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Fluctuations in a cosmology with a spacelike singularity and their gauge theory dual description

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We consider a time-dependent deformation of anti–de Sitter (AdS) space-time which contains a spacelike "singularity"—a spacelike region of high curvature. Making use of the AdS/CFT correspondence we can map the bulk dynamics onto the boundary. The boundary theory has a time dependent coupling constant which becomes small at times when the bulk space-time is highly curved. We investigate the propagation of small fluctuations of a test scalar field from early times before the bulk singularity to late times after the singularity. Under the assumption that the AdS/CFT correspondence extends to deformed AdS space-times, we can map the bulk evolution of the scalar field onto the evolution of the boundary gauge field. The time evolution of linearized fluctuations is well-defined in the boundary theory as long as the coupling remains finite, so that we can extend the boundary perturbations to late times after the singularity. Assuming that the spacetime in the future of the singularity has a weakly coupled region near the boundary, we reconstruct the bulk fluctuations after the singularity crossing making use of generic properties of boundary-to-bulk propagators. Finally, we extract the spectrum of the fluctuations at late times given some initial spectrum. We find that the spectral index is unchanged, but the amplitude increases due to the squeezing of the fluctuations during the course of the evolution. This investigation can teach us important lessons on how the spectrum of cosmological perturbations passes through a bounce which is singular from the bulk point of view but which is resolved using an ultraviolet complete theory of quantum gravity.

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I. INTRODUCTION

The AdS/CFT correspondence [1] is a most promising proposal for a nonperturbative definition of string theory. Thus, this correspondence should also have important consequences for early universe cosmology. In fact, over the years there have been several proposals which address the meaning of cosmological singularities in the dual field theory [2–5]. The general idea is the following: consider an asymptotically AdS space-time which is contracting towards a curvature singularity. According to the AdS/CFT dictionary, this may correspond to a dual conformal field theory which lives on the boundary which is in a nontrivial unstable state [2,5] or which has a time dependent coupling [3,4] which becomes small when the bulk singularity is reached. While the bulk theory cannot be used to evolve further in time, it may be possible to track the time evolution in the dual field theory in a controlled fashion. It is then not unreasonable to assume that the dual field theory admits a continuation in time beyond the time $t_B = 0$ when the bulk singularity occurs.

There are several motivations for this investigation. One of the motivations comes from cosmology. Although the inflationary scenario [6] is the current paradigm of early universe cosmology and has been quite successful phenomenologically, it faces conceptual challenges. In

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particular, a robust embedding of large field inflation into string theory has proven to be difficult (see e.g. [7]) [8]. At the same time, it has been realized that there are alternative cosmological scenarios which are at the moment also in agreement with cosmological data. One of these is the "matter bounce" scenario (see [11] for a recent review), a bouncing scenario which begins with a matter-dominated phase of contraction during which the scales which we observe today with cosmological experiments exit the Hubble radius. It was shown in [12,13] the if fluctuations begin in their Bunch-Davies vacuum at past infinity, that the growth of the fluctuations on super-Hubble scales converts the vacuum spectrum into a scale-invariant one for scales exiting the Hubble radius during the matter phase of contraction. Adding a small cosmological constant (of magnitude similar to the one observed today) leads to a small red tilt in the spectrum [14]. The observed spectrum of curvature fluctuations is indeed scale-invariant with a small red tilt (see [15] for the most recent data).

In the context of effective field theory and Einstein gravity, it is difficult to obtain a nonsingular bouncing cosmology. One either needs to postulate that matter violates the "null energy condition" (NEC) during the bounce, or one needs to go beyond Einstein gravity. Examples of the former are adding ghost condensate matter [16] or Galileon matter [17], an example of the latter is Horava-Lifshitz gravity in the presence of nonvanishing spatial curvature [18]. However, it is doubtful whether any of these constructions actually can emerge from an ultraviolet complete theory such as string theory. Hence, it would be very interesting to investigate if the AdS/CFT correspondence leads to a consistent bouncing cosmology [19].

Regardless of the above motivation, it is clearly interesting to investigate what happens to classical spacelike singularities in a complete theory of gravity. In particular, does the holographic correspondence predict a time evolution beyond this "singularity"? Despite a lot of effort, it is not clear whether any of the AdS/CFT models which contain *true singularities* in the bulk admit a smooth time evolution in the dual theory. In the original model proposed in [5] which was based on earlier work of [2], there were some technical problems which indicated that the time evolution past the singularity was not under control [23], mainly due to the backreaction of the fluctuations on the background space-time. There have been attempts to overcome these obstacles [24,25], but the final verdict is still out.

In the works of [3,4,26–28] a bulk dilaton field ϕ had a time dependent (or a null coordinate dependent) boundary condition so that e^ϕ becomes small at some time (or null time), while Einstein frame curvatures become large in the bulk, signifying a singularity. When the singularity is null, the dual theory appears to predict a smooth time evolution, and because of the absence of particle production one expects that the spacetime is smooth in the future. However for backgrounds with spacelike singularities, as in [28] there is no clear conclusion. Even though the background supergravity solution is time symmetric, the issue relates to the effect of fluctuations. In the boundary field theory, the question becomes that of particle production. In [28] it was argued that in the case when the boundary theory coupling hits a zero, the time evolution of each individual momentum mode is in fact singular. However it was not clear what happens when one considers the full field theory. In a regulated version of the theory where the boundary coupling becomes small but does not hit a zero, time evolution is well-defined. However, the energy due to particle production at times after the crunch would be large, and the spacetime will not bounce back to pure AdS even at very late times. On general grounds, one might expect that a black brane is formed [28] [29].

In this paper, we turn to a different aspect of the kind of backgrounds with spacelike singularities studied in [4,27,28] as a result of a time dependent boundary condition for the bulk dilaton. The goal of our study is to include cosmological perturbations in this picture. This is important for at least two reasons. Firstly, it is important to study whether the background is stable against the addition of fluctuations. Secondly, most of the data which we would like to explain in cosmology concerns fluctuations (inhomogeneities in the distribution of galaxies and anisotropies in the temperature of the cosmic microwave background).

We will not explore further the important question whether there are apparently singular AdS cosmologies which lead to a bounce to a relatively empty space. Rather, we will study models of the same type as [4,27,28] which have Kasner singularitites. However we will keep e^ϕ finite (but small) at all times, rather than going to zero, by putting in a cutoff in time, ξ. More specifically, instead of the exact Kasner behavior near the singularity

$$e^\phi(t) = |t|^\alpha \tag{1}$$

we will use

$$e^\phi(t) = |t|^\alpha \theta(|t| - \xi) + \frac{\cosh^2(M\xi)}{\cosh^2(Mt)} \xi^\alpha \theta(\xi - |t|). \tag{2}$$

We do not know of exact solutions with such a cutoff dilaton: we *assume* that these can be constructed. In the presence of such a finite cutoff, the boundary gauge theory is well-defined: our aim to study some aspects of this. In some sense, the spirit of our investigation is similar to that of [31] and [32] (see also [33]) where signatures of a past Kasner singularities in the dual field theory were studied.

We want to determine how the *spectrum* of fluctuations evolves as the system passes through the "singularity". This question is independent of nature of the late time space-time so long as there is a region of normal spacetime near the horizon, which is where the fluctuations are measured.

This question is particularly interesting if any of these models reliably predict a bounce since in this case connections with bouncing cosmologies studied by many cosmologists can be made. There are classes of scenarios where, starting from vacuum perturbations at early times in the contracting phase, a scale-invariant spectrum of fluctuations is generated before the bounce. This occurs both in matter bounce scenarios [12,13] for scales which exit the Hubble radius in the matter-dominated phase of contraction, and also in ekpyrotic models (in the presence of entropy fluctuations) [34,35]. To obtain a connection with observations, the spectrum after the bounce needs to be determined. It has been shown [36] that the form of the spectrum after the bounce can depend on details of the bounce, although in many toy models one finds that on large scales the spectrum is preserved (see e.g. the analysis of [37]). A result concerning the transfer of fluctuations in an ultraviolet complete theory is thus highly desired. However, this question is also of interest for the particular model which is analyzed in this paper, which in all likelihood produces a black brane.

