THEORETICAL AND EMPIRICAL STRATEGIES FOR MANAGING IRRIGATION SUPPLIES RISK: THE CASE OF RIO MAYO IRRIGATION DISTRICT IN SONORA, MEXICO

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ABSTRACT OF DISSERTATION

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Agriculture at the University of Kentucky

By
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and Dr. Robert R. Reed III, Professor of Economics

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This dissertation comprises theoretical and empirical models to manage water supply risk in irrigated agriculture. While irrigation is by itself a strategy to regulate the supply of water for farm use, water systems that depend on surface water sources are still subject to the random inflows that feed their reservoirs. Depending on the size of the reservoir, the demand for irrigation, and the seasonal distribution of inflows, water availability may decrease to levels that severely constraint agricultural production. This dissertation begins with a theoretical examination of on-farm cropping decisions under water endowment risk. However, the analysis is extended to the use of a risk-sharing innovation to transfer the water availability risk outside an irrigation district. Specifically, the design, use, and economic feasibility of an inflow-based derivative are studied for the Rio Mayo irrigation district, located in Northwestern Mexico.

On the theoretical front, the analysis consists of modeling the on-farm economics of hedging against uncertain irrigation endowments. The basic model starts by analyzing the role of crop diversification. As expected, the firm responds to higher degrees of risk, as measured by the variance in the supply of water, by allocating less land towards the water-intensive crops. The underlying motivation in these strategies is the need to avoid the relatively larger reductions in productivity sustained by water-intensive crop portfolios. However, crop diversification comes at the cost of reduced profits. As an alterative to crop diversification, the model is modified to study the role of an institution that transfers water contingent on the states of nature. The extension shows that, under certain conditions, enrolling in such a scheme produces the same profit as under certainty.

In the empirical component of the dissertation, the economics of an inflow-based derivative are examined. The modeling strategy consists of simulating the economic environment and hydrological profile of the Adolfo Ruiz Cortinez Reservoir on the Rio Mayo irrigation district. Specifically, a stochastic dynamic simulation model is developed that captures the intra and inter seasonal risk aspects associated with water risk and water use for irrigated agriculture. The results indicate that the inflow-based derivative is a viable instrument in the terms of affordability (i.e. premiums) and yield effective income protection (i.e. risk reduction).

KEYWORDS: irrigation risk, diversification, water transfers, inflow-based derivative, dynamic stochastic simulation.

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January 19, 2006

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RULES FOR THE USE OF DISSERTATIONS

DISSERTATION

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DEDICATION

This dissertation is dedicated to my mother Lesbia Treminio and to my family in Nicaragua

ACKNOWLEDGEMENTS

First, I thank God for giving me the chance to meet and share my stay in Lexington with the persons mentioned below. Without God and these persons, this dissertation would have remained in the chamber of dreams that are destined to become no more than sparks of imagination.

The seed of this project was planted one day of 1998 in my native Nicaragua when I crossed the path of Dr. Jerry Skees. Previous to this random meeting, the notion of enrolling in a Ph.D. program was not even pre-conceived in my imagination as my myopic view pointed a the Masters degree only. I consider myself privileged for receiving the inspirational guidance, experience, and professionalism of Dr. Jerry Skees. Throughout the program I enjoyed the academic advice and friendship of my major professor, who was fundamental to overcome the different types of challenges I encountered in my way to becoming a Ph.D. graduate. I am highly indebted for the time he devoted and the patience he showed to direct my work. However, his mentoring transcended the frontiers of dissertation co-chair as he also helped me grow into a better professional, and above all, into a better person. Put simply, Dr. Jerry Skees is the most influential person in my academic and professional formation.

Also, this dissertation would not have been possible without the direction and dedication of Dr. Robert Reed III. I express my most sincere appreciation for sharing with me his discipline, high standards and professionalism to conduct research. I consider myself fortunate that Dr. Reed accepted not only to be part of my committee, but also the role of dissertation co-chair. The experience of working under his supervision was fundamental to develop my ability to perform research in theoretical economics. Dr. Reed did an excellent job in fine-tuning my research skills, guiding me not only in the academic duties, but also as a professional economist.

I also express my deepest appreciation to Dr. Carl Dillon for the excellent comments, suggestions, and editing of my work, which were crucial to improve the dissertation. Also, I appreciate the comments and advice from Dr. Glen Blomquist, Dr. Michael Reed and Dr. Ronald Fleming.

My utmost gratitude and respect goes to my beloved wife Meyling Treminio and my son Akssell Jr., who bravely and patiently waited for me to complete this project. This dissertation would not have been possible without their love and unconditional support in every step of the way. With her mental strength and tenacious character, Meyling assumed the great responsibility of leading and supporting a young family in my absence. Without her tremendous sacrifice and continuous encouragement, I would not have been able to complete the Ph.D. program. I thank God for blessing me with such a unique and gorgeous woman. Also, I thank Meyling's grandparents, Claudia Mendoza and Joaquin Treminio, for taking care of Akssell Jr. and providing him with love and affection while I was absent.

For their support, friendship and encouragement I also thank my friends Jorge Figueroa, Jason Hartell, Agus Hudoyo, Ibrahim Bamba, Jean Marc Gondonou, Murali Kanakasabai, Kimathi Miriti, Luis Tercero, Luis Valverde, and Tigran Manukyan. Special thanks to Kenny Burdine and his young family for the friendship, hospitality and companionship they offered me during my stay in Lexington.

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Chapter 1: Introduction

1.1 Problem Statement

Water is perhaps the most precious natural resource in the agricultural regions of Northwestern Mexico. Unfortunately, water is seldom present at the exact place, time, quantity and quality to satisfy the uses it serves in many sector of those regional economies. In regions where rains are scarce, communities rely on reservoir systems to supply populations with water, irrigate large cropping areas, and sometimes generate power. Unfortunately, when the replenishment of reservoirs depends on stream flows from a river or system of rivers, the process is characterized by high degrees of uncertainty. As a result, there are some years when low levels of rainfall will not adequately replenish the reservoir and water shortages will set in. By contrast, in years of excess rain where inflows exceed the storage capacity the hydraulic works will be unable to prevent flooding.

The Rio Mayo irrigation district, just as other large number of systems around the world, is characterized by a highly variable supply of surface water and a relatively abundant endowment of agricultural land. The combination of these characteristics implies that water is the limiting factor of production and that the uncertainty surrounding its supply translates into likewise uncertain streams of income for irrigators. Under extreme circumstances, the trickle-down effects of the uncertainty in water supply extend to issues of food security and rural employment for water-based economies. Furthermore, the water supply uncertainty deters irrigators from making investments in technology that improves the utilization of water at the farm level. Finally, the creditor's willingness to supply loans to irrigators is severely constrained to finance projects that maintain and develop the irrigation infrastructure for the system.

The problem described above extends to all of Mexico. According to the *Comision Nacional del Agua*[1] (CNA), the Mexican authority in charge of regulating the use of water resources, there is disparity between the natural availability of water and the locations where the resource is needed the most. In particular, 64% of the resource

[1] CNA is an autonomous federal agency in Mexico responsible for regulating water uses, overseeing water rights and transactions, and planning the allocation of government funds in the different sub-sectors of the water economy.

availability occurs in the south of the country where only 11% of the land suitable for agriculture is located; whereas in the north 53% of land suitable for agriculture is located, only 7% of the water resources are available (CNA, 2004). Due to the relatively scarcity of water, cropping activities in the most productive lands of the Mexican Northwest depend almost entirely on irrigation. Furthermore, the disproportionate endowment of the water resources is accompanied by rapidly growing populations in the water scarce areas; consequently the country has witnessed serious water conflicts among water users when the replenishment of reservoirs is low.

Although CNA employs elaborated hydrological models that provide some guidance in the reservoir operations for irrigated agriculture, these plans do not include any type of formal financial assistance or water transfer schemes to mitigate economic and financial burden imposed by the uncertain availability of water. The only financial assistance available to farmers is the occasional ad hoc disaster payments disbursed from the state governments. Therefore, a typical irrigator in the Mexican Northwest operates in a risky environment characterized by the random availability of the most limiting factor of production and without access to formal risk transfer markets to hedge against such a risk.

1.2 Research Objectives

Considering the problems stated above, the purpose of this research is to develop a richer understanding of the implications of irrigation risk on production decisions, and to investigate the economics of using risk transfer innovations to hedge against the income losses incurred during periods of water scarcity. To enhance the insights in the problem stated above, this dissertation analyzes, both theoretically and empirically, risk management strategies that could be adopted by irrigators in the Rio Mayo irrigation district in Northwest Mexico. Particularly, the study has two objectives:

1) To study the behavior of the firm when facing input supply risk and examine the roles of output diversification and water transfers as risk management strategies;
2) To evaluate the design and effectiveness of an inflow-based derivative in providing financial protection against the economic burdens that arise with the occurrence of hydrological drought.

This research will be carried out at both a theoretical and an empirical level. Since there are no formal statistical tests performed, specific hypothesis test are not investigated. The first objective will be accomplished by developing a theoretical model of firm behavior under water supply uncertainty. Using comparative statics, the model will enhance the understanding of how the firm can choose between crops with different degrees of dependency on water to hedge against the uncertain availability of the factor input – water. The second objective will be by focusing at the irrigation district level. This objective will be achieved using a dynamic stochastic simulation that includes both economics and hydrology to model the risk environment in the Rio Mayo irrigation district. This empirical application will provide insights into risk-reducing capabilities of the proposed risk transfer mechanism, the willingness to pay for the product, and the premium an irrigator may expect to pay under a hypothetical market.

1.3 Outline of the Dissertation

The research is organized in six chapters. Chapter two provides a survey of the different views on the management in water resource based systems: the technical and the institutional views. The technical view proposes the use of optimization techniques to arrive at optimal plans of water resource utilization, whereas the institutional view presents institutional settings such as water markets, property rights and water transfers to deal with the uncertainty in water availability. This chapter concludes with a brief review on weather derivative instruments that have been proposed as an alternative to more traditional agricultural insurance schemes.

The third chapter presents a theoretical model of a farm facing random availability of water supply. Different from other models in the literature of input supply risk, the model in this chapter is based on the notion that irrigators can select between crops with different levels of water dependency, and thereby have the ability to hedge against the input supply risk. The roles of diversification and socially optimal contingent water transfers are analyzed using comparative statics and numerical simulations.

The fourth chapter develops the conceptual framework for analyzing the economics of water supply risk in the Rio Mayo irrigation district. Specifically it shows how an instrument similar to a weather derivative, but using water inflows into a reservoir as an underlying asset, can be applied in conjunction with reservoir operation rules to generate more effective risk management strategies for irrigators. In addition, some aspects of expected utility theory are reviewed.

The fifth chapter develops an empirical model to design and assess risk management strategies in the Rio Mayo district. Using a dynamic stochastic simulation model of reservoir operations the risk profile of this irrigation district is analyzed. The chapter discusses inflow simulation and validation, the adoption of reservoir operation policies and plantings response functions. The strategies to implement the inflow-based derivative are analyzed using certainty equivalent, risk premium, and several measures of downside risk.

The final chapter includes the conclusion from the theoretical and empirical analysis. In addition, the limitations of the research are explained and some insights into future research are included.

1.4 Contribution to the Literature

This dissertation advances the literature on agricultural risk management by examining theoretically and empirically the problem encountered by farm operators that face uncertain availability of water in the production process. From the theoretical side, this research enhances the understanding of the economics of on-farm management of input supply risk. While previous research has analyzed the impact of this particular type of risk, the firm was restricted to produce a single output. Therefore, by introducing the possibility of two production technologies, this research shows that firms can partially hedge against the random availability of one of the factor inputs by choosing a crop mix plan that is less sensitive to such uncertainty. In addition, the theoretical approach shows that crop diversification itself entails reduced profitability to the firm. More importantly, the analysis suggests that agricultural policy and institutions that transfer resources contingent on the occurrence of droughts could potentially improve the competitiveness of irrigation-based farms. Examples of those policies are water banks, water harvesting, and insurance contracts. In summary, the theoretical research justifies the on-farm economics of using a potential risk-sharing mechanism to improve the competitiveness of irrigated agriculture.

From the empirical side, this research extends the theoretical strategies into innovations related to the weather derivatives literature. In particular, this research represents the first attempt to study the economics of the use of such financial instruments in the management of water supply risk for irrigated agriculture. Furthermore, the results have practical implications for improving the management of irrigation districts that depend on superficial inflows for the provision of irrigation services. The framework used to analyze the Rio Mayo district could be adapted to other irrigation districts in Mexico or in other parts of the world. Furthermore, this research contributes to the already ongoing efforts of international economic development organizations to encourage the use of risk-sharing markets to promote agricultural development in Latin America.

Chapter 2: Literature Review

2.1 Introduction

The evolution of irrigation development has presented different problems for economists and hydrologists throughout its different stages. In the first stage of irrigation development, policies were concentrated on exploring and building new infrastructure for the provision of water since large economic benefits were associated with increasing water access and reliability for agriculture. In the second stage, the focus was on expanding and maintaining infrastructure to satisfy the growing demand for water without forgoing reliability. At later stages, however, developing costs rise increasingly and non-structural strategies are more appropriate to secure the supply of water for agriculture. Benefit cost analysis and capital budgeting were the most important tools applicable to the earlier stages. A mature water economy in the Randall sense (1981), however, demands market-oriented allocating policies and risk management analysis for the efficient and sustainable use of water systems.

In dealing with the risk management problems that arise in the different stages of irrigation development, two traditional approaches have emerged in the literature: the technical and the institutional views. One the one hand, the technical view proposes that the reliability of the water supply for irrigation purposes can be improved by incorporating the random nature of inflows in the operation rules for reservoirs through alternative optimization techniques. On the other hand, the institutional view concentrates on the use of water markets and financial-type contracts to share the risk among different groups of water users. While the technical view deals with risk management at the supply level (i.e. reservoir operation), the institutional view concentrates on the demand side (i.e. water user transactions). Despite their differences both views are compatible and can be combined to gain insights into the benefits from proper risk management strategies in a mature water economy.

The purpose of this literature review is to synthesize the relevant literature on the traditional and technical views on water resources management. The first objective is to identify the methodological advances that stem from the technical view and evaluate their strengths and weaknesses in dealing with water supply risk. The second objective is to describe institutional arrangements, particularly water and insurance markets, that have been proposed and assess their contribution to the management of water supply risk.

2.2 The Technical View

The technical view concentrates in the management of the water supply at the source level, rather than at the user level. It recognizes that uncertainties associated with future hydrological conditions coupled with the need to satisfy multiple demands for water complicate the operational effectiveness of existing reservoir systems. In order to investigate the impact of uncertainty and prescribe operation rules, authors have relied on the techniques of stochastic dynamic programming, chance-constrained programming, and stochastic simulation. Policy prescriptions derived from the technical view models are perhaps the most practical guides available for chief engineers in charge of reservoir operations.

2.2.1 Reservoir Operation and Irrigation Planning

The problem of reservoir operation and irrigation planning is a classical example of inter-temporal optimization problems. The dynamic notion of inter-temporal planning problems is concentrated in the inter-temporal tradeoffs of benefits. The analysis considers certain type of decisions that impact decisions or state of the system in future periods during the planning horizon. For example, in a reservoir system if the rate of water releases exceeds the rate of reservoir replenishment, then the availability of the water stock declines over time. Thus, in this type of problems, optimization is executed over a stream of benefits, denoted r_t, that occur as a result of a decision x_t made at different periods of time, denoted t. The total number of time periods determines the planning horizon, given by T. The preference for benefits associated with different time periods is embedded in the discount rate, denoted by β. The objective of a reservoir planner is to maximize the discounted net benefits by allocating the stock of a resource into different periods of time, taking into account the fact that resource use in the present influences resource availability in the future.

In the deterministic version the assumption is that the planner has complete information; thus, the problem is to find the optimal sequence of choices that maximize the objective function, given a set of constraints and assuming that the value of the water stock at the end of the planning horizon is given by $F(x_{T+1})$[2]. The natural process of reservoir replenishment, or motion equation, is given by the function $g(\bullet)$. Specifically, the planner solves the following problem:

$$\text{Maximize } \sum_{t=1}^{T} \beta^{t-1} r_t(w_t, x_t) + \beta^T F(w_{T+1}) \tag{2.1}$$

Subject to

$$w_{t+1} - w_t = g(w_t, x_t) \text{ where } t = 1,2,...,T \tag{2.2}$$

$$w(1) = w_t \tag{2.3}$$

In reality, however, the planner does not have complete information. For instance, the rate of reservoir replenishment is stochastic and the stock of water at any given period is also stochastic, the soil moisture at the beginning of the season is influenced by precipitation, and so is the water demand. Therefore, in the stochastic version, the planner's knowledge about the future is expressed in probabilities, p_t, of occurrence of states of nature, k_t. In the case of reservoir operation models, the most important source of uncertainty is that of water stock or resource availability at any given period of time. Such uncertainty is reflected in the stochastic version of the motion equation, $h(\bullet)$. Thus, the stochastic version of the inter-temporal problem is as follows:

$$\text{Maximize } \sum_{t=1}^{T} \beta^{t-1} \sum_{k} p_t(k_t) r_t(w_t, x_t, k_t) + \beta^T F(w_{T+1}) \tag{2.4}$$

Subject to

$$w_{t+1} - w_t = h(w_t, x_t, k_t) \text{ where } t = 1,2,...,T \tag{2.5}$$

[2] A problem with finite horizon and known final value of the stock is described for illustration purpose only. In other problems the value of the stock at the end of the planning horizon might be unknown or the planning horizon might be infinite.

$$\sum_k p_t(k_t) = 1 \tag{2.6}$$

$$w(1) = w_t \tag{2.7}$$

Optimal control theory is the mathematical foundation for the solution of inter-temporal optimization problems. In the literature on water systems management, stochastic dynamic programming (SDP) and sequential stochastic dynamic programming (SSDP) have been the main methods used to solve models of the inter-temporal optimization. A brief description of each method and its applications to water system management follows.

2.2.2 Stochastic Dynamic Programming

SDP has usually been the preferred technique to integrate risk considerations in the calculation of operating rules for single-reservoir and multiple-reservoir systems (Stedinger, Sule and Loucks, 1984; Lasbadie, 2004).[3] The approach is based on Bellman's principle of optimality (1957): "an optimal policy is one in which, whatever the initial state and initial condition, the following decisions must constitute an optimal policy in relation to the state resulting from the initial decision." In particular, let $V_t(w_t)$ denote the optimal value of the water stock in stage t, which results from making the decisions $x_t^*, x_{t+1}^*, ..., x_T^*$ in the remaining planning horizon. Thus, the SDP methodology consists of breaking down the planning horizon into T single-stage optimization problems and using the recursive nature of the problem to solve the following:

$$V_t(x_t) = \max_{x_t}\left\{\sum_k p_t(k_t) r_t(w_t, x_t, k_t) + \beta V_{t+1}(w_t + h(w_t, x_t, k_t))\right\} \tag{2.8}$$

subject to

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[3] Applications of DSP in the operation and planning of reservoir systems can be found in the works of Dudley, Howell and Musgrave (1971a, 1971b, 1972); Dudley (1972, 1988a, 1988b); Dudley and Burt (1972), Mawer and Thorn (1974); Sobel (1975); Dudley and Musgrave (1988); Rao, Sarma and Chander (1990); Dudley and Hearn (1993).

$$V_{T+1}(w_{T+1}) = F(w_{T+1}) \tag{2.9}$$

$$\sum_k p_t(k_t) = 1 \tag{2.10}$$

$$w(1) = w_t \tag{2.11}$$

The solution algorithm consists of solving first the last stage, T, of the planning horizon. Then, proceed by sequentially solving the remaining stages in a backward fashion until the initial stage is reached; that is stages $T-1, T-2, ..., 1$. For instance, if there were three time periods ($t = 1, 2, 3$), the planner begins the algorithm by allocating x_3 units of the w_3 stock available for allocation in the last period to maximize the value function V_3, taking into account that he knows the value of the final stock of water w_4 by looking at the function $V_4 = F(w_4)$. The optimal value of x_3^* represents the best strategy the planner can adopt in the last period. Consequently, once the value function V_3^* is optimized, the planner can move back to the second period and pick x_2^* to determine the optimal value of V_2^*, given the stock of water at the beginning of the second period w_2 and the equation that connects the second and third periods. Finally, just like in the second period, the planner decides on strategy x_1 given the initial water stock $w(1)$ and link between the first and second periods. Thus, the example illustrates that no matter what period and state the planner starts with, he/she must proceed in an optimal fashion to the next period in order to solve the inter-temporal problem.

2.2.3 SDP Applications to Water Systems

Dudley, Howell and Musgrave use DSP to model decisions in reservoir operation in the short (1971a), medium (1971b), and long run (1972). First, they consider the short-term problem of deriving an optimal irrigation schedule within one season when the soil moisture at the beginning of the season is random, given a deterministic level of water in the reservoir. In the second paper, the authors study the medium-term problem of finding the optimal acreage under irrigation to plant at the beginning of the season while incorporating uncertainty in reservoir inflows. Finally, they extend their analysis to the long run by determining the best size of an irrigation district under uncertain water demand and supply conditions. Dudley (1972) individually presents a more complete paper in which he integrates the three planning horizons developed in the papers described above. These series of papers demonstrate the usefulness of SDP in planning the sustainable development of water resource systems by incorporating different horizons and risks in different state variables.

In a similar paper, Dudley and Burt (1973) use an integrated intra-seasonal and inter-seasonal SDP to analyze the management and design of a reservoir for irrigation purposes. The first choice variable for the management dimension of the problem is the optimal irrigable acreage to plant at the beginning of any given season. Given the plantings choice, a subsequent decision in the management dimension is to decide how to adjust such plantings for the uncertainty associated with reservoir inflows. In other words, a second decision variable is to decide the irrigable acreage to be abandoned from further irrigation if the reservoir inflows during a given season do not generate enough water to satisfy the water requirements of the initial plantings choice. In the design dimension, the authors optimal consider the capacity of the reservoir, given the nature of the management decisions of planting and abandonment. From the Dudley and Burt framework it is possible to derive tradeoff curves between expected benefits of the system and their variability. In general, reservoir operations aimed at high degrees of reliability induce operating policies that result in severe reductions in plantings.

Using SDP in conjunction with simulation analysis, Dudley (1988a) studies the simultaneous planning of reservoir operations and farming activities by a single decision maker. Using plant-growth simulation models and weather data, Dudley is able to characterize the high variability of reservoirs inflows and the impacts of water supply in crop yield. Similar to Dudley and Burt (1993), Dudley (1988a) concludes that expanding the crop plantings generates more revenues while also increasing the variance of revenue. Therefore, the planner faces a tradeoff between higher revenues and higher variance. The tradeoffs may be the cause of disagreements between different stakeholders in the irrigation district since difference risk preferences will lead to different production and reservoir operation plans. This disagreement motivates the need for alternative risk management instruments, especially as water becomes a more scarce resource.

In an effort to update previous research, Dudley (1988b) and Dudley and Hearn (1993) use SDP in conjunction with plant growth and weather simulators to introduce the effects of a random water demand into the information available to irrigation planners. In an effort to model separately the decisions in farm management and reservoir operation, both papers develop versions of SDP models for each type of decision. In the farm management DSP, uncertainty enters through the probabilistic nature of reservoir releases. Reservoir releases, in turn, are the strategies produced with irrigation management DSP, which is based on the uncertainty of both reservoir inflows and crop water demands. Therefore, the combination of both versions of the SDP captures the interrelated decision-making process at the source of water supply and at the point of water use. The results show that for a given size of the irrigation district, the mean revenue and the variance of revenue are lower than in the case that both the reservoir and the farm management decisions are made by the same decision maker.

From the literature, one can gain insights about the use of DSP in water resources planning and operations. One of the advantages of SDP is the ability to disaggregate a large inter-temporal optimization problem into several individual optimization problems. Such advantage makes it is possible to model different planning horizons, which range from multiple stages within one hydrological season to multiple years in the planning of irrigation land development. Therefore, SDP is especially useful for planners that consider different time horizons in their planning strategies.

Despite the fact that DSP is adequately suited to model the inter-temporal decisions about storing and releasing water under an uncertainty environment, the method has some weaknesses. The major disadvantage is the computational costs the methodology requires to solve the problem. In particular, the SDP method suffers from the curse of dimensionality, which refers to the explosive growth in the size of the model as the number of state and choice variables increase. In order to keep the size of the model within a manageable range, researchers have to restrict the number of state or control variables and states of nature. For example, the random nature of reservoir inflows is introduced by segmenting the density function into discrete levels and assigning respective probabilities. Hence, in order to prevent the curse of dimensionality, the segmentation of the density function in discrete levels has to be relatively small.

Moreover, in order to avoid the exponential growth in the problem constraints, most of the models reviewed in this work limit the number of crops to one or two.

Another weakness that is usually mentioned in the literature is the intrinsic assumption that the planner has a perfect foresight of the discrete inflows associated with a given probability. Efforts have been made to correct this weakness in the methodology by incorporating scenarios of future reservoir inflows, rather than probabilistic measures. Examples of these efforts include the following: sampling stochastic dynamic programming (Kelman et al, 1990), recursive stochastic programming (Blanco and Flichman , 2002), value-iteration dynamic programming (Mawer and Thorn, 1974), and forward dynamic programming (Teixeira and Marino, 2002).

Despite the usefulness of SDP to derive optimum reservoir operation policies, it exercises no direct control over the probability of water supply failure or shortages. Since the usual objective of SDP is to maximize the present value of net expected benefits, the strategies derived might create more water shortages, relative to irrigation demand, than what farm managers are willing to tolerate. Askew (1974a, 1974b) proposes using chance constraints or imposing a reliability constraint into DSP models, so that the maximization problem takes into account the risk of failure. [4] Askew shows that if a penalty is imposed each time the system fails to satisfy demand, then operation policies generated by the DSP model will be amended towards lower target releases that produce acceptable levels of risk.

Alternatively, Askew (1975) proposes using a variable discount rate or risk premium, different from the economic discount rate, in conjunction with risk premiums to generate operating policies that deal with the uncertainty in inflows. Particularly, Askew establishes that in order to reduce the probability of failure in the optimal policies derived with SDP, the variable rate at which future expected benefits are discounted must be lower, so that releases are reduced at tolerable levels of safety. Furthermore, the amount by which the risk premium rate needs to be reduced depends on the severity of the reliability constraint imposed on the DSP. Askew shows that the use of a risk

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[4] Chance-constrained programming is discussed in the next subsection of this chapter.

premium, in the form of a variable discount rate, is equivalent to the use of the penalty costs proposed in previous work (1974).

2.2.4 Chance-Constrained Programming

Since risk consideration in the analysis of water resource systems is usually concentrated on the random nature of reservoir inflows, it requires modeling risk in the constraints of a resource allocation problem (i.e. RHS risk in mathematical programming jargon). In the particular case of irrigation planning, random reservoir inflows translate into random availability of the most important resource used in crop production. Charnes and Cooper (1963, 1973) introduced chance-constrained programming (CCP) as an alternative method to linear programming to handle uncertainty in the availability of resources. The method is based on the notion that the decision maker is willing to accept a given level of risk, expressed as a percentage of time, at which a given resource constraint is satisfied (or violated).

Consider the resource allocation problem in which the planner maximizes objective function z by choosing the number hectares of the *jth* crop to be irrigated X_j. The net return per hectare of the *jth* crop is denoted by c_j and the requirement (demand) of the *ith* resource per hectare of the *jth* crop planted is a_{ij}. The variable w_i represents the availability of the *ith* resource. The CCP formulation is presented in equations 2.12 and 2.13. The conventional interpretation of the chance constraint involving the random resource w_i is that such constraint is satisfied α percent of the time. In other words, the chance that the constraint is violated is given by $(1-\alpha)$.

$$Max \sum_j c_j X_j \tag{2.12}$$

Subject to

$$P\left[\sum_j a_{ij} X_j \leq w_i\right] \geq \alpha \tag{2.13}$$

The solution of this problem produces optimal choices X_j^*, which maximize the objective function and guarantee a level of safety for the decision maker. Higher values of α imply lower net revenues; therefore, the choice of α will depend on the planner's willingness trade off lower revenues for higher levels of security.

2.2.5 CCP Applications

In one of the applications to irrigation management, Maji and Heady (1978) use CCP to derive, under conditions of variable reservoir inflows, optimal cropping patterns and a suitable reservoir operation policy for the Mayurakshi irrigation district (India). The farm allocation model combines land, capital, labor and water resources to produce several crops, but only the latter is available in random quantities. The results indicate that as the probability of occurrence are kept within the limit of 10% the crop pattern is more favorable for farm managers growing crops with traditional technology, whom are less prepared to bear risk.

From an irrigation planning perspective, target releases are important information that planners need to transmit to farm managers so that the latter can make sound investment decisions. Eisel (1972) develops a CCP model to derive target releases that must be met with a specified level of certainty. However, uncertainty in reservoir inflows also results in the risk of spills, which can inflict damages to crops due to excess water. Therefore, the probabilistic statement about a desired level of risk is also included in the reservoir spill constraints. The paper concludes by determining tradeoff curves between a higher level of safety and lower levels of income, similar to the ones derived with models using DSP.

The major advantage of CCP is that it allows the decision maker to make statement about the probability at which certain resource constraints are binding. In contrast to DSP, the operation policies derived from CCP maximize the expected net benefits from reservoir releases and crop plantings taking into consideration an acceptable level of risk. Although the policies derived with DSP are optimal, they fail to account for the probability of failure. The DSP oversight of risk of failure might not be desirable, especially in irrigation systems composed of farmers that have little risk-bearing capabilities or when safety-first objectives are considered important for society.

Despite the intuitive approach that CCP brings, the adoption of the CCP method in the analysis of water resource systems has not being as rapid as DSP. The main reason for the slow adoption of CCP is that the methodology is not suitable for modeling inter-annual reservoir management decisions (Fonseca and Flichman, 2002). Furthermore, in the few CCP applications found in the literature, excessively conservative reservoir operation policies are produced. For instance, Loucks and Dorfman (1975) warn reservoir operators that CCP leads to system performance, as measured by the frequency of failures, which can be equally attained using other linear programming models.

2.2.6 Chanced-constrained Dynamic Programming

Given the limitations of both SDP and CCP, Askew (1974b, 1975) proposes a method that combined the capability of CCP to include the risk of failure in reservoir operations, and the capability of DSP to model inter-temporal decisions. Specifically, Askew (1974b) suggests that "if a stochastic dynamic program is developed that has the ability to restrict the probability of given variables taking values outside a fixed range, then it can be said to be capable of handling chance constraints. It is suggested that by analogy with the techniques developed for linear programming, such as stochastic dynamic program could be called 'chanced-constrained dynamic program' (CCDP)." Moreover, CCDP is applicable when future flows into a reservoir are random, and hence future storage volumes and releases are also random, because the method defines limits on the percentage of times at which storage volumes and releases should fall below a target.

However, the solution to CCDP models is highly cumbersome. Askew explains that the flexibility and realism of CCDP comes at the expense of a heavier computational burden. Furthermore, the method does not guarantee a global optimal solution. Therefore, additional applications of CCDP are hard to find in the literature.

2.3 The Institutional View

As regional water economies move from the "expansionary" phase to the "maturing" phase, pressure develops on water resource systems due to the intense competition among different types of users and higher costs in developing new infrastructure for storage and supply (Randall, 1981). As a consequence of such pressure, and exacerbated by the stochastic nature of reservoir inflows, the reliability of water systems experience considerable declines. While the optimization models can assist in making decisions about storage and inter-temporal releases, such models are not sufficient to guarantee efficient and secure allocations of the resource. In order to manage the problems of scarcity and unreliability that characterize a maturing water economy, the institutional view proposes institutional arrangements that enhance the decisions suggested by optimization models to ensure that the resource is distributed to uses that maximize social benefits without exceeding tolerable levels of system-failure risk.

There is considerable literature on the potential of water markets to achieve efficient allocations of the resource that maximize social welfare. However, the development of water markets requires overcoming certain market failures that seriously hinder the realization of market potentials. For instance, the physical and ownership insecurities of water rights encompass a set of constraints that prevent water transfers. Therefore, the foundation of efficient water markets lies in institutional arrangements that guarantee order, flexibility and relative certainty for water users. In the absence of such institutional arrangements, society may not be able to grasp the potential of water markets. Therefore, institutions establish ground rules for resource use such as incentives, information, and penalties that guide water users towards efficient outcomes for society. In the following subsections the following institutional arrangements are reviewed: the doctrine of "prior appropriations," contingent water transfers, and risk sharing in reservoir contents, volume and capacity.

2.3.1 The Doctrine of "Prior Appropriations"

In dealing with the problem of physical insecurity, especially in drought episodes, some water resource systems rely upon the "doctrine of prior appropriation." Under this institutional arrangement, water rights in the system fall into two categories: senior and junior rights. According to the arrangement, the holders of senior rights rank first in the priority of delivery and holders of junior rights are served only after all senior rights have been fulfilled. Therefore, water users can accommodate the uncertainty in water supply by carrying a portfolio of the two types of rights, which can be acquired or sold in a water market. The composition of the portfolio must be flexible enough so that its composition is suitable for adjustments dictated by changing economic and climatic conditions. Thus, the institutional design would combine the “doctrine of prior appropriations” with the notion of water markets to produce a solution for the problems of efficient allocations and physical uncertainty of the resource (Livingston, 1998).

2.3.2 Water Transfers

Despite its potential, the “prior appropriations” doctrine applied in the Western United States has not being designed to produce the most efficient outcomes. Turner and Perry (1997) describe how the design of this institutional arrangement has not taken into account the benefits derived from instream water use in terms of ecosystems sustainability; therefore streams have not been appropriated water rights, except in years with abundant inflows. Given this problem of the “prior appropriations” doctrine, Turner and Perry develop a model to characterize water transfers from agriculture to instream use recognizing the uncertainty in the water supply. They estimate the value of water in agriculture and compare it to a potential market price that would compensate farmers for leasing their water rights to instream use. Uncertainty plays a significant role in estimating the value of water for agriculture such that failure to consider the random nature of the water flows into the system result in an overestimate of the water volume that farmers would be willing to sell or lease to other users.

Besides ecological use, the hydropower industry has been excluded from water rights in the “prior appropriation” system of the Western United States. Hamilton, Whittlesey and Halverson (1989) have proposed an interruptible water market between irrigated agriculture and hydropower use. The potential gains from such market might arise from the combination of the premium that power customers place on supply reliability and the low estimated marginal value of water employed in agricultural uses. An interruptible water market between these industries would reallocate water from agricultural use to the higher-valued hydropower use during years of low water flows, thereby increasing average power generation and achieving greater societal benefits from the resource. Such institutional arrangement would increase the physical security of one sector of the water economy by temporarily shifting the resource allocation away from the lowest valued alternative use.

Residential use is another sector within the “prior appropriation” doctrine that suffers from the limited physical security that accompanies junior water rights. Similar to the interruptible market for hydropower generation proposed by Hamilton, Whittlesey and Halverson (1989), Taylor and Young (1995) propose the use of rural-to-urban water transfers to increase the reliability in the supply of water for residential use. The feasibility of both proposals requires that the additional benefits derived from the purchaser of the water transfer exceed the forgone benefits to agriculture and the transaction costs associated with the transfer.

Michelsen and Young (1993) expand the notion of an interruptible water market to a water supply option contract (WSOC) or "dry year option." Under this institutional arrangement, a formal agreement would exist between a farmer, or a group of farmers, and an urban water user to engage in agriculture-to-urban temporary transfers of water, contingent upon previously specified drought measures. On the one hand, the holder of the WSOC, which in this case is the urban user, acquires the right to buy a previously specified stock of water at a previously determined water price, also termed the exercise price. On the other hand, the seller of the option, which in this case is the group of farmers, guarantees the delivery of the stock of water demanded by the holder of the option. Furthermore, the holder of the option must pay an option price, which is a premium above the exercise price.

While the WSOC shares similarities with options encountered in the stock markets and commodity exchanges, Michelsen and Young (1993) discuss three basic features that distinguish a WSOC from other option contracts. First, WSOC entitles the holder with the temporary use of a stock of water and not with the ownership for that stock of the resource, while option contracts in the financial literature give the holder the ownership of the transacted stock or commodity. Second, while in financial options the contract is terminated with the exercise of the option, a WSOC gives the holder the possibility of exercising the contract multiple times during the contract period. Finally, the decision to exercise the WSOC depends on the supply of water while the decision to exercise stock or commodity options is based on price considerations.

