Engineers have long used the theory of elasticity for the analysis of stresses and displacements in soil media. The basic assumption underlying this theory is that the material which constitutes the medium is linearly elastic; i.e., the stress is directly proportional to the strain, so the modulus of elasticity of the material is a constant independent of the magnitude of stress. This is evidently not true for soils as their modulus of elasticity changes with the magnitude of stress. The exact solution of this problem involves nonlinear differential equations which are difficult, if not impossible, to solve. However, it can be solved by the finite element method.

The purpose of this study is to develop a finite element method for computing stresses and displacements in nonlinear soil media. The nonlinear behavior of sand and clay is assumed, and its effect on the distribution of stresses and displacements in an axisymmetric problem is then investigated. The results of this study show that the nonlinear behavior of sand and clay has a large effect on vertical and radial displacements, an intermediate effect on radial and tangential stresses, and a small effect on vertical and shear stresses. Depending upon the depth of the point in question, the stresses based on nonlinear theory may be greater or smaller than those based on linear theory and, at a certain depth, both theories may yield the same stresses. This may explain why Boussinesq's solution of stresses based on linear theory has been applied to soils with greater success, even though soils themselves are basically nonlinear.

The method developed in this study can also be used for determining the stresses and displacements in layered systems. This information is needed for the development of a rational method of pavement design.

Figure 11. A Nonlinear Axisymmetric Median Proposed for this Study, where $E_0 =$ Initial Modulus of Elasticity, $J_1 =$ First Stress Invariant, $J_2 =$ Second Stress Invariant, $\beta =$ Nonlinear Coefficient for Sand, $\alpha =$ Nonlinear Coefficient for Clay, $\nu =$ Poisson's Ratio, and $a =$ Radius of Loaded Area
Figure 12. Vertical and Radial Displacements in Sand — Solid Lines for Linear Medium and Dotted Lines for Nonlinear Medium, Both Determined by Finite Element Method, with Small Circles Indicating Exact Solution for Linear Medium

Figure 13. Vertical, Shear, Radial, and Tangential Stresses in Sand — Solid Lines for Linear Medium and Dotted Lines for Nonlinear Medium, Both Determined by Finite Element Method, with Small Circles Indicating Exact Solution for Linear Medium