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STUDY OF SUPERPLASTIC FORMING PROCESS USING FINITE ELEMENT ANALYSIS

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ABSTRACT OF THESIS

STUDY OF SUPERPLASTIC FORMING PROCESS USING FINITE ELEMENT ANALYSIS

Superplastic forming (SPF) is a near net-shape forming process which offers many advantages over conventional forming operations including low forming pressure due to low flow stress, low die cost, greater design flexibility, and the ability to shape hard metals and form complex shapes. However, low production rate due to slow forming process and limited predictive capabilities due to lack of accurate constitutive models for superplastic deformation, are the main obstacles to the widespread use of SPF.

Recent advancements in finite element tools have helped in the analysis of complex superplastic forming operations. These tools can be utilized successfully in order to develop optimized superplastic forming techniques.

In this work, an optimum variable strain rate scheme developed using a combined micro-macro stability criterion is integrated with ABAQUS for the optimization of superplastic forming process. Finite element simulations of superplastic forming of Ti-6Al-4V sheet into a hemisphere and a box are carried out using two different forming approaches. The first approach is based on a constant strain rate scheme. The second one is based on the optimum variable strain rate scheme. It is shown that the forming time can be significantly reduced without compromising the uniformity of thickness distribution when using the proposed optimum approach. Further analysis is carried out to study the effects of strain rate, microstructural evolution and friction on the formed product. Finally the constitutive equations and stability criterion mentioned above are used to analyze the forming of dental implant superstructure, a modern industrial application of superplastic forming.

Keywords: Finite Element Simulation, Superplastic Deformation, Microstructural Evolution, Optimum Forming, Ti-6Al-4V

Pushkarraj Vasant Deshmukh

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STUDY OF SUPERPLASTIC FORMING PROCESS USING FINITE ELEMENT ANALYSIS

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STUDY OF SUPERPLASTIC FORMING PROCESS USING FINITE ELEMENT ANALYSIS

THESIS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the College of Engineering at the University of Kentucky

By

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Lexington, Kentucky

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2003

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TO MY PARENTS
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CHAPTER 1

INTRODUCTION

Superplastic materials are a unique class of polycrystalline solids that have the ability to undergo very large, uniform tensile elongations prior to failure. Elongations in excess of 200% usually indicate superplasticity. The low flow stresses and high sensitivity of flow stress to strain rate are the main aspects of superplastic deformation. Fine and equiaxed grain size, forming temperature greater than half the absolute melting temperature of the subject material, and controlled strain rate, are the main requirements for superplasticity. The optimum value of strain rate varies with the type of material, but is usually very low, e.g. 1.0E-3–1.0E-5 /s.

Titanium alloys (Ti-6Al-4V, Ti-6Al-2Sn-4Zn-2Mo), aluminum alloys (5083, 7475) and magnesium alloys (ZK60, AZ31) are typical examples of metallic superplastic materials that have been developed and are increasingly being used to produce complex shapes. Some composites and ceramics are also known to behave superplastically. Superplastic forming (SPF) is a near net-shape forming process which offers many advantages over conventional forming operations including low forming pressure due to low flow stress, lower die cost, greater design flexibility, and the ability to shape hard metals and form complex shapes. However, low production rate due to slow forming process and limited predictive capabilities due to lack of accurate constitutive models for superplastic deformation, are the main obstacles to the widespread use of SPF. This factor has restricted the growth of applications of Superplastic alloy to low volume production industries like the aerospace industry.

1.1 STATEMENT OF PROBLEM

Superplasticity is exhibited in materials only in a narrow range of forming conditions with optimum values unique to each material. This factor makes it essential to determine the pressure loading history in order to maintain the maximum strain rate near the optimum value throughout the whole forming process. Temperature and thickness distribution are other important parameters that need careful consideration. Thus in order to manufacture a part successfully, there is a need to utilize simulation tools, which can accurately predict the SPF
deformation and help to optimize the SPF parameters. Present research in superplastic forming is concentrated in two major fields

- Development of constitutive equations to model large deformation and microstructural evolution in superplastic forming.
- Optimization of the complete SPF process in order to improve the production rate.

Most of the previous research in SPF was carried out based on uniaxial loading conditions and assuming isotropic behavior. Moreover, the Von Mises yield criterion is usually employed to extend the uniaxial models to multiaxial loading conditions through definition of effective stress and strain. However recent results indicate a strong degree of deformation induced anisotropy to be present in superplastic alloys. Present research in superplastic forming is concentrated towards the development of constitutive relation which represents the anisotropic material behavior of superplastic alloys, accounting for micro structural evolution and material hardening due to grain growth.

Pressure control schemes have been introduced recently in various commercial finite element tools. These keep track of the maximum plastic strain rate in the model, and adapts in a rapid way in order to maintain the value of the strain rate within the optimum region. These control schemes can be applied in various ways i.e. constant strain rate control in the free forming region, constant strain rate control in the die entry region, maximum variable strain rate control and strain rate gradient control. However these strain rate control schemes do not help in reducing the time required for the SPF process. It has been observed that developing an optimum forming loading profile using a variable strain rate path results in significant improvement in the formability of superplastic alloys, however there is a need to device a failure criterion that would help in generating the optimum strain rate profile.

The previous work highlights the importance of a constitutive equation which accurately predicts superplastic material behavior and methods by which the forming process can be optimized. In this research finite element analysis of superplastic deformation is carried out based on the constitutive equation modeled within the continuum theory of viscoplasticity with an anisotropic yield function and a microstructure-based overstress function [1-7]. Grain growth and cavitation are incorporated in the model. An optimized pressure-time profile is developed for Superplastic forming operations based on the optimum variable stain rate forming path that was generated from a multi-scale stability criterion, taking into account both geometrical
(macroscopic) and microstructural features including grain growth and cavitation [4]. The effects of void fraction, grain size, and strain rate sensitivity on the stability of superplastic deformation are examined.

1.2 RESEARCH OBJECTIVES

The primary focus of this research is to apply the finite element techniques to analyze and optimize the superplastic forming operation. ABAQUS, a general purpose finite element program is used to carry out the computational analysis. The major research objectives are:

1. Optimize the superplastic forming process, based on the on failure criteria, developed from the optimum variable stain rate forming path [4]. This failure criteria accounts for both geometric instabilities and microstructural aspects including grain growth and cavitations.
2. Study the effect of initial grain size and the coefficient of friction between the sheet and the die, on the superplastic forming operation.
3. Analyze the anisotropic behavior of eutectic Pb-Sn alloy and compare the results with the experimental data.

1.3 OVERVIEW OF THE THESIS

The contents of the dissertation are described as follows. Chapter 2 gives a theoretical background on properties of superplastic alloys, the process used to create complex parts using these alloys and a general finite element formulation of a problem with respect to large deformation. The literature relevant to this research is also categorized and critically reviewed. Chapter 3 describes the formulation of the finite element problem specific to this research. Various aspects of the problem such as modeling, constitutive material model, boundary conditions and the general loading aspects are described here. The results obtained from the analyses of superplastic forming of Titanium alloy are presented and discussed in Chapter 4. The analyses of eutectic Pb-Sn alloy are carried out in Chapter 5. Chapter 6 provides the summary of the present work and discusses the possible areas of future research involving optimization of superplastic forming.
CHAPTER 2

BACKGROUND

2.1 SUPERPLASTICITY

Superplasticity is characterized by low flow stress and high sensitivity of the flow stress to strain rate. A simple form of constitutive equation for superplastic material is given by:

$$\sigma = C_1 \dot{\varepsilon}^m$$

where $\sigma$ is the flow stress, $\dot{\varepsilon}$ is the strain rate, $C_1$ is a constant and $m$ is the strain rate sensitivity index. A typical stress/strain rate relation for superplastic materials is shown on logarithmic scales in Figure 2.1.

The sigmoidal shape can be divided into three regions where different microstructural mechanisms are believed to dominate the deformation behavior. Superplasticity occurs only in region II, where strain rate sensitivity index, $m$, has high values at moderate strain rates, accompanied by very large elongations. The variation of $m$ with strain rate is shown in Figure 2.1. For superplastic behavior, $m$ would be greater than or equal to 0.3 and for the majority of superplastic materials $m$ lies in the range of 0.4 to 0.8. The presence of a neck in a material subject to tensile straining leads to a locally high strain rate and, for a high value of $m$, to a sharp increase in the flow stress within the necked region. Hence the neck undergoes strain rate hardening which inhibits its further development. Thus high strain rate sensitivity confers a high resistance to neck development and results in the high tensile elongations characteristic of superplastic materials. The deformation process in region II is not very well understood and there is no one mechanism that can describe the deformation in this region. However it is believed that grain boundary sliding accompanied by diffusion or dislocation glide and climb is the dominant mechanism. In this region, the crystallographic texture becomes less intense due to limited dislocation activities within the grain. In region III, the deformation mechanism is dominated by conventional recovery controlled dislocation creep (power-law creep). Deformation within this region leads to the observation of slip lines and to the development of high dislocation densities within the grains. Crystallographic texture within the material is increased and significant grain elongation occurs during deformation. The deformation mechanisms in region I are the subject of
controversy. Suggested behavior range from threshold stress (low \(m\) values were observed) to diffusion controlled flow (high \(m\) values were observed).

![Graph showing variation of strain rate with strain rate sensitivity index \((m)\).](image)

Figure 2.1: Variation of strain rate with strain rate sensitivity index \((m)\).

The mechanical behavior of superplastic materials is very sensitive to both temperature and grain size. In general, increasing the temperature or decreasing the grain size of the material has a similar effect on the variation of flow stress with strain rate. Increasing the temperature decreases the flow stress, particularly at the lower strain rates corresponding to the transition from Region II to Region I. The maximum strain rate sensitivity has been found to increase with increasing temperature and the strain rate of maximum \('m'\) moves to higher strain rates.
2.2 SUPERPLASTIC BLOW FORMING

Blow forming is a pressurized forming process which is widely used to produce complex shapes using superplastic alloys. The various steps involved in the blow forming process are shown in Figure 2.2. In this process the sheet is tightly clamped around its periphery and gas pressure is applied on its surface. An inert atmosphere is required in the forming chamber, and argon gas is generally used for both pressurization and maintenance of protective atmosphere. Predetermined pressure-time profile is used to achieve complete adaptation of the metal sheet to the die surface at a controlled rate of deformation.

During the initial stage of deformation the sheet is not in contact with the die. Deformation in this stage is concentrated at the pole. Consequently greatest strain occurs in this region during this stage. Once the pole comes in contact with the surface of the die, the material is locked due to friction. This prevents further deformation. The remaining free region continues to deform until complete contact with the die occurs. The corners of the die are usually the last to be filled, causing greater strain to occur in these regions, consequently theses regions are more prone to failures.

![Figure 2.2: Superplastic blow forming process.](image)

2.3 FINITE ELEMENT METHOD

Finite element method is a numerical procedure for analyzing a wide range of problems that are too complicated to be solved satisfactorily by classical analytical methods. Since the early 1950’s to present, enormous advances have been made in the application of the finite element method to solve engineering problems. Finite element method models a structure as an assemblage of small parts (elements). Each element is of simple geometry and therefore is much easier to analyze than the actual structure.

The general steps involved during finite element method with respect to metal forming analysis using commercial software, are listed below [8-11].
1. Create the model of the die and sheet assembly. Based on the feasibility and the shape of the formed product, choice has to be made between a 3-D model, 2-D model, axisymmetric model or a symmetric model. This is essential to obtain accurate results with low computational cost. Complex assemblies can be generated using specialized modeling software’s and the model can be imported into the finite element analysis software as an IGES file.

2. Divide the body into an equivalent system of finite elements with associated nodes and choosing the most appropriate element type to model most closely the actual physical behavior. The accuracy of the results is greatly dependent on the size of the elements. The finer the mesh, greater is the accuracy; however this increases the computational time. Commercial computer programs, called preprocessors, help in generating a mesh.

3. Formulate the properties of each element.

4. Select the material model to be associated with the die and the sheet. This is essential to determine the strain/displacement and stress/strain relationship during the formulation of the problem.

5. Assign the initial boundary conditions. This involves constraining all the degrees of freedom of the nodes associated with the clamped portion of the sheet and applying the symmetric boundary conditions.

6. Apply the loads associated with the complete finite element model. In the case of metal forming it involves releasing the plunger, over the clamped sheet. This is usually done by assigning a fixed displacement to the plunger or releasing it with an initial velocity.

7. Solve the problem and interpret the results.

The advantages associated with finite element analysis are its ability to model irregular shaped bodies, handle general load conditions, handle unlimited number and kinds of boundary conditions, alter the finite element model relatively easily and cheaply, include dynamic effects and handle nonlinear behavior existing with large deformation and nonlinear materials. These advantages make it an ideal tool for the analysis of superplastic metal forming process.
2.4 PREVIOUS WORK

Superplastic forming involves carrying out the process at an optimum strain rate, which lies in the superplastic region. A great deal of research has been done in order to develop a pressure algorithm, which would optimize the forming process by reducing the forming time and maintaining uniform thickness distribution in the formed part. Finite element technique has played a major role in carrying out the trial and error analysis in order to reach at the desired results. FEA is a powerful tool that allows one to keep track of the intricate die geometries and various other parameter that play an important role during sheet metal forming. This chapter gives an overview of the past and present research, which is directly related, with the use of finite element analysis in superplastic forming.