The problem is that water transfers in the forms described above seldom occur. Two of the most common reasons for the low number of water transactions include transaction costs and risk aversion. One the one hand, Young (1986) partially explains the absence of many transactions by the high transaction and conveyance costs associated with water transfers. In the absence of third-party effects, the economic rationale for a water transaction exists only after two conditions are met: first, if after assessing the cost of all the alternatives sources of supply, the purchaser concludes that the least-cost choice is purchasing the water from another user. Second, if the direct benefits of the purchaser exceed the sum of direct benefits forgone by the seller and the transaction and conveyance costs incurred in the transfer. Under this scenario, a high volume of transactions is expected when both transaction and conveyance costs are low.

On the other hand, Howitt (1998) incorporates the role of risk aversion. Basically, under the uncertainty about quantity of water available and its value, risk-averse sellers and buyers will avoid committing to a water market. One the supply side, buyers will be afraid of undervaluing their water rights and prefer short-term leases that have the option to return the water to their original low value use. On the demand side, purchasers will be afraid of internalizing the long-term risk associated with the high cost of short-term leases and will prefer permanent purchases. In other words, risk-averse sellers will be afraid of selling their rights too cheaply, whereas buyers will be afraid of the uncertain water availability in thin spot markets to satisfy their highly inelastic demand.

2.3.3 Content, Volume, and Capacity Sharing

While the "priority appropriation" doctrine allows individuals to manage their own portfolio of water rights, and hence their risk exposure, other institutions aim at equal risk sharing among the water users. Dudley (1988b and 1988c) discusses the following three different types of property rights systems to share water from a single reservoir among users: content, volume and capacity sharing. While content sharing refers to allocation of water from a single reservoir in a specific point in time, volume sharing expands the concept to sharing in the current reservoir contents plus future inflows into the reservoir. In contrast, capacity sharing is a system of water rights in which each user is allocated a percentage share of reservoir capacity and a percentage share of net inflows from the reservoir.

While they seem related concepts, the timing and the uncertainty in the water inflows create important distinctions among the institutions of volume, content and capacity sharing. For instance, if the reservoir would be full at the beginning of the growing season and no further inflows would be expected during the rest of the season, then content, volume and capacity sharing would be equivalent. However, when more inflows are expected after the beginning of the growing season, the distinction among the three concepts becomes apparent. With additional inflows during the season, when the volume sharing institution is in place, water users have rights not only to the content of the reservoir at the beginning of the season, but also to any future volume produced by inflows that occur during the ongoing season. Notice that under both volume and content sharing, the volume available over the whole season will depend on the aggregate demand for water at the beginning of the season. In contrast, capacity sharing yields more security to water users in the form of a more clearly defined property right since such an institution allows each water user to manage his percentage of capacity in an individual fashion, just like managing a mini-reservoir, controlling his own releases, storages and future inflows. The major advantage of capacity sharing is that the individual allocation of water would be independent from the aggregate demand for water at the beginning of the season. Dudley (1988c) argues that in addition to increasing the physical security of the resource, reservoir capacity sharing would foster water markets and thereby produce more efficient allocations of water in the system. To date, none of these proposals have been implemented.

2.4 Agricultural Insurance and Weather Derivatives

The exposure to risk in economic activities has led to the development of traditional risk-sharing devices such insurance and futures markets. However, these traditional devices have little applicability in the management of water supply risk. On the one hand, experience with insurance schemes around the world indicates that such programs are expensive to implement, and often plagued with problems of moral hazard and adverse selection (Skees and Reed, 1986; Turvey, 2001). For instance, in the US where the agricultural sector has a long tradition of tapping into agricultural insurance markets, the federal government spends considerable resources in the form of subsidies to encourage farmers' participation in the schemes. On the other hand, futures markets can be used only to protect against unfavorable changes in output price. While price variability is also an important source of risk in irrigated agriculture, the source of risk dealt with in this work is water supply risk. While the former entails risk in the output supply side, the latter is implies risk in the input supply side. Therefore, neither agricultural insurance nor futures markets satisfy the need for a risk-sharing device in water resource systems.

2.4.1 Weather Derivatives

Motivated by the need to manage weather risk, and bolstered by the deregulation of energy markets in the US, the weather derivative (WD) has emerged as a new instrument in the set of risk-sharing devices available in the economy. Conceptually, a WD is a contract between two parties that stipulates how payments will be exchanged between the parties as determined by an index of the respective weather variable in a specific location (Stoppa and Hess, 2003; Zeng, 2000). The size and direction of the payment exchange, and any premium payment involved, will depend upon the type of contract (i.e. puts, calls, and swaps). Alternatively, Agarwal (2002) defines a derivative as "a financial instrument whose value depends on the values of other more basic underlying variables." The underlying variable could be either a traded asset (e.g. stocks) or weather index.

While weather derivative contracts (WDC) share common characteristics with agricultural insurance contracts, they also have very important differences. The most important distinction lies in the mechanism that triggers the payments. Different from indemnity payments from insurance contracts, the payoff obtained from a WDC does not require inspection and loss adjustment, but relies on a calculation based on the parameters of the contract. This distinction has two important implications in the writing of WDC. First, waiving inspection and loss adjustment substantially reduces the transaction costs associated with WDC in comparison to the more traditional insurance contracts. Second, since the underlying index is the basis for the payoff from a WDC, moral hazard problems that occur in traditional agricultural insurance schemes are considerably reduced. The caveat is that weather derivatives carry some basis risk, which is the risk that arises when the losses incurred by the insured do not perfectly correlate with the weather index. In other words, weather contracts might result in situations in which the contract does not pay despite the physical occurrence of damages.

A WDC is designed by selecting the following parameters (Zeng, 2000; Leggio and Lien, 2002; Alaton, Djehiche and Stillberger, 2002): contract type (e.g. call, put or swap); contract period; definition of the indexed weather variable underlying the contract, denoted by I; definition of the strike or threshold that triggers payments, S; the tick size, k, in a linear payment scheme, or the constant payment L in a binary payment scheme; location where the underlying weather variable is to be measured; and the premium, denoted by ρ.

In terms of contract type, the most basic forms in which WDC are structured are: calls, puts and swaps. Call and put contracts are similar to an insurance policy: the buyer (insured) pays a premium in exchange for guaranteed payment from the seller (insurer), based on the specification of the contract. Thus, in a call contract the buyer and the seller agree on a contract period and a weather index that serves as the basis for the contract. In exchange for a premium paid by the buyer at the beginning of the contract, the seller will pay the buyer a monetary amount, denoted by P_{call}, if the index exceeds the contracted strike at the end of the contract period. If at the end of the contract the index does not exceed the strike, the seller keeps the premium and the buyer receives no payment. The formula for this European-type of weather option is given by

$$P_{call} = k \max\left(0, I - S\right) \tag{2.14}$$

when the payment structure is linear, and

$$P_{call} = L \text{ if } I - S > 0; \; P_{call} = 0 \text{ if } I - S \leq 0 \tag{2.15}$$

when the payment structure is binary.

The put contract is similar to the call contract, except that in the former the seller will pay the buyer a monetary amount at the end of the period if the index falls below the strike level. The formula for payoff of the European-type put is given by

$$P_{put} = k \max\left(0, S - I\right) \tag{2.16}$$

when the payment structure is linear, and

$$P_{put} = L \text{ if } S - I > 0;\ P_{put} = 0 \text{ if } S - I \leq 0 \tag{2.17}$$

when the payment structure is binary.

In determining the strike and the period parameters, the objective is to select the contract period that maximizes the correlation between the index and potential losses incurred due to the weather variable. In other words, the contract design aims at depicting the variation in the volumetric measure of interest to the insured with the definition of the weather index. In studies about WDC applicability in agriculture, for example, the aim is at depicting changes in output with a rainfall index.[5] In the case of energy markets, the objective is to use the index to depict the changes in the seasonal demand for electricity or natural gas. The volumetric changes in the demand for energy, as measured by temperature indexes, considerably influence the revenue and bills of firms and consumers.

Finally, Agarwal (2002) outlines the following desirable features in the selection of the underlying weather variable: reliability of its value and existence, and recognition of its role in decision making; historical relevance, data availability and correlation to the hedging needs; monitoring and measuring amenability; suitability to modeling; and existence of a suitable pricing methodology.

2.4.2 Applicability of WDC to Irrigated Agriculture

Most applications today are concentrated in the natural gas, oil and electricity sectors. However, pilot programs have also been designed for the agricultural sector in Canada, Morocco and the US[6]. The appeal of the derivative contracts lies in their flexibility to hedge against volumetric risk, rather than price risk, without having to incur the higher transaction costs and information asymmetry problems that characterize the traditional insurance schemes (Stoppa and Hess, 2003; Mueller and Grandi, 2000). In the case of irrigated agriculture, the major source of production risk is reservoir inflows,

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[5] Application of rainfall-based derivatives is described in Turvey, 2001; Stoppa and Hess, 2003; Hao, Hartell and Skees, 2004; Martin, Barnett and Coble; 2001.
[6] See Turvey (2001).

which is a volumetric variable. Therefore, irrigated agriculture seems an industry naturally amenable to the application of WDs.

Up to this date, the only proposals that consider the use of a weather derivative contract in irrigated agriculture are presented in the work of Skees and Zeuli (1999). In their conceptual paper the authors propose to use a rainfall index contract to hedge against the variability in water levels on a reservoir. In their application to the Blowering reservoir in Australia, the authors are able to explain 70% of the variation of the water levels in the reservoir using a rainfall index based on three rainfall stations surrounding the reservoir area. However, correlating rainfall to the storage level implies two caveats. First, the storage variable is the outcome of reservoir management decisions[7], and as such it is subject to manipulation. Without taking into account the operation rules of the reservoir, Skees and Zeuli (1999) are unable to link the rainfall index with the profits or losses of irrigated agriculture. In that sense, their approach does not address the issue of moral hazard and insurable losses in the irrigation district. Second, the data requirements for designing those contracts are very high because a large number of rainfall stations need to be examined to determine a correlation that can be trusted by both sellers and buyers of the contract. Besides overcoming the challenging of missing observations in rainfall stations, establishing the rainfall-storage correlation is not always a successful endeavor.

In a similar proposal, Agarwal (2002a, 2002b) favors the design of a derivative contract using the water table as the underlying index. While a water table index provides a very strong correlation with the soil water contents and the availability of underground water, the author fails to recognize the fact that water tables are subject to man-made changes. Although part of the variation on the level of water in aquifers, similar to surface reservoirs, is directly explained by the variations in the replenishing rate from water that filters to the soil from rainfall and snowmelt, the underground water stock is also subject to management-induced variations[8]. In the particular case of irrigated agriculture, there is evidence that the unregulated pumping of the underground water

[7] Some variables that represent the outcome of a decision are subject to the problem of moral hazard, as described in the insurance literature. Moral hazard means that decision makers may incur in actions after buying insurance so as to increase either/both the probability of a loss or/and the magnitude of the loss.

might lead to the over-exploitation of aquifers. Therefore, the use of the water table as the underlying index of the weather contract might be subject to moral hazard problems and diminish the feasibility of the contract.

2.5 Summary

The random nature of water supply represents a major source of risk in irrigated agriculture and has been the subject of many research reports in the agricultural economics literature. Proposals to deal with this risk fall into two different, but complementary views: the technical view and the institutional view. While the technical view's main focus is the optimal management of the resource at the supply side through reservoir operation rules; the institutional view broadens the spectrum of policy action by seeking rules that link the supply and demand for water, particularly in resolving water shortages. Despite their apparent differences, researchers have combined the methodological developments of the technical view with institutional arrangements to provide general frameworks in the study of water allocation when the availability of the resource is uncertain.

Researchers in the technical view have developed methodologies to manage the random component of stream flows in the design and operation of reservoir. For instance, SDP is a powerful tool that allows decision makers to solve the inter-temporal dimension and derive operation rules. The optimal rules equalize the marginal benefit of current usage with the discounted expected marginal benefits of future resource usage, taking into account that current usage impacts the availability of water in the future. Alternatively, planners can develop probabilistic models using CCP, taking into account the random availability of the resource and allocating resources accordingly. The major contribution of the technical view is that it provides an array of techniques that provide planners with useful insights into the economic impact of random inflows and incorporate risk management components in the operation rules of a reservoir.

However, the application of such operation rules only limits the consequences of water supply shortages to a certain extent. While it is true that a reservoir is by nature a risk management tools that allows planners to store water for future use, there are two

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[8] See previous footnote.

kinds of costs associated with this alternative. One the one hand, when water is the limiting resource or binding constraint in the production process, high opportunity costs are associated with water left idle in the reservoir for future use. Operators strive for a delicate balance between conservative operation rules and high opportunity costs. On the other hand, high transaction costs are incurred in storing water because water evaporates and leaks. Furthermore, storage of water creates the risk of flooding in the event that higher than expected inflows are accumulated during the replenishment season. The point is that operation policies derived from DSP, CCP and CCDP do not insulate the system from the economic consequences of uncertain reservoir inflows. Therefore, there is a clear need for other mechanisms that supplement operating rules in the management of water supply risk.

The institutional view proposes the use of market-based arrangements such as water transfers, and definition of water right systems that establish rules to allocate the scarce and uncertain resource efficient manner. A water market that allows flexible transfers of water to the highest valued uses, especially in times of drought, has the potential to allocate resources efficiently. However, the underlying base for the successful implementation of a water market is a system of water rights. In order to deal with the uncertain availability of water several water right systems have been proposed. For instance the prior appropriations doctrine assigns water rights with different security clauses that clearly establish which rights are to be fulfilled first in the event of water shortages. Conversely, water rights could be designed such that the risk of water supply is equally shared among water users through volume sharing of a reservoir.

Despite their potential, very few water markets have been established. The major challenges in implementing such institutions include high transaction costs associated with water transactions and third-party effects associated with the physical transfer of water. For example, the physical losses during the conveyance of water might exceed the potential benefits of the transaction. In addition, water markets achieve efficient allocation of water only when the proper compensation procedures exist to resolve third-party effects or externalities imposed by water transfers.

Alternative risk-sharing institutions that compliment the operation rules recommended by the technical view include agricultural insurance and weather derivatives. Traditionally, agricultural insurance schemes usually protect farmers, including those in irrigated agriculture, against yield losses caused by multiple perils. The experience with such schemes indicates they are expensive to run, often financed by government subsidies, and plagued with problems of asymmetric information. In the literature of agricultural insurance there is no evidence of insurance markets being used to protect against the economic impact imposed by water shortages in irrigated agriculture.

In lieu of expensive agricultural insurance schemes, weather derivatives could be used in the management of water supply risk in irrigated agriculture. In spite of the popularity they have encountered in the energy markets of the US, very few applications can be found in the agricultural sector. Up to date, only two proposals are found in the literature regarding the use of derivative contracts in the hedging of water supply risk in irrigated agriculture. The studies, however, do not include a feasibility analysis from the producer's point of view (i.e. potential demand). Neither the policies acknowledge the contributions of the technical view in the development of operation rules for reservoirs with uncertain inflows.

Chapter 3: A Theoretical Model of Production under Input Supply Uncertainty

3.1 Introduction

The traditional theory of production postulates that firms maximize profits by their choice of inputs, implicitly assuming that firms have complete access to all factors at posted market prices. However, the supply of some inputs may not be deterministic. This is a pervasive risk in markets for natural resources, particularly water markets. As a result of the variability in input supply, a firm's levels of profits will certainly fluctuate. Moreover, in many cases, firms must commit land and capital before knowing the availability of other factors of production. The absence of complete markets for natural resources suggests a role for government intervention. In particular, public policies may be adopted which provide insurance or alternative risk-sharing mechanisms.

Although there is a clear role for government intervention, it is first important to study the degree to which firms may be able to implement their own strategies to hedge against input supply risk. To be specific, I examine the role of output diversification. In contrast to previous research, I express this problem as a resource allocation problem with portfolio characteristics. In this manner, I determine how the degree of risk and returns affect the choice of factor inputs and risk management strategies. For example, in the case of farmers that depend on uncertain irrigation supplies, one strategy might be converting to crop portfolios that depend less on water.

I develop a model in which a risk neutral firm is faced with the objective of maximizing expected profits when the availability of one of the inputs is uncertain. The firm's production possibilities are constrained by the endowment of two inputs: land and water. While the endowment of land is known with certainty, the supply of irrigation water during a given season is uncertain (i.e. known in probabilistic terms). Different from previous work in the uncertain input supply literature, the proposed framework allows the firm to use output (i.e. crop) diversification to hedge the risk of uncertain input (i.e. water) supply. Production possibilities consist of two types of crops that differ in terms of their intensity of water use. In addition, these crops differ in terms of their market price. The risk-return tradeoff is such that allocating land to the water-intensive crop potentially generates a higher return (the water-intensive crops sells at a high relative price), but the additional contribution to the firm's revenues is accompanied by a relatively higher degree of uncertainty. Since the risk-return profiles of the crops are different, the firm has the ability to hedge against the risk in water supply by planting more of the less water-intensive crop (also referred as the water-saving crop in this chapter). Thus, the profiles will play a role in the allocation of resources.

The present analysis indicates that the firm responds to higher degrees of risk, as measured by the variance in irrigation supplies, by allocating less land towards the water intensive crop. However, if the supply of water is more likely to be available in larger quantities, more land will be allocated to the crop with a higher relative price. The underlying motivation in these strategies is the need to avoid the relatively larger reductions in productivity sustained by water intensive portfolios. For instance, when the portfolio consists only of crops with relatively large water use, the productivity losses experienced during water stress periods outweigh the productivity gains accrued in periods of plentiful irrigation. Thus, the role of output diversification is to anchor these losses by including crops whose productivity is less sensitive to variation in the supply of water. Finally, in the analysis of government intervention, I find that a contingent water-transfer contract can potentially reduce the degree of water supply risk. In fact, the optimal contract renders a deterministic supply of irrigation water.

From a policy perspective, the findings suggest that while farmers can diversify output to mitigate the output losses due to the shortage of a critical input, there are some costs associated with this policy. The potential role of government intervention in the provision of this input can be seen envisioned in two aspects. First, policies that aim at increasing the probabilities of receiving water supply, canal lining for example, will result in a shift towards more profitable crop portfolios. Since farmers already pay for the risk in the form of forgone profit, they could actually share in the costs (WTP) of improving the water conveyance system. Second, policies that aim at stabilizing the supply of irrigation water will promote plantings of higher valued crops. However, in order to determine the optimal level of water supply stabilization, reservoir operators should endogenize the behavior of irrigators. In other words, the policy should stabilize the supply of water at a level that internalizes the irrigator’s costs and benefits of operating under a less uncertain environment.

The impact of input supply uncertainty has been studied in previous work. However, in contrast to my approach, those papers have examined how the governmental provision of inputs creates inefficiencies in factor markets. For instance, Linz and Martin (1982) find that when facing a limited set of risk management choices, firms will develop an over-ordering bias. Further evidence of this bias, even under the absence of government intervention, can be found in Turnovsky (1971) and Martin (1981, 1984). According to the literature, firms will hoard factor inputs to minimize the variability in the supply or guarantee minimum levels of availability. In the particular case of water markets, Beare, Bell and Fischer (1998) and Howitt (1998) have observed that farmers will attempt to hoard water rights to secure a more stable supply of the resource; consequently reducing the possibility of water trade that leads to a more efficient use.

In addition, previous work finds that as a consequence of input supply risk, the expected profits will be lower than a certainty equivalent case. Therefore, input supply uncertainty imposes an opportunity cost on the affected firms. The works of Rodman (1972) and Martin (1981) indicate that the opportunity cost is directly related to the complementarities between the certain and uncertain inputs. When the technology of the firm is characterized by high degrees of complementarities between the certain and the uncertain input, the overall profitability will be more sensitive to the pervasive uncertainty in the supply of the latter input.

The main separation of this work from previous literature is as follows: In this model the firm uses two production technologies, which allow for output diversification as a risk management strategy. In contrast, previous research has studied the over-ordering effect as the main risk management strategy. An exception is the work of Feder (1980) who analyzes the choice between two crops that differ in terms of their yield variability and input requirements. However, Feder simplifies the problem by assuming that the certain crop can be viewed as a fixed financial return from growing this crop, thus ignoring complementarities between farm inputs. Different from Feder, the production functions of the firm I study are characterized by different degrees of complementarities between the certain and the uncertain input. Furthermore, I model diminishing marginal returns to all the factors of production, which motivates the role of output diversification as a risk management strategy.

One additional feature that distinguishes this framework is the presence of an input whose supply is deterministic, but that is quasi-fixed in nature (e.g. land). The presence of this input in the production functions implies that the firm commits resources previous to the resolution of uncertainty. In contrast, both Linz and Martin (1982) and Martin (1981, 1984) allow firms to choose their certain inputs after the uncertainty in the random input is resolved. In this sense, this framework is similar to the work of Moschini and Lapan (1992) who analyze the presence of quasi-fixed inputs, but in the context of output price uncertainty.

The remainder of the chapter is as follows. Section 3.2 describes the economic environment that the firm faces. Section 3.3 presents results in a benchmark scenario in which the input supply is deterministic. Section 3.4 examines the impact of uncertainty using comparative static analysis along with numerical simulations. Section 3.5 studies a social planner's problem and the effectiveness of contingent water transfer contracts. The proofs of the most significant results are presented in the Appendix.

3.2 Model Description and Notation

The output of each crop is given by a separate Cobb-Douglas production function. The output of the first crop is denoted by y_1 while the output of the second crop is given by y_2. The production of each crop depends on both water and land. Let w_i and l_i represent the units of water and land applied to crop *i*. In addition, output of the each crop is given by:

$$y_i = w_i^{\beta_i} l_i^{\alpha_i} \text{ where } i = 1, 2 \tag{3.1}$$

Both crops exhibit diminishing returns to each input. In the case of water, I impose the following restriction $0 \le \beta_1, \beta_2 \le 1$. However, I assume that crop 1 is the water-saving crop; therefore, I also have that $0 \le \beta_1 < \beta_2 < 1$. After normalizing the price of the water-saving crop to 1, p represents the relative market price of the water-intensive crop. The model features a potential risk-return tradeoff: crop 2 is a riskier crop because it uses more of the uncertain input, but it has a higher return since I assume $p > 1$. Finally, the restriction $0 \le \alpha_1 \le \alpha_2 < 1$ is necessary to model diminishing marginal productivity of land.

Furthermore, the firm is endowed with L units of land and W units of water to be allocated. The allocations of water and land between both crops are constrained by the initial resource endowments as described in equations (3.2) and (3.3).

$$W = w_1 + w_2 \tag{3.2}$$

$$L = l_1 + l_2 \tag{3.3}$$

3.3 Benchmark Case: Production under Certainty

This section studies a benchmark case, in which the amount of the water endowment is deterministic and uniformly distributed across the land, such that an equal volume of water is applied to hectare. [9] In section 3.4 below, I study the impact of uncertainty on the allocation of land to each crop.

The problem of the firm consists of maximizing profits by choosing the proportion of the land endowment, x, to be allocated to the water-saving crop. Since the firm owns the land and water it uses, there are no input costs are associated with input use; therefore, maximizing revenues is equivalent to maximizing profits.

The firm's problem can be written as:

$$\underset{x}{Max} = w^{\beta_1}(xL)^{\alpha_1} + pw^{\beta_2}((1-x)L)^{\alpha_2} \tag{3.4}$$

The FOC for the choice of x is given by:

$$w^{\beta_1}(xL)^{\alpha_1 - 1} = pw^{\beta_2}((1-x)L)^{\alpha_2 - 1} \tag{3.5}$$

The marginal conditions in equation (3.5) indicate that the profit-maximizing allocation of land occurs when the marginal value product (MVP) across both crops is the same. The left-hand side represents the MVP of land planted with crop 1 while the right-hand side represents the MVP of land planted with crop 2.

[9] The assumption of uniform allocation of water is necessary to obtain closed-form solutions.

Lemma 1 *Let both crops be equally intensive in land use (i.e., $\alpha_1 = \alpha_2 = \alpha$). In addition, assume the water-saving crop is independent of water (i.e., $\beta_1 = 0$).*[10] *Then, under a uniformly allocated water endowment, the closed-form solution for the profit-maximizing proportion of land allocated to crop 1 is:*

$$x^* = \frac{1}{1 + \left(pw^{\beta_2}\right)^{\frac{1}{1-\alpha}}} \tag{3.6}$$

By equation (3.6), the firm will respond to an increase in the price of the water-intensive crop by reducing the share of land allocated to crop 1. Similarly, due to the higher market value of the water-intensive crop, the firm will reduce plantings of the water-saving crop when both crops use land more intensively and there is more water available. Intuitively, since the land intensity of both crops is the same, increasing the land intensity coefficient raises the return to the water-intensive crop by a relatively greater proportion. The same argument applies to increases in the water endowment and the water-intensity of the water-intensive crop. Therefore, increasing the land intensity, the water endowment, and the share of water in crop 2, escalates the opportunity cost for the land allocated to the water-saving crop.

3.4 Introducing Uncertainty in Input Supply

3.4.1 The Stochastic Environment

In this section, the availability of water is unknown prior to the allocation of land. Specifically, there are two states of nature: "high" and "low," denoted by w_h and w_l, respectively. The "high" state of nature depicts a water-abundant scenario and occurs with probability q; the "low" state of nature depicts a water-scarce scenario and occurs with probability $(1-q)$. Also, let w represent a "reference" endowment of water and ϕ a proportionality factor. Specifically, the firm receives water endowments $w_h = \phi w$ and

[10] The assumption $\beta_1 = 0$ provides tractability to the model. I later present numerical results when $\beta_1 > 0$. Also, the restriction of equal land use intensity in both crops ($\alpha_1 = \alpha_2$) renders the analysis to be clearer.

$w_l = (1-\phi)w$. Please refer to Figure 3.1 for a graph describing the distribution function governing the availability of water.

The implications of different values of the parameters ϕ and q in the distribution function for water supply deserve further discussion. First, note that restricting ϕ to values greater than 0.5 implies that the supply of water in the "high" state is greater than the water supply in the "low" state. Second, when the value of ϕ is 0.5, the realizations of water supply in the "high" and "low" states are the same; therefore, the availability of water is deterministic and corresponds to the benchmark case considered in the previous section. Finally, the distribution of water exhibits a mean-preserving spread when q takes the value of 0.5.

Using the distribution function for water described above, the expected value and variance of the uncertain water supply are:

$$E[\tilde{w}] = w[q\phi + (1-q)(1-\phi)] \tag{7}$$

$$Var[\tilde{w}] = w^2\left\{\left[q\phi^2 + (1-q)(1-\phi)^2\right] - [q\phi + (1-q)(1-\phi)]^2\right\} \tag{8}$$

Please refer to appendix A to see how in the case of a mean-preserving spread, the variance of the uncertain water supply is increasing in the parameter ϕ.

The analysis proceeds by presenting two versions of the firm's problem. In the first version, it is assumed that the water-saving crop is completely independent of the water supply ($\beta_1 = 0$)[11]. As in the previous section, the assumption allows us to obtain closed-form solutions. In this manner, I can clearly examine how the degree of risk affects the allocation of land to each crop. I relax this assumption later and show the impact of risk by numerical computations.

[11] Feder (1980) studies the case in which the "safe" output is completely independent of the uncertain input and yields a constant return. In contrast to Feder, I model diminishing marginal returns to land for the "safe" crop. Furthermore, in the numerical results below, I consider that both crops require some amount of water for production to occur.

3.4.2 The Analytical Results

Given the environment described above, the problem of the firm consists of finding the optimal proportion of land to be allocated to each type of crop to maximize expected profits. A representative firm solves the following problem:

$$\max_x E(\pi) = q\{(xL)^{\alpha_1} + p(\phi w)^{\beta}((1-x)L)^{\alpha_2}\} + (1-q)\{(xL)^{\alpha_1} + p((1-\phi)w)^{\beta}((1-x)L)^{\alpha_2}\} \tag{3.9}$$

The optimal choice of x is implied by:

$$(xL)^{\alpha_1 - 1} = p((1-x)L)^{\alpha_2 - 1}\{w^{\beta}[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}]\} \tag{3.10}$$

The marginal conditions of profit maximization require that the proportions of land devoted to each crop equally contribute to expected profits. In equation (3.10), the left-hand side represents the marginal revenue of land allocated to the water-saving crop, which is known with certainty. In contrast, the right-hand side represents the expected marginal revenue from allocating land to the water-intensive crop. Thus, the optimal choice of x equalizes the safe return from the water-saving crop with the risky return from the water-intensive crop.

Lemma 2 *Suppose both types of crops are equally intensive in land use (i.e., $\alpha_1 = \alpha_2 = \alpha$) and the water-saving crop is independent of water (i.e., $\beta_1 = 0$),*[12] *the expected profit-maximizing allocation of land is given by:*

$$x^{**} = \frac{1}{1 + p^{\frac{1}{1-\alpha}}\{w^{\beta}[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}]\}^{\frac{1}{1-\alpha}}} \tag{3.11}$$

[12] See footnote 3.

In order to capture the impact of risk, I examine how the parameters ϕ and q affect the amount of land allocated to crop 1.

Proposition 1 *The expected profit-maximizing proportion of land allocated to the water-saving crop will be* ***increasing*** *in* ϕ*. Thus, in response to greater risk, the firm will shift resources from the water-intensive crop to the water-saving crop.*

As described above, ϕ represents the spread of the distribution of water around the mean. That is, higher values of ϕ induce a higher degree of risk, as measured by the variance, by increasing the difference between the expected realizations of water in the "high" and "low" states of nature. Then, proposition 1 establishes that even the risk-neutral firm will react to increased degrees of risk by shifting its land allocation towards the water-saving crop. In the absence of risk aversion, such risk management behavior can be explained as an effort to avoid the adverse effects of the "low" state on the water-intensive crop. When the technology of the firm exhibits diminishing marginal returns to the factors of production, there will be a tendency to allocate more resources to the crop whose marginal productivity is less affected during the "low" state of nature. Alternatively, the risk-neutral firm trades off the higher, but uncertain, revenues generated from the water-intensive crop for the lower, but certain, revenues generated from the water-saving crop. In conclusion, output diversification plays an important role in the management of input supply risk, even in the absence of risk aversion.

Proposition 2 *Under uncertain water supply conditions, the optimal proportion of land allocated to the water-saving crop will be* ***increasing (decreasing)*** *in* q *when* $\phi < 0.5$ *(*$\phi > 0.5$*).*

The role of q lies in describing the expectations about the different states of nature the firm faces. Proposition 2 suggests that the firm will increase (decrease) the allocation of land to the water-saving crop when there is a higher probability of occurrence of the "low" ("high") state of nature. Similar to the analysis of the effect of ϕ, risk aversion is not required to engage in the management of water supply risk through diversification. Diversification plays a role in risk management by giving the firm more flexibility to decide how its marginal productivity will be affected in the "low" state of nature. In particular, firms will avoid the relatively larger loss of output from the water-intensive crop by shifting land towards the water-saving crop.

To capture the impact of risk on the level of input use, expressions for the profit-maximizing allocations of land in equations (3.6) and (3.11) are compared. However, for the comparison to be valid, the water endowment in the deterministic setting is equal to the expected realization under uncertainty.

Corollary 1 *Suppose the profit-maximizing expression under certainty in equation (6) is evaluated at the expected amount of water provided, as calculated in equation (7). Then, the allocation of land to the water-saving crop is given by:*

$$x^*\Big|_{w=E(\tilde{w})} = \frac{1}{1 + p^{\frac{1}{1-\alpha}}\left\{w^\beta\left[(1-q)(1-\phi)+q\phi\right]^\beta\right\}^{\frac{1}{1-\alpha}}} \tag{12}$$

Comparing equations (3.11) and (3.12) leads to the following proposition

Proposition 3 *The expected profit-maximizing allocation of land to the water-saving crop under uncertainty is* ***higher*** *than its certainty equivalent analog; thus, uncertainty has a noticeable impact in the input choice of the profit-maximizing firm.*

The risk-neutral firm will alter their optimal allocation of resources when one of the inputs is uncertain. Intuitively, firms will hedge the risk by diversifying their output mix and relying more on the crop that does not depend on the uncertain input. The reason why the result does not depend on risk aversion lies in the fact that even risk-neutral firms will try to avoid diminishing marginal returns to its factors of production. Consider the hypothetical case in which the firm allocates all the land to the water-intensive crop. In this scenario, the productivity losses suffered when the "low" state of nature occurs, outweigh the productivity gains, realized when the "high" state of nature occurs.

Therefore, it is beneficial for the firm to mitigate the potential losses by diversifying the crop portfolio.

The firm's profit-maximizing behavior with respect to the exogenous factors (price, land intensity, water intensity and water supply) will be identical to the behavior under certainty conditions. Although water is uncertain, the higher expected benefit of producing the water-intensive crop prevails as an economic incentive to shift production away from the water-saving crop when any of the exogenous factors is increased. From Lemmas 1 and 2 the following corollary follows:

Corollary 2 *Regardless of the risk environment, the firm will respond to increases in the price and water intensity of the water-intensive crop, land intensity, and water allotment by expanding the production of the water-intensive crop.*

3.4.3. Numerical Results

Now suppose that the water-saving crop is not independent of the realization of water (i.e. $\beta_1 \neq 0$). In this version of the problem, it is possible to derive a closed-form solution for the firm's choice, however the complexity of the expression limits the analytical tractability required to characterize the impact of risk. Therefore, the additional insights to be gained from this version of the problem rely on the use of numerical simulations.

When the water-saving crop depends on the water supply (i.e., $\beta_1 \neq 0$), the expected profit-maximizing allocation of land is

$$x^{**} = \frac{1}{1 + p^{\frac{1}{1-\alpha}} \left\{ w^{\beta_2} \left[(1-q)(1-\phi)^{\beta_2} + q\phi^{\beta_2} \right] \right\}^{\frac{1}{1-\alpha}} \left\{ w^{\beta_1} \left[(1-q)(1-\phi)^{\beta_1} + q\phi^{\beta_1} \right] \right\}^{\frac{1}{\alpha-1}}} \tag{3.13}$$

As a result of relaxing the assumption about the water dependency of crop 1, it is not possible to sign the partial derivatives with respect to the parameters ϕ and q. Thus, firm's behavior is elicited by means of a numerical simulation. I consider the following benchmark parameters: $\alpha = 0.5$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $w = 2.1$, $p = 1.5$, $q = 0.5$ and $\phi = 0.5$. As described above, $\phi = 0.5$ depicts the scenario of a deterministic supply of water.

According to equation (3.7), the deterministic value of water is equal to 1.05. Under the benchmark parameters, the firm allocates 30.35% of the land endowment to the water-saving crop.

Starting from the benchmark case, the parameters ϕ and q are individually increased to illustrate how different degrees of risk impact the profit-maximizing choice of land. On the one hand, increasing ϕ above the 0.5 value implies that the spread of the distribution is wider, which results in a riskier environment for the firm. On the other hand, increasing q results in a higher probability of operating under the "high" state of nature. Please refer to Figures 3.2 and 3.3 for the graphs that show the results of the numerical simulations.

As demonstrated in Section 3.3, the variance, and therefore the risk, associated with the water endowment is increasing in the parameter ϕ when $0.5 < \phi < 1$. Intuitively, the distribution of the random variable widens under higher values of ϕ. A wider distribution implies a greater differential between the realizations of water in the "high" and "low" states of nature, which is tantamount to a greater degree of risk for the firm. Figure 3.2 depicts the impact of widening the distribution when a mean-preserving spread is in effect (q=0.5). The allocation of land to the water-saving crop expands as a result of an increase in the risk inherent in the water supply. The firm mitigates the exposure to higher degrees of risk by reducing the plantings of the water-intensive crop, thereby accepting a lower profit level. The simulation results are consistent with the analytical results stated in Proposition 1.