As is clear from previous work, the gauge theory is in a highly excited state as one approaches the region of weak 't Hooft coupling, possibly in a coherent state. In such a situation, we expect that we can learn a lot from the *classical limit* of the Yang-Mills theory. We therefore study the time evolution of small fluctuations around the background across the region of weak coupling. Such small fluctuations are related, by the AdS/CFT correspondence to bulk fluctuations. The kind of fluctuations we are interested in are those which are given by correlation functions on a fixed radial slice on AdS, close enough to the boundary. As we discussed, the spacetime in the future might contain black holes (branes). However so long as the space-time near the boundary is smooth and normal, we can use the AdS/CFT dictionary to translate boundary fluctuations to bulk fluctuations.

In this paper we take a preliminary step towards computing the transfer of cosmological fluctuations from before the beginning to after the end of the high curvature bulk regime. We begin with a given spectrum of cosmological perturbations in the contracting phase of the bulk, while the bulk is still weakly coupled. At the time when the bulk becomes strongly coupled (and, correspondingly, the boundary conformal field theory becomes weakly coupled) we map the fluctuations onto fluctuations of the gauge fields in the boundary theory. We then evolve the fluctuations to the future in the weakly coupled region on the boundary. In our classical approximation, it is now straightforward to find the fate of these fluctuations at late times. The third step of our analysis is the reconstruction of bulk fluctuations from the boundary data to the future of the bulk singularity. Note that we are interested in fluctuations on scales of current cosmological interest. These scales are infrared modes from the point of view of the physics which we are considering. Specifically, the wavelength of the modes we are interested in is larger than the Hubble radius at the times between $-t_b$ and t_b when we evolve the fluctuations on the boundary.

Our main result is that the momentum dependence of classical fluctuations for momenta much smaller than the cutoff ($k\xi \ll 1$) does not change after crossing the weak 't Hooft coupling region, while their amplitudes change by $O(1)$ factors. This means that the spectrum of fluctuations of the dilaton field near the boundary also have this behavior. In particular, if we start out with a nearly scale invariant spectrum with a red tilt (as is the case for models of matter bounce), the spectrum right after the "bounce" will remain the same. While this result is shown for the bulk dilaton, we conjecture that the result we obtain will directly apply to the evolution of the gravitational wave spectrum, the reason being that a test scalar field in a cosmological background obeys the same equation of motion as that of the amplitude of a particular polarization state of gravitational waves.

This paper is organized as follows. In Sec. II, we review the proposed generalization of the AdS/CFT correspondence to a time-dependent background [3,4]. Section III consider a bulk scalar field and its dual in the boundary gauge theory. As it turns out, the time-dependence of the background scalar field induces a time-dependence of the mass of the gauge field. In Sec. IV we study the evolution of the gauge field given the time-dependence of the coupling constants induced by the nontrivial scalar field in the bulk. Since we are interested in eventually computing linear fluctuations in the bulk, we will focus on linear perturbations of the boundary field. This leads to a dramatic simplification of the analysis. We can work in Fourier space. Each Fourier mode obeys an ordinary differential equation which is analogous to the equation which cosmological fluctuations in a time-dependent background obey in the context of standard general relativistic perturbation theory. Hence, we can use the accumulated knowledge about the evolution of cosmological fluctuations in time-dependent backgrounds to solve for the evolution of the linear boundary gauge field perturbations through the time point $t = 0$ where the bulk theory becomes singular. Since the boundary theory is weakly coupled near the bulk singularity, the computations done in the context of the boundary theory remain under control. At large positive times (when the bulk theory becomes weakly coupled) we then reconstruct the bulk scalar field using boundary-to-bulk propagators. This is discussed in Sec. V where we also extract the power spectrum of the scalar field fluctuations at late times and relate it to the initial spectrum before the bulk singularity. We discuss and summarize our results in the final section.

II. TIME-DEPENDENT ADS BACKGROUND AND CFT DUAL

The original Maldacena conjecture is a duality between a type IIB string theory on $\mathrm{AdS}_5 \times S_5$ and a conformal field theory, a supersymmetric Yang-Mills (SYM) $\mathcal{N} = 4$ large N $SU(N)$ gauge theory, living on the boundary of AdS_5 [1]. The two dimensionless quantities on the bulk side are R/l_s, where R stands for the AdS radius and l_s is the string length and the string coupling constant g_s. These are related to the two dimensionless quantities in the SYM theory, the Yang-Mills coupling g_{YM} and the rank of the gauge group N by

$$\frac{R^4}{l_s^4} = 4\pi g_{\mathrm{YM}}^2 N \qquad g_s = g_{\mathrm{YM}}^2. \tag{3}$$

The string coupling g_s is of course given by the bulk dilaton field φ

$$g_s = \exp\langle\varphi\rangle. \tag{4}$$

There are two particularly noteworthy aspects of this correspondence. Firstly, it relates a gravitational theory (the bulk theory) to a nongravitational field theory on the boundary. From this point of view, the challenge of quantizing gravity suddenly takes on a completely new view. Secondly, the duality is a strong coupling—weak coupling duality. The bulk dilaton provides both a measure of the couplings in the bulk and in the boundary. However, it is precisely when the bulk theory becomes strongly coupled that the boundary theory becomes weakly coupled. In particular we can consider $N \gg 1$ so that bulk quantum effects are small, but $g_{\mathrm{YM}}^2 N \ll 1$ so that the boundary theory is weakly coupled—this would correspond to bulk curvature scales of the order of string scale, signifying that stringy effects become important in the bulk. This can be achieved, e.g. by having $g_s = g_{\mathrm{YM}}^2 \ll 1/N$ for some fixed large N.

This suggests an interesting avenue to address the question of resolution of cosmological singularities. One possible way to do this is to consider a time dependent boundary condition of the bulk dilaton [3,4]. By the standard AdS/CFT correspondence the dual field theory living on the boundary now has a time dependent coupling. A cosmological singularity corresponds to a divergence of the gravitational coupling and thus to a region where conventional approaches to quantizing gravity will fail. However, by the AdS/CFT correspondence the bulk theory is dual to a nongravitational theory on the boundary, and the bulk singularity corresponds to a point in time when the boundary theory becomes weakly coupled. Thus, the usual principles of field theory quantization should be applicable, and one has to deal with a weakly coupled field theory which, however, is time dependent.

The first step is to consider the low energy limit of the bulk type II string theory, namely type II supergravity, and to focus on the bosonic sector of this theory. The second step is to allow for a time dependence in the bulk fields. The bulk fields involve the ten-dimensional metric, the dilaton φ and a five form F_5. The ansatz for such solutions is given by a nontrivial metric of the form

$$ds^2 = \frac{R^2}{z^2}[dz^2 + \tilde{g}_{\mu\nu}(x^\mu)dx^\mu dx^\nu] + R^2 d\Omega_5^2 \tag{5}$$

where $d\Omega_5^2$ is the standard metric on a unit 5-sphere. The surface $z = 0$ is the boundary of the space-time. Note that the metric on a constant z slice is a function of the coordinates x^μ only. This also holds for the dilaton field $\varphi(x^\mu)$. Finally, the five form is given by

$$F_{(5)} = \omega_5 + \star_{10}\omega_5. \tag{6}$$

As shown in [4], these bulk fields satisfy the full ten-dimensional equations of motion provided

$$\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi, \tag{7}$$

where $\tilde{R}_{\mu\nu}$ is the Ricci tensor of the metric $\tilde{g}_{\mu\nu}$, and where the dilaton φ obeys the Klein-Gordon equation

$$\partial_A(\sqrt{-g}g^{AB}\partial_B\varphi) = 0. \tag{8}$$

In the above, g is the determinant of the full metric g_{AB}, and the indices A, B run over all five space-time dimensions.

$\mathrm{AdS}_5 \times S_5$ is of course a special case of a space-time described by the metric (5). In this case, the metric $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, the Minkowski metric, and the dilaton is constant. The radial coordinate z runs from $z = 0$ at the boundary to $z = \pm\infty$ at the Poincaré horizons.