2.4.1 Use of finite element analysis in sheet metal forming

Tikkaya [12] and Mattison [13] presented an overview of the present use of finite element analysis in sheet metal forming industry. ‘Virtual Production’ i.e. the use of numerical analysis techniques in present day industry; has reduced the trial and error procedure followed earlier. The pressure on the modern day industry is for continuous improvement which leads to new products being developed very frequently.

The current industrial requirement from numerical analysis of sheet metal forming is broadly classified as:

- Time reduction
- Cost reduction
- Increase in product quality

For these three requirements to be fulfilled it is expected that

- The simulation tool is able to model various process and operation of sheet metal forming.
- The simulation tool is very user friendly
- There is an CAD-FEM-CAD interface
- The analysis is very efficient
- The various outputs required like stress, strain, thickness distribution, failure modes, etc. should be computed easily.
- And there should be various models present to represent the material behavior accurately.
The various methods of finite element analysis used in sheet metal forming are

**Quasi-Static implicit approach**

This was the very first method used in the simulation of metal forming process. This method enables full static solution of the deformation problem with convergence control. In this case, theoretically the time increments size can be large but practically however it is reduced by the contact conditions present between the sheet and die and the computer time increases almost four times with increase in elements. Another disadvantage of an implicit approach is the singularities of the stiffness matrix at bifurcation points, such as instabilities at wrinkling initiation.

**Dynamic explicit approach**

Dynamic explicit method uses a central time differentiation scheme. The time step is calculated on the basis of

$$\Delta t \approx \frac{L}{\sqrt{\frac{E}{\rho}}}$$

where $L$ is the length of the element, $E$ is the Young’s Modulus and $\rho$ is the density. This is approximately equal to the time required for a bending or compression wave to travel through the smallest element in the mesh. For a typical sheet forming analysis this time step may be as small as 2E-7. This can lead to a very large number of time steps based on the process time, which would make the dynamic explicit method infeasible. To avoid this numerous tricks are applied, one among these is to increase the density of the material, which increases the allowable time step size, thus reducing the total number of time steps required for the process. The additional body force introduced due to this is taken over by the rigid die, due to the high surface to volume ratio.

Wang et al. [14] carried out a comparison between the static implicit and dynamic explicit methods for FEM simulation of sheet forming process in order to determine the optimum method for sheet metal forming.

Two typical sheet metal forming operations were carried out

- Box deep drawing.
- Hydroforming of a flat sheet.

For the box deep drawing process implicit analysis took 104 incremental steps whereas the explicit analysis was carried out with 21000 increments with the scaling of mass and punch
speed. For a punch stroke of 78 mm both the implicit and the explicit methods showed identical thickness distribution in the formed part.

In the case of hydro-forming of flat sheet significant difference was observed between the geometric shapes obtained by implicit and explicit procedures. In case of explicit dynamic analysis the deformation was delayed due to the inertial effect of mass scaling. Thus decreasing the mass did not help in reducing the time, as further calculations were required to get the desired shape. However it was found that using slower loading speeds helps in reducing the inertial effects.

It was concluded that both static implicit and dynamic procedures are successful in analysis of sheet metal forming. For tool driven problems the artificial scaling methods can be applied without significantly changing the analysis result. But in case of force driven problems this method shows very little merit for reducing the total computational time of FE analysis. For such analysis implicit methods are more beneficial.

2.4.2 Optimization of superplastic forming process

In order to maintain constant strain rate during superplastic blow forming Dutta and Mukherjee [15] developed a simple pressure-time equation based on biaxial stress conditions, for relating the required gas pressure to material properties such as flow stress, strain rate and the geometric properties of the sheet.

Carrino et al. [16] carried out research for the optimization of industrial superplastic forming process using a finite element method. The aim was to interface finite element programs with sub-routines in order to determine the thickness distribution of the formed part and to predict the optimum-loading curve.

In general, constitutive equation for superplastic material is given by:

$$\sigma = \phi(\varepsilon, \dot{\varepsilon}, m)$$

where $\sigma$ is the flow stress, $\varepsilon$ is the strain and $\dot{\varepsilon}$ is the strain rate.

Ignoring, the grain size and hardening parameters we have

$$\sigma = k \dot{\varepsilon}^m$$

where $k$ is the material constant and $m$ is the strain rate sensitivity index.

On the basis of membrane theory, i.e. stress is directly proportional to pressure, a time load curve was defined
Knowing all the quantities at time $t$, it was possible to foretell the pressure at time $t + \Delta t$ that will produce an averaged maximum strain rate.

$$
\sigma = k \dot{\varepsilon}^m
$$

Pressure and strain rate are known at time $t$. Assuming $\frac{t+\Delta t}{t} \dot{\varepsilon}_{\text{max}}^\text{opt}$ equal to $\dot{\varepsilon}_{\text{opt}}$ the preceding option can be used to predict $\frac{t+\Delta t}{t} p$. Thus if the value of maximum strain rate at time $t$ is greater than optimum strain rate, the pressure evaluated at time $t + \Delta t$ will be less than the pressure at time $t$ and vice versa.

Using the above optimum pressure profile, a finite element simulation was performed to bulge a superplastic sheet made of Ti-6Al-4V. The temperature at which forming cycle took place was 880°C. The material properties were taken as $k=5267.62$ s m N/mm², $m=0.85$, $\dot{\varepsilon}_{\text{opt}} = 1.5E-4$ 1/s and coefficient of friction = 0.1. It was observed that by using pressure control algorithm the forming time was reduced from 2400 s to 1680 s.

Hambl et al. [17] integrated a similar pressure-cycle control algorithm to keep track of the maximum strain rate, during the forming analysis and adapt itself in order to keep maintain the desired strain rate.

Xing et al. [18] devised a 3-D membrane shell formulation for the finite element simulation of superplastic forming with microstructure variation. An adaptive control algorithm was presented to calculate the back pressure and bulging pressure to control cavity growth and distribution and to maintain the optimum deformation mode.

Johnson et al. [19] developed a constitutive relation which represented the superplastic behavior of Ti-6Al-4V at 900C. It included the static and dynamic grain growth effect, to take into account strain hardening due to micro structural development. Based on the grain size changes an optimum deformation strain rate path was generated. They used this stability criterion
to carry out finite element analysis of SPF and the results obtained showed a considerable reduction in the deformation time along with uniform thickness distribution in the formed path.

Ding et al. [20] analyzed superplastic blow forming process of thin sheet and obtained a stable deformation path that reduced production time. Several control schemes were developed to generate a pressure profile to maintain the strain rate in the sheet at the optimum target value:

1. Constant strain rate control in the free forming region.
2. Constant strain rate control in the die entry radius region.
   In this scheme constant strain rate is imposed on a certain node in the die entry radius region at the initial stage of forming. After the sheet is in full contact with the die entry radius region, the control is switched back to controlling the free forming region with constant strain rate.
3. Variable strain rate control in free forming region.
4. Maximum strain rate gradient control.
   In this case the strain gradient is evaluated in each element. If this gradient is larger than the allowable value, the pressure is reduced and vise versa.
5. Maximum variable strain rate control.
   Here the strain rates at all the nodes are checked and the forming pressure for the next time step is evaluated according to the node with maximum strain rate. Thus the strain rate at all the nodes is less than or equal to the target strain rate throughout the deformation history.

The numerical results showed that it was possible to reduce the forming time or improve the thickness distribution of the formed sheet, but it was difficult to achieve both at the same time. The variable strain rate control in the free forming region was the most efficient scheme, but since the analysis was based on a uniaxial state of stress it needs to be further extended to biaxial state of stress.

Ding et al. [21] further extended his research from uniaxial state of stress to biaxial state of stress, which is a dominant stress state in superplastic sheet blow forming process.

The condition for the onset of localized thinning of the thin sheet is derived with the assumption that the localized necking initiates along the direction perpendicular to the major principle stress direction. This method is applied to the prediction of onset of localized necking in superplastic material (Ti-6Al-4V), a family of strain rate paths for various in plane strain ratio is determined, providing a basis for optimizing the deformation process. Numerical analysis is carried out to
develop the variable strain rate path for Ti-6Al-4V superplastic alloy, based on the developed instability criterion. Two types of loading controls were used:

1. Controlling loading path to keep the strain rate at a constant value of 5E-4 /s, which is the optimum superplastic strain rate.
2. Controlling the loading path such that the strain rate follows the variable strain rate path for uniaxial loading.

During a finite element analysis of tensile test using the above two types of load it was found that for the first case it took 25 minutes for a strain of 66% and some localized deformation was observed in this method. In the second case for the same time a strain of 88% was obtained with uniform distribution. It was concluded that the forming time was significantly reduced as compared to the traditional forming process with constant strain rate, without affecting the thickness profile.

Khraisheh et al. [22] carried out a study in order to determine an optimum pressure profile for superplastic forming in order to reduce the forming time and maintain the desired thickness distribution. The material selected was Pb-Sn eutectic alloy, which exhibits superplastic characteristics at room temperature. Circular sheets having diameter 7.62 cm and thickness 0.127 were used. The experimental setup consisted of pressure control systems, which regulated the air fed into the forming chamber. The advantage of such a system was that user specified pressure varying with time can be applied with high accuracy. The equation derived by Dutta and Mukherjee [15], was used to control the forming pressure while maintaining a constant effective strain rate at the pole. Three sets of experiments were conducted.

- Four sheets having thickness 0.127 were formed until rupture at a constant effective strain rate of 1.0E-4, 3.0E-4, 6.5E-4 and 1.0E-3 /s.
- Two sheets having thickness 0.15 cm were formed at a constant strain rate of 1.0E-4 and 5.0E-4 for 450 s.
- Based on the above two results an optimum pressure profile was constructed and a sheet was formed using this profile.

In the first case as per expectations samples formed with low strain rates took more time to rupture, the dome height obtained was greater and the thickness distribution was more uniform. It is seen from here that in order to obtain better thickness distribution more forming time is required It was also seen that just before the time when the peak pressure is applied rapid
thinning takes place in and around the region of the dome and ultimately the sheet ruptures at peak pressure. In the second case forming was carried out for 450 seconds, which was the time just before rapid thinning took place. The thickness profile obtained was almost uniform. This suggests that one could speed up the forming at the initial stages and slow it down in the later stages without effecting the thickness distribution. In the last case optimum superplastic strain rate range was identified as $\dot{\varepsilon}_1 \ldots \dot{\varepsilon}_n$. Then the pressure profiles under constant strain rates were generated for each strain rate $\dot{\varepsilon}_1 \ldots \dot{\varepsilon}_n$ as $P_1(T) \ldots P_n(T)$. For each pressure profile, time at which peak pressure occurred is determined. Then a limiting time equal to 70% of the peak time is selected, in order to avoid thinning. Based on this an optimum pressure profile was generated. By using this pressure profile the previously achieved bulge height was obtained with 40% reduction in forming time.

Huang et al. [23] and Yong et al. [24] carried out research in order to determine the initial sheet thickness distribution in order to achieve the desired final thickness distribution in the formed part. They treated the design of the initial sheet thickness distribution as an optimization problem. Huang investigated two optimization methods i.e. the gradient search method and the proportional control method. The optimization criteria were the minimization of the mean square error $Y$.

$$Y = \frac{1}{n} \sum_{i=1}^{n} (t_i - \bar{t})^2$$

where $n$ is the number of nodes, $t_i$ is the thickness of $i^{th}$ node and $\bar{t}$ is the required thickness.

Yong et al. used the addition/subtraction method. In this process a trial and error procedure is applied in order to arrive at the optimum value of initial blank thickness. The above-proposed methods were applied to determine the thickness distribution of the blank in order to achieve optimum thickness distribution in the formed part. Using this optimized blank superplastic forming was carried out and it was observed that the forming time was reduced considerably and uniform thickness distribution was achieved in the formed part. However the above techniques require machining or chemical milling process for the preparation of the initial blank, which increases the process cost considerably.
2.4.3 Failure analysis of superplastic forming

Most of the research carried out in superplastic forming describes the plastic flow during forming by the isotropic von Mises flow rule. However it has been found that superplastic materials exhibit a strong degree of anisotropy and hence there is an ever increasing need to develop a constitutive equation to accurately model the deformation process.

Khraisheh [25] addressed one of the major problem faced in superplastic forming i.e. Failure to predict the thinning and rupture of the formed part due to lack of accurate constitutive relations which describe the actual superplastic deformation.

Most of the previous research was carried out assuming isotropic condition with uniaxial loading, but since forming involves multi-axial loading conditions, the existing models have limited predictive capabilities. Due to these limitations, usually low pressure is applied in order to avoid premature failure, thus prolonging the forming time. This paper took into account the failure characteristics of Pb-Sn superplastic sheet subjected to gas pressure forming. The superplastic sheet was formed using four different strain rates (1.0E-3, 6.5E-4, 3.0E-4, 1.0E-4 /s), and the thinning and failure characteristics in each case were observed. It was found that for low strain rate forming, failure occurred as a result of formation of tiny holes around the pole of the dome. This failure was related to the nucleation and void growth during deformation. The failure mode in the forming in the sheet formed at high strain rate was in the shape of cracks around the dome of the sheet. Further experiment was carried out with the help of user specified forming pressure profile, in order to maintain the optimum strain rate at the pole region of the dome where maximum thinning took place during forming. It was observed that the time when the peak pressure was applied was the time when failure occurred. As expected the forming time for samples with low strain rate was greater than those with higher strain rates. Taking this into account it is possible to predict the approximate time of failure. From this study it was concluded that a failure criteria can be generated that depends on the material parameters that remained unchanged with strain rate.