Before proceeding to discuss the impact of q, notice that this parameter has no impact when ϕ is equal to 0.5 because the latter condition implies that the firm receives the same amount of water in both states of nature. Therefore, it is necessary to modify the benchmark value of ϕ to capture the impact of q. The values of ϕ meaningful for this model are those falling within the range $0.5 < \phi < 1$ because they indicate that the amount of water received in the "high" state is relatively larger. In Figure 3.3 the value of ϕ is set at 0.6, but the rest of benchmark parameter remain the same. Hence, the firm responds to higher probabilities of operating under the "high" state of nature by reducing the plantings of the water-saving crop. By planting more of the water-intensive crop, the firm takes advantage of the higher probability of the “high” state, thereby improving the chances of generating higher expected profits.

Figure 3.4 demonstrates how expected profits depend of the different degrees of risk. The most important result is that profits reach a maximum when the value of ϕ is 0.5, which implies that maximum expected profits equal maximum deterministic profits when the supply of water is deterministic. Furthermore, any deviation from the deterministic case, in other words $\phi \neq 0.5$, results in expected profits lower than the deterministic profit. Therefore, it is possible to conclude that risk has an adverse impact on the firm’s profitability. Consistent with the analysis in the previous section, profits tend to decrease for values of ϕ greater than 0.5 because of the greater uncertainty associated the amount of water available. For example, as the value of ϕ increases to 0.9, profits decline by 9% when compared to the deterministic case.

3.5 Introducing Contingent Input Transfer Products

As demonstrated in the previous section, input supply risk lowers expected profits. Specifically, in order to hedge against uncertain availability of water, the firm allocates more land to the crop, which uses water less intensively, but yields lower revenues. This adverse impact introduces the possibility that government-sponsored contractual arrangements could alleviate the input supply risk. From a policy perspective, the social planner’s actions might find it welfare-enhancing to stabilize the firm’s income for at least two reasons: first, the physical security of the resource might foster water trading in the water economy (this aspect is not treated in this chapter), which subsequently leads to a more efficient use of the resource; and second, the agricultural sector might benefit from producing higher valued crops with a stable supply of irrigation.

This section considers a social planner who intervenes in the supply of the uncertain input to maximize the representative firm’s expected profits. I assume that the social planner has complete information about the nature of the distribution function for uncertain water supply. The social planner determines the strategy of storing a fraction of the firm’s water endowment in the “high” state of nature to deliver it during the “low” state of nature. Moreover, the nature of the water transfer is such that the amount that the social planner delivers during the “low” state of nature is exactly the same as the amount stored during the "high" state of nature. Implicit in this contractual arrangement is the assumption that neither physical losses nor transaction costs are associated with the planner's actions to store and transfer water.

The firm's problem under the contingent water transfer contract is formally stated as follows:

$$\max_x E(\pi) = q\{(xL)^{\alpha_1} + p(\phi w - t)^{\beta}((1-x)L)^{\alpha_2}\} \\ + (1-q)\{(xL)^{\alpha_1} + p((1-\phi)w + t)^{\beta}((1-x)L)^{\alpha_2}\} \quad (3.14)$$

where t is the amount of water transferred between states of nature. The FOC of the firm's problem is stated as:

$$(xL)^{\alpha_1 - 1} = p((1-x)L)^{\alpha_2 - 1}\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi) + t]^{\beta}\} \quad (3.15)$$

The profit-maximization condition in equation (3.15) requires the firm to equate the safe return from the water-saving crop with the risky return from the water-intensive crop. The difference between the profit-maximization conditions presented in equations (3.10) and (3.15) lies in the water transfer allowed between states of nature, which in principle decreases the uncertainty in the return for the water-intensive crop. In equation (3.15), the size of t is exogenous to the firm and is determined by the social planner.

Lemma 3 *Suppose both types of crops are equally intensive in land use (i.e., $\alpha_1 = \alpha_2 = \alpha$) and the water-saving crop is independent of water (i.e., $\beta_1 = 0$),[13] the profit-maximizing allocation of land under the structure of the contingent transfer contract is given by:*

$$x^{***} = \frac{1}{1 + p^{\frac{1}{1-\alpha}}\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi) + t]^{\beta}\}^{\frac{1}{1-\alpha}}} \quad (3.16)$$

[13] See footnote 3.

From the perspective of the social planner, equation (3.16) contains relevant information about the behavior of the firm under uncertainty conditions. It is clear from the above expression that the firm's behavior to changes in the exogenous factors is identical to the cases studied in equations (3.6), (3.11) and (3.12). However, of particular importance in the context of contingent water transfer contracts is the reaction of the firm to changes in the size of the transfer.

Corollary 3 *Under the contingent transfer contract, the optimal proportion of land allocated to the water-saving crop will be* ***decreasing (increasing)*** *in* t *when for* $t < \bar{t}$ *(* $t > \bar{t}$ *).*

The firm responds to increases in the contingent water transfer by shifting production towards the water-intensive crop; consequently, from the firm's perspective is desirable to engage in such a contractual arrangement. However, expansion of the proportion of land allocated to the water-intensive crop under the transfer scheme increases at a decreasing rate, due to the decreasing marginal returns to water. Therefore, there is a value of transfer of the resource in which it does not have any impact on the allocation of land. For such a value, denoted by $\bar{t}$, the firm will be indifferent regarding its participation in the contractual scheme.

Given the optimal choice of x under the contractual arrangement in equation (3.16), it is possible to derive the indirect profit function. The social planner can obtain all the information required about the firm's behavior in the indirect profit function, which from the planner's perspective is the firm's reaction function to any level of transfer stated in the contract. In other words, the problem is to find the level of transfer that maximizes the social benefits derived from transferring water between states of nature, which is stated formally as follows:

$$\begin{aligned}\max_t E\left(\pi^{***}\right) = q\left\{\left(x^{***}L\right)^{\alpha_1} + p(\phi w - t)^{\beta}\left((1-x^{***})L\right)^{\alpha_2}\right\} \\ + (1-q)\left\{\left(x^{***}L\right)^{\alpha_1} + p((1-\phi)w + t)^{\beta}\left((1-x^{***})L\right)^{\alpha_2}\right\}\end{aligned} \quad (3.17)$$

From the FOC of the problem in equation is possible to derive the social marginal benefit (SMB) and social marginal cost (SMC) functions of the transfer amount. These expressions are reported in Appendix A. The socially optimal amount of water transfer, denoted as t^s, is that which equalizes the social marginal benefit and the social marginal cost.

Proposition 4 *The socially optimal amount of water transfer is given by:*

$$t^s = \bar{t} = \frac{w(2\phi - 1)}{2} \tag{3.18}$$

There are two important observations to make about the amount of the transfer to be provided. First, since the socially optimal transfer of water is the same as $\bar{t}$, the planner will optimally choose to transfer water up to the point in which marginal transfers of water will not have any impact on the firm's crop portfolio. Second, at $t = t^s$, the firm is guaranteed a certain amount of water, regardless of the state of nature that occurs. In other words, the social contract eliminates uncertainty over the water supply. As a result, the firm becomes indifferent as to which state of nature actually occurs.

In the spectrum of the uncertainty faced by the firm, the social contract introduces an environment tantamount to the deterministic case. Specifically, the choice of land and expected profits are exactly the same as those obtained in the deterministic case. The equality occurs as a direct result of assuming that the planner's actions do not result in storage losses and transaction costs. If this assumption is relaxed, and depending on the size of the transaction costs and storage losses, the firm might have profits that lie in between the deterministic case and the uncertainty case.

3.6 Conclusion

This chapter has analyzed the economic effects of input supply uncertainty and designed a government intervention program to reduce such uncertainty. The results demonstrate that when the production possibilities include two crops that are different in their dependency on the uncertain resource, the firm manages the input supply risk by diversifying the crop portfolio. Specifically, for greater degrees of risk, the firm responds by increasing the allocation of resources to the crop that depends less on the uncertain factor. Moreover, the results show that diminishing marginal returns to the factors of production alone, rather than risk aversion, explain why the firm engages in crop diversification. Thus, I have illustrated that the over-ordering effect expressed in previous research may not be as strong when firms have the ability to hedge against uncertain input supply by diversifying production. In the case of agriculture, in particular, policies that encourage crop diversification potentially reduce the need to hoard water rights, and thereby promote the development of efficient water markets.

As I have shown, irrigators will respond to increases in the probability of operating under "high" states of nature by shifting land towards the more profitable crops. In terms of public policies, the government may not have to actually intervene in the provision of the service to affect this probability. For instance, policies that improve the water conveyance system, like canal lining, should have potential returns by decreasing the risk of water shortages. In other non-agricultural markets, the role of the government might be to implement legislation that foster the investment of alternative sources of supply, which will greatly improve the chance of operating under a "high' states of nature.

In terms of risk-sharing effectiveness, although crop diversification provides some cushion against the adverse impact of input supply risk, it does not guarantee full protection. Indeed, diversification comes at the cost of reduced profits. This failure to fully protect against input supply risk motivates the potential role for government intervention to stabilize the supply of the uncertain input. This chapter proposes a contingent water transfer scheme in which a social planner saves a proportion of the water supply during the "high" state of nature and releases the same proportion during the "low" state. Since the expected marginal value of water is higher during the "low" state, then the firm benefits by transferring a proportion of the lower valued realization in the "high" towards the "low" state. In the absence of transfer costs and storage physical losses, the proposed intervention scheme guarantees a full risk-sharing solution.

Further extensions to this research include evaluating the effectiveness of the intervention scheme allowing for transfer costs and physical losses during storage. In addition, other types of contracts could be explored. For instance, instead of transferring the uncertain input, the government could transfer profits. Moreover, to the extent that the risk is not correlated, extending the analysis so that firms engage in contingent transfers would yield a number of important insights.

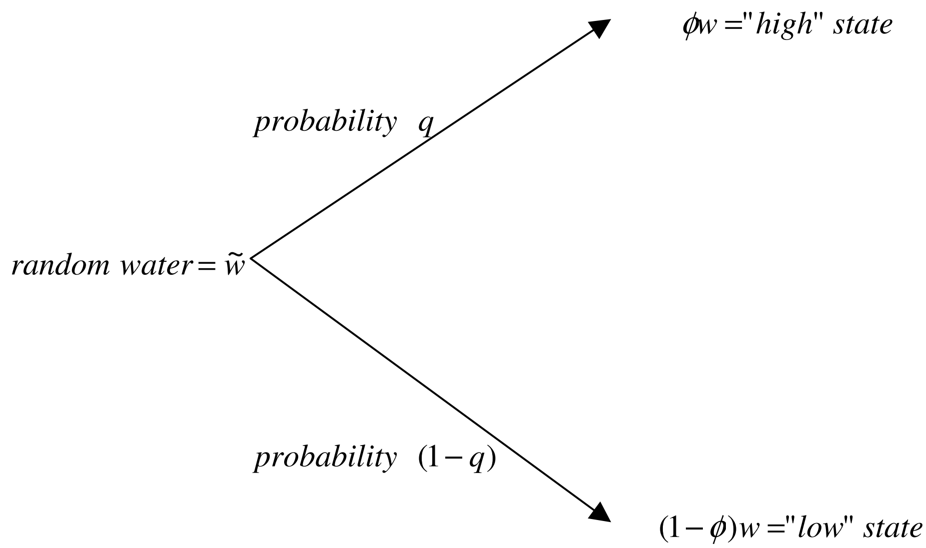

Figure 3.1. Graphical Representation of the Distribution of Water Supply

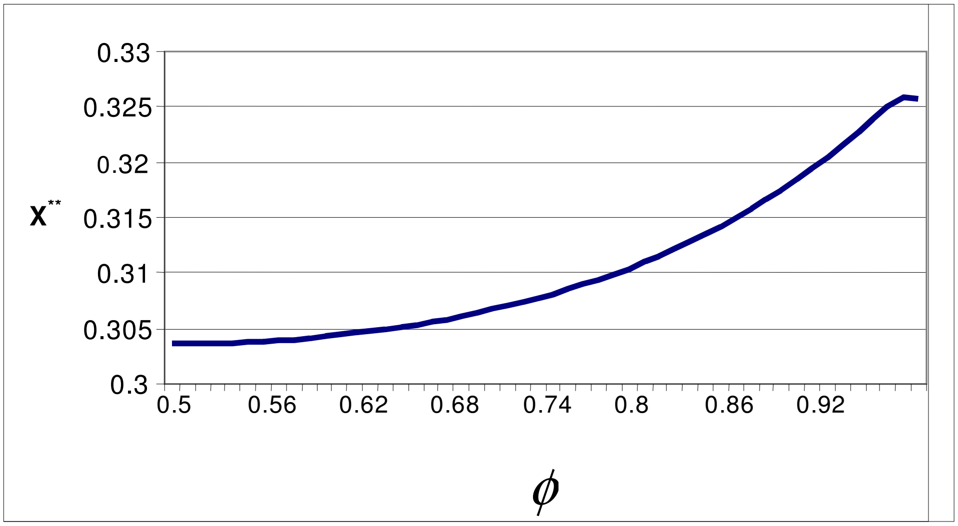

Figure 3.2 Land Allocation to Water-Saving Crop Under Several Degrees of Risk

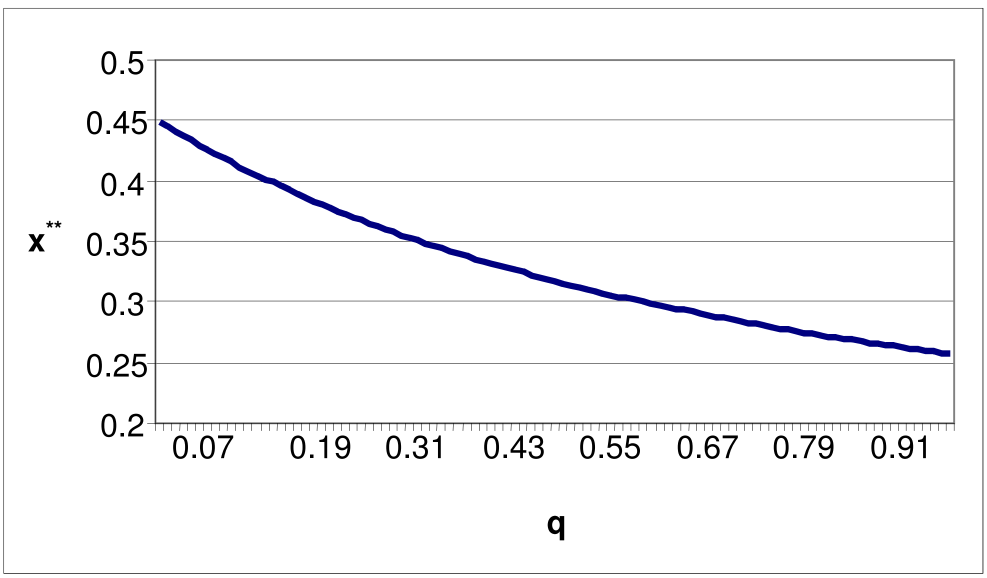

Figure 3.3 Land Allocation to the Water-Saving Crop for Several Probabilities of the "High" State of Nature

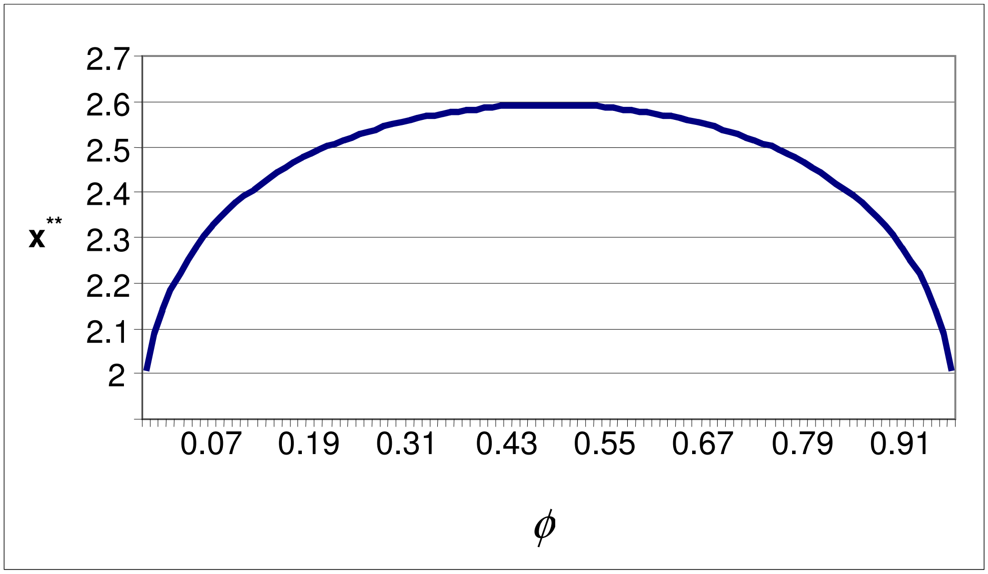

Figure 3.4 Expected Profit Levels Under Several Degrees of Risk

Chapter 4: Economics of Irrigation and Risk Management

4.1 Introduction

In spite of the useful insights gained with the development of reservoir management models and water institutions to deal with the problem of physical uncertainty of the water resource, these strategies only offer partial solutions to the mitigation of the adverse impacts of water scarcity episodes. One the one hand, the reservoir operation rules derived with the optimization techniques reviewed in Chapter 2 provide useful probabilistic information into the different expected states of nature and the consequences of adopting an optimal release strategy. While a reservoir can be operated to accomplish certain risk reducing objectives (for example: stabilization of releases, minimization of floods, and minimization of downside risk), the operation rules do not completely insulate water users from the costs imposed by episodes of inadequate reservoir inflows.

On the other hand, the market-based institutions, described in Chapter 2, are not available to farmers in Mexican irrigation districts. Furthermore, government intervention, through CNA, still plays a major role in irrigation water allocation decisions. In the particular case of Rio Mayo, although some short-term leases of water rights have been reported, these markets are not fully developed and the government policy towards the sub-sector implicitly favors social equity considerations over economic efficiency goals, which further slows down market development. Finally, the common property characteristics of the *ejido* land tenure system, which comprises half of the hectares of the irrigation district, creates a further institutional barrier for trading water.

The objective of this chapter is to present the economic principles that guide the efficient allocation of water resources and the management of irrigation risk. The conceptual model developed in this chapter particularly examines the decision making that takes place at the reservoir management level and at the water user level. The expected utility theory is reviewed and applied to the case of uncertain irrigation supplies

4.2 The Conceptual Model

4.2.1 Water User Allocation Sub-Model

Consider an irrigation district where water is the limiting factor of production. Furthermore, the land and climate conditions are suitable for growing two cropping seasons within a year. Let $y_i(w_i)$ be the number of hectares that can be irrigated in season i with water application w_i. The response functions exhibit diminishing marginal returns to water, therefore I assume that $\frac{\partial y_i}{\partial w_i} > 0$ and $\frac{\partial^2 y_i}{\partial w_i^2} < 0$; where $i = 1$ represents the Fall-Winter season (FW) and $i = 2$ represents the Spring-Summer season (SS). The returns and variable costs associated with each irrigated hectare are denoted by p_i and vc_i. However, irrigation in the SS implies additional costs[14] in the form of storage spills, high evaporation rates, and conveyance losses due to the higher temperatures in the summer. Denote the costs associated with water usage in SS by the loss function e, which increases linearly with the number of hectares irrigated in SS (i.e. $\frac{\partial e}{\partial w_2} > 0$ and $\frac{\partial^2 e}{\partial w_2^2} = 0$). Assuming the irrigator is provided an endowment of water, denoted by W, at the beginning of the year, the problem consists in maximizing the total profits π by the choice of water applications w_i. Formally, the problem is stated as follows:

$$\underset{w_1, w_2}{Max}\ \pi \tag{4.1}$$

subject to

$$\pi = \{p_1 y_1(w_1) - vc_1 y_1(w_1) + p_2 y_2(w_2) - vc_2 y_2(w_2) - e(w_2)\} \tag{4.2}$$

$$W = w_1 + w_2 \tag{4.3}$$

The Lagrange set up for optimization problem is:

$$\underset{w_1, w_2, \lambda}{Max}\ \pi = \{(p_1 - vc_1) y_1(w_1) + (p_2 - vc_2) y_2(w_2) - e(w_2)\} + \lambda [W - w_1 - w_2] \tag{4.4}$$

The FOCs yield

$$\frac{\partial \pi}{\partial w_1} = (p_1 - vc_1)\frac{\partial y_1}{\partial w_1} - \lambda = 0 \tag{4.5}$$

$$\frac{\partial \pi}{\partial w_2} = (p_2 - vc_2)\frac{\partial y_2}{\partial w_2} - \lambda - \frac{\partial e}{\partial w_2} = 0 \tag{4.6}$$

$$\frac{\partial \pi}{\partial \lambda} = W - w_1 - w_2 = 0 \tag{4.7}$$

From the FOCs the following condition for profit maximization emerges:

$$(p_1 - vc_1)\frac{\partial y_1}{\partial w_1} + \frac{\partial e}{\partial w_2} = (p_2 - vc_2)\frac{\partial y_2}{\partial w_2} \tag{4.8}$$

For a given annual endowment of water, the irrigator maximizes profits by choosing applications that equate the MVP of water usage in FW and SS, accounting for the marginal costs associated with water usage with the latter season. Put differently, the marginal cost of using one more unit of water in the SS season is equal to the productivity gains forgone from producing in the FW season plus marginal cost in the form of water losses incurred from storing water to be used in SS.

Please refer to Figure 4.1 for a depiction of the optimal choice of the water application between FW and SS seasons. The horizontal axis measures the proportional application of water to each cropping season. Notice that applications for FW season read from right to left, and that w_1 represents the usage of water in FW. Thus, the choice of water application for the SS season can be inferred from the horizontal axis by deducting w_1 from W. Notice also that both the MVP(FW) and MVP(SS)-e curves exhibit diminishing marginal returns to water applications, and that the latter is adjusted for the e loss function associated with water use in SS. The optimal level of water applications occurs when the MVP(FW) and MVP(SS)-e curves intersect at w_1^*.

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[14] For simplicity, assume evaporation, conveyance, and storage losses do not occur in FW. In reality, they
Footnotes continued on the next page

Given the optimal choices, the indirect profit function can be expressed by

$$\pi^* = \{(p_1 - vc_1)y_1(w_1^*) + (p_2 - vc_2)y_2(w_2^*) - e(w_2^*)\} + \lambda^*[W - w_1^* - w_2^*] \quad (4.9)$$

In addition, from the indirect profit function it is possible to see how the shadow price of water, denoted by λ^*, depends on the size of the water endowment and other factors that influence the net returns to irrigators:

$$\frac{\partial \pi^*}{\partial W} = \lambda^* = f(W, p_1, p_2, vc_1, vc_2, e) \quad (4.10)$$

However, given the diminishing marginal returns to water in both cropping seasons, additional increases in the size of the water endowment will yield less than proportional increases in profits. In other words, the shadow price of water is decreasing in the size of the endowment. This relationship is expressed by:

$$\frac{\partial^2 \pi^*}{\partial W^2} = \frac{\partial \lambda^*}{\partial W} \leq 0 \quad (4.11)$$

This relationship is important because irrigators will not receive the same endowment of water every year. Thus, in years when the annual endowment of water is small, the shadow price of water will be relatively high. In this sense, the shadow price of water serves to measure the relative scarcity of the resource.

Furthermore, the impact of that uncertainty is such that when the size is very small, there is a higher opportunity cost for the water used in the SS season. The higher shadow price of water arises from two sources: first, if the per-hectare return from FW hectares is higher than for the SS, the profit maximizing conditions predict that more water will be applied during FW (price effect). Second, given the smaller size of the endowment, the opportunity costs of water lost due to the evaporation is relatively higher;

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exist, however their magnitude is not as substantial as in the SS season.

thus, the irrigator will have less incentive to store water for the SS season (evaporation effect). The combination of both effects implies that when facing smaller water endowments, the irrigator will find it more costly to grow crops in SS.

Please refer to Figure 4.2 for an illustration of the effect of a smaller water endowment on the irrigator's inter-seasonal allocation of water. Under normal conditions, the marginal productivity of water in SS is higher, thus the $MVP_W(SS)$-e_W curve (dashed line) intersects the MVP(FW) curve at the level of w_{1w}^*. When the endowment of water decreases, the marginal productivity of water declines, as depicted by the $MVP_D(SS)$-e_D curve (solid line). Under water-scarce conditions the optimal allocation of water to the FW season is given by w_{1d}^*, which is higher than w_{1w}^*.

Naturally, the profit function is increasing in the size of the water endowment. Therefore, the random water endowment causes a certain degree of uncertainty over the irrigator's profits. In particular, in the absence of formal risk management arrangements and institutions, the irrigator will decide to forgo some or all of the plantings of the SS season when confronted with water scarcity. For this reason, the opportunity cost of water supply uncertainty can be measured by the reductions in profits derived from plantings in the SS season.

4.2.2 The Reservoir Operation Sub-model

The water endowment underlying the water user allocation model is the outcome of another layer of decision making. Indeed, annual reservoir releases are the most important decision variable in the problem faced by reservoir operators. As presented in Chapter 2, there are two frameworks under which the reservoir management model can be studied. For instance, some authors have combined the problems faced by the water user and the operator, assuming that the reservoir operator also owns the irrigable land holdings. Whereas this approach is useful to simplify the analytics of the optimization problem, it renders results with limited practical implication because rarely the two layers of decision making are vested in one individual. In a different but more realistic approach, the reservoir operator and the water user can be considered separate decision makers. In the latter setting, it is possible to incorporate cases in which the behaviors of the different decision makers are driven by different economic objectives and criteria.

Regardless of the type of decision maker, the problem of operating a reservoir for irrigation purposes consists of determining a release rule that dictates the amount of water made available to irrigators, given a certain storage level and expected future inflows into the reservoir. Fundamentally, the reservoir operation model is an inter-temporal optimization problem under uncertainty. Thus, the release rule must achieve a balance between aggressively maximizing the plantings in the irrigation district and minimizing the probability of incurring shortages or floods.

Consider a reservoir operator that designs the release rule with the objective of maximizing the expected benefits derived from water use in irrigated agriculture. In this case, one important piece of information the operator needs to obtain is the process that drives the irrigator's inter-seasonal allocation of water within the agricultural year. In particular, the shadow value of water provides information about how the firm values the size of the endowment of water implied at any point in time. The operator could maximize the profits of the irrigator, taking into account the profit-maximizing behavior described in the water user sub-model. The problem can be stated as:

$$\underset{w_t}{Max}\left\{\sum_{t=1}^{T}\beta^{t-1}\sum_{t}p_t(k)\pi_t^*(W_t,\varepsilon,k_t)+\beta^T\pi_T^*(W_T,\varepsilon)\right\} \tag{4.12}$$

Subject to

$$W_t = f(S_t,\mathrm{E},k_t) \tag{4.13}$$

$$\sum_k p_t(k_t)=1 \tag{4.14}$$

Where π_t^* represents the maximum annual profit that the irrigator can achieve by following the optimizing behavior derived in the water user sub-model. Notice that ε represents relative prices, costs of production and evaporation rates at the farm level. In addition, W_t represents the annual releases from the reservoir for irrigation purposes. In turn, the annual releases are a function of the following factors: initial storage of the reservoir, denoted by S_t; the annual stochastic inflow realized during the state of nature k; and the structural characteristics of the reservoir, which are represented in the variable E. In principle, the resulting release rule will attempt to equalize the expected inter-annual shadow price of water, recognizing that the marginal benefits accrued from water use in the present should be equal to the expected marginal benefit of saving water for future use.

Despite the existence of optimization techniques that solve the operator's problem, the optimized rule still carries some level of risk. One the one hand, the operator might choose a release rule that guarantees a small, but constant endowment of water that would irrigate a smaller fraction of the irrigable area. This situation would eliminate the risk of facing water shortages at the irrigator's level, but at the expense of reducing the expected profits over time. Under the conservative release rule high levels of storage would be maintained in order to avoid the adverse effects of inflow-deficient states of nature.

The risk-return tradeoff of reservoir operations is depicted in Figure 4.3. The horizontal axis measures the volume of water released from the reservoir. The PVI curve represents the expected value of profits; and the VAR curve the variance, generated from different reservoir release rules. The PVI is expected to be concave because of the diminishing marginal returns associated with water applications to parcels located farther from the reservoir. First, the operator might adopt an aggressive policy, denoted by W_a and point B in Figure 4.3, which increases the average endowment of water made available to the irrigator. Although such policy would irrigate a higher proportion of the irrigable area, it would commit to operating on a generally emptier reservoir. Consequently, the more aggressive policy will increase the risk that inflow-deficient states of nature will lead to reductions in the irrigated area. Alternatively, the operator might adopt a more conservative policy, represented by W_c and point A in Figure 4.3, which results in a lower proportion of the irrigable land supplied with water, but with less probabilities of incurring losses due water scarcity.

4.3 Managing Water Supply Risk Using a Derivate Instrument

The stochastic nature of the reservoir inflows implies that a constant release of water (i.e. no risk) from the reservoir can be guaranteed only by sacrificing substantial levels of expected profits. The operator faces a tradeoff between increasing the expected profits to the irrigator and increasing the probability of incurring water shortages. Therefore, this type of risk calls for a more complete set of decision variables, outside water use and release decisions, for the operator and the irrigator. As Dudley (1988) puts it "why use stored water as an insurance medium when it evaporates and its presence in the reservoir increases reservoir spills." Dudley suggests that storing output and revenues, rather than the uncertain input, should also be included in the set of decision variables of reservoir operation models.

In this chapter, I propose a derivative instrument with inflows as the underlying variable to transfer the water availability risk outside the irrigation district. In essence, an inflow-based derivative is a contingent contract that promises payment to the buyer based on the difference between the realized value of an underlying index - in this case reservoir inflows - and a previously agreed strike value. Since reservoir inflows is a volume variable, which directly impacts agricultural output, and derivative contracts are particularly suited for protecting against volumetric risks, the instrument is particularly suited for protecting against the variations in reservoir inflows.

If the strike level corresponds to a volume of inflows that the irrigator associates with maximum tolerable losses, then a put contract that pays when inflows fall below the strike would offset the potential reduction in profits experienced due to water shortages. From a risk management perspective, the derivative instrument would stabilize the annual income of the irrigator. This would be accomplished by giving up a relatively small stream of income (i.e. the premium) to prevent the larger losses that occur when the irrigation supplies are below the norm. In this case, the indemnity payment would replace part of the income that is not generated due to the reduced plantings. Moreover, the operator of the reservoir does not have to use water as an "insurance medium" and can adopt a more aggressive release rule, which without the derivative would impose an intolerable degree of risk for the irrigator. In other words, insurance would serve to engage in a riskier operation policy that yields higher returns for the irrigation district.

In principle, the use of the derivative would reduce the variance of income. Depending of how the additional returns generated with a more aggressive release rule compare to the stream of income given up to engage in the contract, the expected present value of profits would be the same or lower than without the derivative. For instance, Figure 4.4 depicts the scenario when the release rule remains the same after using the instrument. In this case, the present value of the expected profit will be relative lower, as depicted by the curve PVI_H (dashed line). However, the reduced income will be accompanied with a lower variance, as depicted by the VAR_H curve (dashed line). The value of the derivative is higher when an aggressive policy is into effect. In contrast, the value of the derivative is highly reduced when a conservative operation policy is implemented. However, the magnitudes of the variance reduction and the levels of expected profits will depend on the type of contract and operational rule in place. In the next chapter, the empirical application for the case of Rio Mayo will provide insights into the use and value of the inflow-based derivative.

4.4 The Economics of Using an Inflow-based Derivative

While the benefits of an inflow-based derivative are very clear, the elicitation of the willingness to pay for such an instrument is not straightforward. There are two factors that influence the willingness to pay for a particular design of the instrument: the premium of the contract and the risk aversion of the decision maker.

4.4.1 The Premium (Actuarial Price)

From the decision maker's perspective the premium is the price to be paid to reduce the risk imposing by a random event. Paying the premium at all time periods is tantamount to exchanging relatively infrequent large losses for constant small losses. Therefore, the premium can be also viewed as a fixed cost for the decision maker. However, the payoff in the form of an indemnity from the contract will be a random variable and will vary according to the triggers and other parameters included in the design. In principle, the design should aim at supplementing income when the irrigation district faces a high shadow price of water due to scarcity.

Thus, while the cost of the instrument is fixed at all times, the benefits will vary depending on the random realization of reservoir inflows. In other words, for different levels of inflows, the contract may have net positive or negative income contributions to production income. This argument is further illustrated with the help of graphical analysis in Figure 4.5. In the graph the two sources of income are represented. The right side graph represents the payoff structure from a put contract that pays when the random realization of inflows falls short of the W_l level (strike). The line that runs through the point G, N and L represent the linear payoff structure of the derivative. The maximum indemnity, denoted by point G, is paid when the level of water is zero. The payoff decreases until it reaches zero in point L. In addition, the graph describes the premium level, measured by the distance between 0 and P, which has to be paid independently of the realization of water. Under this hypothetical contract, the net income from hedging with the contract will be positive for points to the left of N, or for realizations of water below the W_n threshold.

In the left side of the Figure 4.5 the total income of the irrigation district is represented. The INC_P curve (solid line) represents the income from production in the absence of the hedging mechanism (i.e. without derivative). In contrast, the INC_{PH} curve (dashed line) represents the combination of production and hedging income. The points G, N, and L from the hedging income graph are also represented in the production income graph. The three points convey important information about the impact of the hedging mechanism on the profit function. In particular, when the level inflows is W_n, the hedging contract breaks even, and consequently, it is only at this level of inflows that income with and without hedging mechanisms are the same (i.e. INC_P and INC_{PH} intersect). For levels of inflow below W_n income will be higher when production income is supplemented with hedging income (i.e. INC_{PH} lies above INC_P). For points to the left of W_n, the difference between the solid and dashed lined in the right graph reflects the net positive indemnities from the hedging mechanism in the right-hand side graph. For instance, point G in both graphs indicates that when the inflow level reaches its minimum, the contact pays its maximum indemnity, which makes total income higher than production income. However, for levels of inflow beyond W_n, the INC_P curve lies above the INC_{PH} because the derivative contract payoffs are negative, so the production income is reduced. Notice that for levels of water to the right of W_l, the INC_P and INC_{PH} curves will be parallel to each other, the distance between them will be proportional to the premium paid for the derivative contract.

Following the basic principle of insurance, the irrigation district gives up revenue when the marginal value of water is low (i.e. points to the right of N in the left graph). In turn, supplementary income from contract is received when the marginal value of water is high (i.e. points to the left of N in the left graph). As showed in mathematical analysis of

Chapter 3, even in the absence of risk aversion, hedging mechanisms have the potential to reduce the risk profile faced by irrigators.

4.4.2 Risk Aversion

The other economic aspect of using an inflow-based derivative to hedge against water supply risk is the degree of risk aversion required by the decision maker to buy the contract. Economists have exploited the expected utility framework to analyze problems involving decision making under uncertainty. According to expected utility theory, decision makers are able to rank gambles or lotteries based on measures of their expected utility.[15] Depending on how the decision maker ranks his/her gambles, he/she might be risk averse, risk lover, or risk neutral. In particular, risk aversion is a behavior in which decision makers always refuse fair bets (Nicholson, 1998). Moreover, risk aversion is also implied when the utility function exhibits diminishing marginal utility of wealth. Furthermore, decision makers exhibiting risk aversion will be willing to pay something to avoid gambles. Arrow (1965) developed this analytics and actually refers to what decision makers are willing to pay to avoid gambles as a risk premium. The Arrow graphic is reproduced in Figure 4.6.

Suppose an irrigator with a concave utility function for the level of water delivered to his land parcel. Suppose also for simplicity that there are two states of nature that can occur with equal likelihood. The high state of nature, denoted by W_h is characterized by a large endowment of water that allows him to fully till his land parcel. In contrast, the low state of nature, denoted by W_l, only brings a very limited amount of water that does not allow the irrigator to employ all his land. Under this environment, the mean endowment of inflow expected by the irrigator is $\overline{W}$. In the figure it is possible to see that if this irrigator were offered with an endowment of water equal to the expected value of the water endowment, he would prefer the hypothetical amount to the gamble. In the figure, this is illustrated by the fact that $U[E(W)] > E[U(W)]$. Notice also that there is a level of water, denoted by W_c, which provides the irrigator with the same level of

[15] Expected utility relies on the property that the utility function expresses the utility of a gamble as the expectation of the utility from its outcomes. Varian (1992) describes certain axioms that such utility function must conform in order to exhibit such an important property of the expected utility theorem.

utility yielded by the gamble. Put differently, when the utility function is evaluated at W_c, the following is true $U[W_c] = E[U(W)]$. This implies that the irrigator is willing to pay a maximum amount, namely $\overline{W} - W_c$, to avoid the gamble in the endowments of water. In the literature, W_c is referred as the certainty equivalent, and the amount $\overline{W} - W_c$ is known as the risk premium.