Time-dependent deformations of this background were considered in [3,4,26–28,30]. Specifically, we shall consider a background of the form [28] obtained by introducing a time-dependent dilaton background and adjusting the metric of $\tilde{g}_{\mu\nu}$ such that the Einstein equation (7) and the Klein-Gordon equation (8) remain satisfied. We will mainly deal with a solution where the boundary metric looks like that of a Friedmann universe

$$\tilde{g}_{\mu\nu}dx^\mu dx^\nu = -dT^2 + a^2(T)\delta_{ij}dx^i dx^j, \tag{9}$$

with T being the cosmic time and $a(T)$ the scale factor. The dilaton and scale factor were taken to be

$$a \sim |T|^{1/3}, \qquad \varphi = \frac{2}{\sqrt{3}}\ln\frac{|T|}{R}. \tag{10}$$

This corresponds to an early contracting phase leading to a "big crunch" singularity at $T = 0$ followed by an expanding cosmology. In conformal time t, the four dimensional part of the metric is

$$\tilde{g}_{\mu\nu} dx^\mu dx^\nu = 2|t|[-dt^2 + \delta_{ij} dx^i dx^j], \tag{11}$$

while the dilaton is given by

$$\varphi = \sqrt{3} \ln \frac{|t|}{R}. \tag{12}$$

If the singularity can be resolved by mapping the dynamics to the boundary, we will have a stringy realization of a bouncing scenario, though one would not expect a perfect bounce. As mentioned in the introduction we do not know yet if this indeed happens.

The dilaton profile (12) leads to a diverging string coupling e^φ at early and late times: this would seem to require incorporation of bulk quantum corrections. However it turns out that one can obtain solutions which have bounded values of the coupling e^φ at all times, and whose behavior near the “singularity” is identical to the above. Such a solution is given by the metric [27]

$$ds^2 = \frac{R^2}{z^2}\left[dz^2 + |\sinh(2t)|\left\{-dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_2^2\right\}\right] \tag{13}$$

and a dilaton

$$e^{\varphi(t)} = g_s |\tanh(t/R)|^{\sqrt{3}}. \tag{14}$$

Near $t = 0$ the dilaton profile goes over to that in (12). The metric (13), whose four-dimensional part is a FRW metric with constant negative curvature does not, however, become (11) as $t \to 0$. The difference between the two, however, become increasingly unimportant as $t \to 0$ where the bulk stress tensor is dominated by the time derivative of the dilaton rather than the spatial curvature.

A sketch of the space-time we are considering is given in Fig. 1. The vertical axis at $z = 0$ is time, the horizontal axis at $t = 0$ represents the AdS radial coordinate. The Poincaré horizons are at 45°. While we will consider the solutions (12) and (11) it is useful to think of this as embedded in the solutions (14) and (13) with a bounded dilaton. Then at early times the Yang-Mills coupling is $g_{\rm YM}^2 = g_s$, and we will always consider $g_s \ll 1, N \gg 1$ with $(g_s N) \gg 1$. Thus the early time evolution is governed by the bulk supergravity equations. The time t_b is defined as the time when the ’t Hooft coupling of the boundary theory becomes $O(1)$, i.e.

$$t_b/R \sim (g_s N)^{-1/\sqrt{3}}. \tag{15}$$

Thus, in the region $t > -t_b$ and $t < t_b$ the bulk gravity theory is weakly coupled. For $-t_b < t < t_b$ the bulk curvatures grow large and stringy effects becomes important, while the boundary field theory is weakly coupled, although in the presence of a time dependent coupling.

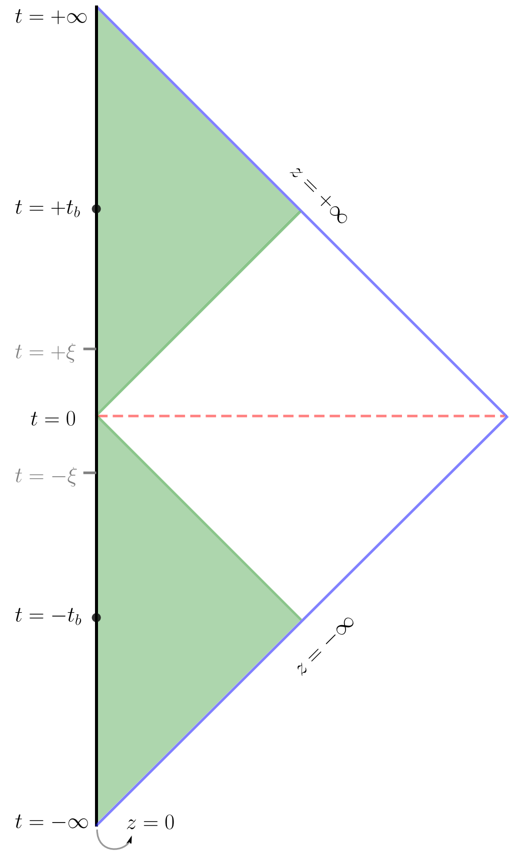

FIG. 1. Conformal diagram of the background space-time. The vertical axis at $z = 0$ is time t, the horizontal direction at $t = 0$ represents the coordinate z. The lines at 45° are the Poincaré horizons at $z = \pm\infty$. If there were no deformation of AdS, the region drawn would correspond to the Poincare patch of AdS. We are considering a deformed space-time in which the bulk gravity is strongly coupled between $t = -t_b$ and $t = t_b$, and singular at $t = 0$. At the same time the boundary gauge theory becomes weakly coupled for t between $-t_b$ and t_b. Hence, after the time $-t_b$ the evolution on the boundary becomes tractable in perturbation theory. On the future side of the bulk singularity, the boundary theory remains tractable perturbatively until the time t_b when the bulk theory becomes weakly coupled again at the cost of the boundary theory becoming strongly coupled. At that time we can reconstruct the bulk information (at least in the vicinity of the boundary) from boundary data (see e.g. [38–42]). As we will see, to study the evolution of the boundary fluctuations we need to impose a cutoff at $t = \pm\xi$.

Note that the solutions considered above have a nontrivial boundary value of the dilaton and a boundary metric which is conformal to flat space. However there are no subleading normalizable pieces of these bulk fields. From the dual field theory point of view, it thus appears that we have nontrivial time dependent sources, but a trivial response. This indicates that such cosmologies correspond to some nontrivial initial states. However the nature of this state in terms of the gauge theory variables is not known.

There is another feature of these solutions which deserves mention. The singularity at $t = 0$ extends from the boundary to the bifurcation point of the Poincare horizon at $z = \infty$. This means that there is in fact a singularity at any finite Poincare time which is infinitely far from the boundary. Presumably this feature has something to do with the nature of the initial state. However, as pointed out in [31], this bifurcation point singularity can be resolved by lifting the solution to one higher dimension and embedding the solution in a higher dimensional soliton solution.

In the following sections we are interested in evolving spectator massless bulk scalar field (which will be in fact taken to be the dilaton itself) perturbations from the past weakly coupled bulk region to the future weakly coupled bulk region by mapping the state onto the boundary at time $t = -t_b$, evolving on the boundary to positive times, and using boundary-to-bulk reconstruction techniques to recover fluctuations in the bulk.

III. THE BULK THEORY OF A TEST SCALAR IN A CONTRACTING UNIVERSE

To begin with, we consider a test scalar field of mass m living in the bulk space-time (in the following section we will take this test scalar field to be the dilaton itself). Its action takes the form

$$S_\varphi = -\int d^5x\sqrt{-g}(g^{MN}\partial_M\varphi\partial_N\varphi + m^2\varphi^2), \tag{16}$$

where g^{MN} is the metric of the five-dimensional deformed AdS space-time

$$ds_5^2 = \frac{R^2}{z^2}[dz^2 + \tilde{g}_{\mu\nu}(x^\mu)dx^\mu dx^\nu] \tag{17}$$

with the four dimensional part $\tilde{g}_{\mu\nu}$ given by (9). Varying the action with respect to the scalar field yields the following equation of motion:

$$\ddot{\varphi} + 3H\dot{\varphi} - a^{-2}\varphi_{,ii} + m^2\frac{R^2}{z^2}\varphi - \varphi_{,zz} + \frac{3}{z}\varphi_{,z} = 0, \tag{18}$$

where $H = \dot{a}/a$ is the Hubble parameter, and a dot denotes the derivative with respect to cosmic time T.

