with as well. Thank you for sharing them.” Even though Student L did not reach resolution by posting worked out solutions to her three problems, she reached integration by confirming the answers of her classmates.

When all of the student posts were combined in the Defend discussions, 38.5% of the posts reached integration. Integration was the highest occurring stage of cognitive presence in the Defend discussions, followed by triggering event (26.8%) and resolution (25.4%). As follow-up responses students could answer one of the unsolved math problems or they could answer all three unsolved math problems. Although some students chose to answer all three of their classmates’ problems, the discussion prompt created a possibility of more integration posts per initial post than in the Apply
discussions. More students could reach integration more often in the Defend discussions than in the Apply discussions.

Resolution: Apply your solution. In the Apply discussions, the discussion prompt intended students to reach resolution in their initial post because it asked students to post a real life scenario of a math concept. Therefore, all students could reach resolution in one post. They were supposed to post three times during the week, so the other two posts could address inquiries from their classmates or instructor, or they could respond to one of their classmate’s initial post. Although only twenty percent of the total posts in the Apply discussion met resolution, over sixty-three percent of students reached resolution in their initial posts. Students who reached resolution in their initial post provided an explanation of applying a real-life situation to a math concept. For example, Student D explained how a linear equation could be applied to a real life scenario.

I am in need of repairing our backyard fence. I don’t currently know the dimension of the yard exactly or how much space I want between upright posts. When I know the perimeter, I will want to space them equally and buy only the supplies I need in order to recreate the fence. So, let’s say it is 100 yards and I want 8 feet between posts and four planks in each section. So, I know I will need 48 planks of 8’ length and four of 4’ length to finish the remaining space after 96’ post. The x factor is the cost per plank. Depending on the quality of wood I want to buy the total expense can vary. If my budget for the entire project is $500, then my equation looks like this…$48x + 4y = 500. This is how I will determine the expense of each piece of plank.
Of the remaining thirty-seven percent of students who did not reach resolution in the initial post, there was only one student who progressed to resolution within her discussion thread. This instance occurred when the instructor responded to the initial post, asking a specific question about the students’ real life scenario. Student D’s initial post met the integration stage because she posted that driving under the speed limit was a real life example of using negative numbers. Instructor M replied in the exploration stage, “Hi. How would you use negative numbers for “Driving under the speed limit”?“ Student D responded in the resolution stage, “Say the speed limit is 45 mph and you are going 40 mph, then you are driving -5 under the speed limit.”

Additionally, there is only one instance in all of the Apply discussions where a student did not reach resolution in the initial post, but he eventually reached resolution in a classmate’s discussion thread. Student J’s initial post only reached the integration stage, “…A real life example where you may need to solve a linear equation is if you were working on a cash register and you needed to consider individual items by hand.” His post was close to reaching resolution, but it only provided a real life situation without an explanation. Two days later Student J responded to his classmate, Student T, when she develops a word problem about banking that incorporates a linear equation. Student T posts,

At his local bank Seth deposits his weekly paycheck of $450.00. He would like to deposit 200 into his checking account and 75 in to his savings account the rest he would like to have back in cash. What is the amount Seth will receive in cash? Student J read the post which ended in a question and applied it to his real life.
Hi. When I go to the bank a lot of times I would sign the deposit, or with drawl slip and let the Teller fill out the rest of the information. Now after this math course I might be encourage to fill out the different transaction slips myself. Also, with money being the mode of currency that we need on earth it is important to learn how to use it as a good steward...Thanks

Therefore, the evidence suggests that either students (1) reached resolution in the initial post or (2) progressed to resolution when an instructor asked a specific question about the real life scenario. Although the data provided evidence of students not reaching resolution when posed with questions or comments from an instructor or classmate, the only instance where a student progressed to resolution occurred when the instructor asked a specific question about the real life scenario. However, there were occurrences of instructors and students posting responses to the real life scenario in an initial post, but their responses either went unanswered or answered with other stages of cognitive presence.

Although ninety percent of the posts that met resolution occurred as the initial post, resolution posts also occurred when students who had already posted in the resolution stage in their initial post responded to classmates who commented on their real life scenario. For example, Student J’s initial post met resolution because he explained a real life scenario, “Weight loss – when we talk about losing weight, we all talk about negative integers. Example, I lost five pounds (-5)”. His classmate, Student Z, responded in the exploration stage, “Your job sounds pretty interesting and complex. It seems that signed numbers play a big role in your career as they do many others. Signed numbers
are a vital part of everyday life in more ways than we notice them to be.” Student J responded in the resolution stage,

Hey – thanks for the response! Yes, my job uses several negative integers throughout the business. For example, when looking at mechanical drawing of parts, sometimes the dimensions reference a tolerance. These tolerances could be two negative numbers or a negative number and a positive number, but the purpose is to give you a specification or range to hold the dimension to…

Additionally, a few students posted in the resolution stage when responding to a classmate whose initial post reached the resolution stage. For example, Student D posted “A real life example would be working towards a goal. If the goal is to have X amount of money, and you have Y amount of savings, what is needed to reach the goal?” One of his classmates, Student S followed up.

I think my saving has a few more x’s in it, lol. Converting a money goal into a linear equation is a good example. You can even take it further and throw in something such as the amount earned each week so it would take so many weeks, so: a – b = c and c/25 = d. a is the money goal, b is the amount in savings, c is the amount needed to save, 25 is the amount saved each week, d is the amount of weeks till you reach your goal. Lots of uses for linear equations when it comes to money. Good example.

Lastly, two students posted in the resolution stage when responding to an instructor’s initial post. Instructor A posted a historical review of the metric system as the first post in the discussion forum.
You folks did a nice job with the recipes - probably appreciated the new cooking ideas more than the math. That's alright. But the actual algebra is going to actually start picking up these last 3 weeks. Regarding the encounter with the math professor, he may know something we don't about the future increased use of the metric system in the U.S. It was first promoted in this country (nationally) back in the '70s, but continues to meet public opposition as the only standard of measurement. Companies have accommodated it so that they could do business with foreign countries, but they typically list measurements using both standards.

Two students replied to the instructors’ post in the resolution stage because they extended his example to a scenario in their own life. Student K replied,

That's interesting Prof A. I work for a company that manufactures cement and we have numerous styles of measurements but the most two common for the cement powder is Short Tons and Metric Tons. All our final numbers are converted to metric (ST x 0.0972). Now I understand why, we are a French owned company. So thanks for that history nugget.

Student O also replied,

I can see why people oppose the metric system, but as I learn how to do this, I can see some benefits. Like counting ounces in a can of soup. It's good to know especially if you’re counting calories.

There was not any recognizable pattern to discern why or under what circumstances a student did not post in the resolution stage. However, this data provided evidence that students only post in the resolution stage under five circumstances: (1) in the initial post when the discussion prompt asked them to create a real life application of
a math concept, (2) when a classmate asked them to provide a better explanation of their real life scenario, (3) as a response to a classmate whose initial post asked a question as part of their real life scenario, (4) when the instructor asked them a specific question about their real life scenario, and (5) in response to the instructor who posted a historical viewpoint of the math concept.

Resolution: Defend your solution. In the discussion prompt for the Defend discussions, students were prompted to reach resolution in their final posts for the week. They were asked to post three different unsolved math problems, let their classmates solve them, and then respond to them explaining what was correct or incorrect. They were also supposed to post their solutions, showing their work and their answers, by the end of the week. Individually, twenty-nine percent of students reached resolution within their Defend discussion thread. However, when all of the student posts were combined in the Defend discussions, twenty-five percent of the posts reached resolution.

