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THICKNESS DESIGN PROCEDURE
FOR PORTLAND CEMENT CONCRETE PAVEMENTS

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SYNOPSIS

Past experience in Kentucky indicated that thickness designs using portland cement concrete best agreed with criterion used in the Portland Cement Association's design method for 2 or 3 million EAL's or less. For EAL's greater than 3 million, past experience best agreed with criterion developed from the AASHO Road Test. Research reported herein indicates the two criteria become asymptotic at approximately 2.5 million EAL's. The merger of the criteria is presented, and the combined criterion is coupled with the principal of equal work as defined in classical physics to produce thickness design curves for portland cement concrete pavements.

Research has indicated that a general conic equation can be used to satisfactorily estimate the work, as calculated by the Chevron N-layered program, at the bottom of the portland cement concrete slab under an 18-kip single axleload. The transition from a tensile strain to a work criterion is presented.

The thickness of portland cement concrete varied up to a maximum of approximately 0.15 inch for the same CBR and design EAL when the thickness of crushed-stone base was varied from 3 to 6 inches. Therefore, the design thickness of the portland cement concrete slab is relatively insensitive to changes in thickness of the crushed-stone base. Resulting thickness design curves are presented for a concrete elastic modulus of 4.2 million psi (typical of Kentucky conditions).
INTRODUCTION

One aspect of pavement design is the selection of the thickness of the pavement and its various components sufficient to support vehicular loadings and to transfer those loadings through successive layers of the pavement -- surface, base, and subgrade -- to the soil on which the pavement rests. If the load is excessive or if the supporting layers are not sufficiently strong, repeated applications of the vehicular loadings will cause rutting and cracking that ultimately lead to a complete structural failure of the pavement. The structural design scheme suggested in this report provides a procedure by which the load-carrying capabilities of a portland cement concrete slab and of the soil upon which the slab rests are not exceeded.

Generally, pavements often are referred to as rigid (portland cement concrete) or flexible (asphaltic concrete), depending upon pavement stiffness relative to the subgrade. This distinction became artificial as the magnitude and frequency of loads became greater and as pavements correspondingly became thicker and stronger.

The primary difference between types of pavements is the manner in which they distribute vehicular loads to the subgrade. A portland cement concrete pavement, because of its high modulus of elasticity for all ranges of temperature, tends to distribute the load over a relatively large area, and a major portion of the structural capacity is provided by the slab itself. As a result, a major factor in the design of portland cement concrete pavements is the structural strength of the concrete. Uniformity of support is also a major factor, but minor variations in subgrade strength have little influence upon the structural capacity of the pavement.

The load-carrying capacity of asphaltic concrete pavements derive from the load-distributing characteristics of a layered system. Such pavements may consist of a series of layers, or a single layer, with the strongest materials typically at or near the surface. The strength of such pavements lies in the buildup of layers to distribute the load over the subgrade. The design thickness of the pavement is influenced appreciably by the strength of the subgrade.

To determine the thickness of a portland cement concrete pavement from design charts and tables, it is necessary to know the EAL's (equivalent axleloads) and the CBR (strength) of the subgrade. Normally, traffic volumes are estimated in connection with needs studies and in the planning stages for all new routes and for major improvements of existing facilities. Whereas the anticipated volume of traffic is an important consideration in the geometric design of a highway facility, composition of the traffic stream in terms of axle weights and lane distributions is essential to the structural design of pavements. Traffic volumes used for EAL computations should therefore be reconciled with other planning forecasts of
traffic. Even though predictions of traffic volumes may be reasonable, estimates of EAL's are also dependent upon
predictions of vehicle types and loadings over the design
life.

Computation of EAL's involves estimates of the total
number of vehicles during the design life and of multiplying
factors for various vehicle types, loading configurations,
and loads to convert traffic volumes to EAL's. Ideally,
yearly increments of EAL's could be calculated and summed.
This approach would permit consideration of anticipated
changes in legal weight limits, changes in styles of cargo
 haulers, and changes in routing.

The first set of thickness design curves for asphaltic
concrete pavements for use in Kentucky (1) was
developed, in 1948. Extensive laboratory tests of soils were performed
using several methods, and the CBR method was chosen as the
basis for evaluating soil strength. Performance correlated
best with minimum laboratory CBR's. Traffic histories were
estimated for the pavements. Those pavements with
approximately the same traffic histories were grouped, total
pavement thickness was plotted versus CBR, and data points
were coded "good" or "bad", depending upon the performance
of the pavements. Best-fit curves were drawn to separate
the failed from the unfailed. The 1948 curves were based on
failure boundaries and performance envelopes.

By 1957, a need to update the 1948 curves and to extend
the curves to increased traffic loadings became apparent.
Another extensive series of field tests and analyses
resulted in the 1959 Kentucky design curves (2). Those
curves were in use prior to the AASHO Road Test, were
verified by field tests, and had precedence over the AASHO
thickness design system that was developed later.

