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MINIMUM COST DESIGN OF WATER DISTRIBUTION SYSTEMS

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ABSTRACT

The objective of this study was to develop the analytical tools and procedures for minimum cost design of water distribution systems. Both analog and digital means of carrying out pressure and flow calculations were developed. As a result of this effort, digital programs for pressure and flow calculations in water distribution systems were written and have been widely distributed to practicing engineers. One procedure is based on a direct solution of the basic system equations using a linearization scheme and has several advantages over conventional techniques such as the Hardy Cross method. These include avoiding the need to initially balance the network and an assured convergence of the procedure.

Using this tool a procedure was developed for selecting pipe diameter which will result in a minimum cost design within the prescribed constraints. The method of steepest ascent and dynamic programming concepts were used to carry out the optimization. This procedure applies to closed loop systems without internal pumping. However, this work provides a basis for extending the concepts to more generalized water distribution systems.

KEY WORDS: water distribution, optimization, piping systems, network design, economic efficiency
ACKNOWLEDGEMENTS

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INTRODUCTION

The objective of the study as outlined in the original proposal was, "to investigate the various functional constraints controlling the design of water distribution systems and develop analytical methods and digital computer routines which can be utilized to design a water distribution system at minimum cost". It was determined early in this investigation that the available means for the hydraulic analysis of water distribution systems did not lend themselves well to a minimum cost analysis. Therefore, a considerable effort was made to develop the analytical tools for pressure and flow calculations which could be incorporated into a minimum cost analysis of water distribution systems.

A promising technique which was investigated was the use of analog simulation for system hydraulics which could be incorporated into a digital-analog scheme to carry out the cost minimization. Techniques for carrying out an analog simulation of pipe system hydraulics on a standard analog computer were developed and reported by the principal investigator (1). It was intended to use the analog computer to model the hydraulics and to use analog to digital conversion and a digital computer to compute the cost. It was felt that with such a model an effective directed search could be undertaken to determine the optimum design of the pipe system. However, it turned out that the necessary equipment for the development of this concept was not available to this project. Since some rather expensive equipment is involved this approach had to be abandoned.

A major effort was devoted to the development of an analytical procedure for hydraulic analysis based on linearization of the basic non-linear system equations. The purpose of developing this approach was that it appeared to offer a method
for handling pipe system hydraulics which could be more easily incorporated into a minimum cost study. A scheme which directly solved the basic equation after linearizing the non-linear terms was developed. A publication is available describing this phase of the study (2). This method of hydraulic analysis referred to as the linear method offered distinct advantages over the conventional Hardy Cross and Newton Raphson methods which are generally used. Therefore, some additional effort was made developing the linear method for generalized situations. As a result of this, a general computer program has been developed and made available to engineers working in this field. Over one hundred and fifty engineering firms have acquired this program.

Finally, using the linear method for hydraulic analysis a computer program has been developed for minimum cost design of closed loop water distribution systems. This work was primarily the effort of C. O. Charles and is documented in his Ph.D. dissertation (3). In this program the method of steepest ascent and concepts of dynamic programming were employed to formulate a procedure which would select the optimum set of pipe diameters for a closed loop system.

RESEARCH PROCEDURES

The entire project is concerned with the development of the basic elements of an analytical model to be employed for minimum cost design. This entails the conception, formulation and testing of certain analytical procedures. In most cases either analog or digital computer programs were the end product of this effort. The usual procedures for formulating, debugging and testing computer programs were employed.
RESULTS

Analog simulation of pipe system hydraulics

This phase of the investigation has been completely documented in Reference 1 and is available through that publication. As previously stated, however, the necessary digital-analog equipment was not available to develop a technique for optimum design using analog simulation.

