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Quantum Quench and Scaling of Entanglement Entropy

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Quantum quench and scaling of entanglement entropy

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ABSTRACT

Global quantum quench with a finite quench rate which crosses critical points is known to lead to universal scaling of correlation functions as functions of the quench rate. In this work, we explore scaling properties of the entanglement entropy of a subsystem in a harmonic chain during a mass quench which asymptotes to finite constant values at early and late times and for which the dynamics is exactly solvable. When the initial state is the ground state, we find that for large enough subsystem sizes the entanglement entropy becomes independent of size. This is consistent with Kibble–Zurek scaling for slow quenches, and with recently discussed "fast quench scaling" for quenches fast compared to physical scales, but slow compared to UV cutoff scales.

1. Introduction

The behavior of entanglement of a many-body system that undergoes a quantum quench has been a subject of great interest in recent times. When the quench is instantaneous (i.e. a sudden change of the hamiltonian), several results are known. Perhaps the best known result pertains to the entanglement entropy (EE) of a region of size l in a $1+1$ dimensional conformal field theory following a global instantaneous quench, $S_{EE}(l)$. As shown in [1], $S_{EE}(l)$ grows linearly in time till $t \approx l/2$ and then saturates to a constant value typical of a thermal state – a feature which has been studied extensively in both field theory and in holography. Generalizations of this result to conserved charges and higher dimensions have been discussed more recently [2–4]. The emphasis of these studies is to probe the *time evolution* of the entanglement entropy.

In physical situations, quantum quench has a finite rate, characterized by a time scale δt, that can vary from very small to very large. When the quench involves a critical point, universal scaling behavior has been found for correlation functions at *early* times. The most famous scaling appears for a global quench which starts from a massive phase with an initial gap m_g, crosses a critical point (chosen to be e.g. at time $t=0$) and ends in another massive phase. For *slow quenches* (large δt), it has been conjectured that quantities obey Kibble–Zurek scaling [5]: evidence for this has been found in several solvable models and in numerical simulations [6,7]. Such scaling follows from two assumptions. First, it is assumed that as soon as the initial adiabatic evolution breaks down at some time $-t_{KZ}$ (the Kibble–Zurek time) the system becomes roughly diabatic. Secondly, one assumes that the only length scale in the critical region is the instantaneous correlation length ξ_{KZ} at the time $t=-t_{KZ}$. This implies that, for example, one point functions scale as $\langle \mathcal{O}(t)\rangle \sim \xi_{KZ}^{-\Delta}$, where Δ denotes the conformal dimension of the operator $\mathcal{O}$ at the critical point. An improved conjecture involves scaling functions. For example, one and two point correlation functions are expected to be of the form [8–14]

$$\begin{aligned}\langle \mathcal{O}(t)\rangle &\sim \xi_{KZ}^{-\Delta}\, F(t/t_{KZ})\\ \langle \mathcal{O}(\vec{x},t)\mathcal{O}(\vec{x}',t')\rangle &\sim \xi_{KZ}^{-2\Delta} F\left[\frac{|\vec{x}-\vec{x}'|}{\xi_{KZ}}, \frac{(t-t')}{t_{KZ}}\right]\end{aligned} \tag{1}$$

Some time ago, studies of slow quenches in AdS/CFT models have led to some insight into the origin of such scaling without making these assumptions [15].

For protocols in relativistic theories which asymptote to constant values at early times, one finds a different scaling behavior in the regime $\Lambda_{UV}^{-1} \ll \delta t \ll m_{phys}^{-1}$, where Λ_{UV} is the UV cutoff scale, and m_{phys} denotes any physical mass scale in the problem. For example,

$$\langle \mathcal{O}(t)\rangle \sim \delta t^{d-2\Delta} \tag{2}$$

where d is the space–time dimension. This "fast quench scaling" behavior was first found in holographic studies [16] and subse-

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quently shown to be a completely general result in any relativistic quantum field theory [17]. The result follows from causality, and the fact that in this regime linear response becomes a good approximation. Finally, in the limit of an instantaneous quench, suitable quantities saturate as a function of the rate: for quench to a critical theory a rich variety of universal results are known in $1+1$ dimensions [18].