Once the risk aversion concept is being established, the next issue is to measure the degree of risk aversion that drives individuals to accept certain gambles reject others. In other words, a measure of risk aversion is convenient to understand how the decision makers rank scenarios with different degrees of risk. The literature proposes several measures of risk aversion. Two of the most commonly used measures are the absolute and relative measures of risk aversion, proposed by Pratt (1964) and Arrow (1965, 1970). Both measures are based on the shape of the utility function of decision makers. In other words, they are based on the fact that risk-averse individuals exhibit diminishing marginal utility of wealth. Thus, the concavity of the utility function (i.e. $U''(W) < 0$) is exploited to infer conclusions about attitudes towards risk.

The absolute measure of risk aversion, denoted by $r(W)$, is given by $-\frac{U''(W)}{U'(W)}$. The absolute measure refers to risk aversion with respect to changes in the overall level of wealth. In other words, it conveys information about the amount of wealth an individual is willing to expose in a gamble, given a certain level of wealth. In contrast, the relative risk aversion measure, denoted by $rr(W)$, is computed by $-W\frac{U''(W)}{U'(W)}$. The relative measure refers to risk aversion with respect to proportional changes in the level of wealth. More specifically, it yields the percentage of wealth an individual is willing to expose in a gamble. Depending on the shape of the utility function, the measures of risk aversion can increase, decrease or remain constant when the individual's wealth increases. In other words, as the decision maker becomes wealthier, he might be willing to increase, decrease or keep a constant level of gambles.

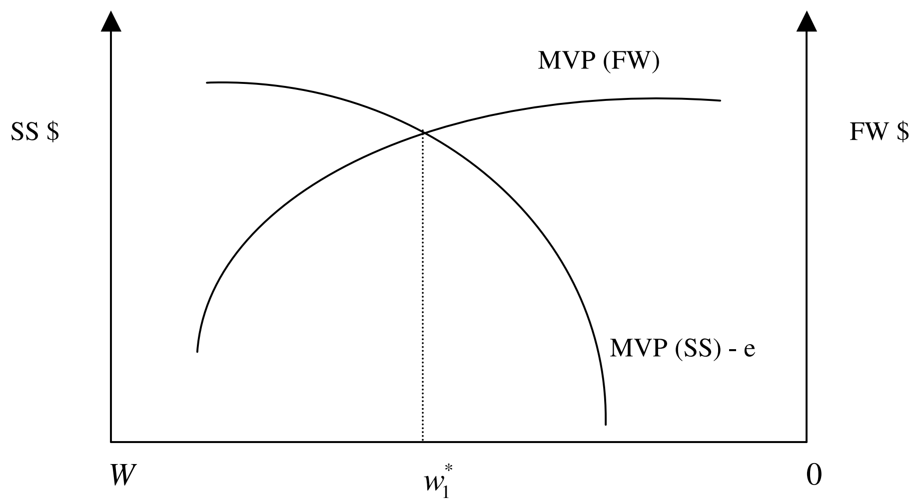

Figure 4.1 Inter-seasonal Water Applications in "Wet" Year

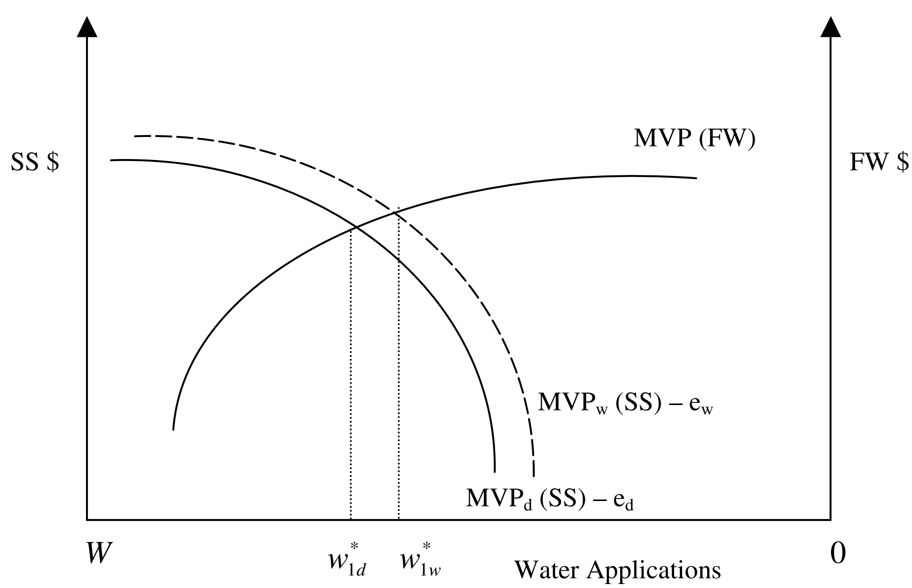

Figure 4.2 Inter-seasonal Water Applications in "Dry" Year

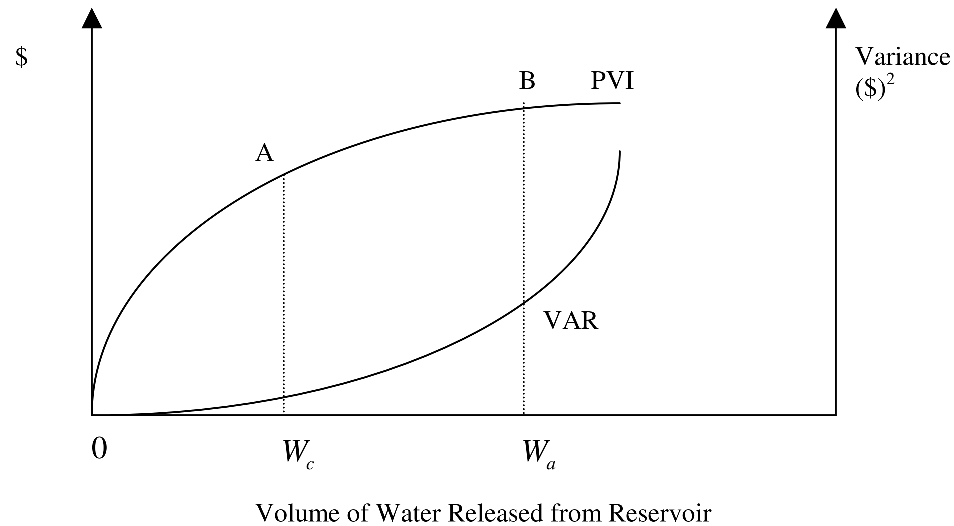

Figure 4.3. Risk Profile of Alternative Operation Policies

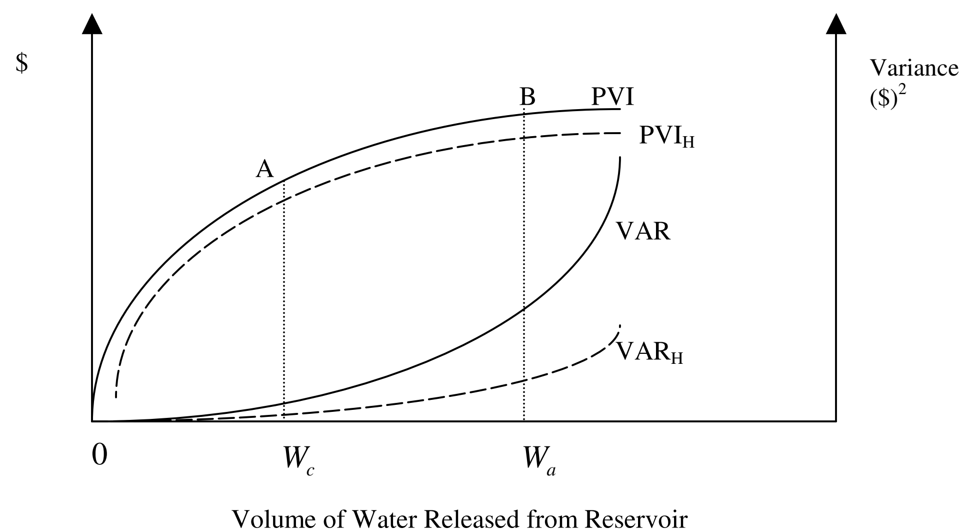

Figure 4.4 Risk Profiles of Alternative Operation Policies with an Inflow Hedge

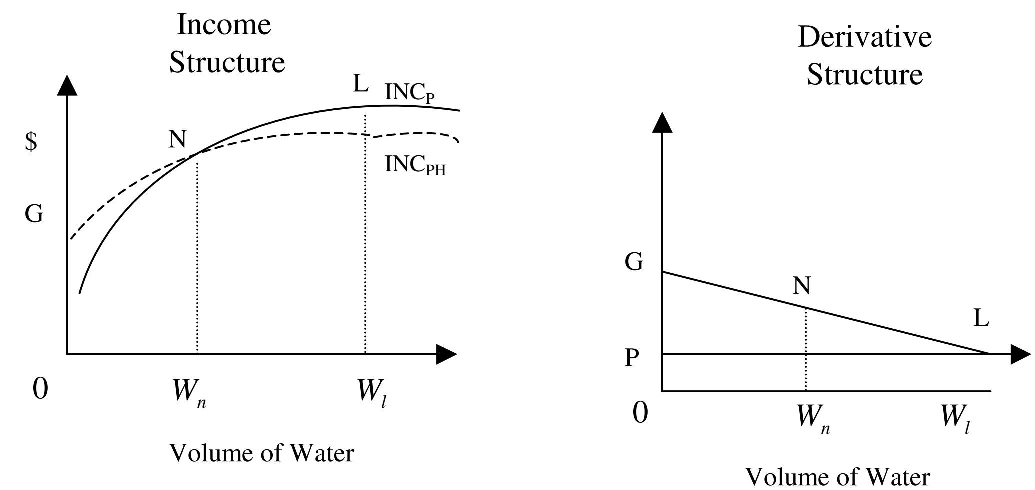

Figure 4.5 Impact of a Derivative in Production Possibilities

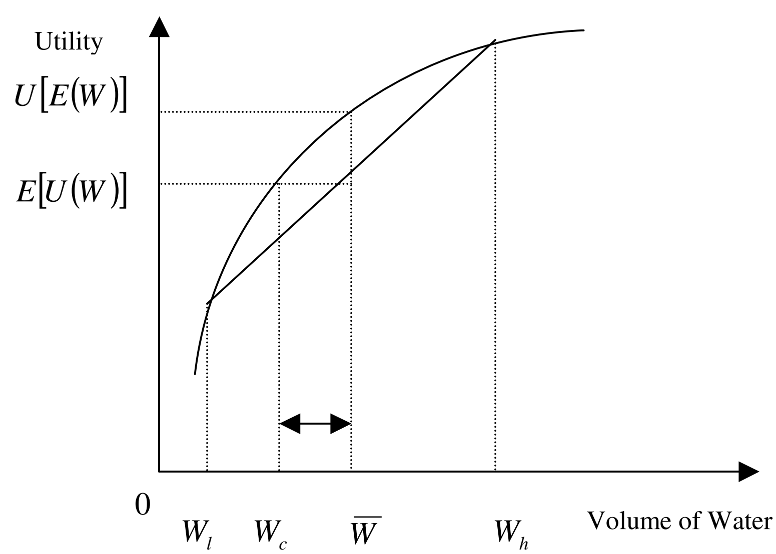

Figure 4.6. Expected Utility and Risk Aversion

Chapter 5: Empirical Simulation of Irrigation Risk Management

In this chapter I develop a dynamic stochastic simulation model of Rio Mayo Irrigation District. The simulation is based on the economic principles established in Chapter 4 in conjunction with some ideas reviewed in Chapter 2 regarding the technical view on water resources management. The model aims to show the feasibility and effectiveness of an inflow-based derivative to manage irrigation risk in a district that depends mainly on surface water to satisfy irrigation needs. Empirical distributions of an income measure for Rio Mayo irrigators are developed using a simulation procedure. In addition, relative and downside measures of risk are used in conjunction with certainty equivalents for ranking risky scenarios and developing efficient sets of hedging strategies.

5.1 Background Information on the Rio Mayo Area[16]

5.1.1. Location

The Rio Mayo irrigation district, also known as No. 038 in CNA's inventory of irrigation districts, is located in the southern part of the state of Sonora, 27°54'' north and 109°36'' west of the Greenwich meridian. The district includes an area of 98,598 ha suitable for irrigated agriculture. In the north, the area is bounded by mountains that comprise part of the Sierra Madre mountain range, and on the southwest by the Gulf of California. The closest cities to the district are Navojoa, Huatabampo and Etchojoa. Please refer to Figure 5.1 for a map of the location. The regional climate is desert-like, characterized by deficient humidity during all the seasons. Moreover, mean temperature and rainfall in the area are 23°C (23°F) and 260 mm (10.24 in), respectively. More importantly, most of the precipitation occurs between July and October, although occasionally cold fronts bring some rain between December and January. For water users in the Mayo Valley the July-October period is critical because it determines the level of replenishment of the reservoir. High levels of solar evaporation and extreme hot weather also preclude significant plantings during the rainy season of July-October; increasing the need for a reservoir to hold water until temperatures are more suitable to crop production.

[16] This information was obtained from the CNA report (2004) and personal communications with SRL management on a field trip to the area in April, 2005.

5.1.2 Source of Water Supply

The main source of water supply for the irrigation district is the watershed of the Mayo River (hence the name Rio Mayo), which covers an approximate area of 11,000 km^2. The river extends for approximately 350 km and averages 1000 million m^3 of stream inflows into the reservoir. The hydraulic work used to secure the flows from the river is the ARC reservoir, also known as Mocuzari. The ARC reservoir was built in 1955 and its infrastructure consists of an earth-filled structure 81 m high above the river bed, 775 m long, and 10 m wide at the crest, and 440 m wide at the base. After an expansion project in 1968, the storage capacity increased from 1,100 million m^3 to 1,300 million m^3. However, the silting that occurs through the years has reduced the capacity. According to the inventory of reservoirs of CNA, the ARC reservoir is classified as a mid-size reservoir in Mexico[17].

5.1.3 Cropping Patterns

The production activities during the agricultural year are divided into two cropping seasons. During the first season, which runs from October to April also referred as Fall-Winter (FW), farmers grow wheat (the main crop of the region), maize, safflower, potato, and other minor crops. In the second season, which runs from February to October or Spring-Summer (SS), the main crops are cotton, sorghum, and maize. In addition, a small fraction of the irrigated land is dedicated to perennial crops. Figures 5.2 and 5.3 present the hectares cultivated in both seasons in the last decade.

The data illustrates two important characteristics about Rio Mayo. First, since water is the limiting factor of production, only when the reservoir has adequate levels of storage is that cropping occurs in both seasons[18]. For instance, Figure 5.3 illustrates how the hydrological drought of the recent years has forced farmers to give up entirely the SS season. In fact, the last time the SS carried a considerable number of hectares was in the year 1995 when the reservoir had storage levels that surpassed the historical mean for April (See Figure 5.7). Second, the data shows how the striking difference between the crop portfolios corresponding to each season. While the FW portfolio is well defined and

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[17] Of the 51 reservoirs in Mexico, the Adolfo Ruiz Cortines ranks 25th in size. The largest system is over 10,000 million m^3 and the smallest is around 270 million m^3 (CNA, 2004).

carries wheat as the dominant crop, the SS portfolio does not have a definite pattern and seems to play a secondary role in the cropping activities.

5.1.4 Farmer Organization

There are approximately 11,717 irrigators in the Mayo Valley, which in turn fall into two categories: private property owners and *ejidatarios*. Private property owners typically operate larger farms and grow cash crops. There are 3,867 irrigators falling into the private property category and owning nearly 47% of the land. In contrast, *ejidatarios* own smaller parcels of land and produce mainly for household consumption. *Ejidatarios* make up 7,850 irrigators and own 53% of the land. The mean landholding in the former group is 11.9 ha and 6.5 for the latter. Please refer to Table 5.1 for more details.

All farmers are organized in a water user association known as SRL, which stands for *Sociedad de Responsabilidad Limitada*, or limited responsibility association in Spanish. In turn the SRL represents 16 irrigation modules of different sizes and water conveyance efficiencies (Table 5.1). The roles of the SRL include: procuring water released by CNA from the ARC reservoir; planning the delivery of water to each irrigation module; collecting fees from modules to cover the operation and maintenance of the irrigation district; and represent farmers in all activities related to projects and investments. At a lower organization level, the irrigation module is responsible for the following activities: collecting irrigation plans of each individual farmer; collecting fees on behalf of the SRL from individual farmers; and distributing water to each parcel.

5.1.5 Water Allocation Decisions

Although the operation of irrigation districts as been decentralized in Mexico, CNA still play a dominant role in the allocation of irrigation water supplies. The decision-making progress is highly complex and many parties participate including: the SRL, CNA and the Watershed Council, which includes civil authorities such local governments. However, the ultimate decision maker in this negotiation process is CNA in Mexico City. The most important steps of the process are outlined below.

At the beginning of the agricultural year (October), the SRL as a collective group develops an annual irrigation plan. This plan is the product of the decision-making

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[18] In Rio Mayo irrigators depend 100% on irrigation for their cropping activities; rain-fed irrigation is virtually non-existent.

process among the representatives of the 16 irrigation modules in a general assembly. The SRL closely monitors the main operation variables of the reservoir on a daily basis, thus they have complete information about water availability for the current year. This information allows the SRL to have a clear idea of how to develop a cropping plan that has high chance of being approved by CNA.

In the next round of decision making, CNA studies the irrigation plans and decides if the plan is feasible or not. In particular, CNA employs a simulation-optimization program called OPTIMA, which produces a release rule for a given volume in reservoir storage as of October 1. The release prescribed by OPTIMA is a minimum level of release because they release additional supplies throughout the year taking into account the probabilities of the worst year in records and the inflows that accumulate within the FW season.

After the plan is approved, CNA releases water from the reservoir according to a schedule that is jointly determined by CNA, the SRL, and the irrigation modules. First, CNA delivers water to SRL in the two main canals of the irrigation district. In turn, the SRL delivers water to each irrigation module. Property rights are generally tied to the parcel of land and water is delivered on a pro-rata basis given detailed farm plans that are adjusted based on available water. Finally, from the irrigation module delivery point, water is supplied to the individual irrigator.

Throughout the delivery system water losses occur due to evaporation and seepage. The greatest losses occur at the module level and SRL level due to the unlined canals used to convey water. Please refer to Table 5.2 for the water conveyance efficiency ratios at different delivery points in the system. These ratios indicate the proportion of the water that actually reaches the next delivery point. For instance, in FW the ratio is 90%, meaning that for each m^3 of water that is delivered from the reservoir, only 90 cm^3 are actually captured in point at which the SRL takes control over water. In the last row of Table 5.2 the overall conveyance efficiency ratios show that in FW only delivered from the reservoir actually reaches the land parcels, while in SS almost 60% is lost in the conveyance process. Although no data is available to compare with other irrigation districts, such low ratios indicate that water losses in the district are quite substantial.

5.2 Data Description and Analysis

The first step to understand the decision-making process in Rio Mayo is outlining how the cropping activities are developed within an agricultural cycle. The cycle is divided into two seasons: Fall Winter (FW) and Spring Summer (SS). The FW season starts in October 1 and ends in March 30. The SS season corresponds to the period between April 1 and September 30. In their planning process during the FW season, the SRL and CNA observe the storage in the ARC reservoir as of October 1 to determine the agricultural releases for the current season. After the harvest from the FW activities have concluded and additional inflows have been accrued in the reservoir, the same process occurs in April 1 to determine releases for the SS season.

CNA and the SRL provided the hydrologic and economic data relevant to this study. The hydrological data includes monthly records of reservoir inflows, storage, and releases. The quality of the hydrological data is highly acceptable since the authorities have recorded daily observations for more than 50 years. The officers of the SRL post those daily observations in a bulletin board for irrigators' planning purposes. The descriptive statistics of these series are given in Table 5.3. The mean inflows accumulation to the reservoir over one agricultural year is approximately 1000 million m^3. The mean annual agricultural releases are 825 million m^3. This release number does not account for municipal water use (around 20 million m^3), reservoir spills, and evaporation losses in the reservoir. In addition, the history of seasonal plantings indicates that mean annual plantings (i.e. including FW and SS) have been around 101,000 hectares per year. However, as depicted in Figure 5.5, in the last 5 years total annual plantings have experienced a downward trend due to the low accumulation of water in the reservoir. The decline has been so severe that in the year 2003-2004 annual plantings were only 50% of the historical mean.

Although no particular test is performed at this point, the large decline in plantings is intuitively explained in part by the low accumulation of inflows within the same five-year period. Figure 5.4 illustrates how the total annual inflows registered in the past three years are among the lowest levels experienced since the reservoir started operations. In fact, the period 2001-2004 is the only time, since inflows have been recorded, in which "back-to-back-to-back" worst-case scenarios materialized for reservoir operators in the Mayo Valley. In addition, Figures 5.6, 5.7, and 5.8 provide information on the operation of the ARC reservoir. In particular, notice in the recent years the striking declines in the reservoir contents in both October and April, along with the dramatic declines in releases. To provide irrigation water during the last 5 years, CNA and the SRL have been quite aggressive in releasing volumes of water that almost equal the inflows accumulated within the same period. Such behavior naturally led to the depletion of the stock of water in the reservoir. These observations corroborate the conclusions encountered in the literature of water resources (Chapter 2) that optimal release rules do not insulate irrigators from the random nature of inflows. However, in the case of Rio Mayo the problem is exacerbated by the political economy of irrigation, which results more aggressive operation policies in the reservoir.

The next step is to test some relationships that help one understand the decision making in the Mayo Valley. Thus, some questions are addressed. First, what is the relationship between reservoir inflows and releases to the agricultural sector? This question is very important because the released volume is the most important resource endowment for irrigators.[19] To address this question, a simple OLS regression was used to explain releases. The first explanatory variable is the level of reservoir storage as of October 1 of each year. This stock variable provides information about the certain availability of water at the beginning of the agricultural year; therefore I expect a positive sign in its coefficient. The second variable is the inflows into the reservoir in the period that falls between October and April. This variable has a twofold importance: one the one hand, it allows irrigators to carry out supplementary irrigation for the crops already established in the FW season; on the other hand, it replenishes the reservoir for future irrigation (and therefore releases) in the SS season. Please refer to Table 5.4 to see the results of the regression.

The results suggest that October storage and FW inflows explain about 85% of the variation in releases. More importantly, it confirms that the storage level of the

[19] Reservoir releases account for 80% of the total water supply for the irrigation district. The rest is extracted from aquifers. Although aquifers could be used as buffer stock to mitigate the water shortages, we have left aside this issue for three reasons. First, the data on underground pumping is not readily available. Second, according to water users, farmers prefer to use the surface supplies because of its lower cost (i.e. no pumping required). Third, data on the cost of pumping is not readily available.

reservoir as of October 1 of each year is not only statistically significant, but also significant in magnitude. On average, holding all other things constant, for an additional million m^3 held in storage, more than half (55%) of that volume will be released for irrigation purpose throughout the agricultural year. Second, although the inflows that occur in the October-March period lead to higher levels of releases, the relationship between these variables is not linear. The OLS regression indicates that a quadratic form seems to provide a good approximation of the effect of FW inflows on annual releases. More importantly, it shows that additional FW inflows result in more releases in the current year, but the positive impact decreases as the level of FW inflows increases. From the OLS regression I conclude that any release rule in place at the ARC reservoir must be based on two variables: beginning-of-year storage and October-April inflows.

Finally, time series of inflows data needs to be tested for autocorrelation. From the perspective of an insurance provider it is desirable that the time series does not show signs of strong autocorrelation because that makes the pricing of the instrument more challenging. The purpose of this exercise is to determine if the insurance provider needs to adjust the premium for the possibility of “back-to-back” drought periods. The presence of autocorrelation might prove difficult for the insurer because of indemnities would have to be paid in consecutive years. In order to visualize and test for autocorrelation I use the correlogram and the BJ statistic. Please see Figure 5.9 to see the correlogram. The resulting autocorrelation coefficient (first lag) indicates the presence of first lag autocorrelation. Although the tests indicate statistical significance, the magnitude of the coefficient is small. The remaining coefficients are not significant at any level, indicating that the autocorrelation does not prevail after one lagged period. Therefore, it is possible to conclude that insurers would not have any considerable problems with a derivative contract based on the inflows to the reservoir.

However, one must be cautious about the autocorrelation test for inflows because during the last years the “back-to-back” scenario actually occurred. To deal with these scenarios, insurers will typically load the price of the contract. The loading factor can reach up to 50% of the actuarially fair price. In this chapter this issue is addressed by a sensibility analysis on the possible loads that insurers may charge for engaging in a potential contract.

5.3 Institutional Framework for Developing an Inflow-based Derivative

5.3.1 The Reform of 1992

The Mexican agricultural sector witnessed profound institutional reforms in the early 1990s, which placed the country among the most cited examples regarding the decentralization of the water sector. Among the changes that affected the irrigation districts, including Rio Mayo, was the new Water Law of 1992. Under the reformed framework, the federal government handed the operation and management (O&M) of irrigation districts to farmer organizations, including the responsibilities of financing the operation and management costs and allocating water among farmers. In addition, CNA was created to function as an autonomous government agency responsible for planning the use of water resources, including the operation of all the reservoirs of Mexico. Besides changing the administration of the water resources, the reforms changed the property right structure in the water sector. In particular, water concessions (i.e. water rights) were issued to farmers for periods ranging between 20-50 years. This profound decentralization was aimed at paving the road for the establishment of water markets and reducing the government expenses in operating the irrigation districts.

In summary, the reforms have established three stakeholders in the Rio Mayo irrigation district. At the top level, CNA is responsible for operating the ARC reservoir and approving the irrigation plans formulated by farmers. In the middle level, the SRL is responsible for procuring the water released from CNA, collecting fees from water users, and allocating water to irrigation modules. At the next level is the irrigation module that collects O&M fees from individual users and distributes water to the end user.

5.3.2 A Hypothetical Structure for Rio Mayo

The implementation of a risk-sharing market in Rio Mayo poses one important question: how could an insurance provider sell an inflow-based derivative contract in a large irrigation district? Unfortunately, selling contracts on an individual basis implies incurring high transaction costs due to the large number of farmers that would need to be served. However, using a collective group as an intermediary may decrease the number of parties and therefore make the contract more feasible for both the insurer and the farmers.

Fortunately, the reforms introduced in 1992 laid down the foundations for two very important organizations that may act collectively to engage in contractual arrangements on behalf of farmers: the SRL and the irrigation module. Given that the SRL has established a good record for handling the responsibilities of distributing water among water modules and collecting the O&M fees (Naylor, Falcon, and Puente-Gonzalez, 2001), this organization could become the vehicle that represents the demand side for a contract. Rather than establishing contracts on an individual basis, the insurer could write a contract directly with the SRL. In turn, the SRL could collect premiums and distribute indemnities among participating farmers or modules in the same way it collects the O&M fees and distributes water. Alternatively, the irrigation modules could be used as the vehicle that represents individual farmers willing to engage in contractual arrangements with an insurance company. However, the irrigation module may not have the administrative competence that the SRL has acquired during the last 10 years, nor it may have some information advantages that the SRL already possesses. For example, the SRL has elaborated maps of conveyance efficiencies ratios, default risk, and crop portfolios for each one of the 16 irrigation modules.

From a policy perspective, the SRL is a better vehicle due to its already established faculties to administer the allocation of water and operate the irrigation district. Exploiting those faculties under a proper incentive structure, the SRL might make decisions that ensure that water is used more efficiently in the irrigation district, particularly in times of hydrological drought. Under the current system, the SRL shares the quota of water on a pro-rata basis among the irrigation modules. Since irrigation modules have different conveyance efficiency ratios, the pro-rata allocation will yield an efficient solution only when the quota of water is sufficiently large to provide each farmer with his/her full allotment. However, when the water quota is not adequate, the pro-rata system does not guarantee an economically efficient distribution. In times of water scarcity, when the shadow price of water increases, the additional losses incurred by delivering water to the modules with the lowest conveyance efficiency ratios may come at very high opportunity costs.

However, if the SRL would engage in a risk-sharing contract, it would have more flexibility to manage the problems created by water scarcity. More importantly, the SRL actions may lead to more efficient outcomes. For instance, if the SRL were to receive a payment when the inflows to the ARC fall below a pre-determined threshold, it could make some choices that could bring efficiency gains to the district. Specifically, the SRL could offer a monetary compensation, rather than water, to irrigation modules whose conveyance efficiency ratios are very low. That same volume of water could be diverted to irrigation modules that enjoy higher conveyance efficiency ratios. By re-allocating a scarce volume of water to irrigation modules that enjoy higher conveyance efficiency ratios, the SRL virtually augments the supply of irrigation. Thus, for a given volume of water, more output could be produced if water is allocated to modules that waste little water, while indemnity payments are distributed to modules that are less efficient in utilizing water.

In the analysis that follows I consider a framework in which farmers delegate all their decision-making power to the SRL. In this sense, the existence of the SRL may be viewed as an effort from farmers to overcome problems of economies of scale and market failure in the provision of irrigation services. Moreover, the SRL could be considered the outcome of a collective choice from farmers to overcome the factors that cause risk-sharing markets to fail in the agricultural sector of developing countries.

For the reasons stated above, in the analysis that follows the demand for an inflow-based derivative will be analyzed from the perspective of the SRL. Furthermore, the SRL is considered as a risk-averse group that prefers to avoid problems of water crisis that might bring discontent among its members. Although the SRL has proved capable of designing a system that brings benefits to the whole irrigation district, conflicts among irrigators and irrigation modules still persist. Such problems are exacerbated by the water scarcity and the tough decisions the SRL has to make to distribute the limited supplies of water.

5.4 Simulation Methodology

5.4.1 Purpose and Components of Simulation for Risk Analysis

A dynamic stochastic simulation model was developed to empirically estimate the income distributions from alternative risk management strategies in the Mayo Valley. The simulation methodology involves solving the mathematical representation of an economic system for a set of exogenous variables (e.g. alternative policy and risk management strategies). The model is solved a large number of times in order to statistically represent all the possible combination of random variables that have an impact in the system.

Richardson (2003) describes the 5 components that constitute a simulation model: key output variables; intermediate results, tables and reports; equations and computations to obtain the reports; exogenous and control variables; and stochastic variables. The five components are described in Figure 5.10. The key output variables are those that the decision maker considers the most important. For instance, irrigators are interested in annual income; investors might consider the net present value of a project. In addition, there are exogenous variables that decision maker cannot control or change. The exogenous variables may include constraints, response functions, initial endowment of wealth, etc. Control variables are those variables that the decision maker can select to achieve his or her behavioral objective. For instance, irrigators may adopt different release rules to operate the reservoir or investors might choose the composition of a portfolio of projects. However, at the end of the spectrum, there are stochastic variables that are hard to predict and the decision maker is uncertain about their realizations. These stochastic variables are represented by a distribution function and drive the risk component of the simulation model. The ultimate goal of simulation in risk analysis is to estimate distributions of key output variables that result from the combination of stochastic, control and exogenous variables.

Based on the framework described above, I simulate the operation of the ARC reservoir for irrigating the Rio Mayo irrigation district. In addition to the stochastic component of reservoir inflows, the model features recursive and dynamic components that were reviewed in Chapter 2. The model pyramid for the Rio Mayo model is presented in Figure 5.11.

5.4.2 Simulation Analysis for Inflows

5.4.2.1 Estimation of MVE Distribution

The estimation of the stochastic variable that drives the risk in the simulation model is accomplished through the estimation of a distribution function. In this sense, there are two types of distributions that can be estimated: parametric and non-parametric distributions. If there are good reasons to believe that the true distribution is parametric (i.e. normal, gamma, etc.) the corresponding parameters can be estimated. However, if there is no a priori information about the true distribution, then a non-parametric distribution can be estimated empirically from the data.

The empirical distribution is a non-parametric distribution. Basically, the empirical distribution is used if the random variable can take any value within a finite range and the researcher does not want to impose a particular parametric distribution function. That is, instead of selecting a distribution with a particular parametric form and estimating the parameters, it produces a nonparametric estimate that adapts itself to the data. For a sample of observations $(x_1,...,x_n)$, the empirical distribution is defined as $F_n(x) = \frac{1}{n}\#\{i : x_i \le x\}$. If a sample comes from a distribution in a parametric family (such as a normal distribution), its empirical CDF is likely to resemble the parametric distribution. If not, its empirical distribution still gives an estimate of the CDF for the distribution that generated the data.

In this work, it was determined that developing multivariate empirical (MVE) distributions would be superior to any attempts to fit a parametric distribution for two reasons. The first reason for using a multivariate procedure is due to the existence of positive correlation between the FW and SS inflows. Please refer to the correlation matrix presented in Table 5.8. Ignoring this correlation during simulation might overstate or understate the variation in both series. Richardson (2003) has demonstrated that the MVE distribution preserves the intra-year and inter-year correlation structure in a satisfactory manner.

Second, the specific parametric form of the distribution is unknown. Furthermore, statistical tests in Table 5.5 provide inconclusive evidence for normally distributed multivariate series. According to the skewness test, the assumption of normality can be rejected. However, the kurtosis test indicates that the data are likely to follow a normal distribution. In order to rule out the normality assumption, each series was tested individually for normality. The results of the individual tests are reported in Table 5.6 for FW inflows and in Table 5.7 or the SS inflows. In the case of SS inflows all the tests indicate that the series is not likely normally distributed. In the case of FW inflows, three out of four tests indicate that the distribution is not normal. Since no a priori information exists about the true parametric distribution of the FW-SS stochastic process, the MVE was fitted from the actual data.

The MVE has three parameters to be estimated. The first parameter is the deterministic component of each random variable, which can be obtained by means of an econometric model, a forecast or simply by the mean of the variable in the case that no trend is present. The second parameter is the stochastic component for each random variable. In particular, the stochastic component is a measure of the dispersion around the deterministic component. Finally, the multivariate component for the MVE distribution is the correlation matrix of rank m for m random variables. The correlation matrix must be computed from the stochastic components.

Using SIMETAR, the three components of the MVE distribution were estimated for Rio Mayo. First, the historical series on inflows suggests no evidence for trends in the inflows. Therefore, the deterministic component of the MVE was determined to be the mean inflows for FW and SS. In turn, the stochastic components will be the fractional deviates from the mean and the correlated uniformed standard deviates (CUSD). Finally, a correlation matrix from the fractional deviates is estimated as the multivariate parameter of the MVE. Table 5.8 presents summary statistics of each component. Notice that the correlation between FW and SS inflows is 0.24 and is statistically significant at the 90% confidence level. Using the estimated components, 1000 multivariate observations of inflows for FW and SS were drawn from the empirical distribution. These values are used as the stochastic component of the reservoir operation model.

5.4.2.2 Validation of Estimated MVE Distribution

The results from the simulation are presented in Table 5.9. For FW inflows, the MVE simulation results in mean inflows of 360.2 million m^3, which is slightly below mean of 369.8 million m^3 obtained from the actual historical series. For the SS inflows, the mean values from the simulated and actual data are very close. Table 5.9 also reports information to compare the covariance between the actual (FW and SS) and the simulated series (FWSIM and SSSIM). Overall, the results indicate that the simulation yields values similar to the actual data.

However, it is important to confirm if the simulated data has the same statistical properties of the actual historical data by means of statistical tests. First, the 2-sample Hotelling multivariate means test is used to statistically test if the collective means of a multivariate random sample comes from the same distribution that generated the actual historical data. The H_0 is that the mean vectors from both series are equivalent. Second, the Box-M test is used to statistically test if the covariance of a multivariate random sample comes from the same distribution that generated the historical data. Under the Box-M test the H_0 is that the covariance matrices from the simulated and the historical series are equivalent. Third, the complete homogeneity test is used to simultaneously test that the means and covariance from the simulated and historical data are equivalent. Finally, a correlation matrix test is performed under the H_0 that the correlation matrix of the simulated data is equivalent to the correlation matrix of the historical data. All the tests validate the simulation results at the 99% level of confidence. The results from the validation are presented in Table 5.10.

