NEW ULTRA-LIGHTWEIGHT STIFF PANELS FOR SPACE APERTURES

Jonathan T. Black
University of Kentucky, j.black@uky.edu

Click here to let us know how access to this document benefits you.

Recommended Citation
https://uknowledge.uky.edu/gradschool_diss/390

This Dissertation is brought to you for free and open access by the Graduate School at UKnowledge. It has been accepted for inclusion in University of Kentucky Doctoral Dissertations by an authorized administrator of UKnowledge. For more information, please contact UKnowledge@lsv.uky.edu.
ABSTRACT OF DISSERTATION

Jonathan T. Black

The Graduate School
University of Kentucky
2006
NEW ULTRA-LIGHTWEIGHT STIFF PANELS
FOR
SPACE APERTURES

ABSTRACT OF DISSERTATION

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Engineering at the University of Kentucky

By
Jonathan T. Black

Lexington, Kentucky

Co-Directors:  Dr. Suzanne Weaver Smith, Lester Professor of Mechanical Engineering
Lexington, Kentucky
and  Dr. Jack Leifer, Assistant Professor of Engineering, Trinity University
San Antonio, Texas

2006

Copyright © Jonathan T. Black 2006
ABSTRACT OF DISSERTATION

NEW ULTRA-LIGHTWEIGHT STIFF PANELS
FOR
SPACE APERTURES

Stiff, ultra-lightweight thermal-formed polyimide panels considered in this dissertation are examples of next generation gossamer structures that resolve some of the technology barriers of previous, membrane-dominated gossamer designs while maintaining their low mass and low stowage volume characteristics. The research involved statically and dynamically characterizing and modeling several of these panels to develop validated computer models which can be used to determine the effects of changing manufacturing parameters and scalability. Static characterization showed substantial local nonlinear behavior that was replicated by new physics-based finite element models, and global linear bending behavior that was modeled using classical shell finite elements incorporating effective properties in place of bulk material properties to represent the unique stiffening structure of these panels. Dynamic characterization was performed on individual panels using standard impact hammer and accelerometer testing, enabling successful extraction of several structural natural frequencies and mode shapes. Additionally, the three dimensional time history of the surface of the panels was rendered from video data, and temporal filters were applied to the data to examine the frequency content. These data were also correlated to the shell element numerical models.
Overall, the research contributes to the total knowledge base of gossamer technologies, advances stiff panel-based structures toward space qualification, and demonstrates their potential for use in apertures and other spacecraft.

KEYWORDS: Gossamer, Lightweight Space Structures, Static and Dynamic Characterization, Physics-based Hybrid Modeling, Photogrammetry

Jonathan T. Black

October 30, 2006
NEW ULTRA-LIGHTWEIGHT STIFF PANELS
FOR
SPACE APERTURES

By

Jonathan T. Black

Dr. Suzanne Weaver Smith
Co-Director of Dissertation

Dr. Jack Leifer
Co-Director of Dissertation

Dr. L. Scott Stephens
Director of Graduate Studies

October 30, 2006
RULES FOR THE USE OF DISSERTATIONS

Unpublished dissertations submitted for the Doctor's degree and deposited in the University of Kentucky Library are as a rule open for inspection, but are to be used only with due regard to the rights of the authors. Bibliographical references may be noted, but quotations or summaries of parts may be published only with the permission of the author, and with the usual scholarly acknowledgments.

Extensive copying or publication of the dissertation in whole or in part also requires the consent of the Dean of the Graduate School of the University of Kentucky.
NEW ULTRA-LIGHTWEIGHT STIFF PANELS
FOR
SPACE APERTURES

DISSERTATION

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Engineering at the University of Kentucky

By
Jonathan T. Black
Lexington, Kentucky

Co-Directors: Dr. Suzanne Weaver Smith, Lester Professor of Mechanical Engineering Lexington, Kentucky
and Dr. Jack Leifer, Assistant Professor of Engineering, Trinity University San Antonio, Texas

2006
Copyright © Jonathan T. Black 2006
for Sarah
ACKNOWLEDGEMENTS

The author would like to thank his dissertation committee members, Dr. Kozo Saito and Dr. George Blandford for their assistance, advice, and support; his outside examiner Dr. Lance DeLong for his time, engagement, and outside perspective; and his co-advisors, Dr. Suzanne Weaver Smith and Dr. Jack Leifer. Dr. Smith put a great deal of faith in me the very first time we met by asking me to work on my PhD with her at the University of Kentucky after just a thirty-minute conversation. For that, and the subsequent years of instruction from which I learned a great deal, I am grateful. Dr. Leifer has been a mentor and friend from the time we first met in the summer of 2002. His engagement and excitement for working with me, and his ubiquitous support throughout this process in good times and bad have been invaluable.

The author would like to acknowledge Mr. Larry Bradford of United Applied Technologies, Huntsville, Alabama, for supplying the great variety and number of test articles used in this work, in addition to funding support. This work would not have been possible without his willingness to fulfill virtually all requests and great flexibility in allowing us to follow a research plan that enabled us to achieve additional objectives outside of the contracted effort.

A portion of this work was completed under a NASA Graduate Student Research Program fellowship, grant number NNL04AA21H, Mr. Richard Pappa, advisor. Mr. Pappa of the NASA Langley Research Center advised the author’s Master’s work at NASA Langley and is the reason I have pursued this line of work and field of study. His impact on my work and life through his generous support and belief in me is immeasurable.

The author would like to thank Dr. Brett deBlonk, Dr. Jeff Welsh, and Dr. Scott Erwin of the Air Force Research Laboratory for the tremendous opportunity to work in their unique organization, and Dr. deBlonk especially for his continued collaboration and support.

The author would also like to acknowledge the American Institute of Aeronautics and Astronautics (AIAA) Foundation Willy Z. Sadeh Award in Space Sciences and Space Engineering and Dr. Eligar Sadeh for their generosity in making a portion of this work possible.

The collaboration with and friendship of Dr. Jennie Campbell of Eastern Kentucky University, Mr. Jeff Gagel, Mrs. Kathy Warren, Mr. John Rowe, Ms. Michiko Usui, Mr. Andrew Simpson, Dr. Johné Parker and other valued colleagues, teachers, and friends at the University of
Kentucky made going to work everyday a pleasant experience. The author would also like to acknowledge Dr. Joe Blandino of James Madison University, Mr. Dave Cadogan of ILC Dover, Dr. Danniella Muheim of NASA Goddard Spaceflight Center, and Mr. Tom Jones and Dr. Keith Belvin of NASA Langley for all of their insight and assistance.

The author would especially like to thank his parents, brother, future in-laws, and other friends and family for all of their love and support. It can be said that seminal personal accomplishments are a combination of the best and worst of the person. For all of the energy, dedication, and talent that were required by this dissertation, possibly hubris and certainly a willingness to sacrifice time with and the general wellbeing of loved ones were also necessary. To my fiancée Sarah McKibben, your patience and resolve sustained me and are as much responsible for this work as anything. I am certain that I have done nothing in my life to deserve your love, that it is a divine gift for which I am forever grateful.
TABLE OF CONTENTS

ACKNOWLEDGEMENTS ........................................................................................................ iii
TABLE OF CONTENTS ............................................................................................................. v
LIST OF FIGURES ................................................................................................................... viii
LIST OF TABLES ...................................................................................................................... xv
LIST OF FILES ......................................................................................................................... xvi
NOMENCLATURE .................................................................................................................. xvii
CHAPTER 1. INTRODUCTION .............................................................................................. 1
  1.1 Motivation ...................................................................................................................... 1
  1.2 Goals ............................................................................................................................. 5
  1.3 Objectives ..................................................................................................................... 8
  1.4 Dissertation Overview ................................................................................................. 8
CHAPTER 2. GOSSAMER STRUCTURES ......................................................................... 10
  2.1 Historical Overview .................................................................................................. 10
  2.2 Current Efforts .......................................................................................................... 22
  2.3 Summary ..................................................................................................................... 29
CHAPTER 3. LITERATURE REVIEW ................................................................................ 31
  3.1 Introduction .................................................................................................................. 31
  3.2 Natural Frequency Study ......................................................................................... 31
  3.3 Photogrammetry ........................................................................................................ 34
  3.4 Static Testing and Modeling of Gossamer Structures ................................................ 38
    3.4.1 Introduction .......................................................................................................... 38
    3.4.2 Interferometry ....................................................................................................... 39
    3.4.3 Capacitive and Laser Sensing ............................................................................. 40
    3.4.4 Photogrammetry ................................................................................................. 41
    3.4.5 Static Modeling of Gossamer Structures .............................................................. 44
  3.5 Plate and Shell Equations ............................................................................................ 50
  3.6 Dynamic Testing and Modeling of Gossamer Structures .......................................... 54
    3.6.1 Fundamentals of Modal Testing ........................................................................ 54
CHAPTER 6. DYNAMIC TESTING AND MODELING OF SINGLE PANEL .......... 138
  6.1 Introduction ........................................................................................................... 138
  6.2 Excitation Alternatives .......................................................................................... 139
    6.2.1 Acoustic Excitation .......................................................................................... 139
    6.2.2 Hammer Excitation .......................................................................................... 143
  6.3 Rectangular Panel Testing ....................................................................................... 148
  6.4 Hexagonal Panel Testing ........................................................................................ 157
  6.5 Rectangular Panel Finite Element Modeling and Correlation .............................. 161
  6.6 Hexagonal-Shaped Panel Finite Element Modeling and Correlation .................... 165

CHAPTER 7. DYNAMIC TESTING AND MODELING OF PANEL ARRAY ........ 168
  7.1 Introduction ........................................................................................................... 168
  7.2 Array Testing ......................................................................................................... 170
  7.3 Array Finite Element Modeling and Correlation .................................................... 179
  7.4 Summary and Future Work .................................................................................... 183

CHAPTER 8. SUMMARY AND FUTURE WORK .............................................. 184
  8.1 Detailed Summary ................................................................................................... 184
  8.2 Contributions ......................................................................................................... 186
  8.3 Future Work ........................................................................................................... 187

APPENDICIES .................................................................................................................. 189
  Appendix A: Equipment Specifications ........................................................................ 189
  Appendix B: Number of Nodes in Full-Sized Models Estimation .............................. 191
  Appendix C: ANSYS Batch Files ................................................................................ 192
  Appendix D: Matlab .m Files ....................................................................................... 193

REFERENCES .................................................................................................................. 194

VITA ................................................................................................................................. 210
LIST OF FIGURES

Figure 1.1 – Current space telescopes .................................................................................................................................................. 1
Figure 1.2 – James Webb Space Telescope (JWST) deployed (left) and side (middle) and front (right) views of folded for launch.\textsuperscript{3} Note flat panel array of hexagonal elements .......... 3
Figure 1.3 – Next-generation gossamer structures and potential optics application ................................................................. 5
Figure 1.4 – Possible space applications for thermal-formed thin film polyimide panels ................................................................. 7
Figure 2.1 – Goodyear Inflatable Search Radar Antenna\textsuperscript{8} ........................................................................................................... 10
Figure 2.2 – Goodyear Inflatable Pyramidal Horn\textsuperscript{8} .................................................................................................................. 11
Figure 2.3 – Goodyear Lenticular Inflatable Paragoric Reflector\textsuperscript{8} ......................................................................................... 12
Figure 2.4 – Goodyear Radar Calibration Sphere\textsuperscript{8} .................................................................................................................. 13
Figure 2.5 – NASA ECHO I Balloon\textsuperscript{8} .......................................................................................................................... 14
Figure 2.6 – Inflatable Exo-Atmospheric Object\textsuperscript{8} .................................................................................................................. 15
Figure 2.7 – European Space Agency Contraves Space Division Inflatable Very Long Baseline Interferometry Antenna\textsuperscript{8} .................................................................................................................. 16
Figure 2.8 – European Space Agency Contraves Space Division Telescope Sun Shade Support Structure\textsuperscript{8} .................................................................................................................. 16
Figure 2.9 – European Space Agency Contraves Space Division Land Mobile Communications Reflector Antenna\textsuperscript{8} .................................................................................................................. 17
Figure 2.10 – Large Offset Reflector Structure\textsuperscript{8} .................................................................................................................. 17
Figure 2.11 – Inflatable Antenna Experiment\textsuperscript{8} .................................................................................................................. 18
Figure 2.12 – Deployment sequence of Inflatable Antenna Experience\textsuperscript{13} .............................................................................. 19
Figure 2.13 – Teledesic Blanket Array in deployed and stowed configurations\textsuperscript{8,19} ................................................................. 20
Figure 2.14 – Examples of current membrane optics.\textsuperscript{30-32} Note the high quality of the reflected images in each case ................................................................................................................................. 23
Figure 2.15 – Solar sail concepts for Halley’s Comet fly-by\textsuperscript{20} .......................................................... 24
Figure 2.16 – Four quadrant solar sail concept\textsuperscript{40} .............................................................................................................. 25
Figure 2.17 – Znamya solar sail\textsuperscript{41} .................................................................................................................. 25
Figure 2.18 – Deployed 20 meter solar sail demonstrator\textsuperscript{56,57} ............................................................................................. 26
Figure 4.9 – 3D pictorial view of full geometry (Full Geo) model of hexagonal stiff polyimide panel .................................................................................................................................................................................. 89
Figure 4.10 – 3D pictorial view of reduced geometry models of individual stiffeners using 360 nonlinear springs (left, (a)) and 12 nonlinear springs (right, (b)) .................................................................................................................................................................................. 90
Figure 4.11 – Numerical model results for the three different geometries used ........................................................................................................................................................................................................ 92
Figure 4.12 – Numerical model results showing models with and without negative stiffness ................................................................................................................................................................................................... 93
Figure 4.13 – Force vs. displacement stiffness curves for single and tri stiffeners, loading curves above unloading curves ........................................................................................................................................................................ 94
Figure 4.14 – Individual isolated node inside stiff ultra-lightweight polyimide panel ........................................................................................................................................................................................................ 95
Figure 4.15 – Static compression test setup ......................................................................................................................................................................................................................................................... 96
Figure 4.16 – Force vs. displacement experimental data for individual nodes, loading curves above unloading curves ........................................................................................................................................................................................................ 96
Figure 4.17 – Force vs. displacement experimental data for the quad-node and individual node configurations, loading curves above unloading curves ........................................................................................................................................................................................................ 98
Figure 4.18 – Force vs. displacement experimental data for the ten, quad, and individual node configurations, loading curves above unloading curves ........................................................................................................................................................................................................ 98
Figure 4.19 – Individual node modeled as a single nonlinear spring results ........................................................................................................................................................................................................ 99
Figure 4.20 – Organizational chart of Section 4.4.3 ......................................................................................................................................................................................................................................................... 102
Figure 4.21 – Four nodes modeled as four parallel nonlinear springs results ........................................................................................................................................................................................................ 103
Figure 4.22 – Single and four node results modeled with surrounding nodes ........................................................................................................................................................................................................ 104
Figure 4.23 – Singe node modeled as four parallel springs connected by horizontal springs with stiffness determined iteratively ........................................................................................................................................................................................................ 106
Figure 4.24 – Singe node modeled as ten parallel springs connected by horizontal springs with stiffness determined iteratively ........................................................................................................................................................................................................ 106
Figure 4.25 – Singe node modeled as 19 parallel springs connected by horizontal springs with stiffness determined iteratively ........................................................................................................................................................................................................ 107
Figure 4.26 – Singe node modeled as 31 parallel springs connected by horizontal springs with stiffness determined iteratively ........................................................................................................................................................................................................ 107
Figure 4.27 – Optimal horizontal connection spring stiffness as a function of the size of the single node model from Figure 4.23 through Figure 4.26 ........................................................................................................................................................................................................ 108
Figure 4.28 – Single node, four node, and ten node substructures modeled as nodes loaded plus one level of surrounding, unloaded nodes.................................................................................. 109
Figure 4.29 – Slopes of the single, quad, and ten node experimentally measured data curves... 110
Figure 4.30 – Single, quad, and ten node data modeled with incorporated softening in the defined stiffnesses of the nonlinear springs.......................................................................... 112
Figure 5.1 – Rectangular panel bending test setup ..................................................................... 115
Figure 5.2 – Rectangular stiff ultra-lightweight polyimide panel with relevant loading and measurement points marked ................................................................................................. 115
Figure 5.3 – Long-side supported full panel bending data for the panel manufactured from 0.127 mm (5 mil) Kapton......................................................................................................................... 116
Figure 5.4 – Long-side supported full panel bending data for the panel manufactured from 0.0762 mm (3 mil) Kapton......................................................................................................................... 117
Figure 5.5 – Short-side supported full panel bending data for panels manufactured from both 0.127 mm (5 mil) and 0.0762 mm (3 mil) Kapton, bottom graphs with best-fit line......... 119
Figure 5.6 – Geometric parameter definitions for Equations 5.1, 5.2, and 5.3........................... 121
Figure 5.7 – Virtual tensile test, constant wall thickness (right), sidewall half thickness of top and bottom (left) ........................................................................................................................................... 122
Figure 5.8 – Full panel bending models for long-side supported rectangular panels manufactured from 0.127 mm (5 mil) Kapton............................................................................................................. 123
Figure 5.9 – Deformed shape of the full rectangular panel model manufactured from 0.127 mm (5 mil) thick polyimide film, scale in meters......................................................................................................... 125
Figure 5.10 – Array cantilever bending test setup.......................................................................... 127
Figure 5.11 – Close-up view of clamped edge of array panel showing minimal stiffener crushing.................................................................................................................................................. 128
Figure 5.12 – Leveled 1.905 cm (0.75 in) thick wood piece used to create z = 0 plane............ 129
Figure 5.13 – Example of actual image used in photogrammetry processing of static array test.......................................................................................................................... 130
Figure 5.14 – Photogrammetry software 3D graphic showing point and camera locations ...... 131
Figure 5.15 – Surface fit to experimental cantilevered array data showing deflection in z-direction ........................................................................................................................................... 132
Figure 5.16 – Surface fit to finite element cantilevered array data showing deflection in z-direction ................................................................................................................................ 133
Figure 5.17 – Surface fit to finite element cantilevered array data at 0.675ρ showing deflection in z-direction ................................................................................................................................ 135
Figure 5.18 – Side view of experimental and finite element cantilevered array data................. 136
Figure 5.19 – Edge-view slices of experimental and adjusted density finite element cantilevered array data, experimental data “+” and finite element data solid lines........... 136
Figure 6.1 – Acoustic testing setup............................................................................................. 140
Figure 6.2 – Measured time history data of the acoustically excited corrugated aluminum plate....................................................................................................................................... 141
Figure 6.3 – Comparison of frequency response functions generated using the two different excitation signals................................................................................................................... 142
Figure 6.4 – Impact hammer structure173 .................................................................................... 144
Figure 6.5 – Impulses supplied by soft, medium, and hard hammer tips and their frequency content113 ............................................................................................................................... 145
Figure 6.6 – Example of measured time history data from impact hammer test of rectangular panel ...................................................................................................................................... 146
Figure 6.7 – Example of frequency response function from impact hammer test of rectangular panel, accelerometer attached at point 2, impacting at point 15…………………………………… 147
Figure 6.8 – Example of frequency response function from impact hammer test of rectangular panel, zoomed 0 to 500 Hz, accelerometer attached at point 2, impacting at point 15 ....... 147
Figure 6.9 – Reciprocity test of impact hammer data.................................................................................................................. 148
Figure 6.10 – Impulse dynamic test setup of single rectangular panel........................................ 149
Figure 6.11 – Functional block diagram of dynamic test setup of single rectangular panel ..... 149
Figure 6.12 – Accelerometer attached to bottom of rectangular panel....................................... 150
Figure 6.13 – Measurement, excitation, and attachment points of impulse measurement of rectangular panel................................................................................................................... 151
Figure 6.14 – Consistency diagram for rectangular panel testing using: ERA, Freq range 40 – 130 Hz, Condense: Method EIG, Nr Virtual 3 ................................................................. 153
Figure 6.15 – Sample residue result, dotted line measured FRF, heavy solid line reconstructed FRF, light solid line error ......................................................................................... 154
Figure 6.16 – Experimentally identified first mode shape of rectangular panel at 71.9 Hz
Figure 6.17 – Experimentally identified second mode shape of rectangular panel at 115.2 Hz
Figure 6.18 – Consistency diagram for rectangular panel testing using: ERA, Freq range 117.5 – 300 Hz, Condense: Method EIG, Nr Virtual 3
Figure 6.19 – Experimentally identified third mode shape of rectangular panel at 197.0 Hz
Figure 6.20 – Experimentally identified fourth mode shape of rectangular panel at 244.6 Hz
Figure 6.21 – Measurement, excitation, and attachment points of impulse measurement of hexagon-shaped panel
Figure 6.22 – Consistency diagram for hexagon-shaped panel testing using: ERA, Freq range 30 - 265 Hz, Condense: Method EIG, Nr Virtual 3
Figure 6.23 – Consistency diagram for hexagon-shaped panel testing using: ERA, Freq range 30 – 422.5 Hz, Condense: Method EIG, Nr Virtual 4
Figure 6.24 – Experimentally identified first mode shape of hexagon-shaped panel at 127.7 Hz
Figure 6.25 – Experimentally identified second mode shape of hexagon-shaped panel at 258.1 Hz
Figure 6.26 – Finite element (top) and experimentally determined (bottom) first mode shapes
Figure 6.27 – Finite element (top) and experimentally determined (bottom) second mode shapes
Figure 6.28 – Finite element (top) and experimentally determined (bottom) third mode shapes
Figure 6.29 – Finite element (top) and experimentally determined (bottom) first mode shapes
Figure 6.30 – Finite element (top) and experimentally determined (bottom) second mode shapes
Figure 7.1 – Array dynamic test setup
Figure 7.2 – Actual images used in videogrammetry processing
Figure 7.3 – Functional block diagram of dynamic test setup of array
Figure 7.4 – Impact locations for excitation of first (A) and second (B) mode shapes of a clamped-free beam
Figure 7.5 – Sample time history and power spectral density from second bending mode test
Figure 7.6 – Experimentally determined first mode shape at 3.2 Hz, impact at free end
Figure 7.7 – Sample time history and power spectral density from first twisting mode test...... 174
Figure 7.8 – Experimentally determined second bending mode shape, band pass filter 12 to
23 Hz..................................................................................................................................... 174
Figure 7.9 – Experimentally determined first twisting mode shape, band pass filter 11 to 16
Hz........................................................................................................................................... 175
Figure 7.10 – Experimentally determined second twisting mode shape, high pass filter 30 Hz 176
Figure 7.11 – Time history from Figure 7.5 with low pass filter at 8 Hz and power spectral
density .................................................................................................................................... 178
Figure 7.12 – Filtered time history with linear best fit damping and logarithmic decrement
estimates................................................................................................................................... 178
Figure 7.13 – Filtered time history with first and second order best fit damping estimates...... 179
Figure 7.14 – Finite element (left) and experimentally determined (right) first mode shapes ... 181
Figure 7.15 – Finite element (left) and experimentally determined (right) second mode
shapes..................................................................................................................................... 182
Figure 7.16 – Finite element (left) and experimentally determined (right) mode shapes........ 182
Figure 7.17 – Finite element (left) and experimentally determined (right) fourth mode shapes 182
# LIST OF TABLES

Table 1.1 – Mirror properties of Hubble and the JWST\(^{1-4,6}\) ................................................................. 4
Table 2.1 – Summary of past gossamer programs.......................................................................................... 21
Table 2.2 – Summary of recent gossamer programs...................................................................................... 29
Table 3.1 – Summary of efforts to predict gravity-induced sag in solar sail quality membranes .................................................................................................................................................. 48
Table 4.1 – Material properties for Kapton Type 100 Hn Film\(^{164}\) ................................................................. 87
Table 4.2 – Comparison of model size for the full and reduced geometry models........................................ 90
Table 4.3 – Softening percentages from experimental data and finite element model.............................. 111
Table 5.1 – Defined properties for shell element models of stiff, ultra-lightweight polyimide panels ........................................................................................................................................................................... 123
Table 5.2 – Experimental and finite element data comparison ..................................................................... 137
Table 6.1 – Data acquisition system settings ................................................................................................. 150
Table 6.2 – Experimentally measured structural natural frequencies of the rectangular panel. 156
Table 6.3 – Experimentally measured structural natural frequencies of the hexagon-shaped panel ........................................................................................................................................................................... 161
Table 6.4 – Finite element and experimental structural natural frequencies of the rectangular panel ........................................................................................................................................................................... 162
Table 6.5 – Finite element and experimental structural natural frequencies of the hexagon-shaped panel ........................................................................................................................................................................... 165
Table 7.1 – Experimentally determined structural natural frequencies of array ........................................... 177
Table 7.2 – Finite element and experimental structural natural frequencies of the array ......................... 180
**LIST OF FILES**

<table>
<thead>
<tr>
<th>File Name</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>BlackDis.pdf</td>
<td>9 Mb</td>
</tr>
<tr>
<td>BlackApC.pdf</td>
<td>0.5 Mb</td>
</tr>
<tr>
<td>BlackApD.pdf</td>
<td>1 Mb</td>
</tr>
<tr>
<td>AryMode1.avi</td>
<td>4 Mb</td>
</tr>
<tr>
<td>AryMode2.avi</td>
<td>15 Mb</td>
</tr>
<tr>
<td>AryMode3.avi</td>
<td>11 Mb</td>
</tr>
<tr>
<td>AryMode4.avi</td>
<td>5 Mb</td>
</tr>
</tbody>
</table>
The following is the list of variables that are used in the equations of the dissertation:

- \( A \) = cross-sectional area of the sidewall (beam)
- \( D_{11} \) = first plate bending stiffness term
- \( E \) = modulus of elasticity
- \( E_1 \) = modulus of elasticity along the major axis
- \( E_2 \) = modulus of elasticity along the minor axis
- \( E_{app} \) = apparent modulus of a hollow beam
- \( E_s \) = modulus of elasticity of the solid material from which the honeycomb is manufactured
- \( F \) = axial force applied to the beam
- \( G_{12} \) = shear modulus
- \( H \) = plate or panel thickness
- \( h \) = hexagonal stiffener edge length, see Figure 5.6
- \( I \) = moment of inertia
- \( K_{eff} \) = effective stiffness
- \( K_{horiz} \) = horizontal connection spring stiffness
- \( k \) = individual spring stiffness
- \( L \) = initial beam length
- \( l \) = hexagonal stiffener edge length, see Figure 5.6
- \( m \) = initial beam mass
- \( n \) = total number of springs in the model
- \( R \) = radius of curvature of the shell
- \( r \) = axial aperture radius
- \( t \) = hexagonal stiffener spacing, see Figure 5.6
- \( t_m \) = membrane thickness
- \( X \) = initial exponential curve amplitude
- \( x \) = axial beam displacement
- \( \varepsilon_1 \) = strain along the major axis
- \( \varepsilon_2 \) = strain along the minor axis
\( \rho \) = volumetric density

\( \sigma_1 \) = applied stress in the \( X_1 \) direction

\( \sigma_2 \) = applied stress in the \( X_2 \) direction

\( \nu \) = generic Poisson's ratio

\( \nu_{12} \) = Poisson's ratio along the major axis

\( \nu_{21} \) = Poisson's ratio along the minor axis

\( \theta \) = internal angle of hexagonal stiffeners, see Figure 5.6
CHAPTER 1. INTRODUCTION

1.1 Motivation

In the mid 1990s NASA’s Office of Space Science founded the Origins program to explore two basic questions that scientists and theologians have been trying to answer for centuries: 1) Where did we come from, and 2) Are we alone? To address these rather broad queries, Origins currently has three space telescopes in orbit with a half-dozen more planned to launch before 2015. The Hubble Space Telescope (Figure 1.1(a)) was launched on 24 April 1990, the Far Ultra-violet Spectroscopic Explorer (FUSE) was launched 24 June 1999, and the Spitzer Space Telescope (Figure 1.1(b)) was launched on 25 August 2003.

(a) Hubble Space Telescope image
(b) Artist’s conception Spitzer Space Telescope

Figure 1.1 – Current space telescopes

In the sixteen years since its launch, Hubble has returned the best visible-light images ever taken of deep space and our solar system. The wealth of scientific knowledge gleaned from its images, and more importantly the unprecedented level of popularity with the general public, have seen the observatory’s mission extended four times by repair visits from the Space Shuttle. Its total mission cost is currently listed at $2.1 billion, excluding Shuttle costs.

In addition to its own popularity, Hubble’s success has created a strong base of public support in the United States for other multi-billion dollar space-based astronomy missions.
including the aforementioned Spitzer and FUSE, as well as future telescopes in planning or construction. This support, however, comes at a price: a demand for a continual increase in observatory performance. The James Webb Space Telescope (JWST) shown in Figure 1.2 and scheduled for launch in 2013 promises to meet and exceed these performance demands by studying the earliest galaxies and stars, looking farther into the universe than ever before.

Enhancing the angular resolution and hence performance of the current generation of space telescopes requires increasing primary mirror diameter. The Hubble Space Telescope is currently the most powerful orbital telescope, comprised of a 2.4 meter diameter primary mirror. To achieve a substantial increase in primary mirror diameter over Hubble, two factors must be addressed: launch mass and launch volume. Hubble’s primary mirror consists of ground glass coated with aluminum and magnesium fluoride, and has a mass of 821 kg and areal density of 180 kg/m$^2$. To gather as much light as possible from celestial objects up to 13 billion light-years away, the Webb telescope will have a 6.5-meter diameter primary mirror. It will examine those objects at infrared (0.6- to 28-micron) wavelengths, allowing it to see inside gas clouds and the centers of galaxies. For a mirror of this size to be practical, however, it must have a much lower areal density than the mirror on Hubble and be able to be folded or otherwise stowed to fit into current launch shrouds several times smaller than the diameter of the aperture. For example, the entire James Webb Space Telescope spacecraft will be folded into the launch fairing and deployed on-orbit, as shown in Figure 1.2.$^{14}$
The mirror selected for the Webb telescope is being manufactured by Ball Aerospace and Kodak from lightweight beryllium. Its 18 semi-rigid flat panel segments, shown (in gold) folded inside the launch vehicle in Figure 1.2, will have an areal density of just 15.6 kg/m$^2$ versus the 180 kg/m$^2$ of Hubble.$^{5,6}$ The mass of the deployed 6.5-meter diameter mirror of Webb will be one-third that of Hubble’s, and the total spacecraft mass will be reduced from 11,000 kg to just 6,200 kg. A direct comparison of selected physical parameters can be found in Table 1.1. Although the lightweight mirror technology to be used on the JWST and other orbital observatories is a marked improvement over the rigid, heavy mirror of Hubble, future missions will require even larger apertures. The generation of orbital aperture technology succeeding Webb may make use of ultra-lightweight and inflatable gossamer structures to satisfy increasingly rigorous performance expectations.
Ultra-lightweight and inflatable *gossamer* space structures, designed to be tightly packaged for launch and deploy or inflate once in space, have the potential to inexpensively and significantly enhance the capabilities of orbital telescopes beyond those of the JWST, as well as other planned satellites and spacecraft. Designs under current study use a combination of ultra-thin membranes and lightweight inflatable booms and tori. For the last decade the extreme flexibility of these structures has presented many challenges that have dictated unique approaches to sensing, testing, and modeling. To enable the future launch of apertures tens of meters in diameter, however, next-generation gossamer structures must have increased stiffness over initial designs. This next-generation of structures will combine ultra-low inertias with increased stiffness to constitute a new class of gossamer structures (examples of thermal formed designs are shown in Figure 1.3). Static and dynamic characterization, development of modeling approaches, and measurement method verification must all be undertaken before these structures can be seriously considered for use in space.
1.2 Goals

While the next generation of gossamer structures will be potentially several orders of magnitude more rigid than those of the last decade, they will retain the low mass and low areal density of previous gossamer spacecraft. This unique combination of properties dictate that such materials will be fundamentally different from current aluminum or titanium beams and panels, meaning that a new set of characterization and space qualification tests must be performed. Goals for next generation gossamer structures include the static and dynamic characterization of stiffened polyimide panels (Figure 1.3(d)) – one of the next-generation gossamer materials –
numerical modeling and model validation of those panels, and the development of a characterization methodology specifically suited for next-generation gossamer structures.