Much less is known about the behavior of entanglement and Renyi entropies *as functions of the quench rate* – a key ingredient of universality. This has been studied for the 1d Ising model (and generalizations) with a transverse field which depends *linearly* on time, $g(t) = 1 - \frac{t}{\tau_Q}$ [19,9,20]. The system is prepared in the instantaneous ground state at some initial time, crossing criticality at $t = 0$. The emphasis of [19] and [9,20] is on the slow regime, which means $\tau_Q \gg a$ where a is the lattice spacing, while [21] also studies smaller values of τ_Q. In particular, [19] and [21] studied the EE for half of a finite chain and found that the answer approaches $S_{EE} \sim \frac{1}{12} \log \xi_{KZ}$ after sufficiently slow quenches. This is consistent with the standard assumptions which lead to Kibble–Zurek scaling mentioned above. According to these assumptions, the system evolves adiabatically till $t = -t_{KZ}$ and enters a phase of diabatic evolution soon afterwards. Thus the state of the system at $t = 0$ is not far from the ground state of the instantaneous hamiltonian at $t = -t_{KZ}$. Furthermore when $\tau_Q \gg 1$ in lattice units, ξ_{KZ} is large, and the instantaneous state is close to criticality. In such a state, the entanglement entropy of a subregion of a large chain with N_A boundary points should obey an "area law" $\frac{c}{6} N_A \log(\xi_{KZ})$, where c is the central charge. When the subsystem is half space $N_A = 1$ and for the Ising model the central charge is $c = 1/2$. Similarly, [9,20] studied the EE of a subsystem of finite size l in an infinite 1d Ising model, with a transverse field linear in time, starting with the ground state at $t = -\infty$. The EE close to the critical point for $l \gg \xi_{KZ}$ was found to saturate to $S_{EE} = (\text{constant}) + \frac{1}{6}\log(\kappa(t)\xi_{KZ})$. The factor $\kappa(t)$ depends mildly on the time of measurement and $\kappa(-t_{KZ}) \approx 1$. Once again, this result is roughly that of a stationary system with correlation length ξ_{KZ}, as would be expected from Kibble–Zurek considerations. The factor $\kappa(t)$ is a correction to the extreme adiabatic–diabatic assumption. The paper [21] investigates an intermediate regime of fast quench (as described above). While this paper investigates scaling of S_{EE} as a function of quench rate in the slow regime, there is no similar analysis in the fast regime.

In this letter, we study entanglement entropy for a simple system: an infinite harmonic chain (i.e. a $1+1$ dimensional bosonic theory on a lattice) with a time dependent mass term which asymptotes to constant *finite* values at early and late times. We choose a mass function for which the quantum dynamics can be solved exactly. The use of such a protocol allows us to explore the whole range of quench rates, where the speed of quench is measured in units of the initial gap rather than the lattice scale. We compute the entanglement entropy for a subsystem of size l (in lattice units) in the middle of the quench and find that it scales in interesting ways as we change the quench rate. The dimensionless quantity which measures the quench timescale is $\Gamma_Q = m_0 \delta t$ where m_0 is the initial gap.