Digital programs for the analysis of pipe system hydraulics

A considerable effort was made to develop a digital computer program which would easily handle general water distribution systems in a manner which would lend itself to a minimum cost investigation. This effort resulted in the development of two computer programs. General information pertaining to the development of these programs follows:

The programs will compute steady flow in pipe systems of any arrangement. The system can include pumps, valves, bends, and other minor loss components, storage tanks and source and storage reservoirs. A system of p pipes can be described by the number of junctions, j, the number of closed primary loops, l, and the number of terminal energy points, t, in the system. A junction is simply a point in the system where two or more pipes meet. Any point where flow enters or exits the pipe system is also a junction. A primary loop is a closed loop of pipes in the system which have no other loops within it. A terminal energy point is a point in the system where the fluid energy is known. This is essentially any point where the pressure and the elevation are known. Source or storage reservoirs, pressurized sources, storage tanks and discharge points of known pressure are the most common terminal
energy points. To describe a system the junctions, loops and terminal energy points must be identified. If a terminal energy point and a junction coincide, this point should be identified as a terminal energy point only. If the junctions, loops and terminal energy points are identified with the restriction just stated, the following holds for all pipe systems:

\[ p = j + \ell + t - 1 \]  

(1)

where

- \( p \) = number of pipes
- \( j \) = number of junctions
- \( \ell \) = number of loops
- \( t \) = number of terminal energy points.

In terms of the unknown discharge in each pipe, a number of continuity and energy equations can be written equaling the number of pipes in the system. For each junction a continuity equation equating the flow into the junction to the flow out is written as:

\[ Q_{in} = Q_{out} \quad (j \text{ equations}) \]  

(2)

For each loop the energy equation can be written as follows

\[ \sum h_L = \sum E_P \quad (t \text{ equations}) \]  

(3)

where

- \( h_L \) = head loss in each pipe (including minor loss)
- \( E_P \) = energy put into the liquid by a pump.

If there are no pumps in the loop then the energy equation states that the sum of the head loss around the loop equals zero.

If there are \( t \) terminal energy points, \( t - 1 \) energy equations can be written for paths between any two terminal energy points as follows
where $\Delta E$ is the energy difference between the two terminal energy points. Any path in the pipe system can be chosen between the points. However, care must be taken to avoid redundant paths. The best method to avoid this difficulty is to either choose all paths starting at one source (like 1-2, 1-3, 1-4, etc.) or to use the previous end point for a path as the starting point for the next path (like 1-2, 2-3, 3-4, etc.). Either of these methods will result in $t - 1$ equations with no redundant ones.

These junction loop and path equations constitute a set of simultaneous equations equal to the number of pipes in the system which can be solved for the discharge in each pipe. A direct solution of these simultaneous equations is not possible because of the non-linear terms. Two basic methods of solution were considered.

**Linear method** - For this approach the non-linear terms are linearized giving a set of linear simultaneous equations which can be solved using matrix methods. The linearization is formulated as follows. The line loss is given by:

$$h_{LP} = K_P Q^n$$

where $K_P$ is a pipe line constant and for the Hazen Williams equation employed in the computer analysis is

$$K_P = \frac{4.73 L}{C^{1.852} D^{4.87}}$$  

Here $L$ = line length in ft, $D$ = line diameter in ft and $C$ is the Hazen Williams roughness coefficient. The discharge $Q$ in eqn. 5 is in cfs and the exponent $n = 1.852$.

Minor losses are given by a loss coefficient, $M$, which multiplies the velocity head to give the loss at the component.
This is

\[ h_{LM} = \frac{V^2}{2g} \]  

(7)

where \( V \) is the mean line velocity and \( g \) is the gravitational constant. In terms of the discharge this is

\[ h_{LM} = K_M Q^2 \]  

(8)

where

\[ K_M = \frac{.02517 M}{D^4} \]  

(9)

The pump head is expressed in two ways.

\[ E_P = \frac{Z_P}{Q} \]  

(10)

For this expression the horsepower put into the system by the pump is given as HP and

\[ Z_P = \frac{550 \, HP}{\gamma} \]  

(11)

where \( \gamma \) = specific weight of the liquid (\#/ft\(^3\)). Alternately the pump head can be expressed as

\[ E_P = A + BQ + CQ^2 \]  

(12)

where \( A \), \( B \), and \( C \) are coefficients of a parabolic characteristic curve which defines the pump operation in the vicinity of the operating point. Since this expression is only valid over a specified range it should not be indiscretely employed in an analysis.