By the mid-1960's, computers had been developed with
enough sophistication, speed, and memory capacity to permit
the development of the Chevron N-layer program to analyze
pavements using elastic theory (3). Using that program,
the 1973 Kentucky design curves for asphaltic concrete
pavements (4) were prepared. Experience confirmed the
design curve for pavements with 1/3 of the thickness being
asphaltic concrete and 2/3 being dense-graded aggregate.
Based upon elastic theory, thickness curves were created for
other proportions. Experimental pavements were designed,
constructed, and tested beginning in 1971. Field test data
have been matched with theoretical solutions and have
confirmed both thickness curves and the method of estimating
pavement fatigue caused by traffic loadings.

Design curves and procedures reported in 1981 (5,
6) for asphaltic concrete pavements for application
specifically to Kentucky conditions evolved over several
decades. Design curves have been refined and updated and
correlated with a broad range of field experience and
theoretical analyses.

A similar background and history of design guides for
portland cement concrete pavements in Kentucky is yet to be
developed. Historically, and currently, portland cement
concrete pavements have been designed using the Portland
Cement Association's method (7) or the method derived from the AASHO Road Test (8). For many years, it was felt that construction procedures would not permit small increments of thicknesses of portland cement concrete pavements. Thus, comparable designs of portland cement concrete and asphaltic concrete pavements specifically for the Kentucky environment may not have resulted.

Design curves and procedures reported herein have been developed using concepts and analyses similar to those used to produce the asphaltic concrete design curves. Additionally, the analyses have been refined, based on concepts of work, and curves presented in this report reflect a first approximation of performance experience of portland cement concrete pavements in Kentucky. Because of the similar bases for development, comparable designs for the two generic types of pavements now can be achieved with confidence.

ELEMENTS OF DESIGN PROCEDURE

SUBGRADE SUPPORT

Several procedures were utilized in a 1948 testing program to evaluate the load-carrying capacity of the subgrade and consisted of plate bearing, North Dakota cone, in-place CBR (California Bearing Ratio), and soaked laboratory CBR tests. The best correlations of pavement performance were with the soaked laboratory CBR (1), which differs from the ASTM method in one aspect. ASTM specifies the sample be soaked for three days (9); the Kentucky method allows soaking until swelling ceases. The soaked CBR was the basis for the 1959 curves (2) and still is the basis for the 1981 curves for asphaltic concrete pavements (5). Correlations have been made with the AASHTO soil support scale (10) and with elastic moduli on the basis of field test data and the Chevron N-layer computer program. A literature review (11, 12) indicated that the elastic modulus of clay soils could be estimated from laboratory tests by

\[ E = 1500 \times CBR, \]

in which \( CBR \) = value of California Bearing Ratio obtained by the Kentucky procedure and \( E \) = elastic modulus of the subgrade.

Research indicates the factor "1500" is reasonably valid for clays, but possibly should be different for sands, gravels, rock, etc. Thus, the factor "1500" may be modified and a new scale fitted, but design thicknesses based upon elastic theory remain valid. Soil at the AASHO Road Test site was assigned a soil support value of 3.0. Samples of the soil were obtained and subjected to the Kentucky CBR test procedure. The equivalent Kentucky CBR was 5.2 (10). The same relationships have been used in the development of
thickness design curves for portland cement concrete pavements.

CHARACTERISTICS OF PAVING MATERIALS

Portland Cement Concrete

Young's modulus of elasticity for portland cement concrete may vary from 3 to 6 million psi, depending upon quality and type of aggregate, cement content, and water-cement ratio. Analyses were made using the N-layer program in which the modulus was varied over this entire range on increments of 1 million psi. Final design curves were based upon a modulus of elasticity of 4.2 million psi, which is representative of AASHTO conditions (8).

The 1956 Kentucky standard specifications (13) required the minimum expected strength of paving concrete at 28 days to be 3,500 psi in compression and 600 psi for the modulus of rupture. In 1965, the requirement for the modulus of rupture was lowered to 550 psi and then was deleted and only the compressive strength specified in 1976. The requirement for compressive strength has remained at 3,500 psi. For acceptance purposes, compressive strength has been judged as a more routinely reliable test result. For thickness design purposes, modulus of rupture is of greater significance; therefore, a relationship between compressive strengths and thickness design strengths (modulus of rupture) is important.

The Portland Cement Association suggests that the modulus of rupture is approximately eight to ten times the square root of a concrete's compressive strength (14). For a compressive strength of 3,500 psi, the modulus of rupture would range from 473 to 592 psi. Another relationship indicates that the modulus of rupture is equal to the 0.79th power of the compressive strength. On that basis, the modulus of rupture would be 630 psi for concrete having a compressive strength of 3,500 psi. Still another relationship shows the modulus of rupture to be about 15 percent of the compressive strength (15). Derucher and Heins suggest the modulus of rupture to be 16 percent for 3,000-psi concrete and 14 percent for 4,000-psi concrete (16).