There were a few instances of an initial post reaching resolution, where the student posted three problems and also included the worked out solutions to those three problems. This affected the cognitive presence stage of the follow-up responses. In all of these circumstances, but one, there were not any follow-up responses to the initial post. However, one student who posted three problems and the worked out solutions resulted in follow-up responses which just restated the answers from the initial post. Student B posted three problems and the solutions.

1. \((3x - 4)(2x + 1) = 3x(2x + 1) + (-4)(2x) + (-4)(1) = 6x^2 + 3x - 8x - 4 = 6x^2 - 5x - 4\)

2. \((x + 3y)(2x + y) = 2x^2 + xy + 6xy + 3y^2 = 2x^2 + 7xy + 3y^2\)
3. \((x + y)(x + y) = x(x + y) + y(x + y) = (x * x + y * x) + y(x + y) = (x + y + x^2) + y(x + y) = (x + y + x^2) + y(x + y) = x + y + x^2 + x + y + y^2\)

A classmate responded with just the same answer to the first problem “1. 6x^2-5x-4.”

Another classmate also posted the answers to Student B’s three problems,

1. \((3x - 4) (2x + 1) = 6x^2 - 5x - 4\)
2. \((x + 3y) (2x + y) = 2x^2 + xy + 3y^2\)
3. \((x + y) (x + y) = x^2 + 2xy + y^2\)

Almost every students’ discussion thread reached integration or resolution in the follow-up responses. For example, Student G posted,

1. \((4x + 3) (3x - 2) = \_\_\_\_\_\_
2. \((x + 4) (4x - 14) = \_\_\_\_\_\_
3. \((26x + 6) (6x - 12) = \_\_\_\_\_\_

Student C reached resolution in his follow-up post,

Problem #1: \((4x + 3) (3x - 2)\)

F: \(4x (3x) = 12x^2\)

O: \(4x (-2) = -8x\)

I: \(3(3x) = 9x\)

L: \((3) (-2) = -6\)

\(12x^2 - 8x + 9x - 6 = 12x^2 + 1x - 6\)

Student G replied to Student C in the integration stage, “Yes, that is the answer I got as well, very well planned out with details in the equation. Thank You.” No other students
attempted to answer Student G’s initial three problems. Therefore, by the end of the week Student G posted solutions to all three of her initial problems.

As follow-up responses students could answer one of the unsolved math problems or they could answer all three unsolved math problems. The discussion prompt asked each student to pick one of his or her classmates and solve one of the unsolved math problems. Although some students chose to answer all three of their classmates’ problems, the discussion prompt created a possibility of more resolution posts per initial post than in the Apply discussions. More students could reach resolution more often in the Defend discussions than in the Apply discussions because the discussion prompt created three mathematical situations.

Some discussion threads did not reach resolution or integration. Each student was supposed to show the worked out solutions to the problems in their initial post. Posting their worked out solutions produced resolution. Each student was also supposed to respond to each follow-up response on their unsolved problems, explaining to their classmates what was correct or incorrect about their solutions. Neither of these situations occurred with any predictability. However, the initial posts that went completely unanswered occurred when the student posted his initial post at the end of the week. The initial post was supposed to be posted on day four of the week, and their two follow-up posts were due by the end of the week. Students lost points if they posted late, but the lost points may not have been enough of a disincentive. Regardless, lack of time resulted in discussion threads not reaching integration or resolution.

**No category: Apply your solution.** Although all of the initial posts in the Apply discussions reached a stage of cognitive presence, fifty-six percent of the follow-up
responses did not reach any stage of cognitive presence. The data showed that the discussion prompt contributed to this large number of posts not reaching any category of cognitive presence. The 3.2 discussion prompt asked students to convert a recipe from U.S. measurements into metric units, discuss the importance of signed numbers, and identify a real life scenario that uses negative numbers. The 4.2 discussion prompt asked students to complete even more tasks. It asked students to describe the concept of like terms, explain why \( x + y \) cannot be simplified further, develop a word problem that uses a linear equation, solve the word problem, outline each step to solve it, and post a real life example of using a linear equation.

After eighty-eight percent of students reached integration or resolution in their initial post, fifty-six percent of the follow-up posts were messages that did not demonstrated a stage of cognitive presence. These messages were usually off topic statements or statements of encouragement. For example, Student T’s initial post met the integration stage because she converted a German chocolate cake recipe to metric units, discussed the importance of signed numbers, and identified three real life scenarios that used negative integers, “Temperature; Bills (accounts); Grades”. A classmate responded, “Hi. Your German Chocolate Cake recipe sounds delicious, it's my husband's favorite, I might have to try it.” Student T replied, “Thank you, I chose this recipe because it was one of my aunts favorite recipe, she passed away some time ago and now my mom makes it and it is just as delicious”. Another classmate, Student K, also responded to Student T’s initial post, “Your cake recipe sounds delicious!! Too bad we can't sample each other’s recipes. :) You examples of negative integers are great. I hadn't thought of grades, but that is a perfect example.” A third classmate replied to Student K, “I had the same
thought wishing we could sample each other’s recipes!” Although the initial post met integration and did not lack the opportunity for a classmate or instructor to ask Student T about her real life scenarios, these off topic posts were very common in the Apply discussions.

Out of one hundred fifty-nine total students, one hundred forty (88%) students posted in the Apply discussions at least one response that did not reach any stage of cognitive presence. Out of the nineteen remaining students whose posts reached a stage of cognitive presence every time, only three students' initial post reached the resolution stage of cognitive presence and posted at least two follow-up responses (meeting the expectations of the course requirement). Of those three students, Student S posted in the resolution stage three times in the 4.2 Discussion. Other than his initial post which reached resolution, the other resolution posts occurred when Student S answered a classmate’s question. Student S’s initial post met resolution because he explained one real life example of using a linear equation, “I think that in the kitchen you can use a linear equation to increase or decrease a recipe for the amount you’d like to make.” This example was a common example used by many students in class. Student E responded by asking a question, “Hello, would you like to provide an example of the linear equation that you would use to increase or decrease a recipe? Thanks!” Student S answered Student E’s question, providing a stronger example of resolution,

You take your desired servings and divide it by the recommended servings, such as 4 desired servings divided by 10 recommended servings. This will give you the number to multiply all of the other ingredients against to get the decreases portion needed to complete the recipe. $4/10 = .4$. 2 cups of milk for 10 servings $2 \times .4 =$
.8 cups for a 4 serving portion. 1 tablespoon of vanilla for 10 servings 1 x .4 = .4 tablespoon for a 4 serving portion so on and so on. I guess the linear part of this would be x = y x z.

x = decreased portion, y = recommended portion, z = portion reduction conversion.

Student S posted again in the resolution stage by responding to a classmates initial post. Student D’s initial post met the resolution stage because he provided a real life example of a linear equation, “A real life example would be working towards a goal. If the goal is to have X amount of money, and you have Y amount in savings, what is needed to reach the goal?” Student S responded to Student D’s question, providing a strong example of resolution,

I think my saving has a few more x's in it, lol. Converting a money goal into a linear equation is a good example. You can even take it further and throw in something such as the amount earned each week so it would take so many weeks, so: a - b = c. c/25 = d. a is the money goal, b is the amount in savings, c is the amount needed to save, 25 is the amount saved each week, d is the amount of weeks till you reach your goal. Lots of uses for linear equations when it comes to money. Good example Steve.