The basic energy equation for a loop or a path between terminal energy points is:
\[ \Sigma (h_{LP} + h_{LM}) = \Delta E + \Sigma E_P \]  \hspace{1cm} (13)

Here \( \Delta E \) is the energy difference between the terminal energy points. This equation can be linearized in terms of a flowrate \( Q_i \) in the vicinity of the solution. This is done as follows

\[ h_{LP} = h_{LPi} + \Delta h_{LP} = K_P Q_i^n + nK_P Q_i^{n-1}(Q - Q_i) \]  \hspace{1cm} (14)

\[ h_{LM} = h_{LMi} + \Delta h_{LM} = K_M Q_i^2 + 2K_M Q_i(Q_i - Q) \]  \hspace{1cm} (15)

\[ E_P = E_{Pi} + \Delta E_P = \frac{Z_P}{Q_i} - \frac{Z_P}{Q_i^2} (Q - Q_i) \]  \hspace{1cm} (16)

or:

\[ E_P = A + BQ_i + CQ_i^2 + (B + 2CQ_i)(Q - Q_i) \]  \hspace{1cm} (17)

With these substitutions eqn. 13 can be expressed as a linear function of \( Q \) as

\[ \Sigma (nK_P Q_i^{n-1} + 2K_M Q_i + \frac{Z_P}{Q_i^2}) Q = \]

\[ \Sigma \left( \frac{2Z_P}{Q_i} + (n-1)K_P Q_i^n + K_M Q_i^2 \right) + \Delta E \]  \hspace{1cm} (18)

For the alternate form of the pump head this equation is

\[ \Sigma (nK_P Q_i^{n-1} + 2K_M Q_i - B - 2CQ_i)Q = \]

\[ \Sigma (A - CQ_i^2 + (n-1)K_P Q_i^n + K_M Q_i^2) + \Delta E \]  \hspace{1cm} (19)

Equation 18 (or 19) is employed to formulate an equation for each loop (\( \Delta E = 0 \)) and \( t - 1 \) terminal energy equations which combine with
the j continuity equations to for a set of P simultaneous linear
equations in terms of the flowrate in each pipe.

Path method - The same notation previously defined in the des­
cription of the linear method is used. The basis of this method is
to compute a flow correction \( \Delta Q \) which when added to an initial
set of flowrates (which satisfy continuity) will tend to satisfy
the energy equation for each path. This is

\[
\Sigma(h_{LP} + h_{LM}) = \Delta E + \Sigma E_P
\]

In terms of the initial flowrate \( Q_i \) and the flow correction \( \Delta Q \) these
terms are

\[
h_{LP} = h_{LPi} + \Delta h_{LP} = K_P Q_i^n + nK_P Q_i^{n-1} \Delta Q
\]

\[
h_{LM} = h_{LMi} + \Delta h_{LM} = K_M Q_i^2 + 2K_M Q_i \Delta Q
\]

\[
E_P = E_{Pi} + \Delta E_P = \frac{Z_P}{Q_i} - \frac{Z_P}{Q_i^2} \Delta Q
\]

or

\[
E_P = A + BQ_i + CQ_i^2 + (B - 2CQ_i) \Delta Q
\]

These can be solved to give a flow correction as

\[
\Delta Q = \frac{\Delta E - \Sigma(K_P Q_i^n + K_M Q_i^2 - \frac{Z_P}{Q_i})}{\Sigma(nK_P Q_i^{n-1} + 2K_M Q_i + \frac{Z_P}{Q_i^2})}
\]

or

\[
\Delta Q = \frac{\Delta E - \Sigma(K_P Q_i^n + K_M Q_i^2 - (A + BQ_i + CQ_i^2))}{\Sigma(nK_P Q_i^{n-1} + 2K_M Q_i - B - 2CQ_i)}
\]

8
Using either eqn. 24 or eqn. 25 a flow correction is computed for each path and the flowrates of the pipes in that path are corrected by this amount.