Inasmuch as quality concrete continues to gain strength with time, it may be advocated that the 90-day modulus of rupture should be used for pavement thickness determinations. Figure 1 illustrates the increase in the strengths of paving concretes at the Road Test with aging (17). It is noted that the ultimate flexural strength was approximately 800 psi. The ratio of 90-day strength to 28-day strength is approximately 1.10 to 1.14. For 28-day compressive strengths of 3,500 psi, the 90-day modulus of rupture would range from 496 to 632 psi. It is noted from Figure 1 that the 28-day compressive strengths at the Road Test were approximately 4,500 psi. Current practice for the design of portland cement concrete pavements indicates the modulus of rupture to range from 600 to 750 psi (18).

It has been suggested that the modulus of rupture of concrete can be approximated by dividing the modulus of
elasticity by a factor varying from 6170 to 6535 (19). For a modulus of elasticity of 4.2 million psi, the modulus of rupture would range from 643 to 681 psi. It has been reported (20) that the coefficient of variation of the modulus of rupture for most concrete paving projects is approximately 10 to 15 percent. Using this variation, the modulus of rupture could range from 547 to 783 psi.

Figure 2 illustrates relationships between tensile strain at the bottom of a portland cement concrete pavement and repetitions of 18-kip load equivalencies for various combinations of pavement thicknesses and moduli of rupture. Stresses and strains under an 18-kip single axleload on various thicknesses of portland cement concrete slabs were obtained using the Chevron N-layer program. For a given modulus of rupture, stresses for respective pavement thicknesses were converted to stress ratios, which, when coupled with the "Present Curve" in Figure 3 (Portland Cement Association's criterion), yield the respective number of load repetitions. Data from Figure 4 (discussed later in this report) for 5-, 6.5-, 8-, and 9.5-inch pavements at the Road Test were transferred to Figure 2. Average numbers of repetitions associated with the above pavement thicknesses were calculated. (The 9.5-inch pavement had only one data point and was eliminated for this analysis.) The average modulus of rupture was calculated for each pavement thickness, and the values were 675, 615, and 575 psi, respectively.

The use of a modulus of rupture of approximately 600 psi would seem to be reasonable, but probably conservative. There is little evidence of fatigue in portland cement concrete pavement slabs in Kentucky. There are no indications of "punch-outs" and only a few sites of longitudinal cracking in wheelpaths. Most deterioration has been observed at joints, and that involves a different mode of failure. Serviceability ratings of portland cement concrete pavements may be more indicative of conditions at joints than of slab conditions. The rationality of using a modulus of rupture of approximately 600 psi is dependent upon the use of 3,500-psi paving concrete. It is likely that higher-quality concrete is obtained on most construction projects. However, it seems that designs should be based on the more conservative quality of concrete called for in the specifications. This does mean, though, that the pavement performs during a significant portion of its life at strengths greater than the design value. This condition is paralleled in the design of asphaltic concrete pavements with the minimum laboratory CBR of the subgrade. The use of such "minimum" design parameters introduce an unknown "factor of safety." This factor of safety for asphaltic concrete pavements has been minimized in part by the correlation of pavement performance with minimum laboratory CBR. The correlation of the performance of portland cement concrete pavements with modulus of rupture of the concrete is still to be documented. As will be discussed later, the use of a modulus of rupture of 600 psi seems to allow the merger of the Portland Cement
Association's criterion and the experience of the AASHO Road Test.

Dense-Graded Aggregate

Kentucky is blessed with high-quality limestone and sandstone aggregates that may be crushed to produce a dense-graded product with very low void contents. Dense-graded aggregate has a very low tensile strength, attributable to a small amount of cementation.

The 1968 analyses of asphaltic concrete pavements assigned one modulus value to the dense-graded aggregate without regard to CBR values. Later analyses were made allowing the modulus of the dense-graded aggregate to vary as a function of the moduli of elasticity of the confining layers. Figure 5 (21) illustrates the relationship of dense-graded aggregate modulus as a function of the asphaltic concrete and subgrade modulii (CBR). Figure 6 illustrates a similar concept and relationship, but in terms of the modulus of elasticity of the portland cement concrete slab.

Design curves presented in this report assume the portland cement concrete slab is placed on a high-quality dense-graded aggregate subbase that may vary from 3 to 6 inches in thickness. If designs for pavement systems incorporating other thicknesses of subbase are desired, different design curves would be required.

TRAFFIC STREAM

The traffic on a pavement is a composite of many styles and sizes of vehicles carrying a wide range of loads. Pavements fatigue under loading, whether the load is legal or not. Ignoring the existence of illegal loads results in underdesigned pavements and contributes to "premature failure."

Axle configurations and the load thereon greatly alter the rate of accumulation of fatigue by the pavement system (17). Adding axles to a given vehicle may, or may not, reduce the fatigue of the pavement. A controlling factor is the suspension system for that configuration. One that can distribute the load equally to all axles of the configuration will reduce the fatigue damage to a minimum for that number of axles. So-called "drop axles" have a high probability of causing an uneven distribution of load among the axles in the closely spaced group. If the axle is lowered so almost no load or a large portion of the load is carried by that axle, then the damage caused by the axle group will be severe compared to the equal-loading situation.

