New Easy-Plane CP^{N-1} Fixed Points

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New Easy-Plane $\mathbb{CP}^{N-1}$ Fixed Points

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We study fixed points of the easy-plane $\mathbb{CP}^{N-1}$ field theory by combining quantum Monte Carlo simulations of lattice models of easy-plane SU(N) superfluids with field theoretic renormalization group calculations, by using ideas of deconfined criticality. From our simulations, we present evidence that at small N our lattice model has a first-order phase transition which progressively weakens as N increases, eventually becoming continuous for large values of N. Renormalization group calculations in $4-\epsilon$ dimensions provide an explanation of these results as arising due to the existence of an $N_{\rm ep}$ that separates the fate of the flows with easy-plane anisotropy. When $N < N_{\rm ep}$, the renormalization group flows to a discontinuity fixed point, and hence a first-order transition arises. On the other hand, for $N > N_{\rm ep}$, the flows are to a new easy-plane $\mathbb{CP}^{N-1}$ fixed point that describes the quantum criticality in the lattice model at large N. Our lattice model at its critical point, thus, gives efficient numerical access to a new strongly coupled gauge-matter field theory.

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The emergence of gauge theories in quantum spin Hamiltonians has played an important role in theoretical descriptions of novel magnetic phenomena over the past few decades. Recently, by exploiting advances in simulation algorithms for quantum antiferromagnets [1], the connections between magnetism and gauge theories have facilitated controlled numerical access to otherwise poorly understood strongly coupled gauge theories; the most prominent example is the study of N-component scalar electrodynamics (also called $\mathbb{CP}^{N-1}$) that emerges right at the direct continuous transitions between magnetic and translational-symmetry-breaking "valence bond solid" (VBS) states in SU(N) magnets, a phenomenon popularly called "deconfined critical points" (DCPs) [2].

Two prominent physical systems where the ideas of DCPs apply are lattice superfluids and antiferromagnets, each giving rise to its own variant of the $\mathbb{CP}^{N-1}$ theory. In the context of superfluids (SFs), the appropriate description is a "easy-plane" $\mathbb{CP}^1$ field theory [2,3] which applies to the superfluid SF-VBS critical point in $S = 1/2$ XY models [4]. The easy-plane case was studied intensely initially, since a self-duality suggested that this could be the best candidate for a DCP [5]. Subsequent numerical work has concluded, however, that this transition is first order, both in direct discretizations of the field theory [6,7] as well as in simulations of the quantum antiferromagnet [8]. The easy-plane case is in sharp contrast to the case of antiferromagnets with SU(N) symmetry, where a striking quantitative agreement between detailed field theoretic calculations [9–12] and numerical simulations has been demonstrated [13–15].

The persistent first-order behavior in the XY-like models and its striking difference from the continuous transitions found in the SU(N) symmetric case has been unexplained so far, despite the central role of both systems in our understanding of the DCP phenomenon. In this work, we address this issue by formulating an extension of the easy-plane XY symmetry of SU(2) to general SU(N). Our approach allows for a study of the first-order transition for arbitrary N using both lattice simulations of an easy-plane-SU(N) [ep-SU(N)] model as well as renormalization group (RG) calculations on a proposed easy-plane-$\mathbb{CP}^{N-1}$ [ep-$\mathbb{CP}^{N-1}$] field theory: We find that the first-order transition in the ep-SU(N) models found for $N = 2$ in previous work persists for larger N. A careful analysis, however, shows that the first-order jump quantitatively weakens as N increases. RG ϵ-expansion calculations find that the field theory hosts a new ep-$\mathbb{CP}^{N-1}$ fixed point only for $N > N_{\rm ep}$, suggesting that the transition can eventually become continuous. Consistent with this result, we find that the transition in our lattice model turns continuous around $N \approx 20$. For $N = 21$, we provide a detailed scaling analysis of our numerical data that confirms a continuous transition in a new universality class. Our work clarifies and significantly extends the discussion of the DCP phenomenon in easy-plane magnets and its relation to the symmetric case.