Since we are interested in the spectrum of fluctuations in the three spatially flat coordinates x^i, we first of all extract the x^i dependence by expanding in Fourier modes. The resulting differential equation is a partial differential equation in T and z, and we make a separation of variables ansatz to separate the T and z dependence. More specifically, we write

$$\varphi(T, z, x^i) = \mathcal{T}(T)Z(z)X(x^i), \tag{19}$$

where the $X(x^i)$ are the spatial Fourier modes, i.e. solutions of

$$\nabla^2 X + k^2 X = 0, \tag{20}$$

with solutions which are positive or negative frequency oscillations in the three vector $\mathbf{x}$, where the rescaled temporal field

$$\tilde{\mathcal{T}}(t) \equiv a(t)\mathcal{T}(t) \tag{21}$$

(with t being conformal time defined via $dT = adt$) obeys the equation

$$\tilde{\mathcal{T}}'' + \left(\omega^2 a^2 + k^2 - \frac{a''}{a}\right)\tilde{\mathcal{T}} = 0, \tag{22}$$

(a prime denoting the derivative with respect to t) and the radial function $Z(z)$ obeys the equation

$$Z_{,zz} - \frac{3}{z}Z_{,z} + \left(\omega^2 - \frac{m^2R^2}{z^2}\right)Z = 0. \tag{23}$$

The separation constant ω plays the role of a temporal frequency. The solutions of the radial equation are Bessel functions

$$Z(z) = C_J z^2 J_\nu(\omega z) + C_Y z^2 Y_\nu(\omega z), \tag{24}$$

with

$$\nu = \sqrt{4 + m^2R^2}. \tag{25}$$

The temporal equation (22) takes on the familiar form of the coefficient function of a comoving Fourier mode of a massive scalar field (mass given by ω^2) in an expanding background space-time which undergoes cosmological squeezing (the final term on the left-hand side of the equation). In the case $\omega^2 = 0$ it is also the equation of motion which gravitational waves in an expanding space obey [43,44]. In particular, in the case of infrared modes for which k^2 is negligible, then close to the singularity the mass term is negligible and the squeezing term dominates. In this paper we will, however, not be evolving the fluctuations in the bulk until the singularity, but only to the point in time when the bulk theory ceases to be weakly coupled. Then, we will map them onto the boundary and evolve them with the boundary equations near the bulk singularity.

Our main result does not depend on the initial fluctuation spectrum. As a concrete example, however, we could e.g. take the initial spectrum to be scale invariant for modes whose wavelength is larger than the Hubble radius. During the phase of contraction the Hubble radius is decreasing in

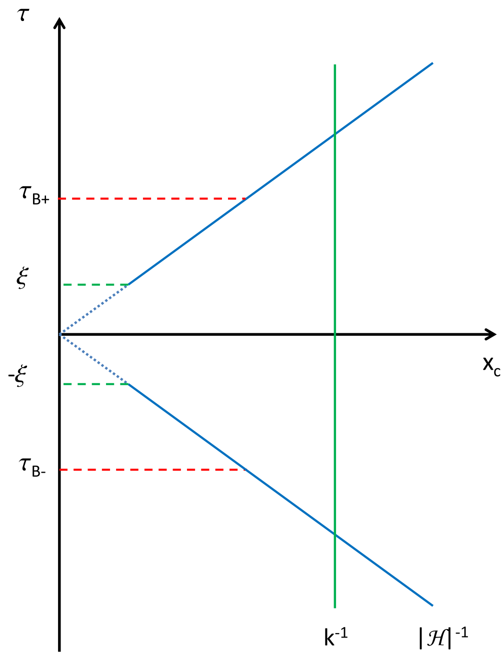

FIG. 2. Space-time sketch of the relevant times and length scales in our deformed AdS_5. The horizontal axis is comoving spatial coordinate (in direction perpendicular to the radial direction). The vertical direction is conformal time. τ_{B+} and τ_{B-} are the times when the coupling constant is 1, the times $\pm\xi$ occur when the bulk curvature reaches string scale. The vertical line represents the wavelength of a mode which we are interested in. Note that the length is larger than the Hubble radius at τ_{B+} and τ_{B-}. The blue lines at 45° indicate the Hubble radius.

comoving coordinates (see Fig. 2). This is motivated by the fact that in several models of "bounce" cosmology (e.g. matter bounce of [12,13] or ekpyrotic [34] scenarios) an initial vacuum spectrum (see e.g. [45]) on sub-Hubble scales gets converted to a scale-invariant one once the scales exit the Hubble radius and undergo squeezing.

We will thus take the initial power spectrum

$$P(k) \equiv k^3 |(\delta\varphi)(k)|^2 \tag{26}$$

on super-Hubble scales to have index $n = 0$ [where we are using the convention for the index used in the cosmology community for gravitational waves, namely $P(k) \sim k^n$].

After discussing the bulk-to-boundary correspondence for the fluctuations we will determine the initial spectrum of perturbations of the boundary gauge fields which are induced by the inhomogeneities of the bulk field discussed above.

IV. THE DUAL BOUNDARY THEORY

A. The deformed dual boundary theory

For a pure AdS bulk, the dual boundary theory is a $\mathcal{N} = 4$ SYM theory. According to the AdS/CFT correspondence, fields in the bulk are related to operators in the boundary theory. For example, the bulk metric $g_{\mu\nu}$ is dual to the boundary energy-momentum tensor $T_{\mu\nu}$. In this paper we are interested in scalar field fluctuations in the bulk and their boundary evolution. Specifically, we will take this scalar field φ to be the dilaton φ. In this case, the bulk scalar field is dual to the trace of the square of the field strength tensor. For an exact AdS bulk, the boundary action is simply

$$S_{\rm YM} = -\frac{1}{4}\int d^4y \frac{1}{g_{\rm YM}^2} {\rm Tr}[F_{\mu\nu}F^{\mu\nu}], \tag{27}$$

with

$$g_{\rm YM}^2 = e^{\varphi} = g_s. \tag{28}$$

Here, we denote the boundary coordinates by y. We use a generalized AdS/CFT correspondence according to which the time-dependent dilaton in the bulk leads to a time-dependent gauge coupling of the boundary theory, i.e. (27) is generalized to be

$$S_{\rm YM} = -\frac{1}{4}\int d^4y e^{-\varphi(t)} {\rm Tr}[F_{\mu\nu}F^{\mu\nu}]. \tag{29}$$

The operator dual to the bulk dilaton is ${\rm Tr}F_{\mu\nu}F^{\mu\nu}$ at large N. This has been carefully derived in Appendix A of [28]. Note that we are always working at large N, but the 't Hooft coupling $g_{\rm YM}^2 N$ can become small.

The bulk solutions we are interested in have, in addition, a boundary metric which is nontrivial and time dependent. However in the slicing chosen in (5) and the time t chosen on the boundary at $z = 0$ as in (11), the metric on which the gauge theory lives is conformally flat. Since the boundary theory is the four-dimensional SYM theory, the conformal factor decouples and the non-trivial effect is that of the time dependent dilaton [46].

What was said so far applies to an unperturbed theory. In the presence of perturbations the correspondence becomes more involved [47]. Each supergravity perturbation has two modes—one normalizable and the other not. The normalizable mode determines the expectation value of the boundary operator, the non-normalizable mode enters as the coupling of the operator in the boundary theory.