The axle configuration is significant in inducing fatigue damage caused by the total load on that vehicle. The number of tires on an axle also causes a variation in fatigue damage. Two single wide tires will cause more damage than four regular tires (22).

Through the years, the size as well as the number of trucks on the highways have been increasing. Development of the interstate system generated more traffic than was
dreamed possible. Increasing the legal load limits greatly increases the rate of fatigue damage.

Traffic Stream Parameters
Traffic volumes may be estimated in a number of ways. Each is dependent upon the type of data base available for analysis. Loads to be supported by a pavement system are related not only to the volume of vehicles but also are dependent upon the distribution of various types of vehicles (and their associated weights). As with traffic volumes, estimates of the proportions of various vehicle types in the traffic stream can be obtained in a number of ways. The distribution of traffic among the various lanes of a multilane facility also is required. Again, these distributions can be obtained in a number of ways.

Axleloadings
An important attribute in determining the equivalent axleloads is the damage or load equivalency factors associated with individual axleloads or with various vehicle types. The load equivalency is a measure of damage to the pavement relative to the damage caused by an 18-kip axleload (Equivalency Factor = 1.0).

Analysis of fatigue data for flexible pavements (22) showed that the equation used to develop thickness design nomographs was a best-fit regression. The same technique was used to investigate the fatigue data for rigid pavements (17). The regression equation and standard deviation were calculated for the relationship between the thickness of portland cement concrete pavements and equivalent 18-kip axleloads (Figure 4) using 1981 Kentucky load equivalency factors from Table 1 (6). The line representing the best-fit regression plus 1.2816 standard deviations (so as to include 95 percent of the data points) is also indicated in Figure 4. Note that an approximate thickness of 9.2 inches is required at 8 million EAL's (for the mean plus 1.2816 standard deviations). This is in reasonably close agreement with design (but not necessarily performance) experience in Kentucky.

FATIGUE ANALYSIS

STRAIN ENERGY
The "work" done by a force when its point of application is displaced is the product of that force (parallel to the direction of movement) and the displacement. When work is done on some systems, the internal geometry is altered in such a way that there is a potential to "give back" work when the force is removed, and the system returns to its original configuration. This stored energy is defined as strain energy. Strain energy per unit volume at a given point in the body is the strain energy density at that point.

If the force acting on a body has varied linearly from zero to $kx$, its average value is $kx/2$, and the work done can
be calculated as the product of the average force and its displacement (23):

\[ \text{Work} = (kx/2) \times kx^2/2. \]

Since this work has not supplied the body with either kinetic energy or gravitational potential energy, and since no work was done against friction, some part of the system must have been supplied with elastic potential energy (or strain energy) \( V \) of

\[ V = kx^2/2. \]

Under an external wheel load, a pavement is stressed and strained from zero to some maximum values, and a measure of "work" can be expressed as

\[ \text{Work} = (1/2) (\text{stress}) (\text{strain}). \]

To calculate work using output from the Chevron N-layered program, it is necessary to obtain an expression for strain energy density.

Strain energy density (inch-pounds per cubic inch) is a function of the Young's modulus of elasticity and Poisson's ratio of the material and the nine strain (or stress) components; however, it is independent of the coordinate system. Stress and strain components, referenced to a local cylindrical coordinate system, for each load can be calculated by the Chevron program (3). The classical equation for strain energy density derived by Sokolnikoff (24) is as follows:

\[
W = \frac{1}{2} \sum \left[ (1/2) \lambda \epsilon_{ij} \epsilon_{ij} + G \epsilon_{ij} \epsilon_{ij} \right] = \frac{(1/2) \lambda \nu^2 + G (\epsilon_{11}^2 + \epsilon_{22}^2 + \epsilon_{33}^2 + 2 \epsilon_{12}^2 + 2 \epsilon_{23}^2 + 2 \epsilon_{31}^2)}{\lambda + 2G},
\]

in which \( W \) = strain energy density, or energy of deformation per unit volume;
\( \epsilon_{ij} \) = \( i,j \)th component of the strain tensor;
\( G = E/[2(1 + \mu)] \), the "modulus of rigidity" or the shear modulus;
\( E \) = Young's modulus of elasticity;
\( \mu \) = Poisson's ratio;
\( \lambda = E\mu/[2(1 + \mu)(1 - 2\mu)] \); and
\( \nu = \epsilon_{11} + \epsilon_{22} + \epsilon_{33} \).

Strain energy density may be calculated (24) using stress components from

\[
W = \frac{-\mu \theta^2/2E + [(1 + \mu)/2E] (\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2) + [(1 + \mu)/E] (\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)}{\lambda + 2G},
\]

in which \( \theta = \sigma_{11} + \sigma_{22} + \sigma_{33} \) and
\( \sigma_{ij} = i, jth \) component of the stress tensor.