Easy-plane model and field theory.—We introduce a family of bipartite ep-SU(N) spin models that are extensions of the quantum XY model to larger N akin to those studied by us recently [8]. They are written in terms of the T_i^a, the fundamental generators of SU(N) on site i:

$$H_{\rm ep} = -\frac{J_{1\perp}}{N}\sum_{a,\langle ij\rangle}{}' T_i^a T_j^{a*} - \frac{J_{2\perp}}{N}\sum_{a,\langle\langle ij\rangle\rangle}{}' T_i^a T_j^a. \qquad (1)$$

The $\sum'$ denotes that the sum on a is restricted to the $N^2 - N$ off-diagonal generators [a sum on all a would give the SU(N) model]. The $\langle ij\rangle$ ($\langle\langle ij\rangle\rangle$) indicates nearest (next-nearest) neighbors on the square lattice which are on opposite (the same) sublattices and in conjugate (the same) representations. $H_{\rm ep}$ is an easy-plane deformation of the

SU(N) J_1-J_2 model [13]; it has a global $U(1)^{N-1} \times S_N$ [we call this ep-SU(N)] in addition to time reversal and lattice symmetries. As we shall show, the model harbors in its phase diagram the SF-VBS transition for all $N > 5$. H_{ep} is Marshall positive for $J_{1\perp}, J_{2\perp} > 0$ [16]; we simulate it with the stochastic series expansion quantum Monte Carlo algorithm on an $L \times L$ lattice at an inverse temperature β [21].

To obtain the effective field theory ($\mathcal{L}_{\text{ep}}$) for Eq. (1), we start with the $\mathbb{CP}^{N-1}$ model ($\mathcal{L}_s$) proposed to describe DCP in antiferromagnets with SU(N) symmetry [2,3] and deform it using an anisotropy operator ($\mathcal{L}_v$), so that $\mathcal{L}_{\text{ep}} = \mathcal{L}_s + \mathcal{L}_v$ with

$$\mathcal{L}_s = \sum_\alpha |(\partial_\mu - ieA_\mu)z_\alpha|^2 + \frac{1}{2}(\vec{\nabla} \times \vec{A})^2 + r\sum_\alpha |z_\alpha|^2 + \frac{u}{2}\left(\sum_\alpha |z_\alpha|^2\right)^2,$$
$$\mathcal{L}_v = \frac{v}{2}\sum_\alpha |z_\alpha|^4, \tag{2}$$

where the z_α are N complex fields coupled to a U(1) gauge field, A_μ. The term v breaks the SU(N) symmetry of $\mathcal{L}_s$ to the same ep-SU(N) symmetry found above for Eq. (1). It is known from the large-N expansion that, for N larger than some finite N_s, in $d = 3$ the $\mathbb{CP}^{N-1}$ field theory $\mathcal{L}_s$ has a finite coupling fixed point (FP) [22,23]. Below, we will address the fate of these FPs for $\mathcal{L}_{\text{ep}}$.

Weakening first-order transition.—We begin with a numerical study of Eq. (1). We have shown [8] that the ep-SU(N) models map to a certain loop model. We can hence calculate two useful quantities to probe the magnetic ordering: the average of the square of the spatial winding number of the loops $\langle W^2 \rangle$ and a normalized magnetic order parameter $m_\perp^2 = N/[(N-1)L^4]\sum_a{}' \sum_{i,j} \tilde{T}_i^a \tilde{T}_j^a$ [where the sum on a is on the off-diagonal generators, i and j are summed on the entire lattice, and $\tilde{T} = T(T^*)$ on the $A(B)$ sublattice], which although off diagonal in the $|\alpha\rangle$ basis can be estimated by measuring a particular statistical property of the loops [16]. We have normalized $m_\perp^2$ so that the maximum value it can take is 1 for all N, allowing for a meaningful comparison across different N.