2. Our setup and quench protocols

The hamiltonian of the harmonic chain is given by

$$H = \frac{1}{2} \sum_{n=-\infty}^{\infty} \left[P_n^2 + (X_{n+1} - X_n)^2 + m^2(t) X_n^2 \right] \tag{3}$$

where (X_n, P_n) are the usual canonically conjugate scalar field variables on an one dimensional lattice whose sites are labelled by the integer n. The mass term $m(t)$ is time dependent. All quantities are in lattice units. In terms of momentum variables X_k, P_k

$$X_n(t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} X_k(t)\, e^{ikn} \qquad P_n(t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} P_k(t)\, e^{ikn} \tag{4}$$

the equation of motion is given by

$$\frac{d^2 X_k}{dt^2} + [4\sin^2(k/2) + m^2(t)] X_k = 0 \tag{5}$$

We are interested in functions $m(t)$ which asymptote to constant values m_0 at $t \to \pm\infty$, and pass through zero at $t = 0$. Let $f_k(t)$ be a solution of (5) which asymptotes to a purely positive frequency solution $\sim e^{-i\omega_0 t}/\sqrt{2\omega_0}$ at $t \to -\infty$, where

$$\omega_0^2 = 4\sin^2(k/2) + m_0^2. \tag{6}$$

A mode decomposition

$$X_k(t) = f_k(t) a_k + f_k^\star(t) a_{-k}^\dagger \tag{7}$$

with $[a_k, a_{-k'}^\dagger] = 2\pi\delta(k - k')$ can be then used to define the "in" vacuum by $a_k|0\rangle = 0$ for all k. The solutions $f_k(t)$ are chosen to satisfy the Wronskian condition $f_k(\dot{f}_k)^\star - (\dot{f}_k) f_k^\star = i$. The state $|0\rangle$ then denotes the Heisenberg picture ground state of the initial Hamiltonian. The normalized wavefunctional for the "in" vacuum state is given by

$$\Psi_0(X_k, t) = \prod_k \frac{1}{[\sqrt{2\pi}\, f_k^\star(t)]^{1/2}} \exp\left[\frac{1}{2} \left(\frac{\dot{f}_k(t)}{f_k(t)} \right)^\star X_k X_{-k} \right] \tag{8}$$

We will choose a quench protocol for a mass function for which the mode functions $f_k(t)$ can be solved exactly. The particular mass function we use is

$$m^2(t) = m_0^2 \tanh^2(t/\delta t) \tag{9}$$

The corresponding mode functions are given by

$$f_k = \frac{1}{\sqrt{2\omega_0}} \frac{(2)^{i\omega_0 \delta t} \cosh^{2\alpha}(t/\delta t)}{E'_{1/2}\tilde{E}_{3/2} - E_{1/2}\tilde{E}'_{3/2}} \times$$
$$[\tilde{E}'_{3/2}\, {}_2F_1(\tilde{a}, \tilde{b}, \frac{1}{2}; -\sinh^2(t/\delta t))$$
$$+ E'_{1/2} \sinh(t/\delta t)\, {}_2F_1(\tilde{a} + \frac{1}{2}, \tilde{b} + \frac{1}{2}, \frac{3}{2}; -\sinh^2(t/\delta t))]$$

where we have defined

$$\begin{aligned}
\omega_0^2(k) &= 4\sin^2(k/2) + m_0^2 \\
\alpha &= \frac{1}{4}[1 + \sqrt{1 - 4m_0^2 \delta t^2}] \\
\tilde{a} &= \frac{1}{4}[1 + \sqrt{1 - 4m_0^2 \delta t^2}] + \frac{i}{2}\delta t \omega_0 \\
\tilde{b} &= \frac{1}{4}[1 + \sqrt{1 - 4m_0^2 \delta t^2}] - \frac{i}{2}\delta t \omega_0 \\
E_{1/2} &= \frac{\Gamma(1/2)\Gamma(\tilde{b} - \tilde{a})}{\Gamma(\tilde{b})\Gamma(1/2 - \tilde{a})} \\
\tilde{E}_{3/2} &= \frac{\Gamma(3/2)\Gamma(\tilde{b} - \tilde{a})}{\Gamma(\tilde{b} + 1/2)\Gamma(1 - \tilde{a})} \\
E'_c &= E_c(\tilde{a} \leftrightarrow \tilde{b}).
\end{aligned} \tag{10}$$