Inspection of Equation 5 shows that the term \( \frac{E}{C^2(1 + \mu)} \) is contained by means of the terms \( \lambda \) and \( G \). Also, it is noted that the strain components are squared. Having calculated strain energy density, "work strain" (22) may be obtained from

\[ \varepsilon_w = \left( \frac{2 W}{E} \right)^{0.5} \]

in which \( \varepsilon_w = \) work strain.

The associated "work stress" is given by \( E \varepsilon_w \).

WORK AS THE BASIS FOR PAVEMENT DESIGN

Two approaches have been used in the development of thickness design curves for portland cement concrete pavements. The first approach (described below in the section entitled STRAIN-WORK STRAIN RELATIONSHIPS) involved a transition from tensile stress and (or) tensile strain to work strain as defined by Equation 7. Initial design curves were developed by graphical techniques. The second approach (described below in the section entitled DIRECT USE OF WORK IN PAVEMENT DESIGN) involved an analyses of work as defined by Equation 6. The fatigue relationship was transformed from a tensile strain to a work basis.

STRAIN-WORK STRAIN RELATIONSHIPS

Interpretations of Work Strain

Admittedly, work strain is not a true strain because Poisson's ratio has not been eliminated prior to taking the square root; however, it is of the same order of magnitude as any of the strain components. Calculating the work strain is a minor effort since all terms of the equations are either required input to, or calculated output of, the Chevron N-layer program (3, 25). Work strain is also the composite, or net effect, of all strain components and thus is an indicator of the total strain behavior. There is a direct correlation between a strain component and work strain.

Some thickness design systems for portland cement concrete pavements are based partially upon tensile strain criteria at the bottom of the concrete slab. The system proposed herein is based upon the concept of "equal work," work strain, and reported laboratory test data wherein measured tensile strains approximated the tangential strain components at the bottom of the concrete slab. The tangential component is generally the largest in magnitude, but the radial component often is nearly as large. Historically, only the tangential component has been utilized because laboratory test data yielded one component of tensile strain. The net effect of all components of
strain (work strain) can be correlated with any component of strain. Thus, design systems based upon one component of strain may be converted to a design system that utilizes the net effect of all component strains. The load equivalency relationships presented herein are based on work strain. All comments above concerning component strains also apply to component stresses.

Equivalency Factors and Road Test Repetitions of Loads

Load equivalency factors were determined, using the 1981 Kentucky load equivalencies, for the various loads applied to the portland cement concrete pavements at the AASHO Road Test. The analyses were based upon output from the Chevron program, and the results are shown in Figure 7.

Stress Criterion

Figure 3 illustrates the relationship between working stress, modulus of rupture, and number of repetitions to failure of an 18-kip axleload. The curve labelled "PCA Present Curve" (26) was used in this analysis and was extrapolated to 10 million repetitions.

Analysis of AASHO Road Test Data

The Chevron N-layer computer program (based on elastic theory) was used to analyze data for the portland cement concrete pavements at the AASHO Road Test. All components of stress and strain at the bottom of the concrete slab were combined using superposition principles to calculate an equivalent value of "work stress" and "work strain." Figure 8 illustrates the direct correlation between work strain and the tensile strain component. While the scatter of data is small about the best-fit line, the wide range in values is a function of the different loads and pavement thicknesses evaluated at the AASHO Road Test.

Young's modulus of elasticity for the concrete slab was varied in the analyses from 3 to 6 million psi on 1-million psi increments. Figure 9 illustrates the relationship between work strain and the thickness of a portland cement concrete slab. Each curve in the figure is for a specific CBR subgrade overlain with 6 inches of crushed stone base. Similar figures were prepared for work stress and tensile strain.

Data from Figure 9 were replotted in the form shown in Figure 10 to facilitate interpolations to obtain the relationship for a CBR 5.2 subgrade. Figure 11a combines the interpolated curves for CBR 5.2 from Figure 10 to obtain the curves for a range of moduli of elasticity of the concrete. Figure 11b illustrates the curves for CBR values shown in Figure 9 and for a modulus of elasticity of 4.2 million psi.

Concept of Equal Work

"Work strain" and "work stress" can be substituted for the single component stress and strain in Equation 4 to obtain
Work = \( \frac{1}{2} \) (work stress) (work strain).

Figures 12, 13, and 14 illustrate the relationships among work stress, work strain, Young's modulus of elasticity, and the product (work) from Equation 8. Note that the lines of equal work are perpendicular to lines of equal moduli of elasticity (see Figure 14). Thus, for a given level of work, strains and stresses vary to maintain a constant value of work. Figures 12 and 13 illustrate the relationships among Young's modulus of elasticity, work stress, and work strain as a function of work given by Equation 8.

**DIRECT USE OF WORK IN PAVEMENT DESIGN**

Strain energy density, or work per unit volume, at a point in a body can be determined from Equation 6 and is dependent upon Young's modulus of elasticity and Poisson's ratio of the material at that point in the body and the squares of the individual strain components, or their sum squared. The Chevron N-layered computer program calculates all nine components of strain, and the program was extended to calculate the strain energy density.