Figure 1.4 shows an example of one of these panels made from Kapton (one type of polyimide manufactured by DuPont), as well as demonstrating that the panels have many potential modular space structures applications in addition to apertures. The ability to customize the thickness, stiffness, mass, and size of the panels, and to connect multiple panels in any configuration may enable their use in solar arrays, flat panel optics, and space habitats, among others. In order to perform optimally in these very different applications, however, the detailed characterization of these panels described below is essential.
Figure 1.4 – Possible space applications for thermal-formed thin film polyimide panels
1.3 Objectives

The specific research objectives of this dissertation are as follows:

1) To perform static testing of stiff, ultra-lightweight panels including small-scale stiffening features, full-sized panels, and panel array assemblies.

2) To perform dynamic testing of stiff, ultra-lightweight panels including full-sized panels and panel array assemblies.

3) To develop and validate finite element models of small-scale stiffening features, full-sized panels, and panel array assemblies.

4) To evaluate approaches for modeling and testing this new class of stiff gossamer structures.

5) To determine the potential use of scaling in designing future spacecraft using this new class of stiff gossamer structures.

1.4 Dissertation Overview

The research program described here yields a detailed understanding of the static and dynamic behavior of stiff, ultra-lightweight panels at multiple scales, validated finite element models that capture that behavior, and a matured characterization methodology specific to this new class of stiff gossamer structures. The a detailed description of the research program, the foundation upon which it was built, and its relevance to the research community comprise this dissertation.

The five objectives listed above are addressed in the subsequent chapters. Chapter 2 presents a brief history of gossamer structures from the 1950s to the present. From this, developments of gossamer technology are better understood. A review of experimental and modeling methods pertinent to this effort is presented in Chapter 3, including photogrammetry and modal testing, among others. Small-scale and large-scale static testing and modeling of the panels is detailed in Chapters 4 and 5, respectively. Dynamic testing of full-sized and large-scale assemblies of panels is detailed in Chapters 6 and 7, respectively. Finally, a summary and description of future work are presented in Chapter 8. Supplementary supporting information
such as references, test equipment specifications, and script files for finite element simulations and mathematical data reduction can be found in the References and Appendices.

Copyright © Jonathan T. Black 2006
CHAPTER 2. GOSSAMER STRUCTURES

2.1 Historical Overview

Chapter 2 in Jenkins (2001) presents a comprehensive summary of past gossamer programs, and is extensively reproduced here. In the late 1950s the Goodyear Company initiated the testing and development of the first structures generally accepted to meet current definitions of gossamer spacecraft. Goodyear investigated several lightweight and inflatable structures, some of which are shown in Figure 2.4 through Figure 2.4. The tests were designed to examine the usefulness of gore panels, metalized thin films, and foam rigidization techniques, among others, in tightly packaged, inflatable high precision systems. Specifications of these structures are listed in Table 2.1, including the size. Note that several of the images include people for scale, especially Figure 2.3.

Figure 2.1 – Goodyear Inflatable Search Radar Antenna
Figure 2.2 – Goodyear Inflatable Pyramidal Horn\textsuperscript{8}
Figure 2.3 – Goodyear Lenticular Inflatable Paragoric Reflector
On 12 August 1960 NASA launched the first of many ECHO balloons (Figure 2.5) from its Wallops Island test facility. The balloons were stowed in a 66.04 cm (26.00 in) diameter container and launched on a Delta rocket into low Earth orbit. Upon achieving orbital altitude, the balloons were inflated to a diameter of 30.48 m (100 ft), at which point an ejection system separated the balloon from the launch vehicle. The balloons were coated with a thin layer of aluminum, which yielded slightly upon inflation. The yielding caused the aluminum to work-harden and rigidize, holding the spherical shape of the balloon. The resulting passive spheres, used for ground radar calibration, orbited for several months and were the “first successful large size, high precision, inflatable space structure on orbit.” The test program continued throughout the 1960s.\textsuperscript{5,8,9}
L’Garde Incorporated tested Inflatable Exo-Atmospheric Objects (IEOs) in the late 1960’s as decoys of the Mark 12 nuclear warhead re-entry vehicle. The IEOs were one to two meters in length, downloaded temperature, pressure, acceleration, and flight telemetry data to the ground in real-time, simulated the pitch and roll characteristics, and radar and infrared signatures of actual re-entry vehicles, and inflated in several milliseconds. The decoys had a carbon fabric outer skin and integrated a “water blanket” directly beneath the outer skin for temperature control. The test program concentrated on investigating miniaturized inflation systems, fully instrumented membrane systems, and used the first shaped inflatable structures.\textsuperscript{5,8,10}
During the 1980s the European Space Agency’s (ESA) Contraves Space Division, located in Switzerland, tested three notable gossamer structures shown in Figure 2.7 through Figure 2.9. The Very Long Baseline Interferometry Antenna (VLBIA), shown in Figure 2.7 consisted of a six-meter, axi-symmetric, multiple gore reflector and an inflatable canopy. A rigidized torus supported the reflector and canopy, and a pre-rigidization surface precision on the order of one millimeter rms was achieved for the 21 GHz RF antenna. The VLBIA tested rigidization techniques, rigidizable materials, alignment and metallization of the inflatable elements, and inflation systems. The Telescope Sun Shade Support Structure, shown in Figure 2.8, consisted of two meter diameter flexible thermal panels and tested inflation systems. The Land Mobile Communications Reflector Antenna, shown in Figure 2.9, consisted of a 10- x 12-meter offset reflector antenna. A surface precision of two millimeters rms was achieved while rigidization of the reflector, canopy, and torus by solar heating was tested. All three structures were classified as technology demonstrators, and were the first large inflatable rigidizable reflector structures. 5,8,9,11
Figure 2.7 – European Space Agency Contraves Space Division Inflatable Very Long Baseline Interferometry Antenna

Figure 2.8 – European Space Agency Contraves Space Division Telescope Sun Shade Support Structure
In the late 1980s L’Garde Inc. and the United States Air Force (USAF) tested the Large Offset Reflector Structure shown in Figure 2.10. The structure consisted of 18 gore panels of six-micron thick aluminized Mylar joined to form a 7 x 9 meter solar concentrator, with a one millimeter rms surface precision. It was also classified as a technology demonstrator, and served as a database for the 1996 Inflatable Antenna Experiment.\textsuperscript{5,8}
On 29 May 1996, the Space Shuttle Endeavour on mission STS-77 launched the Inflatable Antenna Experiment (IAE) shown in Figure 2.11 as a joint L’Garde, NASA, and Jet Propulsion Laboratory (JPL) effort. IAE consisted of a 14 meter diameter inflatable canopy reflector and three 28-meter long inflatable struts. The antenna was released from the Shuttle, which then filmed the deployment. Residual air inside the stowed volume caused a much more rapid and violent deployment than desired (Figure 2.12), but the resulting deployed shape, visible in Figure 2.11, was qualitatively as designed. The unanticipated violence of the deployment demonstrated the resilience and strength of the gossamer materials from which the IAE was manufactured. The experiment demonstrated a capability to deploy high precision inflatable structures in space as well as the benefit of the low cost and ease of manufacturing of gossamer structures in general. Namely, should unanticipated events cause failures on actual missions, replacements or redundant spacecraft are inexpensive to manufacture and launch.\textsuperscript{5,8,12-15}

![Figure 2.11 – Inflatable Antenna Experiment]
In the late 1990s ILC Dover, in cooperation with the USAF and JPL, began testing a Large Solar Array Structure, also called the Teledesic Blanket Array. The array, shown in Figure 2.13, was 3 x 10 meters in size, and was comprised of three inflatable, rigidizable carbon fiber and thermoplastic beams supporting a rigid solar cell substrate. It was intended to produce 9 – 12 kW of power at a power-to-mass ratio of 100 W/kg versus the then standard 45 W/kg. It was expected that the maximum potential of the technology was a power-to-mass ratio of 300 W/kg. The array structure was a technology demonstrator originally intended for flight on STS-107, the final Columbia mission in January 2003. The design was then to be employed on the
Deep Space 4 Champollion mission to land a spacecraft on and study the nucleus of a comet, but the program was cancelled in 1999 due to budget constraints.\textsuperscript{5,8,13,16-18}

Figure 2.13 – Teledesic Blanket Array in deployed and stowed configurations\textsuperscript{8,19}

Table 2.1 below shows a summary of the programs discussed in this section.
<table>
<thead>
<tr>
<th>Date</th>
<th>Program</th>
<th>Test Article Size</th>
<th>Areal Density</th>
<th>Accomplishments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Late 1950s</td>
<td>Goodyear Lenticular Inflatable Paragoric Reflector</td>
<td>12 meter diameter</td>
<td>Data unavailable</td>
<td>First precision inflatable reflector</td>
</tr>
<tr>
<td>Late 1950s</td>
<td>Goodyear Radar Calibration Sphere</td>
<td>6 meter diameter</td>
<td>Data unavailable</td>
<td>Metalization, precision sphere</td>
</tr>
<tr>
<td>August 1960</td>
<td>NASA ECHO Balloon</td>
<td>30.48 meter diameter</td>
<td>85 g/m²</td>
<td>First large inflatable structure in orbit</td>
</tr>
<tr>
<td>Late 1950s</td>
<td>L’Garde Inflatable Exo-Atmospheric Object</td>
<td>1 x 2 meter</td>
<td>Data unavailable</td>
<td>First shaped inflatable structure, millisecond inflation</td>
</tr>
<tr>
<td>Mid 1980s</td>
<td>Contraves IVLBIA</td>
<td>6 meter diameter</td>
<td>330 g/m²</td>
<td>Precision alignment of inflatable elements</td>
</tr>
<tr>
<td>Mid 1980s</td>
<td>Contraves Telescope Sun Shade Support Structure</td>
<td>2 meter diameter</td>
<td>Data unavailable</td>
<td>Flexible thermal panels</td>
</tr>
<tr>
<td>Late 1980s</td>
<td>L’Garde Large Offset Reflector Structure</td>
<td>7 x 9 meter</td>
<td>Data unavailable</td>
<td>Millimeter rms surface precision</td>
</tr>
<tr>
<td>29 May 1996</td>
<td>L’Garde Inflatable Antenna Experiment</td>
<td>14 meter diameter</td>
<td>390 g/m²</td>
<td>First flight of inflatable aperture</td>
</tr>
<tr>
<td>Late 1990s</td>
<td>ILC Dover Teledesic Blanket Array</td>
<td>3 x 10 meter</td>
<td>Data unavailable</td>
<td>Double power to mass ratio</td>
</tr>
</tbody>
</table>
2.2 Current Efforts

Over their lifetime gossamer technologies have, in general, been “push” technologies. To date there has not been a great demand, or “pull,” for the technologies specifically, but instead the general goals of decreasing launch masses and volumes, decreasing manufacturing costs, and simplifying spacecraft have driven their development. However, these are goals which many technologies are capable of addressing. Current efforts continue to focus on advancing and qualifying the technologies with the eventual hope of incorporation into future missions rather than development of specific gossamer structures for specific, funded missions.

The James Webb Space Telescope is a good example of this adoption subsequent to many years of research and development. The sun shield on the current design of the telescope, adopted in the late 1990s, consists of several layers of 22 x 10 meter membranes. Research into using membranes in space, however, began in the late 1960’s with several JPL efforts to fly a solar sail past Halley’s comet. It continued without specific mission funding through the 1980s, when several developments in materials made membranes more viable for space use. Research is progressing at NASA Goddard in coordination with several universities on the specific materials and final configuration of the sun shield.

Aside from the Webb telescope sunshield, another area where gossamer technologies currently have a large potential to contribute is in the area of space apertures. The demand for ever larger telescopes and antennae exists in both the defense and scientific communities, and there has been a steady stream of efforts to use membranes for these apertures. The current incarnations of these efforts involve using doubly curved membranes, canopied and pressurized membrane systems, or membranes attached to semi-rigid substrates to form the aperture shape. Recently, a manufacturing process was developed that is capable of holding membrane thickness variations to sub-optical (approximately $\lambda/20$, where $\lambda$ is the average wavelength of visible light, 735 nm) levels. As is evident from Figure 2.14, much work remains before the support structures of the optical quality membranes matches the ultra-lightweight nature of gossamer structures.
Figure 2.14 – Examples of current membrane optics.\textsuperscript{30-32} Note the high quality of the reflected images in each case.

Aside from membrane manufacturing for apertures, multiple studies are being performed to determine how to simulate, deploy, control, and measure the apertures once in space. The ARISE (Advanced Radio Interferometry between Space and Earth) mission, for example, calls for an array of five telescopes, the largest of which will be 30 meters diameter.\textsuperscript{9,31} While the surface accuracy requirements of interferometric telescopes are not as great as for optical telescopes, the tasks of maintaining a precise doubly curved shape and controlling excitations in 30-m diameter membrane are still very challenging. Several methods of attacking the problems have been proposed and investigated, including electron gun control of a piezoelectric substrate\textsuperscript{34-36} and active boundary control at the torus,\textsuperscript{37,38} but until a viable solution is presented there will be no serious demand for membrane apertures in space. Hence gossamer space apertures remain a push technology.
The other major applications of gossamer technology, aside from apertures, are solar sails and sun shades. Solar sails are propellentless propulsion devices that use the momentum transfer of reflected photons of sunlight or laser light to propel the craft. The force provided by sunlight is small, however; just $9.12 \times 10^{-6} \text{N/m}^2$ at one Astronomical Unit (AU, average distance from the Earth to the Sun). The first proposed use of this technology in the 1960s was as a fly-by mission to Halley’s Comet (Figure 2.15). Current solar sail designs, one of which is shown in Figure 2.16, call for square four quadrant sails 100 to 200 meters on a side, $7 \mu\text{m}$ thick, with areal densities on the order of five g/m$^2$. The large size and low mass of the sail are required to generate positive acceleration from the small propulsive force provided by sunlight. For comparison standard white paper is 100 microns thick with an areal density of 75 g/m$^2$. Though the Russian Space Agency did test the Znanya solar sail in 1999 (Figure 2.17) off the Mir Space Station, the technology was, as of the late 1990s, still not considered space qualified by NASA. To obtain space qualification status, full-scale testing in a relevant space environment is required. Again, without a specific mission to provide funds, there was little interest in performing the costly validation.$^{5,39-45}$

Figure 2.15 – Solar sail concepts for Halley’s Comet fly-by$^{20}$
In 2002 NASA created the In-Space Propulsion (ISP) Program to “… develop in-space propulsion technologies that can enable and/or benefit near and mid-term NASA science missions by significantly reducing cost, mass, and/or travel times.”\textsuperscript{46-48} and “mature a suite of
reliable advanced propulsion technologies that will promote more cost efficient missions through the reduction of interplanetary mission trip time, increased scientific payload mass fraction, and allowing for longer on-station operations. These propulsion technologies will also enable missions with previously inaccessible orbits (e.g., non-Keplerian, high solar latitudes). One of the technologies selected for development was solar sails. ISP funded three main efforts to mature solar sail technology. The first involved the manufacture of three sub-scale models – two 10 meters on a side, one 20 meters on a side – of two competing solar sail designs. The test program culminated in 2005 with the in-vacuum deployment of the 400 meter square sub-scale solar sail shown in Figure 2.18.

The second ISP solar sail effort involved the development of an on-orbit measurement system specific to solar sails. Structures tens of square kilometers in size have never been deployed in space, and, combined with the need to avoid physically attaching measurement devices to the sail membrane to maintain its ultra-low areal density, require a unique metric
system be employed. The third ISP solar sail effort researched numerical and analytical methods for modeling and controlling solar sails. The large size and nonlinear nature of the behavior of such membrane structures requires numerical models with possibly millions of degrees of freedom. Models of this type are extremely computationally expensive, challenging to converge, and are very difficult to experimentally validate.\textsuperscript{37,58-64}

Stiff thermal-formed gossamer structures, such as those shown in Figure 1.3 are new to the gossamer family and have yet to receive significant attention. The University of Kentucky is among the first institutions to investigate this technology and development continues at United Applied Technologies and the Virginia Polytechnic Institute and State University.\textsuperscript{65}

ILC Dover is one current manufactures of gossamer materials, or what they term “innovative softgoods solutions.”\textsuperscript{19} Their products encompass almost all gossamer technologies, including structural and laminated materials, space and other protective suits, planetary entry and landing systems, and unmanned aerial vehicle wings. Examples are shown in Figure 2.19.

![Figure 2.19 – ILC Dover Gossamer Products\textsuperscript{19}](image)

(a) Structural/laminated materials            (b) Space suit                (c) Mars airbag landing system

(d) Unmanned aerial vehicle (UAV) wing                 (e) UAV wing folded
On 12 July 2006 another private manufacturer of gossamer space products, Bigelow Aerospace, launched Genesis I, a 1/3-scale prototype of an inflatable habitat. At 4.4 meters in length and 2.6 meters in diameter, it has 11.5 cubic meters of internal habitable volume. The spacecraft is comprised of Kevlar and Vectran and is intended to orbit for five years to test procedures for maintaining proper internal temperature and pressure, and withstanding impacts from micrometeorites and other space debris. An image of the recently deployed structure is shown in Figure 2.20.66

![Figure 2.20 – Inflated and deployed Genesis I spacecraft](image)

Table 2.2 below shows a summary of the programs discussed in this section.
Table 2.2 – Summary of recent gossamer programs

<table>
<thead>
<tr>
<th>Date</th>
<th>Program</th>
<th>Test Article Size</th>
<th>Areal Density</th>
<th>Accomplishments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002 to present</td>
<td>SRS Optical Membranes</td>
<td>0.70 meter diameter, 8 µm thick</td>
<td>11.5 g/m²</td>
<td>First reliable manufacturing of specular membranes with sub-optical thickness variation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Largest subscale solar sail deployment and static and dynamic characterization</td>
</tr>
<tr>
<td>2002 to 2006</td>
<td>L’Garde Solar Sail Demonstrator</td>
<td>20 meters on a side</td>
<td>no thickness data provided</td>
<td></td>
</tr>
<tr>
<td>Early 1990s to</td>
<td>ILC Dover Mars Airbag Landing System</td>
<td></td>
<td>Data unavailable</td>
<td>Reliable low-cost landing system</td>
</tr>
<tr>
<td>present</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1961 to present</td>
<td>ILC Dover NASA Spacesuits</td>
<td>Custom</td>
<td>Data unavailable</td>
<td>Enabled all NASA extra-vehicular activities</td>
</tr>
<tr>
<td>12 July 2006 to</td>
<td>Bigelow Aerospace Genesis I</td>
<td>4.4 meter length, 2.54 meter diameter</td>
<td>Data unavailable</td>
<td>First privately-funded inflatable habitat demonstrator</td>
</tr>
<tr>
<td>present</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.3 Summary

Ultra-lightweight and inflatable gossamer space structures have the potential to significantly enhance the performance current state-of-the-art technologies and enable unique unconventional designs that significantly out perform the current technologies. Their extremely low areal densities and high packaging ratios offer the possibility of stowing structures hundreds of square meters in size inside current launch shrouds and deploying or inflating them in space. The Goodyear Company was the first to dedicate research and development programs to
gossamer structures beginning in the 1950s. Since then notable achievements have occurred on several continents. Today current research of gossamer technology includes the deployment testing of flight-quality solar sails up to 20 meters on a side, reliable manufacturing processes for optical quality membranes, and small-scale inflatable habitats operating in orbit.

The properties that make gossamer structures so attractive, namely their low mass and extreme flexibility, are also their greatest barriers to implementation. The nonlinear nature of their behavior resulting from the low mass and high flexibility creates testing and modeling challenges that have yet to be overcome. These challenges are expanded upon in the next chapter.

Copyright © Jonathan T. Black 2006
CHAPTER 3. LITERATURE REVIEW

3.1 Introduction

The unique combination of properties that makes gossamer structures so attractive for use in space applications also makes them very difficult to test and model. This chapter details those properties and overviews current and past measurement and modeling methods and their relative success. An analytic study of the dynamic nonlinearities of lightweight space structures is presented first, followed by an abridged discussion of the photogrammetry static measurement technique used throughout the dissertation. Also included are an extensive literature review of past and present static testing and modeling of gossamer structures, a description of the derivation of the plate and shell equations on which the models of the stiff, ultra-lightweight panels are based, an extensive literature review of past and present dynamic testing and modeling of gossamer structures, and a discussion of the techniques used to scale the responses of gossamers structures.

3.2 Natural Frequency Study

Based on the historical structures cited in Chapter 2, a representative gossamer structure consisting of ultra-thin membranes on the order of five microns thick and long booms on the order of 75 meters, can be used as a basis for an analytical analysis. Qualitatively such structures are both extremely light and flexible, qualities that are assets in terms of allowing high packaging volumes and minimizing launch masses and costs. These qualities are not, however, unique to ultra-light and inflatable structures.

To gain insight into what does distinguish gossamer structures from structures comprised of more traditional designs and materials, an examination of dynamic behavior is undertaken here. Dynamic responses can be compared, for example, through the analytical formulation of the natural frequency of a free-free beam in tension. The equation for the first natural frequency, derived from Blevins (pp. 134, 144), is as follows:
where \( \lambda \) is a function of mode number, boundary conditions, and the number of spans, \( P \) is the applied tension load, and all other quantities are defined in Nomenclature. Two terms are significant in Equation 3.1, a bending and a tension term. Here the bending term \( EI \) is constant for a constant cross section and will dominate the response of the beam for large moduli and small length-to-width ratios (where width ~ height). Gossamer beams, however, have much lower moduli than metal beams and much higher length-to-width ratios. Therefore their response is dominated by the tension term \( PL^2/\pi^2 \), which is material independent. Figure 3.1 illustrates the relative magnitudes of the bending and tension terms in Equation 3.1 above for steel \( (E = 200 \, \text{GPa}) \), aluminum \( (E = 70 \, \text{GPa}) \), and Kapton \( (E = 2.5 \, \text{GPa}) \). It shows that as the length-to-width ratio of the beam is increased, the tension term becomes significant regardless of material.

\[
\left. \omega \right|_{p=0} = \frac{\lambda_m^2}{2\pi L^2} \sqrt{\frac{1}{m} \left( EI + \frac{PL^2}{\pi^2} \right)} \quad (3.1)
\]

Figure 3.1 – Bending and tension terms vs. length-to-width ratio for three different materials, taken from Equation 3.1.

Many gossamer structures are manufactured from Kapton or similar materials, and therefore must be treated in all cases as flexible, tension-dominated structures, as shown in Figure 3.1. Traditional space hardware with effective stiffnesses \( (EI) \) less than that of aluminum,
and/or large length-to-width ratios such as the solar arrays on the International Space Station will also enter a regime in which their response will become tension dominated. Therefore it is possible to make the argument that, under the right conditions, they will behave quite similarly to gossamer structures.

As a second example, Glaese et al. (2003) shows that the analytical formulation of natural frequency for a doubly-curved membrane (aperture) also derived from Blevins is as follows:

\[
\sigma_{t,j,\text{shallow\_shell}} = \sqrt{\frac{E}{\rho} \left( \frac{\lambda^4 \nu \epsilon^2}{12(1-\nu^2)R^4} + \frac{1}{R^2} \right)}
\]

(3.2)

where \( r \) is the axial aperture radius, \( R \) is the radius of curvature of the shell, and all other quantities are defined in Nomenclature. Here, as membrane thickness \( t_m \) decreases and membrane area \( \pi r^2 \) increases, the plate bending term \([\lambda^4/(12(1-\nu^2))](t_m^2/r^4)\) decreases, becoming negligible in the gossamer regime of thickness on the order of microns and radii on the order of tens of meters. For membranes of this type, the material-independent membrane or shell-stiffening term is a function of \(1/R^2\) and will therefore dominate the dynamic response of the structure.

Figure 3.2 shows the plate and membrane terms of Equation 3.2 plotted against an \( R/r \) ratio for Ultra Low Expansion (ULE) Glass, from which many current space telescope mirrors are ground; Beryllium, from which the current Spitzer Telescope mirror is and future James Webb Space Telescope mirror will be manufactured; and Kapton. An \( R/r \) ratio of 10 corresponds to an approximate Hubble Space Telescope mirror of 2.4 meters in diameter \((r = 2.4/2 = 1.2)\) and an 11.04 meter radius of curvature \((R)\). The smallest \( R/r \) ratio of 3 would correspond to a very large aperture radius of 10 m and radius of curvature of 30 m. Evident is the fact that for thicker ULE or Beryllium mirrors, the plate bending terms will dominate in the large \( R/r \) regimes of current telescopes. However, as aperture size \( r \) increases more rapidly than radius of curvature \( R \) (moving from left to right along the x-axis), and thickness decreases, even the traditional ULE glass mirrors will exhibit significant influence of the membrane term in the natural frequency. Obviously when considering the ultra-thin Kapton membrane mirrors, the membrane term is always dominant.
Figure 3.2 – Plate and membrane terms vs. radius of curvature to aperture radius ratio for three materials, taken from Equation 3.2.

The tension- and membrane-dominated behavior of gossamer structures is therefore not unique, and may also be present in other lightweight space structures comprised of more traditional materials. These characteristics present many challenges that have led to the development of specialized sensing and modeling technologies to enable characterization of the behavior of these structures, one of which is presented in the next section.

It should be noted that the new stiff ultra-lightweight polyimide panels studied in this dissertation do not fall nicely into one of these categories. While they are comprised of the same Kapton membranes discussed here, they also possess significant bending stiffness, making them unique structures. The conflict between the material similarities to gossamer structures and behavioral similarities to more traditional space hardware re-enforces their distinction as a separate class of next generation gossamer space structures.

3.3 Photogrammetry

The extremely low mass, high flexibility, and large size of gossamer structures have led to the use of a full-field surface measurement technique called photogrammetry for the characterization of static behavior. Photogrammetry is a complex and mature metrology method
that will be described here in abridgement. Section 3.3 is taken almost verbatim from the author’s Master’s thesis, which focused on developing photogrammetry measurement methods specifically for solar sails and other gossamer structures.

Photogrammetry is defined as the science of making three-dimensional measurements from photographs. The majority of these measurements are generated from aerial photographs and used to create topographic surface maps of large areas or land features. Close-range photogrammetry, the technique detailed here, measures objects several orders of magnitude closer to the camera in much greater detail than aerial topography.

The basic photogrammetric process can be broken up into four steps as follows: camera calibration, high contrast imaging, target marking and matching, and bundle adjustment. Each of these steps builds upon the previous to generate high-quality surface measurements.

The first step in the photogrammetric process is the calibration of the cameras. This procedure, described in detail in References 69-72, calculates the focal length (zoom), location of the principal point, radial lens distortion, and decentering lens distortion of each camera. While some of these parameters, such as the focal length of the lens and the principal point location, can be estimated, the calibration process measures them to micrometer precisions. For example, the principal point can be estimated to be at the center of the imager; however imperfections in the lens and internal camera components, slight inaccuracies in the manufacturing and assembly processes, etc. cause this assumption to be inaccurate. Knowledge of the lens distortion and the location of the principal point enables the computer to compensate for any deviation of the recorded image from what would have been recorded by an ideal pin-hole camera.

Using the focal length, principal point location, radial lens distortion, and decentering lens distortion values calculated in the camera calibration process, the photogrammetry software automatically removes any distortions of the images due to those parameters, enabling accurate (up to the object size divided by 100,000) measurements.69

Figure 3.3 and Figure 3.4 show the second step in the photogrammetric process, the taking of high-contrast images. Here, “high contrast sufficient for measurement purposes” is obtained using attached retro-reflective targets, shown in Figure 3.3, and underexposed images, shown in Figure 3.4. The camera flash illuminates the targets, which reflect light back to the camera hundreds of times brighter than a diffuse white surface. The underexposure darkens the rest of the image to the point where only the bright targets are clearly visible, creating a binary
white-on-black image as shown in Figure 3.4. The binary nature of the images permits automatic and accurate detection of the target locations. Note that this condition could also be artificially created with various image processing techniques. Any alteration to the images, however, involves approximations that inherently decrease the accuracy of the measurement.

(a) With camera flash off  
(b) With camera flash on  

Figure 3.3 – Retro-reflective targets

In the third step of the photogrammetry process, multiple binary (high-contrast) images are loaded into the photogrammetry software and associated with the appropriate lens and camera calibration parameters. The targets are marked to sub-pixel accuracy using a centroiding process based on a least squares matching (LSM) algorithm with an elliptical template to account for off-normal viewing angles (see References 71 and 73), and the resulting points corresponding to the exact centers of the targets are matched across the photographs, as shown in Figure 3.5. An algorithm called a “bundle adjustment” is then run (step four) which simultaneously iterates
on the camera locations and orientations from which the photographs were taken – a process called resection – and also calculates the 3D point locations and corresponding precision values – a process called intersection. To obtain these point locations in three-dimensional space, a line is projected from each camera to the point, also shown in Figure 3.5. Projected light rays are infinitesimally wide, so in general the rays from multiple cameras never intersect. However, they do establish the bounds of an intersection region. The intersection region in space is assumed to contain the true point location. This method of calculating point locations requires each target to appear and be marked in at least two images. Note that using more photographs in the photogrammetry process increases the redundancy and hence the accuracy, and the closer the camera locations are to right angles with each other the more accurate the out-of-plane measurement will be.

Figure 3.5 – Matching and triangulation of points\textsuperscript{42}
The final result of the photogrammetric process is a set of 3D points called a point cloud that, with an axis and scale defined, can be exported and measured. Figure 3.6 shows a simple example in which photogrammetry was used to measure the straightness of the seven meter inflatable boom previously shown in Figure 3.4. It shows the curve of the measured locations of the targets on the boom compared with a best-fit straight line. Note that the graph shown in Figure 3.6 is intended only to demonstrate one of many types of possible measurements from photogrammetric data and is not meant to provide any specific results.

![Max Deviation = 0.01m](image)

Figure 3.6 – Exported point cloud and best-fit straight line

3.4 Static Testing and Modeling of Gossamer Structures

3.4.1 Introduction

Development of current gossamer structures relies on detailed characterization of their performance using unique testing and modeling techniques that were either adapted from techniques used to measure similar structures or specifically developed for the particular class of
gossamer structure being analyzed. This section presents an overview of static testing and modeling of past and present gossamer structures.

3.4.2 Interferometry

Membrane mirrors offer the greatest potential of all the gossamer structures to provide dramatically increased resolution over the current, rigid mirror technology. To be useful in optical applications, however, the global figure of a membrane mirror has to be precise to hundreds of nanometers (generally $\lambda/20$). Therefore the critical issue precluding the use of optical quality membranes in apertures is shape control. While many methods of control are possible, the shape of the membrane must be known before it can be controlled. The challenge of accurately measuring the shape of a membrane mirror potentially tens of meters in diameter is daunting, but some progress has been made and is discussed here.

The most standard method of measuring the shape of rigid glass mirrors is by interferometry, and this technique has been applied to optical membranes. Traditional interferometry works by splitting a single beam of light, reflecting one off the surface to be measured, and recombining the beams at an imager. The light waves will constructively or destructively interfere, and produce the familiar fringe pattern which can be used to very accurately measure the shape of the surface off of which the light reflected. This basic premise has been adapted to measure the surfaces of gossamer membrane mirrors.

References 74-77 describe the development and implementation of laser-based interferometry techniques called Multibeam Optical Stress Sensing (MOSS) and Multi-Wavelength Heterodyne Interferometry. Both produce three dimensional shape measurements and were being researched for use in real-time shape control of flexible membrane mirrors. Other techniques rely on white light interferometry and the calculated optical path difference, and have been used successfully to produce three-dimensional shape measurements of membrane apertures.\textsuperscript{78,79,80}

While interferometry has been used quite successfully for the measurement of optical membranes, it is not well-suited for measurement of other much lower precision membrane-based gossamer structures like solar sails, because the response amplitude of interferometry is limited to approximately 300 times wavelengths of light used.\textsuperscript{80} Instead, photogrammetry has
been used almost exclusively to measure the static shape of recent gossamer structures other than optical membranes.