3. Entanglement entropy

Our aim is to calculate the entanglement entropy for a subregion of the infinite chain consisting of l lattice points. This is most conveniently calculated by considering the $2l \times 2l$ matrix of correlators, and the symplectic matrix [4,22]

$$C = \begin{bmatrix} X_{mn}(t) & D_{mn}(t) \\ D_{mn}(t) & P_{mn}(t) \end{bmatrix} \quad J = \begin{bmatrix} 0 & I_{l\times l} \\ -I_{l\times l} & 0 \end{bmatrix}$$

where (m,n) denote lattice sites inside the subsystem and

$$X_{mn}(t) = \langle 0|X_m(t)X_n(t)|0\rangle, \quad P_{mn}(t) = \langle 0|P_m(t)P_n(t)|0\rangle,$$
$$D_{mn}(t) = \frac{1}{2}\langle 0|\{X_m(t), P_n(t)\}|0\rangle \tag{11}$$

The eigenvalues of the matrix iJC then occur in pairs $\pm\gamma_n(t)$ $n = 1, \cdots l$ where $\gamma_n(t) > 0$. The entanglement entropy S is given by

$$S = \sum_{n=1}^{l}\{[\gamma_n(t) + \frac{1}{2}]\log[\gamma_n(t) + \frac{1}{2}] - [\gamma_n(t) - \frac{1}{2}]\log[\gamma_n(t) - \frac{1}{2}]\} \tag{12}$$

In terms of the mode functions (10), the correlators (11) are given by the expressions

$$X_{mn}(t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi}|f_k(t)|^2\cos(k|m-n|)$$
$$P_{mn}(t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi}|\dot{f}_k(t)|^2\cos(k|m-n|)$$
$$D_{mn}(t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi}\mathrm{Re}\left(f_k^*(t)\dot{f}_k(t)\right)\cos(k|m-n|) \tag{13}$$

The correlator $D_{mn}(t)$ is particularly important in the calculations which follow. This is because this quantity is exactly zero when the mass is time independent. For the same reason, this quantity vanishes in the leading order of the adiabatic expansion (i.e. when observables are replaced by their *static* answers with the instantaneous value of the mass), i.e. $f_k^{adia} = \frac{1}{\sqrt{2\omega_k(t)}}e^{-i\omega_k(t)t}$ with $\omega_k(t)^2 = 4\sin^2(k/2) + m^2(t)$.

The computation of correlation functions and the entanglement entropy then involves integrals over momenta k, which are computed numerically. As is well known, numerical integrals with oscillating functions in the integrand are tricky. Numerical methods reduce this to a summation of an alternating series, leading to slow convergence. To confirm our numerical integration, we check the convergence of the integration with varying precision. We perform all calculations with interval sizes $dk = 0.00001, 0.000001$.

4. Regimes of the quench rate

Before we examine these entropies, it is necessary to understand the various regimes of the dimensionless parameter $\Gamma_Q \equiv m_0\delta t$. This can be done by studying the scaling behavior of correlation functions. Since we are interested in the slow as well as the fast regime, we will stay close to the continuum limit where the dimensionless mass $m_0 a$ is small. In the following we express everything in lattice units. Then, Kibble Zurek scaling is expected to hold for these quantities for $\Gamma_Q \gg 1$, while we get the regime of fast quench when $\Gamma_Q \ll 1$, but $\delta t > 1$.