Khraisheh [2] further investigated the yield potential in superplastic forming. He carried out a combined tension/torsion test on Pb-Sn at constant effective strain rate. The yield potential for four different effective strain rates was constructed using the experimental data obtained from the tests. It was seen that the yield surface of Pb-Sn superplastic alloy is anisotropic, especially at
low strain rates. The yield function proposed in this study was capable of describing the initial anisotropic nature of the yield surface and also its evolution.

Taylor et al. [26] carried out a study involving the effect of void size and spacing on the ductility and flow stress of superplastic Pb-Sn. It was found experimentally that for a random distribution of voids and a constant void density, decreasing the void size resulted in an increase in the ductility of the material. They developed an equation relating the failure strain to the void size and density. Similar conclusion was reached by carrying out a FE analysis of a thin sheet with different void distribution.

Khraishi et al. [27] performed numerical and experimental studies on void growth and the parameters affecting it, during superplastic deformation. They concluded that

- Increasing the value of strain rate sensitivity index produced strengthening and decreased the rate of void growth.
- Larger initial void fraction caused accelerated void growth, thus weakening the specimen.
- Multiple voids increased the metal ductility by reducing the extent of necking and its onset.

FE results were in good qualitative agreement with the experimental ones.

Khaleel et al. [28] created a failure criterion for superplastic forming that included the evolution of grain size. He carried out computational analysis to form a long rectangular tray. A pressure algorithm was employed to maintain constant maximum strain rate (1E-3 /s) throughout the forming cycle. The FE analysis results were compared with those obtained by experimentations using enhanced 5083 aluminum alloy and they were found to be in good agreement with each other.

Chandra [29] studied the deformation behavior, of superplastic forming which is closely linked to accommodation mechanism, cavitations and failure process. He outlined the various constitutive models developed over the years at various levels i.e. Macroscopic, mesoscopic and atomistic level. He used the simple power law model, the logarithmic model and the micromechanics model to study the flow stress-strain behavior of Al 5083. Grain boundary structure and sliding at the atomistic level was also considered. It was observed that the superplastic grain boundary sliding is always coupled with migration when grain boundary is subjected to applied shear stress. By calculating the energy associated with it, it was found to be more favorable than the formation of grain boundary cleavage under shear loading conditions.
Li et al. [30-31] generalized the hartz stability criteria in terms of effective strain and effective strain rate, for the analysis of instability and strain concentration during superplastic deformation. They devised a parameter, named as ‘flow localization factor’ ($\xi$) that characterizes the degree of flow localization. If $\xi > 0$ at some point during the deformation, the flow localization will occur at that point until fracturing eventually occurs. A series of superplastic bulge forming experiments were conducted using different forming pressures. The top of the blow formed dome was analyzed and the curves of the flow localization factor at fracture point versus the forming time were plotted. From the curves it was concluded that the localization process can be divided into three stages.

- The short, development stage of localization process, having a high growth rate.
- The steady state where the flow localization factor grows slowly and steadily.
- The final stage when the forming limit is approached. The flow localization is rapid until fracture occurs.

These conclusions can be very helpful in controlling the bulging pressure in superplastic forming, to keep the flow localization factor within the second stage and avoid fracture.

Lin et al. [32-33] established integrated numerical procedures to effectively simulate the superplastic forming process for complex shaped components. They further carried out stability analysis to investigate necking in superplastic material characterized by the sinh-law constitutive equation. They plotted a necking map, by observing the effect of load and strain rate sensitivity parameter on necking, under uniaxial loading conditions. Finite element analysis of superplastic box forming was carried out to understand the non uniform thinning and grain size distribution. It was observed that, for uniaxial loading, increasing the strain rate sensitivity parameter enhances necking, as does increasing load. The FEA results showed that there was a variation in the grain size distribution, in the formed box, due to the spatial variation of strain rate. It was concluded that a higher target strain rate tends to, lead to greater heterogeneity in grain size distribution; this in turn may lead to greater variation in resulting product material properties. However, higher target strain rate and lower values of strain rate sensitivity parameter lead to more uniform thinning in the formed product.

Cheong et al. [34] created a similar necking map based on a step bar model. This step bar model was used to investigate the effect of grain size gradients, geometrical irregularities and deformation rates on the necking of Ti-6Al-4V. The necking map gave an indication of the
necking mode under various combinations of initial grain size grain-size gradients, initial geometrical irregularities and deformation rates.

J. Lin [35] reviewed various constitutive equations for superplastic forming which takes into account micro structural evolution and material hardening due to grain growth during superplastic deformation. Three sets of material models were analyzed by creating subroutines in ABAQUS. These constitutive equations were developed over the years for modeling superplastic behavior (Zhou and Dunne [36], Kim and Dunne [37], Cheong et al. [38], Lin et al. [39]). By using the material constants determined experimentally (Gosh and Hamilton [40], Lin and Yang [41], Cheong et al. [38]) J. Lin carried out finite element analysis for forming a rectangular-section box. From the result obtained he concluded that superplastic forming can be accurately represented by unified constitutive equations with lesser effect of strain rate sensitivity parameter on the thinning of the material. The unified theory accounts for the strain rate sensitivity of the flow stress through the inclusion of grain growth kinetics rather than through the strain rate sensitivity index value only.
CHAPTER 3

FINITE ELEMENT FORMULATION OF THE SHEET METAL FORMING PROBLEM

ABACUS [42], a general purpose finite element program is used to carry out the computational analysis of SPF. ABAQUS includes direct, implicit time integration, using the Hilber-Hughes operator (the Newmark’s method with controllable numerical damping). This implicit software is specifically chosen for the superplastic analysis since it enables a full static solution of deformation problem with convergence control and the time increment size can be defined within practical limits. Fully integrated quadrilateral membrane elements present in the ABAQUS element library are used to mesh the die and the sheet assembly model. A user defined subroutine is used to model the superplastic material behavior. It is based on a failure criterion, which takes into account strain hardening, grain size and void growth during superplastic forming. A macro present in ABAQUS is used to control the pressure at each time step so as to limit the maximum strain rate, during SPF, within a predefined optimum region.

3.1 IMPLICIT AND EXPLICIT ANALYSIS

Implicit and Explicit solvers are two common numerical techniques currently used in FE simulation industry. From the physical point of view the kinetic energy plays an important role in selecting the type of FE technique to be used for the analysis. We can broadly classify the metal forming processes into two types.

1. In quasi-static problem, the kinematic energy is insignificant of the total energy. Superplastic forming falls into this category.
2. In high strain rate phenomena, or purely dynamic processes, the kinetic energy is overwhelmingly dominant. This is the case of processes with a high energetic impact.

3.1.1 Numerical integration in time

The global equation for a FE problem is represented as:

\[ \{F(t)\} = [K]\{d\} + [M]\{\dot{d}\} \]

(3.1)
where \([K]\), \([M]\) and \([F]\) are the global stiffness, mass and force matrices, respectively. \(d\) are the nodal displacements and \(\ddot{d}\) are the nodal accelerations.

Upon discretization of the above equation with respect to time, we can determine the nodal displacements at different time increment for a given system. The general method used is direct integration. There are two classification of direct integration: explicit and implicit. The common explicit and implicit methods are known as the central difference method [9-10] and Newmark’s method respectively.

\[
d(t) \nonumber
\]

\[
d_{i+1} \nonumber
\]

\[
d_{i-1} \nonumber
\]

\[
\Delta t \nonumber
\]

\[
t_i - \Delta t \quad t_i \quad t_i + \Delta t \nonumber
\]

Figure 3.1: Numerical integration.

3.1.2 Explicit finite element method

The central difference method is based on finite difference expression in time for velocity and acceleration at time \(t\) given by

\[
\dot{d}_i = \frac{d_{i+1} - d_{i-1}}{2(\Delta t)} \tag{3.2}
\]

\[
\ddot{d}_i = \frac{d_{i+1} - 2d_i + d_{i-1}}{2(\Delta t)^2} \tag{3.3}
\]

\[
d_{i+1} = 2d_i - d_{i-1} + \ddot{d}_i(\Delta t)^2 \tag{3.4}
\]

From Equation 3.1 we can express acceleration as

\[
\ddot{d}_i = M^{-1}(F_i - Kd_i) \tag{3.5}
\]

From Equations 3.4 and 3.5 we have
Using the displacement $d_{i-1}$, we can use the above equations to determine $d_{i+1}$, $\ddot{d}_{i+1}$, and $\dddot{d}_{i+1}$. The major advantage of using this process is that the matrix inversion of the global mass matrix $[M]$ is not required. However this process is conditionally stable, i.e. the time step for the time integration is subjected to limitation via Equation 3.7.

$$\Delta t \leq 2[(1 + \xi^2)^{0.5} - \bar{\xi}]/\omega_{\text{max}}$$

where $\omega_{\text{max}}$ is the maximum eigen frequency of the system and $\xi$ is the fraction of the critical damping of the highest mode. Thus in order to increase the time step by artificially increasing the punch speed or by artificially increasing the mass density. However such attempts at improving the analysis efficiency result in an increase of inertial effects which affects the accuracy of the solution.

3.1.3 Implicit finite element method

ABAQUS, implicit analysis software, used the Newmark’s direct integration method. Newmark’s equations are given by

$$\ddot{d}_{i+1} = \dot{d}_{i+1} + (\Delta t)[(1 - \gamma)\ddot{d}_i + \gamma \dddot{d}_{i+1}]$$

$$d_{i+1} = d_i + (\Delta t)\dot{d}_i + (\Delta t)^2[(\frac{1}{2} - \beta)\ddot{d}_i + \beta \dddot{d}_{i+1}]$$

where $\beta$ and $\gamma$ are parameters chosen by the user. By multiplying Equations 3.9 by mass matrix $M$ and then substituting Equation 3.5 we obtain

$$K^{'d}_{i+1} = F'_{i+1}$$

where

$$K' = K + \frac{1}{\beta(\Delta t)^2} M$$

$$F'_{i+1} = F_{i+1} + \frac{M}{\beta(\Delta t)^2}[d_i + (\Delta t)\dot{d}_i + (\frac{1}{2} - \beta)(\Delta t)^2 \ddot{d}_i]$$

Using the above equations we can determine the values of $d_{i+1}$, $\dot{d}_{i+1}$ and $\ddot{d}_{i+1}$. Due to the iterative nature of the solution procedure, a successful solution requires the satisfaction of convergence.
criterion at each step. Generally the convergence speed is quite problem dependent and failure to converge results in premature termination of the analysis.

Thus it can be seen that the dynamic explicit method is advantageous for analysis of sheet metal forming where the real time is just a few seconds. It has the characteristic of less memory requirement and greater computer efficiency since the need for consistent stiffness matrix is obviated. However for process such as superplastic forming, this method reveals its inability to reduce the calculations time because of stability requirements on the size of the time step, thus requiring larger number of incremental steps. In addition, when rate sensitive materials are involved, accurate results are extremely difficult to obtain unless a large number of steps are used.

3.2 SURFACE MODELING AND MESH GENERATION

SPF involves modeling complex shaped structural components. It is essential to accurately define an intricate shaped die surface and generate a quality finite element mesh over it, into which the flat sheet metal is deformed. In most cases die surfaces are often topographically irregular and the regular assembly of rectangular patches for modeling these surfaces leaves some non four-sided-holes. Using a consistent expression to represent the non-four-sided surface areas, and generating a quality FE mesh over the regions, are absolutely crucial for carrying out FE simulations of SPF process.

Taking advantage of the symmetry only a quarter of the assembly is considered for the analysis. Usually different types of symmetry may exist in an assembly. These include reflective or mirror, skew, axial and cyclic. Symmetry means correspondence in size, shape, and position of loads; material properties; and boundary conditions that are opposite sides of a dividing line or plane. The use of symmetry allows us to consider a reduced problem instead of the actual problem and the computational time required for the analysis is substantially decreased. One important aspect to be considered during symmetric analysis is to model more than quarter of the die surface. This avoids the elements and nodes on the deformable sheet from sliding off the die.

3.2.1 The superplastic alloy sheet

The superplastic sheet is meshed using quadrilateral membrane elements, of the type M3D4 present in the ABAQUS element library. M3D4 in a term used for 3-Dimensional, 4 node
membrane element. Computer simulations of three dimensional superplastic sheet forming process can be carried out by the finite element method with a membrane element or a shell element. A membrane element is regarded as more preferable rather than a shell element because of the computing efficiency and the easy contact treatment. Membrane elements are sheets in space that carry membrane force but do not have any bending or transverse shear stiffness, so the only nonzero stress component in the membrane are those components parallel to the middle surface of the membrane: the membrane is in a state of plane stress. The bending effect can be neglected because the thickness of the superplastic sheet is negligible as compared to the other sheet dimensions. Membrane elements have three active degrees of freedom i.e. \( u_x, u_y \) and \( u_z \). In geometrically nonlinear analysis the cross-section thickness changes as a function of the membrane strain with a user-defined Poisson’s ratio, \( v \). The top surface of a membrane is the surface in the positive normal direction (shown below) and is called SPOS face for contact definition. The bottom surface is in the negative direction along the normal and is called the SNEG face for contact definition. SPF process involved large-displacement, this may cause buckling to occur if the membrane structure is subjected to compressive loading. Since stress-free flat membrane has no stiffness perpendicular to its plane, out-of-plane loading will cause numerical singularities and convergence difficulties. This may be prevented by loading the membrane element in tension or adding an initial tensile stress. The magnitude of such stress should be such that the final solution is unaffected.