### 3.4.3 Capacitive and Laser Sensing

The most challenging characteristic to analyze that regularly occurs in gossamer structures is wrinkling. Here the term “wrinkling” is used to describe reversible, elastic, out-of-plane deformation of a membrane caused when the compressive stress acting perpendicular to the wrinkles is negative and equal to or greater than the stress required to buckle the membrane.\(^{81}\) Wrinkles are most likely to occur when membranes are tensioned or pre-stressed, and wrinkling almost always causes a performance degradation, even in low-precision membrane structures.\(^{81-83}\) This geometrically nonlinear condition is therefore important to characterize and model if these membranes are going to be used in applications requiring precise shapes such as apertures.

In Jenkins et al. (1998)\(^{82}\) and Dharamsi et al. (2002)\(^{83}\) the surface profiles of horizontally tensioned membranes were measured by capacitance displacement sensors that characterized the entire surfaces of the membranes. Capacitance sensors consist of a shaped core several millimeters in diameter that acts as one half of a capacitor while the conductive surface of the membrane acts as the other. A voltage differential is applied between the sensor and the membrane, and the capacitance of the circuit, which depends on the distance between the membrane and the sensor, is measured. The sensor is very accurate, with a repeatability of 0.01%, has a maximum range of approximately 3 mm, and is totally non-contact. The sensor can be moved over the entire surface to measure the out-of-plane \(z\) displacement over a grid of \(xy\) points, yielding a three dimensional surface profile of the membrane. Wong et al. (2003)\(^{84}\) describes an alternate method in which slices of the out-of-plane displacement of the tensioned membrane were measured using a CCD laser displacement sensor. Also a non-contact method, laser displacement sensors can achieve resolutions on the order of 10 \(\mu m\), and are discussed in more detail in Chapter 4.

Both capacitance sensing and laser displacement sensor measurement techniques accurately characterized wrinkle patterns of membranes which were then useful in evaluating analytical models. However, despite this success, neither technique is an optimal metrology
method for measuring membrane shape. Both techniques are non-contact, with no requirement of physical attachment of components to the structure, but neither is full-field and hence require the sensor to be scanned across the surface, thus allowing potential membrane movement while the sensor is repositioned. The amplitude range of the capacitance sensor is only 3 mm, which may not be sufficient for measuring deflections of large membranes. Finally, both techniques are impractical for use on structures over several meters in size. Therefore photogrammetry was adapted for use in measuring membrane deflection.

3.4.4 Photogrammetry

Photogrammetry has been used to calculate the three-dimensional shape of membrane mirrors as an alternative to interferometry, such as in Lindler and Flint (2004). In Flint et al. (2006), the accuracy of the method was investigated via comparison of two test articles measured both interferometrically and with dot projection photogrammetry. Between 16 and 28 images were used in the photogrammetry measurements. The results showed dot projection photogrammetry can be accurate to 3 µm rms when compared with the “true” shape of the mirror measured with turnkey interferometry systems.

The low mass and high flexibility of membrane-based gossamer structures makes them ideal candidates for measurement with photogrammetry because it is non-contact and full-field, so there is no physical attachment to the structure which may alter the local mass and stiffness of the membrane and the entire structure can be measured simultaneously.

Pappa et al. (2001), Giersch (2001), and Pappa et al. (2003) describe the pathfinder use of photogrammetry in the measurement of a modern gossamer structure. The test article was the 5-meter diameter inflatable parabolic reflector attached to an inflatable Kapton torus, shown in Figure 3.7. The torus has an outer diameter of 6.5 m and a cross-sectional diameter of 0.6 m. The entire structure has a mass of 4 kg. Attached to the rear convex surface of the reflector are approximately 500 retro-reflective targets, and the antenna is supported by three struts on the reverse side.
Multiple images of the reflector, including the one shown in Figure 3.7, were processed in a photogrammetry software package following the procedure described previously. In fact much of the methodology detailed in Section 3.3 was developed in conjunction with this pathfinder measurement. The resulting set of three-dimensional points is also shown in Figure 3.7. The precision of the measurement in the out-of-plane direction was found to be 1:5000, or 1 mm for the 5 m diameter reflector, and the surface was found to have a deviation from a perfect parabola of 1.5 mm rms, comparable to the reflectors discussed in Chapter 2.\textsuperscript{70,71,86}

The measurement shown in Figure 3.7 was full-field, unlike capacitance sensing or laser displacement sensing, and with multiple cameras placed around the structure, virtually instantaneous, allowing no possibility for movement of the structure over the duration of the measurement. However, while the attached retro-reflective targets did not measurably affect the shape of the inflated reflector, they would affect the static sag and wrinkling behavior of solar sail quality membranes less than 10 \( \mu \text{m} \) thick. Therefore, to make the physical attachment of measurement targets to the membrane surface unnecessary, a technique was developed whereby a grid of targets was projected onto the surface. This technique, called dot-projection photogrammetry, was applied to solar sail quality membranes in Pappa et al. (2003),\textsuperscript{68} Black (2003),\textsuperscript{42} and Black and Pappa (2004)\textsuperscript{87} to produce detailed, full-field, totally non-contact
measurements of wrinkle patterns that could then be correlated to very finely-meshed finite element models of the sails.

An example of a dot-projection photogrammetry measurement is shown in Figure 3.8. The top image in Figure 3.8 shows one of the actual images used in the photogrammetry processing in which a grid of approximately 5000 circular dots was projected onto a 0.9 x 0.5 m area of a specular aluminized Kapton solar sail quality membrane. A surface was fit to the resulting set of three-dimensional points (Figure 3.8), showing the visible wrinkle patterns and seam in the membrane. The measurement was calculated to be precise to 0.062 mm, or 1:32,000, and the smallest measurable wrinkle amplitude was approximately 0.25 mm.

![Figure 3.8 – Example of dot projection photogrammetry measurement of solar sail test article](image)

The accuracy of dot-projection photogrammetry measurements was investigated in Flint et al. (2006) and Pappa et al. (2002) in which the technique was compared to interferometric-optical and capacitance-sensing measurements, respectively. In both cases, the method was
found to be capable of measurement precisions comparable to capacitance sensing. Attached-target and dot-projection photogrammetry have since been the primary tools for researchers seeking to validate analytical membrane models and for other purely experimental characterizations of the static behavior of gossamer structures. Pappa et al. (2002) and Pappa et al. (2003) overview the recent progression of photogrammetry methods in measuring gossamer structures, and Meyer et al. (2004) details the use of photogrammetry to examine the effect of reduced gravity on membrane wrinkling.

3.4.5 Static Modeling of Gossamer Structures

3.4.5.1 Sag

A goal of the In-Space Propulsion program development of solar sails, in addition to the testing of 10- and 20-meter test articles described in Section 2.2, was the development of accurate finite element models. To be considered valid, these models were required to reliably predict the wrinkle patterns and induced gravity deflection or sag in solar sail quality membranes. Some recent efforts to model wrinkled and/or sagging membranes, correlated to photogrammetry data, are described here.

Sleight et al. (2005), Taleghani et al. (2005), and Sleight et al. (2006) describe the modeling efforts to reproduce the experimental data gathered in the 10- and 20-meter solar sail deployment tests described in References 53-59. Two different 10 meter sub-scale model solar sails were tested, and therefore two different sets of models were created. One, detailed in Taleghani et al. (2005), modeled the single quadrant triangular membrane and two booms in MSC/NASTRAN with 5435 structural elements and 2856 nodes. The sail was tested at different angles to vertical, and the deflection of the sail surface was measured with a Leica Laser Radar, which scanned the entire surface at a displacement accuracy of 90 µm. Three different angles to vertical were tested and modeled, and the results (experimental vs. predicted) were as follows: 13.18 vs. 13.23 cm; 25.3 vs. 23.98 cm; and 16.91 vs. 19.15 cm. The third set shows the greatest error of 13.25%, and the numerical model alternates between under- and over-predicting the experimental data. A study was then undertaken to determine the effects of altering the elastic modulus and bending stiffness of the membrane. It was found that using 50% of the actual elastic modulus only increased predicted deflection by 0.127 cm.
The test/analysis correlation of the other 10-meter sub-scale model solar sail is presented in Sleight et al. (2005). In this test a full four-quadrant sub-scale model was tested using proximity sensors to measure boom deflection, and photogrammetry of targets attached to the sail to measure gravity-induced membrane deflection (sag). Two different numerical models were constructed, one in NEiNastran and one in ABAQUS. Both models successfully predicted the experimentally-measured shape of the sail under gravity load, but both under-predicted the experimentally-measured deflection amplitude.

Based on the ABAQUS model presented in Sleight et al. (2005), Sleight et al. (2006) describes the test/analysis correlation of the four quadrant 20-meter sub-scale model solar sail test (Figure 2.18). Here the deflection under gravity of the four membranes was measured by photogrammetry using the 768 attached retro-reflective targets. The ABAQUS model of the sail membranes consisted of 45,321 elements and 25,172 nodes. Additionally, the cables used for gravity off-loading of the booms, other boundary conditions, and concentrated masses to account for the attached retro-reflective targets, accelerometers, wires, etc., were also part of the model, as shown in the top image of Figure 3.9. The bottom image of Figure 3.9 shows the side by side comparison of the finite element and experimental results. As seen previously, the model accurately predicts the deformed shape of the membrane, but under-predicts the amplitude by approximately 25%. The authors conclude that: “… the difference in the out-of-plane shape comparison may be attributed to wrinkling of the sail membrane, which reduces the effective modulus of the Mylar sail material and was not accounted for in the analyses.” These results are an excellent example of the challenges presented by the tension- and membrane-dominated responses of gossamer structures discussed in Section 3.2.
Two other recent efforts to predict the gravity-induced sag of solar sail and sunshield membranes are listed in References 24, 25, 90, and 91. Black et al. (2004, 2006)$^{90,91}$ describes two 25.4 µm thick aluminized Kapton membranes, one square and one right triangular, both 1 m on a side, suspended horizontally by attaching the corners of the membranes to an aluminum frame. Dot-projection photogrammetry was used to measure the gravity sag of the membranes,
and these data were compared to simple ANSYS models that excluded the support structure by using fixed boundary conditions at the membrane corners. The models utilized symmetry, and were comprised of 2561 shell elements and 2661 nodes in the square membrane case and 1135 shell elements and 1215 nodes in the triangular membrane case. A -33% maximum discrepancy and a -9% maximum discrepancy between the measured and modeled square and triangular membranes, respectively, were seen. These errors compare favorably to the errors seen in the models of the large solar sail test articles above, indicating that simple finite element models can produce comparable results to the complex models in Sleight et al. (2005), Taleghani et al. (2005), and Sleight et al. (2006). The results also indicate that the most likely cause of the discrepancy between measured and modeled sag was initial membrane slack unaccounted for in the numerical models.

Johnston et al. (2004, 2006) describe efforts to model the gravity sag of a small sun shield test article and a single triangular quadrant from a 2 m, 4-panel sail comprised of 25-µm thick Kapton. Photogrammetry of 130 targets attached to the surface of the triangular membrane was used to measure the deflected shape at several angles to vertical, and these measurements were compared to ABAQUS finite element models consisting of 11,645 shell elements and 11,895 nodes. The Kevlar lines used to attach the membrane to the frame were also modeled so different pre-stress conditions could be analyzed. An average -24% discrepancy was observed for the horizontal case over all pre-loads. These errors again compare favorably to those seen in Sleight et al. (2005), Taleghani et al. (2005), and Sleight et al. (2006), and those of the much simpler models in Black et al. (2004, 2006).

These efforts are presented in tabular form in Table 3.1 to enable direct comparison.
### Table 3.1 – Summary of efforts to predict gravity-induced sag in solar sail quality membranes

<table>
<thead>
<tr>
<th>Name</th>
<th>Test Article</th>
<th>Material</th>
<th>Loading</th>
<th>Finite Element Model</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taleghani et al. (2005)</td>
<td>10-m four-quadrant solar sail system with support members</td>
<td>Aluminized CP-1 (no thickness data provided)</td>
<td>Gravity</td>
<td>MSC-NASTRAN; 5435 structural elements; 2856 nodes</td>
<td>Discrepancy between experimental and predicted values ranged from 0.4% to 13.4%</td>
</tr>
<tr>
<td>Sleight et al. (2006)</td>
<td>20-m four-quadrant solar sail system with support members</td>
<td>Mylar (no thickness data provided)</td>
<td>Gravity</td>
<td>ABAQUS; 45321 membrane elements; 25172 nodes</td>
<td>“The ABAQUS analysis… underpredicted [sic] the sail billow by about 25% compared to the photogrammetry test data.”</td>
</tr>
<tr>
<td>Black et al. (2006)</td>
<td>1-m square membrane; 1-m triangular membrane quadrant</td>
<td>25-µm Kapton</td>
<td>Gravity</td>
<td>ANSYS – 2561 elements and 2661 nodes (square); 1135 elements and 1215 nodes (triangular)</td>
<td>Underpredicts square panel by 33% max and triangular panel by 9% max</td>
</tr>
<tr>
<td>Johnston et al. (2006)</td>
<td>Single triangular quadrant from a 2-m, 4-panel sail; corner attached</td>
<td>25.4 µm Kapton</td>
<td>Gravity</td>
<td>ABAQUS, 11645 shell elements; 11895 nodes</td>
<td>For all preloads, horizontally-oriented case, ABAQUS underpredicted the experimental measurements by average of 24%</td>
</tr>
</tbody>
</table>

#### 3.4.5.2 Wrinkling

In addition to accurately modeling the gravity sag in ultra-thin gossamer membranes, it was also important to characterize their wrinkle patterns. These patterns were measured effectively using dot-projection photogrammetry, laser-displacement sensing, and capacitance...
sensing techniques. The finite element models described in the following paragraphs were intended to be validated using those types of measurements.

Unfortunately the membrane and shell elements used to model gossamer membranes in the common finite element codes do not support either compressive or bending loads, and therefore will not produce out-of-plane wrinkles under only in-plane loading (tension or pre-stress) conditions. Three studies of recent note have attempted to overcome this shortcoming.

Wong and Pellegrino (2002), Wong et al. (2003), Tessler et al. (2003), and Tessler and Sleight (2004) describe efforts to induce out-of-plane wrinkle formation by randomly introducing “out-of-plane geometric imperfections” several orders of magnitude smaller than the expected deflections in the mesh of the membrane. These imperfections initiate membrane buckling under in-plane loading leading to wrinkle formation. While this method was successfully correlated to experimental data, very fine meshes were required in regions where wrinkling occurred, leading to very large model sizes and computation times. This procedure is particularly inefficient in tensioned membranes in which wrinkling only occurs toward the corners of the membrane. Tessler and Sleight (2004) addressed this inefficiency by utilizing a nonuniform mesh in which the center of the tensioned membranes had a much coarser mesh than the corners. In this manner the number of elements used in the model was reduced by half.

Blandino et al. (2002) and Su et al. (2003) describe finite element wrinkles produced by analyzing nodal stresses at each load step. At nodes at which the major stress \( \sigma_{11} \) is positive and the minor stress \( \sigma_{22} \) is negative, out-of-plane displacement was applied to initiate or perpetuate wrinkling. Again a fine mesh was used to accurately model the wrinkles, resulting in a large model of 4,900 elements and 5,041 nodes. The produced wrinkle patterns at the corners of the tensioned membrane compared very well to the experimentally measured pattern, though not necessarily the experimentally measured amplitude.

Leifer and Belvin (2003), Leifer et al. (2003), and Leifer (2005) describe an approach to generating wrinkles that does not require altering the finite element mesh or solution steps. Instead, approximately 30 very small (0.001 N) out-of-plane “inducement” forces were applied randomly to nodes throughout the mesh, and the model was solved. In-plane tension forces were then applied and the model was solved again, producing the desired wrinkles. In Leifer (2005) the results are compared to dot-projection photogrammetry results of a membrane
in shear instead of in tension. The finite element results show identical wrinkle patterns to the experimental data. Once again, however, the model size was large at over 6000 elements.

Black et al. (2004)\textsuperscript{90} describes a method for wrinkle production in which no artificial imperfections, forces, or alterations to the solution steps were required. Gravity was initially applied to simple shell element models of horizontally-suspended 25-\(\mu\)m thick Kapton membranes attached on all sides to a metal frame. The out-of-plane membrane deflection that resulted from the gravity load allowed for the natural formation of wrinkles under simple shear loading. Again the models were on the order of several thousand elements requiring large computation times.

Membrane-based gossamer structures present many challenges to characterization. Measurement techniques must be non-contact to avoid altering the mass and stiffness properties of the membranes and full-field to avoid movement of the membrane over the duration of the measurement. Numerical modeling of static membrane sag and wrinkling require large, finely meshed models that have achieved only limited success. These challenges are attributable to the membrane-dominated behavior of gossamer structures, and are potentially mitigated by the next-generation of stiff gossamer structures.

3.5 Plate and Shell Equations

Stiff, ultra-lightweight polyimide panels tested and modeled here are intended to behave in a more linear plate-bending fashion than other membrane-dominated gossamer structures. Classical plate and Reissner-Mindlin elastic plate equations are detailed in this section upon which the analysis of the panels is based.

The currently-accepted governing equations of lightweight honeycomb core panels treat the panels, in a global sense, as Reissner-Mindlin elastic plates. Developed in the late 1940s and early 1950s, this formulation is based on classical plate theory and is valid under the condition in which the thickness of the panel is at least an order of magnitude smaller than the length and width dimensions.\textsuperscript{97-99} The classical plate governing equations are solved in various boundary and loading conditions in References 100-103, some of which are presented below.

Classical plate theory assumptions from the Kirchoff hypothesis are as follows:
1) Straight lines perpendicular to the mid-surface (i.e., transverse normals) before deformation, remain straight after deformation; 2) The transverse normals do not experience elongation (i.e., they are in-extensible); 3) The transverse normals rotate such that they remain perpendicular to the mid-surface after deformation.\(^{103}\)

For isotropic plates, the differential equation describing the out-of-plane deflection \( w_0 \) as a function of \( xy \) location is:\(^{100-103}\)

\[
D_{11} \left( \frac{\partial^4 w_0}{\partial x^4} + 2 \frac{\partial^4 w_0}{\partial x^2 \partial y^2} + \frac{\partial^4 w_0}{\partial y^4} \right) + k w_0 = q
\]  

(3.3)

where \( k \) is the elastic foundation modulus and all other quantities are defined in Nomenclature or below.

The fourth order differential equation (3.3) for a rectangular plate of dimensions \( a \times b \) can be solved for all boundary conditions according to the Rayleigh-Ritz formulation as follows:

\[
w_0(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{N} C_{m}^{(n)} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}
\]  

(3.4)

where \( N \) is the number of algebraic equations among the \( C_{i}^{(n)} \) for each value of \( n \), and:

\[
C_{m}^{(n)} = \frac{4q_{i}^{(n)}}{m \pi d_{mn}}
\]  

(3.5)

\[
d_{mn} = D_{11} \left( \frac{m \pi}{a} \right)^4 + 2 \hat{D}_{12} \left( \frac{m \pi}{a} \right)^2 \left( \frac{n \pi}{b} \right)^2 + D_{22} \left( \frac{n \pi}{b} \right)^4 + k
\]  

\[
\hat{D}_{12} = (D_{12} + 2D_{66})
\]  

(3.6)

\[
D_{11} = E_1 H^3 / 12(1 - \nu_{12} \nu_{21})
\]  

(3.7)
\[ D_{22} = E_2 H^3 / 12 (1 - \nu_{12} \nu_{21}) \] (3.9)

\[ D_{12} = \nu_{12} E_2 H^3 / 12 (1 - \nu_{12} \nu_{21}) \] (3.10)

\[ D_{66} = G_{12} H^3 / 12 \] (3.11)

\[ \bar{q}_n = q_n - \frac{d^2 M_n^1}{dx^2} + \left( \frac{n \pi}{b} \right)^2 M_n^2 \] (3.12)

where \( M_n^1 \) and \( M_n^2 \) are thermal loads, \( q_n \) is the applied force function, and other all quantities are defined below or in Nomenclature.

For a simple case in which all the edges are simply supported, meaning \( w_0 \) is zero at \( x = 0 \) and \( a \) and \( y = 0 \) and \( b \), thermal loads are zero, and point loading of magnitude \( Q_0 \) is applied at location \( x_0, y_0 \):

\[ q(x, y) = Q_0 \delta(x - x_0) \delta(y - y_0) \] (3.13)

Inserting 3.13 into 3.5 for \( \bar{q}_n \), and inserting 3.5 into 3.4 yields the solution:

\[ w_0(x, y) = \frac{4Q_0}{d_{mn} \pi^4 ab} \sum_m \sum_n \frac{\sin(n \pi x_0 / a) \sin(m \pi y_0 / b) \sin(n \pi x / a) \sin(m \pi y / b)}{(n^2 / a^2 + m^2 / b^2)^2} \] (3.14)

for \( m = 1, 2, 3, \ldots \) and \( n = 1, 2, 3, \ldots \). This solution produces curvature in both \( x \) and \( y \) directions and can be easily solved for any number of \( n \) and \( m \) until converged.

In the tests detailed in Chapter 4 in which the panels are simply supported only on two adjacent sides, such as at \( x = 0 \) and \( a \), and the same point load as above is applied. Applying the previously published procedure described above, the derived load function takes the follow form:

\[ q_m = \frac{2}{a} \int_0^a q(x, y) \sin \frac{m \pi x}{a} dx \] (3.15)
\[ \bar{q}_n = q_m = \frac{2}{a} \int_0^a Q_0 \delta(x - x_0) \delta(y - y_0) \sin \frac{m \pi x}{a} \, dx \]  

(3.16)

where the integral sums along the \( x \)-direction through the variable \( m \), yielding:

\[ q_m = \frac{2Q_0}{a} \delta(y - y_0) \sin \frac{m \pi x_0}{a} \]  

(3.17)

\[ C_m^{(n)} = \left( \frac{4}{m \pi d_{mn}} \right) \frac{2Q_0}{a} \delta(y - y_0) \sin \frac{m \pi x_0}{a} \]  

(3.18)

\[ w_0(x, y) = \sum_{n=1}^\infty \sum_{m=1}^N \left( \frac{4}{m \pi d_{mn}} \right) \frac{2Q_0}{a} \delta(y - y_0) \sin \frac{m \pi x_0}{a} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \]  

(3.19)

The summation from \( n = 1 \) to \( \infty \) has the same effect as integrating from 0 to \( b \) along \( dy \), therefore:

\[ w_0(x, y) = \sum_{n=1}^\infty \sum_{m=1}^N \left( \frac{4}{m \pi d_{mn}} \right) \frac{2Q_0}{a} \sin \frac{m \pi x_0}{a} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \]  

(3.20)

Equation 3.20 is then the derived deflection of a plate simply supported at two adjacent edges under point loading based on the published solutions of the classical plate equations, such as Equation 3.14. Note that there will only be curvature in one direction if Equation 3.20 is solved for all \( x, y \).

Equations 3.14 and 3.20 describe the out-of-plane deflection of classical plates under point loading. However, Reissner-Mindlin elastic plate equations are used as the basis for describing the behavior of honeycomb-core panels because they incorporate shear effects and were therefore found to more accurately predict panel behavior. Solutions to these equations are not presented here, but are based on those discussed above. To modify these equations to account for the honeycomb core, various material properties, such as modulus and Poisson’s ratio, are altered. Based on this principle of generating effective material properties in the
governing equations, finite element models of stiff, ultra-lightweight polyimide panels will use basic shell elements with effective material properties to represent the structural details.

### 3.6 Dynamic Testing and Modeling of Gossamer Structures

#### 3.6.1 Fundamentals of Modal Testing

Modal analysis is defined as the study of the dynamic characteristics of a system or structure. These characteristics are studied through modal testing, which involves the collection and analysis of dynamic data. To understand how and why the process used in Chapter 5 is successful, a brief summary of the theory is presented here. This summary is based on References 104-115.

The dynamic behavior of any structure is described by its equation of motion that takes the form of Equation 3.21 for multi degree of freedom systems.

\[
[m][\ddot{x}] + [c][\dot{x}] + [k][x] = \{f(t)\}
\]  

where \([m], [c], \text{ and } [k]\) are the system mass, damping, and stiffness matrices, \(\{\ddot{x}\}, \{\dot{x}\}, \text{ and } \{x\}\) are the system acceleration, velocity, and position vectors, and \(\{f(t)\}\) is the external force supplied to the system. Equation 3.21 can be solved by the eigensolution of the following reorganized equation assuming negligible damping and harmonic motion of the form \(x_1 = A_1 \sin(\omega t) = A_1 e^{i\omega t} \cdot \)

\[
[k - \omega^2 m][x] = \{f(t)\}
\]  

where the eigenvalues (roots) of \([k - \omega^2 m]\) are the undamped structural natural frequencies of the system and the eigenvectors are displacement vectors called mode shapes unique to each frequency. The structural natural frequencies and mode shapes describe the dynamic behavior of any system, and their extraction is the objective of the experimental modal analysis process.
Structural natural frequencies and mode shapes are experimentally identified through the calculation of frequency response functions. The frequency response function (FRF) $H(\omega)$ of the system can be calculated by relating the known excitation to the known response of the system. Peaks and corresponding phase shifts of the FRF indicate damped structural natural frequencies of the system. Essentially, the FRF is defined as the Fourier transform of the output divided by the Fourier transform of the input, or $H(\omega) = Y(\omega)/X(\omega)$, although it may be computed in several ways depending on the objective, such as to minimize noise, among others.

Once FRFs have been collected experimentally, they are loaded into a modal analysis software package that fits a polynomial to the data in a process called modal parameter estimation (MPE). Several iterative algorithms were developed in the late 1970s and early 1980s to perform MPE on large multi degree of freedom systems. The result of the curve fit is the transfer function of the system, the poles of which correspond to structural natural frequencies. The system response at each pole can then be animated, showing the mode shape that corresponds to the extracted frequency.

3.6.2 XModal

The modal analysis software XModal is exclusively used here to perform the modal parameter estimation. Other similar software is available, including MEScope. Initially a complete set of measured frequency response functions is loaded into the software and the coordinates of the excitation and response points are defined. The curve fitting algorithm is selected, and the data are sieved and filtered as shown in Figure 3.10. Because of the redundancy created by the overdetermination of the symmetric FRF matrix consisting of all measurement and response points, more data than is necessary to perform the curve fitting is loaded into the software. Sieving limits the spectral range of the curve fit, shown in the top graph of Figure 3.10, and filtering limits the temporal range of the curve fit, shown in the bottom graph of Figure 3.10. Filtering is normally necessary to eliminate the high-frequency noise at the end of the impulse response function that is an artifact of the inverse Fourier transform of the FRF. An equation condensation is then performed by limiting the order of the polynomial, and hence the number of structural natural frequencies, to be fit to the data using a parameter called NrVirtual. The selected modal parameter estimation algorithm is then run.
A consistency diagram, such as the one shown in Figure 3.11 is created by XModal using the specified modal parameter estimation algorithm and parameters discussed above. This diagram shows the progress of the curve fitting as a function of iteration, in which as the number of iterations increases along the positive y-axis, as the model converges. The symbols show the transfer function poles fit to the FRFs, with the diamond shape representing the best case in which a pole and vector were both located. Lesser cases are listed in the box to the left of the plot in descending order of desirability. Two FRFs are plotted here indicating that two response points were used in the data collection. They are, however, simply for reference purposes since the MPE algorithm curve-fits to all of the FRFs. From this diagram, the user selects the frequencies at which to animate mode shapes by selecting symbols on the graph, which become the denominator of the transfer function. The software then solves for the corresponding numerator of the transfer function, the residues. The deflection shapes at the selected frequencies can then be animated.
3.6.3 Previous Modal Testing of Gossamer Structures

The development of the Eigensystem Realization Algorithm (ERA) and other iterative time-domain modal parameter estimation techniques in the mid- to late-1970s is the foundation upon which modal testing of gossamer structures, other large space structures, automotive systems, etc. was built. These iterative techniques, and the advances in computer processor speeds that made them feasible, enabled accurate and rapid estimation of the structural natural frequencies, mode shapes, and damping of large, complex, nonlinear, multi degree of freedom (MDOF) structures. Previously developed methods such as substructure and scale model testing had only been marginally successful. Hedgepeth (1981) described critical requirements for making large flexible space structures feasible, including dynamic testing and data analysis methodologies. Juang and Pappa (1987) state that “Active control of flexible structures will require the combined efforts of researchers in both [modal testing and system identification for controls] disciplines,” and present an exhaustive survey of the history of modal testing from the 1940s through the late 1980s.

Large space structures (LSS) are of interest for the obvious reason: in general the larger the system the more powerful it is. This is specifically true for solar arrays and apertures. While more powerful is not always desirable, several projects were in the planning and design phases in the late 1970s and early 1980s for which it was. The maiden flight of the Space Shuttle Columbia in 1981 was supposed to be the dawn of the next great age of human space travel for
NASA. The Space Transportation System (STS) – the official program name – was to be an inexpensive, reusable, heavy lift, large volume, fast turnaround launch vehicle that would allow NASA to construct a space station for permanent human habitation of Low Earth Orbit (LEO). Whatever form the space station was to take, multiple STS flights were going to be required to construct the largest structure ever assembled in space.\textsuperscript{118}

The current International Space Station (ISS) contains the same fundamental components that would have comprised any of the potential space station designs of the 1980s. The ISS is basically several habitable modules attached to very large solar arrays by long truss segments. Most of the modules would have been carried to space fully assembled as they are today, but the trusses and specifically the solar arrays could be deployed or assembled on-orbit. At over 73 meters in length, the recently installed solar arrays on ISS are perfect examples of the large space structures analyzed using the new ERA and other time-domain modal parameter estimation algorithms in the mid-1980s.

The Solar Array Flight Experiment (SAFE) was launched aboard the Space Shuttle Discovery on 30 August 1984 as part of mission STS-41D. NASA mission archives state: “The 102-foot-tall, 13-foot-wide Office of Application and Space Technology (OAST-1) solar wing extended from the payload bay. The wing carried different types of solar cells and extended to its full height several times. It demonstrated large lightweight solar arrays for a future in building large facilities in space such as a space station.”\textsuperscript{119} The OAST-1 array was actually used for four separate experiments: SAFE, the Solar Array Flight Dynamics Experiment (SAFDE), a photogrammetric experiment, and a solar cell calibration experiment. SAFDE and the photogrammetric experiment both sought to characterize the dynamic behavior of the array when fully extended from the shuttle’s cargo bay, as shown in Figure 3.12.
Initially, OAST-1 was only going to be investigated statically. But dynamic tests were eventually added because, “In past vehicles and on-orbit structures, the structural natural frequencies were significantly higher than the control system natural frequencies. However, with large space structures, the structural natural frequencies are so low that the control natural frequency will either be very close to the first natural structural frequency or nested between a pair of the lower natural structural frequencies.”

Twenty-three retro-reflective targets similar to those discussed previously were attached to the array. The requirements for the dynamic metric system (aside from surviving the launch) were to simultaneously track all of the targets to 19 arc seconds accuracy at up to ±45 cm displacement. A laser diode illuminated the targets which were recorded by the camera, both located at the base of the array. The intensity of the diode and the size of the targets were non-uniform, with greater diode intensity and larger targets as distance from the array base increased. This approach yielded images of nearly uniform target size and intensity along the length of the array. The reaction control thrusters on the shuttle provided dynamic excitation, and a time-domain algorithm was used to curve-fit the tracked target data. Four modes were identified in this manner. The identified mode shapes very closely
matched the predicted shapes, but there was some discrepancy between the predicted and measured frequencies.\textsuperscript{117-122}

Other LSS dynamic testing in the 1980s involved ground-based apertures. These tests were initially feasibility tests, and later progressed to analyze the dynamic characteristics of the structures. One such test article was the 15 meter hoop-column array, which consisted of a mesh aperture that folded up like an umbrella for storage. Several tests were performed under vacuum and ambient conditions to validate early linear static and dynamic finite element models. It was found that effective modulus and shape information from the static tests could be used to update the dynamic models and achieve better correlation.\textsuperscript{117,123,124}

By late 1995 NASA had abandoned the American-only space station concepts promoted during the 1980s in favor of a globally cooperative International Space Station. To gain practical experience in the long duration missions and on-orbit docking and assembly that ISS would require, and to build international good-will, NASA began a series of missions in which the Space Shuttle rendezvoused with the Russian Federal Space Agency Mir Space Station. The STS-74 Atlantis mission launched 12 November 1995 was the second such mission, and NASA used the opportunity to again study the dynamic behavior of large, lightweight, flexible solar arrays. Instead of the OAST-1 array that was deployed directly from the shuttle cargo bay in the SAFE experiments a decade earlier, this mission would study previously-installed, functioning arrays on Mir.