Although students and instructors ask questions that go unanswered, this data provided evidence that students are less likely to post messages that do not reach any stage of cognitive presence when an instructor or classmate asked a specific question about their real life scenario. Students did not need to fill the discussion forum requirement of posting at least three responses with posts that did not meet any stage of cognitive
presence when they were cognitively engaged in the discussion threads answering questions from their instructor and classmates.

**No category: Defend your solution.** All of the initial posts in the Defend discussions reached a stage of cognitive presence while only nine percent of the follow-up responses did not reach any stage of cognitive presence. The discussion prompt contributed to this low number of posts not reaching any category of cognitive presence. The structure of the Defend discussion prompt kept students cognitively engaged in the discussion threads because it provided clear direction on what their initial post and two follow-up posts were supposed to include. For the initial post they posted three unsolved math problems. For the second post they posted the solution to one of their classmate’s unsolved math problems. In the third post they posted the solutions to all of their three unsolved math problems.

In the Defend discussions the student posts that did not met any stage of cognitive presence were short statements of encouragement, appreciation, or acknowledge. For example, Student C posted three unsolved math problems as an initial post. A classmate, Student M posted three answers to Student C’s problems. Another classmate, Student S replied by posting a message of encouragement, “You did well on these problems and I see that you identified the multiplier of 12 on problem 2. Nice work.” There were only three off topic posts in the Defend discussions. For example, as a follow-up response Student T posted

I want to say good luck to everyone. This is my last class before I graduate on April 25th! I can't believe I finally made it to this point. Hang in there. At times it will be tough, but boy will it be worth it! Take care everyone.
These posts coded for no category occurred without any consistency. There were only 2.4 no category posts per class and only one no category post for every 3.8 students who participated in the Defend discussions. Students did not need to fill the discussion forum requirement of posting at least three responses with posts that did not meet any stage of cognitive presence because they were cognitively engaged in the discussion threads following the directions in the discussion prompt.

**Instructor Engagement**

Along with cognitive presence, the Community of Inquiry theory recognized the importance of social presence and teacher presence. Although the CoI theory provided a set of categories to code discussion forum responses in order to identify social presence and teacher presence, this research project investigated the relationship of how instructor responses engaged students in developing deeper levels of cognition.

The four discussion forums in the sixteen online classes prompted students to reach the resolution stage of cognitive presence. However, the students’ initial posts may not thoroughly address the discussion prompt. Their initial post may or may not reach the resolution stage. Therefore, this study investigated the presence or absence of the instructor responses to the students, identified themes in what type of response the instructor provided, and whether the students advanced in their stage of cognitive presence.

In order to explore the relationship between the students’ stage of cognitive presence and instructor engagement, the posts by the course instructors were imperative to this analysis. No instructor was removed from any class, so the sixteen instructors facilitated all four discussions. Across the four discussion forums in the sixteen online
classes, this research project coded and analyzed sixty-four discussion forums with 2228 student posts and 267 instructor posts, totaling 2495 messages.

A series of sub-questions guided the analysis. (1) How did the instructors participate in the discussion forums? (2) How did the instructors respond to the students’ posts? (3) How did the instructors engage the students in cognitive presence?

**Instructor Participation.** Across the four discussion forums in the sixteen online classes (sixty-four discussion forums) which generated data for this research study, instructors posted at least one message in thirty-two discussion forums. These thirty-two discussion forums were analyzed in this section for themes regarding instructor engagement. In this sample, there was an average of 8.2 instructor posts per discussion forum and one instructor post per student in each discussion forum. Forty-six percent of the instructor posts occurred in the Apply discussions and fifty-four percent occurred in the Defend discussions. The majority of the instructor posts were follow-up messages to students. However, four instructors posted the first message in the discussion forum. Instructor Z posted an example of converting a recipe from metric units to U.S. units. Instructor J posted a short lecture on the real number system and a short historical perspective on why the U.S. uses the English standard measurements. Instructor A also posted a short historical perspective on the metric system. Lastly, Instructor Z posted an example post containing three worked out math problems, showing students how to factor and solve quadratic equations.

Two instructors only posted one message in their discussion forums. The posts contained an introductory message to the students about the last weeks’ discussion on the metric system and an example post containing three worked out math problems. Six
instructors participated in all four discussion forums in their class. These six instructors posted an average of 9.5 instructor posts per discussion forum and 1.2 instructor post per student in each discussion forum, very similar to the overall average for the thirty-two discussion forums with instructor participation. However, one instructor doubled these averages. Instructor M posted an average of 20.3 posts per discussion forum and 2.2 posts per student in each discussion forum. Instructor M and her students posted messages more often that did not meet any stage of cognitive presence (48% and 33%, respectively) compared to the other instructors and students (34% and 23%, respectively). Additionally, only 65% of her students reached resolution within their discussion thread compared to 72% of the other students. However, Instructor M’s students posted slightly more often in the resolution stage (27%) than the other students (25%)

Instructors participated in the discussions by engaging in different stages of cognitive presence. Overall, 39% of the instructors’ posts did not reach any stage of cognitive presence, while 2% of the instructors’ posts met the triggering event stage, 33% met the exploration stage, 18% met the integration stage, and 9% met the resolution stage of cognitive presence. Additionally, students reached resolution more often in the Apply discussions (70%) than the Defend discussions (33%), where the instructors participated.

**No Category.** More instructor posts did not reach any stage of cognitive presence than the four stages of cognitive presence. The vast majority of those posts either did not ask for a student response or the students’ follow-up response did not reach any stage of cognitive presence. For example, Student S posted three unsolved math problems in her initial post,

Example 1: $2(x + 5) - 7 = 3(x - 2)$
Example 2: \((2y + 1) (y - 1) = (y + 5) (2y - 5)\)

Example 3: \(2x + 4(9) = 50\)

Instructor Z replied, “Very nice examples Student S!” and Student S responded, “Thank you, Lots of studying and redoing!!”

However, in one occurrence the instructor’s no category post was part of a cognitively engaging discussion with the student. Student D posted three unsolved math problems,

Here are the three questions that I propose to the class:

1. \((7x^2 + 14x - 7) - (-6x^3 + 3x^2 + 3x - 18) = \)
2. \((5x + 6)(3x + 3) = \)
3. \(\sqrt{8x} \sqrt{6x} = \)

Instructor Z’s first responded in the exploration stage, “Equations needs to be equal to something. Since the assignment asks you to solve them initially then you should equal them to zero.” Student D reposted,

1. \((7x^2 + 14x - 7) - (-6x^3 + 3x^2 + 3x - 18) = 0\)
2. \((5x + 6)(3x + 3) = 0\)
3. \(\sqrt{8x} \sqrt{6x} = 0\)

Instructor Z responded without any category of cognitive presence, “Yes, thank you.”

However, Student D followed-up with a post in the resolution stage,

As for the answers to the equations though, making the third example equal to zero creates a problem in my solution. The solutions that I would have came up with would be 1. \(6x^3 - 4x^2 + 11x + 11 = 0\)
2. \(15x^2 + 33x + 18 = 0\)
While the first two examples can still be solved based on the zero property, the third would still be a simplified answer with the radical in the answer. Thank you.

Student D responded again, “My mistake, I meant to add in the solutions as well. For example 1, the solution would be -0.67122. For example 2, the solution would be -1. Thank you.” Instructor Z replied,

Student D, in #3 equation what value of x will make the equate true? In #2, based on zero property you will have two solutions: solve for x 5x + 6 = 0 OR solve for x 3x + 3 = 0. First one yields x = -6/5 and second, x = -1. Both are solutions to your #2 equation. #1 seems to me complex and will not follow grouping for CGF method. You would need other methods to solve it which are not used in this class. Try to come up with a simpler example (second degree, a trinomial) that you can solve.