5.4.3 Plantings Response Functions and Income Measure

Establishing the relationship between release volume and number of hectares cultivated is a challenging task for several reasons. First, the time series of plantings reveals that the portfolio of crops grown in the region has not remained constant over time. The reasons for those changes include economic factors (i.e. prices, subsidies, technical change) as well as institutional factors (i.e. water law, land reform, etc). Unfortunately, without a fixed crop portfolio it is very difficult to estimate precisely the irrigation needs. Second, the small data sample does not provide a robust estimation of a response function that characterizes the impact of water on plantings. Due the challenges encountered above, responses functions were carefully imposed on the data and the fit was corroborated with the engineers of the SRL in Rio Mayo. The specific functional form for each response function is provided in table 5.11.

To visually test the accuracy of the response function, fitted plantings were generated using actual release data. Although no specific statistical approach was followed, the FW response function provides a reasonable fit for the FW. For the entire series the correlation coefficient is 0.6, however the correlation improves for the 1987-2002 period to 0.69. In Figure 5.12 the historical and fitted series are depicted for the FW season. Notice that during the last 9 years of data the FW response function provides a remarkably good fit of FW plantings.

In the case of the SS, Figure 5.13 presents the visual correlation between the actual and fitted plantings. The SS response function yields a correlation of 0.86 for the complete series. However, the fitted plantings closely match the actual for the period 1972-1987 as suggested by Figure 5.13 and the correlation coefficient of 0.9. For the most recent period, the response function correlates to the actual plantings, but it overstates the impact of water releases.

There are two important features in the fitted response functions that play an important role in the inter-seasonal allocation of water. First, the curvature of both response functions reflects the fact that there are diminishing marginal returns to the application of water. The diminishing marginal returns are directly related to the distance between the land parcels and the irrigation canals. Specifically, while parcels located closer to the main irrigation canals enjoy high efficiencies of water conveyance, those located at further distances experience low water conveyance efficiency. Thus, for a given volume of water released from the reservoir, more land can be irrigated in areas closer to the irrigation canals than in areas located in the farther boundaries of the irrigation district. The data shows that the inefficiency in FW can be as high as 55%. That is, for an additional million m^3 released from the reservoir, only 55% of this volume is actually received at the farm level. Please refer back Table 5.2 for the estimated efficiency in water conveyance at different points of the distribution system.

The second feature relates to the inter-seasonal conveyance efficiencies. Since the average temperature during the SS season is higher, the conveyance inefficiencies in this season are relatively higher. Therefore, for each unit of water released from the reservoir, more land can be employed in FW than in SS. In other words, the marginal productivity of water in the irrigation district is higher in FW than in SS, once evaporation losses are accounted for.

Due to data limitations, a measure of income expressed in hectares was developed. Examining the production history of the district suggests that wheat is the main crop grown in the FW season. The second most important crops are maize and safflower. In terms of the profitability of these crops, it is important to notice that besides the market price, they receive government supports in the form of a per-hectare subsidy. This makes the relative profitability of the crops in the FW season higher. In terms of the SS season, the crop portfolio is highly variable. The evidence suggests that in SS the preferred crops include cotton and safflower. Unfortunately, it was not possible to obtain prices and costs of production for the SS season since production activities have been suspended during the last three years.

Relying on the opinions from irrigators in Rio Mayo, the SRL , and CAN, I conjecture that the portfolio of crops in the SS season carries a lower value than the FW portfolio. In order to overcome the limited information, the revenues of the district are expressed in hectare equivalents. Specifically, the per-hectare return to the crop portfolio planted in FW is normalized to 1, so that income is given in hectare equivalent units (ha). In turn, the return on the hectares planted in SS is expressed as a fraction of the FW return, specifically 0.7. For instance, if 80,000 ha are planted in FW and 20,000 ha in SS, the corresponding hectare-equivalent income is 94,000 ha (80,000 ha from FW and 14,000 ha from SS). With these computations, an approximate mean monetary return per-hectare in FW can be used to convert the SS hectares into a monetary figure, and consequently obtain annual returns[20].

5.5 The Reservoir Operation Model

The reservoir operation model comprises a series of management and physical characteristics of the ARC reservoir. It features seasonal operation rules, response functions, reservoir size, evaporation rates, and the stochastic element of inflows. A logical representation of the different steps in the model is described in Figure 5.17.

At the beginning of the agricultural year, given a beginning-of-the-year stock of water measured on October 1, the operator releases water according to the release rule for the FW season. This allocation of water is used to grow crops between October and March and the corresponding number of hectares planted is computed using the planting response function for the FW season. In the month of April (April 1), the operator computes the amount of water available for the SS season, tracking the FW releases, any additional inflows accumulated between October and March, and the evaporation losses.

[20] Since data on prices and production costs was not available, the hectare-equivalent measure was used as an indicator of returns to agriculture rather than a monetary measure.

In turn, stock of water in storage as of April 1 and its corresponding releases rule determine if the SS season will be carried out. If carried out, the SS plantings response function is used to compute the number of hectares planted. At the end of the year, two computations are carried out. First, the annual plantings are computed by adding the FW and SS plantings. Second, the end-of-the-year stock of water is computed, by adding inflows received in the April-September period and deducting releases and evaporation losses during the SS season. Consequently, the end-of-the-year stock is the starting stock for the following agricultural period.

5.6 Using Release Rules to Manage Water Supply Risk

The literature review conducted in Chapter 2 concludes that most studied source of risk management for the reservoir operator is the release rule or operation policy. As established in Chapter 4, the release rule can be conservative or aggressive. For a given a volume of storage, an aggressive policy will release more water, on the average, than a conservative policy. While aggressive operation policies yield higher revenues, they also imply a higher level of risk. Therefore, the operator needs to tradeoff higher revenues for lower levels of risk. Unfortunately, the operation policy of the ARC reservoir did not produce satisfactory results as a planning tool.[21] Therefore, in this section I develop a ranking of release rules based on simulation and expected utility theory. Specifically, I use the concept of stochastic efficiency, with emphasis on certainty equivalents and risk premiums (Richardson, 2003).

Three sets of reservoir operation policies are made available to the SRL. Each set consists of two seasonal rules. The FW rule is based on the level of storage in the reservoir as of October 1 of each agricultural cycle. Similarly, the SS rule is based on the stock of water available for irrigation as of April 1. In each of the three sets the FW rule is more aggressive than the SS rule due to the fact that the marginal productivity of water is higher in FW than in SS. The difference between the three sets lies in the degree of "aggressiveness" that each one represents. In particular, policy A is the most aggressive policy in the sense that for a given level of water in the reservoir this rule releases the

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[21] According the CNA, the reservoir is operated without taking into account any economic criteria. The multiple objective function consists in maximizing releases and minimizing spills. In addition, the release
Footnotes continued on the next page

greatest volume of water for both FW and SS. In contrast, policy C is more conservative in both FW and SS. In turn, policy E is a mixed policy in the sense that it is more aggressive in FW than policy C, but less aggressive than policy A in SS. The functional forms for the three sets of release rules are given in Table 5.12.

The results of the simulation are presented in Table 5.13. As expected from Chapter 4, there is a tradeoff between mean income and the variation of income. In this sense, policy A has the highest mean income and the highest variation, whereas policy E has the lowest mean income and the lowest variation. Also notice that the sensitivity of the results is not that large with respect to changes in the "aggressiveness" in the release rules. This further confirms the claim that using the release rule as a hedging mechanism may not be the best risk management policy. In terms of choosing a preferred rule, if the SRL were risk neutral it would prefer Policy option A that has the highest mean hectare-equivalent income. However, it is more likely the SRL is not risk-neutral. In the case of risk aversion, assessing the risk-return tradeoff requires knowing both the probabilities and preferences for outcomes.

In the literature, risky alternatives have been ranked using mean-variance analysis and stochastic dominance. However, these methods result in inconclusive rankings for some types of decision makers. Therefore, I use stochastic efficiency with respect to a function (SERF), proposed by Hardaker et al (2004), as an alternative method. SERF orders a set of risky alternatives in terms of certainty equivalents (CE) for a specified range of attitudes towards risk. More specifically, those risky alternatives with the largest CE[22], within a given range of risk aversion coefficients (RAC), are preferred. The main advantage of this method is that it can compare all the risky alternatives simultaneously, rather than pair wise as in stochastic dominance, and produce a smaller efficient set than that found by stochastic dominance over the same range of risk aversion attitudes.

The empirical PDF of simulated income can be used to identify the risk-return profile produced by each operation policy (see Figure 5.14). To rank those distributions,

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rule is used only to plan the minimum level of release, and further adjustments are made throughout the year.

[22] CE is the certain amount of income that makes a person indifferent between that value and the risky alternative. In other words, the CE is the sure income that yields the same utility as the expected utility of the risky alternative.

the SERF method is used under the assumption of a power utility function[23]. The range of relative risk aversion coefficients (RRAC) used is that suggested by Anderson and Dillon (1992). The range of RRAC goes from 0.5 (i.e. a hardly risk averse individual) to 4 (i.e. an extremely averse individual). The certainty equivalents for the RRAC range are provided in Table 5.14. and Figure 5.15. The risk analysis indicates that the CE for policy A is the largest for relatively modest degrees of risk aversion (i.e. 0.5-1.82 range). However, at the RRAC of 1.78 the decision maker is indifferent between policies A and E. For RRAC over 1.78 policy E is preferred over both A and C. Notice that Policy C is not preferred at any RRAC. In summary, if the SRL is moderately risk-averse (i.e. RRAC is 2) it will prefer policy E over policies A and C to operate the ARC reservoir.

Another informational advantage of the SERF method is that the risk premium[24] that decision makers place over the preferred alternative is readily computable from the absolute difference between the CE values of risky alternatives. In other words, the risk premium can be interpreted as the degree of conviction in the preference of one alternative over the other. In Table 5.15 the risk premium for each RRAC is reported. Notice that for low to moderate levels of risk aversion the SRL is willing to pay a positive amount to move from the inferior policy C to policy A. For instance, at very low level of risk aversion, the SRL would give up 562 ha (0.6% of mean income), but at moderate level of risk aversion the willingness to pay reduces substantially. Alternatively, an extremely risk-averse SRL would pay up 851 ha (1% of mean income) to shift from policy C towards policy E. Although not reported on the table, at the RRAC of 1.78 the SRL would pay the same sum to operate under either policy A or E. In other words, at this particular RRAC the SRL would be indifferent between the dominant policies.

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[23] The analysis is based on a power utility function that exhibits constant relative risk aversion (CARA) such that preferences are unchanged when all the payoffs are converted to a monetary unit by multiplying by the appropriate per hectare returns. Specifically, utility is given by $u = \frac{1}{(1-R_r)} w^{(1-R_r)}$, where R_r refers to the relative risk aversion coefficient.

[24] The risk premium is the amount of money that a risk-averse individual is willing to give up to avoid the risky alternative. Alternatively, the risk premium is the amount of money that an individual has to be paid to accept the risky alternative.

5.7 Components of the Inflow-Based Derivative

There are six elements in the design of a weather derivative: the underlying variable, the accumulation period, the location of measurement, the tick size, the trigger, and the indemnity rules. Each element requires careful choices that ultimately determine the effectiveness of this hedging mechanism.

The underlying variable used in this research is reservoir inflows. Alternatively, one could have used reservoir storage. The problem with reservoir storage is that it is the result of a combination of management and random events. One of the most basic principles of insurance is not insuring management. In contrast, reservoir inflows are a purely random variable and no manipulation can be exercised over it. In addition, just like reservoir storage and with the help of well-understood reservoir operation policies, inflows convey all the information about the scarcity of water and the consequences of major shortfalls.

In terms of the accumulation period, there are two choices. On the one hand, one could design a simple contract whose payoff depends on the accumulation of inflows during one time period. The time period could coincide with the agricultural cycle (i.e. Oct-Sep). On the other hand, the contract could be designed into two time periods: agricultural cycle and 6 months after the agricultural cycle. From the point of view of the SRL, the first 12-month period determines the availability of water for the FW season. The second period is more related to the particular characteristics of inflows in the Mayo Valley. The inflows data show that in the period corresponding to October 1 to March 30, there are events that bring more than expected inflows. Throughout this dissertation, I refer to the higher-than-expected inflows in FW as the "bonus." This event is beneficial for irrigators because it replenishes the reservoir previous to the beginning of the SS season. Thus, when the "bonus" occurs, irrigators are more likely to grow crops in the SS season. The importance of this event is that it partially decreases the need for insurance. Therefore, any contract that aims to protect the income of irrigators grouped in the SRL has to take this event into account to both make premiums more feasible and to correctly price the contract.

The location where the inflows are measured is the reservoir itself. Fortunately, even before the reservoir was built, CNA measured the potential inflows generated by the

Mayo River. Therefore, monthly observations of the inflows were obtained dating back to 1943. More importantly, the SRL and CNA trust in the quality of these measurements, which are essential for the decision-making process.

Since annual income will be measured in hectare equivalents (as explained in section 5.4.3), the tick size will also be expressed in this unit. Specifically, the contract pays 100 ha for each million m^3 below the corresponding strike level. The choice of the trigger is directly related to the choice of time periods. Specifically, under the single time period scheme only one trigger is required, but under two accumulation periods at least three triggers are required. The particular triggers are described in the next section.

In terms of the indemnity rules, I propose a put contract that pays when the accumulation of inflows falls short a given trigger or set of triggers. The general structure of the put contract is stated in equations 5.1, 5.2, and 5.3 below.

Equation 5.1 describes the maximum payoff or indemnity paid by the contract in a given year, denoted by $\overline{P}_t$. This maximum payment is a function of the first trigger and the volume of inflows accumulated throughout the previous time period, denoted by I_C and I_{t-1}, respectively. The indemnity rule is linear and pays only when the I_{t-1} accumulation falls short of the critical level represented by I_C. This component of the indemnity rule is the most basic one and applies to contracts that have one accumulation period or two accumulation periods.

$$\overline{P}_t = TIC \times \begin{cases} 0 & if\ I_{t-1} > I_C \\ \left(I_C - I_{t-1}\right) & if\ I_{t-1} < I_C \end{cases} \tag{5.1}$$

In addition, equation 5.2 represents a discount rule that applies in the case that the contract includes more than one time period. In other words, the actual payment P_t received in a given year is equal to the maximum qualifying payment $\overline{P}_t$ discounted by the factor D. The discount factor is computed according to equation 5.3 and can take any value in the interval [0,1]. In particular, the maximum payment $\overline{P}_t$ is discounted if the volume of FW inflows accumulated in the second time period (i.e. October-March), denoted by $I_{1,t}$ falls within the bounds of the upper and lower triggers, denoted by $I_{\max}$ and $I_{\min}$, respectively. If $I_{1,t}$ falls short of the lower bound trigger, no discount rule applies and the contract pays exactly $\overline{P}_t$. However, if $I_{1,t}$ exceeds the upper bound trigger then the contract makes no payment.

$$P_t = \overline{P}_t * D \tag{5.2}$$

$$D = 1 - \begin{cases} 1 & \text{if } I_{1,t} \geq I_{\max} \\ \left(\dfrac{1}{I_{\min} - I_{\max}} \right) \times \left(I_{\min} - I_{1,t} \right) & \text{if } I_{\min} < I_{1,t} < I_{\max} \\ 0 & \text{if } I_{1,t} \leq I_{\min} \end{cases} \tag{5.3}$$

For example, assume the following parameters: I_c = 725, TIC = 100 ha. Then, if the inflows corresponding to the period agricultural year 2005-2006 were 550 million m^3, then maximum payment would be 17, 500 ha.[25] However, if inflows of 300 million m^3 were registered in the period October 2006 to March 2007, then the payment would have to be discounted[26] to 50% of the maximum payment, which is 8,750 ha.

5.8 Measuring the Effectiveness of the Hedging Instrument

The results of the derivative contract will be assessed for the following criteria: affordability, value for the SRL, and risk reduction effectiveness. The affordability will be assessed according to the price of the instrument. This price carries two components: the expected payoff from the instrument (e.g. expected loss) and a loading factor. The first component is equal to the mean indemnities received by the SRL divided by the maximum plausible indemnity. In a world with no transaction costs and more uncertainty about the distribution function of inflows, the SRL would expect that in the long run the premium payments would be offset by the mean indemnities. However, due to

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[25] $\overline{P} = (725 - 550) \times 100\,ha = 17{,}500$

[26] $D = (1 - (-0.0025) \times (-200)) = 0.5$, supposing $I_{\min}$ = 100 and $I_{\max}$ = 500.

administration costs and uncertainty, the insurer usually charges a "loaded" premium[27]. The load is just a percentage (i.e. mark up) above the fair price of the instrument.

In addition to the typical descriptive statistics, the risk from the simulated income (as measured in hectare equivalent) distributions will be evaluated. In particular, the risk reduction potential of the instrument is examined using the concepts of value-at-risk (VaR) and conditional value-at-risk (CVaR). According to the financial literature, VaR is a measure of the maximum financial loss for a given confidence level in a specific time horizon. For example, for a specified confidence level β, VaR is simply the loss that is exceeded over the time period with probability $1-\beta$. In this empirical application, VaR is expressed in terms of hectare equivalent income and is computed directly from the empirical CDF of simulated income, not losses. Therefore, in this application VaR is interpreted as the minimum income that is exceeded over time with probability $1-\alpha$. Moreover, in a slight diversion from the financial literature, my approach consists of finding the probability that hectare-equivalent income falls short of a minimum threshold[28]. Since I fix the VaR threshold, the instrument will be risk-reducing if the VaR probability is smaller under the corresponding hedging strategy. In other words, the probability that income realizations falls short of the 75,000 ha threshold should be lower under the risk management policy than without intervention.

While VaR provides information about the probability of experiencing income reductions beyond a specified threshold, it does not provide information about the magnitude of those reductions. Therefore, I use the notion of conditional value at risk (CVaR) to measure these expected reductions. In the financial literature CVaR is a currency-denominated measure of the significant unfavorable changes in the value of a portfolio. For example, for the confidence level β, there is a VaR that can be computed. CVaR is the expected loss that occurs in excess to the VaR threshold with $1-\beta$ probability. In the current application, CVaR measures the mean income that falls short of the 75,000 ha threshold. Similar to the VaR measure, the instrument will be risk

[27] The fair price or premium is computed using the historical or burn-rate method. This is based on the assumption that the history of the data accurately represents the risk to be analyzed.

reducing if the expected income below the 75,000 ha threshold is smaller under the hedging strategy than without intervention. In other words, a smaller CVaR measure is equivalent to less downside risk exposure.

Next, I discuss the necessity of introducing a double-trigger contract to hedge the inflows risk, as opposed to a single trigger. In order to make this necessity clear, I compare the effectiveness of the contract with and without the discounting rule. A single-trigger contract would be simpler to implement as it could be sold at the beginning of the agricultural year (October) and the indemnity payment would be received at the end of the year (September). The put contract would pay only if inflows fall short of the strike level. However, with this design the SRL would not take advantage of the natural hedging mechanism offered by the reservoir and the inflows during the FW season. If the "bonus" inflows are sufficiently large, irrigators in the SRL have the ability to carry out productive activities during the SS season, which increases their annual income. The results from the base case scenario (no derivative) are presented in Table 5.16.

Using the response function specifications described in Table 5.11 and the reservoir operation policy E in Table 5.12, the simulation produces a mean hectare-equivalent income of 92,338 ha. The variability around the mean, as measured by the CV, is 15.9%. According to the VaR measure, there is a 13.9% chance that the annual income is equal or less than 75,000 ha. In addition, the CVaR measure indicates that the mean income falling short the VaR threshold is 9,385 ha. The maximum and minimum levels of income, without hedging, are 54,178 ha and 115,545 ha, respectively.

5.9 Developing and Ranking Alternative Risk Management Strategies

Using the formulas in equations 5.1 and 5.3, alternative hedging strategies were simulated. To simulate the maximum indemnities from equation 5.1, the strike levels range from 600 to 800 million m^3, in 50 units increments, for a total of 5 strikes. In turn, to simulate the discount factor for equation 5.3, 4 sets of parameters were selected, including an alternative in which no discount is applied to payments (i.e. single-strike contract). From the combination of these parameters, 20 alternative hedging strategies

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[28] This threshold is similar to a safety-first criterion in the sense that income falling short of the threshold is not tolerable by irrigators. In other words, I assume farmers already now the VaR threshold, but they are interested in the probability of income falling of that threshold (i.e. α).

were designed (4 parameter sets x 5 strikes). Specifically, 5 strategies include contracts in which payments are not discounted, while the remaining 15 strategies include contracts with discount factors as in equation 5.3. The difference in the structure of these two sets of contracts is presented to test the effectiveness of the double-trigger contract in taking advantage of the "bonus" inflows that occur in the FW season.

The empirical strategies in Table 5.17 are labeled according to their respective parameters and strikes in the following format: (strike, I_{min}, I_{max}). For instance, strategy 21 (800, 200, 500) indicates that in the first accumulation period a maximum potential indemnity occurs if annual inflows in the first year fall below 800 million m^3 (strike). However, the potential indemnity is modified according to the parameters in the discount rule of equation 5.3. Specifically, if FW inflows in the following year are: equal or more than 500 million m^3 (I_{max}), no indemnity is paid; less or equal to 200 million m^3 (I_{min}), the maximum potential indemnity is paid; at some level between I_{min} and I_{max}, a fraction of the potential payment is paid.

The strategies in Table 5.17 characterize the trade-off between expected income and risk across the spectrum of strategies. For instance, the base strategy (no derivative) produces the highest mean income (92, 338 ha) but also represents second riskiest strategies (13.9% VaR and 9,385 ha CVaR). In contrast, strategy 10 provides a mean income of 90,773 ha, and secures the lowest VaR measure with only a 4.7% probability of income falling below the 75,000 ha threshold and with expected shortfall below the same threshold averaging only 3,272 ha. Also, notice that sometimes CV does not provide a meaningful measure of risk exposure; thus, VaR and CVaR measures are used to account for downside risk, which is a most relevant criterion for risk-averse decision makers. For instance, according to the CV measure, the "riskiest" scenario is the base strategy (no derivative). However, according to the VaR and CVaR measures the "riskiest" scenario is produced by strategy 6. Finally, in the last column of Table 5.17 the loaded insurance premium, as a percentage of the maximum possible indemnity, is reported for each strategy. For example, strategy 21 implies that the SRL can expect to pay a price of 6,560 ha, which is equivalent to 8.2% of the maximum indemnity of 80,000 ha for this particular strategy. All the prices reported in Table 5.17 are "loaded" or marked up by 50% above the actuarially fair price, which is closer to what an insurance provider would charge to account for other factors such as uncertainty, required rate of return, and overhead costs.

If the SRL were an organization composed of risk-neutral individuals, the best criteria for ranking would be the expected income of each alternative. In the case of risk-loving irrigators they would choose strategies that imply a high degree of downside and high potential returns in the upside. However, the most likely and interesting scenario is that irrigators are risk averse. In this case, irrigators will give up high potential returns in exchange of a lower degree of downside risk. Under the assertion that most individuals exhibit some degree of risk aversion, and supposing that Rio Mayo irrigators are not different, the cases of risk-preferring behavior will not be addressed in this study.

Since risk-averse individuals assess their risky prospects by examining both the probabilities and the preferences for outcomes, information on both aspects is needed to produce a ranking of the 21 alternative hedging strategies (including the base case). First, the probabilities for outcomes are estimated from the simulated income empirical distribution produced by each strategy. In turn, the preference for outcomes is approximated using a power utility function due to certain desirable and convenient properties[29]. Using both pieces of information, the SERF method is used to rank the strategies and compute measures of conviction and utility gains.

Based on expected utility theory, the SERF method assumes the decision maker maximizes his/her expected utility and ranks risky prospects according to the certainty equivalents across a range of RACs. In particular, the stochastic efficiency analysis yields an efficient set over the range of RACs. For the Rio Mayo strategies, the CE values in the efficient set are reported in Table 5.18. For instance, under the base case strategy (no derivative), the CE is 91,732 ha for a hardly risk-averse SRL. In other words, a sure sum of 91,732 ha provides the same utility as the expected income of 92,338 ha. Table 5.18 also reports how as the degree of risk aversion increases there is a greater preference for a risk-free alternative, as predicted by expected utility theory. Following the example

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[29] Specifically, utility is given by $u = \frac{1}{(1-R_R)} w^{(1-R_r)}$, where R_r refers to the relative risk aversion coefficient. In addition to exhibiting CRRA, this function exhibits the desirable property of diminishing absolute risk aversion, which is applicable given that the payoffs are defined in terms total wealth (Hardaker et al, 2004).

above for the base strategy, as the SRL becomes extremely risk-averse, the CE value declines from 91,732 ha to 87,121 ha. That is, when the SRL is highly risk-averse it would be indifferent between a sure sum of 87,121 ha and the expected income of 92,338 ha that the base alternative yields.

The SERF efficient set is constructed by finding strategies with the highest CE values within a range of RRAC. In Table 5.18 and Figure 5.16, the strategies with the highest CE values within the 0.5-4 range of RRAC include the following: 1 (base), 12 (600-100-400), 13 (650-100-400), and 14 (700-100-400). In particular, using Anderson and Dillon (1992) scale[30] of risk aversion one can conclude that if the SRL exhibits risk aversion that ranges from very low to normal, it would prefer the base strategy. In other words, within the 0.5-1.8 range the highest CE values are produced by the strategy 1 (base strategy), consequently eliminating the demand for the inflow-based derivative[31]. However, as the RRAC increases from 1.81 to 2.4 (i.e. normal risk-averse), strategy 12 (600-100-400) yields the highest CE values, thus the demand for the derivative becomes effective at this particular subset of risk aversion. As the degree of risk aversion increases within the 2.4-4.0 RRAC range, strategies 13 (650-100-400) and 14 (700-100-400) enter the efficient set.

In addition to the CE, the SERF analysis provides a risk premium[32] for each one of the hedging strategies (Table 5.19). More importantly, this risk premium provides an approximation of the utility gain, expressed in income units, that a risk-averse SRL can achieve by adopting the corresponding strategy. Since the income profile of each strategy has already being adjusted for the cost of buying the contract (i.e. figures under last column in Table 5.17), the SERF risk premium represents the amount, which in excess to the contract price, the SRL is willing to pay to engage in the contract. For instance, a

[30] Anderson use the following scale for RRACs: 0.5 for hardly risk averse; 1.0 for normal or somewhat risk averse; 2.0 for modest or rather risk averse; 3.0 for very risk averse; and 4.0 for extremely risk averse.
[31] This is partially explained by the fact that throughout the analysis a load of 50% over the fair price of the derivative is assumed. If the load factor would be eliminated, in which case the price of the derivative would be actuarially fair, irrigators would prefer to buy insurance even at very low degrees of risk aversion. In the next section, a sensitivity analysis will be performed to isolate the impact of the loading factor on the demand for insurance.
[32] The risk premium from the SERF analysis is interpreted different from the "loaded" insurance premium. The former refers to the amount of income a decision maker is willing to pay to obtain a certain amount of income and not face a gamble. The latter, in turn, refers to an actuarial computation of the expected insurance indemnities as a proportion of the maximum payable indemnity.

moderate risk-averse SRL with a RRAC of 2.1 is willing to pay 127 ha, in excess to the 1,298 ha "loaded" insurance premium, to hedge with strategy 12 (600-100-400). In other words, because the risk-return profile of strategy 12 (600-100-400) provides a higher expected utility than the base case scenario, the SRL is willing to pay a total of 1,425 ha (i.e. 1298 for the "loaded" premium and 127 ha in excess) to engage in the contract. However, since the insurance company only charges the actuarially fair price (865 ha) plus a 50% "load" (432 ha), the remaining 127 ha represent the net utility gain accrued to the SRL. As predicted by expected utility theory, at higher degrees of risk aversion the SRL puts more value on the safer outcomes; therefore, higher levels of net utility gains are observed, for a given strategy, for the higher RRACs in Table 5.19.

In addition, Table 5.19 reports the risk premium across strategies for a given RAC. This information is useful to provide a measure of the degree of preference of one strategy over others as measured by the corresponding utility gains. For example, at the RRAC of 2.1, strategy 12 (600-100-400) is dominant because the SRL would pay only 82.6 ha to operate under strategy 13 (650-100-400) and would have to be compensated with 46.9 ha (a utility loss) to operate under strategy 14 (700-100-400). In other words, the dominant strategy is the one with the highest risk premium. Within the efficient set, the utility gains range from 36.6 ha for a moderately risk-averse (RAC = 1.8125) SRL interested in strategy 12 (600-10-400) to 1,056 ha (1.14% of mean income) for an extremely risk-averse SRL (RRAC = 4) preferring strategy 14 (700-100-400).

As the efficient set is described in Table 5.18 and Figure 5.16 notice that the effective demand for the inflow-based derivative starts at moderate levels of risk aversion. In fact, a moderately risk-averse SRL (1.81-2.39 range of RRAC) will prefer strategy 12 (600-100-400); a very risk-averse SRL will prefer strategy 13 (650-100-400); and an extremely risk-averse SRL will prefer strategy 14 (700-100-100). In other words, as the SRL becomes more risk averse, it will give up alternatives with higher expected payoffs for alternatives that entail lower downside risk.

The economic outcomes of adopting each of the strategies in the efficient set are directly observed from Table 5.17. For instance, one can see that by preferring the base strategy (no derivative), an SRL that exhibits slight to normal risk aversion would prefer the expected income of 92,338 ha and downside risk of 13.9% and 9,385 ha as measured by VaR and CVaR, respectively. In turn, a moderately risk-averse SRL by choosing strategy 12 (600-100-400) prefers to pay an insurance premium of 2.2 % (1,298 ha) for a double-triggered contract that yields an expected income of 91,905 ha and reduces the downside risk to 11.3% and 7,929 ha as indicated by the VaR and CVaR measures. On the final strategy of the risk efficient set, an extremely risk-averse SRL is willing to pay a price of 8.2 % (6,560 ha) to engage in strategy 14 (700-100-100) that produces a mean income of 91,313 ha and reduces the VaR and CVaR measures to 7.1% and 5,045 ha, respectively.

5.10 Insurance Loading and Willingness to Pay

In this section I perform sensitivity analysis with respect to the loading factor that the SRL can accept in a hypothetical contract. In principle, if the SRL were offered an actuarially fair insurance premium, the price of the insurance would be equivalent to the expected indemnities paid by the contract. In other words, in the long run the SRL would receive back the premium payments in the form of indemnities. In the insurance industry, however, the supplier of insurance and derivative products charges a mark up above the actuarially fair premium to cover other factors including profitability, uncertainty about the risk covered, and to protect against large indemnities in the first years of the program scheme. If the inflow-based derivative would be offered at the actuarially fair price, the SRL would find it beneficial at very low degrees of risk aversion or even at risk neutrality. Therefore, one way to gauge the demand for the derivative is to apply several "loads" to the premium and observe the corresponding break-even RRAC (BRRAC). BRRAC is the level of risk aversion in which two risky alternatives are equally preferred in the efficient set (McCarl 1998). In principle, as the loading factor is increased, the BRRAC that makes the SRL indifferent between hedging and base strategy will also increase. Eventually, the BRRAC will be very large so that only extremely risk-averse individuals will show interest in the hedging instrument. Consequently, the maximum load can be interpreted as an upper bound of the maximum premium that can be charged to SRL. Please refer to Table 5.20 for a sensitivity analysis with respect to the loading factor. Notice that the sensitivity analysis is based on strategy 12 (600-100-400) because even with a 50% loading factor, the SRL does not have to be extremely risk-averse in order to engage in such strategy. Therefore, strategy 12 (600-100-400) has the greatest potential to absorb higher mark ups.

The sensitivity analysis indicates that at an actuarially fair price the SRL would be indifferent between the derivative and the base scenario at the RRAC of 0.00425. In other words, given the fair price, even a risk neutral group of irrigators would buy the hedging mechanism. Basically, operating under the hedging rule generates the same expected income (92,338 HA) as operating without the hedge, but it decreases the VaR measure to 10.6% and the CVaR measure to 7,421 ha. Therefore, a risk-neutral SRL seizes the opportunity of obtaining a similar mean profit in addition to benefiting from a reduced degree of downside risk.

However, as the load factor is increased above the fair price, the degree of risk aversion at which the hedging instrument becomes appealing to the SRL increases. Since loading the price implies that the SRL pays more than what they get back in the long run, the expected income is reduced. Hence, only risk averse agents who give up higher income for lower downside risk will show interest in the instrument. For instance, as the load factor is increased to 50% above the fair price, the demand for insurance is effective when the SRL exhibits moderate risk aversion (BRRAC=1.86). At this particular loading risk-neutral the SRL obtain the same level of expected utility of income with or without the hedging strategy. As the load factor reaches 100% of the fair price, only a very risk-averse SRL (BRRAC =3.35) would show any interest in the derivative. Finally, when the price is increased 250% over the actuarially fair price, only the extremely risk-averse SRL (BRRAC = 4.69) would effectively demand the instrument. To summarize, a loading factor of 50% seems to be the maximum premium that the insurance provider can charge before the SRL decides not to buy the insurance.

The sensitivity analysis indicates that at the actuarially fair premium of 1.4%, the risk reduction measures and expected income are at its maximum. As the premium is loaded these measures start to decrease. The highest feasible load at the 75,000 ha threshold is 2.5. This load should make the insurance contract viable for the insurer while still making it affordable for the SRL.

5.11 Conclusion

The chapter examines the risk environment of the Rio Mayo irrigation district in Sonora and developed a stochastic dynamic simulation model of the ARC reservoir. A major motivation was to examine the potential value of a risk transfer product that would pay when inflows of water are well below normal. To accomplish this objective the bi-variate empirical distributions inflows in the FW and SS were used in conjunction with plantings response functions. With this framework an inflow-based derivative was designed to transfer the risk of having water shortfalls. Given the complexity of this particular system (i.e., having 2 distinct planting seasons and the opportunity for 'bonus' inflows during the FW season), the results indicate that the best design is a double trigger contract that discounts payments for occurrence of "bonus" inflows in each FW season. This design is feasible because it takes advantage of the natural hedging for annual income risk in the irrigation district due to extra inflows that allow for more income by unanticipated planting in the SS season. The risk transfer product is also investigated in a feasible fashion with loaded premium rates and at levels that remain below a 10% premium rate benchmark.

Although the irrigation district is modeled under the assumption that it is a single production unit, in reality the district is a collection of decision units. However, the institutional characteristics of the SRL as a collective group indicate that the implementation of a contract of this type could be feasible. The SRL could be used as an intermediary between the irrigators and the insurance company that sells the contract. In this sense, the role of the SRL would be to collect the premium from the individual contribution of its members and distribute the indemnities accordingly. Since the SRL is a group of the same farmers, it already possesses informational advantages in terms of the impact of water shortages on each individual. For instance, we know that farms that are located closer see their supply see water curtailed at a lower proportion than marginal farms located in the most distant areas from the canal. The SRL could implement rules that differentiate the relative size of the premium to be paid or the indemnities to be received. Furthermore, the indemnity payment could be used to organize more efficient compensatory policies for winners versus losers when there are shortages.

In the same line of thought, one could conceive that indemnity payments during water scarcity periods would provide additional liquidity to the system that would spur water market transactions. Those farmers willing to buy water will now have more cash to confront the incremented marginal price of water due to its relative scarcity. Thus, in some fashion we can see that this contract would not only mitigate the losses to the irrigation district as a whole, but also would encourage the development of water markets that lead to an efficient use of the resource. Further research on this aspect is suggested.