The STS-74/MIR Photogrammetric Appendage Structural Dynamics Experiment was intended to demonstrate that, building on the success of SAFDE, photogrammetric measurement of articulating, nonoptically targeted, flexible solar arrays and appendages was a viable, low-cost measurement option for ISS. “The Photogrammetric Appendage Structural Dynamics Experiment (PASDE) was developed to mitigate the technical risk and cost associated with on-orbit measurement of solar array and other flexible appendage structural responses for the ISS program. The experiment objectives were to demonstrate photogrammetric structural response measurement methods for solar arrays using video cameras without the use of optical targets, to provide engineering data on solar array designs similar to those expected to be used on the ISS, and to verify that routine on-orbit spacecraft operational events provide sufficient excitation for structural response testing.”\textsuperscript{125}
PASDE was part of the planning for ISS and was designed to test the effectiveness of photogrammetry as a metric system to provide the needed on-orbit data after the arrays were installed to validate numerical models. In space, dynamic data was normally collected by strain gauges or accelerometers, signal conditioned, and transmitted to Earth for analysis using ERA or other modal parameter estimation techniques. While those same techniques were planned for use on truss elements, the expense and added mass of the wires, power, signal conditioning, electronics, etc. was considered to be too expensive and impractical on future ISS solar arrays 73 meters in length. The experimental equipment required for PASDE consisted of only three small Hitchhiker canisters installed in the shuttle cargo bay pointing up at the Kvant-II solar array on Mir, as shown in Figure 3.13. The canisters are denoted by red cylinders inside the shuttle bay. Each canister contained two video cameras, one focused at the root of the solar array and one focused at the tip, and each camera could record at 30 frames per second for 115 minutes.

![Figure 3.13 – STS-74 Shuttle/Mir configuration](image)

Preparation for PASDE involved assembling and testing the Hitchhiker canisters and calibrating the cameras on the ground prior to launch. During the mission, five maneuvers were performed by Mir and Atlantis to excite in- and out-of-plane motion in the solar array. Additionally, the cameras recorded the behavior of the array during other mission operations.
such as docking, day-to-night terminators, attitude maneuvers, etc. Following the return of the shuttle, the cameras were re-calibrated.

The Russian-built array was constructed without the retro-reflective targets used in optical tracking on OAST-1, so other methods of defining points were used. Line-tracking involved tracing straight edges in the images. The intersection of two of these lines was considered a target that could be tracked through the image sequences. A cross-correlation method was also used that tracked the movement of a target using multiple cameras to determine its location. The photogrammetry collinarity equations were used to triangulate the three-dimensional locations of the targets in space, and these locations were tracked through the image sequences to produce time histories of all the points.

An ERA analysis was then run on the gathered temporal data to extract structural natural frequencies, mode shapes, and damping of the array. Five modes were identified between 0.1 and 0.5 Hz: two global modes of the Mir/Shuttle system and three flexible body modes of the arrays including two out-of-plane modes and a single in-plane mode. These results illustrate the effectiveness of both photogrammetry and ERA in flight testing, as well as other time-domain techniques in the characterization of large, flexible structures. The measured array exhibited five very closely-spaced modes that time-domain curve fitting techniques are particularly good at distinguishing. The photogrammetry data generated without attached targets or measurement devices from three small self contained canisters was full-field, accurate, three dimensional, and sensitive, as most of the motion captured was less than 2.54 cm peak to peak.\textsuperscript{125,126}

Early testing of what would be considered modern gossamer structures began in the late 1990s. These structures were comprised of ultra-thin membranes and inflatable rigidizable composite booms and struts, and dynamic data were required to validate numerical models. “Due to the uniqueness of the structures being tested, experiments conducted to date generally have required as much effort in developing the test methods as in acquiring test results. Test procedures used for traditional aerospace structures are often not applicable.”\textsuperscript{86} Therefore new dynamic testing procedures were developed in some of the following pathfinder experiments.

The Jet Propulsion Laboratory (JPL) and NASA Langley Research Center conducted a series of tests from approximately 1999 to 2003 to develop a database of the structural characteristics of inflatable rigidizable tubes. Several dozen cylindrical hollow tubes were tested statically and dynamically, most with length over diameter ratios of 10, and several much higher.
Tube diameters were all between 10.16 and 15.24 cm (4 and 6 inches). Dynamic testing was conducted under fixed-free boundary conditions using impact hammers and miniature accelerometers to determine the effective elastic and shear moduli of the tubes. Like the stiff, ultra-lightweight panels tested below, the rigidized tubes were sufficiently stiff as to be amenable to this type of impact testing.\textsuperscript{86,127,128}

To demonstrate that excitation could be actively controlled using embedded piezoactuators, several Macro Fiber Composite (MFC) actuators were embedded in composite inflatable rigidizable tubes identical to those described above. A laser vibrometer was used to measure the free decay vibration of the tubes from impacts. It was found that active MFC actuators increase the damping of the first bending mode from 0.4\% to 8\%.\textsuperscript{86,129,130}

The five-meter diameter parabolic reflector in Figure 3.7 was tested statically using photogrammetry of attached retro-reflective targets to determine the accuracy of the reflector shape. Development was undertaken, subsequent to the successful static characterization of the reflector shape, to perform the photogrammetry triangulation over sequences of images, generating a set of time histories of three dimensional points. This process of applying photogrammetry to video sequences was eventually called videogrammetry.\textsuperscript{70,86} To baseline the videogrammetry measurements, dynamic data were collected at each of the retro-reflective targets using a laser vibrometer. The structure was excited by an electrodynamic shaker attached to the aluminum end caps of the composite support struts.\textsuperscript{131}

Several other membrane-based, torus-supported apertures in the form of solar thermal concentrators and antenna were dynamically tested at NASA Marshall Space Flight Center using laser vibrometry. Most of these were tested in fixed-free boundary conditions in which the metal end caps of the composite rigidizable support struts were mechanically secured to the ground. Shakers were attached to the metal end caps to provide excitation. A laser vibrometer was selected as the measurement device in these tests because of the non-contact nature of the measurements. The major drawback to the laser vibrometer is that it is single point and not full-field.\textsuperscript{5,86,132-135}

In addition to pressurized membrane aperture structures, simple, flat, tensioned membranes were also dynamically tested in the form of scale models of the Next Generation Space Telescope (renamed the James Webb Space Telescope) sunshield. A 1.5 x 1.3 m 1/10 scale model of the sunshield consisted of four layered membranes 13 microns (0.5 mil) thick.
The model was suspended from the top of a vacuum chamber by a long metal rod which was excited by a shaker both inside and outside the chamber depending on chamber size. The resulting motion of the membranes was recorded by a laser vibrometer outside of the chamber measuring through a glass porthole in the chamber wall. Deployment tests of scale models up to 1/2 the size of the actual structure were also conducted.  

3.6.3 Current Dynamic Testing of Gossamer Structures

Most of the recent modal tests of gossamer structures were completed using either videogrammetry or laser vibrometry. As stated above, videogrammetry is the process of applying photogrammetry to synchronized video sequences. The photogrammetry processing is applied at each video frame, generating time histories of a set of three-dimensional points. Videogrammetry is a non-contact, full-field measurement technique. Laser vibrometry is also inherently non-contact, but analyzes single points on the structure instead of the structure as a whole. The resolution of a videogrammetry measurement is identical to that of a photogrammetry measurement, but because digital video cameras are generally lower resolution than their static counterparts, the measurements also tend to have lower resolution. In general, most videogrammetry measurements will have a resolution of 1:1000, or 1/1000 the size of the object being imaged. The resolution of laser vibrometers is not dependant on the structure, but is a property of the laser. These systems generally have a resolution of 10 µm. Because the result of a videogrammetry measurement is a three-dimensional time history of a set of points, the technique is inherently three dimensional, meaning only one set of data is required to analyze in- and out-of-plane motion. Conversely a laser vibrometer can only measure one degree of freedom, along the laser beam.

When measuring the lightest and largest gossamer structures, solar sails and sunshields comprised of ultra-thin membranes potentially thousands of square meters in size, attaching objects directly to the membranes, including small retro-reflective targets, substantially alters the local mass and stiffness and invalidates the data. Therefore when using videogrammetry or laser vibrometry to characterize the dynamic behavior of such gossamer membrane structures, a technique of projecting targets onto the membranes was developed. Despite the mass savings, several factors complicate the use of dot-projection photogrammetry and
videogrammetry. The first involves the projection of a target field over the thousands of square meters of surface area of the membranes. Two notable solutions to this problem were developed. One involved doping the membrane material with a dye that fluoresces when exposed to a particular wavelength of incident laser light. A laser could create any type of grid on any part of the membrane, and change the grid or grids over the course of the mission. The other solution was to use the reflection of stars off the specular surface of the membrane as targets. The second complication with using dot-projection videogrammetry is that the targets are not attached to the surface, meaning the measurement technique is no longer three dimensional. Only motion along the ray of light can be measured, identical to a laser vibrometry measurement.

A third complication related to the use of videogrammetry in general, not only dot-projection videogrammetry, is that only the response of the structure is measured. To create frequency response functions that allow complete characterization of the structural natural frequencies, mode shapes, and damping of the test article, simultaneous measurement of the excitation supplied to the system and the system response is necessary. While it is possible to independently measure the excitation and create separate FRFs, it is difficult to reliably synchronize two different metric systems. Conversely the single data acquisition system used in laser vibrometer, impact hammer, and other more standard dynamic testing automatically synchronizes the input and response measurements and creates FRFs in a single step. The temporal videogrammetry data can be filtered and animated to view the motion of the structure at particular frequencies, called Operating Deflection Shapes (ODS), but these cannot be considered fundamental dynamic properties of the system.

The next logical question to arise challenges the usefulness of operating deflection shapes and their corresponding frequencies. Blandino et al. (2003), Black (2003), and Black and Pappa (2004) directly compared structural natural frequencies and mode shapes extracted from FRFs measured with laser vibrometry to operating deflection shapes and the corresponding frequencies extracted from temporal filters and Fourier transforms of the time histories measured with dot-projection videogrammetry. The test article was one triangular quadrant of a small four square meter four-quadrant solar sail test article (same configuration as the solar sails in Figure 2.16 and Figure 2.18). To facilitate the dot-projection technique, diffuse white 0.1-mm thick drafting Vellum was used in place of actual gossamer aluminum-coated Kapton or Mylar.
membranes. The projected target field that was recorded by the video cameras was also used as the measurement grid of the scanning laser vibrometer. Excitation was applied by a shaker attached to the tip of one of the two booms from which the quadrant was suspended. The right-triangular Vellum quadrant was suspended at three attachment points at the corners of the triangle. The right-angle corner was attached to the sail hub, and the two other corners were attached to the tips of hollow steel rods (OD 0.47 cm, ID 0.30 cm) or booms extending from the hub.

The FRFs generated by the laser vibrometer showed five structural natural frequencies occurring between 1 and 5 Hz, as did the operating deflection dot-projection videogrammetry measurement. The discrepancy in the measurements, with the structural natural frequencies taken as actual, was calculated to be 5.14%, 1.15%, 3.89%, 1.36%, and 0.021%, respectively. This comparison indicates that although not measuring frequency response functions, the frequencies corresponding to the operating deflection shapes obtained using dot-projection videogrammetry measurements are extremely close to the actual structural natural frequencies of the structure. The comparison of the mode shapes to the operating deflection shapes showed them to be identical, as seen in fifth mode shapes shown in Figure 3.14. Therefore the authors concluded that despite the fact that no FRFs are calculated, dot-projection videogrammetry gives a very accurate picture of the dynamic behavior of membrane-based gossamer structures and the technique could be very useful in future gossamer structure investigations or missions.

(a) Laser vibrometry shape at 4.74 Hz          (b) Videogrammetry shape at 4.75 Hz

Figure 3.14 – Fifth mode of Vellum solar sail quadrant
Many other dynamic tests of membrane-based and inflatable rigidizable gossamer structures have occurred in recent years. Slade et al. (2002, 2003)\textsuperscript{140,141} compared modal testing of a single quadrant of a four-quadrant square solar sail (Figure 2.16 and Figure 2.18) 2 meters on a side in ambient and vacuum conditions. The quadrants were manufactured from 25.4-micron (1 mil) thick aluminized Kapton, and small retro-reflective targets were attached to their surfaces. In these experiments, non-contact measurement (laser vibrometer) and non-contact excitation were used. A shaker was the excitation source, but instead of being attached directly to a boom tip via a stinger, a small permanent magnet was attached to the end of the stinger, very close to, but not in contact with, the boom tip. In this manner the boom could be excited without the parasitic mass or stiffness of the stinger and shaker. Ambient results showed several structural modes of the booms, and only one membrane mode, whereas the vacuum results showed multiple, closely-spaced, low-frequency membrane modes. These low-frequency membrane modes were not seen under ambient conditions due to the apparent mass of the air surrounding the ultra-thin membranes, thus illustrating one of the key challenges of testing gossamer structures on Earth as opposed to in space. And these tests were on very small, four square meter test articles. The difficulty of apparent mass only magnifies as test articles are increased toward full-scale structures hundreds or thousands of square meters in size.

Adetona et al. (2003)\textsuperscript{142} describes the modal testing of a flat, specular gossamer membrane suspended in a hexapod comprised of rigidizable graphite-epoxy composite tubes. A hexapod is a circular torus (here 3.8 meters in diameter) from which the membrane is suspended, supported by six struts. The hexapod test article was suspended vertically, excited by a shaker attached to a hard plastic end cap of one of the tubes comprising the torus, and the structure and membrane were measured with a laser vibrometer. Several structural natural frequencies and mode shapes occurring between 1 and 300 Hz were identified using ERA and yielded a complete picture of the dynamic behavior of the entire structure, including membrane, torus, and struts.

Leifer et al. (2006)\textsuperscript{143} describes videogrammetry testing to measure the in-plane dynamic behavior of a small optical quality membrane. A 12.7-\(\mu\)m thick polyester film coated with 0.2 \(\mu\)m of aluminum was attached with silicone adhesive to a 2-mm thick copper ring with a 76.45 mm (3.01 in) inner diameter and a 117.7 mm (4.64 in) outer diameter. A Krypton Fluoride Excimer laser etched a series of 0.25-mm diameter circular dots on the surface of the membrane.
by removing 5% of the aluminum coating. Removal of this small amount of coating did not impact the dynamic response of the membrane but was visible to the naked eye, therefore attaching or projecting targets onto the membrane was not required. The copper ring to which the etched membrane was adhered was attached to a shaker that provided in-plane excitation. Videogrammetry was used to track the motion of the etched targets (0.25-mm diameter circular dots), and plotting the distance between two points on the membrane as a function of time revealed several in-plane structural natural frequencies of the membrane.

The tests described in References 144-151 used adaptations of previously discussed dynamic testing methods to characterize pressurized tori. In Griffith and Main (2000, 2002) a 1.8-meter diameter pressurized Kapton torus was tested at different internal pressures using an impact hammer modified to spread the applied impulse out over a wider area by attaching a small aluminum plate to the hammer head instead of a traditional tip, and a small accelerometer to measure its response. Excitation using a tip would have caused local buckling. The first three in- and out-of-plane mode shapes were observed at pressures above 0.8 psig, but despite the modifications to the hammer, it was still unable to globally distribute the excitation energy, leading to poor coherence.

A similar 1.8-meter diameter torus comprised of a 0.15-meter diameter Kapton tube pressurized to 0.5 psi was tested in Ruggiero (2002), Park et al. (2002), Ruggiero et al. (2003), Sodano et al. (2003), and Ruggiero et al. (2004). In these tests, the piezoactuator Macro Fiber Composite (MFC) patches discussed above were used as both the actuators and sensors. Multiple MFC patches were attached to the surface of the torus at various intervals from each other, and structural natural frequencies, out-of-plane mode shapes, and damping were obtained from the data. These tests were also performed with a membrane mirror attached to the torus to determine the boundary interactions of the two structures. Finally, in addition to excitation and measurement, the MFC actuators were used to control the structure, obtaining up to a 70% reduction in the amplitude of vibrations.

The work upon which the testing described in Chapter 5 is based is presented in Song et al. (2006). Similar to the other tori described above, the test article shown in Figure 3.15 has a 1.8 m ring diameter and a 0.20 m tube diameter. It is unique, however, in that it is self-supporting under ambient gravity loading, meaning that the structure is not pressurized. A regular pattern of convex hexagonal domes 8 mm side-to-side and 3.5 mm high is formed into
the Kapton membrane from which the torus is constructed. These domes give the torus its knurled appearance and provide the stiffness that enables the structure to be self-supporting.

Based on the success of the testing methods described in Griffith and Main (2000, 2002),\textsuperscript{144,145} the same modified impact hammer was initially investigated as the excitation method. However, “In this project the unpressurized torus structure is far too flexible to be excited with any solid contact excitation: it simply crumpled under the force.”\textsuperscript{151} Totally non-contact methods of excitation and measurement were therefore used in the form of acoustic excitation provided by the speaker and laser displacement sensor shown in Figure 3.15. In- and out-of-plane frequency response functions were measured at multiple points around the torus, and the data were loaded into the previously discussed XModal modal analysis software. Structural natural frequencies, mode shapes, and damping were successfully extracted from the data for the first three in- and out-of-plane modes, all occurring below 23 Hz.\textsuperscript{151} The similarity of this test article to the stiff, thermal-formed panels investigated here and the success of this acoustic testing method lead to its attempted implementation in the dynamic testing described in Chapter 5.
### 3.6.4 Current Dynamic Modeling of Gossamer Structures

The majority of dynamic data produced by finite element models are generated by performing a modal analysis of the model used to predict the static behavior of the gossamer structure. The dynamic response of several of the static models discussed in Section 3.3.3 is therefore detailed here along with three examples of models expressly created to examine the dynamic behavior of gossamer structures.

The models of the ten meter subscale solar sail deployment tests described in Sleight et al. (2005) and Taleghani et al. (2005), and the model of the twenty meter sub-scale solar sail deployment test described in Sleight et al. (2006) and shown in Figure 2.18 were all run in modal analyses in their respective finite element software packages and compared to the gathered modal test data. Modal analyses of the models of the single triangular quadrant of a four quadrant sail ten meters on a side and two support booms in Taleghani et al. (2005) were undertaken in both NASTRAN and ANSYS. The experimental vs. NASTRAN structural natural frequencies and error for the first five modes are as follows: 1.4 vs. 1.46 Hz, 4% error; 1.82 vs. 2.43 Hz, 34% error; 2.23 vs. 2.67 Hz, 20% error; and 2.82 vs. 3.46 Hz, 23% error. The structural natural frequencies solved for in ANSYS were slightly lower than those from NASTRAN, and all of the measured and predicted mode shapes were identical.

In Sleight et al. (2005), the dynamic behavior of the four booms of the ten-meter four-quadrant sub-scale solar sail was simulated without the attached sail membrane in both the NASTRAN and ABAQUS models. The first two structural natural frequencies were experimentally determined to occur at 0.6 and 0.8 Hz. The NASTRAN model predicted the first two structural natural frequencies to occur at 0.86 and 1.79 Hz, over-predicting the experimental frequencies by 43.3% and 123.8%, respectively, while the ABAQUS model predicted the first two structural natural frequencies to occur at 0.92 and 1.99 Hz, over-predicting the experimental frequencies by 53.3% and 146.8%, respectively. The experimentally-determined and predicted mode shapes were identical.

In Sleight et al. (2006) the dynamic behavior of the twenty-meter four-quadrant sub-scale solar sail Figure 2.18 was analyzed in ABAQUS from the static finite element model. The first three structural natural frequencies were experimentally determined using laser vibrometry of the attached retro-reflective targets on the booms and surface of the sail to occur at 0.829, 1.31, and 1.45 Hz. The finite element model predicted the first three natural frequencies to occur
at 0.83, 1.56, and 1.24 Hz. The predicted mode shapes were matched to the experimentally-measured shapes, leading to this unusual sequence in which the second predicted structural natural frequency occurs at a higher frequency than the third. The discrepancies between the experimental and predicted structural natural frequencies are -1.3%, +19.1%, and -13.9%, respectively. These three tests again demonstrate the difficulties in modeling gossamer structures. Even with a program such as ISP dedicated to advancing solar sails toward flight-ready status by creating extremely accurate models of the structures that could be scaled up to predict how the structures would behave in space, 20% errors are common. These modeling issues led the ISP metrology solar sail team to declare that finite element models accurate to within 10% of actual would be considered valid and ready for flight simulations and mission design.

Models that were created expressly to predict dynamic behavior are described in References 37, 58, 142, and 152-156. Adetona et al. (2003) briefly describes an I-DEAS finite element model of the 3.8 meter hexapod structure described in Section 3.3.6. This model consisted of approximately 1000 elements. Beam elements were used to model the inflatable rigidizable composite epoxy tubes of which the six struts and torus were comprised and the cables attaching the membrane to the torus, and plate elements were used to model the specular membrane. The I-DEAS model was run in NASTRAN using a Lanczos eigenvalue extraction method to calculate the structural natural frequencies and mode shapes of the hexapod. The model predicted 208 structural natural frequencies between 0 and 100 Hz, only two of whose corresponding mode shapes matched experimentally determined shapes. The second mode at 1.67 Hz was successfully correlated.

Wang and Johnson (2002) describes efforts to simulate the inflation deployment of coiled and Z-folded 152.4-µm thick polyethylene tubes with a diameter of 9.7 cm. The modeling approach was originally developed to simulate airbag inflation, and used LSDYNA to model the inflation gas. An example of the results of the inflating Z-folded tube is shown in Figure 3.16.
In Lore and Smith (2005)\textsuperscript{152} and Lore (2004),\textsuperscript{153} a technique used extensively in the automotive industry to extract dynamic information from models potentially millions of degrees of freedom in size was applied to models of thermal-formed gossamer structures such as the torus shown in Figure 3.15. This technique, called Automated Multi-Level Substructuring (AMLS)\textsuperscript{154} “uses many levels of substructuring, defined automatically by the numerical characteristics of the assembled matrices of the finite element model. These substructures typically do not correspond to physical substructures, but instead provide dynamic mathematical substructures that are assembled to define the global response solution subspace.”\textsuperscript{152} The nature of the method is amenable to parallel or distributed processing, and for large models, is much more computationally efficient than traditional Lanczos eigenvalue extraction. The studies found that AMLS produced considerable time savings over a Lanczos solver for a 500,000 degree of freedom model from which 275 modes were extracted.

Glaese et al.(2003)\textsuperscript{37} compared the modal analyses of three different finite element software packages of a 0.5 meter-diameter self-supporting 51-μm thick spherical polyimide membrane shell. The baseline model against which the others were compared was an MSC/NASTRAN model consisting of 8000 elements and 8001 nodes. This model was validated using the analytical formulation described in Section 3.1. The first NASTRAN structural natural
frequency occurring at 413.8 Hz, under-predicting the analytically determined value of 422 Hz by approximately 2%. Both methods showed a very high modal density around 430 Hz. The STAGS model was much larger than the NASTRAN model, consisting of 36,000 degrees of freedom, and showed several modes occurring between 200 and 400 Hz and many modes occurring around 430 Hz, as in the two previous methods. ANSYS modeled a slightly thicker shell and predicted the first structural natural frequency to occur at 346 Hz, a 13.6% discrepancy from the baseline NASTRAN model. The differences in the results were attributed to “subtle difference in model geometry or boundary conditions.”

Smith et al. (2005)\textsuperscript{155} describes the development of control simulations as part of the ISP solar sail program. Assuming that a torque could be applied to the sail hub to control the out-of-plane motion of the booms, a two-dimensional boom model was developed analytically and used to validate a comparable ANSYS finite element model comprised of beam and point-mass elements. A complete model of the sail was then created according to Sleight and Muheim (2004)\textsuperscript{156} consisting of four booms and four triangular membranes. Similar control simulations were also run with this model. The results show that the extreme flexibility of long, lightweight booms creates a situation in which any control torque applied to the sail hub produces an oscillation of the entire structure. That motion can be eliminated after a period of time, however, meaning control torque applied to the central hub of the sail is a viable method of maneuvering and controlling the dynamic response of solar sails.

3.7 Scaling

3.7.1 Introduction

To effectively design and test solar sails and other gossamer structures, ground experiments must be performed. Since it is impractical to test full-size sails on the order of 10,000 square meters in size in a laboratory, scale models must be used. If true similitude scaling were used, however, all of the length dimensions would be scaled equally, meaning the thickness of the membrane would have to be reduced by the same factor as the length and width. And since the manufacture of membranes significantly thinner (ten or twenty times, based on the required scale factor for the length and width dimensions) than 5.0 microns is currently
impossible, partial scaling must be used in which the thickness of the membrane is held constant. This method, referred to as constant-thickness scaling, was developed in Greschik et al. (1998)\textsuperscript{157} using an “engineering” approach is compared here to new scaling laws derived through the law method.\textsuperscript{158}

3.7.2 Scaling Laws Assumptions

The obvious first assumption of the constant-thickness scaling method is that the membrane thickness of the scale model is equal to that of the full-scale prototype. Therefore, where volume normally would have been thought of as a length ($\ell$) cubed in which each length dimension is multiplied by the same scaling factor $\lambda$, here it is assumed to be a length squared times a membrane thickness $t_m$, which is held constant, as follows:

$$
Volume = \ell^3 \rightarrow \ell^2 t_m \tag{3.25}
$$

Mass is then derived as follows:

$$
Mass = \rho \ell^3 \rightarrow \rho \ell^2 t_m \tag{3.26}
$$

where $\rho$ is the volumetric density of the membrane material.

The new scaling laws are derived by relating the forces governing the system. For a vibrating membrane system in ambient $1g$, the governing forces are assumed to be the gravitational force $F_g$, the damping force of air, the applied excitation force $F_a$, and the tensile force $F_t$. $F_a$ and $F_t$ are applied quantities and are therefore derived through the scaling laws. The damping force of air is comprised of viscous $F_v$ and inertial $F_i$ forces. Via dimensional analysis, the three forces used in the derivation of the scaling laws are as follows:\textsuperscript{159}

$$
F_g = mg = \rho_m \ell^2 t_m g \tag{3.27}
$$

$$
F_v = \mu \ell v \tag{3.28}
$$
\[ F_i = ma = \rho_m \ell^3 \frac{\ell}{t^2} = \rho_a \ell^2 v^2 \]  

(3.29)

where \( m \) is mass, \( g \) is the acceleration due to gravity, \( \rho_m \) is the volumetric density of the membrane, \( \mu \) is the viscosity of air, \( v \) is the velocity of the membrane, \( a \) is acceleration, and \( \rho_a \) is the volumetric density of air.

### 3.7.3 Scaling Laws Derivation

The three governing forces in Equations 3.27, 3.28, and 3.29 above are related to form two scaling laws (pi-numbers) as follows:

\[ \pi_1 = \frac{F_v}{F_g} = \frac{\mu v}{\rho_m \ell^2 t_m g} = \frac{\mu v}{\rho_m \ell^2 t_m g} \]  

(3.30)

\[ \pi_1 = \pi_1' \rightarrow \frac{\mu v}{\rho_m \ell^2 t_m g} \rightarrow \frac{\mu v'}{\rho_m' \ell^2 t_m' g'} \]  

(3.31)

\[ \pi_2 = \frac{F_i}{F_g} = \frac{\rho_a \ell^2 v^2}{\rho_m \ell^2 t_m g} = \frac{\rho_a v^2}{\rho_m \ell^2 t_m g} \]  

(3.32)

\[ \pi_2 = \pi_2' \rightarrow \frac{\rho_a v^2}{\rho_m \ell^2 t_m g} \rightarrow \frac{\rho_a' v'^2}{\rho_m' \ell^2 t_m' g'} \]  

(3.33)

where superscript prime indicates the scale model, and \( \pi_1 \) and \( \pi_2 \) are the scaling laws. According to the similitude condition of the law approach of scale modeling, the pi-number for the prototype and the pi-number for the scale model must be the same and are therefore set equal to each other in Equations 3.31 and 3.33. However, from prototype to model, the acceleration due to gravity, the viscosity of air, the density of air, the density of the membrane material, and the
material thickness all do not change. Therefore \( g = g', \mu = \mu', \rho_a = \rho_a', \rho_m = \rho_m', \) and \( t_m = t'_m. \) These quantities divide out, and Equations 3.31 and 3.33 then become:

\[
\pi_1 = \pi'_1 \rightarrow \frac{v}{\ell'} = \frac{v'}{v} = \frac{\ell'}{\ell} \tag{3.34}
\]

\[
\pi_2 = \pi'_2 \rightarrow v^2 = v'^2 \rightarrow \frac{v'}{v} = 1 \tag{3.35}
\]

The two derived laws above obviously conflict, which will be discussed later, but consider that conceptually, the larger the membrane the lower the frequencies at which the modes occur.\(^{87,157}\) Assuming, however, that the air around the membrane is in turbulent flow due to the large size of the membranes, the inertial force will dominate the damping and the viscous force can be neglected.\(^{159}\) While experimental validation is needed for this assumption, only \( \pi_2 \) will be used to derive the quantities discussed below.

### 3.7.4 Derived Quantities

All of the major dimensions, forces, and results must be scaled according to a general scale factor \( \lambda. \) Using \( \pi_2 \) and \( \lambda, \) the following quantities were defined:

\[
\frac{\ell}{\ell'} = \lambda \rightarrow \ell' = \lambda^{-1} \ell \tag{3.36}
\]

\[
m = \rho g \ell^2 t_m \rightarrow m' = \lambda^{-2} m \tag{3.37}
\]

\[
v' = v \tag{3.38}
\]

To scale applied forces and the measured dynamic responses, time is scaled using velocity as follows:
where $T$ is time. Inserting 3.39 into $\pi_2$ (3.35, 3.38):

$$\pi_2 = \frac{\ell'/T'}{\ell/T} = 1 \rightarrow T' = \lambda^{-1}T$$

(3.40)

The period $T_o$ and frequency $f$ of the dynamic responses and the applied forces are derived from time (3.40) as follows:

$$T_o' = \lambda^{-1}T_o$$

(3.41)

$$f = t^{-1} \rightarrow f' = \lambda f$$

(3.42)

$$a = \ell/t^2 \rightarrow a' = \lambda a$$

(3.43)

$$F = ma \rightarrow F_{a,t}' = \lambda^{-1}F_{a,t}$$

(3.44)

Other material properties and derived quantities (work, strain, stiffness, etc.) can be scaled as a function of the quantities listed above. All of these newly derived quantities agree with the results of the scaling derivation using a gradient differentiation engineering method in Greschik et al. (1998). This approach involved scaling length and mass as above, assuming a generic displacement $u$ scaled by the same factor $\lambda^{-1}$ as the length, differentiating this displacement function to generate velocity and acceleration relationships, and finally calculating force and time.