Student D did not respond to Instructor Z’s last question. However, only an hour and a half elapsed from the instructor’s first response to her last response. This almost synchronous conversation demonstrated that a no category instructor post does not have to impede the progression of cognitive presence. However, the timeliness of the posts probably played a more important role in keeping a cognitively engaged conversation going. Timely responses helped Student D progress in cognitive presence.

**Triggering Event.** Only six instructor posts met the triggering event stage of cognitive presence. Two of them elicited two student responses. In response to a student’s initial post, which reached resolution, Instructor M directed his student’s
attention back to answering the discussion prompt. “Great post! Your example of the real-life scenarios are relevant for anyone. Be sure to reference your assigned readings and cite any sources you used.” The student responded, “Thank you, Professor M! Have a great evening!” Secondly, Instructor P posted in the triggering event stage when she noticed an error in one of her students’ initial problems: “Very good. In equation 1 I think you meant $x^2 - 4 = 0$.” Her student responded, “Yes, I did I noted that below. I forgot to go back and change it after I googled how to do it!” The data showed that instructor posts in the triggering event stage did not play an important or consistent role in the progression of cognitive presence.

**Resolution.** When instructors posted in the resolution stage in the Defend discussions, they posted solutions to the students’ unsolved math problems. When Student C posted her initial post of three unsolved math problems,

Solve using the FOIL method:

1. $(4x + 1) (3x + 5)$
2. $(5xy + 1) (2xy - 5)$
3. $(2x + 5) (5x - 5)$

Instructor M responded,

Hi there, Student C. I will do #2.

$(5xy + 1)(2xy – 5)$

F: $5x^2y^2$

O: $-25xy$

I: $2xy$

L: $-5$
\[ 5x^2y^2 - 25xy + 2xy - 5 = 5x^2y^2 - 23xy - 5 \]

Student C replied to the instructor, “Your x and y factors didn’t show. Here is my answer. 10x^2y^2 – 23xy – 5.” Instructor M got the answer wrong, no one caught the mistake, and no one learned from it. Although Instructor M posted in the resolution stage, less than half of her students reached resolution.

Also, by only approaching the discussion forum as a student would approach it, Instructor M did not correct the mistakes her students made. For example, Student C posted an unsolved problem in his initial post “3. (3x^3 + 3)(x + 5)”. Another student attempted his problem,

Student C, I will take problem #3.

\[(3x^3 + 3)(x + 5)\]

First: \((3x^3 \times x) = 4x^3\)

Inner: \((3x^3 \times 5) = 15x^3\)

Outer: \((x \times x) = x^2\)

Last: \((x \times 5) = 5x\)

Now add them all together. \(4x^3 + 15x^3 + x^2 + 5x = 19x^3 + x^2 + 5x\)

This answer was wrong, and no one corrected his mistakes.

Additionally, when instructors posted worked out solutions, the posts were never followed by resolution posts. For example, Student T posted three unsolved problems.

1. \((x + 7) (x - 9)\)

2. \((4x + 6) (3x + 5)\)

3. \((x - 7) (x - 8)\)
Student S replied, “1. \((x + 7)(x - 9) = x(x - 9) + 7(x - 9) = x^2 - 9x + 7x - 63\).” Instructor C replied to Student S, “Hi, Student S. You need to combine like terms. So, \((x + 7)(x - 9) = x(x - 9) + 7(x - 9) = x^2 - 9x + 7x - 63 = x^2 - 2x - 63\). Thank you.” Student S responded, “I see where I messed up at, oops!” Student S did not redo and repost his problem or try another problem. He acknowledged his mistake, but the instructor cannot be sure if he really understood it. Instructor C could have responded, “Hi, Student S. You need to combine like terms. So, \((x + 7)(x - 9) = \ldots\) Now try this one: \((x - 3)(x + 4)\).” In that case Student S would have been given an opportunity to apply and test his new knowledge to a new problem.

In response to an Apply discussion prompt, Student A reached resolution by converting a meat loaf recipe to metric units and identifying three real-life scenarios that used negative numbers.

1. Negative integers can show a loss in a sale. For example: A shirt costs $50.00 and sells for $40.00, which would be a -$10.00 loss.

2. Negative integers are also used to show temperatures. For example: -8 ° would be 8 degrees below zero.

3. Negative integers can also be represented in money. For example: You get paid $400 per month. $200 goes to rent, $25 goes to gas, $50 goes to food, $125 goes to insurance, and $25 goes to utilities. How much would you have left? -$25.00 (You wouldn't have enough to cover the bills).

Instructor C responded in the resolution stage,

Thank you Student A. Good examples. In U.S., we measure the temperature the degree in Fahrenheit. But in my country, Korea, we measure the temperature the
degree in Celsius. In U.S., we measure the weight in pounds, but in my country we measure the weight in kilograms. In U.S., we measure the distance in miles, but in my country we measure the distance in kilometers. It is interesting to have the unit conversion but it would be more effective if we can use the same unit.

Student A only responded in the triggering event stage by assessing his state of knowledge, “I like how you broke down your examples. We use negative and positive integer more than I thought, great job.” Instructor C’s resolution post did not promote a resolution post from his student. Instructors posting in the resolution stage did not lead to students reaching resolution.

Integration. Instructors posting in the integration stage also did not lead to students reaching resolution. When instructors posted in the integration stage, most of the time they acknowledged correct answers. However, even when instructors acknowledged incorrect answers, the follow-up posts did not lead to resolution. For example, Student R posted three problems in a Defend discussion,

1. $\sqrt{(9x)} \sqrt{(3x)}$
2. $x^{12}/x^{10}$
3. $(x + 5) (x^2 + 9x - 4)$

Student K solved them,

1. $3x\sqrt{2}$
2. $x^2$
3. $x^3 + 9x^2 + 41x - 20$

Student R responded, “The #1 and #3 are incorrect. You were very close on both of them.” Instructor J also replied, “I think #1 may be a typo? And, as Student R said, #3 is
very close!” Even though Student R and Instructor J both told Student K that his answers were wrong, Student K did not correct his mistakes and repost his solutions. Overall, instructors posting in the resolution and integration stage hindered student learning and did not lead to students progressing in stages of cognitive presence.

**Exploration.** However, sometimes when instructors posted in the exploration stage, the data showed that students progressed in cognitive presence. For example, Student K posted three problems,

1. $x^2 \times \sqrt{49}$
2. $3x^2 + x$
3. $4x + [2x + x]^2$

Instructor P responded, “Student K, Those are expressions not equations. An equation will have an = sign.” Student K replied, “Thank you, Instructor P, I will post again.” He reposted his problems and also posted the solutions a few days later.

More importantly, when instructors asked questions, students progressed to resolution. For example, Student K posted three problems,

1. $7\sqrt{3} \sqrt{12}$
2. $5\sqrt{2} + \sqrt{18}$
3. $8/7 + \sqrt{3}$

Student R solved the first problem, posting in the integration stage, “Student K, I’m not sure if we’re supposed to do them all so I did your first equation. It was a little tricky so hopefully I did it right. 1. $7\sqrt{3} \sqrt{12} = 42.”$ Instructor J responded in the exploration stage by asking Student R a question, “Student R, can you show how you got this? That might
help others understand how to approach the problem.” Student R responds, “$7\sqrt{3\sqrt{12}} = 7\sqrt{(3 \times 12)} = 7\sqrt{36} = 7 \times 6 = 42$,” meeting resolution.