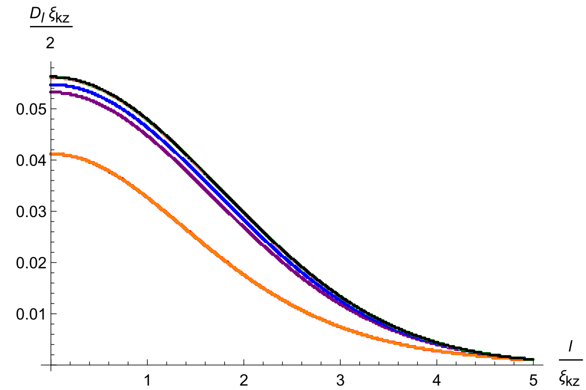

Fig. 1. (Color online.) Rescaled correlator $\frac{1}{2}\xi_{KZ}D_{mn}(t=0)$ as a function of $|m-n|$. The various values of Γ_Q, ξ_{KZ} are in different colors: On top of each other we see Red: ($\Gamma_Q = 1, \xi_{kz} = 400$) and Orange: ($\Gamma_Q = 1, \xi_{kz} = 500$), then Purple: ($\Gamma_Q = 5, \xi_{kz} = 400$) and Blue: ($\Gamma_Q = 10, \xi_{kz} = 400$) and again on top of each other: Green: ($\Gamma_Q = 100, \xi_{kz} = 400$), Pink: ($\Gamma_Q = 100, \xi_{kz} = 500$) and Black: ($\Gamma_Q = 500, \xi_{kz} = 500$).

For slow quench, the usual Landau criterion leads to a Kibble Zurek time and the instantaneous correlation length at this time to be $t_{KZ} = \xi_{KZ} = \sqrt{\delta t/m_0}$, and we expect e.g. the following leading answers at $t = 0$: $X_{mn}(0) \sim \xi_{KZ}^0 F_X(|m-n|/\xi_{KZ})$, $P_{mn}(0) \sim \xi_{KZ}^{-2}F_P(|m-n|/\xi_{KZ})$, $D_{mn}(0) \sim \xi_{KZ}^{-1}F_D(|m-n|/\xi_{KZ})$, where F_X, F_P, F_D are smooth functions. We indeed find this behavior for $\Gamma_Q \geq 100$. For example, Fig. 1 shows $\xi_{KZ}D_{mn}(0)$ as a function of $|m-n|$ for various values of Γ_Q. Clearly for large enough Γ_Q the points fall on top of each other, showing that the above scaling behavior holds. The results for $X_{mn}(0)$ and $\xi_{KZ}^2 P_{mn}(0)$ are also consistent with the above expectations.

In the fast quench regime $\Gamma_Q \ll 1$ the analysis of [17] leads to the following scaling relations for coincident correlators

$$X_{nn} \sim (\text{const}) + (\delta t)^2 \quad P_{nn} \sim (\text{const}) + (\delta t)^0 \quad D_{nn} \sim \delta t \tag{14}$$

We found this behavior for small $\Gamma_Q < 0.1$. For correlation functions X_{mn}, P_{mn}, D_{mn}, with the constant mass values subtracted, we expect these scalings to hold for large $|m-n| < \delta t < m_0^{-1}$, while for $\delta t < |m-n| < m_0^{-1}$ they should become independent of δt [17]. We indeed see this behavior for sufficiently small Γ_Q. This is most clearly seen for D_{mn} since the constant mass value of this quantity is exactly zero.

5. Scaling of entropies

Now that we have identified the slow and fast quench regimes, we go ahead and explore possible scaling properties of the entanglement entropy S_{EE}. This quantity is studied in detail at $t = 0$. (which is the middle of the quench) for both fast and slow quenches.

First, we consider the fast quench regime, $\Gamma_Q \ll 1$. We have computed the difference of the entanglement entropy at $t = 0$ from its value at $t = -\infty$,

$$\Delta S_A = S(t=0) - S(t=-\infty) \tag{15}$$

for various subsystem sizes l as a function of Γ_Q. Fig. 2 shows the results for $\xi_i = 600$ as a function of Γ_Q. When l is smaller than the initial correlation length $\xi_i = m_0^{-1}$ this quantity depends on l for a given Γ_Q. However for l sufficiently large compared to ξ_i we find that the l-dependence saturates.