For each method the following information must be available before the hydraulic analysis can be made. For each line the length, diameter and the Hazen Williams Roughness coefficient must be known. This latter parameter is available in handbooks and depends on the type and condition of the pipe. Valves, bends, meters, etc. are included in the analysis by determining the minor loss coefficient for the components. The minor loss coefficient is defined as a constant which multiplies the velocity head in the line to give the head loss at that component. In many cases a standard value for this coefficient is given in various references. This coefficient can also be easily determined if discharge-head loss data is available for the component. Several components can be included in a line by summing their minor loss coefficients.

Pumps can be included in two ways. The useful horsepower (or kilowatts) which the pump puts into the system may be specified. Alternately the coefficients of a parabolic characteristic curve may be specified. This curve represents the pump head-discharge relationship as shown below.

\[ E_p = A + BQ + CQ^2 \]
where A, B, and C are coefficients of the fitted curve. If this representation of the pump is used, however, the solution must yield a discharge in the normal range of operation or the solution will be invalid. This is because the characteristic curve is not valid outside that range.

For each junction the external inflow or outflow is specified and the elevation of the junction is known.

Program description and users options - The programs were developed for use by practicing engineers and were offered to engineers on several bases. Material on the programs has been provided to over 150 engineering firms and individuals. The following brief release provided information to potential users:

Two programs have been developed at the University of Kentucky which will analyze pressure and flow in any pipe system and are available to potential users. These programs are written in FORTRAN IV, G Level and a users guide with source program listings and examples has been prepared. A brief description of the programs follow.

I Program based on linearized system equations -

This program utilized a new procedure for pipe systems analysis which has several advantages over conventional methods. Because this method simultaneously computes the flow in each pipe, the convergence is very fast (usually 3-4 trials to very high accuracy regardless of size of the system). Also, convergence is assured. Initial flowrates are not assumed and changes in flow system demand only require a change in data pertaining to that demand. However, since matrix methods are employed a computer of sufficient storage must be available to use this method for a system of n pipes approximately (n x (n+35) dimensioned storage locations must be available. The IBM 360-65 computer at the University of Kentucky, for example, will handle systems up to 220 pipes with its present storage capacity and without using additional disc storage. The procedure is fast. A 37 pipe system
can be analyzed in about 10 seconds while a 125 pipe system takes 2 minutes and 15 seconds on the University of Kentucky computer.

II Program based on loop and path flow adjustments -

This program is essentially an extension of a loop balancing method similar to the Hardy Cross technique to any type of pipe flow system with pumps, valves, etc. included. It does require as input data initial flowrates which satisfy continuity. In rare cases the Hardy Cross procedure does not produce convergence and this situation could occur. However, for most situations the program produces a fast accurate solution. In addition much less storage is required so a large system can be analyzed with a computer of limited storage capacity.

Basic features of both procedures are:

1. Any piping configurations can be analyzed (closed loop networks, tree systems of combinations).
2. Flow units of CFS, GPM, MGD or SI units (M^3/s) can be used.
3. Pump, valves and other lossy components, and storage tanks can be included in the pipe system.
4. Pressures and hydraulic grades at indicated points in the system are output in addition to head changes at pumps, valves and in lines.
5. Data preparation for both programs is straightforward and very similar and allows any number of changes in system parameters (pipe sizes, pump characteristics, flow demands, etc.) to be investigated in a single computer run.

The programs are available to interested users in one of the following ways:

1. Attend two day short course at the University of Kentucky - announcement attached. This is the best means of gaining the necessary experience for using the programs effectively and is especially recommended for persons not presently using computers for hydraulic analysis. All material and computer source programs are provided for participant. Some post-course consultation and
a post-course laboratory problem chosen by the participant provide additional aid in implementing the programs.

2. Participate in users course on a correspondence basis. This is primarily for users who have an interest in developing the capability of using the programs but cannot attend a short course. It is desirable that the participant have some background in the use of computers. All material and source programs are provided for the participant in addition to problems. The data for these problems are coded and returned for computer processing. In addition the participant may code and submit data for an additional problem over a period of a year which is of interest to him. Systems of up to 50 pipes will be processed as part of the course (larger systems require a nominal additional charge for computing expenses). Consultation regarding the application of the programs to pipe systems will be provided by phone or mail.