In one instance, a student initially posted in the integration stage, but eventually met resolution. This occurred in an Apply discussion when the instructor responded to her initial post, asking specific questions about the students’ real life scenario. Student D’s initial post met the integration stage because she posted that driving under the speed limit was a real life example of using negative numbers. Instructor M responded by asking a specific question, “Hi Student D, How would you use negative numbers for ‘Driving under the speed limit’?” Student D explained, “Say the speed limit is 45 mph and you are going 40 mph, then you are driving -5 under the speed limit.” Because Instructor M asked a specific question about her real life example, Student D answered by connecting the mathematical concept to her example. If Instructor M had not asked the question, Student D would not have made the mathematical connection or reached resolution.

In the same Apply discussion another student reached resolution in her initial post because, along with converting a cookie recipe, she provided enough explanation of how negative numbers were used in three real life scenarios,

1. A negative or positive balance in your bank account.
2. Below zero or negative temperatures.
3. An increase (positive) or decrease (negative) in weight loss.

Although Student B showed resolution by applying a contextually specific solution to negative numbers, Instructor M extended Student B’s cognitive presence by asking about
her recipe conversions. “Student B, how do you convert 3 cups of cooking oats to 709.77 milliliters?” Student B responded by defending her calculations, meeting resolution.

Instructor M, Thank you for responding. 1 cup = 236.588ml. In order to see how many milliliters are in 3 cups you would multiply 236.588 by 3, giving you 709.764.

236.588 * 3 = 709.764 milliliters. If rounded, it should actually be 709.76, because 4 is less than 5 so you would not round up.

Moreover, instructors asked students to correct their mistakes. In Student K’s initial post he met the integration stage because he did not explain this three real life scenarios. He also forgot to convert his pancake recipe from U.S. measurements to metric units.

Pancakes:

- 1 ½ (12oz) cups flour
- 1 ¼ (10oz) cups milk
- 1 egg
- 3 ½ (0.61oz) teaspoons baking powder

b. In your opinion, what is the most important function of signed numbers?

Positive or negative.

c. Identify three real-life scenarios that call for the use of negative integers? 1.

Weight fluctuation. 2. Balancing checkbook. 3. Temperature.

Instructor J responded by focusing on the recipe conversion, instead of the real life scenarios.
Student K, in the past I would always use balancing the checkbook when teaching signed numbers in a traditional class. Everyone seems to be able to relate when the reference is to money. Also, in part "a" you are to convert the English measurements, like cups, to a METRIC unit. Why not give it another try?

Because the instructor asked him to repost his initial post, Student K reposted it. The pancake recipe was correctly converted to metric units.

Pancakes:
- 1 ½ (0.35 liters) cups flour
- 1 ¼ (0.3 liters) cups milk
- 1 egg
- 3 ½ (0.017 liters) teaspoons baking powder

His repost still did not reach resolution because he still did not explain the three real life scenarios, and he did not explain how he converted U.S. measurements to metric units.

But if Instructor J had not identified the conversion misconception, Student K would have continued to believe that ounces were a metric unit of measurement.

However, not every question asked by an instructor was answered. Student A posted her initial post with a cookie recipe conversion and an explanation of three real life scenarios using negative numbers, meeting the resolution stage,

1. Temperature: 2 below zero = -2°.


While Student A was logged into the course, she also posted a response that did not reach any stage of cognitive presence to a classmate. Knowing that she only needed one more
post to fulfill the discussion forum requirement, she logged in to the course the next day.

She probably noticed that her instructor had responded to her initial post earlier that day.

   Student A, you did a great job with that recipe; those chocolate chip cookies sound great right about now!! Well, I am a chocolate-lover, so I am up for chocolate chip cookies at any time. You also did a good job with your description of the use of signed numbers. The first two examples of the use of negative numbers that you provided were good examples, however, the third example seemed to be an example of a positive integer instead of a negative integer. How could you change that to where it would be an example of a negative number?

   Hang in there and continue to work hard.

Instead of answering her instructor’s question about her bank deposit scenario, she decided to post a response that did not reach any stage of cognitive presence to another classmate. Although an instructor can try to lead a student to resolution, students may choose not to engage in cognitive presence.

   Furthermore, generic instructor questions do not lead students to deeper stages of cognitive presence. In Student R’s initial post meeting resolution, she posted a brownie recipe conversion and explained three real life scenarios that used negative numbers.

   Temperature, a bank account withdrawal, and weight loss are three real-life scenarios that use negative integers. Temperature falls below zero, making a withdrawal from your bank account would show a negative on your bank statement, and you use a minus sign when calculating how much weight you have lost.
Instructor J responded by asking questions, posting in the exploration stage, “Student R, Good job on this! Was this a recipe you use all the time? Do your measuring cups have any metric units on them?” Instructor J’s questions did not engage her student in high stages of cognitive presence because they were not about the mathematical content or application. Student R responded,

Most of the time I buy a box of brownies and follow the instructions on the box. One day though I will make homemade brownies and hopefully they will turn out right. My measuring cup does have both American and metric units on it. One day I think for fun, I will make 2 of something, one following U.S. measurements and the other metric and see if they both come out the same. I think that would be a neat experiment.

Although Student R showed an understanding that the two units would equate to different amounts of ingredients, this exchange did not engage him in deeper stages of cognitive presence.

Additionally, instructors asking specific questions about the mathematical content did not always lead to resolution. Student J only reached integration in his initial post because he did not explain how his real life scenarios called for the use of negative numbers.

1. When the bottom line of a P&L is not in the positive.
2. When tallying a really good golf score.
3. When you are looking at construction notes and need to know how much wood you are needing to complete your project.
Instead of asking Student J to explain how his real life scenarios called for the use of negative numbers, Instructor M asked about his recipe conversion, “Hi Student J, How did you convert 1/2 c of chopped nuts to 118.287 milliliters?” Instructor M’s question could have lead Student J to defend his calculation. However, Student J answered honestly,

Hey professor, I will not lie to you. There was a conversion chart that was on the recipe site. All you would do was type in the amount of the ingredient and there would be 8 different types of measurement that you could use. I actually thought about just using milliliters for the enter chart, but thought doing a variety would be fun as well.

A classmate responded to Student J,

I saw that site also. I did not use it. I worked it out long hand and I would be willing to be that I convert some things wrong, but I tried. Especially the apples in the pie. I mean, actually, how do you really convert apples because I would think that it would depend on the size of the apples. At least you were honest about the site.

Students were more likely to progress in cognitive presence when an instructor asked a specific mathematically related question, but an instructor’s question leading to a student’s resolution was not a consistent pattern.

**Instructor Non-Participation.** Out of the sixty-four discussion forums used to generate data in this research study, instructors did not participate in thirty-two discussion forums. The number of posts was similar between the discussion forums where instructors did not participate (n = 1108) and the discussion forums where instructors did
participate (n = 1120). In the discussion forums where instructors did not participate, only 42% of the students reached resolution, compared with 54% in the discussion forums where the instructors participated. Consistent with the discussion forums where the instructors participated, students reached resolution more often in the Apply discussions (59%) where the instructors did not participate than the Defend discussions (26%).