Namely, the data for large l/ξ_i fits to the following result for small Γ_Q

$$\Delta S = c\,\Gamma_Q^2, \tag{16}$$

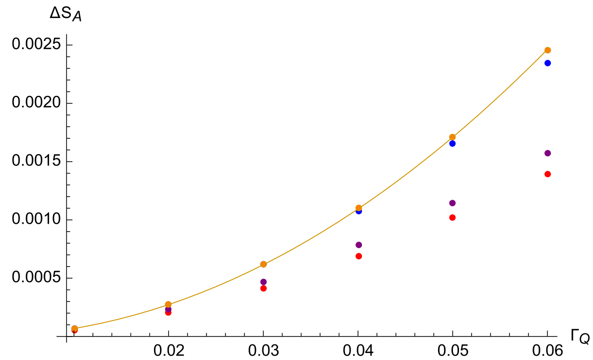

Fig. 2. (Color online.) ΔS_A at $t=0$ as a function of Γ_Q for fast quench. All the data are for $\xi_i = 600$. Different colors correspond to different values of l. Red, Purple, Blue, Green and Orange have $l = 5, 10, 100, 1000, 2000$ respectively. The data for $l = 1000, 2000$ are almost on top of each other.

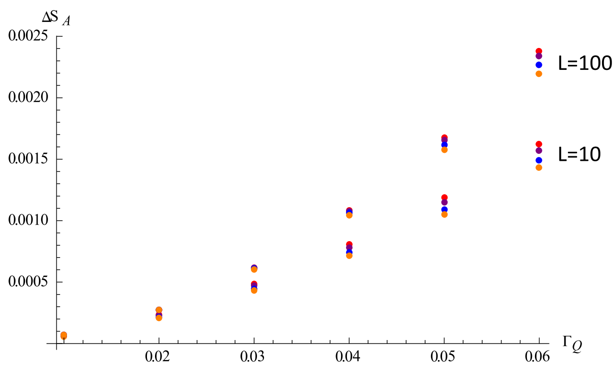

Fig. 3. (Color online.) ΔS_A at $t=0$ as a function of Γ_Q for fast quench. The data are for $l = 10$ and $l = 100$. Different colors correspond to different values of ξ_i. Red, Purple, Blue, and Orange have $\xi_i = 500, 600, 800, 1000$ respectively.

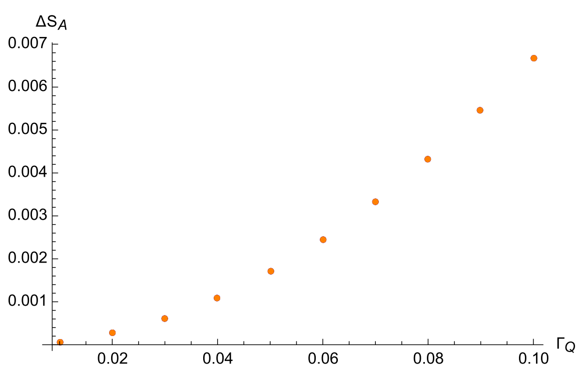

Fig. 4. (Color online.) ΔS_A at $t=0$ as a function of Γ_Q for fast quench. All the data are for $l = 2000$. We plotted this for different values of ξ_i. Red, Purple, Blue, and Orange have $\xi_i = 500, 600, 800, 1000$ respectively. However the data are basically on top of each other.

where the constant c is independent of l and ξ_i and approximately equal to 0.67.

In fact, for small l/ξ_i, ΔS_A does depend on ξ_i for a given Γ_Q and l, as shown in Figs. 3 and 4. These are plots of ΔS_A as a function of Γ_Q for a given l, and different values of ξ_i. Fig. 3 has $l = 100$ with $\frac{l}{\xi_i}$ ranging from 0.1 to 0.2: the result has a clear dependence on ξ_i. Fig. 4 has $l = 2000$ with $\frac{l}{\xi_i}$ ranging from 2.0 to 4.0 – the data for various values of ξ_i are right on top of each other.