Table 5.1: Organizational Characteristics of Rio Mayo Irrigation District

Module	Efficiency* (%)	Area	Ejidatarios		Private	
			Users	Area (Ha)	Users	Land (Ha)
1	74.9	7,898	653	4,073	430	3,825
2	74.6	8,637	819	6,098	225	2,538
3	75	5,214	430	4,164	81	1,049
4	70.3	5,468	513	2,762	337	2,706
5	68.3	5,289	321	1,390	600	3,898
6	75.4	7,875	472	2,303	374	5,572
7	72.9	8,130	1,132	6,220	63	1,911
8	79	3,766	246	2,372	92	1,395
9	77.2	6,063	286	2,755	91	3,318
10	77.4	5,154	355	2,874	93	2,280
11	80.9	5,168	364	3,408	38	1,760
12	84.6	7,032	95	488	183	6,544
13	80.8	6,545	240	2,775	97	3,769
14	76.5	4,797	508	2,460	249	2,337
15	70.7	6,086	1,055	5,158	144	928
16	59.4	3,925	361	1,757	770	2,167
Total		97,047	7,850	51,050	3,867	45,997

Source: SRL of Rio Mayo Irrigation District
* Annual weighted average efficiency

Table 5.2: Water Conveyance Efficiency Ratios of Rio Mayo Irrigation District

	Fall-Winter	Spring-Summer
From Reservoir to SRL	90%	77%
From SRL to Module	83%	75%
From Module to Farm	73%	67%
Overall (Reservoir to Farm)	55%	39%

Source: CNA, SRL

Table 5.3: Descriptive Statistics of Hydrological and Economic Characteristics of Rio Mayo Irrigation District

	Inflows[a]	Release[a]	FW Storage[a,b]	SS Storage[a]	Annual Plantings[c,d]	FW Plantings[c,e]	SS Plantings[c,e]
Mean	1,013.19	825.43	743.48	488.81	101.05	75.97	28.19
Standard Error	64.79	28.90	32.39	45.13	3.65	1.54	4.07
Median	914.50	824.70	685.00	415.39	99.24	78.86	20.62
Standard Deviation	453.52	202.30	226.74	315.92	215.48	8.41	22.30
Minimum	454.73	440.88	314.80	25.11	51.81	49.72	0
Maximum	2,511.51	1,240.41	1,206.32	1,124.56	142.46	85.64	72.66
Count	49	49	49	49	35	30	30
Confidence Level[f]	130.27	58.11	65.13	90.74	7.40	3.14	8.32

Notes: Data collected from CNA and SRL
a: inflows, releases and storage are measured in million m^3 (1 cubic meter = 0.0008107 acre foot = 35.315 cubic foot) for the period 1955-2004
b: storage as of October 1 (beginning of agricultural cycle)
c: production measured in thousand hectares (1 hectare = 2.47 acre)
d: Period 1969-2003
e: Period 1973-2003
f: 95.0%

Table 5.4: OLS Regression Results Dependent Variable: Annual Releases (Mean[a] 825)

Variable	Coefficient	t-ratio	Mean[a]
Constant	245.89	4.45	
October Storage	0.55	8.22	741
Fall-Winter Inflows	1.07	7.12	375
(Fall-Winter Inflows)^2	-0.0007	-5.021	
Adjusted R^2	0.85		
F-statistic	62.63		
Durbin-Watson statistic	2.15		
Observations	33		

Source: Own computations from data for period 1970-2003.
a: measured in million m^3.

Table 5.5 Tests for Normality of the Multivariate FW-SS Inflows

Criterion	Test Value	Critical Value	p-Value	Conclusion*
Skewness	15.27	9.49	0.0042	Reject H_0
Kurtosis	0.53	1.96	0.2961	Fail to Reject H_0

H_0: The data are multivariate normally distributed.
* 95% confidence level.

Table 5.6 Normality Tests for FW Inflows Distribution

Criterion	Test Value	p-Value	Conclusion*
Shaphiro-Wilks	0.93	0.016073	Reject H_0
Anderson-Darling	0.83	0.029951	Reject H_0
Cramer-von Mises	0.13	0.044615	Reject H_0
Chi-Squared	11	0.275709	Fail to Reject H_0

H_0: The data are normally distributed.
* 95% confidence level.

Table 5.7 Normality Tests for SS Inflows Distribution

Criterion	Test Value	p-Value	Conclusion*
Shaphiro-Wilks	0.84	0.00001133	Reject H_0
Anderson-Darling	2.85	0.00000027	Reject H_0
Cramer-von Mises	0.50	0.00000233	Reject H_0
Chi-Squared	31.08	0.00028624	Reject H_0

H_0: The data are normally distributed.
* 95% confidence level.

Table 5.8 MVE Distribution Components for Simulation

Variable	Simulation Components				
	Deterministic	Stochastic		Multivariate	
	Mean	Correlated Deviate As % of Mean	CUSD	Correlation Matrix FW	SS
FW	369.8	-0.7	0.2	1	0.2
SS	664.5	-0.4	0.1	0.2	1

Notes: CUSD refers to the Correlated Uniform Standard Deviates. The CUSD change for each iteration in the simulation, and they are the basis for the MVE distribution.

Table 5.9 Statistical Properties for Simulated and Historical Data

Variable	Means	Covariance	
		Historical Series	
		FW	SS
FW	369.8	111,631.6	18,934.3
SS	664.5	18,934.3	53,971.7
		Simulated Series	
		FWSIM	SSSIM
FWSIM	360.2	87,207.6	19,468.5
SSSIM	662.9	19,468.5	58,768.8

Note: FW and SS represent the historical inflow accumulations corresponding to the periods of October-March and April-September, respectively. FWSIM and SSSIM represent the MVE simulated inflows corresponding to the periods of October- March and April-September, respectively.

Table 5.10 Distribution Comparisons of Simulated and Historical Data

	Test Value	Critical Value	P-Value	Conclusion*
2 Sample Hotelling T^2 Test[a]	0.03	9.58	0.984	Fail to Reject H_0
Box's M Test[b]	1.21	11.34	0.750	Fail to Reject H_0
Complete Homogeneity Test[c]	1.19	15.09	0.946	Fail to Reject H_0
Correlation Matrix Test[d]	0.19	2.69	-	Fail to Reject H_0

* 99% level of confidence.
[a]H_0: Mean vectors are the same.
[b]H_0: Covariance matrices are equivalent.
[c]H_0: Mean vectors and covariance matrices are equivalent, respectively.
[d]H_0: correlation matrices are equivalent.

Table 5.11: Parameters for Seasonal Plantings Response Functions

	Fall-Winter hectares	Spring-Summer hectares
Constant	-19,000	-12,000
Fall-Winter Release	380	
$(\text{Fall-Winter Release})^{1.5}$	-8.8	
Spring-Summer Release		245
$(\text{Spring-Summer Release})^{1.5}$		-5.6

Source: own computations with SRL corroboration

Table 5.12: Parameters for the Seasonal Operation Policies for the ARC Reservoir

Variable	Reservoir Operation Policies					
	A (aggressive)		C (conservative)		E (mixed)	
	FW	SS	FW	SS	FW	SS
Constant	300		187		200	
Storage as of October 1	-0.0005		-0.0005		-0.0005	
$(\text{Storage as of October})^{1.5}$	0.021		0.02		0.021	
Storage as of April 1		0.8		0.6		0.5

Source: own computations and corroborated with SRL

Table 5.13: Simulation Output from Two Sets of Release Rules

Policy	Mean Income	Standard Deviation	Coefficient of Variation	Minimum Income
A (aggressive)	93,198.7	17,081.7	18.3	46,207.8
C (conservative)	92,516.8	15,791.2	17.1	52,074.5
E (mixed)	92,338.9	14,718.5	15.9	54,178.5

Table 5.14 Certainty Equivalents for Efficient Set of Operation Policy

RRAC	Reservoir Operation Policies		
	A (aggressive)	C (conservative)	E (mixed)
0.50	**92,373.1**	91,810.5	91,732.0
0.64	**92,126.1**	91,599.9	91,552.0
0.79	**91,876.4**	91,387.2	91,370.8
0.93	**91,624.0**	91,172.5	91,188.3
1.08	**91,369.0**	90,955.7	91,004.5
1.22	**91,111.4**	90,737.0	90,819.5
1.37	**90,851.1**	90,516.3	90,633.4
1.52	**90,588.4**	90,293.7	90,446.0
1.66	**90,323.3**	90,069.2	90,257.6
1.81	90,055.7	89,843.0	**90,068.0**
1.95	89,785.8	89,614.9	**89,877.4**
2.10	89,513.7	89,385.1	**89,685.8**
2.25	89,239.4	89,153.6	**89,493.2**
2.39	88,963.0	88,920.5	**89,299.7**
2.54	88,684.6	88,685.9	**89,105.2**
2.68	88,404.3	88,449.8	**88,909.9**
2.83	88,122.2	88,212.2	**88,713.8**
2.97	87,838.3	87,973.3	**88,516.9**
3.12	87,552.8	87,733.2	**88,319.3**
3.27	87,265.8	87,491.8	**88,121.0**
3.41	86,977.4	87,249.3	**87,922.2**
3.56	86,687.7	87,005.8	**87,722.7**
3.70	86,396.9	86,761.3	**87,522.7**
3.85	86,105.0	86,516.0	**87,322.3**
4.00	85,812.1	86,269.9	**87,121.4**

Note: Bold values indicate maximum CE at the given RRAC.

Table 5.15 Risk Premium for Efficient Set of Operation Policies (with respect to C)

RRAC	Reservoir Operation Policies	
	A (aggressive)	E (mixed)
0.50	**562.60**	-78.50
0.64	**526.20**	-47.90
0.79	**489.20**	-16.40
0.93	**451.50**	15.80
1.08	**413.30**	48.80
1.22	**374.40**	82.50
1.37	**334.80**	117.10
1.52	**294.70**	152.30
1.66	**254.10**	188.40
1.81	212.70	**225.00**
1.95	170.90	**262.50**
2.10	128.60	**300.70**
2.25	85.80	**339.60**
2.39	42.50	**379.20**
2.54	-1.30	**419.30**
2.68	-45.50	**460.10**
2.83	-90.00	**501.60**
2.97	-135.00	**543.60**
3.12	-180.40	**586.10**
3.27	-226.00	**629.20**
3.41	-271.90	**672.90**
3.56	-318.10	**716.90**
3.70	-364.40	**761.40**
3.85	-411.00	**806.30**
4.00	-457.80	**851.50**

Note: Bold values indicate maximum risk premium at the given RRAC.

Table 5.16: Base Case Scenario Simulation Results (No Hedging)

Risk Measure	Result
VaR[a] Probability (%)	13.9
Coefficient of Variation (%)	15.9
CVaR[a,b]	9,385
Expected Annual Income[b]	92,338.9
Minimum Income	54,178.5
Maximum Income	115,545

Source: own computations from simulations.
a: Evaluated at the 75,000 threshold.
b: Measured in hectare equivalent income.

Table 5.17 Summary and Risk Statistics of Hedging Strategies

Number	Strategy[a]	Mean Income[b]	SD[c]	CV[d]	VaR[e]	CvaR[f]	Price[g]
1	base	92,338.9	14,718.5	15.94	13.9	9,385.0	-
2	600-000-000	91,474.9	13,306.2	14.55	12.4	8,667.5	4.3
3	650-000-000	90,934.0	12,796.0	14.07	11.3	7,945.7	6.5
4	700-000-000	90,275.0	12,434.0	13.77	11.8	8,364.0	8.8
5	750-000-000	89,503.0	12,305.3	13.75	13.0	9,188.2	11.3
6	800-000-000	88,585.0	12,378.8	13.97	14.2	9,922.7	14.1
7	600-100-500	91,862.1	13,321.8	14.50	11.0	7,716.1	2.4
8	650-100-500	91,566.2	12,633.3	13.80	9.5	6,748.7	3.6
9	700-100-500	91,201.2	11,920.4	13.07	6.8	4,832.2	4.9
10	750-100-500	90,773.9	11,249.4	12.39	4.7	3,272.1	6.2
11	800-100-500	90,257.4	10,585.8	11.73	5.4	3,779.9	7.8
12	600-100-400	91,905.5	13,409.8	14.59	11.3	7,929.2	2.2
13	650-100-400	91,640.2	12,768.7	13.93	9.9	7,037.0	3.2
14	700-100-400	91,313.5	12,107.8	13.26	7.1	5,045.0	4.4
15	750-100-400	90,931.2	11,486.1	12.63	5.4	3,786.2	5.6
16	800-100-400	90,467.0	10,871.4	12.02	5.8	4,069.7	7.0
17	600-200-500	91,801.4	13,208.5	14.39	11.1	7,795.8	2.5
18	650-200-500	91,465.0	12,475.5	13.64	9.5	6,746.9	3.7
19	700-200-500	91,049.0	11,733.7	12.89	7.0	4,964.2	5.1
20	750-200-500	90,560.4	11,069.8	12.22	5.8	4,055.4	6.6
21	800-200-500	89,971.2	10,454.7	11.62	6.6	4,617.5	8.2

Notes:

a: Each strategy is coded according to the following format (strike, I_{min}, I_{max}). Please refer to equations 5.1, 5.2, and 5.3 for more explanation on the parameters of each contract.

b: Measured in hectare equivalents.

c: Standard Deviation measured in hectare equivalents

d: Coefficient of Variation (in %).

e: Value at Risk evaluated at the 75,000 ha threshold (in %).

f: Conditional Value at Risk at the 75,000 ha threshold in hectare equivalents.

g: Insurance premium with a 50% mark up, expressed as a percentage of the maximum indemnity (i.e. strike x tick). Example: for a tick of 100 ha, strategy 21 has a premium of 8.2 % or 6,560 ha, computed as (price x strike) x tick = (0.082 x 800) x 100.

Table 5.18 Certainty Equivalents for Efficient Set of Hedging Strategies

RRAC	Risk Management Strategies[a]			
	Base	600-100-400	650-100-400	700-100-400
0.50	**91,732.0**	91,412.0	91,194.9	90,913.5
0.64	**91,552.1**	91,267.4	91,064.9	90,796.9
0.79	**91,370.8**	91,122.6	90,934.9	90,680.4
0.93	**91,188.3**	90,977.5	90,804.9	90,564.0
1.08	**91,004.5**	90,832.3	90,674.9	90,447.8
1.22	**90,819.6**	90,686.9	90,545.0	90,331.6
1.37	**90,633.4**	90,541.5	90,415.2	90,215.7
1.52	**90,446.1**	90,395.9	90,285.5	90,099.9
1.66	**90,257.6**	90,250.3	90,156.0	89,984.3
1.81	90,068.1	**90,104.7**	90,026.6	89,868.9
1.95	89,877.5	**89,959.1**	89,897.4	89,753.8
2.10	89,685.9	**89,813.5**	89,768.5	89,638.9
2.25	89,493.3	**89,668.1**	89,639.8	89,524.4
2.39	89,299.7	**89,522.7**	89,511.4	89,410.1
2.54	89,105.3	89,377.6	**89,383.3**	89,296.1
2.68	88,910.0	89,232.6	**89,255.6**	89,182.5
2.83	88,713.8	89,087.8	**89,128.2**	89,069.2
2.97	88,517.0	88,943.3	**89,001.2**	88,956.3
3.12	88,319.4	88,799.1	**88,874.6**	88,843.7
3.27	88,121.1	88,655.2	**88,748.4**	88,731.6
3.41	87,922.2	88,511.7	**88,622.7**	88,619.9
3.56	87,722.7	88,368.5	88,497.5	**88,508.6**
3.70	87,522.7	88,225.8	88,372.7	**88,397.8**
3.85	87,322.3	88,083.4	88,248.5	**88,287.4**
4.00	87,121.4	87,941.6	88,124.8	**88,177.5**

Notes: Bold values indicate maximum CE at the given RRAC.

a: Each strategy is coded according to the following format (strike, I_{min}, I_{max}). Please refer to equations 5.1, 5.2, and 5.3 for more explanation on the parameters of each contract.

Table 5.19 Risk Premium for Efficient Set of Hedging Strategies (with Respect to Base)

RRAC	Risk Management Strategies[a]		
	600-100-400	650-100-400	700-100-400
0.50	-320.0	-537.1	-818.5
0.64	-284.6	-487.1	-755.1
0.79	-248.2	-435.9	-690.4
0.93	-210.8	-383.4	-624.3
1.08	-172.2	-329.6	-556.8
1.22	-132.6	-274.6	-487.9
1.37	-91.9	-218.2	-417.7
1.52	-50.2	-160.6	-346.2
1.66	-7.3	-101.7	-273.3
1.81	**36.6**	-41.5	-199.2
1.95	**81.6**	20.0	-123.7
2.10	**127.7**	82.6	-46.9)
2.25	**174.8**	146.6	31.1
2.39	**223.0**	211.7	110.4
2.54	272.3	**278.1**	190.8
2.68	322.6	**345.6**	272.5
2.83	374.0	**414.4**	355.3
2.97	426.4	**484.2**	439.3
3.12	479.8	**555.2**	524.4
3.27	534.1	**627.3**	610.5
3.41	589.5	**700.5**	697.7
3.56	645.8	774.7	**785.9**
3.70	703.0	850.0	**875.0**
3.85	761.1	926.2	**965.1**
4.00	820.2	1,003.4	**1,056.1**

Note: Bold values indicate maximum risk premium at the given RRAC

a: Each strategy is coded according to the following format (strike, I_{min}, I_{max}). Please refer to equations 5.1, 5.2, and 5.3 for more explanation on the parameters of each contract.

Table 5.20: Sensitivity Analysis for Loading Factor for Strategy 12 (600-100-400)

Risk Measure	Base	Load Factors			
	-	1	1.5	2	2.5
BRRAC		0.00425	1.86	3.35	4.69
Premium (%)		1.4	2.2	2.9	3.6
VaR[a] (%)	13.9	10.6	11.3	12.2	12.9
CV (%)	15.9	14.5	14.6	14.6	14.7
CVaR[b,a]	9,385	7,451	7,929	8,553	9,024
Expected Annual Income[b]	92,338	92,338	91,905	91,473	91,040

Source: own computations
a: Evaluated at the 75,000 threshold.
b: Measured in hectare equivalent income.

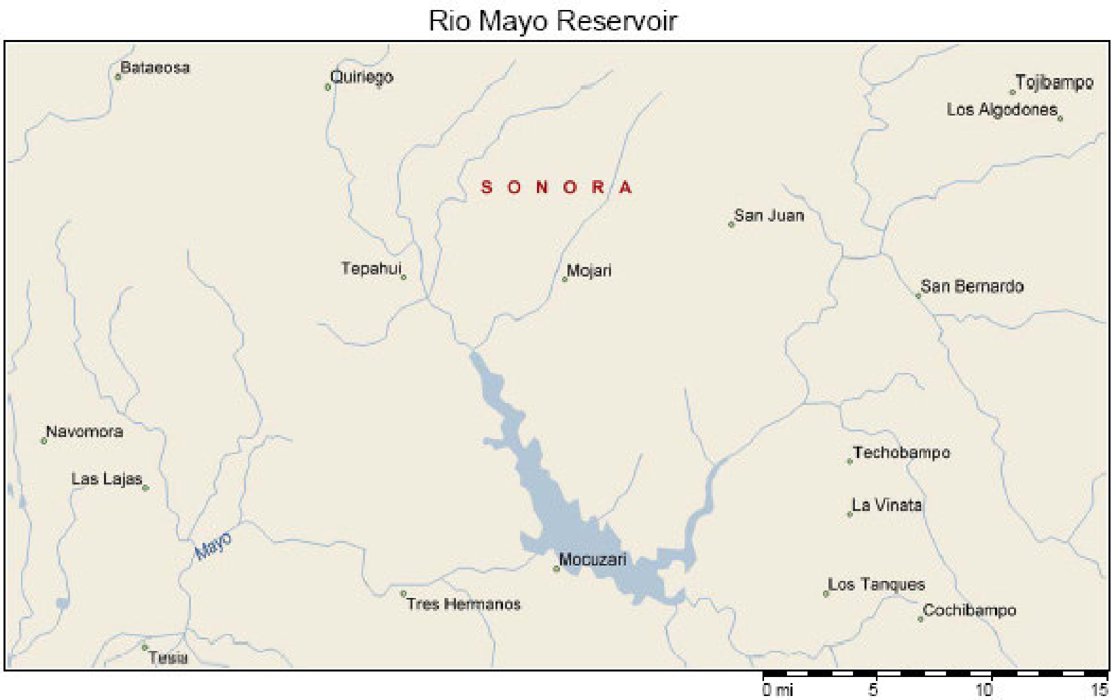

Figure 5.1: Adolfo Ruiz Cortinez Reservoir

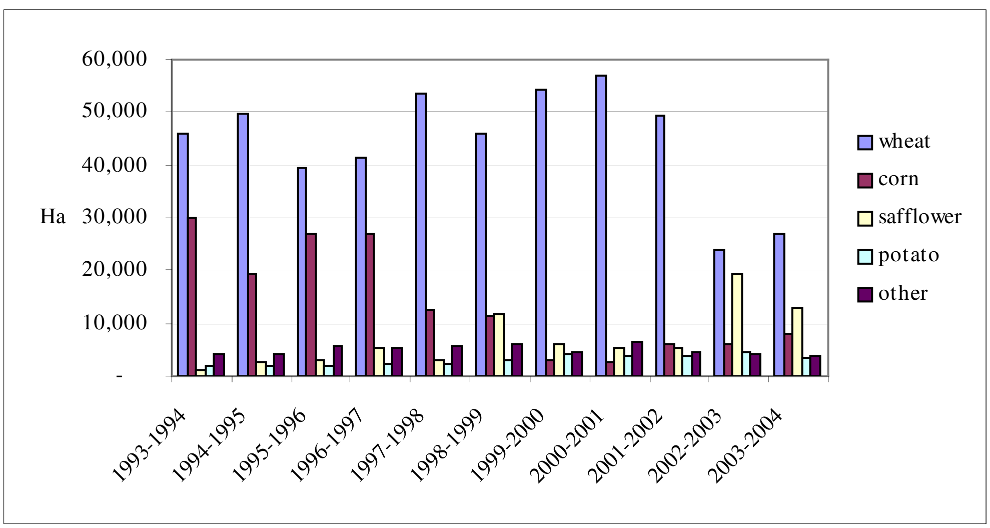

Figure 5.2: Rio Mayo Fall-Winter Cropping Patterns, 1994-2004

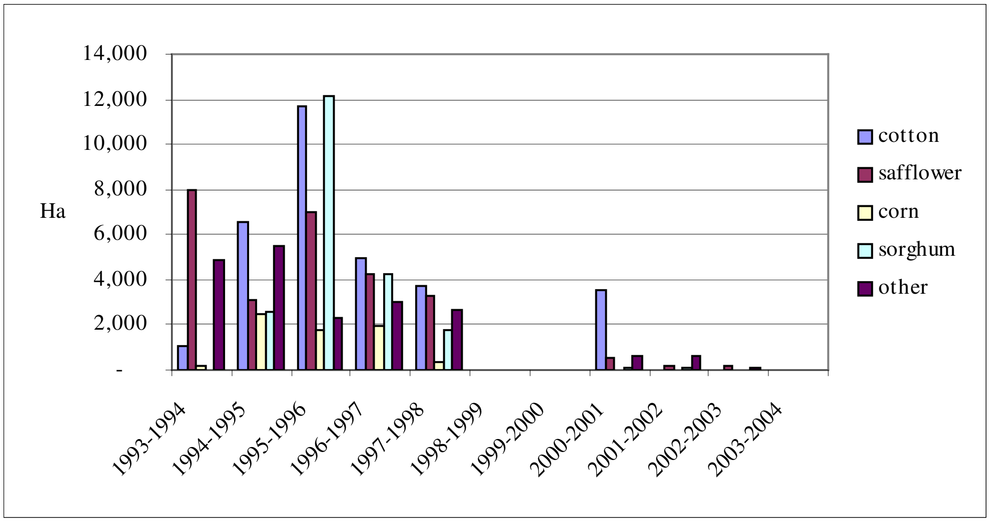

Figure 5.3: Rio Mayo Spring-Summer Cropping Patterns, 1994-2004

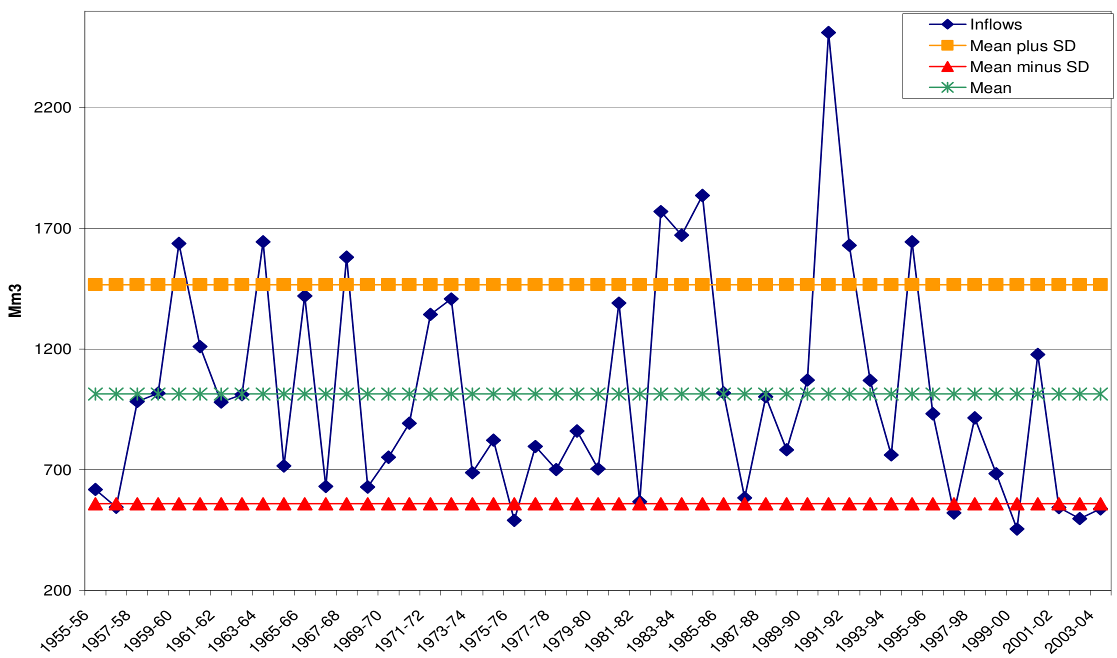

Figure 5.4: Inflows to the ARC Reservoir, 1955-2004

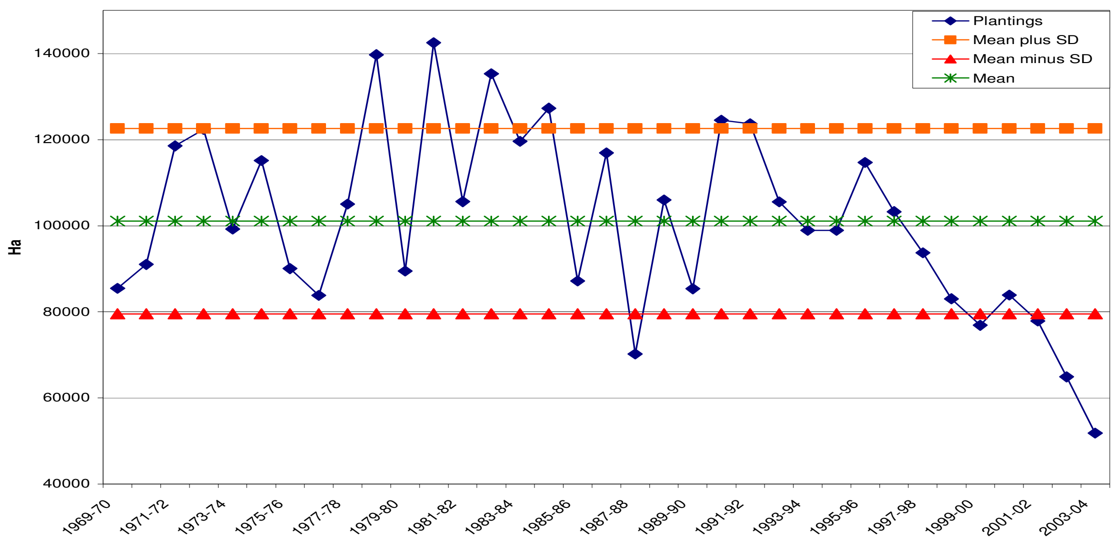

Figure 5.5: Total Annual Plantings in the Rio Mayo Valley, 1969-2004

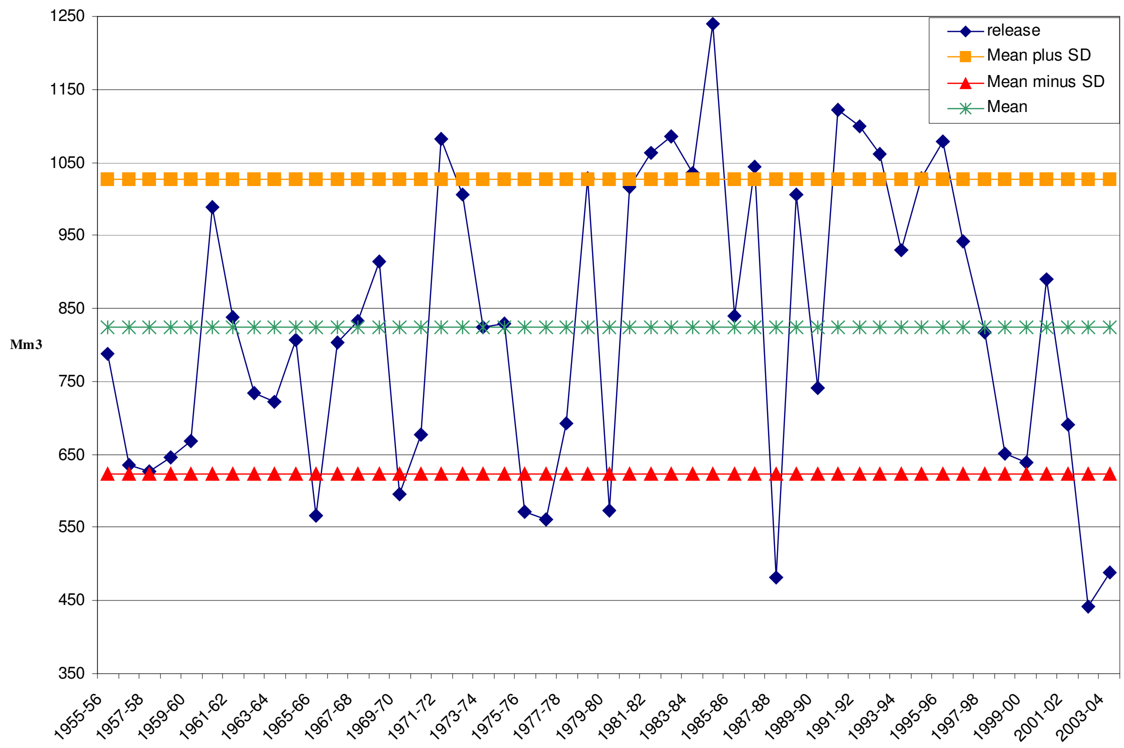

Figure 5.6: Releases for Agriculture from the ARC reservoir, 1955-2004

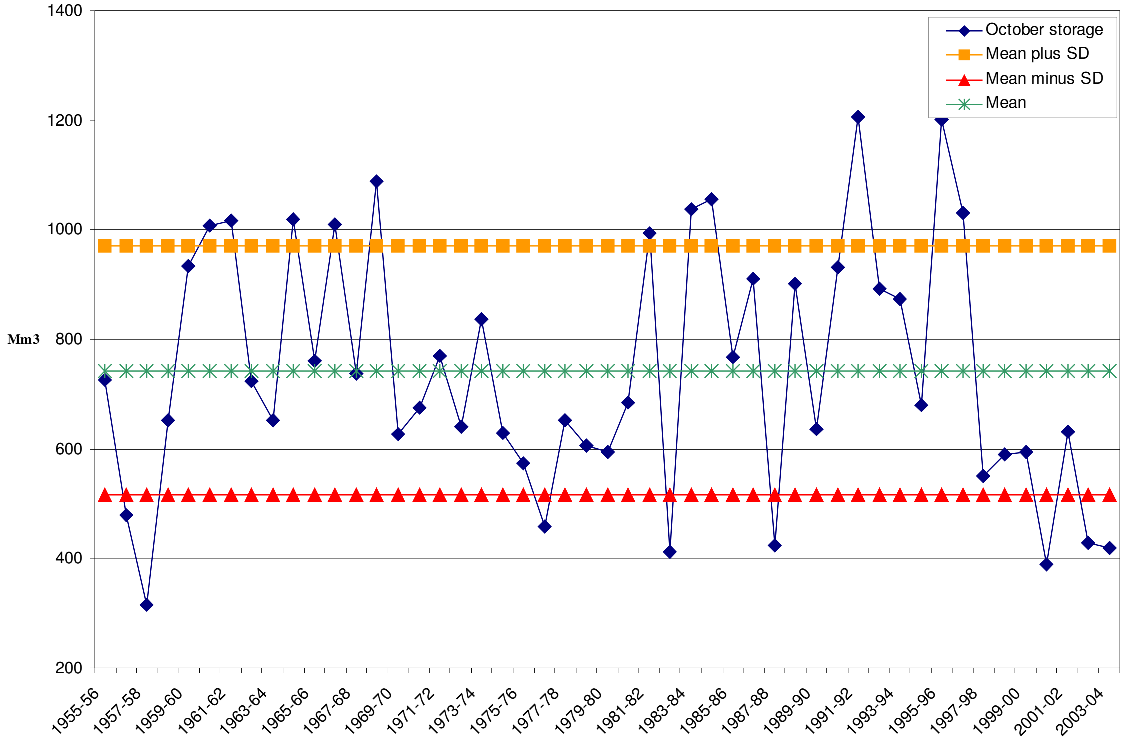

Figure 5.7: Storage Volumes as of October 1 in the ARC Reservoir, 1955-2004

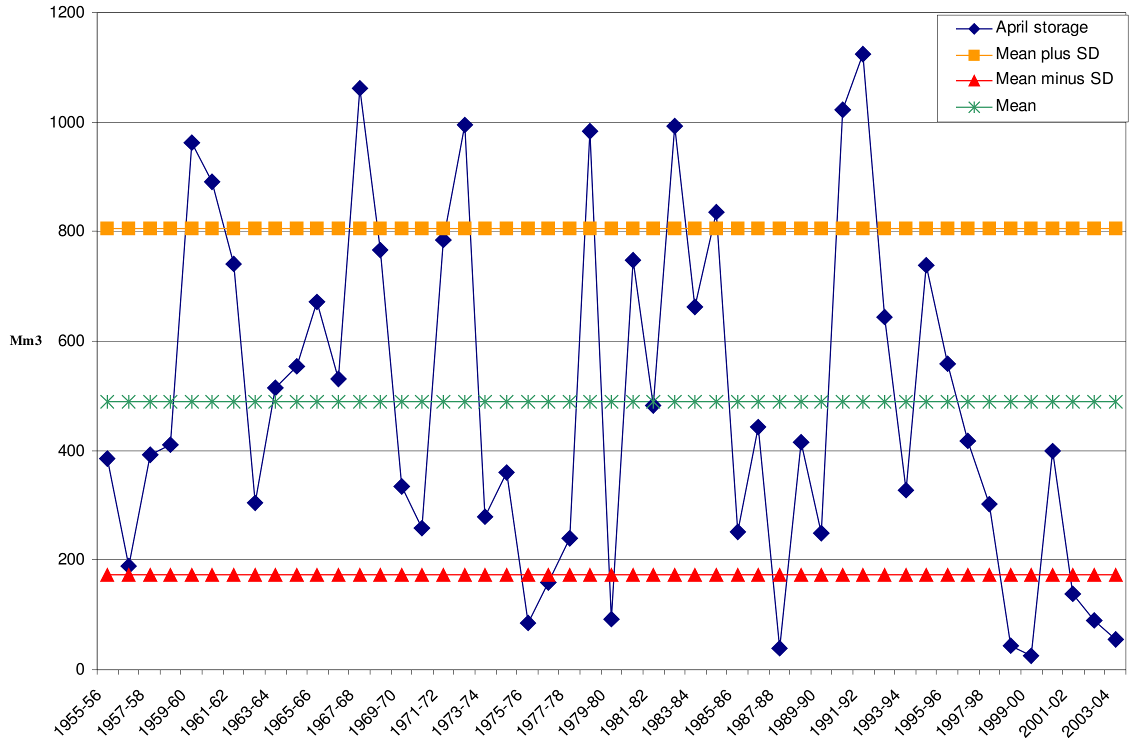

Figure 5.8: Storage Volumes as of April 1 in the ARC Reservoir, 1955-2004

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Box-Pierce Statistic =     8.9038      Box-Ljung Statistic  =    9.7904
Degrees of freedom   =         10      Degrees of freedom   =        10
Significance level   =      .5413      Significance level   =     .4591
* => |coefficient| > 2/sqrt(N) or > 95% significant.
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Lag |  Autocorrelation Function     |Box/Prc|  Partial Autocorrelations
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
  1 | .360*|           |****        |  6.36*| .360*|            |****
  2 | .094 |           |*           |  6.80*| .063 |            |*
  3 |-.026 |          *|            |  6.83 |-.097 |          * |
  4 | .110 |           |*           |  7.42 | .226 |            |**
  5 |-.029 |          *|            |  7.47 |-.131 |          * |
  6 | .063 |           |*           |  7.66 | .175 |            |**
  7 | .069 |           |*           |  7.89 | .121 |            |*
  8 |-.047 |          *|            |  8.00 |-.220 |         ** |
  9 |-.131 |          *|            |  8.84 |-.125 |          * |
 10 | .035 |           |*           |  8.90 | .209 |            |**
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Figure 5.9. Correlogram for Inflows Times Series