**Summary of the Findings**

The qualitative data analysis investigated how students demonstrated cognitive presence in an online discussion forum. Cognitive presence was coded according to five categories of the CoI theory: triggering event, exploration, integration, resolution, and no category (Garrison, Anderson, & Archer, 2000). Students’ stage of cognitive presence was explored by analyzing discussion forums in the online classes. Across the four discussion forums in the sixteen online classes, this research project coded and analyzed sixty-four discussion forums with 2228 student posts and 267 instructor posts, totaling 2495 messages.

**Research Question 2**: How did online college mathematics students demonstrate cognitive presence in a discussion forum?

- In the Apply discussions
  - Messages that did not demonstrate any stage of cognitive presence were very common. Most of these posts contained messages that were tangential conversations of the discussion prompt.
  - Students were less likely to post messages that do not reach any category of cognitive presence when an instructor or classmate asks them a specific question about their real life scenario.
• The triggering event posts were not imperative for progression toward integration or resolution.

• Exploration posts that lead to higher stages of cognitive presence occurred more often when students or instructors asked specific question about their real life scenario.

• Integration posts only lead to resolution when an instructor asked a specific question about the real life scenario.

• The discussion prompt created the possibility of students reaching resolution in the initial post.

• Individual students reached resolution more often, although less posts overall reached resolution.

• In the Defend discussions
  • Messages that did not demonstrate any stage of cognitive presence were almost non-existent. The discussion prompt kept students cognitively engaged because it provided clear direction on what their posts were supposed to include.
  • Exploration posts that lead to higher stages of cognitive presence occurred more often when students or instructors asked specific question about the math problems.
  • Timely responses helped students progress in cognitive presence.
  • The discussion prompt created the possibility of more students reaching integration and resolution more often.
• Individual students reached resolution less often, although more posts overall reached resolution.
CHAPTER 6: DISCUSSION

The purpose of this study was to determine whether there were differences in cognitive load, instructional efficiency, and learner involvement between online and face-to-face college algebra courses. Additionally, the purpose of this study was to determine how online students demonstrate cognitive presence in online discussions, and whether there was a relationship between cognitive load and cognitive presence in the online college algebra courses. The convergent parallel mixed methods design combined the strengths of both forms of research to understand cognitive load and cognitive presence. Specifically, the mental effort measure (Paas, 1992) was used to test the Cognitive Load Theory which predicts that task characteristics influence the cognitive load for students with similar learner characteristics. Concurrent with this data collection, qualitative discussion forums explored cognitive presence for students in online college algebra courses. This chapter will examine the conclusions of the quantitative and qualitative results for cognitive load and cognitive presence. It will also identify the limitations of the study and recommendations for further research.

Discussion of Research Question #1

The quantitative results of this study revealed differences in effectiveness between the online and face-to-face math courses. The quantitative results also revealed the relationship of the relative efficiency of online and face-to-face math courses.

Research Question 1

How did instructional effectiveness and efficiency vary between online and face-to-face college mathematics courses?
Effectiveness – Performance

The results of this study revealed two statistically significant differences in effectiveness between the online and face-to-face students. First, the face-to-face students earned higher final course grades (93%, A-) than the online students (88%, B). Secondly, the face-to-face students earned higher homework grades (96%, A) than the online students (92%, A-). Research has shown mixed results when comparing the effectiveness of distance education to face-to-face education. In Bernard et al.’s (2004) meta-analysis distance education was more effective than face-to-face education when measuring achievement, but face-to-face education was more effective than distance education when measuring student attitude and retention. In Allen et al. (2004) meta-analysis distance education was more effective than face-to-face education when measuring exams and course grades. In their meta-analysis Zhao et al. (2005) found that distance education was more effective than face-to-face education when measuring student satisfaction, grades, student attitudes and beliefs, and student participation. However, face-to-face education was more effective than distance education when measuring student perception of learning and metacognition (Zhao et al., 2005).

The results of this study revealed that face-to-face students earned higher homework grades and final course grades than the online students, which is inconsistent with meta-analyses from Bernard et al. (2004), Allen et al. (2004), or Zhao et al. (2005). However, none of those studies restricted literature into their analyses with the same population as this research study. They accepted literature into their analyses, which compared distance education and face-to-face courses. Distance education is the delivery of education through any type of non face-to-face method, including correspondence.
classes, video classes, and online classes. Although their meta-analyses can give a broad picture of non face-to-face course, this research study investigated differences in cognition between fully online and face-to-face courses.

The demographics of the participants in this research study is also inconsistent with the Bernard et al. (2004), Allen et al. (2004), or Zhao et al. (2005) meta-analyses. The average age of a student in this research study was 37 years old. Sixty-two percent of the students were female, 64% Caucasian, 29% Black, and 49% married. The results of this study may be situated more appropriately with research investigating the non-traditional adult student. Moreover, although this study found statistically significant differences between online and face-to-face students in homework grades and final course grades, the differences may not be practically significant. The difference between an A and A- on the homework or between an A- and B in the course is not significant enough to influence any decisions about online education or instructional design. Students earning an A- on the homework and a B in the course provided evidence of understanding the necessary content to advance to the next math course.

Additionally, part of the significant difference in final course grades was attributed to the significant difference in homework grades. Homework accounted for 30% of the final course grades, so the significant difference in final course grades was attributed to other components of the course. The tests accounted for 40% of the final course grades, but this study did not reveal any significant differences in test scores. The final 30% of the final course grades was attributed to the discussion forums. Online students earned those points for participation in the discussion forums. Face-to-face students earned those points for attendance and participation in their face-to-face class.
The majority of the significant difference in final course grades was attributed to the differences in points earned in the discussion forums, which was not a reflection on cognitive load or cognitive presence.

**Effectiveness – Cognitive Load**

Measures of performance or effectiveness alone cannot explain all of the cognitive learning processes students experience during learning experiences. The results of this study revealed that there was not any statistically significant differences in cognitive load between students taking online and face-to-face college mathematics courses. When combined with performance the average face-to-face student earned higher grades than the average online student while exerting similar cognitive loads. This combination of understanding performance with cognitive load is relative efficiency, which is discussed in the next section. Although measures of effectiveness and cognitive load can be important to determine the power of different educational experiences, a meaningful integration of that effectiveness and cognitive load should be given in context of its associated efficiency (Paas & van Merrienboer, 1993).

**Relative Efficiency**

Paas and van Merrienboer’s (1993) relative efficiency approach combined measures of mental effort with task performance in order to compare two different instructional methods. According to their model, if one instructional method produced learners with the same performance as learners in another instructional method but the learners in the first method exerted a lower cognitive load, the first instructional method is more efficient. Additionally, the first instructional method is more efficient if it produced learners with higher performances compared to learners in another instructional
method when all of the learners exerted similar cognitive loads. The results of this study demonstrated that the face-to-face math course was slightly more efficient because it produced learners who exerted similar cognitive load as learners in the online course but the learners in the face-to-face earned higher performance score. Figure 10 shows the graph of the relative efficiency of the face-to-face and online students in this research study.

Although the performance scores, cognitive load scores, and relative efficiency scores were not statistically significantly different, the face-to-face students earned statistically significant higher final course grades and higher homework grades. The results of this study conclude that the face-to-face course was slightly more efficient than the online course. The online course was slightly less efficient because students earned lower performance scores while exerting a similar cognitive load as the face-to-face course. It created a slightly heavier “use of limited cognitive-processing capacity” (Sweller, 1998, p.261) than the face-to-face course. Further research should be conducted comparing the relative efficiency of various learning environments or environmental characteristics.
Learner Involvement

Research has shown that complex learning tasks or environments impact motivation (Frankola, 2001; Spencer & Usher, 2007; Tsai, 2012) and motivation impacts learning (Bernard et al., 2004; Kanfer & Ackerman, 1989; Tsai, 2012; Wadsworth et al., 2007; Zhao et al., 2005). Additionally, Paas et al. (2005) argued “the amount of mental effort invested in a certain learning task can be considered a reliable estimate of the learner’s motivation or involvement in that task” (p.28). Since a learner may appear to be involved in a task but may actually not be engaged, Paas et al. (2005) developed a computational approach to learner involvement which combined measures of cognitive

Figure 10. Relative Efficiency of Face-to-Face and Online Mathematics Courses, p > .05.
load with task performance. When learner involvement is high, more cognitive load effort is likely to also be used, which will most likely result in a higher performance (Paas et al., 2005).