3. Obtain material only. This is primarily for users who are already using a digital computer for hydraulics problems. Users guides, program listings and examples will be sent for both programs. Complete details of the programs, program listings and examples are provided in the users manuals (4, 5).

Program for optimum design of water distribution networks.

A major effort to develop a programmable procedure for minimum cost design was made. Details of this effort are included in a Ph. D. thesis (3). The salient points of this effort will be covered in this report.

Problem definition - In designing a hydraulic network distribution system, the engineer has not only to meet the demands at particular points in the system, but also should do so within specified constraints and at the least possible cost. For this study the geometrical configuration of the network is prescribed. The cost is a function of diameter and flow and the constraints can be
regarded as of three types: (i) hydraulic (Kirchhoff laws), (ii) pressure and (iii) diameter. There are also different classes of constraints within each type. For example some pressure constraints are of the type that the pressure must be greater than or equal to a minimum while another constraint is that the pressure must not exceed some maximum value. The diameter constraints are normally of two kinds; the first is that no diameter should be less than a certain minimum, the other that the diameters should be available on the market. This becomes necessary since pipe diameters are made commercially in certain discrete sizes. The problem is therefore to find a set of pipe diameters to satisfy all the constraints at the least cost. The method used to do this is a combination of steepest descent (ascent) and dynamic programming.

Since the cost function involves flow in pipes, it is necessary to calculate the flow quickly. Flow is also important in the calculation of pressure since pressure is a function of flow. To compute flow quickly the method of linear analysis which was developed for this purpose is employed.

Problem formulation - Any problem of optimization has essentially two characteristics (1) a cost function and (2) one or more constraints. For the hydraulic network these are described as follows:

(1) Cost function -- The cost function used for network optimum design is divided into two parts: (a) Capital and (b) operation and maintenance costs. For the capital cost the result of the regression analysis performed by Linaweaver and Clark is used. This analysis was carried out on pipe line data for oil, gas and water pipe lines and gives a relationship between the variables, diameter, D, in inches and the capital cost, in dollars, per mile. This relationship is given by

\[ \text{Capital Cost} = 1890 \times D^{1.29} \text{ per mile} \]

or

\[ \text{Capital Cost} = 0.3581^{1.29} \text{ per foot} \]
The correlation coefficient is 0.98 according to the article.

At the time of this survey the Engineering News Record Construction Cost Index (ENRI) was 877. This enables the capital cost relationship to be updated by the ratio, PRESENT ENRI/877. The procedure described is used herein for the capital cost portion of the cost analysis. Alternate schemes could be used to express the capital cost as some continuous function of pipe diameter.

The capital expenditure is usually incurred at the time of construction of the project, and is paid back over the life of the project. During that time, the value of money is determined by the rate of interest, i% per annum. In order to spread the capital cost evenly over the whole life of the project, it is necessary to multiply the initial cost by a capital recovery factor (crf) where:

\[
\text{crf} = \left[ \frac{i}{100} \left(1 + \frac{i}{100}\right)^{nn} \right] \div \left[ \left(1 + \frac{i}{100}\right)^{nn} - 1 \right]
\]

where \(nn\) = life of project in years.

Thus in a system of \(m\) pipes the annual capital cost is

\[
\sum_{i=1}^{m} 0.358 \frac{L_i}{D_i} 1.29 \text{(crf)} \frac{(\text{PRESENT ENRI})}{877}
\]

where \(L_i\) = length in feet of \(i\)-th pipe.

The operation and maintenance portion of the cost function is obtained by first equating the pumping power required for each pipe to that of an equivalent number of kilowatt-hours and then multiplying the number of kilowatt-hours by the corresponding unit cost. The power utilized in a pipe is related to the corresponding head loss. The Hazen-Williams empirical expression for head loss is used. This expression is \(H_L = KQ^{1.8518}\)

where

\[
K = \frac{4.77L(12)^{4.87}}{C1.8518D^{4.87}}
\]

(26)
and \( C \) = roughness coefficients.