Figure 3.2: Membrane Element (M3D4).
3.2.2 The die surface

In order to obtain optimum and reliable convergence, it is essential that the rigid surface representation is smooth. ABAQUS allows for re-constructing complex shaped 3D rigid surface with Bezier triangular patches based on triangular elements generated over the surface. The number of patches required to define the die surface can be reduced by specifying the normal at every vortex point of the mesh.

3.3 CONTACT AND FRICTION

The contact problem during the SPF process is complex. Contact between the sheet and the die is highly nonlinear because of its asymmetry, where at a position in space a node is either free or rigidly constrained depending on an infinitesimal change of position normal to the die surface. The boundary conditions dramatically change as a result of change in the contact region, which evolves continually and unpredictably [42].

An extended version of classical isotropic Coulomb friction model provided in ABAQUS is used during the present SPF analysis. ABAQUS defines contact between two bodies in terms of two surfaces that may interact; these surfaces are called a “contact pair”. For each node on the first surface (the “slave” surface) ABAQUS attempts to find the closest point on the second surface (the “master” surface) of the contact pair where the master surface’s normal passes through the node on the slave surface. This is illustrated in Figure 3.3. The interaction is then discretized between the point on the master surface and the slave node. During the analysis it is essential that rigid surface must always be the master surface and the deformable bodies must be the slave surface.

Figure 3.3: Contact and initial discretization [42].
If during iteration a slave node is found to have penetrated the master surface by more
than a specific distance, ABAQUS abandons the increment and tries again with a smaller
increment size. This distance is known as HCRIT. The default value of HCRIT is the radius of
the sphere that circumscribes a characteristic surface element face.

3.4 LOADING AND BOUNDARY CONDITIONS

Superplasticity is exhibited by materials only in a narrow strain rate range with an
optimum value unique to each material. This factor makes it essential to determine the pressure
loading history in order to maintain the maximum strain rate near the optimum value throughout
the whole forming process.

This control formulation is implemented in ABAQUS by means of solution-dependent
amplitude. The applied pressure $P$ is to be varied throughout the simulation to maintain the strain
rate $\dot{\epsilon}$ at a predetermined value $\dot{\epsilon}_{op}$. During an increment, ABAQUS calculates $\gamma_{max}$, the ratio of
the equivalent strain rate to the target optimal strain rate for any integration point in a specified
element set. This element set is selected on the basis of the control scheme used i.e. constant
strain rate control in the free forming region, constant strain rate control in the die entry region,
maximum variable strain rate control or strain rate gradient control.

$$\gamma_{max} = \frac{\dot{\epsilon}}{\dot{\epsilon}_{op}}$$

Assuming all quantities are known at increment $n$, the pressure algorithm is developed as
follows:

\[
\begin{align*}
\text{if } \gamma_{max} < 0.2, & \quad \text{then } P_{r+1} = 2.0P_r \\
\text{if } \gamma_{max} > 3.0, & \quad \text{then } P_{r+1} = 0.5P_r \\
\text{if } 0.2 \leq \gamma_{max} < 0.5, & \quad \text{then } P_{r+1} = 1.5P_r \\
\text{if } 0.5 \leq \gamma_{max} < 0.8, & \quad \text{then } P_{r+1} = 1.2P_r \\
\text{if } 0.8 \leq \gamma_{max} < 1.5, & \quad \text{then } P_{r+1} = P_r \\
\text{if } 1.5 \leq \gamma_{max} \leq 3.0, & \quad \text{then } P_{r+1} = 0.5P_r \\
\end{align*}
\]

where $P_{r+1}$ is the new pressure value corresponding to the iteration $r+1$ and $P_r$ is the old pressure
value corresponding to the iteration $r$. Although the controlling algorithm is simple and relatively
crude, it helps to obtain the desired pressure time profile at a low computational cost.
The pressure profile obtained at each increment during the FE analysis is applied on the free forming region of the sheet. The clamped region of the sheet is represented by constraining all degrees of freedom of the nodes along the periphery of the sheet. Symmetric boundary conditions are applied to the nodes on the plane of symmetry. At the plane of symmetry the displacement in the direction perpendicular to the plane must be zero. The die surface is completely fixed with respect to all degrees of freedom.

3.5 CONSTITUTIVE EQUATION

The constitutive model used in this study is based on the continuum theory of viscoplasticity with internal variables. Details on the development of the model are given elsewhere [1-7]. The simplified 1-D form of the generalized model is given by:

\[
\dot{\varepsilon} = \frac{C_i}{d^p} \left[ \left( \frac{\sigma}{(1 - f_a)} \right)^m - \sigma_a \right]^n + C_{ii} \left( \frac{\sigma}{(1 - f_a)} \right)^n
\]

where \(\dot{\varepsilon}\) is the strain-rate, \(\sigma\) is the flow stress, \(\sigma_a\) is the threshold stress, \(m\) is the strain rate sensitivity index, \(n\) is the stress exponent, \(p\) is the grain growth exponent, \(d\) is the average grain size, \(f_a\) is the area fraction of cavities and \(C_i\) & \(C_{ii}\) are material constants. Equation 3.13 accounts for microstructural evolution if \(d\) and \(f_a\) are updated during deformation, which can be simply achieved by introducing evolution equations for grain size and cavitation.

3.5.1 Grain Growth Equation

The grain growth model employed here is similar to the one used by Hamilton et al [1], where both the static and deformation-enhanced (dynamic) growth are taken into account. The static grain growth is assumed to follow the kinetics of particle stabilized growth rates and is used to account for thermal exposure during SPF process. Clark and Alden [43] proposed a model for the deformation enhanced grain growth kinetics, which assumes that the grain boundary mobility is increased due to an increase in the grain boundary vacancy concentration resulting from grain boundary sliding. The static \((\dot{d}_s)\) and dynamic \((\dot{d}_d)\) grain growth mechanisms are assumed to be independent and the total grain growth rate \((\dot{d})\) is given by:
\[
\dot{d} = \dot{d}_s + \dot{d}_D = \frac{k_s}{d_s} + \frac{k_d \tau \dot{\varepsilon}}{d_s} \left(1 - \exp\left(-\frac{t}{\tau}\right)\right)
\]

(3.14)

where \( t \) is time, \( k_s, g, k_d \) and \( \tau \) are material constants.

3.5.2 Void Growth Equation

Most superplastic alloys develop internal cavitation during deformation [44-45]. Cavitation is the phenomenon of internal void formation, which generally occurs within metallic materials during secondary working and superplastic deformation. Excessive cavitation not only causes premature failure, but also imposes significant limitations on the industrial use of superplastically formed parts. Cavity growth is a result of diffusion-controlled mechanism or plasticity controlled mechanism. Diffusional cavity growth rate is stress-dependent and drops sharply after a rapid growth rate. Eventually, void growth rate during superplastic deformation is dominated by plastic flow of the surrounding matrix. Because of the large deformation associated with superplasticity, we only consider void growth that is dominated by plasticity-controlled mechanism and is given by Equation 3.15 [46]:

\[
f_a = f_{ao} \exp(\psi \varepsilon)
\]

(3.15)

where \( f_{ao} \) and \( f_a \) are the initial and instantaneous area fraction of voids, respectively, and \( \psi \) is the void growth parameter which is a function of the strain rate sensitivity index \( (m) \).

3.5.3 Stability Criterion

The amount of stable and uniform deformation is limited by the onset of localized necking and cavitation. The condition for stable deformation as defined by Hart [47] is given by:

\[
\left(\frac{d\dot{A}}{dA}\right)_p \leq 0
\]

(3.16)

where \( d\dot{A} \) is the variation in the area increment rate and \( dA \) is the variation in cross-section area. Assuming that the stress is a function of strain and strain rate only (not accounting for microstructural aspects), Hart derived a stability criterion for a uniaxial loading case, which has the following form:

\[
\gamma + m \geq 1
\]

(3.17)
where $\gamma$ is the strain hardening exponent and $m$ is the strain rate sensitivity index. Incorporating the modified constitutive Equation 3.13 along with the evolution Equations 3.14 & 3.15 into the framework of Hart’s analysis, a new stability criterion accounting for both geometrical instabilities and microstructural aspects is developed:

$$\gamma' + m' + \zeta' \geq 1$$

$$\gamma' = \frac{\partial}{\partial \varepsilon} \left( \frac{\partial \varepsilon}{\partial d} \right)_{\sigma,f} ; \quad m' = \frac{\sigma}{\dot{\varepsilon}} \left( \frac{\partial \varepsilon}{\partial \sigma} \right)_{d,f} ; \quad \zeta' = \frac{\psi * f_{a}}{\dot{\varepsilon}} \left( \frac{\partial f_{a}}{\partial f_{a}} \right)_{\sigma,d}$$

(3.18)

where $\gamma'$ represents strain hardening due to grain coarsening, $m'$ represents strain rate sensitivity and $\zeta'$ represents cavitation.

### 3.6 WRITING USER SUBROUTINES WITH ABAQUS

ABAQUS provides users with an array of user subroutines that allows them to adapt ABAQUS with particular analysis requirements. The above constitutive equations are implemented into the FE solver ABAQUS through user defined subroutine CREEP.

![Figure 3.4: Detailed flow of ABAQUS/Standard [42].](image)
Figure 3.4 shows the basic flow of data and actions from the start of an ABAQUS analysis to the end of a step. CREEP subroutine is used to define time dependent viscoplastic material behavior. The user subroutine must define the increment of inelastic strain, as a function of stress and the time increment. Other variables such as grain growth and cavitations are defined as solution dependent state variables (SDV). SDV’s are values that can be defined to evolve with the solution of the analysis. The SDV’s are initialized using the SDVINI subroutine and its evolution is calculated within the CREEP subroutine. The interface to user subroutine CREEP is:

```
SUBROUTINE CREEP (DECRA, DESWA, STATEV, SERD, EC0, ESW0, P, QTILD,
1 TEMP, DTEMP, PREDEF, DPRED, TIME, DTIME, CMNAME, LEXIMP, LEND,
2 COORDS, NSTATV, NOEL, NPT, LAYER, KSPT, KSTEP, KINC)
C
INCLUDE 'ABA_PARAM.INC'
CHARACTER*80 CMNAME
C
DIMENSION DECRA (5), DESWA (5), STATEV (*), PREDEF (*), DPRED (*),
1 TIME (2), COORDS (*)

User coding to define the stress strain relationship.

RETURN
END
```

The variables to be defined are:
- DECRA (1): Deviatoric creep strain increment.
- DESWA (1): Volumetric strain increment.
- DECRA (5): $\partial \Delta \varepsilon / \partial \Delta \sigma$.

The variables passed in for information are:
- QTILD: Effective stress
• TIME (2): Value of total time at the end of the increment.
• DTIME: Time Increment.
• NOEL: Element number.
• NPT: Integration point number.
• KSTEP: Step number.
• KINC: Increment number
CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 SUPERPLASTIC BULGE FORMING

The finite element model for simulating superplastic forming of a Ti-6Al-4V sheet into a hemisphere is shown in Figure 4.1. The free forming region of the sheet is 76.2 mm in diameter, with a 3.9 mm flange around it, and the initial sheet thickness is 1.98 mm.

Taking advantage of the symmetry, a quarter of the blank is modeled using 885 quadrilateral elements (M3D4). These elements are fully integrated bilinear membrane elements. The sheet is clamped along the circumference and symmetric boundary conditions are applied along the axis of symmetry. The flat initial configuration of the membrane model is entirely singular in the normal direction, unless it is stressed in biaxial tension. This problem is overcome by applying a very small initial biaxial stress on the surface of the sheet. The material parameters used in the constitutive models are for Ti-6Al-4V at 900 °C and are given in [1, 19]. In addition, the experimental work of Hamilton et al. [1] was used to calibrate these models.

Two approaches are used in the finite element analysis. The first approach uses a forming pressure profile based on a constant strain rate at the pole of the sheet. Four different strain rates of 1E-4, 5E-4, 1E-3, and 1E-2 (1/s) were used covering the superplastic region range. The second approach uses an optimum forming pressure profile based on variable strain rates derived from the multi-scale failure criterion described in Chapter 3. In all simulations, forming is carried

Figure 4.1: Finite element model for superplastic bulge forming :a) Geometry of the assembly, b) FE Mesh used for the analysis.
out till the bulge height was equal to the radius of the sheet. The forming pressure profiles for all simulations are shown in Figure 4.2. It is observed that higher gas pressure is required for higher target strain rate forming. This is because of the higher flow stress associated with higher strain rate. The variation of the dome height with forming time for all simulations is shown in Figure 4.4. The sheet thickness variation with time at the pole of the bulge, for various strain rates is shown in Figure 4.5. The forming time required and the pole thickness obtained at the end of deformation for all simulations are listed in Table I. The thickness distribution of the formed sheets along a radial line passing through the pole is shown in Figure 4.6. The bulge profile obtained at various strain rates is shown in figure 4.7. It is seen that forming at a lowest strain rate (1E-4 (1/s)) generates a more uniform curvature of the bulge compared to the profiles obtained at higher strain rates and the formed sheet shows the most uniform thickness distribution. However, it took 11380 seconds to form the dome having a height of 3.81 cm as compared to 140 seconds required for forming at a strain rate of 1E-2 /s. The above results show that conventional methods of superplastic forming can help obtain products having good structural integrity; however at the cost of large forming time.