Although the performance scores, cognitive load scores, and learner involvement scores were not statistically significantly different, the results of this study conclude that the face-to-face course demonstrated slightly more learner involvement than the online course. The face-to-face courses experienced slightly more learner involvement because their performance scores were higher than the online course, even though the cognitive load scores were not different. However, the online course was graphed on the line, E= 0, of learner involvement, meaning that the online course did not produce high or low learner involvement. Figure 11 shows the graph of learner involvement in the face-to-face and online students in this research study.

This approach to learner involvement has not been vetted or confirmed by cognitive load theorists or educational researchers. Therefore, the results on learner involvement concluded in this study should be taken as a preliminary investigation into learner involvement between online and face-to-face courses. Further research should be conducted using this approach to learner involvement and more established approaches to learner involvement.
Figure 11. Learner Involvement of Face-to-Face and Online Mathematics Courses

Technology Effect

One purpose of this study was to research whether there were differences in cognitive load, instructional efficiency, and learner involvement between online and face-to-face college algebra courses. This study did not produce strong results that supported one environment over another environment. A reason for this outcome is that both environments were heavily supported by technology. Any differences in cognitive load could have been mitigated by technology. This study uncovered three examples of how technology could have mitigated differences in cognitive load.
First, the technology, where students completed their homework and tests, allowed them to start and finish the homework and tests at their own convenience within an extended time frame. When a learner is asked to complete a complex task (high intrinsic cognitive load) where the learner is given extended time to complete the task (low extraneous cognitive load), there remains a high germane cognitive load for schema construction. The extended time effect could have enabled learners to focus on the complex task without the time pressure of traditional task completion requirements.

Secondly, students could also redo individual homework problems that were incorrect, and they could retake any test to improve their score within a certain time frame. When a learner is asked to complete a complex task but is also allowed to make corrections on the task, this may decrease intrinsic cognitive load, which creates a high germane cognitive load for schema construction. The corrections effect could have enabled learners to focus on the complex task without the assessment pressure of traditional task requirements.

Lastly, when students were exposed to new mathematical concepts, both learning environments supported the students with immediately accessible multimedia learning aids, videos, animations, and tutorial help. Therefore, students were not actually completing the learning activities alone, processing information through their own schemas. The help features embedded in the technology may provide a similar effect as the worked example effect.

The worked example effect occurs when some conventional problem-solving tasks are replaced with worked examples because it enables learners to focus on the problem states and solution states. Learners can induce generalized solutions or schemas.
However, learners are not forced to study them (Cooper & Sweller, 1987; Paas & van Merrienboer, 1994b; Sweller et al., 1998; Sweller & Cooper, 1985; Zhu & Simon, 1987). When a learner is asked to complete a complex task (high intrinsic cognitive load) where the learner is given help (low extraneous cognitive load), there remains a high germane cognitive load for schema construction. The help effect could have enabled learners to focus on the complex task with the assistance of extra help features.

**Discussion of Research Question #2**

The results of this study revealed many quantitative and qualitative findings, which explained how students demonstrate cognitive presence in a discussion forum. The findings uncovered relationships between cognitive presence and discussion forums that prompt for resolution and relationships between cognitive presence and instructor engagement in a discussion forum.

**Research Question 2**

How did online college mathematics students demonstrate cognitive presence in a discussion forum?

**Cognitive Presence and Discussions that Prompt for Resolution**

The results of this study uncovered relationships between cognitive presence and discussion forums that prompt for resolution. The Apply and Defend discussions produced different consequences of how students demonstrated cognitive presence. Even though all of the discussions prompted for resolution, there were not as many resolution posts as were expected. Integration posts occurred more often than expected, but so did posts with no cognitive presence.
The Apply discussions prompted students to solve and apply a solution as part of their initial post. Because the Apply discussions prompted students to reach resolution in their initial post, it did not create an expectation of cognitive presence in the follow-up posts. Overall, the posts were more likely to reach exploration or contain messages that did not demonstrate any stage of cognitive presence. They were also less likely to reach integration or resolution. However, more individual students reached resolution in the Apply discussions than in the Defend discussions.

The Defend discussions prompted students to post three math problems as their initial post, then pick a couple classmates to solve their math problems. Because the Defend discussions prompted students to reach resolution in their follow-up posts, it created the expectation and possibility of more resolution posts per initial post than in the Apply discussions. More students could reach resolution more often, and students had less opportunity for off topic posts than in the Apply discussions. Overall, the posts were less likely to contain messages that did not demonstrate any stage of cognitive presence. They were also more likely to reach integration or resolution. However, fewer individual students reached resolution in the Defend discussions than in the Apply discussions.

The results of this study inform educational research which seeks to know “what are the question formats that make provision for constructive mathematical responses?” (Engelbrecht & Harding, 2005). Overall, both the Apply and Defend discussions engaged students in cognitive presence. However, the two discussion types engaged students in cognitive presence through different strategies. When students were prompted to apply their solution to a real world scenario or to provide a defense of their solution, they demonstrated high stages of cognitive presence in their online discussion posts. Most
students reached resolution in their initial posts, but they were often not cognitively present in their follow-up posts. The strategy for the Apply discussion prompt was to ask students to reach resolution (the highest stage of cognitive presence) in their initial post, but then no encourage students to reach resolution in their follow-up posts.

The unintended outcome of the Apply discussions was that not as many posts reached high stages of cognitive presence, as compared to the Defend discussions. This may have occurred because students were not graded on the quality of their follow-up posts. If they met the discussion prompt requirement (apply their solution), they could post two follow-up messages that discussed anything they wanted.

When students were prompted to initially post three unsolved problem, wait for a classmate to solve them, then respond by defending their solution, students also demonstrated high stages of cognitive presence in their online discussion posts. Most students initially posted in the triggering event stage, but reached resolution in their follow-up posts. Unlike the Apply discussions, where students were asked to reach the highest stage of cognitive presence in their first post, the strategy for the Defend discussions encouraged students to progress through the stages of cognitive presence.

The unintended outcome of the Defend discussions was that not as many individual students reached resolution, as in the Apply discussions. This may have occurred because students were not graded on the quality of their follow-up posts. If they met the discussion prompt requirement (post three unsolved math problems), and also posted two follow-up messages, then they could have skipped posting the solutions to their math problems without missing any points.
Even though this study only investigated cognitive presence in the online discussion forums, the results of this study may add to the Community of Inquiry research that investigates social presence in online discussions. Students posted more messages that did not demonstrate any stage of cognitive presences in the Apply discussions than in the Defend discussions. However, even though the students often did not demonstrate cognitive presences in their follow-up posts, the posts may demonstrate social presence. Social presence is defined as “the ability of participants in the Community of Inquiry to project their personal characteristics into the community” (Garrison et al., 2000, p.89). Common indicators of social presence include emotions, risk-free expression, and encouraging collaboration (Garrison et al., 2000). Therefore, even though the posts that did not demonstrate any cognitive presence could be seen as a distraction a successful learning experience, these posts are actually very important. They function to support and facilitate critical thinking by the learners.