Recent studies of flexible pavements showed that strain energy density at the top of the subgrade provided the most sensitivity for fatigue analyses, and the next most sensitive location was at the bottom of the asphaltic concrete layer. Therefore, the top of the subgrade was investigated first for portland cement concrete pavements. Figure 15 shows the relationship between unit work (equal and opposite to strain energy density) and thickness of the portland cement concrete slab. As expected, the amount of work was significantly decreased for an increase of 1 inch of slab thickness. However, a tenfold increase in subgrade modulus (CBR) produced virtually no change in the amount of work for the same slab thickness. Thus, it was concluded that the top of the subgrade was not the most sensitive location for thickness design purposes.

Figure 16 illustrates the relationship between work at the bottom of the slab and the thickness of the slab. A 1-inch increase in the thickness of the pavement significantly decreased the amount of work caused by an 18-kip single axleload. However, an increase in the subgrade modulus also resulted in a significant reduction in work. Therefore, for thickness design purposes, the most sensitive location was assumed to be at the bottom of the portland cement concrete slab. The curves in Figure 16 are for a wide range of realistic moduli of the materials involved and for a wide range of thicknesses of portland cement concrete pavement, but for a fixed thickness of 6 inches of crushed-stone base. Analyses also were made for the same range of variables and crushed-stone bases 3, 4, and 5 inches thick.

The shape of the family of curves in Figure 16 suggested that they could be expressed by the general conic equation. Earlier research of the behavior of asphaltic concrete pavements indicated a similar conclusion and resulted in an excellent agreement between solutions from
the Chevron program and the fitted general conic equation. Thus, the same approach was attempted for portland cement concrete pavements. Matrix analyses were used to determine the numerical values of the coefficients and constant in the conic equation

\[
\log(\text{Work}) = a(X1)^2 + b(X1)(X2) + c(X2)^2 + d(X1)(X3) + e(X3)^2 + f(X2)(X3) + g(X4)^2 + h(X1)(X4) + i(X2)(X4) + j(X3)(X4) + k(X1) + m(X2) + n(X3) + p(X4) + q,
\]

in which \(X1\) = thickness of portland cement concrete, inches, \(X2\) = thickness of crushed-stone base, inches, \(X3\) = modulus of elasticity of portland cement concrete, million psi (for a modulus of 3 million psi, enter as 3.0), \(X4\) = modulus of elasticity of subgrade,ksi, and \(a - q\) are coefficients and the constant determined by matrix analyses (see Table 2).

The accuracy of the fitted equation can be seen in Figure 17. Equation 9 and the coefficients in Table 2 produced quite accurate results for a range of CBR values from 3 to 30. Sensitivity analyses revealed that the family of curves, Figure 16, was best expressed by a parabolic form of the conic equation. Equation 9 can be solved using programmable desk-top calculators. Table 3 gives the coefficients for specific combinations of crushed-stone thicknesses and moduli of portland cement concrete and the subgrade.

**FATIGUE CRITERIA**

**STRESS-RATIO FATIGUE CRITERION**

The combination of a fixed value for Young's modulus of elasticity, modulus of rupture, and the stress-ratio versus repetitions relationship for the curve labeled "PCA Present Curve" in Figure 3 permits the development of a tensile strain versus repetitions criterion. The exact position of a curve is a function of the specific combination of values for the moduli of rupture and of elasticity.

The Portland Cement Association's and the AASHTO methods of selecting thicknesses for concrete pavements are based on limiting the tensile strain (or stress) in the slab to some tolerable level. The design method developed by the Portland Cement Association was based upon model and full-scale tests (7). The Arlington Tests were conducted in the 1940's and would be applicable to lower-volume roads in today's highway environment. Conversely, the AASHO Road Test was conducted in the early 1960's and had high volumes of trucks over a relatively few years.

Inspection of Figure 7 indicates that the strain criterion for 10 million repetitions of an 18-kip single axle load would be too low and result in slab thicknesses unreasonably thick using the Portland Cement Association's design method. Conversely, the AASHO criterion would result...
in slab thicknesses that would be too thin in the lower-volume ranges. For a modulus of elasticity of 4.2 million psi, combined with the mean plus 1.2816 standard deviation line from Figure 8, thicknesses at specific numbers of repetitions were converted to tensile strain using the relationships among tensile strain, thickness, and a CBR of 5.2. Those points fix the relationship for the AASHO portion of Figure 7. There should be a value of the modulus of rupture that would permit the strain criterion of the Portland Cement Association to become tangent to the AASHTO strain criterion. Furthermore, the literature on the characteristics of concrete discussed previously suggests the value of the modulus of rupture should have a value between 475 and 680 psi. Figures 2 and 7 indicate the modulus of rupture is approximately 600 psi when the two criteria merge into one smooth curve.