In order to reduce the forming time the analysis was repeated using the optimum pressure profile. Using this approach the forming time obtained was 880 seconds, significantly reduced from 11380 seconds, and the uniformity of the thickness distribution was maintained. These results are graphically shown in Figure 4.8, where the thickness distribution of the sheet at the end of deformation is shown. In order to quantify the uniformity of thickness distribution, the thinning factor is calculated and shown in Table II. Thinning factor is defined as the ratio between the thickness at the pole and the average sheet thickness [48]. Higher thinning factor indicates more uniform thickness distribution. Due to the symmetric nature of the model, the average thickness was obtained by dividing the sum of the thicknesses at equidistant points along the radius of the dome by the number of points. The results summarized in Table II clearly highlight the benefits obtained by using the optimum forming pressure profile. The thinning factor for the sheet that was formed at a strain rate of 1x10^{-4} s^{-1} is approximately 0.77. The thinning factor for the sheet that was formed using the optimum forming pressure profiles is about 0.7, slightly less than 0.77. However, the forming time was considerably reduced from 11380 to 880 seconds.
Figure 4.2: Pressure-Time profiles at different target strain rates, for superplastic bulge forming of Ti-6Al-4V alloy.

Figure 4.3: Stress-strain curves at different target strain rates, for superplastic bulge forming of Ti-6Al-4V alloy.
Figure 4.4: Variation of bulge height with time at different target strain rates, for superplastic bulge forming of Ti-6Al-4V alloy.

Figure 4.5: Variation of sheet thickness at the pole with time, at different target strain rates, for superplastic bulge forming of Ti-6Al-4V alloy.
Figure 4.6: Variation of sheet thickness along the radius of the dome at different target strain rates, for superplastic bulge forming of Ti-6Al-4V alloy.

Figure 4.7: Bulge profile at different target strain rates, for superplastic bulge forming of Ti-6Al-4V alloy
Table I. Summary of the results obtained, during bulge forming analysis, at various strain rates

<table>
<thead>
<tr>
<th>Strain rate:</th>
<th>Forming time</th>
<th>Dome height</th>
<th>Pole thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1E-4/s</td>
<td>11380 s</td>
<td>3.81 cm</td>
<td>0.0719 cm</td>
</tr>
<tr>
<td>5E-4/s</td>
<td>2079 s</td>
<td>3.81 cm</td>
<td>0.0697 cm</td>
</tr>
<tr>
<td>1E-3/s</td>
<td>1070 s</td>
<td>3.81 cm</td>
<td>0.066 cm</td>
</tr>
<tr>
<td>1E-2/s</td>
<td>140 s</td>
<td>3.81 cm</td>
<td>0.048 cm</td>
</tr>
<tr>
<td>Optimum</td>
<td>882 s</td>
<td>3.81 cm</td>
<td>0.065 cm</td>
</tr>
</tbody>
</table>

Table II. Thinning factor calculated in the formed hemisphere, at various strain rates

<table>
<thead>
<tr>
<th>Strain rate:</th>
<th>Pole thickness</th>
<th>Average thickness</th>
<th>Thinning Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1E-4/s</td>
<td>0.0719 cm</td>
<td>0.0924 cm</td>
<td>0.778</td>
</tr>
<tr>
<td>1E-2/s</td>
<td>0.0481 cm</td>
<td>0.0948 cm</td>
<td>0.507</td>
</tr>
<tr>
<td>Optimum</td>
<td>0.0647 cm</td>
<td>0.0922 cm</td>
<td>0.7017</td>
</tr>
</tbody>
</table>

Figure 4.8: Sheet thickness distribution at different target strain rates, for superplastic bulge forming of Ti-6Al-4V alloy.

A] 1E-4 (1/s)  
B] Optimum (1/s)  
C] 1E-3 (1/s)  
D] 1E-2 (1/s)
4.2. VERIFICATION OF DUTTA AND MUKHERJEE’S PRESSURE EQUATION

Superplastic forming is highly sensitive to strain rate, thus it is essential to generate an accurate pressure-time profile to maintain the desired strain rate, during deformation. Dutta and Mukherjee [15] devised a relationship based on biaxial stress conditions to maintain a constant strain rate during superplastic blow forming of a hemisphere.

\[
P = 4 \frac{S_0}{a} \dot{\sigma} \exp(-\dot{\varepsilon})[\dot{\sigma} \exp(-\dot{\varepsilon}) (1 - \dot{\sigma} \exp(-\dot{\varepsilon}))]^{1/2}
\]

- \(P = \text{Pressure}\)
- \(t = \text{Time}\)
- \(\dot{\sigma} = \text{Effective Stress}\)
- \(S_0 = \text{Initial Sheet Thickness}\)
- \(a = \text{Die Radius}\)
- \(\dot{\varepsilon} = \text{Effective Strain rate}\)

The pressure profile obtained using the above equation is compared with the pressure profile generated by using Abaqus, during the bulge forming analysis of titanium alloy at a constant strain rate of 1E-4 (1/s). The initial sheet thickness and the die radius used for the analysis and the calculation of pressure profile are 1.98 cm and 3.81 cm respectively.

Figure 4.9 shows that it is essential to update the effective stress continuously in Dutta and Mukherjee’s pressure equation, in order to account for strain hardening. The pressure profile obtained without considering strain hardening, i.e. keeping the effective stress constant, generates a very low pressure curve.

For a given strain rate, Dutta and Mukherjee’s equation (with strain hardening) overestimates the pressure required to maintain a constant strain rate during superplastic bulge forming. This can be attributed to the fact that in Dutta and Mukherjee’s model the thickness variation along the dome profile occurring during the deformation was not taken into account, resulting in overestimation of the pole thickness. This leads to an over estimation of the pressure required in order to maintain the given rate of deformation. This effect is more predominant at higher strain rates as there is greater localized thinning at the pole. Secondly, in Dutta and Mukherjee’s model, the radius of curvature is assumed to decrease continuously with time, during deformation. This overestimates the decrease of thickness after the radius of curvature equals the radius of the sheet, thus resulting in the drop in pressure eventually, as can be seen in Figure 4.9.

The pressure profile obtained from Mukherjee’s equation is integrated with ABAQUS to carryout a blow forming analysis of a circular sheet into a hemisphere having a bulge height...
equal to the radius of the sheet. The values of strain rate obtained during the deformation at the pole using Mukherjee’s pressure profile were compared with the values of strain rate obtained by using ABAQUS predicted pressure profile. It is seen from Figure 4.10 that using Mukherjee’s pressure profile the resulting strain rate obtained was in the region of 1.1E-4 /s, slightly higher than the desired strain rate of 1E-4 /s. However the strain rate profile obtained was more uniform than that obtained by using ABAQUS predicted pressure profile.

The previous hemispherical blow forming analysis was repeated for a constant strain rate of 5E-4, 1E-3 and 1E-2 /s. For superplastic forming involving higher strain rates Mukherjee’s model failed to maintain a constant strain rate. This can be observed in Figures 4.12, 4.14 and 4.16. The pressure profiles generated by ABAQUS at a strain rate of 1.0E-2, 1.0E-3 and 5E-4 /s is compared with those obtained by Dutta and Mukherjee’s model in Figures 4.11, 4.13 and 4.15 respectively. The time required to reach a dome height of 3.81 cm, during the above analysis is listed in Table III.

Dutta and Mukherjee’s analytical model is applicable only to hemispherical blow forming process. The pressure profiles obtained using Dutta and Mukherjee’s model, were used earlier for research in superplastic forming. However, advances in finite element analysis tools such as ABAQUS have helped to replace analytical models. FEA is a preferred choice for generating pressure profiles mainly because of its accuracy and compatibility with complex shapes.

Table III. Comparison of the forming time required using finite element analysis and Dutta and Mukherjee’s equation.

<table>
<thead>
<tr>
<th>Strain Rate (1/s)</th>
<th>$\dot{\varepsilon}$ : 1E-4</th>
<th>$\dot{\varepsilon}$ : 5E-4</th>
<th>$\dot{\varepsilon}$ : 1E-3</th>
<th>$\dot{\varepsilon}$ : 1E-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dome height (cm)</td>
<td>3.81 cm</td>
<td>3.81 cm</td>
<td>3.81 cm</td>
<td>3.81 cm</td>
</tr>
<tr>
<td>Forming time (s)</td>
<td>ABAQUS 11380 s</td>
<td>2079 s</td>
<td>1070 s</td>
<td>140 s</td>
</tr>
<tr>
<td>Forming time (s)</td>
<td>Mukherjee’s Model 11000s</td>
<td>1648 s</td>
<td>768 s</td>
<td>50 s</td>
</tr>
</tbody>
</table>
Figure 4.9: Comparison between ABAQUS generated pressure profile and the pressure profile generated using Mukherjee’s analytical equation for $\dot{\varepsilon} = 1E-4$ (1/s).

Figure 4.10: Strain rate profile obtained using ABAQUS and Mukherjee’s model $\dot{\varepsilon} = 1E-4$ (1/s).
Figure 4.11: Comparison between ABAQUS generated pressure profile and the pressure profile generated using Mukherjee’s analytical equation for $\dot{\varepsilon} = 5 \times 10^{-4}$ (1/s).

Figure 4.12: Strain rate profile obtained using ABAQUS and Mukherjee’s model $\dot{\varepsilon} = 5 \times 10^{-4}$ (1/s).
Figure 4.13: Comparison between ABAQUS generated pressure profile and the pressure profile generated using Mukherjee’s analytical equation for $\dot{\varepsilon} = 1E-3$ (1/s).

Figure 4.14: Strain rate profile obtained using ABAQUS and Mukherjee’s model $\dot{\varepsilon} = 1E-3$ (1/s).
Figure 4.15: Comparison between ABAQUS generated pressure profile and the pressure profile generated using Mukherjee’s analytical equation for $\dot{\varepsilon} = 1E-2$ (1/s).

Figure 4.16: Strain rate profile obtained using ABAQUS and Mukherjee’s model $\dot{\varepsilon} = 1E-2$ (1/s).
4.3 SUPERPLASTIC BOX FORMING

Box forming model consists of a sheet 64 cm long and 44 cm wide with a thickness of 0.3175 cm. This sheet is placed over a box die having a depth of 20 cm. The sheet is modeled with the help of 704 quadrilateral membrane elements. 231 Rigid elements of the type R3D3 are used to model the female die. The female die is extended along the axis of symmetry to avoid the contacting nodes from sliding off the master surface. The finite element model of the die and the sheet assembly is shown in Figure 4.17. Coulomb’s coefficient of friction is defined between the sliding surfaces of the die assembly. The sheet is clamped along the circumference and symmetric boundary conditions are applied along the axis of symmetry.

![Figure 4.17: Finite element model for superplastic box forming: a) Geometry of the assembly b) FE Mesh of the initial and deformed sheet.](image)

The material parameters used in the constitutive models are for Ti-6Al-4V at 900 °C and are given in [1, 19]. In addition, the experimental work of Hamilton et al. [1] was used to calibrate these models.

Two approaches are used in the finite element analysis. The first approach uses a forming pressure profile based on a constant strain rate at the pole of the sheet. Four different strain rates of 1E-4, 5E-4, 1E-3, and 1E-2 /s were used covering the superplastic region range. The second approach uses an optimum forming pressure profile based on variable strain rates derived from the multi-scale failure criterion described in Chapter 3. In all simulations, forming is carried out till the sheet takes the shape of the die. Target strain rate is maintained by controlling the deformation of the corner node which lies in the region of maximum deformation.
The forming pressure profiles for all simulations are shown in Figure 4.18. The pressure profiles contained three distinctive stages.

- There was a gradual increase in pressure for approximately 15% of the forming time.
- The pressure remained constant for approximately 40% of the time thereafter.
- Finally during the later half of the forming time there was a gradual increase in the required pressure.

The rise in the pressure within the first stage is due to the rapid increase in stresses, mainly due to grain growth and isotropic hardening within the sheet material. Gradually, due to the balance between the effects of thinning and hardening within the materials, the pressure becomes steady. Finally as the sheet comes in contact with the bottom surface of the die, the deformation is restricted locally and the effective area on which the gas pressure is applied becomes smaller. Thus to maintain the target strain rate within the sheet, greater gas pressure is required. It is observed that higher gas pressure is required for higher target strain rate forming. This is because of the higher flow stress associated with higher strain rate. Figure 4.21 shows the variation of Von Mises stress with strain observed during the box forming analysis of Ti-6Al-4V.

The deviation of the actual strain rate from the target strain rate is seen in Figure 4.20. Some variation is observed due to crude nature of the strain rate control algorithm integrated within ABAQUS. It is also difficult to maintain the exact target strain rate due to varying geometric property of the entire component. The thickness variation over the deformed sheet is shown in Figure 4.19. As expected, greater localized thinning is observed in the sheet formed at a higher target strain rate. The most uniform thickness distribution was achieved using the target strain rate forming of 1E-4 (1/s). However it took 14150 seconds for the complete box forming operation as compared to 189 seconds required by the target strain rate forming of 1E-2 (1/s).

In order to reduce the forming time without affecting the thickness distribution of the formed part, the analysis was repeated using the optimum, variable target strain rate profile. This variable strain rate profile is derived from the multiscale failure criterion described in Chapter 3 and is shown in Figure 4.22. Using this approach the forming time obtained was 2860 seconds, significantly reduced from 14150 seconds, and the uniformity of the thickness distribution was maintained. These results are graphically shown in Figure 4.23, where the thickness distribution of the sheet at the end of deformation can be observed. Table VI lists the time required for the
complete box forming process at various strain rates with the maximum and minimum thickness achieved in the deformed sheet.

