SUPPORTING INFORMATION FOR:

Gutowski, T., S. Jiang, D. Cooper, G. Corman, M. Hausman, J-A. Manson, T. Schudeleit, K. Wegener, M. Sabelle, J. Ramos-Grez, D. P. Sekulic. 2017. Note on the Rate and Energy Efficiency Limits for Additive Manufacturing. *Journal of Industrial Ecology.*

Summary

This supporting information includes (i) derivation of energy efficiency equation 4, and maximum rate for filament extrusion equation 5 and (ii) derivation of scaling law for filament extrusion.

Derivation of energy efficiency equation 4, and maximum rate for filament extrusion equation 5

Assume that the energy efficiency of the process is the ratio of the minimum energy required to raise the temperature of the powder to the final melt temperature, divided by the nominal actual energy used for this task which can be provided by two separate heating mechanisms; 1. The laser with power P, and 2. The heated chamber at temperature T_c. Using relevant efficiencies for the laser, the electric grid and the heated chamber, as η_{laser}, η_{grid} and $\eta_{chamber}$ one gets,

$$\eta_{energy} = \frac{m\left(c\left(T_f - T_c\right) + \gamma + c\left(T_c - T_{amb}\right)\right)}{Pt \cdot \frac{1}{\eta_{laser}} \cdot \frac{1}{\eta_{grid}} + mc\left(T_c - T_{amb}\right) \cdot \frac{1}{\eta_{chamber}} \cdot \frac{1}{\eta_{grid}}} \quad \text{A1}$$

Using equations 1 and 2 this can be rewritten as,

$$\eta_{energy} = \frac{1 + \frac{(T_c - T_{amb})}{(T_f - T_c) + \gamma/c}}{\frac{1}{\alpha \cdot \eta_{adiabatic}} \cdot \frac{1}{\eta_{laser}} \cdot \frac{1}{\eta_{grid}} + \frac{(T_c - T_{amb})}{(T_f - T_c) + \gamma/c} \cdot \frac{1}{\eta_{chamber}} \cdot \frac{1}{\eta_{grid}}} \quad \text{A2}$$

Substituting values for the relevant parameters from Tables 1 and 2 for steel and aluminum powders, this can be approximated as

$$\eta_{energy} \cong \alpha \cdot \eta_{adiabatic} \cdot \eta_{laser} \cdot \eta_{grid} \qquad \text{A3}$$

Note that because of the high chamber temperature used (relative to the melt temperature) for polymer powder printing, a different result is obtained. Additionally, one could add additional power requirements in an alternative derivation.

Derivation of scaling law for filament extrusion

Referring to Fig 4, assume a constant wall temperature and that the thermal resistance is dominated by conduction in the cylindrical polymer filament of diameter D = 2R, and length L, where L>>D. The constant wall temperature heat transfer condition was interpreted as an equivalent convective heat transfer with infinitely large heat transfer coefficient. And further assuming that the filament is melted and ready to print when the dimensionless temperature has gone from a value of 1.0 to 0.1 at the centerline of the filament. This corresponds to transient heat conduction when the dimensionless time, the Fourier Number Fo, obtains a value of 0.5 [Lienhard 2011], that is,

$$Fo \equiv \frac{\alpha t}{R^2} = \frac{kt}{\rho c R^2} \cong 0.5 \qquad \text{A4}$$

Hence the time to bring mass $\rho L \pi R^2$ to the onset of melting is approximately,
$t \cong \frac{\rho c R^2}{2k}$, The resulting mass process rate is then given by

$$\dot{m} = 2\pi \frac{k}{c} L \qquad \text{A5}$$

Reference:

J. H. Lienhard and J. H. Lienhard, (2011) *A heat transfer textbook*. Mineola, N.Y.: Dover Publications