3.7.5 Scaling Law Conflict

The conflict in $\pi_1$ and $\pi_2$ must be resolved to determine the true scaling law. Experiments must therefore be performed to determine which law governs the scaling by testing
two or more membranes of different scale. If the responses agree with one or the other of the pi-
numbers, then that number is considered validated and used as the governing scaling law. More
likely, however, the measured responses will not fully agree with either of the pi-numbers, in
which case some relaxation of the laws must be performed to accurately predict the responses.
In Equation 3.33 it was assumed that $\rho_a = \rho_a'$ and $\rho_m = \rho_m'$. While the density of the membrane
material is necessarily constant on both scales, air is a compressible gas, and hence its density
can be changed. Conducting the experiment in a vacuum chamber would allow $\rho_a$ to be changed, altering the prediction of the response as follows:

$$\pi_2 = \frac{\rho_a v^2}{\rho_a h g} \rightarrow v \sim \sqrt{\frac{\rho_m}{\rho_a}}$$

where $\rho_a$ is no longer a constant. According to Equation 3.45, velocity is a function of the
density of air, meaning $\rho_a$ can be altered to generate the $v$ predicted by $\pi_2$. In this manner the
discrepancy in the pi-numbers can be resolved.

### 3.7.6 Scale Model Testing

In Jenkins (2001), scale models of the NASA Shooting Star experiment, similar in
design to the Inflatable Antenna Experiment shown in Figure 2.11 and Figure 2.12, were tested
on NASA’s reduced gravity aircraft. Like the volumetric density of air was relaxed in Equation
3.45, here the acceleration due to gravity (typically constant in a 1g environment) was relaxed.
A 1 x 0.5 m test article comprised of a torus and struts manufactured from membrane material
was deployed in the reduced gravity environment. “Results of the scale-model tests showed
surprisingly similar qualitative behavior to the full-scale IAE experiment during the inflation
process. … Microgravity deployment experiments on the KC-135 clearly demonstrate inflated
structure deployment phenomena and consequently may be useful in model validation.”

Applying the constant thickness scaling laws derived above, the stiff, ultra-lightweight
panels tested below can also be used to predict the behavior of full-scale prototype structures.
According to the discussion in Section 3.4, the tessellating honeycomb structure of the panels
can be accounted for by using “effective moduli” in the traditional elastic plate equations and
shell elements. The constant thickness scaling laws defined above should also therefore be valid for these panels, and only the length and width dimensions of the panels would have to be scaled by factor $\lambda$. There should be no need to modify the internal dimensions of the hexagonal stiffener structures themselves. Once this assumption is validated experimentally by testing two panels of identical construction but scaled length and width dimensions, the panel behavior characterized below could be used to predict the behavior of individual panels or arrays of any dimensions.

Copyright © Jonathan T. Black 2006
CHAPTER 4. MICRO-SCALE STATIC TESTING AND MODELING

4.1 Next-Generation Gossamer Structures Characterization Methodology

Because of the challenges associated with testing and modeling gossamer structures, it was decided that an alternate characterization approach should be developed for the previously unstudied next-generation stiff, ultra-lightweight polyimide panels studied here. Instead of attempting to create extremely accurate and detailed numerical models validated through exhaustive and difficult testing of these large, flexible structures, here another approach is pursued in which the intractability of very large models and large-scale testing are purposely avoided.

The methodology involves recognizing that the behavior of large, lightweight space structures is of interest on two very different scales that have different detail and precision requirements. One is a small, local, micro scale in which it is necessary to understand the detailed mechanics of the structure to enable attachment to other structures or components on the spacecraft, survivability from debris impacts, fine actuation for high-precision structures, etc. The other is a large, global, macro scale in which it is necessary to understand the behavior of the panel structure, array of attached panels, or the spacecraft as a whole. The micro scale can require precisions and resolutions ranging from millimeters down to nanometers in the case of optics, while the macro scale involves resolutions ranging from centimeters to the size of the panel structure itself and approaching the assembled spacecraft size, possibly kilometers.

The static characterization discussed in this chapter and the next – the first ever performed on these new panels – is therefore comprised of two different types of tests driven by two different general goals. The first determines the fundamental structure governing the local, micro-scale static behavior of the panels. This structure is examined through local compression testing, the behavioral mechanics of which are captured in physics-based hybrid finite element models. The second type of test determines the global, macro-scale behavior of the panels as whole structures and of an array of panels through bending tests. These behaviors are captured in simple shell element finite element models in which effective material properties are used that reflect the unique construction and behavior of the honeycomb panels.

Characterization and modeling of the panels on the local and global scales allows for a comprehensive understanding of the behavior of the panels using a methodology specifically
suited to these unique next-generation gossamer structures. This understanding can then be used to perform design trade studies to determine the effects on final panel performance of altering the design specifications to which the panels are manufactured, thus allowing panels to be designed and manufactured to optimally fulfill their specific intended roles.

4.2 Terminology

To ensure clarity and facilitate the discussions that will follow, specific and detailed terminology that will be used throughout the rest of the dissertation when referring to the tested panels or any subsets, substructures, or groups thereof, are described here.

4.2.1 Polyimide Panel Design

Stiff, thermal-formed, ultra-lightweight polyimide panels are part of a new class of gossamer structures that seek to address some of the measurement and modeling challenges of previous, nonlinear, membrane and tension dominated structures. Shown in Figure 4.1 are two examples of different geometries of these panels, both comprised of two layers of thermal-formed, 127-µm thick Kapton polyimide film. The two membranes were formed in a process similar to sheet-metal stamping that created regular and repeating hexagonal indentations 9.525 mm deep. The bottoms of the indentations of the two membranes were then bonded together (back-to-back) to produce the 257 x 152 x 19.05 mm regular hexagon-shaped honeycomb panel and the 270 x 225 x 19.05 mm rectangular honeycomb panel in Figure 4.1. A close-up view of the honeycomb structure is shown in Figure 4.2. The tessellating thermal-formed indentations shown in Figure 4.2 significantly increase the stiffness of the individual polyimide membranes versus unformed membranes.
While the thermal-forming process gives an individual membrane some bending stiffness, it is still extremely compliant and cannot support its own weight. By attaching the bottoms of the indentations of two membranes to each other to form “stiffeners”, a neutral plane is created through the center of each symmetric panel and, in bending, the top layer is in tension and the bottom layer is in compression. Because the stiffeners are all connected by the polyimide membrane material, the panels behave similar to laminated composite plates, in which bending stiffness is a function of the cube of plate (panel) thickness $H$, as follows:

$$D_{11} = \frac{EH^3}{12(1 - \nu^2)}$$ (4.1)
where all quantities are defined in Nomenclature. This method of manufacture therefore produces self-supporting panels with measurable bending stiffness that by definition the membranes from which they are manufactured do not possess, equal in mass to only twice that of a polyimide membrane of identical area plus adhesive.

To further increase bending stiffness or provide greater functionality, a membrane or other film can be attached to the surface of the panels with the honeycomb acting as a substrate. In this configuration bending stiffness would be a function of both the $D_{11}$ plate bending term in Equation 4.1 and the tensile strength of the attached surface film. Attachment of an optical-quality membrane would enable the panels to be used telescope apertures due to the inherent stiffness of the honeycomb structure.

4.2.2 Definitions

- **Stiffener**: The unit formed by adhering the bottoms of two hexagonal indentations together, shown in Figure 4.3. The stiffener is tessellated in a honeycomb pattern to form panels.
- **Node**: The unit formed by the junction of three stiffener side walls, shown in Figure 4.4.
- **Panel**: Honeycomb structure comprised of tessellating hexagonal stiffeners formed by adhering two thermal-formed polyimide membranes back-to-back, shown in Figure 4.1.
- **Array**: Set of panels joined together, shown in Figure 4.5.

![Figure 4.3 – Top view schematic and 3D pictorial view of a stiffener](image)
4.3 Stiffener Testing and Modeling

4.3.1 Experimental Data

As an initial step in the characterization of the new stiff, ultra-lightweight polyimide panels, static compression tests were performed on individual hexagonal stiffeners to investigate their use as the fundamental structure governing the static behavior of the panel at the local,
micro scale. A micropositioner applied downward displacement to an attached plate in specific intervals called *steps*, uniformly compressing a single stiffer resting on an Ohaus Explorer Pro balance with a measurement precision of 0.0001 N. The force was recorded from the balance at each compression (displacement) interval. This setup is shown in Figure 4.6 and detailed specifications are listed in Appendix A.

At most displacement steps in the single stiffener compression tests, significant drift was observed in the force data. After the plate was stepped down, the force measured by the balance would spike, and then drift down. The final settling time for each data point was between 15 and 30 minutes. Two types of force/displacement data were therefore taken, one in which force drift was minimized by using displacement steps in rapid succession and one in which the force was allowed to settle at each displacement step. The combination of the prescribed micropositioner steps and the force data from the balance enabled the construction of force vs. displacement graphs, or stiffness curves, shown in Figure 4.7.

![Figure 4.6 – Static compression test setup](image)

*left image is side view, right image is top view*
Figure 4.7 – Force vs. displacement stiffness curves for same stiffener (left, (a)) and two stiffeners (right, (b))

Figure 4.7(a) shows the different results obtained using the static testing approaches – rapid loading and slow loading. The slow loading approach, which allowed for the force measurement to settle before each value was recorded, shifts the stiffness curve to the right and slightly down. This settling is similar to a stress relaxation phenomena, however instead of stress dissipation through creep, here it is possible that part of the stiffener is buckling under the compressive load. In contrast, no creep, hysteresis, or plastic deformation of the stiffeners is observable in Figure 4.7(a), in which the data for the two rapid data tests of the same stiffener follow almost exactly the same curve. The slow and rapid data sets were taken in succession using the same stiffener test article, and the experimental setup was not adjusted or altered between data sets.

Figure 4.7(b) shows the test results for two separate stiffeners measured in identical manners. The stiffness curves are similar in shape, but slightly offset from each other. The behavior of a single stiffener consists of multiple regions of widely varying stiffness and one region of negative stiffness. If loading was confined to a single force-displacement region, the stiffness could be approximated with a linear relation. For loading over multiple regions or the entire force-displacement range, however, a complex multi-linear or nonlinear approximation is necessary. And while numerical modeling of nonlinear stiffness values is relatively straightforward, the region of negative stiffness, corresponding to the negatively sloped regions of Figure 4.7, is more complicated.
Based on the results and careful examination of this experiment, it has been concluded that the negative slopes seen in Figure 4.7 do not correspond to a static behavior in the material. Instead it is believed that they are a product of the method in which the tests were performed, using specified displacements instead of forces. This conclusion is consistent with what was observed during the compression tests, where in the negative stiffness region of data, the curved chamfer region at the bottom of the hexagonal indentations that were attached to form the stiffeners (see Figure 4.3) was buckling. Therefore in static loading in which the displacement of the stiffeners is not constrained, after the applied force exceeds the bucking force of the chamfers (the peak seen around 1.5 N, 0.5 mm in Figure 4.7) they will completely “snap through” the buckling process immediately and reach the final positive stiffness seen in the right side of the graphs, thereby skipping over the region of negative stiffness.

Negative stiffness is a complex phenomenon, the mechanics of which will not be explored further here. It may be addressed in the future using adaptations of modeling methods that have previously been developed such as those presented in Muheim and Johnson (2003)\textsuperscript{162} and Zong and Crisfield (1996).\textsuperscript{163}

4.3.2 Finite Element Data

Numerical models of individual stiffeners were constructed in the ANSYS 8.0 finite element software. The following material properties were used for Kapton polyimide film:

<table>
<thead>
<tr>
<th>Material property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus, $E$</td>
<td>2.5 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio, $\nu$</td>
<td>0.34</td>
</tr>
<tr>
<td>Volumetric Density, $\rho$</td>
<td>1420 kg/m$^3$</td>
</tr>
</tbody>
</table>

Initially, a fully representative geometric model of a stiffener was created using four node, six degrees of freedom per node shell elements specifically suited for large strain nonlinear applications.\textsuperscript{165} The model of the individual stiffener is shown in Figure 4.8 and the model of the entire panel in Figure 4.9. Visible in the individual stiffener model is the curved chamfer region.
whose buckling is believed to be responsible for the negative stiffness seen in the experimental compression tests.

A method of attacking convergence difficulties when modeling gossamer structures is to use very fine meshes in regions of geometric nonlinearities such as membrane wrinkling. A similar approach was used here to address the buckling the curved chamfer region. A very fine mesh is visible in the region indicated in Figure 4.8.

While finer meshes do aid in convergence, they also create extremely large models that are very computationally intensive. The single stiffener model shown in Figure 4.8, refined to have only one element along the depth of the stiffener walls to minimize model size, contains 3,072 nodes and 18,432 degrees of freedom. When replicated 60 times and connected with additional material to form the full model of the full regular hexagon-shaped panel in Figure 4.1, the node and element limits of the software were exceeded. Figure 4.9 was created using stiffeners with a coarser mesh of only two elements along the chamfer arc instead of the four elements used in the model in Figure 4.8. This reduction still produced a full model containing 53,423 elements, 48,361 nodes, and 290,166 degrees of freedom.
Despite the fine mesh of the fully representative geometric model, it failed to accurately predict the known displacements when run in compression using the forces measured during the experimental compression tests (modeling results will be discussed in more detail later in the section). A method of hybrid modeling using the experimental data and nonlinear spring elements was then used to both reduce model size and improve accuracy.

Nonlinear springs were connected between the bottom of the sidewalls of the stiffener and the base on the neutral axis, replacing the curved chamfer region. Two types of models were created in this manner, one in which the sidewalls and base had meshes that were identical to that of the full-geometry model with a total of 360 springs placed along the entire bottom edge of the sidewalls (Figure 4.10(a)); and one in which a coarser mesh on the sidewalls and base was used and springs were placed only at the corners of the sidewalls yielding a total of 12 springs (Figure 4.10(b)). These reduced hybrid models not only much more closely matched the experimental data, but were also much less computationally intensive.

Table 4.2 shows the reduction in model complexity.
Figure 4.10 – 3D pictorial view of reduced geometry models of individual stiffeners using 360 nonlinear springs (left, (a)) and 12 nonlinear springs (right, (b))

Table 4.2 – Comparison of model size for the full and reduced geometry models

<table>
<thead>
<tr>
<th>Model</th>
<th>No. of Nodes</th>
<th>% of Full Geo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Geometry</td>
<td>3072</td>
<td>100</td>
</tr>
<tr>
<td>360 Nonlinear Springs</td>
<td>1030</td>
<td>33.5</td>
</tr>
<tr>
<td>12 Nonlinear Springs</td>
<td>521</td>
<td>17.0</td>
</tr>
</tbody>
</table>

The reduced models produced better agreement with the experimental data (as will be shown later in the section) and significant computational savings. In addition, the 12 nonlinear spring model shown in Figure 4.10(b) allowed better understanding of the behavior of the sidewalls than was obtained using the fully-representative geometric model because of the use of three elements along the depth of each wall.

The stiffness of each of the nonlinear springs was defined so that the total effective stiffness $K_{eff}$ of the modeled stiffener matched that measured in the experimental test of Stiffener 1 shown in Figure 4.7(a). The rapid data set from Figure 4.7(a) was used here, and rapid data sets will be used in all models from this point forward because of the more consistent and repeatable data collection process. Each nonlinear spring in the models shown in Figure 4.10 has another identical spring on the opposite side of the base (neutral axis) against which it pushes in compression, creating a parallel condition. Each set of two parallel springs is then in series with the other sets of parallel springs around the entire edge of the stiffener (Figure 4.10(a)) or just at the corners (Figure 4.10(b)). The equation for the stiffness $k$ of each individual spring is
therefore based on fundamental relationships governing the behavior of springs in parallel and series, as follows:

\[ K_{\text{eff}} = n \left( \frac{1}{k} + \frac{1}{k} \right)^{-1} = n \left( \frac{2}{k} \right)^{-1} = \frac{kn}{2} \]  \hspace{1cm} (4.2a)

\[ k = 2K_{\text{eff}} / n \]  \hspace{1cm} (4.2b)

where \( n \) is the total number of springs in the model.

The stiffness of the nonlinear springs was defined by entering into the model the force vs. displacement data shown in Figure 4.7(a). The spring stiffness in between each point entered into the model was assumed to have a constant slope (slope = stiffness). In this manner up to 20 discrete piecewise-linear stiffnesses could be defined for each set of nonlinear spring elements.

Equation 4.2b was implemented at each point by replacing \( K_{\text{eff}} \) with the entered force value for each of the points, yielding the stiffness for each individual spring element (see Appendix C for code).

4.3.3 Test/Analysis Correlation

The numerical models were constrained to zero displacement in the \( z \)-direction at the base of the stiffeners, matching the experimental conditions. Force was then applied either along the entire length of the top edge of the sidewalls, as was the case for the reduced geometry model of 360 nonlinear springs spread along the entire bottom edge of the sidewalls, or only point forces at the corners, as was the case for the reduced geometry model of 12 nonlinear springs at the corners of the stiffener. The fully-representative geometric model (Full Geo) was run for both loading conditions. A nonlinear, large-displacement static analysis was run using the arc-length method, which automatically divided the load into non-uniform sub-steps and applied each sub-step in such a way that total model convergence was maintained. The results of the simulations obtained using each of the three numerical models are shown in Figure 4.11, along with the experimental Rapid Data curve from Figure 4.7(a) that was used to define the stiffnesses of the nonlinear springs.
To elucidate the discussion of the results of the simulations, the ways numerical simulations can fail should be clearly defined as follows: they can fail to converge under a particular loading condition or at a particular load step; they can rapidly diverge or bifurcate; they can fail to accurately approximate the experimental data; etc. Some of the failure modes can be caused by complications with the creation of the model itself, with the applied boundary conditions, or with the selected solution method, among others. Several of these methods of failure are discussed below.

Figure 4.11 shows that, surprisingly, the fully-representative geometric model (Full Geo) does a very poor job of replicating the experimental data. And for the case most representative of the experimental data (in which the force is spread along the entire top of the sidewalls), the model diverges so rapidly it is not even visible on the graph. The Full Geo model with only point loads applied at the corners of the stiffener converges for a large range of force values relative to the other models, bifurcating around 2.5 N, but does not show clearly-defined regions of different slopes (stiffnesses), as is seen in the rapid and slow experimental data curves.

The reduced models incorporating nonlinear springs are much better at replicating the experimental data than the full-geometry model. All of the reduced models have two clear
regions of unique stiffness, with some converging at higher forces than others. They do not, however, exactly follow the Rapid Data curve used to define the stiffness of the nonlinear springs. Also of note is the fact that the 360-spring model that used twice the stiffness and half the applied force as the other models produced exactly the same results as the 12 spring model, while the 360 spring model with stiffness and applied force equal to the other models diverged sooner.

An attempt was made to converge the reduced geometry models past the 1.0 N loading beyond which they all bifurcated in the simulations shown in Figure 4.11. The stiffness values of the nonlinear springs were altered to only reflect that portion of the experimentally measured stiffness before (to the left of) the negative stiffness region. Figure 4.12 shows that when the negative stiffness region is removed from the 360-spring model, it does converge above the 1.0 N loading condition, however there is a large discontinuity in the stiffness curve. After following the 12 spring curve for a short time, the curve jumps to a region of similar stiffness, through a large displacement region of higher average compliance. This discontinuity occurs when the springs compress completely, causing the top and bottom walls of the stiffeners to push directly against each other.

![Diagram](image)

**Figure 4.12** – Numerical model results showing models with and without negative stiffness
4.3.4 Multiple Stiffener Experimental Data

Despite the inability of the individual stiffener models to achieve an accurate approximation of the negative stiffness region seen in the experimental data, compression testing of the smallest substructure comprised of multiple stiffeners was performed to determine if it would present the same negative stiffness challenge seen in the single stiffener data. The result, seen in Figure 4.13, shows that indeed not only is the negative stiffness behavior present in the tri-stiffener substructure, but it is even more pronounced than in the single stiffener data. Therefore the conclusion reached in the previous subsection – that models of the individual stiffener that include shell elements are not acceptable models of the fundamental structure governing the static behavior of the panel on a local, micro scale – remains valid, and was not pursued any further.

Figure 4.13 – Force vs. displacement stiffness curves for single and tri stiffeners, loading curves above unloading curves

4.3.5 Summary

Fully representative geometric shell element models and reduced shell element models including hybrid nonlinear springs of single stiffeners were not able to accurately replicate the experimental stiffener compression data. These results lead to the final conclusion that the approach of modeling the single stiffener as the fundamental structure governing the static
behavior of the panels on a local, micro scale is invalid. Based on the results of the parameter variation analysis, it is further concluded that no single area, density, modulus, or structural change will enable these models to approximate the negative stiffness buckling region of the experimental data. Therefore an alternate modeling approach is explored in the next section.

4.4 Nodal Testing and Modeling

4.4.1 Experimental Data

As an alternative to assuming the hexagonal stiffener to be the fundamental structural unit governing the static behavior of the panel on a local, micro scale, the individual node formed by the junction of three stiffener side walls of the stiff, thermal-formed, ultra-lightweight polyimide panel was selected for evaluation in this role. Such nodes have been illustrated schematically in Figure 4.4, and their physical location in the polyimide panel is shown Figure 4.14.

![Figure 4.14 – Individual isolated node inside stiff ultra-lightweight polyimide panel](image)

Using the test setup in Figure 4.15, compression tests on single nodes isolated on the top and bottom of the panel (Figure 4.14) were performed and force vs. displacement plots were generated (Figure 4.16) in which the slopes of the curves at each data point corresponds to the stiffness of the individual node. Each of the tested nodes was located as shown in Figure 4.14, internally in the panel, several stiffeners away from the edge. Figure 4.16 shows that the behavior of a single node consists of several regions of widely varying stiffnesses and one region
of negative stiffness, comparable to the behavior of a single stiffener. It can also be seen from Figure 4.16 that different nodes behave in similar manners. Note that the steepest slopes, which occur at an applied displacement of 0.5 mm, are similar and approximately linear. Variation of the onset and range of negative stiffness among the nodes differentiate their responses. No discernable pattern to the variation was observed, and is likely caused by random imperfection in the manufacturing process.

Figure 4.15 – Static compression test setup

Figure 4.16 – Force vs. displacement experimental data for individual nodes, loading curves above unloading curves
To validate numerical models detailed in the next subsection, compression tests were also performed on the smallest substructure of the full panel, consisting of multiple adjacent nodes (schematic shown in Figure 4.17). This testing required isolating the three nodes surrounding the single isolated node shown in Figure 4.14 and loading all four simultaneously within the panel, yielding static compression data on the four node or “quad-node” configuration. The resulting force vs. displacement plots are shown in Figure 4.17, in which the slopes of the curves at each applied displacement corresponds to the stiffness of the combined quad-node configuration. Expanding this approach a step further, Figure 4.18 shows the resulting force vs. displacement plots for the compression tests of the substructure consisting of all of the nodes immediately surrounding the quad-node substructure, or ten-node configuration. Figure 4.17 includes the single-node result and Figure 4.18 includes both the single-node and quad-node results, demonstrating that the combined stiffness of four nodes is approximately 2.5 times that of the corresponding individual node and the combined stiffness of ten nodes is approximately 5 times that of the corresponding individual node. No creep, hysteresis, or plastic deformation of the nodes is observable in Figure 4.17 or Figure 4.18, in which the data for the two quad-node and ten-node configurations is shown to be repeatable. The minor difference in the results of the quad-node tests at larger displacements is likely the result of slight misalignment of the panel, compression plate, and balance, and not caused by any plastic deformation resulting from the first compression test.
Figure 4.17 – Force vs. displacement experimental data for the quad-node and individual node configurations, loading curves above unloading curves

Figure 4.18 – Force vs. displacement experimental data for the ten, quad, and individual node configurations, loading curves above unloading curves
4.4.2 Finite Element Data

To model the results from the individual isolated node testing, a single nonlinear spring element with stiffness defined to identically match the experimental data was used. As was the case for the combination shell and nonlinear spring element stiffener models discussed in Section 4.3, the stiffness was defined by entering data points from the Single Node curves on the force vs. displacement graphs shown in Figure 4.17 and Figure 4.18 into the model. The spring stiffness in between each point was assumed to have a constant slope (slope = stiffness). In this manner up to 20 discrete piecewise-linear stiffnesses could be defined for the nonlinear spring element. The spring was fully constrained to match the experimental conditions, and a nonlinear, large displacement static analysis was run to generate the data shown in Figure 4.19.

![Diagram showing applied displacement](image)

Figure 4.19 – Individual node modeled as a single nonlinear spring results

In Figure 4.19, the experimental Single Node curve (replotted from Figure 4.17 and Figure 4.18) was entered into the finite element model to define the stiffness of the nonlinear spring. This curve is closely followed by several sets of finite element data. All models show multiple stiffness regions, including negative stiffness (except the Force Input model), indicating that the physics-based hybrid single nonlinear spring model is capable of accurately replicating the results of the experimental testing. One of these curves, the case in which force was applied to the top of the spring instead of displacement as in all of the other models (Force Input), does
not match the data at the negative stiffness region. Instead, it validates qualitative observations from the previous subsection that the negative stiffness region is actually a dynamic buckling phenomenon in which the curved chamfer regions buckle at the bottom of the indentations forming the stiffeners. When the applied force in the model reached the region on the curve around 0.7 mm at which the slope decreases approaching the inflection point at the onset of the negative stiffness region, the model jumped over the artificial negative stiffness to the point at which the chamfers were completely buckled and the side walls on either side of the neutral surface were in direct contact with each other, identified as region “A” in the graph. This was also seen in the results presented previously in Figure 4.12. If force was the controlled variable in the experimental testing instead of displacement, the negative stiffness would not have existed because, as in this model, when the force reached the point at which the buckling of the curved chamfer region began, the entire region would dynamically buckle and the next data point would be the positive stiffness region on the far right side of the curve in which the side walls of the stiffeners are in direct contact.

The two finite element curves that do not match the experimental Single Node curve represent different, unsuccessful methods for defining the stiffnesses of two nonlinear springs in series to model the individual node. The third method for defining the stiffnesses of two nonlinear springs in series exactly matches the entered Single Node experimental data curve. These models were created to demonstrate that, by placing multiple nonlinear springs in series, the behavior through the thickness of the panel could be obtained.

4.4.3 Test/Analysis Correlation

This section is summarized in Figure 4.20 as an overview of the correlations efforts. All of the models presented here represent incremental steps toward identifying and validating a model of the fundamental structure governing the static behavior of the panel on a local, micro scale. The process of developing a valid model was not straightforward. Initially, the validated single-node model was used to construct a quad-node model matching the geometry in the quad-node substructure experimental tests. The stiffness of each spring was defined to match the experimental Single Node curve for the individual (also called single) node test shown in Figure 4.17, Figure 4.19, and Figure 4.21. The resulting model of four springs in parallel behaves as a
linear combination of the individual stiffnesses of the springs, identically matching fundamental spring theory, and responds to compressive loading at four times the entered Single Node experimental data curve (Figure 4.21). Unfortunately the experimental data of four nodes in compression yielded a curve that was only 2.5 times as stiff as the single node data (Figure 4.17 and Figure 4.21), thereby invalidating the model of the fundamental structure governing the local, micro-scale static behavior of the panel as a single, independent node in isolation. Also note in Figure 4.21 that when the Quad Node experimental data was entered into the model instead of the Single Node experimental data by dividing the Quad Node stiffness by four and applying the result to each of the four nonlinear springs, the model identically matches the entered experimental data. An alternative approach for modeling the fundamental structure governing the micro-scale static behavior of the panel was therefore required. This approach is summarized in Figure 4.20.
Figure 4.20 – Organizational chart of Section 4.4.3
The model of the fundamental structure governing the micro-scale static behavior of the panel as a single, independent node was updated based on the results above and qualitative observations of the panel. It was observed that a single node in compression is not isolated from the rest of the panel as assumed above, but interacts with the three surrounding nodes (see Figure 4.14). This interaction was modeled by horizontally connecting the single vertical nonlinear spring with three other vertical nonlinear springs at coordinates corresponding to the three surrounding nodes (middle column in Figure 4.20), as shown in Figure 4.22. Each vertical spring stiffness was defined to be 1/4 that measured in the single node rapid data test, and the stiffness of the three horizontal connecting springs $K_{horiz}$ was iteratively determined. Vertical compressive displacement was applied to the center spring, and a nonlinear, large displacement static analysis was run to generate the data shown in Figure 4.22.
Figure 4.22 – Single and four node results modeled with surrounding nodes

Figure 4.22 shows that the model of a single node as four vertical parallel springs connected horizontally exactly matches the inputted Single Node curve. As the next step in the finite element model validation, the four node configuration was modeled as four vertical springs connected horizontally to each other and the six nodes surrounding them as shown in Figure 4.22, each with stiffness defined as 1/4 of the Single Node experimental data (middle column in Figure 4.20). Vertical displacements were applied to the central four nodes of the resulting ten spring model in a large displacement, nonlinear analysis, the results of which are also plotted in Figure 4.22. The ten spring model very closely matches the measured quad-node data, and a horizontal connection spring stiffness $K_{\text{horiz}}$ of $1 \times 10^7$ N/m was found to provide the best approximation (the curves of models in which $K_{\text{horiz}}$ was less than $1 \times 10^7$ N/m are not shown because of their low initial slopes and resulting poor approximations of the experimental data curve). These data serve as a validation of the modeling approach that individual nodes in the panel behave as an interaction of the node to which the displacement is applied and the surrounding unloaded nodes.
To study how the horizontal connection spring stiffness changes as a function of the size of the model of the fundamental structure governing the static micro-scale behavior of the panel (single-node model), four analyses were run (right column in Figure 4.20). The first, shown in Figure 4.23, models the single node as in Figure 4.22, four parallel horizontally connected vertical springs with stiffness defined to be 1/4 that measured in the single node test. Multiple horizontal connection spring stiffnesses were used, with $1 \times 10^{10}$ N/m determined to be optimal by exhibiting the smallest deviation from the experimental data in this configuration.

The second analysis modeled a single node as a vertical spring connected horizontally to the two levels of surrounding unloaded nodes, yielding a total of ten parallel vertical springs with stiffness defined to be 1/10 that measured in the single node test. Multiple horizontal connection spring stiffnesses were used, with $1 \times 10^{12}$ N/m determined from Figure 4.24 as optimal in this configuration. The third analysis modeled a single node as a vertical spring connected horizontally to the three levels of surrounding unloaded nodes, yielding a total of 19 parallel vertical springs with stiffness defined to be 1/19 that measured in the single node test. Multiple horizontal connection spring stiffnesses were used, with $1 \times 10^{13}$ N/m determined from Figure 4.25 as optimal in this configuration. And the final analysis modeled a single node as a vertical spring connected horizontally to the four levels of surrounding unloaded nodes, yielding a total of 31 parallel vertical springs with stiffness defined to be 1/31 that measured in the single node test. Multiple horizontal connection spring stiffnesses were used, with $1 \times 10^{13}$ N/m determined from Figure 4.26 as optimal in this configuration.
Figure 4.23 – Single node modeled as four parallel springs connected by horizontal springs with stiffness determined iteratively

Figure 4.24 – Single node modeled as ten parallel springs connected by horizontal springs with stiffness determined iteratively
It can be seen from Figure 4.23 through Figure 4.26 that as the size of the model of the fundamental structure governing the micro-scale static behavior of the panel increases, it is less sensitive to changes in the horizontal connection spring stiffness. The study also reveals that as
As stated previously, from qualitative observations of single nodes in compression it is evident that the first level of surrounding three unloaded nodes experiences deflection as the single node at the center is loaded, but there is no observable interaction with other nodes in the structure. The fundamental structure governing the micro-scale static behavior of the panel was therefore selected from these observations and the above study as the nodes being loaded and one level of surrounding, unloaded nodes, as was used in the data in Figure 4.22.

This model should also quantitatively replicate the measured data based on the comparison of experimentally measured stiffness of different substructure sizes in Figure 4.18. The model of the fundamental structure governing the static behavior of the panel on a local, micro scale yields 4, 10, and 19 spring models for the single-, quad-, and ten-node substructures, respectively. The stiffness of each vertical nonlinear spring is therefore defined as 1/4 of that measured in the Single Node data, as was used in models in Figure 4.22 and Figure 4.23. The
total stiffness of the quad-node model consisting of ten parallel vertical springs is then 10/4 that measured in the Single Node experimental data, and the total stiffness of the ten-node model consisting of 19 parallel vertical springs is 19/4 that measured in the Single Node experimental data. Figure 4.18 shows that the measured stiffness of the quad-node substructure is approximately 2.5 times that of the single node, exactly matching the 10/4 factor calculated above. The measured stiffness of the ten-node substructure in Figure 4.18 is approximately 4 times that of the single node, approximately matching the 19/4 factor calculated above.