Table IV. Summary of the results obtained, during box forming at various strain rates

<table>
<thead>
<tr>
<th>Strain Rate (1/s)</th>
<th>1E-4</th>
<th>5E-4</th>
<th>1E-3</th>
<th>1E-2</th>
<th>Opt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forming Time (s)</td>
<td>14150</td>
<td>2910</td>
<td>1440</td>
<td>189</td>
<td>2860</td>
</tr>
<tr>
<td>Original Thickness (cm)</td>
<td>0.318</td>
<td>0.318</td>
<td>0.318</td>
<td>0.318</td>
<td>0.318</td>
</tr>
<tr>
<td>Maximum Thickness (cm)</td>
<td>0.265</td>
<td>0.267</td>
<td>0.269</td>
<td>0.298</td>
<td>0.269</td>
</tr>
<tr>
<td>Minimum Thickness (cm)</td>
<td>0.109</td>
<td>0.109</td>
<td>0.104</td>
<td>0.077</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Figure 4.18: Pressure-Time profiles at different target strain rates for superplastic box forming of Ti-6Al-4V alloy.
Figure 4.19: Variation of sheet thickness with time in the region of maximum deformation, at different target strain rates, for superplastic box forming of Ti-6Al-4V alloy.

Figure 4.20: Strain rate in the region of maximum deformation during the analysis at different target strain rates, for superplastic box forming of Ti-6Al-4V alloy.
Figure 4.21: Von Mises stress-strain curves at different target strain rates, for superplastic box forming of Ti-6Al-4V alloy.

Figure 4.22: Optimum Variable strain rate path for superplastic forming Ti-6Al-4V alloy.
Figure 4.23: Sheet thickness distribution in the box formed at different target strain rates.
4.4 EFFECT OF INITIAL GRAIN SIZE ON SUPERPLASTIC FORMING PROCESS

One important property of superplastic alloys is a fine-grained microstructure. Thus superplastic forming analysis requires a microstructural based constitutive equation to accurately represent the alloy under consideration. In the previous section of this work an initial grain size of 4 microns was considered. During the present analysis the initial grain size is increased to 8 microns and its effect on the forming process is studied in detail.

Figures 4.24, 4.25, and 4.26 shows the gas-pressure histories obtained from the analysis using an initial grain size of 4 microns and 8 microns at a strain rate of 1E-4 (1/s), 1E-3 (1/s) and 1E-2 (1/s) respectively. In all the cases it is seen that the pressure required in maintaining a constant strain rate increases with an increase in grain size. This is because there is greater hardening due to coarser average grain size in the formed sheet material. Highest gas pressure is required, for the grain size of 8 microns, just before the forming is completed. This is because the last part of the sheet that comes into contact with the die, the three sided corner surface of the die, possesses the greatest thickness and large grain size (largest grain growth). Thus large gas pressure is required to form the material at that specified target strain rate.

The variation of sheet thickness, with the increase in the initial grain size is shown in Figure 4.27. It is seen that there is an adverse effect of grain size on the thickness of the formed sheet. Greater localized thinning is observed with an increase in the initial grain size. Since the largest strain occurs at the corner of the formed box, the greatest grain growth also occurs in this region and, as a result, causes the most severe localized thinning. The maximum grain growth was 6.38 microns for the analysis having the initial grain size of 4 microns and 8.50 microns for the analysis having the initial grain size of 8 microns. The grain size gradient obtained after the complete box forming process is shown in Figure 4.28.

For the target strain rate of 1E-4 (1/s) the forming time is increased by 5.3%, with an increase in the grain size from 4 microns to 8 microns. This was very less as compared with an increase of 25.7% and 15.9% using a target strain rate of 1E-3 (1/s) and 1E-4 (1/s) respectively. Table V lists the time required for the complete box forming process using an initial grain size of 4 microns and 8 microns.
Table V. Comparison of the results obtained using different initial grain sizes, during superplastic forming.

<table>
<thead>
<tr>
<th>Grain size</th>
<th>( \dot{\varepsilon} : 1E-4 ) (1/s)</th>
<th>( \dot{\varepsilon} : 1E-3 ) (1/s)</th>
<th>( \dot{\varepsilon} : 1E-2 ) (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forming time (s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 ( \mu )</td>
<td>14150</td>
<td>1440</td>
<td>189</td>
</tr>
<tr>
<td>8( \mu )</td>
<td>14900</td>
<td>1810</td>
<td>219</td>
</tr>
<tr>
<td>Original Thickness (cm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 ( \mu )</td>
<td>0.318</td>
<td>0.318</td>
<td>0.318</td>
</tr>
<tr>
<td>8( \mu )</td>
<td>0.318</td>
<td>0.318</td>
<td>0.318</td>
</tr>
<tr>
<td>Maximum Thickness (cm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 ( \mu )</td>
<td>0.265</td>
<td>0.269</td>
<td>2.98</td>
</tr>
<tr>
<td>8( \mu )</td>
<td>0.282</td>
<td>0.306</td>
<td>0.318</td>
</tr>
<tr>
<td>Minimum Thickness (cm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 ( \mu )</td>
<td>0.109</td>
<td>0.104</td>
<td>0.077</td>
</tr>
<tr>
<td>8( \mu )</td>
<td>0.104</td>
<td>0.077</td>
<td>0.061</td>
</tr>
</tbody>
</table>

Figure 4.24: Variation of pressure profile with time due to change in initial grain size of the sheet, during deformation at a constant strain rate of 1E-4 (1/s).
Figure 4.25: Variation of pressure profile with time due to change in initial grain size of the sheet, during deformation at a constant strain rate of 1E-3 (1/s).

Figure 4.26: Variation of pressure profile with time due to change in initial grain size of the sheet, during deformation at a constant strain rate of 1E-2 (1/s).
<table>
<thead>
<tr>
<th>Initial Grain Size: 4 Microns</th>
<th>Initial Grain Size: 8 Microns</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{\varepsilon} = 1E - 4 ) (1 / s)</td>
<td>( \dot{\varepsilon} = 1E - 4 ) (1 / s)</td>
</tr>
<tr>
<td>( \dot{\varepsilon} = 1E - 3 ) (1 / s)</td>
<td>( \dot{\varepsilon} = 1E - 3 ) (1 / s)</td>
</tr>
<tr>
<td>( \dot{\varepsilon} = 1E - 2 ) (1 / s)</td>
<td>( \dot{\varepsilon} = 1E - 2 ) (1 / s)</td>
</tr>
</tbody>
</table>

Figure 4.27: Resulting thickness distribution (cm) in the formed sheet, for an initial grain size of, a) 4µ and b) 8µ.
Figure 4.28: Resulting microstructure of the formed sheet, using an initial grain size of, a) 4 µ and b) 8µ.
4.5 EFFECT OF FRICTION ON SUPERPLASTIC FORMING PROCESS

Various mechanisms such as adhesion, mechanical interaction of surface asperities and ploughing of one surface by asperities on the other, are recognized mechanisms of friction [49]. According to the Coulomb’s friction model, the two contacting surfaces carry shear stresses up to a certain magnitude across their interface before they start sliding. The critical shear stress \( \tau_c \) at which the sliding starts is a fraction of the gas pressure applied over the sheet surface. This fraction is known as the coefficient of friction \( \mu \).

The basic form of Coulomb’s friction model is utilized in the superplastic box analysis to understand the effect of friction on the sheet thickness distribution and the pressure profile generated, for a given target strain rate. The analysis is carried out at three different target strain rates of 1E-4 (1/s), 1E-3 (1/s) and 1E-2 (1/s) and the coefficient of friction is varied from 0.05 to 4 for each analysis. The pressure profiles obtained are given in Figure 4.29, 4.30 and 4.31 and the final thickness distribution is shown in Figure 4.32.

In the superplastic box forming process, the sheet is clamped along its circumference while the rest of it flows into the die cavity, driven by gas pressure. Contact first occurs at the flange area and the sheet gradually stretches and slides over the die surface. Thinning of the sheet under predominant tensile stress takes place against friction resistance from the contact between the sheet and the die surface. A gradual increase in the pressure causes the critical shear stress to reach the value of shear flow stress \( k \) of the alloy. When \( \tau_c = \mu = k \) sticking occurs, and subsurface flow takes place. This adversely affects the uniformity of deformation and thickness distribution in the formed component.

As discussed previously, the strain rate during box forming is maintained near the target value by controlling the deformation of the sheet at the region of maximum straining i.e. the corner region of the die. Due to an increase in the coefficient of friction greater localized thinning takes place in this region as seen in Figure 4.32. The pressure required to maintain the desired strain rate is reduced. This neutralizes the increase in pressure due to greater frictional resistance. Thus the maximum value of pressure required to form the complete box is unaffected with the increase in the coefficient of friction during superplastic forming as seen from figures 4.29, 4.30 and 4.31. The total time required for the complete box forming operation at various strain rates is listed in Table VI.
Table VI. Comparison of the results obtained, using different coefficient of friction during superplastic forming.

<table>
<thead>
<tr>
<th>Coefficient of Friction (µ)</th>
<th>Forming time (s)</th>
<th>Original Thickness (cm)</th>
<th>Maximum Thickness (cm)</th>
<th>Minimum Thickness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficient of Friction (µ)</th>
<th>1E-4 (1/s)</th>
<th>1E-3 (1/s)</th>
<th>1E-2 (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>14500</td>
<td>1500</td>
<td>190</td>
</tr>
<tr>
<td>0.4</td>
<td>17300</td>
<td>1750</td>
<td>224</td>
</tr>
<tr>
<td>0.05</td>
<td>0.318</td>
<td>0.318</td>
<td>0.318</td>
</tr>
<tr>
<td>0.4</td>
<td>0.318</td>
<td>0.318</td>
<td>0.318</td>
</tr>
<tr>
<td>0.05</td>
<td>0.274</td>
<td>0.275</td>
<td>0.301</td>
</tr>
<tr>
<td>0.4</td>
<td>0.311</td>
<td>0.312</td>
<td>0.315</td>
</tr>
<tr>
<td>0.05</td>
<td>0.105</td>
<td>0.097</td>
<td>0.069</td>
</tr>
<tr>
<td>0.4</td>
<td>0.081</td>
<td>0.077</td>
<td>0.058</td>
</tr>
</tbody>
</table>

Figure 4.29: Variation of pressure profile with time due to change in coefficient of friction between the sheet and the die, during deformation at a constant target strain rate of 1E-4 (1/s).
Figure 4.30: Variation of pressure profile with time due to change in coefficient of friction between the sheet and the die, during deformation at a constant target strain rate of 1E-3 (1/s).

Figure 4.31: Variation of pressure profile with time due to change in coefficient of friction between the sheet and the die, during deformation at a constant target strain rate of 1E-2 (1/s).
Figure 4.32: Resulting thickness distribution (cm) in the formed sheet with change in coefficient of friction, a) 0.05 and b) 0.4.
4.6 APPLICATION OF SUPERPLASTIC FORMING: DENTAL IMPLANT

Recently, superplastic forming has started making inroads into the medical field for the fabrication of medical devices and implants. Figure 4.33 show two examples of dental parts made of Ti alloys, produced by SPF [50]. Because of the complexity of shapes associated with these applications, SPF offers unique capabilities over conventional forming processes.

![Figure 4.33: Application of SPF in medical industry: a) Partial upper denture, b) Dental implant superstructure](image)

Because medical implants and specialized medical devices are usually custom-made, production speed (the main obstacle of SPF) is of secondary importance. This makes the medical field one of the most promising areas in utilizing SPF. In addition, the need for biocompatible metals like titanium alloys, which are hard to form by conventional methods, make SPF a unique technique to form such materials into very complicated shapes, like the two examples shown in Figure 4.33.

For SPF to be accepted as an efficient forming technique, accurate models of deformation and failure are needed to design optimum forming practices using FE analysis. Recently, Khraisheh et al developed a generalized constitutive model for superplastic deformation, in which both microstructural evolution and anisotropy of the material are taken into account. In addition, a multiscale failure criterion was developed and then used to optimize the superplastic forming of Ti-6Al-4V alloy [1-7].

For illustration purposes, and to show the capabilities of the model and the failure criterion, we simulate the superplastic forming of a dental implant superstructure made of Ti-6Al-4V alloy, similar to the one shown in Figure 4.33. The sheet is formed onto the die using
pressurized gas. Two approaches are used in the finite element analysis. The first approach uses a forming pressure profile based on a constant strain rate in the sheet. The second approach uses an optimum forming pressure profile derived from the multiscale failure criterion developed earlier.

The finite element analysis is carried out using ABAQUS [42]. The superplastic forming assembly consists of a sheet and a die, as shown in Fig. 4.34. Taking advantage of the symmetry, only one half of the assembly is modeled.

![Figure 4.34: The die and sheet assembly for forming dental implant superstructure.](image)

The sheet is clamped along the circumference, and symmetric boundary conditions are applied along the axis of symmetry. The flat initial configuration of the membrane model is entirely singular in the normal direction, unless it is stressed in biaxial tension. This problem is overcome by applying a very small initial biaxial stress on the surface of the sheet. Contact between the rigid die surface and the deformable superplastic sheet is defined using Coulomb friction model, with a coefficient of friction $\mu = 0.2$. The constitutive equations for the material have been implemented using a user-defined subroutine. The constitutive model used is specifically created to take into account the microstructural behavior of Ti-6Al-4V alloy. In order to carry out the forming analysis for a given target strain rate, ABAQUS uses a built in pressure control algorithm to obtain the load curve. The algorithm automatically adjusts the applied gas pressure to control the rate of deformation. The analysis is completed when the sheet takes the shape of the die.
The analysis was carried out using a target forming strain rate of 5E-4 /s (first approach), and using optimum variable strain rate path (second approach). The initial sheet thickness is 2mm. The optimum variable strain rate path is based on the failure criteria devised by Thuramalla and Khraisheh [3-4]. The optimum forming path is designed such that the deformation speed is initially large and as deformation continues the speed of deformation is reduced to avoid localized deformation. This optimum forming practice allows us to obtain uniform thickness distribution in the formed part with considerable saving in the forming time. In both approaches, the sheet is formed to the same depth level. The shape of formed sheet is shown in Figure 4.35.