FATIGUE ANALYSIS

Figure 7 illustrates the relationship between tensile strain in the "xx" direction and 18-kip EAL's and the merger of the limiting strain criteria used in the Portland Cement Association's and AASHTO design methods. For Kentucky conditions, portland cement concrete was assumed to have a Young's modulus of elasticity of 4,200,000 psi. The relationship between tensile strain and work is given by

\[
\text{Work} = 0.5 \cdot \exp[0.1534709969 + 1.010508693 \cdot \log(\text{exx})^2],
\]

in which Work = opposite of strain energy density at the bottom of the portland cement concrete slab, inch-pounds per cubic inch, \( E \) = Young's modulus of elasticity of the concrete, psi, and \( \text{exx} \) = tensile strain in the "xx" direction at the bottom of the concrete slab.

Converting tensile strain in Figure 7 using Equation 10 to a criterion based upon "work" resulted in the relationship shown in Figure 18 and is identical to the term "work" used in Equation 9.

Figure 7 illustrates the merger of Portland Cement Association and AASHTO Road Test criteria at two million 18-kip single axleloads. The criteria curve for equivalent axleloads of two million or less is given by

\[
\log Y = a + b \cdot \log X + c(\log X),
\]

in which \( X \) = number of 18-kip equivalent axleloads, \( Y \) = tensile strain, work strain, or work (whichever is being used) at the bottom of the portland cement concrete pavement, and \( a, b, c \) = coefficients of regression (see Table 4).

For equivalent axleloads greater than two million, the criteria curve is defined by

\[
\log Y = e + f \cdot \log X,
\]
in which \( e \) and \( f \) = coefficients of regression (see Table 4).

THICKNESS DESIGN CURVES

DEVELOPMENT OF DESIGN CURVES

Equation 9 and values in Table 2 were used to construct the thickness design curves shown in Figure 19. When the CBR, thickness of the crushed-stone base, and modulus of the portland cement concrete were fixed, the thickness of the slab was determined. Analyses of resulting thicknesses for a given CBR and EAL indicated a maximum difference of 0.15 inch when the thickness of the crushed-stone base was varied from 3 to 6 inches. The usual difference was between 0.05 and 0.10 inch (low to high EAL's, respectively).

This variation in the required thicknesses is well within construction tolerances and thus deemed to be not worth the trouble to have a separate set of thickness design curves for each thickness of crushed-stone base. The rigidity of the portland cement concrete slab apparently overshadowed the effects due to decreased base thicknesses (at least in the range of crushed-stone base thicknesses investigated). Thus, the required thicknesses of portland cement concrete slab were averaged for the various thicknesses of dense-graded aggregate base to obtain design curves shown in Figure 19. Figure 19 can be used to determine thickness designs for portland cement concrete pavements using 18-kip EAL's calculated by applying 1981 Kentucky load equivalency factors for asphaltic concrete pavements.

CURRENT DESIGN PRACTICE

Historically, the controlling total thickness of an asphaltic concrete pavement in Kentucky has been 23 inches on a CBR 7 subgrade for 8 million 18-kip EAL's. The 23 inches consists of 7.7 inches of asphaltic concrete over 15.3 inches of dense-graded aggregate. Figures 9, 11b, and 18 were used to determine the design thicknesses for rigid pavements on a CBR 3 subgrade (and 6 inches of dense-graded aggregate) using the 1981 Kentucky load equivalency factors for asphaltic concrete pavements.

The thickness of portland cement concrete pavements for interstate-type highways historically have been taken to be 10 inches over 6 inches of dense-graded aggregate at 8 million 18-kip EAL's calculated using flexible equivalency factors. Figure 19 shows that a CBR 3 subgrade and 8 million EAL's require a 9.9-inch slab. Kentucky's toll road designs have been associated with 4 million 18-kip repetitions, and the design thickness was taken to be 9 inches of portland cement concrete over 4 inches of dense-graded aggregate. For the CBR 3 curve in Figure 19, the required portland cement concrete pavement would be 8.8 inches. Thus, the two designs of the toll roads are very similar.
OVERLAY THICKNESS DESIGNS

Thickness design curves in Figure 19 may be used to design fully bonded portland cement concrete overlays only on rigid pavements. However, this requires a method of evaluating the conditions of the existing rigid pavement and determining the equivalent effective thickness of new (unfatigued) portland cement concrete having a modulus of elasticity of 4.2 million psi. The rigid overlay thickness is the difference between that thickness required to support the future expected traffic and the "behavioral" thickness of the existing pavement.

FUTURE RESEARCH

A rational method for designing asphaltic concrete overlays for rigid pavements is needed. The design scheme should provide designs for portland cement concrete pavements that might be fragmented into various particle sizes or left intact. Two construction projects (I 71 in Gallatin County and I 64 in Shelby County) may provide much insight into the behavior of fragmented and overlaid pavements.

Method(s) to evaluate the existing condition and value of portland cement concrete pavements need to be further refined. Current research is just beginning to develop potential methodologies. However, phenomena related to the behavior of portland cement concrete slabs and joints are not yet fully understood.