In addition to being physically accurate, this model also has the benefit of being the smallest. And while this model appeared valid in Figure 4.22, an additional attempt was made to match data from the compression testing of the ten-node substructure of the full panel in Figure 4.18 and is shown in Figure 4.28.

Figure 4.28 – Single node, four node, and ten node substructures modeled as nodes loaded plus one level of surrounding, unloaded nodes

Figure 4.28 shows that, after a very brief initial period below 0.25 mm, the model of the ten-node substructure as 19 vertical springs in parallel connected horizontally does not accurately replicate the experimental data. It can be also be seen from the experimental data in Figure 4.28 that as the tested substructure size increases, a decrease in slope or softening occurs in the stiffest (second) region of the force vs. displacement curves after 0.5 mm, in which the quad-node data
has a lower slope than the single-node data, and the ten-node data has a lower slope than the other two. To study this softening phenomenon in more detail, the slopes of the three experimental data curves in Figure 4.28 are plotted in Figure 4.29.

![Figure 4.29 – Slopes of the single, quad, and ten node experimentally measured data curves](image)

The decrease in slope, or softening, of the stiffest region of force vs. displacement data before the negative stiffness region can be clearly seen in Figure 4.29. Indicated in the figure are the points at which the slopes of the data curves change from positive to negative, indicating the transition to the negative-stiffness region. The occurrence of the inflection points later in the data curve as the substructure size tested increases means that the physics-based hybrid technique used to model the data should reflect this softening phenomena. Table 4.3 shows the softening as a percentage change in the x-axis location of the inflection points.

110
The softening phenomenon seen in the experimental data was incorporated into the models by applying softening percentages to the entered Single Node experimental data curve used to define the stiffnesses of the nonlinear springs, the results of which are shown in Figure 4.30. The model results in Figure 4.30 use the optimal softening percentages determined iteratively that most closely matched the finite element to the experimental data curves. These values are also shown in Table 4.3. The optimal values for the softening determined independently in the finite element models are quite comparable to the experimentally measured values – seen by comparing the Percent Change from Single Node and FE Optimal Percent Change columns, and the Slope Inflection Point and FE Optimal Slope Inflection Point columns in Table 4.3. Therefore, without knowledge of the experimentally-determined softening percentages, the finite element models came very close to the actual values measured in the experimental testing. It should be noted that the horizontal connection spring stiffness $K_{hori}$ was held constant at the optimal value for the single node model determined in Figure 4.23 of $1 \times 10^{10}$ N/m in all of the models. It is now therefore possible to use only experimental data in the creation of substructure models with limited iterations or alterations of the models from one substructure to the next, reflecting only the measured softening in the data.

<table>
<thead>
<tr>
<th>Substructure Size</th>
<th>Slope Inflection Point (mm)</th>
<th>Percent Change from Single Node</th>
<th>FE Optimal Percent Change</th>
<th>FE Optimal Slope Inflection Point (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Node</td>
<td>0.450</td>
<td>n/a</td>
<td>n/a</td>
<td>0.450</td>
</tr>
<tr>
<td>Quad Node</td>
<td>0.600</td>
<td>133.33</td>
<td>150.00</td>
<td>0.675</td>
</tr>
<tr>
<td>Ten Node</td>
<td>1.200</td>
<td>266.67</td>
<td>260.00</td>
<td>1.170</td>
</tr>
</tbody>
</table>

Table 4.3 – Softening percentages from experimental data and finite element model
Figure 4.30 – Single, quad, and ten node data modeled with incorporated softening in the defined stiffnesses of the nonlinear springs

### 4.4.4 Summary

Figure 4.30 shows that the fundamental structure governing the local, micro-scale static behavior of the panel modeled as the loaded nodes attached to one level of the surrounding unloaded nodes is valid at multiple substructure sizes. The first part of the modeling methodology described in Section 4.1 is therefore satisfied by the validation of a physics-based reduced hybrid finite element model of the local micro-scale mechanics of the stiff, ultra-lightweight polyimide panels. This model can now be used to satisfy any analysis requirements to examine the local behavior and performance of these panels in tasks such as fine actuation and connection to other spacecraft components, among others.

While the nonlinear spring model of an individual node exhibits a substantial reduction in model size versus the smallest combined shell element and nonlinear spring model of an
individual stiffener – 8 nodes, 24 degrees of freedom (no rotational dofs in spring models) vs. 521 nodes, 3,126 degrees of freedom – it is still impractically large for use on a global, macro scale. For example, a nonlinear spring model of a one square meter panel would be comprised of approximately 63,000 finite element nodes and 189,000 degrees of freedom, and a nonlinear spring model of a 100 square meter panel would be comprised of over 700 million finite element nodes and over 2 billion degrees of freedom (see Appendix B for estimation). Models of this size are clearly impractical for use on a global, macro scale, therefore simpler models specifically designed for large-scale applications are developed and validated in the next section pursuant to the new next-generation gossamer structures characterization methodology.

Copyright © Jonathan T. Black 2006
CHAPTER 5. MACRO-SCALE STATIC TESTING AND MODELING

5.1 Panel Testing and Modeling

5.1.1 Experimental Data

In addition to investigating the fundamental structure governing the micro-scale static behavior of the new stiff thermal-formed ultra-lightweight polyimide panels, investigation of the global, macro-scale static behavior of the panels was undertaken in the form of panel bending tests. Two rectangular panels of identical geometry to the one shown in Figure 4.1 were tested using the setup shown in Figure 5.1. One panel was manufactured from 0.127-mm (0.005-in) thick Kapton and the other was manufactured from 0.0762-mm (0.003-in) thick Kapton. Each panel was supported along parallel edges by the edge-support dowels that were fully constrained to approximate a simple two-dimensional line contact support along the panel edges. Several different loads were applied to the center of each panel, and the displacement of two points along the center line of the panel were recorded for each load by the Keyence laser displacement sensors capable of resolving displacements greater than 10 μm, and a data acquisition system, all shown in Figure 5.1 (exact specifications in Appendix A). The nodes – defined as in the previous subsection as junctions of the sidewalls of the honeycomb and illustrated in Figure 4.4 – that supported the applied loads and the nodes at which displacement was measured are shown in Figure 5.2. Each rectangular panel was tested under two support conditions: one in which the long sides of the panels were supported by the edge support dowels and the short sides were free; and one in which the short sides of the panels were supported by the edge support dowels and the long sides were free. In the long-side supported configuration, displacements were measured at points L1 and L2; and in the short-side supported configuration displacements were measured at points S1 and S2, shown in Figure 5.2.
Figure 5.1 – Rectangular panel bending test setup

Figure 5.2 – Rectangular stiff ultra-lightweight polyimide panel with relevant loading and measurement points marked

To mitigate the possibility of systemic bias arising from mechanical hysteretic effects caused by repeated bending in the same direction or misalignments, three different groups of full panel bending tests were undertaken for each of the long-side supported and short-side supported test configurations. After several initial sets of data were taken, the panels were rotated 180°, switching the positions of the measurement points, and flipped upside down so bending occurred
in the opposite direction. Figure 5.3 and Figure 5.4 show the results of the experimental bending tests for the stiff, ultra-lightweight panels manufactured from 0.127-mm (0.005-in or 5-mil) thick and 0.0762-mm (0.003-in or 3-mil) thick Kapton, respectively. In both of the figures, the upper-left graph shows the 0° or initial data group, the upper-right graph shows the data from the same panels rotated 180°, the bottom-left graph shows the data from the same panels flipped upside down, and the bottom-right graph shows all of the data in the previous three graphs plotted together with fit trend lines. Pt’s 1 and 2 in the graphs in Figure 5.3 and Figure 5.4 correspond to measurement points L1 and L2 in Figure 5.2, with the rotation or flip accounted for.

![Graphs showing bending test results for stiff, ultra-lightweight panels manufactured from Kapton](image)

Figure 5.3 – Long-side supported full panel bending data for the panel manufactured from 0.127 mm (5 mil) Kapton
Figure 5.4 – Long-side supported full panel bending data for the panel manufactured from 0.0762 mm (3 mil) Kapton

Figure 5.3, Figure 5.4, and Figure 5.5 show the deflection of each measured point as a function of the applied mass. They demonstrate that in bending the panels behave, in general, in a linear fashion, regardless of their orientation. Figure 5.3 and Figure 5.4 also show that Point 2 consistently demonstrates greater deflection than Point 1. Because this disparity is present regardless of orientation, and taking into account that Point 1 is slightly closer to the center of the panel than Point 2 (Figure 5.2), it is likely that the panels under point loading deform into a saddle configuration, with curvature in both $x$ and $y$ directions rather than a simple trough configuration as predicted by classical plate bending theory. For this reason two trend lines are fit to the data shown in the lower right graphs in Figure 5.3 and Figure 5.4. The upper line fits the deflection of Point 2 and the lower Point 1. These lines will be compared to the results of the physics-based hybrid finite element models in the next section.
Comparing Figure 5.3 and Figure 5.4 demonstrates the effect of varying the thickness of the polyimide film from which the panels are manufactured. The figures show the bending test results from rectangular panels that are identical in every way with the exception of the thickness of the Kapton polyimide film. As expected, the panel manufactured from the thinner 0.0762 mm film in Figure 5.4 exhibits greater deflection under the same loading than the thicker 0.127 mm film panel in Figure 5.3. But while the overall panel stiffness of the 0.0762 mm panel is shown to be less than the 0.127 mm, it still behaves in a similar linear fashion for the same range of loads.

Figure 5.5 provides a side-by-side comparison of the two different rectangular panels in bending with their short sides supported. The graphs on the left side of the figure show the deflection of Points 1 and 2, corresponding to S1 and S2 in Figure 5.2, of the panel manufactured from 0.127-mm thick film and the graphs on the right of the figure show the deflection of the same points of the 0.0762-mm thick film panel. The plot scales are identical in order to allow a direct comparison of the panels’ deflection. As expected, the panel manufactured from the thinner 0.0762 mm film exhibits greater deflection under the same loading than the thicker 0.127 mm film panel. The data all lie along straight lines, meaning that regardless of film thickness or deflection, the panels still behave in a linear manner for this range of applied loads. In addition, Figure 5.5 lacks any indication of disparity in deflection between Point 1 and Point 2. While it is still likely that the panels exhibit a saddle configuration under point loading as in the long-side supported tests, because the points measured are exactly symmetric, Figure 5.2 single trend lines are fit to the data in the bottom graphs.
Figure 5.5 – Short-side supported full panel bending data for panels manufactured from both 0.127 mm (5 mil) and 0.0762 mm (3 mil) Kapton, bottom graphs with best-fit line

5.1.2 Finite Element Data

As discussed previously, the prohibitively large size and limited success of large, geometrically accurate, finely-meshed shell element models to numerically simulate the behavior of ultra-lightweight gossamer-class space structures,\textsuperscript{59-61,152} and the demonstrated linear quasi-static macro-scale behavior of panels in the previous chapter together led to an alternate approach here. Following the methodology of developing simple, accurate, but inherently separate models of gossamer structures on the micro and macro scales, comparatively coarsely-meshed, single-layered shell element models of the panel geometries tested above are described below. The material properties of the solid shell elements were defined to capture the behavior of the honeycomb geometry and unique construction of the panels, yielding simple yet accurate
models that can easily applied to all of the experimental cases by modifying the global, macro-
scale geometry.98

The currently accepted governing equations of lightweight honeycomb core panels treat
the panels, in a global sense, as Reissner-Mindlin elastic plates. This formulation is based on the
classical plate equations solved for selected boundary conditions in Section 3.5. To apply the
Reissner-Mindlin solid plate equations to honeycomb core panels, the various material properties
such as shear and rotational moduli and Poisson’s ratios must be properly calculated. The
derivations of the methods for calculating these properties for honeycombs are listed in Gibson

To approximate the full panel bending data from Figure 5.3, Figure 5.4, and Figure 5.5,
simple finite element models were constructed from four-node shell elements possessing six
degrees of freedom per node. A simple rectangle matching the length and width dimensions of
the rectangular panel in Figure 4.1, Figure 5.1, and Figure 5.2 was created and meshed with a
single layer of shell elements of thickness equal to that of the panel, 40 elements per side.
Material properties were defined as regular repeating hexagonal honeycomb panels using
Equations 5.1-5.3 below:166,167

\[
\frac{E_1}{E_s} = \frac{\sigma_1}{\epsilon_1} = \left(\frac{t}{\ell}\right)^3 \frac{\cos \theta}{(h/\ell + \sin \theta) \sin^2 \theta}; \quad \frac{E_2}{E_s} = \frac{\sigma_2}{\epsilon_2} = \left(\frac{t}{\ell}\right)^3 \frac{(h/\ell + \sin \theta)}{\cos^3 \theta}
\] (5.1)

\[
\frac{G_{12}}{E_s} = \left(\frac{t}{\ell}\right)^3 \frac{(h/\ell + \sin \theta)}{(h/\ell)^2 (1 + 2h/\ell) \cos \theta}
\] (5.2)

\[
\nu_{12} = -\frac{\epsilon_2}{\epsilon_1} = \frac{\cos^2 \theta}{(h/\ell + \sin \theta) \sin \theta}; \quad \nu_{21} = -\frac{\epsilon_1}{\epsilon_2} = \frac{(h/\ell + \sin \theta) \sin \theta}{\cos^2 \theta}
\] (5.3)

where \(t, h, \ell, \) and \(\theta\) are defined in Figure 5.6.
However, honeycomb panels consisting of regular hexagons \((h = \ell, \theta = 30^\circ)\) are isotropic, therefore:\(^{166,168,169}\)

\[
\frac{E_1}{E_s} = \frac{E_2}{E_s} = 2.3 \left(\frac{t}{\ell}\right)^3; \quad \frac{G_{12}}{E_s} = 0.57 \left(\frac{t}{\ell}\right)^3 = \frac{1}{4} \frac{E}{E_s}; \quad \text{and} \quad \nu_{12} = \nu_{21} = 1
\] (5.4)

Because a Poisson’s Ratio of 1 creates a situation in which a division by 0 occurs, such as in Equation 4.1, the Poisson’s Ratio in the finite element model was set to 0.98.

Unique to these panels and not reflected in the above equations, the side walls of the hexagonal voids (stiffeners) that create the honeycomb pattern are not solid Kapton polyimide material, but are instead hollow. To incorporate this variation into the existing model without modeling individual cell walls, \(E_s\) in the equations above was modified by performing a “virtual” tensile test on the hollow sidewalls of the stiff thermal-formed thin film polyimide panel using Equation 5.5, the equation for a beam in tension:\(^{102,170}\)

\[(EA/L)x = F\] (5.5)

The value of \(A\) was set to match the cross sectional area of the hollow rectangular sidewalls of the stiffeners in which the thickness of the polyimide film, 0.127 mm (5 mil) was the wall thickness used, and the value of \(E\) was set to that of Kapton. The values of \(x\) were solved for a range of values of \(F\), engineering stress \(F/A\) was plotted versus engineering strain \(x/L\), and the slope of the line was calculated, as shown in Figure 5.7. This value, called the apparent modulus of a hollow beam, or \(E_{app}\), was used instead of \(E_s\) in Equation 5.4.

Figure 5.6 – Geometric parameter definitions for Equations 5.1, 5.2, and 5.3
Figure 5.7 – Virtual tensile test, constant wall thickness (right), sidewall half thickness of top and bottom (left)

5.1.3 Test/Analysis Correlation

The full panel shell models, created using the properties listed in Table 5.1, were constrained to have zero \( z \) displacement along the long edges, loading was applied to the center of the panels, and a nonlinear, large-displacement static analysis was run. The results are plotted in Figure 5.8 along with the best-fit trend lines from the experimental panel bending data in Figure 5.3. Two different apparent moduli (\( E_{app} \)) were used, the smaller of which was calculated using hollow cross-section sidewalls half as thick as the top and bottom, and the greater of which was calculated setting all of the walls of the hollow beam to equal, 0.127 mm thickness (Figure 5.7). Clearly the two values chosen for the \( E_{app} \) approximation represent upper and lower bounds between which the true value of \( E_{app} \) lies; a different setpoint for \( E_{app} \) between the upper and lower bounds would have further improved the fit of the numerical results through the experimental data.
Table 5.1 – Defined properties for shell element models of stiff, ultra-lightweight polyimide panels

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel Dimensions</td>
<td>25.0825 x 26.67 x 1.905 cm</td>
</tr>
<tr>
<td>Panel Mass</td>
<td>38.3 g</td>
</tr>
<tr>
<td>Volumetric Density, $\rho$</td>
<td>30.0545 kg/m³</td>
</tr>
<tr>
<td>Areal Density</td>
<td>0.57 kg/m²</td>
</tr>
<tr>
<td>Number of elements</td>
<td>1600</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>1681</td>
</tr>
<tr>
<td>Stiffener Spacing, $t$</td>
<td>0.3175 cm</td>
</tr>
<tr>
<td>Internal Stiffener Angle, $\theta$</td>
<td>30°</td>
</tr>
<tr>
<td>Stiffener Edge Length, $\ell$</td>
<td>1.27cos($\theta$) cm</td>
</tr>
<tr>
<td>Poisson’s Ratio $xy$, $\nu_{xy}$</td>
<td>0.98</td>
</tr>
<tr>
<td>Poisson’s Ratio $xz$, $\nu_{xz}$</td>
<td>0.34</td>
</tr>
<tr>
<td>Poisson’s Ratio $yz$, $\nu_{yz}$</td>
<td>0.34</td>
</tr>
<tr>
<td>Apparent Modulus, $E_{app}$</td>
<td>$1.3318 \times 10^8$, $2.3179 \times 10^8$ Pa</td>
</tr>
<tr>
<td>Major and Minor Axis Moduli, $E_x = E_y$</td>
<td>$2.3094 E_{app} (t/\ell)^3$</td>
</tr>
</tbody>
</table>

![Five mil Full Panel Bending, Long Side Supported](image)

Figure 5.8 – Full panel bending models for long-side supported rectangular panels manufactured from 0.127 mm (5 mil) Kapton
It was noted in the data in Figure 5.3 above that the two measurement points in the long-side supported full-panel bending test configuration were not symmetric. Point 1 was located closer to the center of the panel than Point 2 (L1 and L2 in Figure 5.2), and the fact that it deflected less than Point 2 in the data in Figure 5.3 indicated that the deformed shape of the panel under point loading was a saddle shape. The data plotted in Figure 5.8 for Point 1 and Point 2 from the finite element model data were also taken from asymmetric points, with Point 1 slightly closer to the center of the panel than Point 2. The finite element results show the same trends as the experimental data, and closely match the experimental results, validating the modeling philosophy of using simple shell elements to model the panels on a global, macro scale and negating the need for detailed models of the nonlinear behavior of individual nodes in compression.

The deformed panel from the finite element analysis is shown in Figure 5.9, clearly detailing the saddle configuration under point loading.
Table 5.1 also lists the areal density of the panels for comparison to the space aperture materials discussed in Chapter 1. Table 1.1 lists the areal densities of ultra-low expansion (ULE) glass and Beryllium as 180 kg/m² and 15.6 kg/m², respectively. The James Webb Space Telescope’s Beryllium primary mirror therefore represents an order of magnitude improvement in areal density over the Hubble Space Telescope’s ULE glass primary mirror. Then at 0.57 kg/m², a space telescope with a primary mirror consisting of the new stiff, ultra-lightweight
polyimide panels studied here would represent an order of magnitude improvement in areal density over Webb.

5.1.4 Summary

Figure 5.8 shows that simple coarsely-meshed shell element models can be used to approximate the behavior of stiff, thermal-formed polyimide panels on a global, macro scale. Therefore the new methodology for characterizing next-generation gossamer structures dictating simple, accurate, and separate models of the micro- and macro-scale behavior of these panels is shown to be feasible and effective at describing their behavior at the two most important operational scales.

An example of a trade study to determine the effect on panel stiffness as a function of polyimide film thickness was also shown to be effective. More studies of this type may be performed to determine the effects of varying other manufacturing parameters such as stiffener spacing, stiffener size, and panel thickness on other macro- or micro-scale performance metrics of the panels such as mass per unit area and fundamental frequency, allowing them to eventually be specifically designed to perform optimally for each specific application. Finite element models such as the one described above can also be used, once validated, in these studies.

To demonstrate that the shell element global models are effective at any size ranging from a single panel to the size of the full structure, an array of attached panels is tested and modeled in the next section.

5.2 Array Testing and Modeling

5.2.1 Experimental Data

In addition to investigating the local static behavior of the panels by testing and modeling stiffeners and nodes, and the global static behavior of the panels by testing and modeling full panels, the static behavior of an array of rectangular ultra-lightweight stiff polyimide panels was also investigated. This investigation validates the use of the simple, coarsely-meshed macro-
scale shell element models developed in the previous section on a larger scale ranging up to the full size of the structure.

The 78.74 x 23.495 x 1.905 cm (31 x 9.25 x 0.75 in) array of ultra-lightweight stiff rectangular panels shown in Figure 4.5 was cantilevered horizontally as shown in Figure 5.10. To ensure consistent panel properties and mechanics along the full length of the array, the portion of the array underneath the clamped angle bracket was varied until the array could be secured with minimal stiffener crushing and no panel buckling. Transitioning from crushed or buckled to unloaded stiffeners along the edge of the clamped portion of the array would involve a change in material properties and static mechanics that would have to be taken into account in the finite element models. This unnecessary complexity was therefore avoided.

Through trial-and-error, clamping of the top 5.715 cm (2.25 in) of the array was found to be optimal to minimize stiffener crushing while still effectively cantilevering the array. This portion was therefore covered under the steel angle bracket and clamped to the edge of the workstation with standard C-clamps, shown in Figure 5.10 and Figure 5.11. Figure 5.11 shows that, while slight crushing of the stiffeners did occur, the C-clamps were not so tight as to crush the entire row of stiffeners underneath the angle bracket, meaning that only one set of material properties was necessary and identical shell elements could be used in the clamped and free regions of the array.

Figure 5.10 – Array cantilever bending test setup
The wood piece, seen beside the array in Figure 5.10, was used to create the $z = 0$ plane from which the vertical $z$ deflection of the array under gravity loading was measured. To ensure an accurate measurement, the wood piece was leveled, as shown in Figure 5.12. Circular retro-reflective targets were attached to the top-facing surface of the array and to the wood piece whose thickness of 1.905 cm (0.75 in) was identical to that of the array. The retro-reflective targets appear as bright white circles in Figure 5.10 through Figure 5.13.
Five images of the array were captured with calibrated (see Section 3.3) Olympus E20n 4.9 Megapixel cameras set to a 9 mm focal length, an aperture of f/8, a 1/80 sec exposure time, and infinity focus. Each image had an approximate resolution of 0.3 mm/pixel. One of the images used in the measurement is shown in Figure 5.13. Note how the retro-reflective targets appear as bright white circles on the dark, underexposed background created by the small aperture and short exposure time. Similar to street signs and other reflectors, the retro-reflective targets reflect most incident light directly back to its source. In this case, the light from the camera flash is reflected back to the camera instead of scattered evenly in all directions, meaning the targets appear much brighter than the diffuse surfaces to which they are attached.
The five images were then loaded into the Photomodeler photogrammetry software package and associated with the appropriate camera calibration parameters that allowed for the removal of image distortions cause by lens curvature and camera aberrations and imperfections (see Section 3.3). The targets were marked to sub-pixel accuracy (approximately 1/10 pixel) using an automatic least-squares matching algorithm\textsuperscript{42,137,138,171,172} and referenced across the images to the marks corresponding to the same targets. A triangulation algorithm was run which simultaneously and iteratively solved for the actual three-dimensional (3D) locations of the targets and camera positions. The final product of the photogrammetry process was a set of 3D points corresponding to the locations of the imaged retro-reflective targets. These points were then used to determine the shape of the surface of the array to which they were attached. A much more detailed description of the process is presented in Section 3.3.

Figure 5.14 shows a 3D graphic generated by the photogrammetry software showing the resulting 3D points along with the 3D locations from which the images were taken (camera locations). The points on the wood piece, visible in Figure 5.13 and appearing in an “L” shape above and to the left of the larger set of points on the array in Figure 5.14, were used to scale and rotate the data. The targets along the length and width of the wood piece, were used to create the
$xy (z = 0)$ plane and scale the project. A surface was then fit to the data, and is shown in Figure 5.15.

Figure 5.14 – Photogrammetry software 3D graphic showing point and camera locations
Figure 5.15 shows the deflection under gravity of the cantilevered array, measured as displacement in the z-direction. The array appears to have deflected uniformly, with no evident twist. At the tip (free end), the deflection was measured via photogrammetry to be -19.54 mm.

5.2.2 Finite Element Data

Based on the methodology presented previously, a simple finite element model of the array was created using 1.905-cm (0.75-in) thick shell elements each with four nodes and six degrees of freedom at each node. The material properties were defined to be identical to those discussed in the previous section, $E_{app}$ was set to 231.79 MPa calculated in the previous section, and density was set to the actual mass of the array, 0.0814 kg, divided by the actual volume of the array. A 78.74 x 23.49 cm (31 x 9 in) rectangle matching the dimensions of the array was created and meshed with 140 shell elements and 209 nodes, the upper 23.49 x 5.715 cm (9 x 2.25 in) of which was constrained to be fixed, meaning all six degrees of freedom (three translational,
three rotational) of all of the nodes in that area were set to zero. Uniform gravity was applied in the \(-z\)-direction, and a nonlinear, large-displacement static analysis was run using the arc-length method, which automatically divided the load into non-uniform sub-steps and applied each sub-step in such a way that total model convergence was maintained (see Appendix C for batch file).

A surface was fit to the finite element nodal solution of the \(z\) displacement excluding the clamped nodes constrained to zero displacement in all degrees of freedom, and the result is plotted in Figure 5.16. Figure 5.16 shows the deflection under gravity of the modeled cantilevered array as displacement in the \(z\)-direction at an identical scale to the experimental data in Figure 5.15. Again, the array appears to have deflected uniformly, with no evident twist. At the tip, the deflection is -27.99 mm versus the -19.54 mm deflection measured experimentally. This large over-prediction of the experimental data led to modification of the model.

![Figure 5.16](image)

Figure 5.16 – Surface fit to finite element cantilevered array data showing deflection in \(z\)-direction

As has been discussed, the developed methodology for characterizing these new gossamer space structures requires simple models that accurately represent their structural
mechanics. The optimum balance between fixing the end of the array and minimizing crushing of the stiffeners was found to be securing the top 5.715 cm of the array. While this minimal crushing simplified the finite element model because only one set of material properties was required, it also meant that the C-clamps used were not fully tightened. This possible clamp looseness, the micro-scale nonlinear stiffness behavior of the panel, and experimental uncertainty make it impossible that the clamp was ideal in the experimental data set. It is therefore likely that some motion at the base of the array did occur. To correct for this inconsistency between experimental setup and finite element model, the density used in the defined material properties was decreased until a suitable match with the experimental data was achieved. 67.5% of actual density was found to produce excellent agreement with the measured deflection results, and was used in all of the finite element results to follow. This density change equals a change in mass of the panel from 81.4 to 55.0 g. Note that to avoid this unsettlingly large mass change, modulus could be altered as an alternative, but density was chosen here because of the extensive effort to calculate and measure the modulus of the panels in Section 5.1.2 and Figure 5.8. In addition, other model refinements and modeling techniques could be investigated such as more detailed modeling of the boundary conditions and analysis of the internal stress fields, to generate models that do not require material property alterations. The density alteration used here demonstrates that the models can accurately replicate the experimental data. Other refinements are left to future work.

A surface was fit to the finite element nodal solution of the $z$ displacement excluding the clamped nodes constrained to zero displacement in all degrees-of-freedom, and the result is plotted in Figure 5.17. Figure 5.17 shows the deflection under gravity of the modeled cantilevered array as displacement in the $z$-direction at an identical scale to the experimental data in Figure 5.15. Again, the array appears to have deflected uniformly, with no evident twist. At the tip, the deflection is -18.93 mm versus the -19.54 mm deflection measured experimentally.
5.2.3 Test/Analysis Correlation

In addition to the three-dimensional surface data shown in Figure 5.15 and Figure 5.17, the experimental and finite element data were also plotted on the same graphs in Figure 5.18 and Figure 5.19 to facilitate a direct comparison of the results. In Figure 5.18 side views of the two data sets are shown as slices along the long axis of the panel, directly down the middle. The z scale is shown identical to those in Figure 5.15 and Figure 5.17. This figure shows excellent agreement of the finite element model at 67.5% density and the experimental data along the entire length of the array.

In Figure 5.19, edge view comparisons of the two data sets are shown of slices along the width of the array at 1/4, 1/2, and 3/4 of the length, and at the array tip. At this magnified z scale, it shows that the finite element data slightly under-predicts the experimental data. For each slice, the error is shown in Table 5.2. At the tip there is only a discrepancy of 3.1% between the experimental and finite element data.
Figure 5.18 – Side view of experimental and finite element cantilevered array data

Figure 5.19 – Edge-view slices of experimental and adjusted density finite element cantilevered array data, experimental data “+” and finite element data solid lines
Table 5.2 – Experimental and finite element data comparison

<table>
<thead>
<tr>
<th></th>
<th>1/4 Length</th>
<th>1/2 Length</th>
<th>3/4 Length</th>
<th>Tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured z Displacement at</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Midpoint (mm)</td>
<td>-1.87</td>
<td>-6.94</td>
<td>-13.48</td>
<td>-19.49</td>
</tr>
<tr>
<td>Finite Element z Displacement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>at Midpoint (mm)</td>
<td>-1.07</td>
<td>-5.82</td>
<td>-12.33</td>
<td>-18.93</td>
</tr>
<tr>
<td>Percent Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-42.8</td>
<td>-16.1</td>
<td>-8.5</td>
<td>-2.9</td>
</tr>
<tr>
<td>Measured Average z Displacement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finite Element Average z</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Displacement (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured Twist Angle (deg)</td>
<td>0.14</td>
<td>0.24</td>
<td>0.33</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Also visible in Figure 5.19 are cross-sectional shapes that were not seen at the larger scales in the previous figures. A slight bowing is visible in the finite element data, becoming less pronounced as distance from the fixed edge increases, and a slight twist in the experimental data becomes more pronounced as distance from the fixed edge increases. For each slice, the angle of twist is shown in Table 5.2, but in all cases is well below 1.0 degree. The twist could be caused by slight misalignments in the clamp setup, or the manner in which the three rectangular panels were joined to create the array, among other possibilities.

5.2.4 Summary

Figure 5.18, Figure 5.19, and Table 5.2 show excellent agreement between the simple, coarsely-meshed shell element models and the experimental photogrammetry data of the cantilevered array of joined rectangular panels. These data, combined with the results presented in Section 4.5 show that these types of models are valid at two different global scales. Once validated at single panel level, the methodology then dictates, and this section shows, that these same models can be used to predict the global, macro-scale behavior of an array of any geometry and size comprised of joined panels.
CHAPTER 6. DYNAMIC TESTING AND MODELING OF SINGLE PANEL

6.1 Introduction

The dynamic behavior of individual panels was studied through experimental modal analysis with impact hammer excitation. This testing revealed several structural natural frequencies and mode shapes of the rectangular and hexagon-shaped new stiff, ultra-lightweight polyimide panels shown in Figure 4.1. Several recent dynamic tests of other ultra-lightweight gossamer space structures upon which this work is partially based were discussed in Section 3.5. Of these tests, those that involve thermal-formed or otherwise stiffened membrane structures are detailed in References 146-153.

These unique next-generation stiff gossamer structures present unique testing challenges. Song et al. (2006)\textsuperscript{151} states: “In this project the unpressurized torus structure is far too flexible to be excited with any solid contact excitation: it simply crumpled under the force. Therefore, impact hammers and shakers simply will not work as methods of excitation. In order to overcome this drawback, acoustic excitation from a loudspeaker was used to excite the torus.” Based on the successful characterization of the dynamic behavior of the test article described in Song et al. (2006)\textsuperscript{151} and its similarities to the new stiff, thermal-formed polyimide panels investigated here, a nearly identical acoustic test setup was initially employed.