![Figure 4.35: The deformed sheet obtained during SPF of dental implant superstructure.](image)

The forming pressure-time profiles according to both approaches are shown in Figure 4.36. Initially the pressure required to maintain the desired strain rate is considerably low, but later when the sheet comes in contact with the die, the required pressure increases significantly due to friction. Figure 4.37 shows the displacement and thickness distribution of the formed sheet, using the constant strain rate pressure profile and the optimum strain rate forming pressure profile. For the same displacement, more uniform thickness distribution is obtained by using the optimum strain rate profile. In order to quantify the thickness distribution, we use the thinning factor to study the uniformity of the deformed sheet. The thinning factor used here is defined as the ratio between the minimum sheet thickness and the average thickness of the sheet, similar to the one used by Cornfield and Johnson [48]. Higher thinning factor is desirable, as it reflects more uniform thickness distribution. A summary of the finite element analysis results are shown in Table VII.

It is also important to note that the forming time of the sheet using a target strain rate of 5E-4 (1/s) is more than twice the time required to form the sheet using an optimum strain rate profile. The results clearly indicate that the optimum procedure presented here not only reduces
the forming time significantly, but also maintains (and even improves) the uniformity of the deformed sheet.

Table VII. Summary of the results obtained during the finite element analysis of dental implant superstructure.

<table>
<thead>
<tr>
<th>Result</th>
<th>Strain rate: 5E-4 (1/s)</th>
<th>Optimized strain rate path</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forming time (s)</td>
<td>2600</td>
<td>1220</td>
</tr>
<tr>
<td>Min. thickness (mm)</td>
<td>0.79</td>
<td>0.8</td>
</tr>
<tr>
<td>Average thickness (mm)</td>
<td>1.34</td>
<td>1.35</td>
</tr>
<tr>
<td>Thinning factor</td>
<td>0.589</td>
<td>0.593</td>
</tr>
</tbody>
</table>

Figure 4.36: Pressure-Time profiles obtained at different target strain rates during superplastic forming of dental implant superstructure.
Figure 4.37: Displacement and thickness distribution in the deformed sheet for a target strain rate of a) 5E-4 (1/s) and b) optimum strain rate path (1/s).
CHAPTER 5

FINITE ELEMENT SIMULATION OF EUTECTIC Pb-Sn ALLOY

5.1 INTRODUCTION

In this Chapter the bulge forming performance of Pb-Sn superplastic sheet is analyzed under different target strain rate forming conditions. The results obtained using finite element analyses are compared with the experimental data obtained by Khraisheh [22]. The finite element model used is similar to the hemispherical model described in Chapter 4. The constitutive model developed by Khraisheh et al., [5] is used in this analysis. In this material model, superplastic deformation is modeled within the framework of the continuum theory of viscoplasticity; with the general associated flow rule given by:

\[ \dot{D_p} = f \frac{\partial J}{\partial \sigma_{ij}} \]  

where \( \dot{D_p} \) is the plastic strain rate tensor, \( f \) is the overstress function, \( J (\sigma - \alpha) \) is a positive scalar-valued function of the state variables having the dimensions of stress, and \( \sigma \) is the Cauchy stress tensor. Here we use the overstress function that describes the characteristics of superplastic materials, and takes the microstructural evolution into account [5.2]:

\[ f = \frac{C_i (J - (K_0 + R))^{\frac{1}{m}}}{d^p} + C_{ii} J^n \]  

where \((K_0 + R)\) is a reference stress whose variable part \((R)\) represents the isotropic hardening, \( d \) is the average grain diameter, \( p \) is the grain size exponent, \( m \) is the strain rate sensitivity index, \( C_i \), \( C_{ii} \) and \( n \) are material constants.

Anisotropic Yield Function

The following anisotropic yield function is defined in reference to the axes \( x_i \) (\( i=1, 2 & 3 \)), and it will be employed in this model [5, 6]:

\[ J = \left[ \frac{3}{2} (S - a)(S - a) + c_1 (M(S - a))^2 + c_2 (N_1(S - a))^2 + c_3 (N_2(S - a))^2 \right]^{\frac{1}{2}} \]  

\[ N_1 = a_i \otimes a_i; \quad N_2 = a_2 \otimes a_2; \quad M = \frac{J}{2} [a_i \otimes a_i + a_2 \otimes a_2] \]

63
where $S$ is the deviatoric part of Cauchy stress tensor, $c_1$, $c_2$ and $c_3$ are material constants (for $c_1=c_2=c_3=0$, the anisotropic yield function reduces to von Mises isotropic yield function). $a_1$, $a_2$ and $a_3$ ($a_3 = a_1 \times a_2$) are orthonormal vectors along the axes of anisotropy $x'_i$ ($i=1, 2 & 3$). The directions of $a_1$ & $a_2$ are defined by the angle $\phi$ between $a_1$ and $x_1$, measured positive counterclockwise from the $x_1$ direction. $M$, $N_1$ and $N_2$ are directional tensors, expressed in terms of the angle $\phi$.

**Grain Growth Equation**

The grain growth model employed here is similar to the one used by Hamilton et al [1], where both the static and deformation-enhanced (dynamic) growths are taken into account. The static grain growth is assumed to follow the kinetics of particle stabilized growth rates and is used to account for thermal exposure during SPF process. Clark and Alden [43] proposed a model for the deformation enhanced grain growth kinetics, which assumes that the grain boundary mobility is increased due to an increase in the grain boundary vacancy concentration resulting from grain boundary sliding. The static and dynamic grain growth mechanisms are assumed to be independent and the total grain growth rate is given by:

$$
\dot{d} = \dot{d}_s + \dot{d}_D = \frac{k_s}{d^g} + \frac{k_d \dot{\varepsilon}}{d^g} \left(1 - \exp \left(-\frac{-t}{\tau}\right)\right)
$$

[5.4]

where $d$ is the mean grain diameter, $k_s$, $g$, $k_d$ and $\tau$ are material constants.

**Evolution Equations for the Internal Variables**

Evolution equations for the internal variables (isotropic hardening $R$, and kinematic hardening $\alpha$) similar to those used for viscoplastic materials will be used here [5]. They include hardening, static recovery and dynamic recovery terms; and are given by the following set of equations:

$$
\dot{\alpha} = H \ddot{\varepsilon} - C_D \ddot{\varepsilon} \alpha - C_S \alpha (h(\alpha))^{a-1}
$$

[5.5]

$$
\dot{R} = H \ddot{\varepsilon} - C_D \ddot{\varepsilon} R - C_S R^a
$$

[5.6]

where $H$ is the hardening coefficient, $C_S$ is the static recovery coefficient, $C_D$ is the dynamic recovery coefficient, $\ddot{\varepsilon}$ is the effective strain rate, and $a$ is a constant. The term $h(\alpha)$ is a scalar function of the internal stress tensor, having the form of the anisotropic yield function. All the material parameters are listed in Table VIII.
Table VIII. List of the material parameters used in the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s$</td>
<td>0.0412</td>
</tr>
<tr>
<td>$G$</td>
<td>3.9</td>
</tr>
<tr>
<td>$k_d$</td>
<td>6.0</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1300 s</td>
</tr>
<tr>
<td>$d_0$</td>
<td>5.0 microns</td>
</tr>
<tr>
<td>$H$</td>
<td>80 MPa</td>
</tr>
<tr>
<td>$C_D$</td>
<td>15 MPa</td>
</tr>
<tr>
<td>$a$</td>
<td>2.2</td>
</tr>
<tr>
<td>$K_{I}$</td>
<td>1.8944</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>-5.683</td>
</tr>
<tr>
<td>$K_{III}$</td>
<td>4.2625</td>
</tr>
<tr>
<td>$C_s$</td>
<td>$0.006(-K_{II})^{a-1}(2K_{I}\sqrt{K_{III}})^{1-a}$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>2.9</td>
</tr>
<tr>
<td>$c_2$</td>
<td>3.0</td>
</tr>
<tr>
<td>$c_3$</td>
<td>3.1</td>
</tr>
<tr>
<td>$\phi$</td>
<td>45°</td>
</tr>
<tr>
<td>$m$</td>
<td>0.5</td>
</tr>
<tr>
<td>$n$</td>
<td>5.5</td>
</tr>
<tr>
<td>$K_0$</td>
<td>$1.8K_{I}^{0.5}$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>$d_0^{-\gamma}(890K_{I}^{0.75})^{-l/m}$</td>
</tr>
<tr>
<td>$C_{II}$</td>
<td>$4.408\times10^{12}K_{I}^{-3.25}$</td>
</tr>
<tr>
<td>$p$</td>
<td>$k_1+k_2\dot{\varepsilon}+k_3\dot{\varepsilon}^2$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>0.269</td>
</tr>
<tr>
<td>$k_2$</td>
<td>-500.0</td>
</tr>
<tr>
<td>$k_3$</td>
<td>-3000000.0</td>
</tr>
</tbody>
</table>
5.2 RESULTS AND DISCUSSIONS

Bulge forming analysis of a Pb-Sn sheet, having an initial thickness of 1.27 mm, was carried out till a dome height of 38.1 mm at a constant target strain rate of 1E-4, 3E-4, 6.5 E-4 and 1E-3 1/s. The forming time required was 12155, 4200, 2068 and 1322 seconds, respectively. The forming pressure profiles for all simulations are shown in Figure 5.1. As expected, greater gas pressure is required for higher target strain rate forming due to the higher flow stress associated with it. The plot of Von-Mises stress with strain, at various strain rates is shown in Figure 5.2. The variation of the dome height and thickness with forming time for all simulations is shown in Figure 5.3 and Figure 5.4 respectively. The forming time required and the pole thickness obtained at the end of deformation for different target strain rate forming are listed in Table IX. The thickness distribution of the formed sheets along a radial line passing through the pole is shown in Figure 5.5. It is seen that the thickness distribution obtained in the deformed sheet at lower strain rate (1E-4 /s) is more uniform compared to the thickness distribution obtained at higher strain rates. The thinning factor calculated for each analysis is listed in Table IX. The results clearly show that in order to achieve more uniform deformation greater forming time is required.

Table IX. Summary of the results obtained, during superplastic forming of Pb-Sn alloy, at various strain rates.

<table>
<thead>
<tr>
<th>Strain Rate (1/s)</th>
<th>$\dot{\epsilon}$ : 1E-4</th>
<th>$\dot{\epsilon}$ : 3E-4</th>
<th>$\dot{\epsilon}$ : 6.5E-3</th>
<th>$\dot{\epsilon}$ : 1E-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dome height</td>
<td>38.1 mm</td>
<td>38.1 mm</td>
<td>38.1 mm</td>
<td>38.1 mm</td>
</tr>
<tr>
<td>Forming Time</td>
<td>12155 s</td>
<td>4200 s</td>
<td>2068 s</td>
<td>1322 s</td>
</tr>
<tr>
<td>Initial Thickness</td>
<td>1.27 mm</td>
<td>1.27 mm</td>
<td>1.27 mm</td>
<td>1.27 mm</td>
</tr>
<tr>
<td>Pole Thickness</td>
<td>0.3347 mm</td>
<td>0.3087 mm</td>
<td>0.2959 mm</td>
<td>0.2948 mm</td>
</tr>
<tr>
<td>Average Thickness</td>
<td>0.6022 mm</td>
<td>0.6065 mm</td>
<td>0.6090 mm</td>
<td>0.6093 mm</td>
</tr>
<tr>
<td>Thinning Factor</td>
<td>0.56</td>
<td>0.51</td>
<td>0.49</td>
<td>0.48</td>
</tr>
</tbody>
</table>
Finite element simulations were further carried out, up to the bulge height obtained at failure, during the experiments of blow forming of Pb-Sn sheets. These FE results were then compared with the experimental results obtained. It is seen that the forming time predicted using finite element analysis was very large as compared to the time that is actually measured during the experiments. For example, during forming at a target strain rate of 1E-4 1/s, the time required during experiments for obtaining a bulge height of 4.36, was 2135 s, whereas the time predicted using finite element analysis was 15500 s. This value is more than 725 % greater than the actual time. By considering plane stress and balanced biaxial stretching conditions at the pole of the dome, and employing the von Mises relationship for isotropic material, the thickness of the sheet at the pole at any given time during deformation is given by Dutta and Mukherjee (D-M) [15] as

\[ t_p = t_o \exp(-\dot{\varepsilon}T) \]  

This analytical model also predicts failure at much longer forming time then what is actually measured experimentally. These results are summarized in Table X. Thus it is seen that the finite element analysis tools have the theoretical predictive capabilities, but in order to accurately predict thinning and failure, there is a need to make modifications in the FE tools so that it can help in simulate the practical forming process.