Previously, we found that the SF-VBS transition is first order for $N \le 5$ [8]. In Fig. 1 we present data that show the first-order behavior persists as N is increased up to $N = 10$. A hitherto unanswered but important question is whether the first-order jump weakens as N increases. We find evidence in favor of this assertion, since the histogram peaks get closer as N is increased. Beyond $N \approx 16$, we have found no evidence for double-peaked histograms. To carry out a more quantitative analysis, which has been popular in the study of the DCPs [6], we turn to $\langle W^2 \rangle$ (which is related to the spin stiffness as $\beta\rho_s$). At a first-order transition, one expects a linear divergence of $\langle W^2 \rangle$ as one approaches the phase transition, since ρ_s stays finite. Any sublinear behavior

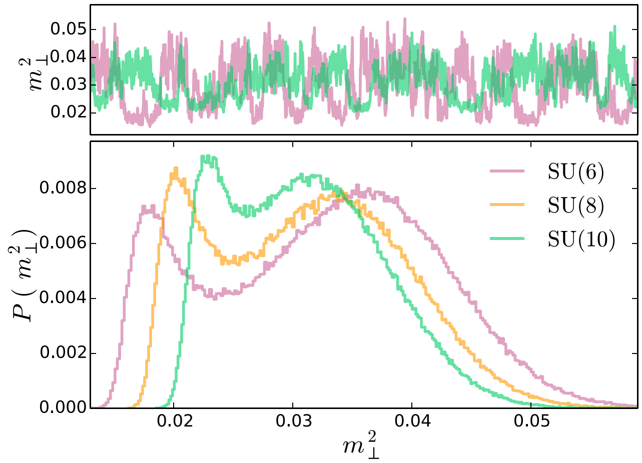

FIG. 1. First-order transitions for moderate values of N. The upper panels shows Monte Carlo histories (arbitrary units) of the estimator for $m_\perp^2$ for $N = 6$ and 10. The bottom panel shows histograms of $m_\perp^2$ taken at $L = 50$ for $J_2/J_1 \equiv g = 0.250, 0.876, 1.58$ for $N = 6$, 8, 10, respectively, clearly showing double-peaked behavior. The double-peaked behavior persists in the $T = 0$ and thermodynamic limit [16].

indicates that the transition is continuous, since ρ_s vanishes in the thermodynamic limit [24]. In Fig. 2, we present a study of the crossing of $\langle W^2 \rangle$. We find clear evidence for the expected linear behavior at moderate values of N. As N is increased beyond about $N \approx 16$, we find a very slow growth of $\langle W^2 \rangle$ inconsistent with linear behavior but consistent with what has been found in SU(N) models, where the transition is believed to be continuous [24,25]. This study provides clear evidence that the first-order jump decreases as N increases, possibly becoming continuous.

RG analysis.—The weakening of the first-order SF-VBS transitions at larger N raises important questions: Is the transition first order for all N, or does it become continuous beyond some finite N_{ep}? If the transition becomes continuous, is it a new universality class of an ep-$\mathbb{CP}^{N-1}$, or is the anisotropy irrelevant resulting in $\mathbb{CP}^{N-1}$ criticality for the easy-plane models?

To answer these questions, we compute the RG flows of Eq. (2) in $4 - \epsilon$ dimensions. We will work in the critical plane where $r = 0$, the r operator being strongly relevant at the tree level will continue to be relevant in the ϵ expansion. To leading order [assuming u, v, and e^2 are $\mathcal{O}(\epsilon)$], we find the following RG equations:

$$\frac{de^2}{d\ln s} = \epsilon e^2 - \frac{N}{3}e^4,$$
$$\frac{du}{d\ln s} = \epsilon u - (N+4)u^2 - 4uv - 6e^4 + 6e^2 u,$$
$$\frac{dv}{d\ln s} = \epsilon v - 5v^2 - 6uv + 6e^2 v, \tag{3}$$

which for $v = 0$ reduce to the well-known RG equations for the $\mathbb{CP}^{N-1}$ model [22,26]. Given the relevance of r, a