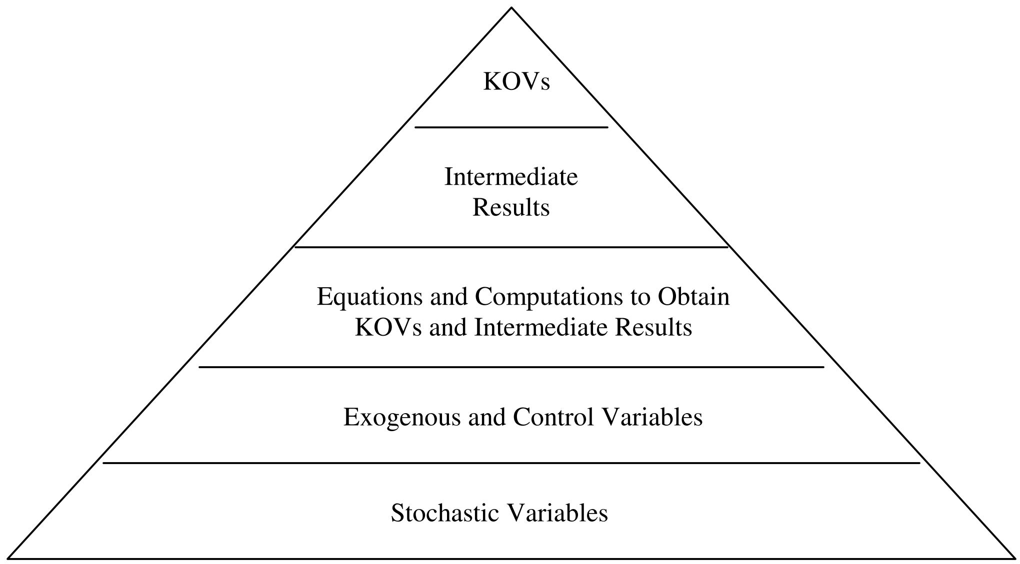

Figure 5.10 Stochastic Modeling Pyramid

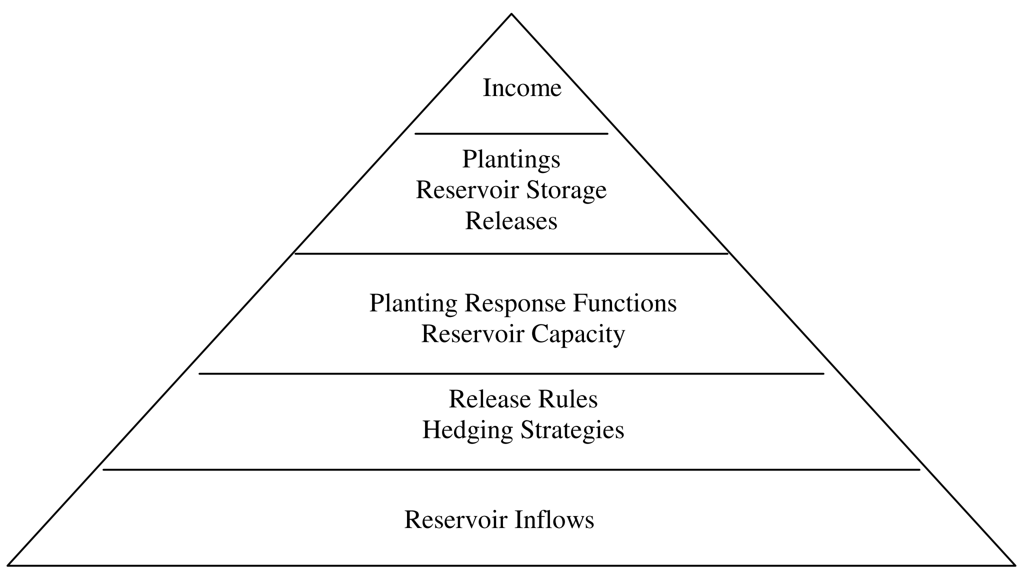

Figure 5.11 Rio Mayo Model Pyramid

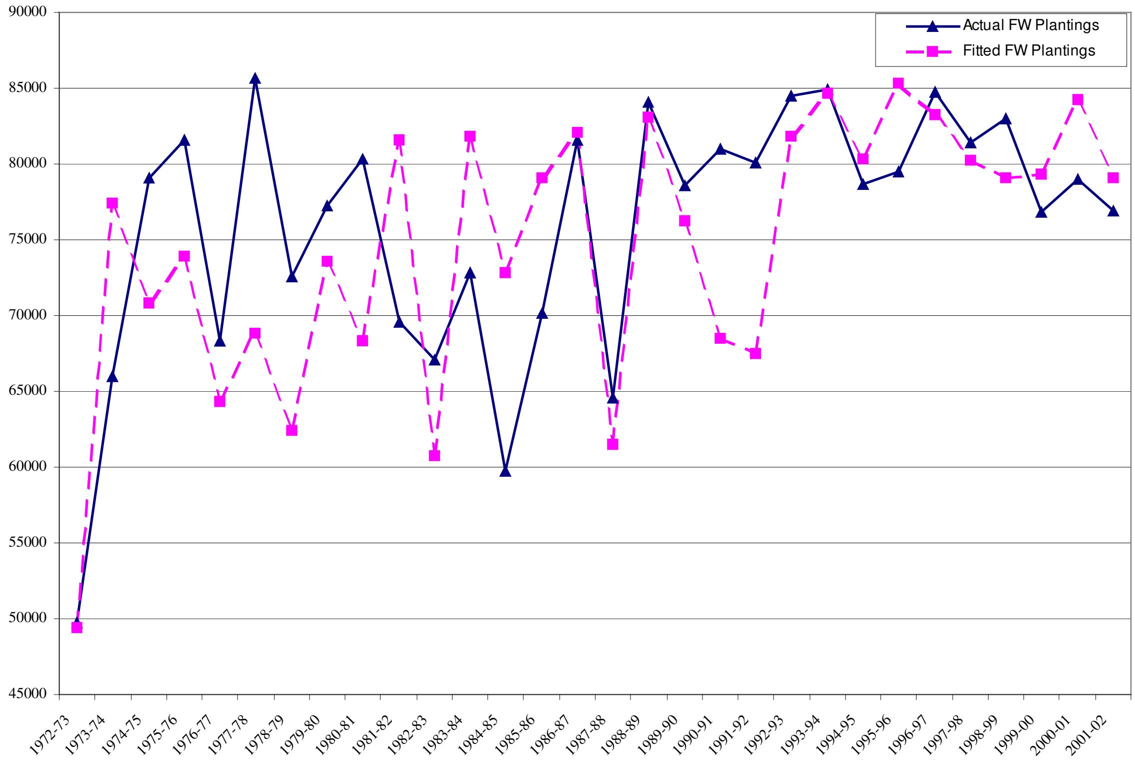

Figure 5.12: Actual vs. Fitted FW Plantings, 1973-2002

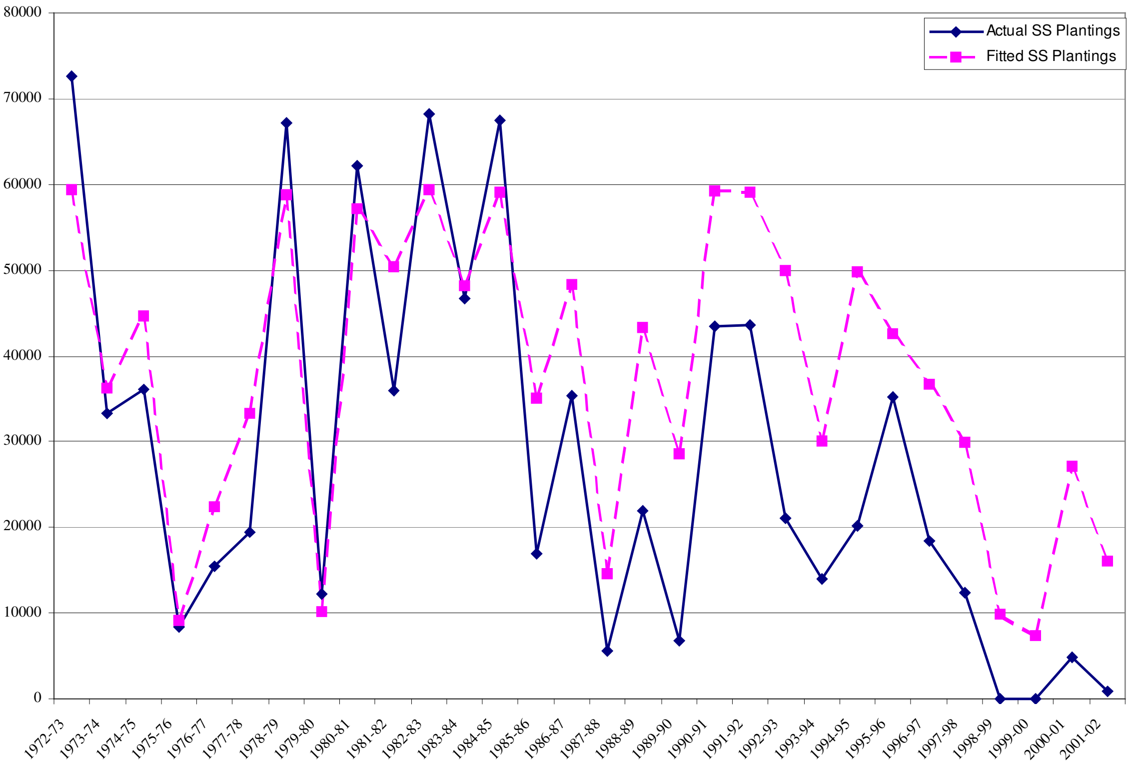

Figure 5.13. Actual Vs. Fitted SS plantings, 1973-2002

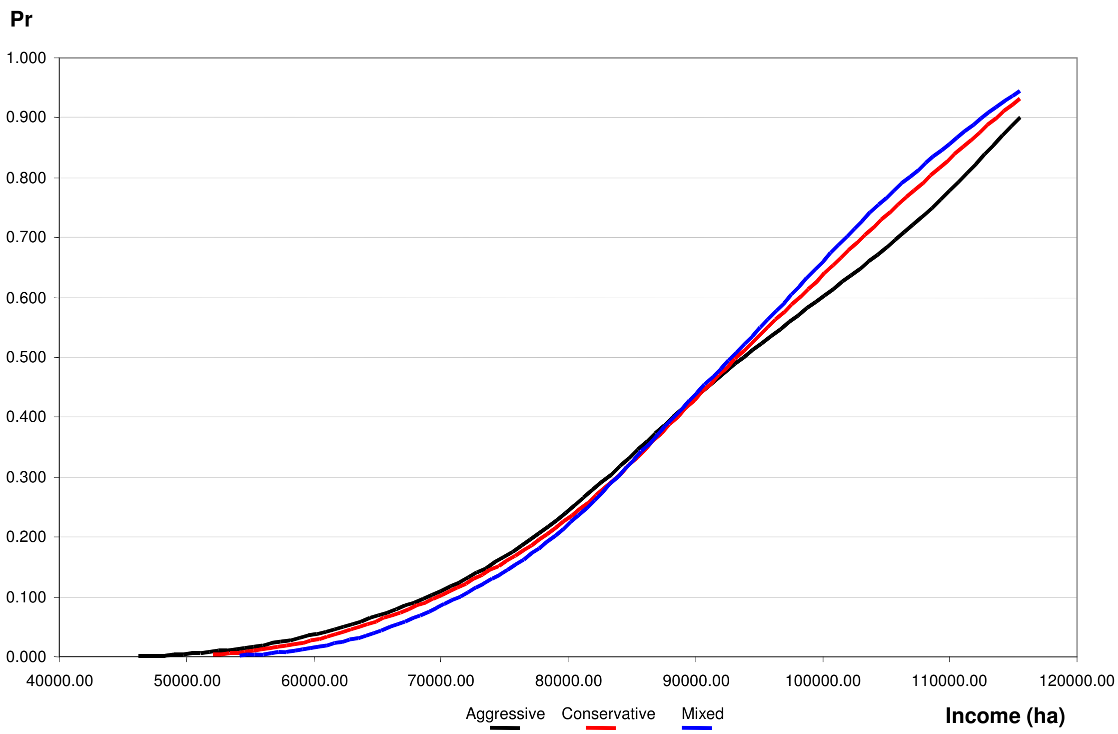

Figure 5.14 Simulated Empirical Distribution of Income

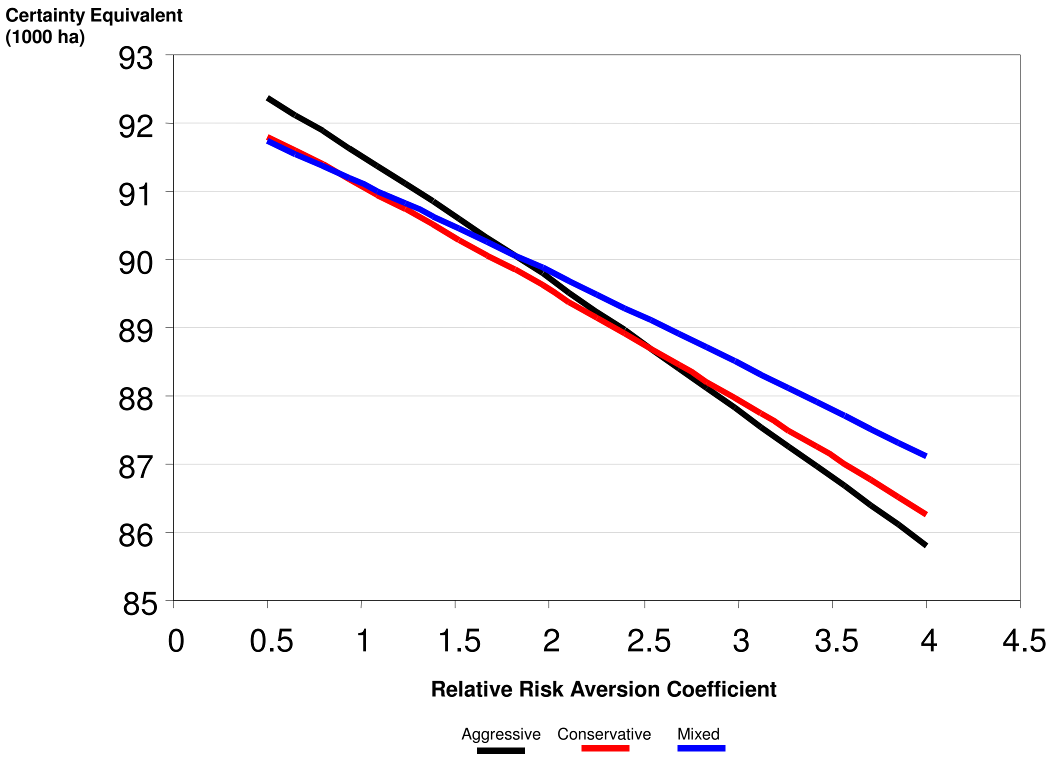

Figure 5.15 Certainty Equivalents Under Alternative Release Rules

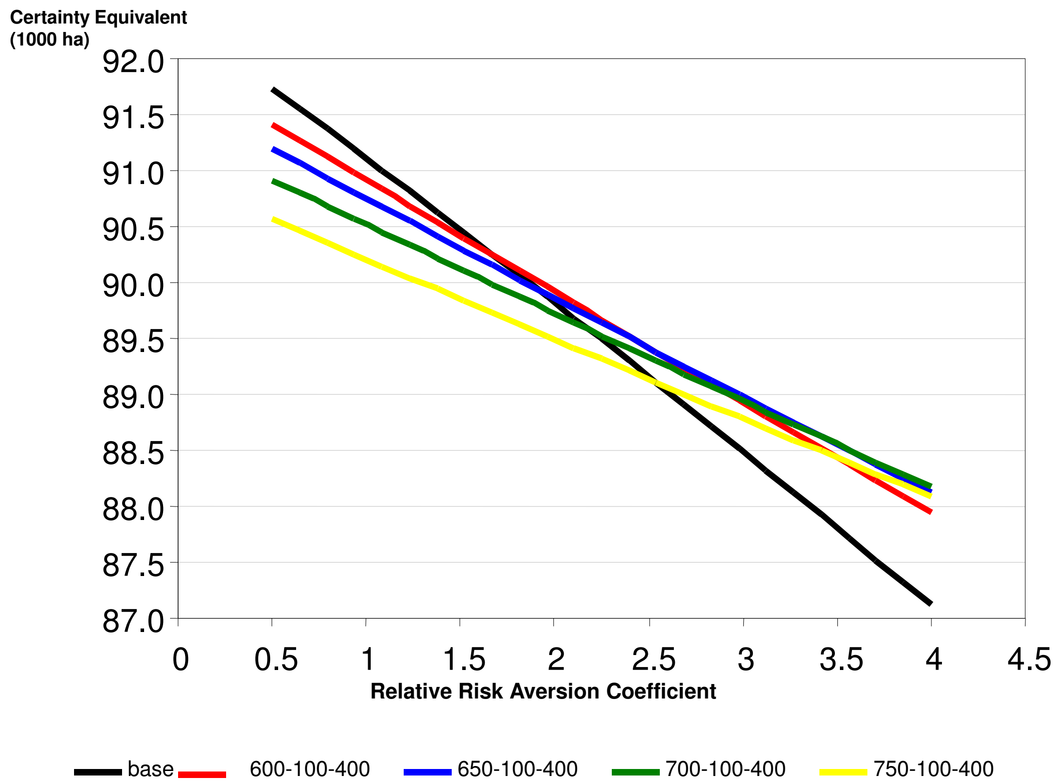

Figure 5.16 Efficient Set of Hedging Strategies under SERF

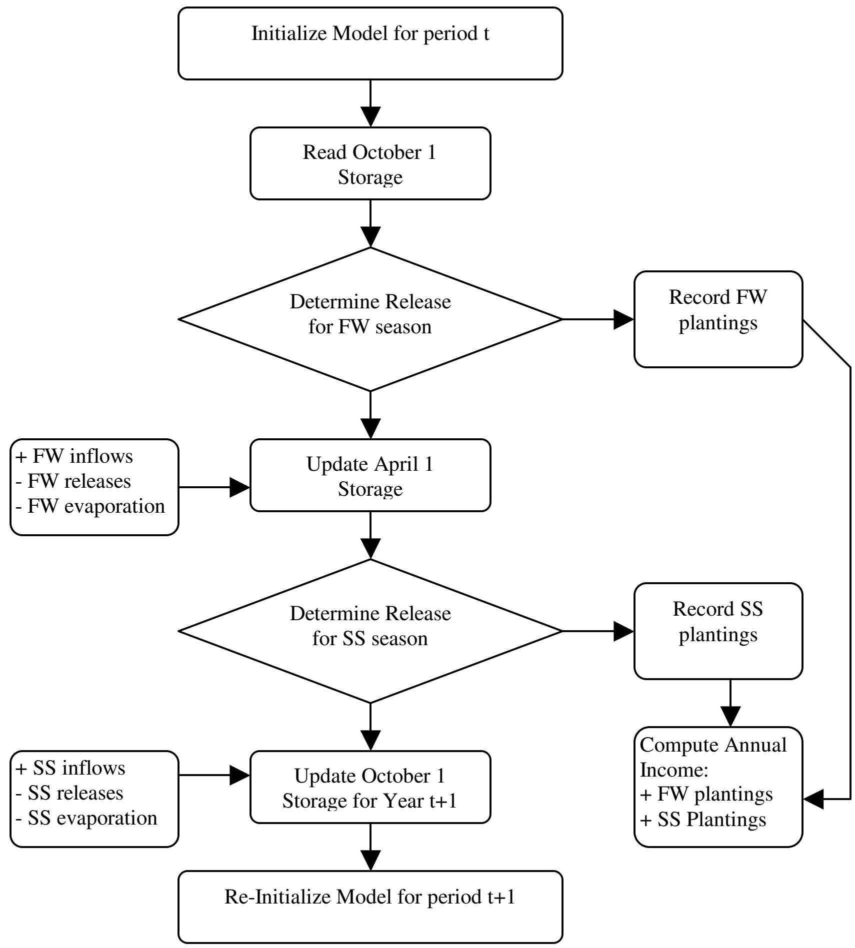

Figure 5.17 The Reservoir Operation Model

Chapter 6: Conclusion

6.1 Introduction

This dissertation focuses on uncertain irrigation supplies first at the firm level using comparative statics, and then at the aggregate level of an irrigation district using a dynamic-stochastic simulation model in the case of the Rio Mayo in Mexico. In the theoretical section, I explore the roles of output diversification and contingent water transfer for individual decision makers. In the empirical work, I study the design and effectiveness of an inflow-based derivative that is designed to transfer the risk of water shortfalls out of the irrigation district. While the theoretic model gives irrigators the possibility to self-insure by their choice of output and studies the intervention of a social planner in managing this type of risk, the empirical component studies how the stochastic profile of irrigators' wealth changes when a hedging instrument is used. By using risk aversion assumptions for the collective group of farmers within an irrigation district, the willingness to pay for a risk transfer products is also empirically investigated.

Expected utility theory predicts that when irrigators have the ability to self-insure (as in Chapter 3) the demand for risk-hedging instruments (as in Chapter 5) is reduced. However, the reduction depends on the degree of risk aversion of the irrigator. Since including all these aspects of decision making in a single model results in high complexities with little explanatory power, each component of the problem was examined separately.

6.2 Conclusions from Theoretical Strategies

In the theoretical model, I analyze the economic effects of input supply uncertainty and design a government intervention program to reduce such uncertainty. The results are consistent with portfolio theory in the sense that when the production possibilities include two crops that are different in their dependency on the uncertain resource, the firm manages the input supply risk by diversifying the crop portfolio. Specifically, for greater degrees of risk, the firm responds by increasing the allocation of resources to the crop that depends less on the uncertain factor. Moreover, the results show that diminishing marginal returns to the factors of production, besides risk aversion, explain why the firm engages in crop diversification. Thus, the theoretic model presented in Chapter 3 illustrates that the over-ordering effect expressed in previous research may not be as strong when firms have the ability to hedge against uncertain input supply by diversifying production. The policy implications for this theoretical result is that policies that encourage crop diversification potentially reduce the dependence on the uncertain input in irrigated agriculture, and thereby help mitigate the risk exposure of firms. Furthermore, diversification can also be viewed in the allocation of water across cropping seasons within an agricultural year (i.e. FW and SS) as they also represent different water productivities and returns.

As it was shown in Chapter 3, irrigators will respond to an increase in the probability of operating under "high water" states of nature by shifting land towards the more profitable crops. In terms of policy implications, the government may not have to directly intervene in the provision of the water service to affect this probability. For instance, policies that foster investments to improve and maintain the water conveyance system, like canal lining, should have potential returns by decreasing the risk of water shortages. When seepage and evaporation losses are reduced, irrigators will see their water endowments "augmented" and their chances of operating in the "high water" state of nature increased, thereby providing incentives to move towards more water-intensive and profitable crop portfolios.

In terms of risk-sharing effectiveness, although crop diversification provides some cushion against the adverse impact of input supply risk, it does not guarantee full protection. Indeed, diversification comes at the cost of reduced profits. This failure to fully protect against input supply risk motivates the potential role for government intervention to stabilize the supply of the uncertain input. This dissertation proposes a theoretical contingent water transfer scheme in which a social planner saves a proportion of the water supply during the "high" state of nature and releases the same proportion during the "low" state. Since the expected marginal value of water is higher during the "low" state, then the firm benefits by transferring a proportion of the lower valued realization in the "high" towards the "low" state. In the absence of transfer costs, storage and physical losses, the proposed intervention scheme guarantees an improved risk-sharing solution.

6.3 Conclusions from Empirical Strategies

In the empirical sections, the risky environment of the Rio Mayo irrigation district was studied by developing a stochastic dynamic simulation model. The main risk management strategy studied in Chapters 4 and 5 is a derivative-type instrument to hedge against the production losses imposed by water shortages. While irrigation districts can design reservoir operation policies that suit their tolerance for water shortage risk, the outcome from these polices seldom provides adequate protection against periods of low inflows. Therefore, there is a case for studying the implementation of a risk-sharing mechanism that transfers some of the risk outside of the irrigation district.

To study the impact of the inflow-based derivative, MVE distributions were fitted from historical inflows data to reproduce the history in the irrigation district in 1000 years. Using plantings response functions and several sets of releases rules (i.e. reservoir operation policies), the distribution of an indicator of annual income (i.e. hectare equivalents) was estimated using an empirical procedure. In this framework, several designs of the inflow-based derivative were developed and evaluated. The results indicate that for the particular case of Rio Mayo irrigation district and the ARC reservoir, the best design is a double-triggered contract that discounts payments for occurrence of “bonus” inflows in each FW season. For instance, referring to Table 5.17, by using the simple contract in strategy 3 (650-00-00) or the complex contract in strategy 12 (600-100-400) the SRL reduces the probability of income falling below a 75,000 ha threshold to 11.3% (versus 13.9% without hedging), and reduces the expected reduction beyond the same threshold to approximately 7,900 ha (versus 9, 385 ha without hedging); however, the SRL would have to pay a 6.5% insurance premium (i.e. 4,225 ha) for strategy 3, which is more expensive than the 2.2% insurance premium (i.e. 1,298 ha) required to hedge with strategy 12.

Moreover, using SERF an efficient set of empirical strategies was developed over a given range of RACs. The importance of the efficient set of strategies is that it is possible to compare how the risk aversion influences the SRL’s choice of strategies. For Rio Mayo the efficient set comprises the following strategies (Please refer to Table 5.18): no hedging, for low risk aversion; strategy 12 for normal risk aversion; strategy 13 for higher risk aversion; and strategy 14 for extreme levels of risk aversion. Although the exact degree of risk aversion of the SRL is unknown, the importance of the SERF analysis is that it reduces the number of risk management strategies to a small number and identifies each strategy to the corresponding risk tolerance; therefore, the exercise provides policy makers and stakeholders a first approximation about the most feasible options and the cost and benefits of implementing them.

Also, from the sensitivity of loading factors in Chapter 5 it is possible to conclude that for this instrument to be feasible from the demand side, the loading factor needs to be restricted in the 50-100% range. If the competition level among insurance suppliers is high, the actual loading factor will be pushed down to a level closer to the actuarially fair rate of 50%; on the contrary, if there only a few insurance suppliers, the lack of competition may result in a loading rate of 100%.

6.4 Future Research and Limitations

Further extensions of the theoretical strategies may include evaluating the effectiveness of the contingent water transfer scheme allowing for transfer costs and physical losses during storage. While the absence of those costs render a result in which socially optimal transfers guarantees results equivalent to the case of complete certainty, the relaxation of this assumption will render a more realistic transfer level that leaves some degree of uncertainty in the system. In addition, other types of contingent contracts could be explored. For instance, instead of transferring the uncertain input across states of nature, the government could transfer profits or revenues. That is, the government could use a tax system that refunds the amount of profit collected (during the "high" realizations of water) when the "low" state of nature occurs. Moreover, to the extent that the risk is not positively correlated, extending the analysis so that multiple firms mutually engage in contingent transfers would yield a number of important insights.

Another limitation of the theoretical strategies is the fact that decisions are based on the assumption of risk neutrality. Despite that assumption, the model yields very important insights about the role of output diversification in hedging against water supply risk. Basically, the concavity of the production functions are exploited to generate an environment in which the firm finds it optimal to plant less of the crops that are heavily dependent on water and yield a higher economic return. Thus, the model features the tradeoff between higher mean income and increased variance. More importantly, the firm responds to increased levels of risk by diversifying more its portfolio, which is the rational behavior expected.

In the empirical side, inflow-based derivative was based on the idea that the SRL as a collective buys the contract from an insurance agent. This assumption was made to avoid the more complex scenario in which irrigators purchase their contracts individually. However, the fact that the SRL could purchase the contract to protect its members is a viable and plausible application of the contract. In the visit to Rio Mayo it was observed that farmers are strongly adhered to the SRL and they trust in the collective agreements that the organization achieves. For instance, after CNA operators deliver water to the main canals, the SRL is the only organization responsible for distributing water to all the farmers. This is a great responsibility in the district due to the conflicts that arise among irrigators within an irrigation module or even among irrigations modules. It was my observation that farmers trust the decisions of the SRL as a collective group and that the SRL has been very effective in solving conflicts arising from water usage. Another instance in which the SRL has proved to be effective in collecting resources from farmers, and in executing activities that serve the common good, is the improved recovery of O&M costs. At the beginning of each agricultural cycle, and based on the availability of water, the SRL collects payments from individuals farmers to finance the drainage and operation of the canals that supply all farmers with water. If these canals are left unattended silting deteriorates their quality and, consequently, evaporation and seepage losses problems increase for all farmers.

Based on those observations, the simplified decision making that the empirical model is based upon is justified. The SRL could be used as an intermediary between the irrigators and the insurance company that sells the contract. In this sense, the role of the SRL would be to collect the premium from the individual contribution of its members and distribute the indemnities accordingly. Since the SRL is a group of the same farmers, it already possesses informational advantages in terms of the impact of water shortages on each individual. For instance, they know that farms that are located closer see their supply see water curtailed at a lower proportion than marginal farms located in the most distant areas from the canal. The SRL could implement rules that adjust the relative size of the premium to be paid or the indemnities to be received according to the distance between irrigation modules and distribution canals.

However, it is also plausible that contracts could be designed at a more disaggregated level. In the visit to Rio Mayo, some representatives of irrigation modules suggested that the module approach might be desirable from the point of view of their members. Additional research regarding the impact of inflow-based derivatives on water markets is recommended. In principle, when farmers can substitute water for indemnity payments as presented in chapter 4, the incentives to use water might change. For instance, if the module approach is implemented, irrigation modules located at a closer distance to the irrigation canals and enjoying higher rates of water delivery efficiency might be willing to use their indemnity payments in dry years to purchase or lease the water rights of those modules located farther from the canals. Due to the different efficiencies of water delivery, these trades might occur among modules located very distant (less efficient) from the main canals and those located a shorter distances (more efficient). The indemnity payments would introduce more liquidity into the water market and perhaps more water trade could occur. It is suggested that more research should be conducted about the impact of the inflow-based derivative of water markets in the Rio Mayo.

The empirical model also has some limitations with respect to the data. For instance, the model does not include measures of the shadow price of water in the irrigation district. Without reliable data on crop prices and costs of production it is difficult to reach conclusions about the economic return to irrigated agriculture in the Mayo Valley. In addition, the O&M costs were not included in the model due to lack of availability. Updating the study with this information should provide more reliable results than those outlined in chapter 5.

Appendix A

A. Proof of proposition 1

Following the FOC in equation 5, assuming both crops are equally intensive in land use ($\alpha_1 = \alpha_2 = \alpha$) and considering that the water-saving crop is completely independent of water ($\beta_1 = 0$), the FOC are given by:

$$(xL)^{\alpha-1} = pw^{\beta_2}\left((1-x)L\right)^{\alpha-1} \quad \text{(1A)}$$

Simplifying the expression leads to

$$x^{\alpha-1} = pw^{\beta_2}(1-x)^{\alpha-1} \quad \text{(2A)}$$

Raising both sides of 2A to the $\frac{1}{\alpha-1}$ power leads to

$$x = \left(pw^{\beta_2}\right)^{\frac{1}{\alpha-1}}(1-x) \quad \text{(3A)}$$

Expanding the right hand side of 3A and grouping factors involving x in the left hand side leads to

$$x + x\left(pw^{\beta_2}\right)^{\frac{1}{\alpha-1}} = \left(pw^{\beta_2}\right)^{\frac{1}{\alpha-1}} \quad \text{(4A)}$$

Factoring the left hand side of 4A leads to

$$x\left[1+\left(pw^{\beta_2}\right)^{\frac{1}{\alpha-1}}\right] = \left(pw^{\beta_2}\right)^{\frac{1}{\alpha-1}} \quad \text{(5A)}$$

Solving for x in 5A leads to

$$x = \frac{\left(pw^{\beta_2}\right)^{\frac{1}{\alpha-1}}}{1+\left(pw^{\beta_2}\right)^{\frac{1}{\alpha-1}}} \quad \text{(6A)}$$

Dividing the numerator and the denominator of equation 5A by $\left(pw^{\beta_2}\right)^{\frac{1}{\alpha-1}}$ leads to

$$x = \frac{1}{1 + \left(pw^{\beta_2}\right)^{\frac{1}{1-\alpha}}} \quad (7A)$$

B. Variance is increasing in ϕ

From equation 8, the partial derivative with respect to ϕ is

$$\frac{\partial Var[\tilde{w}]}{\partial \phi} = \frac{w^2\left\{\left[q\phi^2 + (1-q)(1-\phi)^2\right] - \left[q\phi + (1-q)(1-\phi)\right]^2\right\}}{\partial \phi} \quad (8.1A)$$

$$\frac{\partial Var[\tilde{w}]}{\partial \phi} = w^2\{[2q\phi - 2(1-q)(1-\phi)] - 2[q\phi + (1-q)(1-\phi)](2q-1)\} \quad (8.2A)$$

In order to simplify the proof and be consistent with the assumption about a mean-preserving spread, suppose $q = \frac{1}{2}$

$$\frac{\partial Var[\tilde{w}]}{\partial \phi} = w^2\left\{\left[2\frac{1}{2}\phi - 2\left(1 - \frac{1}{2}\right)(1-\phi)\right] - 2[q\phi + (1-q)(1-\phi)](1-1)\right\} \quad (9A)$$

which reduces to

$$\frac{\partial Var[\tilde{w}]}{\partial \phi} = w^2\{\phi - (1-\phi)\} \quad (10A)$$

$$\frac{\partial Var[\tilde{w}]}{\partial \phi} = w^2\{2\phi - 1\} \quad (11A)$$

Taking into account that in the distribution function the “high” state of nature, denoted w_h, results from restricting the ϕ proportionality factor to values greater than 0.5 proves that the expression in equation 11A is greater than zero, and therefore the variance is increasing in the ϕ proportionality factor.