This result (16) is in agreement with expectations based on [17]. In these papers it was argued that the fast quench scaling

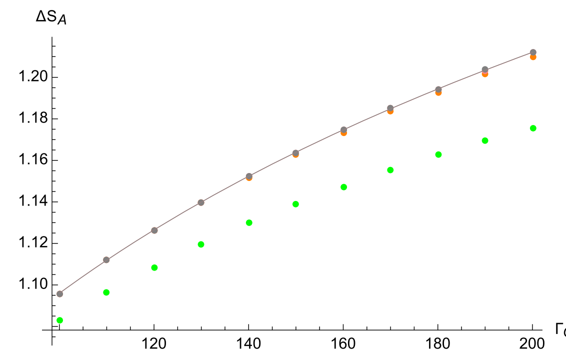

Fig. 5. (Color online.) ΔS_A at $t=0$ as a function of Γ_Q for slow quench. The data are for $\xi_i = 40$ and $\xi_i = 50$ for different values of l. However the points for these two different ξ_i are basically on top of each other. Green, orange, pink and grey correspond to $l = 1000, 2000, 3000, 3500$. The $l = 3000$ and $l = 3500$ values are too close to be distinguishable.

for one point functions follow from the fact that linear response becomes a good approximation in this regime. Even though the scaling behavior of the two point function is complicated (as mentioned above), one would expect that the δt dependence of the matrix elements of C would still be governed by perturbation theory. To lowest order this should be therefore proportional to m_0^2. In the regime of quench rates we are considering, the only other scale is δt. Since the entanglement entropy is dimensionless, we expect that the leading answer is proportional to $m_0^2 \delta t^2$ which is Γ_Q^2. The fact that the result becomes independent of l is also expected since the measurement is made at $t = 0$.

It would be interesting to gain further insight by using the perturbation theory for entanglement entropy (as e.g. developed in [23]).

Let us now consider quench rates slow enough to ensure that the correlation functions discussed above display standard Kibble Zurek scaling. Fig. 5 shows S_{EE} as a function of Γ_Q for a given m_0 various values of subsystem size l.

For large enough l we therefore find that ΔS_A is independent of l and fits the relation

$$\Delta S_A = (\text{constant}) + \frac{1}{6} \log(\Gamma_Q) \tag{17}$$

The constant in (17) is roughly 0.3. This result is consistent with the expectations from Kibble Zurek scenario. More precisely, the state of the system at $t = 0$ is fairly close to the instantaneous ground state at $t = -t_{KZ}$, i.e. the ground state of the system with a fixed (time independent) correlation length ξ_{KZ}. For such a system the entanglement entropy should behave as $\frac{1}{3}\log(\xi_{KZ})$, independent of l for $l > \xi_{KZ}$. Since $\xi_{KZ} = \xi_i \sqrt{\Gamma_Q}$, this is $\frac{1}{3}\log(\xi_i) + \frac{1}{6}\log(\Gamma_Q)$. On the other hand, $S_{EE}(t = -\infty)$ is ground state entanglement entropy of the system at a fixed correlation length ξ_i and therefore behaves as $\frac{1}{3}\log(\xi_i)$ for $l \gg \xi_i$. Thus the KZ scenario predicts that when ΔS_A is expressed as a function of Γ_Q, the dependence on ξ_i should cancel – which is exactly as we find.

In conclusion, we have calculated the entanglement entropy of a subsystem in a harmonic chain in the presence of an exactly solvable mass quench and examined the dependence of this quantity at the middle of the quench on the quench rate. Our nonlinear quench protocol asymptotes to constant values at initial and final times: in this respect it differs significantly from the kind of protocols most commonly studied in the literature, e.g. couplings which behave linearly in time. For quench rates which are fast compared to the initial mass (but slow compared to the lattice scale) our results show, for the first time, that the fast quench scaling established for correlation functions in [17] extend to nonlocal quantities like entanglement entropy. For quench rates slow compared to the initial mass, our results are consistent with the predictions of Kibble–Zurek scenario, and with earlier results in the 1d Ising model.

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