We are interested in evolving linearized bulk fluctuations. This means we will turn on normalizable bulk modes. Hence, we will be evolving the linear fluctuations in the gauge field A_μ on the boundary. The initial conditions for the gauge field fluctuations on the boundary are set by the dilaton perturbations via the linearized version of the correspondence

$$\varphi(y) \to \frac{1}{g_{\rm YM}^2}\langle {\rm Tr}[F^2(y)]\rangle, \tag{30}$$

where the normalizable dilaton fluctuation behaves in the standard fashion as

$$\varphi(z, y) \to z^4 \varphi(y) \tag{31}$$

as one approaches the boundary $z = 0$.

The effects of linear dilaton fluctuations on the evolution of the gauge field fluctuations would be a second order effect in the amplitude of fluctuations. Hence, at linear level in perturbation theory there is no such coupling and the only effect of the dilaton on the gauge field fluctuations is via the dilaton-dependence of the gauge coupling constant at background level.

Given a spectrum of dilaton fluctuations at early times in the bulk, we will evolve them in the bulk until bulk perturbation theory breaks down at time $-t_b$. At that point, we compute the boundary values of the dilaton fluctuations and use them to determine the initial values of the gauge field fluctuations δA_{mu} at that time.

B. Evolution of the boundary fluctuations before the singularity

As we have mentioned, the dilaton field evolves as a function of cosmic time and initially takes a large value, and thus the Yang-Mills coupling is also very large initially which implies that the boundary theory is strongly coupled in the far past. [It is useful to think in terms of the solutions (13) and (14) with bounded dilaton profiles]. At these early times, the dynamics of the whole space-time can be studied making use of the bulk theory since the gravity sector is weakly coupled. As the Universe contracts, the value of e^{φ} decreases and the corresponding 't Hooft coupling of the field theory becomes $O(1)$ at the time $-t_b$ given by (15). This is the time where the gravity approximation begins to fail.

As the bulk space further contracts, the boundary theory becomes weakly coupled as $g^2_{\mathrm{YM}} N < 1$. When $t = 0$ and $\varphi \to -\infty$, the bulk singularity point is reached. While it may appear that the boundary gauge theory is now free, as shown in [28], each momentum mode of the gauge theory displays a singular behavior. We therefore work with the modified dilaton profile (2) with $\alpha = \sqrt{3}$ so that this is a genuine bulk solution for $|t| > \xi$.

We will make a gauge choice

$$A_0 = 0, \tag{32}$$

and also impose an additional constraint

$$\partial^i A_i = 0. \tag{33}$$

The Gauss law constraint is then automatically solved [28].

The form of the action (29) suggests a field redefinition (for an analysis in terms of the original variables see the Appendix)

$$A_\mu \to \tilde{A}_\mu \equiv e^{-\varphi/2} A_\mu. \tag{34}$$

This has two effects. The first is that it introduces a "mass" term for the redefined field. With a dilaton which depends only on time, the effective mass square is given by

$$M^2_{\mathrm{YM}} = \frac{\ddot{\varphi}}{2} - \frac{\dot{\varphi}^2}{4}. \tag{35}$$

The second effect is to bring in a factor of $e^{\varphi/2}$ in front of the cubic interaction term and a factor of e^{φ} in front of the quartic interaction. This might suggest that as $t \to 0$ the nonlinear terms become small and can be ignored. However, as shown in [28] this is incorrect. If one substitutes a general solution of the linearized equations of motion (see below) into the action one finds that the fields $\tilde{A}_\mu$ blow up as $t \to 0$ in such a way that the original field A_μ becomes $O(1)$. Therefore, for arbitrary amplitudes the nonlinear terms cannot be ignored.

However we are interested in the time evolution of *small* fluctuations of the gauge field. The nonlinear terms are then suppressed because the amplitudes are small. In the following we will deal with the linear theory. Effects of nonlinearities will be explored in a future work.

Then the leading terms in the equation of motion for $\tilde{A}_i$ give

$$-\partial_\mu \partial^\mu \tilde{A}_i + M^2_{\mathrm{YM}} \tilde{A}_i = 0. \tag{36}$$

In the background (2) M^2_{YM} becomes

$$M^2_{\mathrm{YM}}(t) = \begin{cases} -\frac{\alpha(\alpha+2)}{4t^2} & \text{if } |t| > \xi \\ -\frac{\alpha(\alpha+2)}{4\xi^2} & \text{if } |t| < \xi. \end{cases} \tag{37}$$

Note that the coefficients in the mass term diverge as $t \to 0$ if $\xi = 0$. This is why the evolution of the fluctuations is nontrivial in spite of the fact that the boundary gauge theory becomes weakly coupled at this time. Note that in terms of the original variables A_μ there is no divergence. But a branch cut in the solutions remains. Working with the rescaled variables has the advantage that the equation of motion is similar to that of a simple quantum mechanics problem in a nontrivial potential, and we can use our knowledge about quantum mechanical scattering problems to find good ways to solve the equation.

In linear theory, each Fourier mode will evolve independently. In fact, we are interested in following modes which early in the contracting phase have a wavelength smaller than the Hubble radius and then exit the Hubble radius at some point in time (i.e. the Hubble radius decreases such that the wavelength becomes larger). Thus, we Fourier transform the gauge field

$$\tilde{A}_\nu(\xi^\mu) = \int_{-\infty}^{\infty} d^3\vec{k} c_A(\vec{k}) \tilde{A}_k(t) \epsilon_\nu e^{i\vec{k}\cdot\vec{\xi}}, \tag{38}$$

where ϵ_ν is the polarization unit vector. The Fourier mode $A_{\tilde{k}}$ then obeys the following equation of motion:

$$\ddot{\tilde{A}}_k + (k^2 + M_{\rm YM}^2)\tilde{A}_k = 0, \qquad (39)$$

which is a harmonic oscillator equation with time-dependent mass. Upon quantization, the Fourier mode can be written as a combination of creation and annihilation operators, and the time-dependence of the mass leads to squeezing of the wave function in the same way that infrared modes of cosmological perturbations and gravitational waves are squeezed on super-Hubble scales in a dynamical cosmological background.

The general solution of a Fourier mode of the gauge field can be written in terms of Bessel functions

$$\tilde{A}_k(t) = \begin{cases} (-t)^{\frac{1}{2}}[\mathcal{D}_J^-(k)J_{\nu_g}(-kt) + \mathcal{D}_Y^-(k)Y_{\nu_g}(-kt)] & \text{if } t \le -\xi, \\ \mathcal{A}\exp[\beta t] + \mathcal{B}\exp[-\beta t] & \text{if } -\xi \le t \le \xi, \\ t^{\frac{1}{2}}[\mathcal{D}_J^+(k)J_{\nu_g}(kt) + \mathcal{D}_Y^+(k)Y_{\nu_g}(kt)] & \text{if } t \ge \xi, \end{cases}$$

with the index

$$\nu_g = \frac{1+\alpha}{2} \qquad \alpha = \sqrt{3}, \qquad (40)$$

and

$$\beta \equiv \sqrt{\frac{\alpha(\alpha+2)}{\xi^2} - k^2}. \qquad (41)$$

For $t < \xi$ the solution of the above mode equation involves two coefficients $\mathcal{D}_J^-$ and $\mathcal{D}_Y^-$ which can be determined by matching the bulk solution and the boundary operator at the surface of $-t_b$. We are interested in modes which start out in the vacuum state early during the phase of contraction, i.e. in their Bunch-Davies [45] state.

On sub-Hubble scales (large values of kt) both modes are oscillating. For small values of kt the modes have very different asymptotics. If $\mathcal{D}_Y^- = 0$ around the moment $-t_b$ then the asymptotic form of the solution is

$$\tilde{A}_k(t) \sim \mathcal{D}_J^- \frac{|t|^{\frac{1}{2}}}{\Gamma_{1+\nu_g}}\left(\frac{|kt|}{2}\right)^{\nu_g}, \qquad (42)$$

i.e. it is in general a decaying mode as $t \to 0$. The second mode scales as

$$\tilde{A}_k(t) \sim -\mathcal{D}_Y^- \frac{\Gamma_{\nu_g}}{\pi}|t|^{\frac{1}{2}}\left(\frac{2}{|kt|}\right)^{\nu_g}, \qquad (43)$$

which is a growing mode which in fact diverges as $t \to 0$. This is a reflection of the fact that the mass term diverges. The physical solution will be dominated by the growing mode unless $\mathcal{D}_Y$ vanishes. But it will not vanish in the general case. In particular, if we were to match the solutions to a Bunch-Davies vacuum, then we would expect the magnitude of both coefficients $D_{J,Y}^-$ in (IV B) to be of the same order.