<table>
<thead>
<tr>
<th>Strain Rate (1/s)</th>
<th>1E-4</th>
<th>3E-4</th>
<th>6.5E-4</th>
<th>1E-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dome Height at Failure (mm)</td>
<td>4.36</td>
<td>4.09</td>
<td>3.96</td>
<td>3.83</td>
</tr>
<tr>
<td>Experimental Forming Time (s)</td>
<td>2135</td>
<td>935</td>
<td>550</td>
<td>195</td>
</tr>
<tr>
<td>FE Analysis Forming Time (s)</td>
<td>15500</td>
<td>4900</td>
<td>2250</td>
<td>1360</td>
</tr>
<tr>
<td>D-M ‘s Analytical Model (s)</td>
<td>17700</td>
<td>5530</td>
<td>2550</td>
<td>1660</td>
</tr>
</tbody>
</table>
Figure 5.1: Pressure-Time profiles at different target strain rates, for superplastic bulge forming of Pb-Sn alloy.

Figure 5.2: Stress-strain curves at different target strain rates, for superplastic bulge forming of Pb-Sn alloy.
Figure 5.3: Variation of bulge height with time at different target strain rates, for superplastic bulge forming of Pb-Sn alloy.

Figure 5.4: Variation of sheet thickness at the pole with time, at different target strain rates, for superplastic bulge forming of Pb-Sn alloy.
Figure 5.5: Variation of sheet thickness along the radius of the dome at different target strain rates, for superplastic bulge forming of Pb-Sn alloy.
CHAPTER 6

SUMMARY AND RECOMMENDATIONS

Finite element analysis is carried out to understand and optimize the superplastic behavior of Ti-6Al-4V. The material subroutine used in the analysis is based on the constitutive model that accurately captures the behavior of the titanium alloy under consideration.

The effect of superplastic forming at various strain rates, on the structural integrity of the formed product is observed. The optimum strain rate profile is integrated with ABAQUS and the results obtained using the optimum loading curve show a marked improvement in the uniformity of thickness distribution in the formed product. Presently the target strain rate utilized for forming Ti-6Al-4V alloy lies in the rage of 1E-4 /s to 5E-4 /s. The total time required, by using the optimized method for the superplastic forming, is reduced by 92% and 58% compared to the process utilizing a target strain rate of 1E-4 /s and 5E-4 /s, respectively.

The analytical model devised by Dutta and Mukherjee to determine the pressure profile for superplastic bulge forming at a given strain rate, is compared with the pressure profile generated by ABAQUS. The results clearly indicate that at higher strain rates Dutta and Mukherjee’s model overestimates the pressure required for the bulge forming. Recent developments in the finite element tools make it a more viable option to determine the constant strain rate pressure profile for superplastic forming.

Superplastic forming requires a fine grain microstructure in the alloy prior to forming. The effects of varying the initial grain microstructure on the forming process are observed. It is seen that there is an adverse effect of greater initial grain size on the thickness of the formed sheet. Greater localized thinning is observed with an increase in the initial grain size. The time required to complete the superplastic forming process is also affected. An increase in the initial grain size increases the time required to form the required product. This effect is predominant at higher strain rates.

Past research has proved that friction plays a vital role in any metal forming operation. However the study involving friction during superplastic forming is limited. In this work the effect of varying the coefficient of friction on the formed product is observed. An increase in friction between the sheet and the die lead to greater localized thinning. However due to the
particular strain rate control scheme utilized during the analysis, the pressure remained unaffected with the change in the friction coefficient during the superplastic forming process.

Finally the material constitutive equation and the optimized pressure profile studied above are used in carrying out analysis of an industrial application of superplastic forming. Medical industry is increasingly utilizing superplastic alloys to make custom made products. Dental implant superstructure is one such application. Upon utilizing the optimum pressure profile, the time required to form the dental implant was reduced by 52%. This process also helped to obtain a uniform thickness distribution in the formed product.

6.1 RECOMMENDATIONS

Present day superplastic forming applications are limited due to the large forming time, lack of accurate constitutive equations to predict the behavior of superplastic alloys and drawbacks in the analytical tools in representing the actual superplastic forming process. Future work should adopt an approach so as to eliminate these drawbacks. The following approach is recommended for analyzing the superplastic forming process.

1. Carry out finite element analysis of uniaxial tensile testing and biaxial forming of the alloy under consideration and compare the results obtained using similar experimental data. On the basis of these results make the desired changes in the finite element model for accurate representation of the material under consideration and to predict the failure of the alloy.

2. Previous work includes the microstructural evolution in the finite element model, but research needs to be done in order to study the effect of variable grain size distribution in the initial blank on the forming characteristics. The region undergoing larger deformation can be processed for finer grain size prior to forming. Finite element analysis can be used to develop a criterion so as to identify the deformation zones and optimum grain size distribution.

3. Analysis is to be carried out to study the effect of void size and spacing on the ductility and flow stress of the material. Experimental data based on void growth and cavitations is required to be compared with the analysis results and the final cavitation model should be incorporated in the finite element tool.
4. Friction plays a very important role during superplastic forming. Detailed experimental data is required to understand the effect of temperature, strain rate and normal pressure on the measured coefficient of friction. Once this data is gathered, it can be incorporated in the finite element code and analysis can be carried out in order to study the effect of variable lubrication on the formed product.

5. Superplastic forming is a constant target strain rate forming process. However during any forming operation, at any given instance, the strain rate varies throughout the deformation zone. The desired strain rate is maintained only in a small region of maximum deformation. Selective grain refinement and selective lubrication can be utilized in order to make certain regions of the deformation zone more ductile than the other, so as to offset the above limitations to a certain extent. Finite element analysis can be carried out to generate standard approach to carry out superplastic forming operations of parts having different shapes and sizes.

6. The pressure control algorithm integrated within Abaqus has a very simple form. A pressure subroutine is required which is highly sensitive to the variation of strain rate in the deformation zone, so that accurate pressure profile can be generated.

7. Once the above steps are completed, the customized finite element tool can be used to design and analyze complex industrial applications of superplastic alloys.
APPENDIX

INPUT FILE FOR THE FINITE ELEMENT ANALYSIS OF SUPERPLASTIC BULGE FORMING OF A HEMISPHERICAL DOME USING ABAQUS

HEADING
FINITE ELEMENT ANALYSIS OF SUPERPLASTIC BLOW FORMING
*PREPRINT, ECHO=YES, HISTORY=YES,MODEL=YES,CONTACT=YES
*RESTART,WRITE,FREQUENCY=30
**DEFINING THE NODES AND ELEMENTS OF SHEET AND DIE
*NODE,INPUT=freenodes-cm.inp,NSET=FREENODES
*NODE,INPUT=constrainednodes-cm.inp,NSET=CLAMPEDNODES
*NODE,NSET=CENTER
  337
*ELEMENT,TYPE=M3D4,ELSET=FREESHEET,INPUT=freeelements.inp
*ELEMENT,TYPE=M3D4,ELSET=CONSTRAINEDSHEET,INPUT=constelements.inp
*ELSET,ELSET=SHEET
  FREESHEET,CONSTRAINEDSHEET
*NSET,NSET=SHEET1
  FREENODES,CLAMPEDNODES
*ELSET,ELSET=EDGE
  315,330,345,360,375,390,405,420,435,450,465,480,495,510,525,
  806,807,808,809,810,811,812,813,814,815,816,817,818,819,820,821,
  822,823,824,825,826
*ELSET,ELSET=CENTER1
  315
*ELSET,ELSET=SQUARE,GENERATE
  301,525
**MATERIAL DEFINATION
*MEMBRANE SECTION,ELSET=SHEET,MATERIAL=TI-6AL-4V
  0.198,
*MATERIAL,NAME=TI-6AL-4V
  ELASTIC
  115E5,0.3
*CREEP,LAW=USER
*DEPVAR
  2,
**BOUNDARY CONDITIONS
*NSET,NSET=EDGEX
  337,
  353,354,355,356,357,358,359,360,361,362,363,364,365,366,
  352,
  592,593,594,595,596,597,598,599,600,601,602,603,604,605,
  606,607,608,609,610,
  577,
  909,924
*NSET,NSET=EDGEY
  1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,
  16,17,18,19,20,21,
  337,338,339,340,341,342,343,344,345,346,347,348,349,350,351,
  877,878
*BOUNDARY
  EDGEX,YSYMM
  EDGEY,XSYMM
  CLAMPEDNODES,1,3
**INITIAL CONDITIONS, TYPE=SOLUTION, USER**

**INITIAL CONDITIONS, TYPE=STRESS**

SHEET, 1, 0.0001

*AMPLITUDE, NAME=PRES, DEFINITION=SOLUTION DEPENDENT 1, 0.001, 10000

*STEP, NLGEOM, unsymm=yes

*STATIC 2.E-3, 1.0,

*DLOAD

FREESHEET, P, 0.1

*EL PRINT, ELSET=SHEET, FREQUENCY=100

S, E

CE,

SINV,

*EL FILE, ELSET=SHEET, FREQUENCY=100

S, E

CE,

SINV,

*EL FILE, ELSET=EDGE, FREQUENCY=11500, POSITION=AVERAGED AT NODES STH

*EL PRINT, ELSET=EDGE, FREQUENCY=11500, POSITION=AVERAGED AT NODES STH

*EL PRINT, ELSET=CENTER1, FREQUENCY=1, POSITION=AVERAGED AT NODES STH

*EL FILE, ELSET=CENTER1, FREQUENCY=1, POSITION=AVERAGED AT NODES STH

*NODE PRINT, NSET=SHEET1, FREQUENCY=100

U,

*NODE FILE, NSET=SHEET1, FREQUENCY=100

U,

*OUTPUT, FIELD, OP=ADD, FREQUENCY=100

*ELEMENT OUTPUT, ELSET=SHEET

S, E

CE,

SINV

*OUTPUT, FIELD, OP=ADD, FREQUENCY=100

*NODE OUTPUT, NSET=SHEET1

U,

*OUTPUT, FIELD, OP=ADD, FREQUENCY=1

*ELEMENT OUTPUT, ELSET=CENTER1

STH

*END STEP

*STEP, INC=500000, NLGEOM, unsymm=YES

*VISCO, CETOL=0.05

0.00002, 11500.0,, 20

*DLOAD, AMPLITUDE=PRES

FREESHEET, P, 0.1

*CREEP STRAIN RATE CONTROL, ELSET=FREESHEET, AMPLITUDE=PRES

0.0001

*NODE FILE, NSET=CENTER, FREQUENCY=1

U,

*OUTPUT, FIELD, OP=ADD, frequency=1

*NODE OUTPUT, NSET=CENTER

U,

*END STEP
USER SUBROUTINE FOR MODELING THE MATERIAL BEHAVIOR OF Ti-6Al-4V

SUBROUTINE SDVINI(STATEV, COORDS, NSTATV, NCRDS, NOEL, NPT, LAYER, KSPT)
  INCLUDE 'ABA_PARAM.INC'

  DIMENSION STATEV(NSTATV), COORDS(NCRDS)
  STATEV(1)=4.0D-4
  STATEV(2)=1.0D0
  RETURN
END

SUBROUTINE CREEP(DECRA, DESWA, STATEV, SERD, EC0, ESW0, P, QTILD, TEMP, DTEMP, PREDEF, DPRED, TIME, DTIME, CMNAME, LEXIMP, LEND, COORDS, NSTATV, NOEL, NPT, LAYER, KSPT, KSTEP, KINC)
  INCLUDE 'ABA_PARAM.INC'

  CHARACTER*80 CMNAME

  DIMENSION DECRA(5), DESWA(5), STATEV(*), PREDEF(*), DPRED(*), TIME(2), COORDS(*)

  DOUBLE PRECISION C1, C2, CS, CD, E, TAU, P1, TRY, M, N, Q, A, B, Y, Z, I, J, K, X1, X3, D1, D2, ST, ST1, T2, T1, AA
  C1=1.3D-18
  C2=2.2D-19
  CS=4.12D-2
  CD=939.0
  E=1.0D-4
  TAU=1.3
  P1=3.0
  N=4.3
  M=0.7
  Q=3.9
  AA=STATEV(2)

  IF(KINC.EQ.AA) THEN
    D=STATEV(1)
  ENDIF

  IF(KINC.GT.AA) THEN
    D1=STATEV(1)*10000
    X1=D1**Q
    X3=-TIME(1)/TAU
    I=CS/X1
    J=(CD*E)/X1
    K=1-EXP(X3)
    D2=(I+(J*K))*DTIME
    D=(D1+D2)*1.0D-4
    STATEV(1)=D
    STATEV(2)=KINC
    IF(NOEL.EQ.315.AND.NPT.EQ.1) THEN
      WRITE(7,*) STATEV(1), STATEV(2), TIME(1), D, QTILD
    ENDIF
  ENDIF
TRY=D**P1
ST=(QTILD)
ST1=(QTILD)
A=((C1/TRY)*(ST**((1/M))))
B=C2*(ST1**(N))
DECRA(1)=(A+B)*DTIME
Y=((C1/TRY)/M)*(ST**((1/M)-1))
Z=N*C2*(ST1**(N-1))
IF(LEXIMP.EQ.1) THEN
DECRA(5)=(Y+Z)*DTIME
ENDIF
RETURN
END
REFERENCES


VITA

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- March 28, 1978 at Saswad, Pune-MH, INDIA

Education

- Bachelor's Degree in Mechanical Engineering (B.E.), Padmabhushan Vasantrao Dada Patil Institute of Technology, Budhgaon, MH, INDIA (2000)

Work Experience

- Worked as a Research Assistant at the University of Kentucky, Department of Mechanical Engineering, (January 2002 - December 2003)

Technical Publications