Despite the previous success of acoustic testing methods in characterizing the dynamic behavior of similar stiff, ultra-lightweight gossamer-class structures, they proved inadequate in the full panel testing described below. The much greater local stiffness of the current panels in comparison to the torus tested acoustically in Song et al. (2006)\textsuperscript{151} enabled more traditional impact hammer testing to be successful here. Resulting natural frequencies and mode shapes were used to validate the simple coarsely-meshed shell element models used to replicate the global, macro-scale static behavior of the panels in the previous chapter.
6.2 Excitation Alternatives

6.2.1 Acoustic Excitation

Acoustic excitation was investigated for characterizing the dynamic behavior of the stiff polyimide panels shown in Figure 4.1. Figure 6.1 shows the experimental setup, including an aluminum plate test article and laser displacement sensing. This type of setup is attractive for dynamic testing of gossamer-class structures because it is non-contact. The majority of gossamer structures to date are membrane-based, and the attachment of excitation or measurement devices sufficiently alters the stiffness and mass properties of the test articles and invalidates the test results. Therefore, the extraction of dynamic information without physically contacting the test article is desirable for this class of structures.

The 30.48-cm (12.00-in) diameter speaker in Figure 6.1 acoustically excites the test article suspended directly in front of it, and the response of that structure is measured with the laser displacement sensor (specifications listed in Appendix A). The laser displacement sensor consists of a point laser emitter and a CCD camera that images the laser spot on the test article and triangulates its displacement from a previously defined reference. In the high-precision mode used here, the sensor is capable of resolving displacements as small as 10 µm. Assuming the reflective properties of the test article allow for sufficient contrast, this sensor and the speaker comprise a totally non-contact measurement system ideal for analyzing ultra-lightweight, membrane-based gossamer structures.
signal generator and recording the voltage input into the speaker, and by the accelerometer attached to the center of the speaker directly measuring the acoustic output, shown in Figure 6.1. A simple structure, a 20.5 x 17.5 x 0.1 cm corrugated aluminum plate as seen in Figure 6.1, was used to compare these two excitation measurement methods and evaluate the efficacy of this test setup.

Figure 6.2 shows sample time histories from the acoustic test of the corrugated aluminum plate shown in Figure 6.1. The top graph is the white noise voltage signal the signal generator supplied to the speaker, the middle graph is acceleration of the center of speaker recorded by the attached accelerometer, and the bottom graph is the displacement of the aluminum plate recorded by the laser displacement sensor. Note in the response of the aluminum plate the low frequency pendulum motion of the plate appears as an underlying sine wave and the high frequency response to the acoustic excitation.
Figure 6.2 – Measured time history data of the acoustically excited corrugated aluminum plate

Figure 6.3 compares the FRF generated by relating the measured response of the aluminum plate and the white noise voltage supplied by the signal generator (SigGen) to the FRF generated by relating the same measured response of the plate and the measured response of the speaker (Speaker). Figure 6.3 demonstrates that two methods of computing the FRF show the same resonance frequencies (see Section 3.5), and that there is no spectral information contained in one that is not seen in the other. Therefore there seems to be little or no difference in the voltage signal entering and leaving the speaker, and either the white noise voltage from the signal generator or the data from the accelerometer could be used to compute the FRFs. The downward trend present in the accelerometer FRF that is not seen in the signal generator FRF is the result of the two different kinds of measured excitation time history data. The accelerometer measures acceleration which is scaled by a factor of $\omega^2$ in the frequency domain when compared to the generic voltage data of the signal generator (or force, etc).
Figure 6.3 – Comparison of frequency response functions generated using the two different excitation signals

Figure 6.3 also shows an inherent problem in using acoustic excitation setup in Figure 6.1, the limited spectral range of the method. The coherence plot on the bottom can be thought of as showing the amount of the measured response caused by the known excitation on a scale from 0 to 1, with 1 indicating all of the response was caused by the excitation. High quality dynamic data should show a coherence close to 1 except at resonance frequencies, when the coherence should dip indicating that the structural response at that frequency is caused by a fundamental mode of the structure and not solely the excitation.\textsuperscript{104-107} The bottom graph in Figure 6.3 shows good coherence close to 1 from approximately 8 Hz to approximately 60 Hz, with dips corresponding to peaks in the magnitude of the FRF and phase angle changes, indicating resonances of the corrugated aluminum plate. At 60 Hz the coherence starts to fall toward zero. As the coherence approaches 0 as frequency increases beyond 60 Hz, there is a corresponding increase in noise in the magnitude and phase of the FRFs. These two simultaneous occurrences indicate that little or no useful dynamic data can be extracted using this particular speaker below 8 Hz or above 60 Hz in which ranges the coherence trends towards or is close to 0.

In spite of the known spectral limitation of the acoustic setup, multiple attempts were made to measure individual panels by suspending them in an identical manner to the aluminum plate, directly in front of the speaker. These attempts revealed two additional limitations that led
to the use of an alternate excitation method. The first limitation was that response amplitudes of the panels above the 10 µm resolution threshold of the laser displacement sensor were very difficult to achieve. The second limitation was that ideal modal parameter estimation methods such as those used in Song et al. (2006) rely on single point excitation. In Song et al. (2006), the test article was an order of magnitude larger than either of the panels tested here, meaning that the speaker could be assumed to be exciting only a single point on the structure. Here, however, the 30.48 cm diameter speaker is slightly larger than the hexagon-shaped panel and approximately the same size as the rectangular panel, meaning that all of the points on the structure were excited simultaneously. While dynamic measurements of this type are possible, the modal analysis software used here relies on FRFs created from individual point excitation. That complication combined with the limited spectral range of the speaker and response amplitude barely above the noise floor of the sensor led to the decision to use an alternate excitation method.

6.2.2 Hammer Excitation

Due to the complications and limitations associated with acoustic excitation, an alternate excitation method was selected in the form of an impact hammer. A modally tuned impact or impulse hammer consists of a head, an extender, a calibrated force sensor, a seismic mass, and an impact tip, as shown in the schematic in Figure 6.4. The extender is used to balance the seismic mass, and the force sensor records the impact between the tip and the structure.
Impact or impulse excitation was not used in the tests described in Song et al. (2006) due to the extreme flexibility of the self-supporting torus. The panels tested here, however, exhibit much greater local stiffness than that torus, and do not crumple under moderate local impacts. They are also stiff enough to enable the attachment of small accelerometers to the panel surface without significantly altering their stiffness properties. Therefore, instead of the non-contact acoustic testing necessary for characterization of other gossamer-class structures, a more traditional modal test was able to be performed in which an impulse hammer excited the panels and their response was measured by attached accelerometers.

As the name implies, the excitation force supplied by the hammer approaches that of an ideal impulse, the amplitude level of which corresponds to the amount of energy supplied to the structure being tested. Impact excitation will excite structural responses over a wide frequency range. Because the velocity of the hand-held hammer impact is difficult to precisely control, mass can be added to or subtracted from the hammer head as shown in Figure 6.4 to give the desired amplitude of excitation. Additionally, the impact can involve different materials affecting the frequency range of the impact excitation. Three examples of impulses supplied by an impact hammer and their respective spectral contents are shown in Figure 6.5.
Figure 6.5 – Impulses supplied by soft, medium, and hard hammer tips and their frequency content

The three impulses shown in the top graph of Figure 6.5 were supplied by three different hammer tips, one soft, one medium, and one hard. The frequency content of the excitation applied by the hammer to the structure is dependant on the stiffness of the contacting surfaces. Since the surface of the structure being tested is usually a property of the structure itself and unalterable, the type of hammer tip is altered to yield the proper frequency range for the testing, shown in the bottom graph of Figure 6.5.

In the impact hammer results described in the next subsection, a hard, steel tipped hammer was used to excite the structure instead of a medium, rubber tip. The steel tip was selected because it provides the greatest frequency range (Figure 6.5) and does not have the slight point at the end of the rubber tip seen in Figure 6.4. The flat steel tip spreads the force of the impact out over the entire surface area of the tip instead of concentrating it at a point that
may have overloaded and damaged a local area of the thin Kapton membrane of which the panels are comprised.

An example of actual measured time history data using the steel tipped impact hammer shown in Figure 6.6. The top graph shows the excitation impulse supplied by the hammer, and the bottom graph shows the response of the structure measured by the attached accelerometer. The excitation impulse appears almost identical to the ideal one shown in Figure 6.5, and there is no evidence of multiple peaks that would indicate multiple hits. The structural response is well above the noise floor of the accelerometer, and exponentially decaying as expected in impact testing.

Figure 6.6 – Example of measured time history data from impact hammer test of rectangular panel

Figure 6.7 shows the FRF created by relating the known excitation to the known response in the top and bottom graphs of Figure 6.6, respectively. Data from twenty impacts at the same point were averaged together here. This FRF is obviously not as clean as those shown in Figure 6.3, a symptom of the panel’s nonlinear behavior (see reciprocity test below), but peaks or dips in the magnitude corresponding to phase shifts and dips in coherence are clearly visible. These indicate that structural natural frequencies can be identified from the data. Also of note is that the coherence is close to 1 for a much greater range of frequencies than the acoustic data presented in the previous subsection, from approximately 0 to 500 Hz (Figure 6.8). Impact
hammer testing should therefore be capable of identifying the structural resonance frequencies and mode shapes of the stiff, ultra-lightweight polyimide panels. These results are detailed in the next section.

Figure 6.7 – Example of frequency response function from impact hammer test of rectangular panel, accelerometer attached at point 2, impacting at point 15

Figure 6.8 – Example of frequency response function from impact hammer test of rectangular panel, zoomed 0 to 500 Hz, accelerometer attached at point 2, impacting at point 15
A reciprocity test was performed to examine the nonlinear behavior of the panel and the quality of the impact hammer measurements, and is shown in Figure 6.9. As will be discussed below, two separate data runs were performed with the accelerometer attached at point 2 in the first and point 19 in the second. One of the FRFs in Figure 6.9 was therefore calculated from the first “a” data run impacting at point 19, creating the condition in which point 2 was the known response and point 19 was the known excitation, and the second was calculated from the second “b” data run impacting at point 2, creating the reciprocal condition in which point 19 was the known response and point 2 was the known excitation. Reciprocity tests of linear structures will show nearly identical frequency response functions. Figure 6.9 reveals that the two reciprocal FRFs follow approximately the same path until around 250 Hz, at which point variation between the two appears. This indicates that the measured data is likely of high quality below 250 Hz but may be questionable above.

Figure 6.9 – Reciprocity test of impact hammer data

### 6.3 Rectangular Panel Testing

The test setup in Figure 6.10, operating as diagramed in Figure 6.11, was used to investigate the dynamic behavior of the rectangular stiff, ultra-lightweight polyimide panel shown in Figure 4.1. The panel was suspended horizontally by tensioning rubber bands attached
to the panel and 1.5 m long monofilament lines. The rubber bands were adhered to the panel with super glue without cutting or otherwise damaging the panel. This suspension method approximated a free-free boundary condition by limiting all rigid body suspension responses of the structure to a frequency range an order of magnitude less than the first structural natural frequency of the panel. In this case the “up-and-down” vertical suspension rigid body mode was counted visually to be approximately 2 Hz and the rocking mode approximately 3 Hz.

Figure 6.10 – Impulse dynamic test setup of single rectangular panel

Figure 6.11 – Functional block diagram of dynamic test setup of single rectangular panel
The hard, steel-tipped impact hammer shown in Figure 6.10 was used to excite the structure and the response was measured by an accelerometer attached to the underside of the panel with super glue, shown in Figure 6.12. The output cables from the hammer and accelerometer were both routed through a rack mounted signal conditioner and recorded by the data acquisition system. The specifications for the data acquisition system software, signal conditioner, accelerometer, and hammer are all listed in Appendix A. Settings for the acquisition software used to collect the data and calculate the FRFs are shown in Table 6.1 below.

![Figure 6.12 – Accelerometer attached to bottom of rectangular panel](image)

Table 6.1 – Data acquisition system settings

<table>
<thead>
<tr>
<th>Setting</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement Bandwidth</td>
<td>0-2000 Hz</td>
</tr>
<tr>
<td>Frame Size</td>
<td>2048</td>
</tr>
<tr>
<td>Number of Averages</td>
<td>20</td>
</tr>
<tr>
<td>Δ Frequency</td>
<td>2.5 Hz</td>
</tr>
<tr>
<td>Window</td>
<td>Exponential</td>
</tr>
</tbody>
</table>

Forty-eight FRFs were collected by striking the top of the panel with the steel-tipped impact hammer at the 24 excitation points (nodes) in Figure 6.13 and measuring the response of the structure with the accelerometer attached to the underside of the panel at the two measurement points. Only one accelerometer was attached to the panel at a time to avoid altering the mass properties of the panel, therefore two separate data runs were taken in which all 24 excitation points were struck, and the accelerometer was moved from one measurement point
to the other in between runs. Each of the 24 excitation points was struck 20 times, and the data was averaged to create a single FRF. This number of averages was selected by observing the change in the computed FRF as more averages were added, and no change was observed beyond 20 averages. One of the 48 computed FRFs is shown in Figure 6.7 and Figure 6.8.

Figure 6.13 – Measurement, excitation, and attachment points of impulse measurement of rectangular panel
All forty-eight FRFs over the entire 0 – 2000 Hz bandwidth were combined and loaded into the modal parameter analysis software X-Modal. A modal parameter estimation was then performed using the eigensystem realization algorithm (ERA), a time-domain, high matrix polynomial order algorithm that is numerically stable and recursively determines the order of the system based on Gram-Schmidt orthonormalization.\textsuperscript{114,115} The first step in the modal parameter estimation process is to sieve the data, limiting the range over which the frequencies are estimated. After the frequency range is selected, the range of the time history is also limited. The parameter limiting the order of the function fit to the data, corresponding to the number of modes to be solved for, called \textit{NrVirtual} in the software, is then selected resulting in equation condensation. All of these processes are described in much more detail in Chapter 3, along with the selection of the structural natural frequencies discussed below.

After all of the parameters were selected, X-Modal created consistency diagrams that were used to determine the resonant frequencies. The consistency diagram used to determine the first two natural frequencies of the rectangular panel is shown in Figure 6.14. Two FRFs are plotted here indicating that two response points were used in the data collection. They are, however, simply for reference purposes since the ERA modal parameter estimation curve-fits to all of the FRFs. Here the analyzed spectral content was sieved to between 40 and 130 Hz and the temporal content was set to include everything from the beginning of the measurement to just before the high frequency noise at the end. The identified resonances are therefore only calculated in the sieved frequency range and not the entire 2000 Hz bandwidth of the measurement. As the number of iterations increases, the model converges and the resonance frequencies are identified as indicated by solid symbols in the figure. These two frequencies represent the first and second structural natural frequencies of the rectangular panel, at 71.9 and 115.2 Hz respectively. Note that both of these frequencies occur above the useful acoustic excitation range.
After the frequencies were identified, a process was run in X-Modal to calculate the residue results, a sample of which is shown in Figure 6.15. Figure 6.15 shows the curve fit to the data over the sieved frequency range, and the error of the fit for this particular point. The deflection shapes corresponding to the selected frequencies at 71.9 and 115.2 Hz were then animated, single frames of which are shown in Figure 6.16 and Figure 6.17. The first mode shape in Figure 6.16 is an edge view of a twisting mode in which adjacent corners of the panel are moving in opposite directions. The second mode shape in Figure 6.17 shows a half sine wave along each edge of the panel, with opposite edges moving in the same direction in phase with each other, adjacent edges moving in opposite directions out of phase with each other, and little or no motion occurring in the center. These clean, clearly-recognizable shapes confirm that the two frequencies selected from the consistency diagram in Figure 6.14 are structural natural frequencies.
Following the successful extraction of the first two structural resonance frequencies and mode shapes, the data was re-processed to determine if higher modes could also be extracted. The spectral information was sieved to between 117.5 and 300 Hz, the temporal content was set to include everything from the beginning of the measurement to just before the high frequency noise at the end, Nr Virtual was set to 3, and the generated consistency diagram used to determine the third and fourth natural frequencies of the rectangular panel is shown in Figure 6.15 – Sample residue result, dotted line measured FRF, heavy solid line reconstructed FRF, light solid line error

Figure 6.16 – Experimentally identified first mode shape of rectangular panel at 71.9 Hz

Figure 6.17 – Experimentally identified second mode shape of rectangular panel at 115.2 Hz
6.18. Again, as the number of iterations increases, the model converges and the modal frequencies are identified as indicated by solid symbols in the figure. These two frequencies represent the third and fourth structural natural frequencies of the structure, and were found to occur at 197.0 and 244.6 Hz respectively.

![Consistency Diagram](image)

Figure 6.18 – Consistency diagram for rectangular panel testing using: ERA, Freq range 117.5 – 300 Hz, Condense: Method EIG, Nr Virtual 3

After the frequencies were identified, the process was run in X-Modal to calculate the residue results. The deflection shapes corresponding to the selected frequencies at 197.0 and 244.6 Hz were then animated, single frames of which are plotted in Figure 6.19 and Figure 6.20. The third mode shape in Figure 6.19 shows half sine waves along the short edges of the panel moving in opposite directions out of phase with each other and full sine waves along the long edges of the panel moving in the same direction in phase with each other. The fourth mode shape in Figure 6.20 is difficult to identify from the individual frame, but appears as a third order bending mode when animated. These clean, clearly-recognizable shapes confirm that the two frequencies selected from the consistency diagram in Figure 6.18 are structural natural frequencies.
Attempts at extracting additional structural resonance frequencies and mode shapes by sieving the data over other frequency ranges, including below 71 Hz and above 250 Hz, increasing the order of the function fit to the data by increasing Nr Virtual, or any other method proved unsuccessful. The four successfully extracted structural resonance frequencies are listed in Table 6.2 below. It should be noted that all four fall within the range of useful data predicted by the reciprocity test in Figure 6.9.

Table 6.2 – Experimentally measured structural natural frequencies of the rectangular panel

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Experimental Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71.9</td>
</tr>
<tr>
<td>2</td>
<td>115.2</td>
</tr>
<tr>
<td>3</td>
<td>197.0</td>
</tr>
<tr>
<td>4</td>
<td>244.6</td>
</tr>
</tbody>
</table>
6.4 Hexagonal Panel Testing

A test setup identical to that in the previous section was also used to investigate the dynamic behavior of the hexagon-shaped polyimide panel shown in Figure 4.1. The panel was again suspended horizontally by tensioning rubber bands attached to the panel and 1.5 m long monofilament lines. The rubber bands were adhered to the panel with super glue without cutting or otherwise damaging the panel. This suspension condition approximated a free-free boundary condition by limiting all rigid body suspension responses of the structure to a frequency range an order of magnitude less than the first structural natural frequency of the panel. In this case the “up-and-down” vertical suspension rigid body mode was counted visually to be approximately 2 Hz and the rocking mode approximately 3 Hz.

The same impact hammer shown in Figure 6.10 that was used to excite the structure in the previous section was used here, and the responses were measured by an accelerometer attached to the underside of the panel with super glue, shown in Figure 6.12. The output cables from the hammer and accelerometer were both routed through a rack mounted signal conditioner and recorded by the data acquisition system. The specifications for the data acquisition system software, signal conditioner, accelerometer, and hammer are all listed in Appendix A. Settings for the acquisition software used to collect the data and calculate the FRFs are shown in Table 6.1.

Twenty-four FRFs were collected by striking the top of the panel at 12 excitation points (nodes) and measuring the response of the structure with the accelerometer attached to the underside of the panel at two measurement points, as shown in Figure 6.21. Only one accelerometer was attached to the panel at a time to avoid altering the mass properties of the panel. Therefore two separate data runs were taken, and the accelerometer was moved from one measurement point to the other in between runs.
All twenty-four FRFs were combined and loaded into the modal parameter analysis software X-Modal. A modal parameter estimation was then performed using ERA, identical to that performed in the previous section. The consistency diagram used to determine the first natural frequency of the hexagon-shaped panel is shown in Figure 6.22. Two FRFs are plotted here indicating that two response points were used in the data collection. They are, however, simply for reference purposes since the ERA modal parameter estimation curve-fits to all of the
FRFs. Here the analyzed spectral content was sieved to between 30 and 265 Hz and the temporal content was set to include everything from the beginning of the measurement to just before the high frequency noise at the end. The identified resonances were therefore only calculated in the sieved frequency range and not the entire 2000 Hz bandwidth of the measurement. As the number of iterations increases, the model converges and the resonance frequency is identified as indicated by a solid symbol in the figure. This frequency represents the first structural natural frequency of the hexagon-shaped panel at 127.7 Hz.

Figure 6.22 – Consistency diagram for hexagon-shaped panel testing using: ERA, Freq range 30 - 265 Hz, Condense: Method EIG, Nr Virtual 3

The consistency diagram used to determine the second natural frequency of the hexagon-shaped panel is shown in Figure 6.23. Here the analyzed spectral content was sieved to between 30 and 422.5 Hz and the temporal content was set to include everything from the beginning of the measurement to just before the high frequency noise at the end. The identified resonances were therefore only calculated in the sieved frequency range and not the entire 2000 Hz bandwidth of the measurement. As the number of iterations increases, the model converges and the resonance frequency is identified as indicated by a solid symbol in the figure. This frequency represents the second structural natural frequency of the hexagon-shaped panel at 258.1 Hz.
After the frequencies were identified, the process was run in X-Modal to calculate the residue results. The deflection shapes corresponding to the selected frequencies at 127.7 and 258.1 Hz were then animated, single frames of which are plotted in Figure 6.24 and Figure 6.25. The first mode shape in Figure 6.24 shows the first twisting mode. The second mode shape in Figure 6.25 is difficult to identify from the individual frame, but appears as a first order billow or breathing mode when animated. These clean, clearly-recognizable shapes confirm that the two frequencies selected from the consistency diagrams in Figure 6.22 and Figure 6.23 are structural natural frequencies.

Figure 6.23 – Consistency diagram for hexagon-shaped panel testing using: ERA, Freq range 30 – 422.5 Hz, Condense: Method EIG, Nr Virtual 4

Figure 6.24 – Experimentally identified first mode shape of hexagon-shaped panel at 127.7 Hz
Attempts at extracting additional structural resonance frequencies and mode shapes by sieving the data over other frequency ranges, including below 30 Hz and above 423 Hz, increasing the order of the function fit to the data by increasing Nr Virtual, or any other method proved unsuccessful. The two successfully extracted structural resonance frequencies are listed in Table 6.3 below.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Experimental Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>127.7</td>
</tr>
<tr>
<td>2</td>
<td>258.1</td>
</tr>
</tbody>
</table>

6.5 Rectangular Panel Finite Element Modeling and Correlation

A simple, coarsely-meshed shell element finite element model identical to that detailed in Section 4.5 was used to replicate the experimental modal analysis data for the rectangular stiff, ultra-lightweight polyimide panel. No boundary conditions or constraints were applied to the model to reflect the free-free test setup and a modal analysis was run, the results of which are listed in Table 6.4. Note that the finite element solution returned the six rigid body modes that are excluded from Table 6.4, and that the table therefore starts numbering at mode seven. For comparison, the experimentally determined structural natural frequencies are also listed next to the finite element frequencies of identical mode shape.
Table 6.4 – Finite element and experimental structural natural frequencies of the rectangular panel

<table>
<thead>
<tr>
<th>ANSYS Mode Number</th>
<th>ANSYS Frequency (Hz)</th>
<th>Experimental Frequency (Hz)</th>
<th>ANSYS $0.75\rho$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>62.659</td>
<td>71.9</td>
<td>72.353</td>
</tr>
<tr>
<td>8</td>
<td>99.884</td>
<td>115.2</td>
<td>115.34</td>
</tr>
<tr>
<td>9</td>
<td>175.13</td>
<td>197.0</td>
<td>202.22</td>
</tr>
<tr>
<td>10</td>
<td>183.39</td>
<td>197.0</td>
<td>211.76</td>
</tr>
<tr>
<td>11</td>
<td>247.46</td>
<td>244.6</td>
<td>285.74</td>
</tr>
<tr>
<td>12</td>
<td>348.41</td>
<td></td>
<td>402.31</td>
</tr>
<tr>
<td>13</td>
<td>435.74</td>
<td></td>
<td>503.15</td>
</tr>
<tr>
<td>14</td>
<td>462.03</td>
<td></td>
<td>533.51</td>
</tr>
<tr>
<td>15</td>
<td>523.62</td>
<td></td>
<td>604.63</td>
</tr>
<tr>
<td>16</td>
<td>562.86</td>
<td></td>
<td>649.94</td>
</tr>
<tr>
<td>17</td>
<td>588.7</td>
<td></td>
<td>679.77</td>
</tr>
</tbody>
</table>

Table 6.4 shows that the finite element model under-predicts the first three experimentally determined structural natural frequencies by 13%, 13%, 11%, and 7%, respectively, and over-predicts the fourth by 1%. The third experimentally determined structural natural frequency is repeated; the ninth and tenth finite element modes correspond to the third experimentally determined mode with one orthogonal to the other.

To achieve better correlation between the finite element and experimental data, the density defined in the model was altered. A finite element model density of 75% of actual was found to produce the best results, listed in Table 6.4, over predicting the first two experimentally determined structural natural frequencies by just 0.7% and 0.09% respectively. This density change equals a change in mass of the panel from 38.3 to 28.7 g. Note that to avoid this mass change, modulus could be altered as an alternative, but density was chosen here because of the extensive effort to calculate and measure the modulus of the panels in Section 5.1.2 and Figure 5.8. Also note that the rectangular panel has a large amount of excess polyimide film (Figure 4.1, Figure 6.10). The density used in the original finite element model was calculated by dividing the measured 38.3 g mass of the panel by the dimensions of tessellating honeycomb structure, not the total dimensions of the film. Only the dimensions of the honeycomb structure
was modeled. Therefore the original model density overestimates the actual density of the modeled portion of the panel. This overestimation could be corrected with geometry changes to the model, addition of the excess material to the model, or removal of the excess material from the panel. In addition, other model refinements and modeling techniques could be investigated such as more detailed modeling of the boundary conditions and analysis of the internal stress fields, to generate models that do not require material property alterations. The density alteration used here demonstrates that the models can accurately replicate the experimental data. Other refinements are left to future work.

Direct comparisons of the first three simulated and experimentally determined mode shapes are shown in Figure 6.26, Figure 6.27, and Figure 6.28. Note that altering the density to achieve better agreement with the experimental data did not affect the shapes shown below. The comparison of first mode shapes in Figure 6.26 show edge views of identical twisting modes in which adjacent corners are moving in opposite directions. The comparison of second mode shapes in Figure 6.27 shows identical half sine waves along each edge of the panels, with opposite edges moving in the same direction in phase with each other, adjacent edges moving in opposite directions out of phase with each other, and little or no motion occurring in the center. The comparison of third mode shapes in Figure 6.28 shows identical half sine waves along the short edges of the panel moving in opposite directions out of phase with each other and full sine waves along the long edges of the panel moving in the same direction in phase with each other.

![Figure 6.26 – Finite element (top) and experimentally determined (bottom) first mode shapes](image)
The comparison of the first three finite element and experimentally determined mode shapes in Figure 6.26, Figure 6.27, and Figure 6.28 show them to be quite similar. Therefore the use of simple coarse-meshed shell element models to approximate the dynamic behavior of rectangular stiff, thermal-formed polyimide panels is valid. To demonstrate that the shell element global models are effective at any size ranging from a single panel to the size of the full structure, an array of attached panels is tested and modeled in the next chapter.
6.6 Hexagonal-Shaped Panel Finite Element Modeling and Correlation

A simple, coarsely-meshed shell element finite element model 14.5 cm on a side was used to replicate the experimental modal analysis data for the hexagon-shaped stiff, ultra-lightweight polyimide panel. No boundary conditions or constraints were applied to the model to reflect the free-free test setup; and a modal analysis was run, the results of which are listed in Table 6.5. Note that the finite element solution returned the six rigid body modes that are excluded from Table 6.5, and the table therefore starts numbering at mode seven. For comparison, the experimentally determined structural natural frequencies are also listed next to the finite element frequencies of identical mode shape.

Table 6.5 – Finite element and experimental structural natural frequencies of the hexagon-shaped panel

<table>
<thead>
<tr>
<th>ANSYS Mode Number</th>
<th>ANSYS Frequency (Hz)</th>
<th>Experimental Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>120.12</td>
<td>127.7</td>
</tr>
<tr>
<td>8</td>
<td>120.24</td>
<td>127.7</td>
</tr>
<tr>
<td>9</td>
<td>247.18</td>
<td>258.1</td>
</tr>
<tr>
<td>10</td>
<td>321.27</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>487.98</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>489.21</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.5 shows that the finite element model under predicts the first two experimentally determined structural natural frequencies by 6%, 6%, and 4%, respectively. The first experimentally determined structural natural frequency is repeated; the seventh and eighth finite element modes correspond to the first experimentally determined mode with one orthogonal to the other.

Direct comparisons of the first two simulated and experimentally determined mode shapes are shown in Figure 6.29 and Figure 6.30. The comparison of first mode shapes in Figure 6.29 shows views of identical twisting modes in which adjacent corners are moving in opposite directions. The comparison of second mode shapes in Figure 6.30 shows entirely different
shapes. In fact, the experimentally determined breathing shape does not match any finite element shape at any frequency.

Figure 6.29 – Finite element (top) and experimentally determined (bottom) first mode shapes

Figure 6.30 – Finite element (top) and experimentally determined (bottom) second mode shapes
The failure to match experimentally determined mode shapes to finite element mode shapes of the hexagon-shaped panel beyond the first mode, and the lack of an array of attached hexagonal-shaped panels to test led to the decision to focus the remainder of the research program on the rectangular panel and array of attached rectangular panels. Additional research is required to fully understand the dynamic behavior of the hexagon-shaped panel and to generate effective models. The residual stresses in the panel may be very different from the rectangular panel, effective material properties may be different, or boundary conditions may have to be altered in the model. These and all other research avenues are reserved for future work.

Copyright © Jonathan T. Black 2006
CHAPTER 7. DYNAMIC TESTING AND MODELING OF PANEL ARRAY

7.1 Introduction

Investigation of the dynamic behavior of the array of joined rectangular panels shown in Figure 4.5 was undertaken in the form of experimental modal analysis of a cantilevered configuration. The array of three joined rectangular panels was cantilevered vertically as shown in Figure 7.1, and impact excitation was used to dynamically excite its first four vibrational modes. The motion of the array following impact was recorded by the two synchronized monochromatic digital video cameras at rates as high as 75 frames per second, also shown in Figure 7.1. The same small, circular, high-contrast retro-reflective targets used in Section 4.3 were attached to the array and table leg, and appear as bright white circles in Figure 7.1. The targets were illuminated by flood lights placed directly behind the video cameras recording their motion so the incident light was reflected directly back into the cameras. Actual images used in the processing are seen in Figure 7.2. Here, the retro-reflective targets appear as bright white circles on a dark, underexposed background created by the small aperture. The test setup operating as diagramed in Figure 7.3.
Figure 7.2 – Actual images used in videogrammetry processing

Figure 7.3 – Functional block diagram of dynamic test setup of array

The synchronized image sequences were then loaded into the Photomodeler photogrammetry software package and associated with the appropriate camera calibration parameters that allowed for the removal of image distortions cause by lens curvature and camera
aberrations and imperfections (see Section 3.3). The targets were marked to sub-pixel accuracy (approximately 1/10 pixel) using an automatic least-squares matching algorithm\textsuperscript{42,137,138,171,172} and referenced across the images to the marks corresponding to the same targets. A triangulation algorithm was run that simultaneously and iteratively solved for the three-dimensional (3D) locations of the targets and camera positions. The final product of the photogrammetry process was a set of 3D points corresponding to the locations of the imaged retro-reflective targets. This process was then applied to all of the synchronized image frames from the two video cameras to yield time histories of all the 3D points.\textsuperscript{42,68,80,87,90,91,139,143,137,172,174} The points on the table leg, visible in Figure 7.1 and Figure 7.2 were used to scale and rotate the data. Specifications of the video system used to capture the data are listed in Appendix A.