C. Determination of the spectrum of the boundary fluctuations at $-t_b$

A key step in our analysis is to extract the spectrum of the boundary gauge field from that of the bulk dilaton. The coefficients $\mathcal{D}_J$ and $\mathcal{D}_Y$ are determined via (30) by taking the limit of the dilaton fluctuations at the boundary. We make this identification at the time $-t_b$ when the coupling constant vanishes. At first sight, we are faced with a puzzle: the right-hand side of (30) is quadratic in the gauge field, the left-hand side is linear in the dilaton. It is thus nontrivial to infer the gauge field fluctuations from the bulk dilaton. The approach we will take is to look for a power law form of the gauge field Fourier modes $A_\mu(k)$ which yields the spectrum of the bulk dilaton fluctuations we are starting with.

Recall that we are interested in perturbations on cosmological scales which are in the far infrared and for which spatial gradient terms can be neglected. Thus, the dominant term for the infrared modes in $\mathrm{Tr}[F^2(\xi)]$ is the term $\dot{A}_i^2$. Inserting the Fourier expansion of $A_i(x)$ yields

$$\dot{A}_i^2 = \int d^3k_1 d^3k_2 \dot{A}_i(k_1)\dot{A}_i(k_2)e^{i(k_1+k_2)x}V, \qquad (44)$$

where V is the normalization volume used to define the Fourier transform.

In writing (44) we have ignored the nonlinear terms which are contained in $\mathrm{Tr}(F^2)$. This is because we are interested in the spectrum of small fluctuations so that the nonlinear terms are suppressed by powers of the amplitude.

We introduce new momenta

$$k_1 = \frac{1}{2}(k+k')$$
$$k_2 = \frac{1}{2}(k-k'). \qquad (45)$$

We write down the Fourier expansion of the dilaton field φ, and insert into (30) and (31) to identify Fourier coefficients. This leads to

$$\varphi(k) = \frac{1}{4}\int d^3k' \dot{A}_i\left(\frac{k+k'}{2}\right)\dot{A}_i\left(\frac{k-k'}{2}\right)V^{1/2} \tag{46}$$

where φ is defined in (31).

One way to find a consistent $A_i(k)$ to give rise to a power law bulk spectrum $\varphi(k) \sim k^{-\gamma}$ is as follows. We can divide this integral into a region R_1 with $k' < k$ and a region R_2 with $k' > k$. In the integral over region R_1 we can set $k' = 0$ to find the approximate result

$$k^3 \dot{A}_\mu^2(k) \sim k^{-\gamma} \tag{47}$$

and hence (making use of the fact that on super-Hubble scales $\dot{A} \sim HA$)

$$A_i(k) \sim k^{-\frac{\gamma+3}{2}}. \tag{48}$$

It is straightforward to check that this is a consistent solution for any $\gamma > 0$. Substituting (48) into (46) we get

$$\varphi(k) \sim \int d^3k' (k^2 - (k')^2)^{-(\gamma+3)} = k^{-\gamma}\int d^4q(1-q^2)^{-\frac{\gamma+3}{2}}. \tag{49}$$

The integral over q is convergent when $\gamma > 0$. In particular, for a scale invariant spectrum $\gamma = 3/2$.

A nonzero value of A_i would lead to nonzero values for operators involving higher powers of the field strength. In this paper, however, we are interested in *small fluctuations*. This means that the effect of operators like $\mathrm{Tr}F^n$ are suppressed.

For cosmological scales we are interested in, the modes are outside of the Hubble radius at the time $-t_b$. Hence, we can use the small argument limit of the Bessel functions. We will assume that at $t = -t_b$ both modes have the same amplitude. Let us denote the total amplitude of $A(k)$ (we are dropping the index i) by $2\tilde{A}$ (which is k-dependent). Then we find the following values of the coefficients D_Y^- and D_J^- before the bounce

$$D_Y^- = \tilde{A}|-t_b|^{-1/2}\left(\frac{|-kt_b|}{2}\right)^{\nu_g}$$
$$D_J^- = \tilde{A}|-t_b|^{-1/2}\left(\frac{|-kt_b|}{2}\right)^{-\nu_g}. \tag{50}$$

At this point we know the equation of motion and the initial conditions for the boundary gauge field at the time $-t_b$ when the boundary theory becomes weakly coupled and when we begin the evolution of the fluctuations on the boundary. The evolution can be followed by determining the coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{D}_J^+$, $\mathcal{D}_Y^+$ by standard matching of the function $\tilde{A}_k(t)$ and its time derivative at $t = \pm\xi$, as detailed in the next section.

D. Evolution of boundary fluctuations through the singularity

It is useful to discuss matching of solutions of the general form

$$\tilde{A}_k(t) = \begin{cases} A_-(k)F_1(t) + B_-(k)G_1(t) & \text{if } t \le -\xi, \\ \mathcal{A}\exp[\beta t] + \mathcal{B}\exp[-\beta t] & \text{if } -\xi \le t \le \xi, \\ A_+(k)F_2(t) + B_+(k)G_2(t) & \text{if } t \ge \xi. \end{cases}$$

Matching $\tilde{A}$ and its time derivative across $t = \pm\xi$ then leads to

$$\begin{aligned} A_+ = \frac{1}{\Delta}\Big\{&\cosh(2\beta\xi)[F_1(-\xi)\dot{G}_2(\xi) - \dot{F}_1(-\xi)G_2(\xi)] \\ &+ \sinh(2\beta\xi)\left[\frac{1}{\beta}\dot{F}_1(-\xi)\dot{G}_2(\xi) - \beta F_1(-\xi)G_2(\xi)\right]\Big\}A_- \\ &+ \frac{1}{\Delta}\Big\{\cosh(2\beta\xi)[G_1(-\xi)\dot{G}_2(\xi) - \dot{G}_1(-\xi)G_2(\xi)] \\ &+ \sinh(2\beta\xi)\left[\frac{1}{\beta}\dot{G}_2(\xi)\dot{G}_1(-\xi) - \beta G_1(-\xi)G_2(\xi)\right]\Big\}B_- \end{aligned} \tag{51}$$

$$\begin{aligned} B_+ = \frac{1}{\Delta}\Big\{&\cosh(2\beta\xi)[F_2(\xi)\dot{F}_1(-\xi) - \dot{F}_2(\xi)F_1(-\xi)] \\ &+ \sinh(2\beta\xi)\left[-\frac{1}{\beta}\dot{F}_1(-\xi)\dot{F}_2(\xi) + \beta F_1(-\xi)F_2(\xi)\right]\Big\}A_- \\ &+ \frac{1}{\Delta}\Big\{\cosh(2\beta\xi)[F_2(\xi)\dot{G}_1(-\xi) - \dot{F}_2(\xi)G_1(-\xi)] \\ &+ \sinh(2\beta\xi)\left[-\frac{1}{\beta}\dot{F}_2(\xi)\dot{G}_1(-\xi) + \beta G_1(-\xi)F_2(\xi)\right]\Big\}B_-, \end{aligned} \tag{52}$$

where we have defined

$$\Delta \equiv F_2(\xi)\dot{G}_2(\xi) - G_2(\xi)\dot{F}_2(\xi). \tag{53}$$

To specialize to the case of interest we need to substitute

$$F_1(t) \equiv (-t)^{1/2} J_{\nu_g}(-kt) \qquad F_2(t) \equiv (t)^{1/2} J_{\nu_g}(kt)$$
$$G_1(t) \equiv (-t)^{1/2} Y_{\nu_g}(-kt) \qquad G_2(t) \equiv (t)^{1/2} Y_{\nu_g}(kt)$$
$$A_\pm = \mathcal{D}_J^\pm \qquad B_\pm = \mathcal{D}_Y^\pm. \tag{54}$$