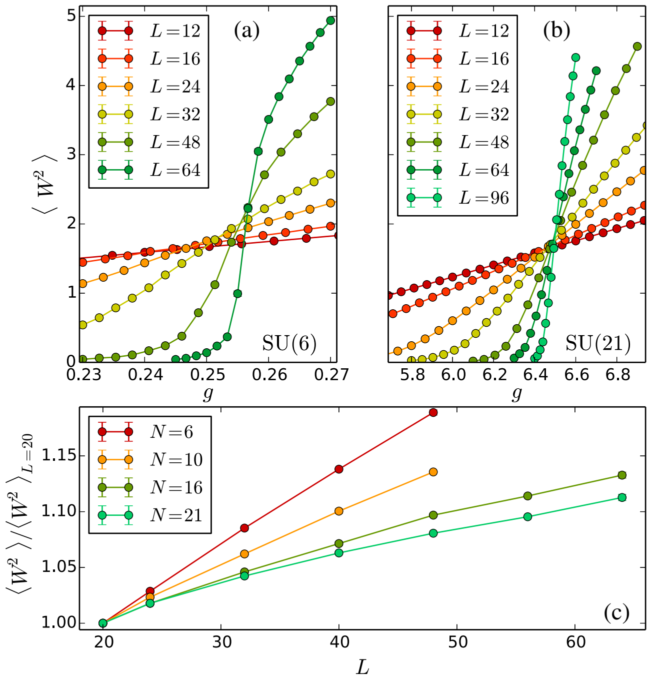

FIG. 2. Scaling of the spatial winding number square $\langle W^2 \rangle$. (a) Crossing for $N = 6$. (b) Crossing for $N = 21$. (c) Value at the L and $L/2$ crossing of $\langle W^2 \rangle$ for a range of N normalized to the crossing value at $L = 20$ for each N. For the smaller N, a clear linear divergence is seen as expected for a first-order transition (ergodicity issues limit the system sizes here). For larger N, a slow growth is observed very similar to what has been studied in detail for the SU(2) case and interpreted as evidence for a continuous transition with two length scales [25], like we have here. The data were taken at $\beta = 6L$, which is in the $T = 0$ regime [16].

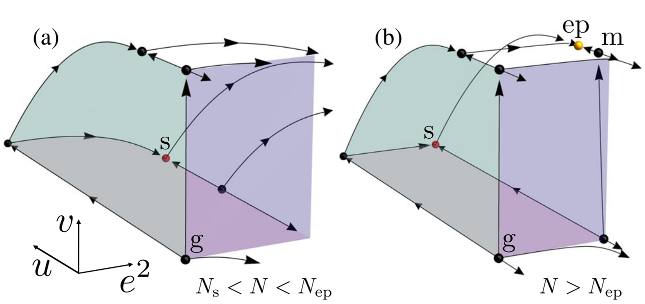

FIG. 3. RG flows of the ep-$\mathbb{CP}^{N-1}$ model for (a) $N_s < N < N_{\text{ep}}$ and (b) $N > N_{\text{ep}}$ at leading order in $4 - \epsilon$ dimensions obtained by numerical integration of Eq. (3). Fixed points are shown as bold dots; we have labeled only those significant to our discussion. The flows in the $v = 0$ plane have been obtained previously [22] and include the "s" fixed point that describes DCP in SU(N) models (red dot). While the flows have many FPs [16], a DCP of the ep-SU(N) spin model must have all three eigendirections in the e^2-u-v irrelevant. For $N < N_{\text{ep}}$ there are no such FPs; there is hence a runaway flow to a first-order transition. For $N > N_{\text{ep}}$ two FPs emerge: "m" is multicritical and "ep" is the new ep-DCP that describes the SF-VBS transition (yellow dot). The Gaussian fixed point at the origin has been labeled "g" for clarity. See [16] for further details.

generic critical point of ep-$\mathbb{CP}^{N-1}$ would be a fixed point of Eq. (3) with all three eigendirections in e^2-u-v space irrelevant. The FP structure and flows of Eq. (3) (shown in Fig. 3) change at two values of N: N_s and N_{ep} with $N_s < N_{\text{ep}}$. For $N < N_s$ [not shown], there are no FPs with $e^2 \neq 0$, and a generic flow runs away to a first-order transition. For $N > N_s$, a $v = 0$ FP "s" appears, which describes the SU(N) DCP phenomenon, but at which v is always relevant. There are two distinct fates of the flow with $v \neq 0$: For $N_s < N < N_{\text{ep}}$ [see Fig. 3(a)], v causes a runaway flow to a discontinuity FP; i.e., the phase transition turns first order. On the other hand, for $N > N_{\text{ep}}$ [see Fig. 3(b)], a new fixed point "ep" appears. At this FP, all eigendirections in the e^2-u-v space are irrelevant, and hence r is the only relevant perturbation. ep hence describes a generic continuous deconfined SF-VBS transition in models of the form of Eq. (1). In the leading order of the ϵ expansion, we have $N_s \approx 183$ [22] and $N_{\text{ep}} \approx 5363$ (independent of ϵ). From previous work on the symmetric case, it is well known that these leading-order estimates are unreliable in $d = 3$:

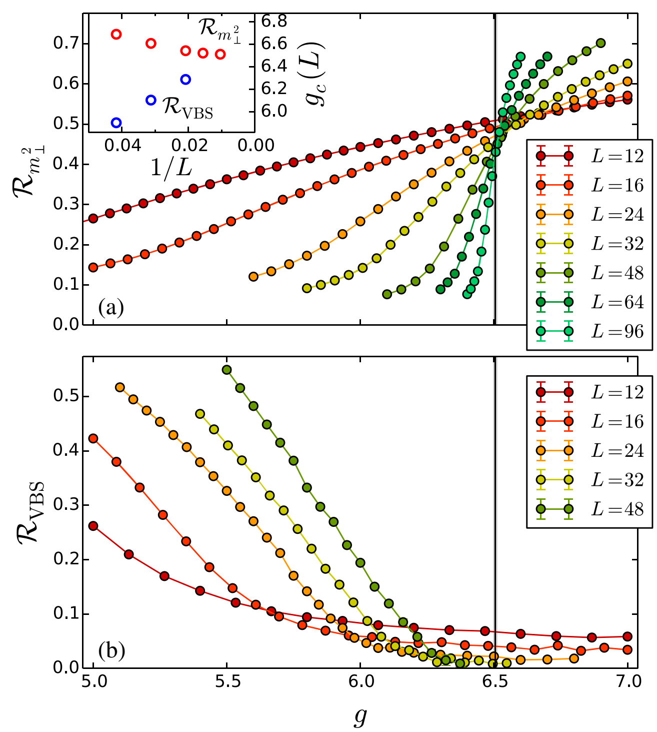

FIG. 4. Correlation ratios close to the phase transition for $N = 21$. (a) The SF order parameter ratio $R_{m_\perp^2}$ shows good evidence for a continuous transition with a nicely convergent crossing point of $g = 6.505(5)$. (b) R_{VBS} shows a crossing point that converges to the same value of the critical coupling. We note, however, that the crossing converges much more slowly (see the text). The inset shows the convergence of the crossings points of L and $L/2$ of SF and VBS ratios. Note their convergence to a common critical coupling indicating a direct transition. The data shown in Figs. 4 and 5 were taken at $\beta = 6L$, which is in the $T = 0$ regime [16].