**Cognitive Presence and Instructor Engagement in a Discussion Forum**

The results of this study uncovered themes in cognitive presence emerging from instructor engagement in online college algebra discussion forums. More instructor posts did not reach any stage of cognitive presence than the four stages of cognitive presence. The data showed that instructor posts in the triggering event stage did not play an important role in the progression of cognitive presence. Additionally, instructors posting in the resolution and integration stage hindered student learning and did not lead to students progressing in stages of cognitive presence. However, instructors posted in the exploration stage of cognitive presence more than the other stages of cognitive presence combined. The exploration posts that
progressed students to higher stages of cognitive presence occurred more often when instructors asked students specific questions about their real life scenario or math problems. Timely responses also helped many students progress in cognitive presence. Student posts reached the exploration and resolution stage of cognitive presence more often when instructors participated in the discussions. Students were less likely to post messages that do not reach any category of cognitive presence when an instructor or classmate asks them a specific question about their real life scenario. Additionally, student integration posts only lead to resolution when an instructor asked a specific question about the real life scenario.

Overall, students progressed in cognitive presence when an instructor asked them a specific question about their math problem or real life scenario in a timely manner. Instructors should post questions to their students that directly ask for an application of their hypothesis or an explanation how they arrived at their hypothesis. When instructors post solutions, explanations, or real life scenarios, students do not progress through to the highest stage of cognitive presence. The results of this study inform educational research which seeks to know “when and how should the instructor intervene, support, and guide students in their work and how can the instructor facilitate productive dialogue among students?” (Engelbrecht & Harding, 2005). This study also adds to the Community of Inquiry research that investigates cognitive presence in online discussions.

**Discussion of Research Question #3**

**Research Question 3**

What was the relationship between cognitive load and cognitive presence in an online college mathematics course?
The results of this study did not uncover any significant relationship between cognitive load and cognitive presence in an online college mathematics course. However, the quantitative findings uncovered a very weak positive linear relationship between cognitive load and the integration stage of cognitive presence. Although this result did not produce a strong linear relationship between cognitive presence and cognitive load, this study was the first attempt to find a relationship between those two theories. Other research methods and measures are needed to further investigate this relationship. In this study the cognitive load data was sourced from the homework, and the cognitive presence data was sourced from the online discussion forums. Even though the same group of students completed the homework and the discussion forums, the students’ amount of mental effort invested in learning the math content did not influence their stage of cognitive presence in the online discussions.

**Limitations of the Study**

1. This study used a convenience sample method as students were not randomly assigned to different groups. Students self-selected into the online or face-to-face modality of the college algebra course.

2. The author of this study was the only researcher to code the data. Not having more than one researcher could affect the reliability of the coding.

3. Whether the college algebra course is taken by online or face-to-face students, it is organized exactly the same and each instructor is expected to follow the predetermined course syllabus. However, this study did not take into consideration the variations in instructor engagement in the face-to-face course.
4. This study generated data from face-to-face math courses and from online math courses. Therefore, the results do not provide insight into a hybrid or asynchronous modality.

5. The cognitive load subjective rating-scale technique was administered within the MyMathLab assignments. Thus, students could complete the assignments at their own convenience and they could redo the assignments within a certain time frame, which could bias the participants’ responses.

**Recommendations for Further Research**

Further research is warranted which investigates cognitive load when students engage with their instructor or their classmates in a conversational learning activity in an asynchronous discussion forum. Cognitive Load Theory researchers have only investigated the cognitive load students experience when they learn in isolation. This study only researched cognitive load when students engage in an online computational learning activity. However, students may also experience cognitive load when they discuss mathematics problems in an asynchronous discussion forum. Students could evaluate their cognitive load as they participate in the discussion forum. Therefore, investigation of the cognitive load students experience when they learn in a community may produce interesting research.

Secondly, further research is also warranted that investigates different environmental characteristics that may influence cognitive load. This study varied the environmental characteristics (online and face-to-face) by controlling the task characteristics and accounting for the learner characteristics. However, this study did not produce strong results that supported one environment over another environment. Since
the environmental characteristics in this study were heavily supported by technology, additional research of other environmental characteristics, such as the extended time effect, corrections effect, and help effect, may produce interesting research.

Further research is also warranted that investigates the relationship between posts that do not demonstrated cognitive presence and posts that demonstrate social presence. This study only investigated cognitive presence in the online discussion forums. The data generated in this study could also be analyzed for social presence and teaching presence. The results of this study lead to more questions about how students demonstrate social presence and its relationship to cognitive presence.

Additionally, further research is warranted which investigates a relationship between cognitive load and cognitive presence. In this study data from were cognitive load and cognitive presence were collected from different sources. Collecting cognitive load data from an asynchronous discussion forum may reveal a relationship with cognitive presence. Additionally, a different perspective of quantifying cognitive presence may reveal a relationship with cognitive load.

Further research is also warranted that investigates this approach to learner involvement (Paas et al., 2005) and more established approaches to learner involvement. This computational approach to learner involvement has not been vetted or confirmed by cognitive load theorists or educational researchers. Therefore, the results on learner involvement concluded in this study should be taken as a preliminary investigation into learner involvement between online and face-to-face courses. Further research into learner involvement and more established approaches to learner involvement will produce stronger results.
Lastly, further research is warranted which investigates the differences in student satisfaction, perceived growth, and other attitudinal factors between face-to-face and fully online mathematics courses. Bernard et al. (2004), Allen et al. (2004), and Zhao et al. (2005) meta-analyses found differences in different measures of effectiveness of distance education to face-to-face education. The results of this study revealed that face-to-face students earned higher homework grades and final course grades than the online students. However, more research into other measure of effectiveness between face-to-face and fully online mathematics courses may produce interesting results.

Conclusions

This research study addressed a gap in the literature for investigating cognitive load over the duration of a face-to-face and online college algebra course and for investigating the relationship between cognitive load and cognitive presence in an online college algebra course. This research uncovered patterns and themes emerging from cognitive presence and cognitive load for students learning mathematics online and face-to-face.

The results of this study revealed that face-to-face students earned statistically significant higher final course grades and homework grades than the online students. The face-to-face math course was slightly more efficient because it produced learners who exerted similar cognitive load as learners in the online course but the learners in the face-to-face earned higher performance score. This study did not produce strong results that support an online learning environmental over a face-to-face learning environment or vice versa.
Online discussion prompts that ask student to apply their solution or defend their solution engaged students in cognitive presence differently. When students were prompted to apply their solution to a real world scenario or to provide a defense of their solution, they demonstrated high stages of cognitive presence in their online discussion posts. When students were prompted to apply their solution to a real world scenario, most students reached resolution in their initial posts, but they were often not cognitively present in their follow-up posts. When students were prompted to provide a defense of their solution, most of the posts demonstrated cognitive presence, but not as many individual students reached resolution.

Students progressed in cognitive presence when an instructor asked them a specific question about their math problem or real life scenario in a timely manner. Instructors should post questions to their students as soon as possible that directly ask for an application of their hypothesis or an explanation how they arrived at their hypothesis. When instructors post solutions, explanations, or real life scenarios, students do not progress through to the highest stage of cognitive presence. Finally, although this study did not find a strong linear relationship between cognitive presence and cognitive load, further research is needed to investigate this possible relationship.
References


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