Since we are always interested in wave numbers small compared to the various time scales t_b and ξ, we can replace the Bessel functions by their small argument values for $k\xi \ll 1$. This yields

$$\begin{aligned} 2\nu\mathcal{D}_J^+ &= \left[-\cosh(2\beta\xi) + \sinh(2\beta\xi)\left(\frac{1}{4\beta\xi}(1-4\nu_g^2) + \beta\xi\right)\right]\mathcal{D}_J^- \\ &\quad + \left(\frac{2}{k\xi}\right)^{2\nu_g}\Bigg[\cosh(2\beta\xi)(2\nu_g - 1) \\ &\quad + \sinh(2\beta\xi)\left(\beta\xi + \frac{1}{4\beta\xi}(1-2\nu_g)^2\right)\Bigg]\mathcal{D}_Y^- \\ 2\nu\mathcal{D}_Y^+ &= \left(\frac{k\xi}{2}\right)^{2\nu_g}\Bigg[\cosh(2\beta\xi)(2\nu_g + 1) \\ &\quad - \sinh(2\beta\xi)\left(\frac{1}{4\beta\xi}(1+2\nu_g)^2 + \beta\xi\right)\Bigg]\mathcal{D}_J^- \\ &\quad + \left[\cosh(2\beta\xi) + \sinh(2\beta\xi)\left(\beta\xi + \frac{1}{4\beta\xi}(1-4\nu_g^2)\right)\right]\mathcal{D}_Y^-. \end{aligned} \tag{55}$$

We now substitute the initial conditions, (50) to calculate $\tilde{A}_k(t)$ at time $t = t_b$. The result is

$$\begin{aligned} \tilde{A}_k(t_b) &= \frac{\tilde{A}(k)}{2\nu_g}\Bigg[2\sinh(2\beta\xi)\left(\beta\xi + \frac{1}{4\beta\xi}(1-4\nu_g^2)\right) \\ &\quad + \Bigg(\cosh(2\beta\xi)(2\nu_g + 1) - \sinh(2\beta\xi) \\ &\quad \times \left(\beta\xi + \frac{1}{4\beta\xi}(1+2\nu_g)^2\right)\Bigg)\left(\frac{\xi}{t_b}\right)^{2\nu_g} \\ &\quad + \Bigg(\cosh(2\beta\xi)(2\nu_g - 1) + \sinh(2\beta\xi) \\ &\quad \times \left(\beta\xi + \frac{1}{4\beta\xi}(1-2\nu_g)^2\right)\Bigg)\left(\frac{t_b}{\xi}\right)^{2\nu_g}\Bigg]. \end{aligned} \tag{56}$$

The significant point about (56) is that in the $k\xi \ll 1$ limit, the expression inside the square bracket becomes independent of the momentum k. This is because $\beta = (\frac{\alpha(\alpha+2)}{4\xi^2} - k^2)^{1/2}$, leading to $\beta\xi \approx \frac{1}{2}\sqrt{\alpha(\alpha+2)}$. The sole momentum dependence comes from the overall factor $\tilde{A}(k)$ which is the amplitude at $t = -t_b$. This means that the spectrum of $\tilde{A}_k$ is the same as $t = -t_b$ and $t = t_b$. The amplitude is however amplified, since for $t_b \gg \xi$ we get

$$\begin{aligned} \tilde{A}_k(t_b) &\approx \frac{1}{4\nu_g}\Bigg[\cosh(2\beta\xi)(2\nu_g - 1) \\ &\quad + \sinh(2\beta\xi)\left(\beta\xi + \frac{1}{4\beta\xi}(1-2\nu_g)^2\right)\Bigg] \\ &\quad \times \left(\frac{t_b}{\xi}\right)^{2\nu_g}\tilde{A}_k(-t_b) \sim \left(\frac{t_b}{\xi}\right)^{2\nu_g}\tilde{A}_k(-t_b). \end{aligned} \tag{57}$$

The amplification is due to squeezing of the perturbation modes while their wavelength is larger than the Hubble result.

This result holds for arbitrary functional form of $\tilde{A}(k)$. In particular, when $\tilde{A}(k)$ is a power law and given by a scale invariant spectrum at $t = -t_b$ the spectral index does not change at $t = t_b$.

This is our main result. In the next section we will argue that this result in the boundary theory implies that the spectrum of cosmological fluctuations on a constant z slice in the bulk is also unchanged as the system goes through $t = 0$.

E. The general result

From the above analysis it is now clear that the final result does not really depend on the details of the time dependence of the boundary coupling. This conclusion agrees with what was found when studying nonsingular bulk cosmological models (see e.g. [11] for a recent review). Consider the equation for the gauge field perturbation (36) with a function $M^2_{\rm YM}(t)$ which is smooth and bounded everywhere. Whenever the Eq. (39) has a regular solution for $k = 0$, it is clear that $\tilde{A}_i$ and its time derivative at $t = t_b$ is related to those at $t = -t_b$ by a Bogoliubov transformation

$$\begin{bmatrix} \tilde{A}_i(k, t_b) \\ \dot{\tilde{A}}_i(k, t_b) \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \tilde{A}_i(k, -t_b) \\ \dot{\tilde{A}}_i(k, -t_b) \end{bmatrix}$$

where the matrix M_{ij} depends on the potential, but not on k. Therefore if the initial conditions are such that

$$\tilde{A}_i(k, -t_b) \sim \dot{\tilde{A}}_i(k, -t_b) \sim B(k) \tag{58}$$

it follows that

$$\tilde{A}_i(k, t_b) \sim \dot{\tilde{A}}_i(k, t_b) \sim B(k) \tag{59}$$

up to factors which do not depend on k. The specific power law enhancement factor in (57) is a feature of the fact that the coupling constant is a power law away from the region of small coupling.

Therefore the momentum dependence of the fields in the future is the same as that in the past for all momenta small compared to all other scales in the problem.

V. RECONSTRUCTION OF THE BULK FROM THE BOUNDARY DATA

Given the amplitude and spectrum of the gauge field fluctuations after the singularity, we can reconstruct the late time spectrum of the bulk dilaton after the bounce. We evolve the boundary data until the time $+t_b$ when the 't Hooft coupling becomes unity.

As we emphasized above, the nature of the entire spacetime at positive times in the presence of these fluctuations is not known and might contain a black brane in the bulk. We will, however, assume that there is a region near the boundary where the curvatures are small enough to enable us to use the AdS/CFT dictionary. In particular we will assume that the bulk dilaton field at a point $(z, \vec{x}, t)$ for small enough z can be reconstructed from the dual boundary operator $\mathcal{O}$ using a bulk boundary map of the form

$$\varphi(z, \vec{x}, t) = \int dt' d^3x' K(\vec{x}', t'|z, \vec{x}, t)\mathcal{O}(\vec{x}', t'), \qquad (60)$$

where the kernel $K(x', t'|z, x, t)$ [where (t', x') are boundary coordinates and (t, x, z) are the bulk coordinates] is nonvanishing only for points within the AdS causal wedge (see Fig. 3) of compact support. Due to the translation symmetry of the background, we assume that the kernel K depends only on the relative spatial separation $|\vec{x}? - \vec{x}|$.

The idea of the reconstruction is depicted in Fig. 3: we consider a wedge sticking into the bulk with base on the boundary. The wedge is centered at the time t_b. We need the boundary points of the wedge to all have time coordinate $t' > \xi$—otherwise the wedge will intersect the singular part of the bulk. This will limit the distance from the boundary to which we can reconstruct the bulk.

As we will see, we will in fact never need the explicit form of the boundary-to-bulk propagator in order to extract the spectrum of bulk dilaton fluctuations. The only property of this propagator which will be used is that it is nonvanishing only within the wedge shown in Fig. 3. In particular as $z \to 0$, one recovers the correspondence (31).