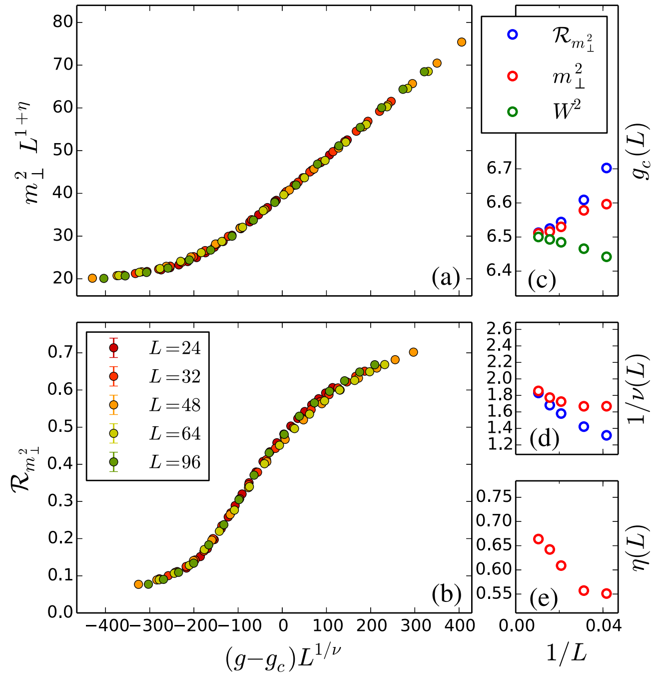

FIG. 5. Data collapse for the SF order parameter at $N = 21$. (a) Finite size data collapsed to $m_\perp^2 = L^{-(1+\eta_{\rm SF})}\mathbb{M}[(g-g_c)L^{1/\nu}]$ with parameters $g_c = 6.511$, $\nu = 0.556$ $(1/\nu = 1.795)$, and $\eta = 0.652$. (b) Collapse of ratio $R_{m_\perp^2} = \mathbb{R}[(g-g_c)L^{1/\nu}]$ with parameters $g_c = 6.518$ and $\nu = 0.582$ $(1/\nu = 1.719)$. The side panels show the convergence of estimates for various quantities from the collapse of L and $L/2$ data: (c) the critical coupling $g_c = 6.505(1)$ from pairwise collapses of $m_\perp^2$ and $R_{m_\perp^2}$, as well as crossings of $\langle W^2\rangle$ (see Fig. 2) for data. Panel (d) shows $1/\nu(L)$, which we estimate to converge to $1/\nu = 2.3(2)$. Likewise, we estimate $\eta(L)$ to converge to $\eta = 0.72(3)$, which is shown in panel (e).

Indeed, in the next-to-leading order, N_s becomes negative for $\epsilon = 1$ [27,28]. Ultimately, the values of $N_{s,\rm ep}$ must be obtained from numerical simulations. Nonetheless, it is expected that the basic structure of fixed points and flows obtained here using the ϵ expansion are reliable. Based on our study, we make the following conclusions: Even in a regime where there is a symmetric fixed point ($N > N_s$), for $N_s < N < N_{\rm ep}$, easy-plane anisotropy will drive the DCP first order. For $N > N_{\rm ep}$, a new FP emerges. Easy-plane anisotropy then results in a continuous SF-VBS transition in a new ep-$\mathbb{CP}^{N-1}$ universality class.

Study of a fixed point.—Having presented evidence from the ϵ expansion that with increasing N the transition should turn continuous and in a new universality class, it is of interest to study the scaling behavior at large N. We will focus on $N = 21$, where we have found no evidence for first-order behavior on the largest system sizes that we have access to. We construct dimensionless ratios $\mathcal{R}_{m_\perp^2}$ and $\mathcal{R}_{\rm VBS}$ which go to 1(0) in their respective ordered (disordered) phases. Figure 4 shows our data for $N = 21$. The large correction to scaling observed in the VBS data is expected: According to the DCP theory, the VBS anomalous dimension $\eta_{\rm VBS} \propto N$, which causes the leading VBS correlation functions to decay very rapidly at this large value of N. This makes it hard to separate the leading and subleading behavior on the available system sizes. Since the SF data show a good crossing, we carry out a full scaling analysis in Fig. 5. The data for both $m_\perp^2$ and $R_{m_\perp^2}$ collapse nicely without the inclusion of corrections to scaling [29,30]. They lead to consistent values of critical couplings and scaling dimensions, lending support for a continuous transition ep-$\mathbb{CP}^{N-1}$ fixed point emerging at large N.

In conclusion, we have studied new lattice models for deconfined criticality with easy-plane SU(N) symmetry. We find persistent first-order behavior in these lattice models at small to intermediate N, in sharp contrast to the continuous transitions found in the symmetric models for the same range of N. As N increases, the first-order easy-plane transition weakens and eventually becomes continuous. Our RG flows provide a way to understand both the first-order and shift to continuous transitions: The easy-plane anisotropy is always relevant at the symmetric $\mathbb{CP}^{N-1}$ fixed point; for $N < N_{\rm ep}$, there is no easy-plane fixed point, and hence the anisotropy drives the transition first order. For $N > N_{\rm ep}$, a new fixed point emerges, resulting in a continuous transition in a new easy-plane-$\mathbb{CP}^{N-1}$ universality class, which is an example of a strongly coupled gauge-matter field theory. Our lattice model provides a sign-free discretization of this field theory that is amenable to efficient numerical simulations. We leave for future work the determination of a precise value of $N_{\rm ep}$, comparisons of the universal quantities with easy-plane large-N expansions, and a comparative study of the scaling corrections between the easy-plane and symmetric cases. It would be of interest to complement our work with studies of field theories such as Eq. (2) using the conformal bootstrap [31].

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