### 7.2 Array Testing

Four different points were impacted to excite the first and second bending and first and second twisting modes of the cantilevered array. For each desired mode, the array was impacted at the point of maximum deflection as follows:\textsuperscript{67} at the free end (point “A” in Figure 7.4) and approximately half way between the clamped and free ends (point “B” in Figure 7.4), both in the middle of the array for the first and second bending modes, respectively, and at the free end (point “A” in Figure 7.4) and approximately half way between the clamped and free ends (point “B” in Figure 7.4), both at the edge of the array for the first and second twisting modes, respectively.
The time history data from the videogrammetry processing was examined for spectral content. Figure 7.5 shows the time history for the impact point in the first bending mode shape test. The top graph of the figure shows the full time history recorded at 37.5 frames per second in which the impact did not occur for over a second after video recording was initiated, and the reduced time history in which the information from before the impact is eliminated to enable the Fourier transform. The point exhibits a decaying harmonic oscillation, as expected from cantilevered impact testing. The power spectral density of the reduced time history (obtained via Fourier transform), shown in the bottom graph of Figure 7.5, shows the frequency content of the signal. As expected, there is a peak at approximately 3.2 Hz, corresponding to the approximately 0.33 second period apparent in the temporal data. Note that in this study of the dynamic behavior of the array, only the spectral content of the response data is examined. The excitation is not incorporated as in the previous chapter in which frequency response functions were examined.
To determine which shape this frequency corresponds to, a surface was fit to all of the points on the array in each synchronized measurement frame, called an epoch, and animated over all of the epochs (see accompanying file AryMode1.avi). The animation clearly shows the first bending mode shape, one frame of which is shown in Figure 7.6. Note that to enhance visualization, the data have been rotated to appear horizontal, and are not presented in their true vertical orientation.
To locate the remaining three desired modes, the spectral information from sample points on the array, such as those shown in Figure 7.5 and Figure 7.7, were examined and the peaks corresponding to different modes were selected. A temporal filter was then applied to the response of all of the points on the array to eliminate spectral content outside of the desired frequency range. These data sets were recorded at 75 frames per second. In Figure 7.8 only motion from 12 to 23 Hz was animated using a band pass 8th order Butterworth temporal filter of the data set in which the exact center of the array (maximum deflection location) was impacted to show the second bending mode shape (see file *AryMode3.avi*). In Figure 7.9 only motion from 11 to 16 Hz was animated using a band pass 8th order Butterworth temporal filter of the data set in which the edge of the array was impacted at the free end to show the first twisting mode shape (see file *AryMode2.avi*). These two frequency ranges were selected to capture as much motion over as wide a spectrum as possible to ensure the desired shape was clearly visible. In Figure 7.10 only motion above 30 Hz was animated using a high pass 8th order Butterworth temporal filter of the data set in which the edge of the array was impacted half way between the free and clamped ends to show the second twisting mode shape (see file *AryMode4.avi*). Because the maximum frame rate of the video cameras was 75 frames per second, spectral information was only available below Nyquist of 37.5 Hz and therefore the exact range of the peak corresponding to the second twisting mode could not be determined. It is still possible, however, to see from the animated shape that second twisting motion is present, indicating that the mode occurs between 37.5 and 75 Hz.
Figure 7.7 – Sample time history and power spectral density from first twisting mode test

Figure 7.8 – Experimentally determined second bending mode shape, band pass filter 12 to 23 Hz
Figure 7.9 – Experimentally determined first twisting mode shape, band pass filter 11 to 16 Hz
The slight divot visible at approximately 1/4 of the panel length from the fixed end in Figure 7.9 and Figure 7.10 and most pronounced in Figure 7.8 results from the glare of the flood lights on the surface of the array seen in the top portion of the array in the left image in Figure 7.2. This glare resulted in inaccuracies in target marking throughout the videogrammetry data processing.

The deflection shapes shown in Figure 7.6, Figure 7.8, Figure 7.9, and Figure 7.10 show that the peaks in the spectral data shown in Figure 7.5 and Figure 7.7 are structural natural frequencies of the array of joined rectangular panels. These frequencies are summarized in Table 7.1.
Table 7.1 – Experimentally determined structural natural frequencies of array

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Experimental Frequency (Hz)</th>
<th>Video System Frame Rate (fps)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.2</td>
<td>37.5</td>
<td>1st bending</td>
</tr>
<tr>
<td>2</td>
<td>13.5</td>
<td>75.0</td>
<td>1st torsion</td>
</tr>
<tr>
<td>3</td>
<td>19.0</td>
<td>75.0</td>
<td>2nd bending</td>
</tr>
<tr>
<td>4</td>
<td>&gt;37.5</td>
<td>75.0</td>
<td>2nd torsion</td>
</tr>
</tbody>
</table>

The temporal data from the impact testing of the array of joined panels can also be used to examine damping. Figure 7.11 shows the time history of the peak shown in Figure 7.5 low-pass filtered to remove all frequency content above 8 Hz. The remaining decaying harmonic oscillation and its corresponding power spectral density show only the first mode behavior at approximately 3.2 Hz. In a linearly damped system, the rate at which the amplitude of the oscillating signal decays is an exponential function of the damping coefficient of the structure as follows.\(^{104,105}\)

\[
y(t) = X \exp(\zeta \omega_n t)
\]  

(7.1)

The damping coefficient \(\zeta\) can be estimated by the logarithmic decrement method as follows.\(^{104,105}\)

\[
\delta = \frac{1}{n} \ln \left( \frac{x_0}{x_n} \right) = \frac{2\pi\zeta}{\sqrt{1 - \zeta^2}}
\]  

(7.2)

where \(x_0\) and \(x_n\) are the amplitudes of the initial and \(n\)th peaks of the harmonic signal. Therefore, from any given experimental decaying signal, such as the one in Figure 7.11, one can estimate the damping of the system. Figure 7.12 shows the results of the logarithmic decrement method applied to the low-pass filtered time history shown in Figure 7.11. As often happens, damping estimates only yield a range of values of \(\zeta\), here from 6.5 to 11%. However, it can also be seen that fitting a linear (first order) exponential curve to the peaks in the data also does not satisfactorily fit the decaying signal.
Figure 7.11 – Time history from Figure 7.5 with low pass filter at 8 Hz and power spectral density

Figure 7.12 – Filtered time history with linear best fit damping and logarithmic decrement estimates
In Figure 7.13, several exponential curves were fit to the peaks in the data, each of a different order. Again, the first order linear curve does not perform satisfactorily, but the second order nonlinear exponential is a good match to the decaying harmonic. This result indicates that the array of three joined rectangular panels exhibits complex nonlinear damping and may explain why the logarithmic decrement method was unsuccessful at providing a single damping estimate.

![Filtered time history with first and second order best fit damping estimates](image)

Figure 7.13 – Filtered time history with first and second order best fit damping estimates

7.3 Array Finite Element Modeling and Correlation

A simple shell element finite element model nearly identical to that detailed in Section 4.6 was used to cantilevered panel array. The same 78.74 x 23.49 cm (31 x 9 in) rectangle was meshed using the same 1.905-cm (0.75-in) thick shell elements. Here, however, the upper 23.49 x 3.175 cm (9 x 1.25 in) instead of the upper 23.49 x 5.715 cm portion of the array was constrained to be fixed, matching the experimental boundary conditions. No load was applied to the model (see Appendix C for batch file). A modal analysis was run, and the results are listed in Table 7.2. For comparison, the experimentally determined natural frequencies are also listed.
Table 7.2 – Finite element and experimental structural natural frequencies of the array

<table>
<thead>
<tr>
<th>ANSYS Mode Number</th>
<th>ANSYS Frequency (Hz)</th>
<th>Experimental Frequency (Hz)</th>
<th>ANSYS Frequency 1.10ρ (Hz)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.389</td>
<td>3.2</td>
<td>3.231</td>
<td>1st bending</td>
</tr>
<tr>
<td>2</td>
<td>14.813</td>
<td>13.5</td>
<td>14.123</td>
<td>1st twisting</td>
</tr>
<tr>
<td>3</td>
<td>21.559</td>
<td>19.0</td>
<td>20.556</td>
<td>2nd bending</td>
</tr>
<tr>
<td>4</td>
<td>30.033</td>
<td>n/a</td>
<td>28.635</td>
<td>1st in plane</td>
</tr>
<tr>
<td>5</td>
<td>48.096</td>
<td>&gt;37.5</td>
<td>45.858</td>
<td>2nd twisting</td>
</tr>
</tbody>
</table>

Table 7.2 shows that the finite element model over-predicts the first three experimentally determined structural natural frequencies by 6%, 10%, and 14%, respectively. The fourth finite element structural natural frequency is an in-plane mode not measured by the experimental data.

To achieve better correlation between the finite element and experimental data, the density defined in the model was altered. This procedure is considered valid because, as in the experimental setup in Section 4.6, to minimize crushing of the stiffeners the C-clamps used were not fully tightened. This possible clamp looseness, the micro-scale nonlinear stiffness behavior of the panel, and experimental uncertainty make it impossible that the clamp was ideal in the experimental data set. It is therefore likely that some motion at the base of the array did occur. To correct for this inconsistency between experimental setup and finite element model, the density used in the defined material properties was increased until a suitable match with the experimental data was achieved. 110% of actual density was found to produce excellent agreement with the measured structural natural frequencies, also listed in Table 7.2, over predicting the first three experimentally determined frequencies by just 1%, 6%, and 8% respectively. This density change equals a change in mass of the array from 81.4 to 89.5 g. In addition, other model refinements and modeling techniques could be investigated such as more detailed modeling of the boundary conditions and analysis of the internal stress fields, to generate models that do not require material property alterations. The density alteration used here demonstrates that the models can accurately replicate the experimental data. Other refinements are left to future work.

Direct comparisons of the first three simulated and experimentally determined mode shapes are shown in Figure 7.14 through Figure 7.17. Note that altering the density to achieve
better agreement with the experimental data did not affect the shapes shown below, and that to enhance visualization the data have been rotated to appear horizontal rather than the actual vertical orientation. Also note that the finite element shapes shown below include the portion of the array underneath the clamp, while the experimental shapes do not. The comparison of first mode shapes in Figure 7.14 show comparable first beam bending modes. The comparison of second mode shapes in Figure 7.15 show comparable first beam twisting modes. The comparison of third mode shapes in Figure 7.16 shows comparable second beam bending modes. The comparison of fourth mode shapes in Figure 7.17 shows an ideal second twisting mode from the finite element simulation and what appears to be a second twisting mode from the experimental data. Because the mode occurred above Nyquist, a specific range of band pass filtering could not be used on the experimental data, therefore more high-frequency noise is present in the displayed shape than in the other experimental shapes, making it difficult to clearly match the experimental and finite element shapes.

Figure 7.14 – Finite element (left) and experimentally determined (right) first mode shapes
The comparison of the three finite element and experimentally-determined mode shapes in Figure 7.14 through Figure 7.16 show them to be well correlated. Therefore the use of simple
coarse-meshed shell element models to approximate the dynamic behavior of an array of joined rectangular panels is valid.

7.4 Summary and Future Work

The excellent agreement between simple, coarsely-meshed shell element models and the static and dynamic data of the rectangular panel and array show that the models are valid at two different global scales. Therefore, once validated at single panel level, the methodology then dictates that these same models can be used to predict the global, macro-scale behavior of an array of any geometry and size comprised of joined panels.

The array statically and dynamically tested in the last two chapters is comprised of three rectangular panels joined together in such a way that the thickness of the Kapton polyimide film at the joint is 127 µm, identical to the film thickness throughout the rest of the array. Panel attachment procedures, however, can be altered to produce joints that are both thinner and thicker than the polyimide film throughout the rest of the panel. In either case, the array would not behave as simply a larger version of the individual panels comprising it, as in the case here. The lesser or greater mass and stiffness at the joints would mean that models would have to be created to reflect the unique mechanical behavior of the array joints. This would require that in addition to validating the model of an individual panel, two models of different-sized arrays of joined panels would also have to be validated to fulfill similitude scaling requirements. Once the array model is proven valid on two different scales, then the combination of material properties of the individual panels and the properties of the joints can be used to predict the global, macro-scale behavior of an array of any geometry and size. This represents an additional step than was used here, where validating the model at two separate scales only required testing an individual panel and a single array because the material properties of the array are uniform throughout.

Copyright © Jonathan T. Black 2006
CHAPTER 8. SUMMARY AND FUTURE WORK

8.1 Detailed Summary

The success of the Hubble Space Telescope has created a strong base of public support in the United States for multi-billion dollar orbital observatories. This enthusiasm has been an asset to NASA in its efforts to launch as many as a dozen additional space telescopes, but it is also accompanied by the demand to increase performance. This demand has led to the utilization of new materials and packaging techniques to enable considerably larger and therefore more powerful telescopes to be feasible, such as the planned James Webb Space Telescope. To launch missions including apertures potentially 30 meters in diameter, however, an entirely new class of space hardware will have to be developed.

One of the candidate technologies that may enable telescopes tens of meters in diameter to be feasible is ultra-lightweight and inflatable gossamer structures. Originally investigated by the Goodyear Company in the 1950s, gossamer structures have seen steady development over the last fifty years. Traditionally ultra-thin membrane and inflatable rigidizable boom based, the extreme flexibility and low mass characteristics of gossamer structures allow them to achieve very high packaging ratios and low launch masses. These characteristics may enable the launch of membrane apertures tens of meters in diameter, solar sails and sun shades hundreds of meters on a side, and solar arrays measured in square kilometers. Chapter 2 presents a history of these structures and an overview of current gossamer research and development programs.

Chapter 3 begins by analytically demonstrating that the main assets of gossamer structures are also the source of their greatest barriers to implementation. The low mass and high flexibility properties that enable the high packaging ratios and large deployed sizes cause significant behavioral challenges that have required the development of unique approaches to testing and modeling that are extensively reviewed in Chapter 3. As an alternative to developing new space qualification procedures, a new class of stiff gossamer structures has been developed that retain the low mass and high packaging ratio characteristics of traditional gossamer structures while significantly reducing their hindering flexibility. The static and dynamic characterization of a member of the new class, stiff, ultra-lightweight, thermal-formed polyimide panels, is the focus of Chapters 4 through 7.
Small-scale testing to analyze the local static behavior of the panels and determine the fundamental structure governing that behavior detailed in Chapter 4 was initially undertaken in the form of individual stiffener testing. These tests revealed nonlinear stiffness behavior, including a region of negative stiffness. Fully geometrically accurate shell element models and reduced finite element models that included nonlinear spring elements were incapable of accurately replicating the experimental data and too large to be practical. As an alternative to the individual stiffener, the node created by the junction of the stiffener sidewalls was investigated as the fundamental structure governing the static behavior of the panel.

The same small-scale static testing procedure used to examine the stiffener was also used in the investigation of individual nodes. The same type of nonlinear stiffness behavior that was seen in the stiffener testing was also seen in individual node and multi-node substructure testing. This behavior was modeled using single nonlinear spring elements with stiffnesses defined to match that measured in the experimental testing. Several variations on this model were investigated in Chapter 4, and the fundamental structure governing the static behavior of the panel on the local, micro scale was validated to consist of the nodes being loaded attached to each other and one level of surrounding unloaded nodes.

The large-scale static behavior of a single panel and array of attached panels was investigated in Chapter 5. Static panel bending tests were performed in which it was evident that the nonlinear behavior present in the small-scale testing was not present at the global, full panel level. The linear full-panel behavior enabled the use of coarsely-meshed modified shell element models to capture the large-scale static panel behavior. The modification involved altering the modulus of the shell elements to reflect the unique honeycomb internal structure of the panels and thermal-formed Kapton polyimide film from which the panels are manufactured. The modified shell element model was also effective at replicating the cantilevered array bending data.

Noncontact excitation and measurement techniques used successfully in the testing of past gossamer structures were investigated in Chapter 6 for use in the dynamic characterization of the panels. It was determined, however, that more standard impact hammer excitation and accelerometer measurement was more successful at extracting structural natural frequencies and mode shapes of the panels than the other techniques. This setup was used to characterize the
dynamic behavior of two different geometries of individual panels, and the same coarsely-meshed shell element models used in the static modeling accurately predicted that behavior.

The dynamic behavior of the array of attached panels was investigated in Chapter 7 through cantilevered impact testing. The response of the panel was recorded and processed using videogrammetry, yielding the 3D time history behavior of the panel. Filtering was applied to the data to remove all motion outside of specific frequency ranges, and in this manner the deflection shapes of the array and corresponding resonant frequencies were extracted. Again, the coarsely-meshed shell element model used in the static modeling accurately predicted the measured dynamic behavior.

The testing and modeling procedures used to statically and dynamically characterize the stiff thermal-formed polyimide panels were developed and validated specifically for the new class of stiff gossamer structures. This methodology states that the behavior of these large and lightweight space structures is of interest at two very different scales. Understanding the fine mechanics at small, local, micro scale and the overall behavior at the large, global, macro scale is required to use these structures effectively in space, but the same model does not have to be used for both scales. The new methodology therefore focuses on each scale separately, testing and modeling each. In this manner simple but accurate tests and models can fully capture the behavior of the panels with much less effort than previous research programs.

### 8.2 Contributions

Specific contributions of this work to the research community are:

1) A database of the nonlinear small-scale static behavior of the panels.

2) Verification that, over the range tested, the small-scale nonlinear static behavior is not present in large-scale, global panel and array testing.

3) Simple physics-based hybrid models are capable of accurately replicating the small-scale nonlinear static behavior of the panels.

4) Simple shell element models modified to reflect the unique internal structure of the panels are capable of accurately replicating the large-scale static and dynamic behavior of the panels.
5) Standard impact hammer and accelerometer testing is capable of extracting structural natural frequencies and mode shapes of the panels.

6) Filtered video data is capable of extracting structural natural frequencies and mode shapes of the panels.

7) Validated testing and modeling methodology for new stiff gossamer structures

8.3 Future Work

Parameter studies are required to understand the effect of varying manufacturing parameters on performance metrics. Manufacturing parameters such as membrane thickness, stiffener spacing, and panel thickness could be varied to determine the effect of each on performance metrics such as panel stiffness, panel mass, and fundamental frequency. In this manner a design tool could be generated to optimally manufacture the panels based on performance criteria for each specific application.

Model refinement is required to address the discrepancy between static and dynamic coarsely-meshed shell element models and the corresponding experimental data. In all cases here, the density was altered to demonstrate that the models were capable of replicating the experimental data, however, this technique is unsatisfactorily arbitrary. Ideal models would not require alteration to match the experimental data. Therefore studies should be undertaken in which boundary conditions are more accurately modeled, internal stress fields are analyzed, model geometry is altered, gravity is accounted for, etc., until all of the finite element models either accurately replicate experimental data or require minimal and predictable alterations of material properties.

Final justification of the proposed methodology in which the new stiff, ultra-lightweight polyimide panels are tested and modeled on two separate scales – one local, micro and the other global, macro – could be realized in the form of overlapping data sets. Local and global models that both accurately replicate the same experimental results should be identified.

The use of the local, micro-scale model to determine the boundary conditions of the global, macro-scale model should be investigated.
Skins or other coatings could be applied to the panels to characterize their effect on panel stiffness. Additionally, optical quality membranes could be attached to the panel surface and attachment and panel manufacturing techniques could be investigated to determine the usefulness of these panels in telescope applications. Actuators, wires, coolant, and other multi-use equipment could be attached to or embedded in the panels and the total system functionality could be investigated.

The type of panel attachment could also be investigated. The array tested here was comprised of three rectangular panels seamlessly attached such that there is no difference in the material properties between the joints and the rest of the array. The method of attachment, however, can be altered to produce joints either more or less stiff than the rest of the array. Additionally, panels could be attached with rivets or bolts. These varied attachment methods should be quantitatively studied for their effect on the final behavior and performance of the array, array stowage, etc.

Other methods of modeling the behavior of the panels, such as using a network of hollow beams, could be investigated.

The constant thickness scaling laws derived in Chapter 3 should be tested using two or more panels or arrays whose length and width dimensions are scaled by the same factor from one panel to the next, and whose thicknesses are equal. Here the only available test articles similar test articles were the rectangular panel and array of attached panels. While the array is the same thickness as the rectangular panel, only the length dimension is scaled properly. With properly-scaled test articles, the validity of the scaling laws could be investigated.

Finally, studies could be undertaken to determine how to accurately predict the static and dynamic behavior of full-scale structures in space.
APPENDICIES

Appendix A: Equipment Specifications

Olympus E-20n Camedia SLR Digital Camera

Color CCD imager: 2560 x 1920 pixels = 4915200 total pixels, 8.704 x 6.582 mm
9 – 36 mm zoom lens
f2.0 – f11
Auto and manual focus
Dynamic Range: 8 bit color

Keyence LK-503 CCD Laser Displacement Sensor

Response time: 1.0 ms (8.0 ms for 8 averages), using 8 averages

<table>
<thead>
<tr>
<th></th>
<th>Long Range Mode</th>
<th>High Precision Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Output</td>
<td>±5V</td>
<td>±10V</td>
</tr>
<tr>
<td>Measurement Range</td>
<td>250-750 mm from sensor</td>
<td>250-450 mm from sensor</td>
</tr>
<tr>
<td>Output Resolution</td>
<td>1 mV</td>
<td>1 mV</td>
</tr>
<tr>
<td>Resolution</td>
<td>50 mm/V = 50 µm/mV</td>
<td>10 mm/V = 10 µm/mV</td>
</tr>
</tbody>
</table>

OHaus Explorer Pro EP4102C Balance

Manual leveling
Auto calibrating
Capacity: 4100g
Resolution: 0.0001N, 0.01g

Zonic Medallion Dynamic Data Acquisition System

Fundamental Acquisition Software 4.0

I/O Tech Wavebook 512

WVBK20A computer interface

PCB Piezotronics Shear Accelerometer
Model Number: A353A16
Voltage Sensitivity: 11.68 mV/g
Frequency Range: 1-10000 Hz
Output Bias Level: 11.2 V

PCB Piezotronics Modally Tuned Impulse Hammer
Model Number: 086C03

PCB Piezotronics Multichannel Signal Conditioner
Model Number: 481A
Number of Channels: 16

Pulnix TM-6710CL Video Cameras
Resolution: 650x480 pixels, 1 Megapixel
Imager: 8 bit Monochromatic CCD
Max frame rate: 120 fps
Progressive line scan camera

IO Industries Video Savant 4.0 frame grabber software
IO Industries DVR Express high speed video capture board
Appendix B: Number of Nodes in Full-Sized Models Estimation

Total length = \((N + (N+1)/8)\) inches where \(N\) is the number of stiffeners.

So, a 1 x 1 m panel = 35 stiffeners on a side (1 m = 39.3701 in, \(35 + 36/8 = 39.5\) in), 35 x 35 = 1225 stiffeners.

Inserting into fit equation above, \(y = -0.02345 \times 1225^2 + 3.208 \times 1225 + 3.435 = 31,256\) nodes – plus one level of surrounding, unloaded nodes.

10 x 10 m panel = 350 stiffeners on a side, 350 x 350 = 122,500 stiffeners.

Inserting into fit equation above, \(y = -0.02345 \times 122500^2 + 3.208 \times 122500 + 3.435 = -351,503,579\) nodes – plus one level of surrounding, unloaded nodes.

NOTE: must multiply number of nodes calculated here by 2 for number of nodes in fe model!
Appendix C: ANSYS Batch Files

See file: BlackApC.pdf
Appendix D: Matlab .m Files

See file: BlackApD.pdf
REFERENCES


Murphy, D., et al., “Progress and Plans for a System Level Demonstration of a Scaleable
Square Solar Sail”, 39th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit,

an Effective Solar Sail Design to TRL 6”, 39th AIAA/ASME/SAE/ASEE Joint Propulsion

Lichodziejewski, D., Derbès, B., Reinert, R., Belvin, K., Slade, K., and T. Mann,

Lichodziejewski, D., Derbès, B., Slade, K., and T. Mann, “Vacuum Deployment and
Testing of a 4-Quadrant Scalable Inflatable Rigidizable Solar Sail System,” 46th
AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference,

Lichodziejewski, D., Derbes, B., Friese, D., and D. Galeana, “Vacuum Deployment of a 4-
Quadrant Scalable Inflatable Rigidizable Solar Sail System”, 40th AIAA/ASME/SAE/ASEE

Lichodziejewski, D., Derbès, B., Sleight, D., and T. Mann “Vacuum Deployment and

Mann, T., Behun, V., Lichodziejewski, D., Derbes, B., and D. Sleight, “Ground Testing a
20-Meter Inflation Deployed Solar Sail,” 47th AIAA/ASME/ASCE/AHS/ASC Structures,

Jenkins, C., Gough, A., Pappa, R.S., Carroll, J., Blandino, J.R., and J. Miles, “Design
Considerations for an Integrated Solar Sail Diagnostics System,” 45th
AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference,


200


111Experimental Modal Analysis, Lecture Notes, Parts I and II, University of Cincinnati Structural Dynamics Research Laboratory, June 2005.


154 Bennighof, J.K., Kaplan, M.F., and M.B. Muller, “Extending the Frequency Response Capabilities of Automated Multi-Level Substructuring,” 41st AIAA/ASME/ASCE/AHS


165 ANSYS Incorporated, Version 8.1 Documentation, Canonsburg, PA 15317.


208


VITA

JONATHAN T. BLACK
PhD Candidate
Department of Mechanical Engineering
University of Kentucky
J.Black@uky.edu

Education

George Washington University   Washington, DC
08/2001 – 07/2003, MS in Mechanical and Aerospace Engineering
Thesis: Photogrammetry and Videogrammetry Methods Development for Solar Sail Structures
Keywords: optical measurement methods, ultra-lightweight structures, structural testing
Advisor: Mr. Richard Pappa
Support: Joint Institute for Advancement of Flight Sciences Graduate Research Scholar Assistantship

University of Illinois at Urbana-Champaign   Urbana, IL
08/1997 – 05/2001, BS in Industrial Engineering with Honors, International Minor in French Studies

Experience

University of Kentucky   Lexington, KY
07/2006 – present, University of Kentucky Dissertation Year Fellow, Dynamic Structures and Controls Laboratory, Department of Mechanical Engineering
07/2004 – 06/2006, NASA Graduate Student Researchers Program Fellow, Dynamic Structures and Controls Laboratory, Department of Mechanical Engineering
08/2003 – 06/2004, Graduate Research Assistant, Dynamic Structures and Controls Laboratory, Department of Mechanical Engineering

Air Force Research Laboratory   Albuquerque, NM
Summer 2004 and 2005, Space Scholars Program, Space Vehicles Directorate, Kirtland Air Force Base

NASA Langley Research Center   Hampton, VA
08/2001 – 07/2003, Graduate Research Scholar Assistant, Joint Institute for Advancement of Flight Sciences, Structural Dynamics Branch

University of Illinois at Urbana-Champaign   Urbana, IL
01/2001 – 05/2001, Class Assistant, Department of Mathematics

Motorola Incorporated   Arlington Heights, IL
05/2000 – 08/2000, Industrial Engineering Intern, Customer Fulfillment Warranty Repair Center, Network Solutions Sector

Molex Incorporated   Lisle, IL
1998 – 2000, Mechanical Engineering Intern (three rotations), Die Engineering Group, Data Communications Division

Research Projects

- Dissertation Year Fellowship, University of Kentucky Graduate School, $24,000 total award, Investigating Lightweight Kapton Panels for use in Orbital Apertures, 07/2006 – 06/2007
- Modular Self-Rigidizing Lightweight Structures, researcher with Mr. Larry Bradford (PI), NASA Small Business Innovation Research (SBIR) Program, Structures and Habitation Subtopic, Phase I, $19,000 total award, 01/2006 – 06/2006
- Space Scholars Program, Air Force Research Laboratory, Dr. Brett deBlonk, Technical Advisor, A Composite Support Structure for a Membrane Mirror, Summer 2004, Deployment Repeatability Testing of Composite Tape Springs for Space Optics Applications, Summer 2005
- Evaluation of Surface Patterns for In-Plane Measurement of Thin-Film Structures, researcher with Drs. Suzanne Smith and Jack Leifer (PI’s), Kentucky Science and Engineering Foundation (KSEF), $53,991 total award, 06/2002 – 12/2004
- Large Inflatable Self-Rigidizing Polymer Film Structures, researcher with Dr. Suzanne Smith and Mr. Larry Bradford (PI’s), NASA Small Business Innovation Research (SBIR) Program, Phases I and II, $70,000 total award, 09/2002 – 09/2004

Awards and Distinctions

- University of Kentucky Dissertation Year Fellowship, 2006/2007, 1 of 12 awardees university-wide
- AIAA Foundation Willy Z. Sadah Award in Space Sciences and Space Engineering, 2004/2005
- Department of the Air Force, Special Act or Service Citation, 08/2004 and 08/2005
- Kentucky Graduate Scholarship, 2003-2006
- AIAA Region I Student Paper Competition, Graduate Division, 2nd Place, 2003
- Tau Beta Pi National Engineering Honor Society Inductee, 1999

Publications

Archival Journal Articles
Leifer, J., Black, J.T., Smith, S.W., Ma, N., and J.K. Lumpp, “Measurement of In-Plane Motion of Thin-Film Structures Using Videogrammetry,” Journal of Spacecraft and Rockets, under review, 06/2006.


**Refereed Conference Proceedings**


Conference Proceedings
Black, J.T., Smith, S.W., and J. Leifer, “Reduced Model Validation of Thermal-Formed Polyimide Panels,” to be presented at the 25th International Modal Analysis Conference, Orlando, FL, 02/2007.


NASA Technical Memoranda


Other


Abstracts and Presentations


Teaching Experience

University of Kentucky Lexington, KY
Fall 2004 and Fall 2005 semesters, Substitute Lecturer (6 one hour lectures), Dynamics (EM313)
Fall 2005 semester, College Teaching (GS610) course, part of Preparing Future Faculty certification program

Kaplan Inc. Lexington, KY
09/2004 – 02/2005, Instructor, SAT Classroom test preparation course

Carnegie Center for Literacy and Learning Lexington, KY
09/2003 – 04/2004, School-After-School Tutoring Program, volunteer one-on-one tutoring

Machen Elementary School Hampton, VA
09/2001 – 05/2003, NASA/Langley Study Buddies Program, volunteer one-on-one tutoring

University of Illinois at Urbana-Champaign Urbana, IL
01/2001 – 05/2001, Class Assistant, Linear Transformations and Matrices (MATH315)
Professional Development

- College Teaching (GS610) course, University of Kentucky, part of Preparing Future Faculty certification program, Fall 2005
- Practical Systems Engineering Principles, presented by F.R. Bourne, Harris Incorporated, 01/2005

Service

Professional

- AIAA Gossamer Spacecraft Program Committee, special project on the history of gossamer programs, 2006
- Session co-chair at 6th Gossamer Spacecraft Forum, 46th AIAA/ASME/ASCE/AHS/ASC Conference on Structures, Structural Dynamics and Materials, Austin, TX, 04/2005
- AIAA Gossamer Spacecraft Program Committee Meeting, assisted Dr. Suzanne Smith (host), Lexington, KY, 10/2004
- AIAA Langley Student Branch Executive Board, Hampton, VA, 2002-2003

Community

- Carnegie Center for Literacy and Learning, Lexington, KY, School-After-School Tutoring Program, volunteer one-on-one tutoring, 09/2003 – 04/2004
- Machen Elementary School, Hampton, VA, NASA/Langley Study Buddies Program, volunteer one-on-one tutoring, 09/2001 – 05/2003
- Tau Beta Pi National Engineering Honor Society, Community Projects Director, 08/1999 – 05/2001

Professional Memberships

- Member: The American Institute of Aeronautics and Astronautics (AIAA)
- Member: The Society of Experimental Mechanics (SEM)
- Member: The American Society of Mechanical Engineers (ASME)
- Engineer in Training, State of Kentucky, #12668