Now we turn to the extraction of the power spectrum of the bulk field φ in the future of the reconstruction time t_b. Our starting point is Eq. (60) which gives the bulk field in terms of the known boundary data. To extract the spectrum of fluctuations, we need to take the Fourier transform of $\varphi(z, x, t)$ in the spatial hyperplane perpendicular to the AdS radial coordinate axis z

$$\tilde{\varphi}(z, k, t) = V^{-1/2} \int d^3x \varphi(z, x, t) e^{ik\cdot x}, \qquad (61)$$

where the tilde symbol indicates the Fourier transform.

We now insert the reconstruction formula (60) into the above. Introducing the new coordinates

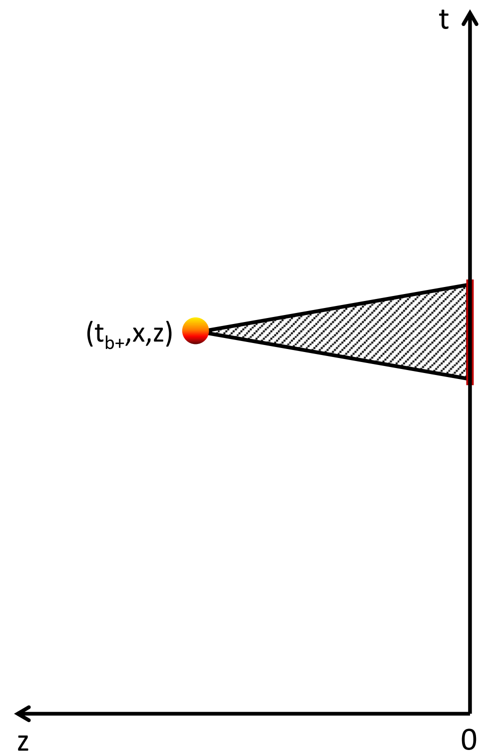

FIG. 3. Sketch of the reconstruction process for bulk operators in the future of the singularity. The vertical axis is time, the horizontal axis indicates the radial coordinate z. The solid line is the boundary. The bulk field at a point with coordinates (t_b^+, x, z) is given by integrating the boundary data against the boundary-to-bulk propagator. The integration involves data in the shaded region only. The solid curves connecting the bulk point to the boundary are null geodesics.

$$\mathbf{x}' = \mathbf{x} + \mathbf{y}'$$
$$t' = t + s, \qquad (62)$$

the result becomes

$$\tilde{\varphi}(z, k, t) = V^{-1/2} \int dsd^3y' K(y', s|z, t) e^{-i\mathbf{k}\mathbf{y}'} \times \int d^3x e^{i\mathbf{k}(\mathbf{x}+\mathbf{y}')} \mathcal{O}(x + y', t + s). \qquad (63)$$

The final integral simply gives the Fourier mode of the boundary operator

$$\tilde{\varphi}(z, k, t) = \int dsd^3y' K(y', t + s|z, t) e^{-i\mathbf{k}\mathbf{y}'} \mathcal{O}(k, t + s). \qquad (64)$$

We are interested in values of k which are small compared to both the inverse Hubble radius and the AdS radius. Assuming that the propagator K has support within the AdS causal wedge (see Fig. 3) of the bulk point, then for all values of the boundary coordinate $\mathbf{y}'$ for which K does not vanish we have $\mathbf{k}\mathbf{y}' \ll 1$, and one has approximately

$$\tilde{\varphi}(z,k,t) \simeq \int ds d^3y' K(y',s|z,t)\mathcal{O}(k,t+s). \quad (65)$$

The k- dependence of the bulk fluctuation is therefore completely determined by the k-dependence of the expectation value of the operator $\mathcal{O}$, which—in our framework—is in turn determined by the k-dependence of the gauge field fluctuations $A_k(t)$. We have seen that the spectral index of the latter does not change when we cross the "singularity". The time integral in (65) has an extent which is roughly of the same order as z. Therefore for z small enough, the k dependence of $\tilde{\varphi}(z,k,t)$ is basically given by the k-dependence of the gauge field fluctuations. Hence, we find that the spectral index of fluctuations close enough to the boundary does not change when matching across the "singular" region (singular in quotation marks because we have cut off the actual singularity). On the other hand, the amplitude of the spectrum changes by a factor $\mathcal{F}$ given by

$$\mathcal{F} = \left(\frac{t_b}{\xi}\right)^{2\nu_g}. \quad (66)$$

In particular, the bulk spectrum on a small z slice is chosen to be scale invariant, i.e. $\varphi(k) \sim k^{-3/2}$ the spectrum continues to be scale invariant.

VI. CONCLUSION AND DISCUSSION

In this paper we have studied the evolution of linearized test field fluctuations in a deformed AdS/CFT cosmology. The deformed AdS background which is the basis of our study is obtained by introducing a nontrivial time-dependence for the dilaton and choosing the metric such that the bulk supergravity equations of motion are satisfied. The background begins with a contracting phase which approaches a bulk singularity at time $t = 0$. As the singularity is approached, the background becomes highly curved. At the time $-t_b < 0$ the gravitational theory becomes strongly coupled. However, the dual gauge theory which lives on the boundary becomes weakly coupled. After the singularity, the bulk expands, and after some time $t_b > 0$ the bulk theory once again becomes weakly coupled.

Our goal is to compare the spectrum of fluctuations of the bulk field in the far past ($t < -t_b$) and in the far future ($t > t_b$). Specifically, we consider fluctuations in the dilaton field. We evolve the fluctuations in the bulk until $t = -t_b$, map them onto the boundary at that time, infer the spectrum of the boundary gauge field fluctuations at this time and then evolve the boundary gauge field fluctuations forward in time, past the singularity, until the time $t = t_b$. At that time, we reconstruct the bulk dilaton field using boundary-to-bulk propagators which are nonvanishing only in a 'AdS causal wedge, and compute the spectrum of the fluctuations.

Since the boundary fluctuations blow up at the time $t = 0$ in spite of the fact that the boundary theory is weakly coupled, we need to smooth out the singularity. We, therefore, modify the dilaton between $-\xi < t < \xi$, where ξ is a cutoff scale and then match the boundary fluctuations in a standard fashion.

Our main result is that the spectral index of the dilaton fluctuations near the boundary is the same in the far past and the far future. While we do not have the tools to map out the future space-time, we believe that our result will have important implications for pre big bang scenarios of cosmology.

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APPENDIX: ANALYSIS IN TERMS OF THE ORIGINAL VARIABLES

The kinetic energy term of the gauge fields on the boundary is not of canonical form. This was the motivation for rescaling the gauge field. The rescaling factor, however, diverges at $t = 0$. Hence, we might worry that the divergence of the rescaled gauge field $\tilde{A}_\mu$ which we found is a result of this rescaling, and that the evolution in terms of the original variables A_μ might be better behaved.

In fact, the equation of motion for the fluctuation of the original variable is

$$\left(-\partial_t^2 + \frac{\sqrt{3}}{t}\partial_t + \partial_i\partial^i\right)A_j = 0, \qquad \text{(A1)}$$

which has Fourier mode solutions

$$A_j(kt) = t^{\nu_g}(c_+ J_{\nu_g}(kt) + c_- Y_{\nu_g}(kt)). \qquad \text{(A2)}$$

The first mode goes to zero at $t = 0$ whereas the second mode approaches a finite value. Hence, there is indeed no divergence in the solutions. However, the first mode has a branch cut at $t = 0$. Hence, matching conditions are still required in order to evolve the solutions from negative to positive values of t.

In the spirit of the AdS/CFT correspondence it would be nice not to have to impose any cutoffs in the matching calculation. This could have been expected since the gauge theory becomes free at $t = 0$. Indeed, there is a matching of the two modes at $t = 0$ for which there is no particle production at all (in terms of the rescaled variables this matching corresponds to having the derivative of $\tilde{A}_{mu}$ match to its inverse between $t = \xi$ and $t = -\xi$). This matching, however, does not correspond to what is done in standard quantum mechanics problems and it misses the particle production which is expected on physical grounds.

Hence, it appears that a matching prescription is needed. An explicit calculation shows if the matching prescription is taken to be the same as the one we used for the rescaled variables, that then the matching calculation in terms of the original variables leads to the same result as that obtained using the rescaled field.

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