No investigation has been made of the effects of nonuniform axleload distributions within tandem and tridem assemblies on portland cement concrete pavements. The effect of nonuniform loading on asphaltic concrete pavements was significant, but the impact on portland cement concrete pavements is not yet evaluated.

A cursory series of tests were made on I 71 to study the effects of time of day (temperature distribution within the pavement) upon the behavior of portland cement concrete slabs. At several joints, Road Rater data revealed that the slab was resting on the four corners and arching diagonally across the slab and along the joint. Thus, corner failures or breaks may be a function of a heavy load in the center of the slab being supported only by the corners of the slab that rest upon the foundation. This condition needs additional field studies and theoretical analyses to determine whether the thickness design methodology should be predicated on conditions other than center-of-slab loadings.

An analysis of quality-control data for paving concretes might assist in selecting a more definitive value for the modulus of rupture to use in pavement design. The performance of portland cement concrete pavements also should be correlated with the modulus of rupture.

Joints in portland cement concrete pavements introduce special problems and considerations into the design of rigid pavements. The effects of joints on thickness design, both for original construction and for overlays, is not known.
The design, construction, performance, and maintenance of joints are subjects for study.

REFERENCES


25. Warren, H.; and Dieckmann, W. L.; "Numerical Computations of Stresses and Strains in a Multiple-Layered...

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### Table 3. Coefficients for Parabolic Equation for Specific Combinations of Variables

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<thead>
<tr>
<th>CBR (MILLION)</th>
<th>3 INCHES DGA</th>
<th>6 INCHES DGA</th>
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<td></td>
<td>C</td>
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<td></td>
<td>40.0</td>
<td>1.8283895</td>
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For given values of parameters, Equation 9 reduces to the form \( AX^2 + BX + C = 0 \), in which \( X \) = thickness of portland cement concrete. Coefficient A is 0.00600052 for all thicknesses of DGA and moduli of the concrete. Coefficient B is the same for all EAL values for a given CBR.
### TABLE 4. COEFFICIENTS IN EQUATIONS 11 AND 12

<table>
<thead>
<tr>
<th>REGRESSION</th>
<th>TENSILE STRAIN</th>
<th>WORK STRAIN</th>
<th>WORK</th>
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<tr>
<td>COEFFICIENTS</td>
<td>$\varepsilon_{xx}$</td>
<td>$\varepsilon_w$</td>
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<tr>
<td>a</td>
<td>$-3.8833629$</td>
<td>$-3.770701$</td>
<td>$-1.2191827$</td>
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<tr>
<td>b</td>
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<td>c</td>
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<td>$-0.038985048$</td>
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<tr>
<td>e</td>
<td>$-2.6870602$</td>
<td>$-2.5618267$</td>
<td>$1.1985659$</td>
</tr>
<tr>
<td>f</td>
<td>$-0.26982863$</td>
<td>$-0.27266418$</td>
<td>$-0.54532835$</td>
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</tbody>
</table>
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<table>
<thead>
<tr>
<th>AXLE CONFIGURATION</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-Tired Single</td>
<td>-3.540112</td>
<td>2.728860</td>
<td>0.289133</td>
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<tr>
<td>Front Axle</td>
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<tr>
<td>Four-Tired Single</td>
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<td>Rear Axle</td>
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<tr>
<td>Eight-Tired</td>
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<td>Tandem Axle</td>
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<td>Twelve-Tired</td>
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<tr>
<td>Tridem Axle</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\log (\text{Load Equivalency Factor}) = a + b \log(\text{Load}) + c [\log(\text{Load})]^2
\]

Load = Total load (in kips) on axle configuration
### Table 2. Regression Coefficients for General General Conic Equation

<table>
<thead>
<tr>
<th>COEFFICIENT</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
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<td>a</td>
<td>0.600052 E-02</td>
</tr>
<tr>
<td>b</td>
<td>-0.267029 E-03</td>
</tr>
<tr>
<td>c</td>
<td>0.505066 E-02</td>
</tr>
<tr>
<td>d</td>
<td>0.110149 E-02</td>
</tr>
<tr>
<td>e</td>
<td>0.100708 E-01</td>
</tr>
<tr>
<td>f</td>
<td>0.241089 E-02</td>
</tr>
<tr>
<td>g</td>
<td>0.349686 E-03</td>
</tr>
<tr>
<td>h</td>
<td>-0.196695 E-03</td>
</tr>
<tr>
<td>i</td>
<td>0.356436 E-04</td>
</tr>
<tr>
<td>j</td>
<td>0.152409 E-03</td>
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<tr>
<td>k</td>
<td>-0.248306</td>
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<tr>
<td>m</td>
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</tr>
<tr>
<td>n</td>
<td>-0.165527</td>
</tr>
<tr>
<td>p</td>
<td>-0.281134 E-01</td>
</tr>
<tr>
<td>q</td>
<td>0.156250 E-01</td>
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