C. Proof of proposition 2

Imposing the condition that both crops are equally intensive in land use, or $\alpha_1 = \alpha_2 = \alpha$; and considering the case in which the water-saving crop is completely independent of water $\beta_1 = 0$, the FOC reported in equation 10 become:

$$(xL)^{\alpha-1} = p((1-x)L)^{\alpha-1}\left\{w^\beta\left[(1-q)(1-\phi)^\beta + q\phi^\beta\right]\right\} \quad (12A)$$

Simplifying the expression leads to

$$x^{\alpha-1} = p(1-x)^{\alpha-1}\left\{w^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\} \tag{13A}$$

Raising both sides of 13A to the $\frac{1}{\alpha-1}$ power leads to

$$x = (1-x)\left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}} \tag{14A}$$

Simplifying the expression leads to

$$x + x\left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}} = \left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}} \tag{15.1A}$$

$$x\left[1 + \left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}}\right] = \left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}} \tag{15.2A}$$

Solving equation 15.2A for x leads to

$$x = \frac{\left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}}}{1 + \left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}}} \tag{16.1A}$$

Dividing the numerator and denominator in right hand side of equation 15A by

$\left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}}$ leads to

$$x^{**} = \frac{1}{1 + \left\{pw^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}} \tag{16.2A}$$

D. Proof of Proposition 4

First, let

$$u = 1 + p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[\left(1-q(1-\phi)^{\beta} + q\phi^{\beta}\right)\right]\right\}^{\frac{1}{1-\alpha}} \tag{17A}$$

Such that

$$x^{**} = \frac{1}{u} \tag{18A}$$

Then,

$$\frac{\partial x^{**}}{\partial \phi} = \frac{\partial x}{\partial u}\frac{\partial u}{\partial \phi} = -\frac{1}{u^2}\frac{\partial u}{\partial \phi} = -\frac{1}{\left(1 + p^{\frac{1}{1-\alpha}}\left\{w^\beta\left[(1-\phi)^\beta(1-q) + q\phi^\beta\right]\right\}^{\frac{1}{1-\alpha}}\right)^2}\frac{\partial u}{\partial \phi} \tag{19A}$$

Using equation 17A, the partial derivative $\frac{\partial u}{\partial \phi}$ is computed as follows:

$$\frac{\partial u}{\partial \phi} = p^{\frac{1}{1-\alpha}}\frac{\partial\left\{w^\beta\left[(1-\phi)^\beta(1-q) + q\phi^\beta\right]\right\}^{\frac{1}{1-\alpha}}}{\partial \phi} \tag{20.1A}$$

Computing the partial derivative on the right hand side of yields 20.1A yields

$$\frac{\partial u}{\partial \phi} == \frac{p^{\frac{1}{1-\alpha}}}{1-\alpha}\left\{w^\beta\left[(1-\phi)^\beta(1-q) + q\phi^\beta\right]\right\}^{\frac{\alpha}{1-\alpha}} w^\beta \frac{\partial\left\{w^\beta\left[(1-\phi)^\beta(1-q) + q\phi^\beta\right]\right\}}{\partial \phi} \tag{20.2A}$$

Computing the partial derivative in the right hand side of 20.2A yields

$$\frac{\partial u}{\partial \phi} == \frac{p^{\frac{1}{1-\alpha}}}{1-\alpha}\left\{w^\beta\left[(1-\phi)^\beta(1-q) + q\phi^\beta\right]\right\}^{\frac{\alpha}{1-\alpha}} w^\beta \beta\left[q\phi^{\beta-1} - (1-q)(1-\phi)^{\beta-1}\right] \tag{20.3A}$$

Substituting equation 20.3A into equation 19A yields

$$\frac{\partial x^{**}}{\partial \phi} = -\frac{\beta w^\beta p^{\frac{1}{1-\alpha}}\left\{w^\beta\left[(1-\phi)^\beta(1-q) + q\phi^\beta\right]\right\}^{\frac{\alpha}{1-\alpha}}\left[q\phi^{\beta-1} - (1-q)(1-\phi)^{\beta-1}\right]}{(1-\alpha)\left(1 + p^{\frac{1}{1-\alpha}}\left\{w^\beta\left[(1-\phi)^\beta(1-q) + q\phi^\beta\right]\right\}^{\frac{1}{1-\alpha}}\right)^2} \tag{21A}$$

The sign of the expression in equation 21A depends on the sign of the expression $\left[q\phi^{\beta-1} - (1-q)(1-\phi)^{\beta-1}\right]$. The proof relies on the assumption of a mean-preserving spread in the distribution of water, which occurs when the parameter $q = 1/2$. Using that restriction leads to:

$$\left[\frac{1}{2}\phi^{\beta-1} - \left(1 - \frac{1}{2}\right)(1-\phi)^{\beta-1}\right] = \frac{1}{2}\left[\phi^{\beta-1} - (1-\phi)^{\beta-1}\right] \tag{22A}$$

After imposing the mean-preserving spread, the sign of $\frac{\partial x^{**}}{\partial \phi}$ depends on the sign of the expression $\left[\phi^{\beta-1}-(1-\phi)^{\beta-1}\right]$. Particularly, $\left[\phi^{\beta-1}-(1-\phi)^{\beta-1}\right]<0 \Rightarrow \frac{\partial x^{**}}{\partial \phi}>0$. Therefore, values of ϕ for which $\left[\phi^{\beta-1}-(1-\phi)^{\beta-1}\right]<0$, implicitly represent the conditions under which the sign of $\frac{\partial x^{**}}{\partial \phi}$ in equation 21A is positive. Therefore, the proof requires the following:

$$\left[\phi^{\beta-1}\right]<\left[(1-\phi)^{\beta-1}\right] \tag{23A}$$

The proof proceeds by squaring raising both sides of the inequality in 23.A to the $\frac{1}{\beta-1}$ power as follows:

$$\left[\phi^{\beta-1}\right]^{\frac{1}{\beta-1}}>\left[(1-\phi)^{\beta-1}\right]^{\frac{1}{\beta-1}} \tag{24A}$$

Simplifying the inequality leads to

$$\phi>(1-\phi)\Rightarrow 2\phi>1\Rightarrow \phi>\frac{1}{2} \tag{25A}$$

Therefore, as long as volume of water received during the "high" state of nature, w_h, is greater than the volume received in the "low" state of nature, w_l, the sign of the derivative $\frac{\partial x^{**}}{\partial \phi}$ is positive.

E. Proof of Proposition 5

Taking into account equations 17A and 18A, then the partial derivative of x^{**} with respect to q is given by:

$$\frac{\partial x^{**}}{\partial q}=\frac{\partial x}{\partial u}\frac{\partial u}{\partial q}=-\frac{1}{u^2}\frac{\partial u}{\partial q}=-\frac{1}{\left(1+p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}\right)^2}\frac{\partial u}{\partial q} \tag{26A}$$

Referring to equation 17A, the partial derivative $\frac{\partial u}{\partial q}$ is given by:

$$\frac{\partial u}{\partial q}=p^{\frac{1}{1-\alpha}}\frac{\partial\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}}{\partial q} \tag{27.1A}$$

Computing the derivative in the right hand side of 27.1A leads to:

$$\frac{\partial u}{\partial q}=\frac{p^{\frac{1}{1-\alpha}}}{1-\alpha}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{\alpha}{1-\alpha}}w^{\beta}\frac{\partial\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}}{\partial q} \qquad (27.2A)$$

Computing the partial derivative in the right hand side of 27.2A yields

$$\frac{\partial u}{\partial q}=\frac{p^{\frac{1}{1-\alpha}}}{1-\alpha}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{\alpha}{1-\alpha}}w^{\beta}\left[\frac{\partial q\phi^{\beta}}{\partial q}+\frac{(1-q)(1-\phi)^{\beta}}{\partial q}\right] \qquad (27.3A)$$

Which leads to

$$\frac{\partial u}{\partial q}=\frac{p^{\frac{1}{1-\alpha}}}{1-\alpha}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{\alpha}{1-\alpha}}w^{\beta}\left[\phi^{\beta}-(1-\phi)^{\beta}\right] \qquad (27.4A)$$

Substituting equation 27.4A into equation 26A yields

$$\frac{\partial x^{**}}{\partial q}=-\frac{w^{\beta}p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{\alpha}{1-\alpha}}\left[\phi^{\beta}-(1-\phi)^{\beta}\right]}{(1-\alpha)\left(1+p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}\right)^{2}} \qquad (28A)$$

The sign of equation 28 depends only on the sign of the expression $\left[\phi^{\beta}-(1-\phi)^{\beta}\right]$. Particularly, $\left[\phi^{\beta}-(1-\phi)^{\beta}\right]>0\Rightarrow\frac{\partial x^{**}}{\partial q}<0$ Thus, the proof proceeds as follows:

$$\left[\phi^{\beta}\right]>\left[(1-\phi)^{\beta}\right] \qquad (29A)$$

Raising both sides of the inequality in 29A to the $\frac{1}{\beta}$ leads to

$$\left[\phi^{\beta}\right]^{\frac{1}{\beta}}>\left[(1-\phi)^{\beta}\right]^{\frac{1}{\beta}} \qquad (30A)$$

Simplifying the inequality leads to

$$\phi>(1-\phi)\Rightarrow 2\phi>1\Rightarrow\phi>\frac{1}{2} \qquad (31A)$$

Therefore, as long as volume of water received during the "high" state of nature, w_h, is greater than the volume received in the "low" state of nature, w_l, the sign of the derivative $\frac{\partial x^{**}}{\partial q}$ is negative.

F. Proof of proposition 6

In order to simplify the proof, the mean-preserving spread feature of the distribution function is imposed on equations 11 and 12. Hence, evaluated at $q = 1/2$, equation 11 is:

$$x^{**}\Big|_{q=\frac{1}{2}} = \frac{1}{1+\left\{pw^{\beta}\right\}^{\frac{1}{1-\alpha}}\left\{\frac{1}{2}\left[(1-\phi)^{\beta}+\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}} \tag{32A}$$

Equation 12 evaluated at $q = 1/2$ becomes

$$x^{*}\Big|_{w=E(\tilde{w}),\,q=1/2} = \frac{1}{1+\left\{pw^{\beta}\right\}^{\frac{1}{1-\alpha}}\left\{\left(\frac{1}{2}\right)^{\beta}\right\}^{\frac{1}{1-\alpha}}} \tag{33A}$$

Comparing equations 32A and 33A permits finding conditions under which $x^{**}\Big|_{q=1/2} > x^{*}\Big|_{w=E(\tilde{w}),\,q=1/2}$. The inequality holds if the denominator in equation 33A is larger than the denominator in equation 32A. Therefore, the proof proceeds as follows:

$$\left\{\left(\frac{1}{2}\right)^{\beta}\right\}^{\frac{1}{1-\alpha}} > \left\{\frac{1}{2}\left[(1-\varphi)^{\beta}+\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}} \tag{34.1A}$$

Raising both sides of the inequality in 34.1A to the $(1-\alpha)$ power leads to:

$$\left[\left\{\left(\frac{1}{2}\right)^{\beta}\right\}^{\frac{1}{1-\alpha}}\right]^{1-\alpha} > \left[\left\{\frac{1}{2}\left[(1-\varphi)^{\beta}+\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}\right]^{1-\alpha} \tag{34.2A}$$

which leads to

$$\left(\frac{1}{2}\right)^{\beta} > \frac{1}{2}\left[(1-\varphi)^{\beta}+\phi^{\beta}\right] \tag{34.3A}$$

or

$$2\left(\frac{1}{2}\right)^{\beta} > (1-\varphi)^{\beta} + \phi^{\beta} \tag{34.4A}$$

The only situation in which the inequality in 34.4A does not hold is when $\phi = \frac{1}{2}$ [33], which yields the maximum value of the expression $(1-\varphi)^{\beta} + \phi^{\beta}$, as proved by evaluating the expression in at $\phi = \frac{1}{2}$ as follows:

$$2\left(\frac{1}{2}\right)^{\beta} \equiv \left(\frac{1}{2}\right)^{\beta} + \left(\frac{1}{2}\right)^{\beta} \tag{34.5A}$$

Consequently, $x^{**} > x^{*}\big|_{w=E(\tilde{w})}$ when the distribution exhibits a mean-preserving spread, $q = \frac{1}{2}$, and when there is uncertainty in the model, that is $\phi \neq \frac{1}{2}$.

G. Proof that x^{**} is Decreasing β

Taking into account equations 17A and 18A, then the partial derivative of x^{**} with respect to β is given by:

$$\frac{\partial x^{**}}{\partial \beta} = \frac{\partial x}{\partial u}\frac{\partial u}{\partial \beta} = -\frac{1}{u^2}\frac{\partial u}{\partial \beta} = -\frac{1}{\left(1 + p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q) + q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}\right)^2}\frac{\partial u}{\partial \beta} \tag{35A}$$

Referring to equation 17A, the partial derivative $\frac{\partial u}{\partial \beta}$ is given by:

$$\frac{\partial u}{\partial \beta} = p^{\frac{1}{1-\alpha}}\frac{\partial\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q) + q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}}{\partial \beta} \tag{36.1A}$$

Computing the derivative in the right hand side of 36.1A leads to:

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[33] Consider the function, $f = (1-\phi)^{\beta} + \phi^{\beta}$, whose maximum is found when the partial derivative $f_{\phi} = \beta\left[\phi^{\beta-1} - (1-\phi)^{\beta-1}\right] = 0$. The partial derivative equals zero when evaluated at $\phi = 1/2$. Therefore, for values of $\phi \neq 1/2$, the value of the function f is less than $2\left(\frac{1}{2}\right)^{\beta}$.

$$\frac{\partial u}{\partial \beta} = \frac{p^{\frac{1}{1-\alpha}}}{1-\alpha}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{\alpha}{1-\alpha}} \frac{\partial\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}}{\partial \beta} \tag{36.2A}$$

Computing the partial derivative in the right hand side of 36.2A yields:

$$\frac{\partial\{\bullet\}}{\partial \beta} = w^{\beta}\left\{(1-q)(1-\phi)^{\beta}\left[\ln(w)+\ln(1-\phi)\right]+q\phi^{\beta}\left[\ln(w)+\ln(\phi)\right]\right\} \tag{36.3A}$$

Substituting 36.3A into 36.2A leads to

$$\begin{aligned}\frac{\partial u}{\partial \beta} = \frac{p^{\frac{1}{1-\alpha}}}{1-\alpha}&\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{\alpha}{1-\alpha}} \\ &\times w^{\beta}\left\{(1-q)(1-\phi)^{\beta}\left[\ln(w)+\ln(1-\phi)\right]+q\phi^{\beta}\left[\ln(w)+\ln(\phi)\right]\right\}\end{aligned} \tag{36.4A}$$

Substituting equation 36.4A into equation 35A yields

$$\begin{aligned}\frac{\partial x^{**}}{\partial \beta} = -&\frac{w^{\beta}p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)\left(1+p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}\right)^{2}} \\ &\times\left\{(1-q)(1-\phi)^{\beta}\left[\ln(w)+\ln(1-\phi)\right]+q\phi^{\beta}\left[\ln(w)+\ln(\phi)\right]\right\}\end{aligned} \tag{37A}$$

The sign of equation 37A depends upon the sign of the expression $\left\{(1-q)(1-\phi)^{\beta}\left[\ln(w)+\ln(1-\phi)\right]+q\phi^{\beta}\left[\ln(w)+\ln(\phi)\right]\right\}$. If the expression is positive, then the derivative $\frac{\partial x^{**}}{\partial \beta}$ is negative. In order to sign this expression, the mean-preserving spread characteristic of the distribution is imposed by evaluating the expression at $q = \frac{1}{2}$. The expression simplifies to:

$$\frac{1}{2}\left\{(1-\phi)^{\beta}\left[\ln(w)+\ln(1-\phi)\right]+\phi^{\beta}\left[\ln(w)+\ln(\phi)\right]\right\} \tag{38A}$$

The proof proceeds by finding conditions under which the expression $\left\{(1-\phi)^{\beta}\left[\ln(w)+\ln(1-\phi)\right]+\phi^{\beta}\left[\ln(w)+\ln(\phi)\right]\right\}$ is positive. The two conditions required to ensure that the expression is positive are the following: first, restricting ϕ to values different than $\frac{1}{2}$, or considering the case in which the realizations of water during the "high" and "low" states of nature are different; and second, assuming relatively large values of the water endowment.

H. Proof that x^{**} is Decreasing in α

Taking into account equations 17A and 18A, then the partial derivative of x^{**} with respect to α is given by:

$$\frac{\partial x^{**}}{\partial \alpha} = \frac{\partial x}{\partial u}\frac{\partial u}{\partial \alpha} = -\frac{1}{u^2}\frac{\partial u}{\partial \alpha} = -\frac{1}{\left(1+p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}\right)^2}\frac{\partial u}{\partial \alpha} \quad (39A)$$

Referring to equation 17A, the partial derivative $\frac{\partial u}{\partial \alpha}$ is given by:

$$\frac{\partial u}{\partial \alpha} = p^{\frac{1}{1-\alpha}}\frac{\partial\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}}{\partial \varepsilon} \quad (40.1A)$$

Computing the derivative in 40.1A leads to:

$$\frac{\partial u}{\partial \alpha} = \frac{\partial p^{\frac{1}{1-\alpha}}}{\partial \alpha}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}} + \frac{\partial\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}}{\partial \alpha}p^{\frac{1}{1-\alpha}} \quad (40.2A)$$

Further computing the partial derivatives in the expression in 40.2A yields:

$$\frac{\partial u}{\partial \alpha} = \frac{\ln(p)p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}}{(1-\alpha)^2} + \frac{\ln\left(w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right)p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}}{(1-\alpha)^2} \quad (40.3A)$$

Substituting 40.3A into 39A leads to

$$\frac{\partial x^{**}}{\partial \alpha} = -\frac{p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}\left\{\ln(p)+\ln\left(w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right)\right\}}{(1-\alpha)^2\left(1+p^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right\}^{\frac{1}{1-\alpha}}\right)^2} \quad (41A)$$

The sign of equation 41A depends upon the sign of the expression $\left\{\ln(p)+\ln\left(w^{\beta}\left[(1-\phi)^{\beta}(1-q)+q\phi^{\beta}\right]\right)\right\}$. If the expression is positive, then the derivative

$\frac{\partial x^{**}}{\partial \alpha}$ is negative. The two conditions required to ensure that the expression is positive are the following: assuming relatively large values of price and the water endowment.

I. Deriving the Optimal Choice of the Water-saving Crop when $\beta_1 \neq 0$

$$\max_x E(\pi) = q\left\{(\phi w)^{\beta_1}(xL)^{\alpha_1} + p(\phi w)^{\beta_2}((1-x)L)^{\alpha_2}\right\} + (1-q)\left\{((1-\phi)w)^{\beta_1}(xL)^{\alpha_1} + p((1-\phi)w)^{\beta_2}((1-x)L)^{\alpha_2}\right\} \quad (42A)$$

In order to simplify the notation, assume that $\beta_1 = \beta$ and that $\beta_2 = \lambda\beta$, where $\lambda > 1$.The factor λ serves to distinguish between the water intensities of the water-saving and the water-intensive crops, such that higher values of λ reflect a relative higher water intensity of the water-intensive crop. In addition, the problem is simplified assuming both crops are equally intensive in land use (i.e. $\alpha_1 = \alpha_2 = \alpha$). Under these modifications, the FOC for the choice of x is stated as follows:

$$q\left\{(\phi w)^{\beta}(xL)^{\alpha-1} + p(\phi w)^{\lambda\beta}((1-x)L)^{\alpha-1}\right\} + (1-q)\left\{((1-\phi)w)^{\beta}(xL)^{\alpha-1} + p((1-\phi)w)^{\lambda\beta}((1-x)L)^{\alpha-1}\right\} = 0 \quad (43A)$$

After algebraic manipulation, the FOC is re-written as follows:

$$(xL)^{\alpha-1}\left\{w^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\} = p\,((1-x)L)^{\alpha-1}\left\{w^{\lambda\beta}\left[(1-q)(1-\phi)^{\lambda\beta} + q\phi^{\lambda\beta}\right]\right\} \quad (44A)$$

Raising both sides of the expression in 44A to the $\frac{1}{\alpha-1}$ power and dividing both sides by L yields the following:

$$(x)\left\{w^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}} = (1-x)p^{\frac{1}{\alpha-1}}\left\{w^{\lambda\beta}\left[(1-q)(1-\phi)^{\lambda\beta} + q\phi^{\lambda\beta}\right]\right\}^{\frac{1}{\alpha-1}} \quad (45A)$$

Let a represent $\left\{w^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}}$, and b $\left\{w^{\lambda\beta}\left[(1-q)(1-\phi)^{\lambda\beta} + q\phi^{\lambda\beta}\right]\right\}^{\frac{1}{\alpha-1}}$, such that the expression in 44A turns into

$$ax = b(1-x)p^{\frac{1}{\alpha-1}} \quad (46A)$$

Solving for x and substituting a and b for their respective expressions leads to

$$x = \frac{p^{\frac{1}{\alpha-1}}\left\{w^{\lambda\beta}\left[(1-q)(1-\phi)^{\lambda\beta} + q\phi^{\lambda\beta}\right]\right\}^{\frac{1}{\alpha-1}}}{\left\{w^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}} + p^{\frac{1}{\alpha-1}}\left\{w^{\lambda\beta}\left[(1-q)(1-\phi)^{\lambda\beta} + q\phi^{\lambda\beta}\right]\right\}^{\frac{1}{\alpha-1}}} \quad (47A)$$

dividing the numerator and denominator of 47A by the expression $p^{\frac{1}{\alpha-1}}\left\{w^{\lambda\beta}\left[(1-q)(1-\phi)^{\lambda\beta} + q\phi^{\lambda\beta}\right]\right\}^{\frac{1}{\alpha-1}}$ leads to the equivalent expression:

$$x^{***} = \frac{1}{1 + p^{\frac{1}{1-\alpha}}\left\{w^{\lambda\beta}\left[(1-q)(1-\phi)^{\lambda\beta} + q\phi^{\lambda\beta}\right]\right\}^{\frac{1}{1-\alpha}}\left\{w^{\beta}\left[(1-q)(1-\phi)^{\beta} + q\phi^{\beta}\right]\right\}^{\frac{1}{\alpha-1}}} \quad (48A)$$

J. Deriving the Optimal Choice Of x under the Contingent Transfer Contract

Assuming the water saving-crop is independent of water ($\beta_1 = 0$) and that both crops are equally intensive in land use ($\alpha_1 = \alpha_2 = \alpha$), the FOC in equation 15 become:

$$(xL)^{\alpha-1} = p((1-x)L)^{\alpha-1}\left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)+t]^{\beta}\right\} \quad (49A)$$

Raising both sides of the expression in 49A leads to the $\frac{1}{\alpha-1}$ power leads to

$$(xL) = p^{\frac{1}{\alpha-1}}((1-x)L)\left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{1}{\alpha-1}} \quad (50A)$$

Dividing both sides of 50A by L leads to

$$x = (1-x)p^{\frac{1}{\alpha-1}}\left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{1}{\alpha-1}} \quad (51A)$$

expanding the right hand side of equation 51A and grouping the terms that involve x in the left hand side leads to

$$x\left\{1 + p^{\frac{1}{\alpha-1}}\left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{1}{\alpha-1}}\right\} = \left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{1}{\alpha-1}} \quad (52A)$$

solving for x leads to the optimal choice of under the contingent transfer program

$$x = \frac{\left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{1}{\alpha-1}}}{1 + p^{\frac{1}{\alpha-1}}\left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{1}{\alpha-1}}} \quad (53A)$$

Dividing the numerator and denominator of expression 53A by $\left\{q(\phi w-t)^{\beta}+(1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{1}{\alpha-1}}$ leads to the equivalent expression

$$x^{***}=\frac{1}{1+p^{\frac{1}{1-\alpha}}\left\{q(\phi w-t)^{\beta}+(1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{1}{1-\alpha}}} \tag{54A}$$

K. Proof that x^{***} is Decreasing in t

First, let

$$v=1+p^{\frac{1}{1-\alpha}}\left\{q(\phi w-t)^{\beta}+(1-q)[(1-\phi)w+t]^{\beta}\right\}^{\frac{1}{1-\alpha}} \tag{55A}$$

such that

$$x^{***}=\frac{1}{v} \tag{56A}$$

Then,

$$\frac{\partial x^{***}}{\partial t}=\frac{\partial x}{\partial v}\frac{\partial v}{\partial t}=-\frac{1}{\left(1+p^{\frac{1}{1-\alpha}}\left\{q(\phi w-t)^{\beta}+(1-q)[(1-\phi)w+t]^{\beta}\right\}^{\frac{1}{1-\alpha}}\right)^{2}}\frac{\partial v}{\partial t} \tag{57A}$$

referring to equation 55A, the partial derivative $\frac{\partial v}{\partial t}$ is computed as follows:

$$\frac{\partial v}{\partial t}=\frac{p^{\frac{1}{1-\alpha}}}{1-\alpha}\left\{q(\phi w-t)^{\beta}+(1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{\alpha}{1-\alpha}}\frac{\partial\left\{q(\phi w-t)^{\beta}+(1-q)[(1-\phi)w+t]^{\beta}\right\}}{\partial t} \tag{58.1A}$$

After computing the partial derivative of the right hand side of 58.1A, the expression becomes:

$$\frac{\partial v}{\partial t}=\frac{\beta p^{\frac{1}{1-\alpha}}}{1-\alpha}\left\{q(\phi w-t)^{\beta}+(1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{\alpha}{1-\alpha}}\left\{(1-q)[(1-\phi)w+t]^{\beta-1}-q(\phi w-t)^{\beta-1}\right\} \tag{58.2}$$

Substituting 58.2A into 57A leads to:

$$\frac{\partial x^{***}}{\partial t} = -\frac{\left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)+t]^{\beta}\right\}^{\frac{\alpha}{1-\alpha}}\left\{(1-q)[(1-\phi)w+t]^{\beta-1} - q(\phi w - t)^{\beta-1}\right\}}{(1-\alpha)\beta^{-1}p^{\frac{1}{\alpha-1}}\left(1+p^{\frac{1}{1-\alpha}}\left\{q(\phi w - t)^{\beta} + (1-q)[(1-\phi)w+t]^{\beta}\right\}^{\frac{1}{1-\alpha}}\right)^{2}} \quad (59A)$$

The sign of the partial derivative in equation 59A depends upon the sign of the expression $\left\{(1-q)[(1-\phi)w+t]^{\beta-1} - q(\phi w - t)^{\beta-1}\right\}$. In particular, when the expression is positive, then partial derivative $\frac{\partial x^{***}}{\partial t}$ is negative. The proof proceeds by imposing the conditions that ensure a mean-preserving spread in the distribution is in effect, namely $q = \frac{1}{2}$. Using this restriction, the condition for $\frac{\partial x^{***}}{\partial t}$ to be negative is the following:

$$\frac{1}{2}[(1-\phi)w+t]^{\beta-1} > \frac{1}{2}(\phi w - t)^{\beta-1} \quad (60.1A)$$

which simplifies to

$$[(1-\phi)w+t]^{\beta-1} > (\phi w - t)^{\beta-1} \quad (60.2A)$$

raising both sides of 60.2A to the $\frac{1}{\beta-1}$ power leads to

$$\left\{[(1-\phi)w+t]^{\beta-1}\right\}^{\frac{1}{\beta-1}} < \left\{(\phi w - t)^{\beta-1}\right\}^{\frac{1}{\beta-1}} \quad (60.3A)$$

which is equivalent to

$$[(1-\phi)w+t] < (\phi w - t) \quad (61A)$$

which after algebraic manipulation 56.2 leads to

$$t < \frac{w(2\phi-1)}{2} \quad (62A)$$

The proportionality factor plays an important role in the inequality presented in 62A. There are two meaningful ranges of values that the parameter ϕ can take and that have implications for the inequality. First, when $\phi = \frac{1}{2}$, the realization of water is known with certainty, and the right hand side of the inequality reduces to zero. In order for the inequality to hold, the value of t would negative when $\phi = \frac{1}{2}$. Since negatives values of t are meaningless, such is not relevant in this study. The second case is when $1 > \phi > \frac{1}{2}$, and this results in values that make the inequality meaningful because the right hand side is positive. In conclusion, the one of the conditions for the $\frac{\partial x^{***}}{\partial t}$ to be negative is that $\phi > \frac{1}{2}$.

On the other hand, consider the following value of t:

$$\bar{t} \equiv \frac{w(2\phi - 1)}{2} \tag{63A}$$

At the value of $\bar{t}$ the derivative $\frac{\partial x^{***}}{\partial t}$ is zero, which implies that for such value of t, the transfer exerts no effect in the behavior of the firm.

L. Deriving the Socially Optimal Transfer

Assuming that both crops are equally intensive in land use (i.e. $\alpha_1 = \alpha_2$), the FOC corresponding to the problem stated in equation 17 are as follows:

$$\begin{aligned}&\alpha\left(x^{***}\right)^{\alpha-1}\frac{\partial x^{***}}{\partial t} - \alpha\left(1-x^{***}\right)^{\alpha-1}\frac{\partial x^{***}}{\partial t}\,p\left[q(\phi w - t)^{\beta} + (1-q)[(1-\phi)w + t]^{\beta}\right] \\ &- \ p\beta q(\phi w - t)^{\beta-1}\left(1-x^{***}\right)^{\alpha} + p\beta(1-q)[(1-\phi)w + t]^{\beta-1}\left(1-x^{***}\right)^{\alpha} = 0\end{aligned} \tag{64A}$$

Regrouping the terms based on their signs elicits the SMB and SMC curves in the following equations

$$\begin{aligned}SMB_t = {} & \alpha\left(1-x^{***}\right)^{\alpha-1}\left(-\frac{\partial x^{***}}{\partial t}\right)p\left[q(\phi w - t)^{\beta} + (1-q)[(1-\phi)w + t]^{\beta}\right] \\ & + p\beta(1-q)[(1-\phi)w + t]^{\beta-1}\left(1-x^{***}\right)^{\alpha}\end{aligned} \tag{65A}$$

and

$$SMC_t = \alpha x^{\alpha-1}\frac{\partial x^{***}}{\partial t} - \ p\beta q(\phi w - t)^{\beta-1}\left(1-x^{***}\right)^{\alpha} \tag{66A}$$

The social optimal level of the transfer, denoted by t^s, occurs where these curves intersect. Therefore, the proof proceeds by conjecturing that expressions in 65A and 66A are equivalent when evaluated at the value of $\bar{t} = \frac{w(2\phi-1)}{2}$ and imposing the conditions that invoke the mean-preserving spread of the distribution of water. Therefore, the proof consists of showing that $SMB_t\big|_{\substack{t=\bar{t} \\ q=1/2}} = SMC_t\big|_{\substack{t=\bar{t} \\ q=1/2}}$. Taking into account the result in expression 63A, which states that $\left.\frac{\partial x^{***}}{\partial t}\right|_{\substack{t=\bar{t} \\ q=1/2}} = 0$ simplifies the proof, such that

$$SMB_t\Big|_{\substack{t=\bar{t}\\ q=1/2}} - SMC_t\Big|_{\substack{t=\bar{t}\\ q=1/2}} = \frac{\left(1-x^{***}\right)^{\alpha} p\beta}{2}\left\{\left(\frac{w}{2}\right)^{\beta-1} - \left(\frac{w}{2}\right)^{\beta-1}\right\} = 0 \qquad (67A)$$

Therefore, the conjectured value of $\bar{t} = \frac{w(2\phi-1)}{2}$ complies with the necessary conditions for a socially optimal value of the contingent transfer. The next step is to confirm that the conjectured value of the transfer also complies with the sufficient conditions for a maximum. The SOC for the planner's problem is:

$$\begin{aligned}\frac{\partial^2 R}{\partial t^2} &= \alpha\left(\Gamma x^{\alpha-2} + \frac{\partial^2 x}{\partial t^2}x^{\alpha-1}\right) - p\alpha\left(\Phi\frac{\partial^2 x}{\partial t^2}(1-x)^{\alpha-1} - \Gamma\Phi(1-x)^{\alpha-2}\right)\\ &\quad - p\alpha\left(\beta\left\{(1-q)\left[(1-\phi)w+t\right]^{\beta-1} - q(\phi w - t)^{\beta-1}\right\}\frac{\partial x}{\partial t}(1-x)^{\alpha-1}\right)\\ &\quad - p\beta q\left(\Psi(\phi w - t)^{\beta-1} - \Omega(\phi w - t)^{\beta-2}\right)\\ &\quad + p\beta(1-q)\left[\Psi\left[(1-\phi)w+t\right]^{\beta-1} + \Omega\left[(1-\phi)w+t\right]^{\beta-2}\right]\end{aligned} \qquad (68A)$$

where $\Gamma = (\alpha-1)\frac{\partial x}{\partial t}\frac{\partial x}{\partial t}$, $\Phi = q(\phi w - t)^{\beta} + (1-q)\left[(1-\phi)w+t\right]^{\beta}$, $\Omega = (\beta-1)(1-x)^{\alpha}$, and $\Psi = \alpha(1-x)^{\alpha-1}\left(-\frac{\partial x}{\partial t}\right)$.

By imposing the result that $\frac{\partial x^{***}}{\partial t}\Big|_{\substack{t=\bar{t}\\ q=1/2}} = 0$, some elements in the SOC become zero, namely: $\Gamma\Big|_{\substack{t=\bar{t}\\ q=1/2}} = 0$ and $\Psi\Big|_{\substack{t=\bar{t}\\ q=1/2}} = 0$. Therefore, the SOC reduces to:

$$\begin{aligned}\frac{\partial^2 R}{\partial t^2}\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}} &= \alpha\left(\frac{\partial^2 x}{\partial t^2}\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\left(x\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\right)^{\alpha-1}\right) - p\alpha\left(\Phi\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\frac{\partial^2 x}{\partial t^2}\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\left(1-x\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\right)^{\alpha-1}\right)\\ &\quad - \frac{p\beta}{2}\left(-\Omega\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}(\phi w - \bar{t})^{\beta-2}\right) + \frac{p\beta}{2}\left(\Omega\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\left[(1-\phi)w+\bar{t}\right]^{\beta-2}\right)\end{aligned} \qquad (69.1A)$$

rearranging and substituting some terms leads to

$$\begin{aligned}\frac{\partial^2 R}{\partial t^2}\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}} &= \alpha\frac{\partial^2 x}{\partial t^2}\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\left(\left(x\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\right)^{\alpha-1} - p\Phi\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\left(1-x\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\right)^{\alpha-1}\right)\\ &\quad + \frac{p\beta}{2}\Omega\Big|_{\substack{t=\bar{t}\\ q=\frac{1}{2}}}\left(\left(\phi w - \tfrac{(2\phi-1)}{2}w\right)^{\beta-2} + \left[(1-\phi)w + \tfrac{(2\phi-1)}{2}w\right]^{\beta-2}\right)\end{aligned} \qquad (69.2A)$$

where $x\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}} = \frac{1}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}$, $\Omega\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}} = (\beta-1)\left(\frac{p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}\right)^{\alpha}$ and $\Phi\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}} = \left(\frac{w}{2}\right)^{\beta}$. Upon further substitution and algebraic manipulation, equation 69.2A becomes

$$\begin{aligned}\frac{\partial^2 R}{\partial t^2}\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}} &= \alpha\frac{\partial^2 x}{\partial t^2}\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}}\left(\frac{1}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}\right)^{\alpha-1}\left(1-p\left(p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)\right)^{\alpha-1}\left(\frac{w}{2}\right)^{\beta}\right)\\ &+ p\beta(\beta-1)\left(\frac{p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}\right)^{\alpha}\left(\frac{w}{2}\right)^{\beta-2}\end{aligned} \tag{69.3A}$$

which is equivalent to

$$\begin{aligned}\frac{\partial^2 R}{\partial t^2}\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}} &= \alpha\frac{\partial^2 x}{\partial t^2}\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}}\left(\frac{1}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}\right)^{\alpha-1}\left(1-p\left(p\left(\frac{w^{\beta}}{2}\right)\right)^{-1}\left(\frac{w}{2}\right)^{\beta}\right)\\ &- p\beta(1-\beta)\left(\frac{p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}\right)^{\alpha}\left(\frac{w}{2}\right)^{\beta-2}\end{aligned} \tag{69.4A}$$

which is equivalent to

$$\frac{\partial^2 R}{\partial t^2}\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}} = \alpha\frac{\partial^2 x}{\partial t^2}\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}}\left(\frac{1}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}\right)^{\alpha-1}(0) - p\beta(1-\beta)\left(\frac{p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}\right)^{\alpha}\left(\frac{w}{2}\right)^{\beta-2} \tag{69.5A}$$

Therefore, the sign of the SOC depends on

$$\frac{\partial^2 R}{\partial t^2}\Big|_{\substack{t=\bar{t}\\q=\frac{1}{2}}} = -\,p\beta(1-\beta)\left(\frac{p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}\left(\frac{w^{\beta}}{2}\right)^{\frac{1}{1-\alpha}}}\right)^{\alpha}\left(\frac{w}{2}\right)^{\beta-2} < 0 \tag{69.6A}$$

Therefore, the value of $\bar{t} = \frac{w(2\phi-1)}{2}$ is the social optimal level of the water transfer and complies with the necessary and sufficient conditions for a maximum.

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