The power lost in the pipe line is related to \( H_L \) in the following way:

\[
\text{Power loss} = \frac{H_L Q \gamma}{550} \text{ horse power (where } Q \text{ is in cfs and } \gamma \text{ is in the specific weight of the liquid in Lb/ft}^3)\
\]

Inserting the head loss equation in this expression gives:

\[
\text{Power loss} = \frac{KQ^{2.8518}}{550} (62.4) \text{ horse power (assuming water is the liquid)}.
\]

The final annual cost due to maintenance and operation in a pipe can be expressed as:

\[
KQ^{2.8518} \frac{(62.4)}{550} (0.746)(365 \times 24)c \text{ per year}
\]

where \( c \) = unit cost of electricity in \$ per KwH. Thus the total cost function \((R)\) is

\[
R = \sum_{i=1}^{m} 0.358 L_i D_i^{1.29} (\text{crf})(\text{Present ENRI/877}) + \sum_{i=1}^{m} K_i Q_i^{2.8518} \frac{(62.4)}{550} (0.746)(365 \times 24)c = R \quad (27)
\]

The factor \((365 \times 24)\) assumes that the system is in operation for the whole year. If this is not the case, then this factor can be replaced by the anticipated number of hours in the year that the system will be in operation.

(2) Constraints -- The primary constraint is one which requires the flows to obey basic hydraulic relationships involving continuity of flow at junctions and head losses in the individual loops.

Another type of constraint, dealing with pressures, assumes alternate forms. The system must be designed for a maximum
value of pressure which must not be exceeded.

There also may be a minimum pressure required for each junction which is necessary to maintain acceptable system performance. An example of such a minimum is that required by fire-fighting activities for which standards have been developed by National Board of Fire Underwriters (NBFU) (33). It is assumed that though there are variations in minimum pressure, there is none in the maximum.

Acceptable pipe diameters are also constrained. NBFU recommends that the diameter of street mains should not be less than 6-inches. This is again a fire-protection provision where the primary consideration is to obtain an acceptable quantity of water. In addition, diameters available on the commercial market are discrete and not continuous. A 6-inch pipe may be available, while 6.25-inch usually is not. It is therefore necessary that the sizes selected for design must be commercially available.

(3) The mathematical model -- The problem is summarized as follows:

The cost function to be minimized is

\[ R = \sum_{i=1}^{m} 0.358 L_i D_i^{1.29} \left( \frac{\text{Present ENRI}}{877} \right) \text{crf} + \]

\[ \sum_{i=1}^{m} K_i Q_i^{2.8518} \left( \frac{62.4}{550} \right) \left( 0.746 \right) (365 \times 24) c \]

Subject to the following constraints:

(i) pressure and flow obey basic hydraulic relationships
(ii) \( p \leq \text{ABMAX} \) where \( p \) is pressure and \( \text{ABMAX} \) is absolute maximum pressure
(iii) \( p \geq \text{ABMIN} \) where \( \text{ABMIN} \) is the absolute minimum pressure allowed
(iv) \( p_A \geq P_A \) where \( p_A \) is pressure at junction A and \( P_A \) is the minimum allowed at junction A
(v) \( D \geq \text{DMIN} \) where \( \text{DMIN} \) is the absolute minimum size diameter allowed

(vi) \( D \in (d_1, d_2, d_3, \ldots, d_n) \) where \( d_1, d_2, \ldots, d_n \) are the available commercial size diameters.

**Method of Solution** The law of continuity is expressed in terms of flow \((\Sigma Q_i = 0)\) while the "head loss" or loop equations \((\Sigma K_i Q_i^n = 0)\) are functions of flow and pipe properties. The Hazen-Williams line loss expression is used herein. Thus from (26)

\[
K = \frac{4.77L(12)^{4.87}}{C^{0.8518}D^{4.87}}
\]

which is referred to as the loss coefficient. Flow is given by

\[
Q = Av
\]  

(28)

where \( A \) is the cross-sectional area and \( v \) the velocity

It can be demonstrated that the differential \( dQ \) is given by

\[
dQ = \frac{2Q}{D} dD + A dv
\]  

(29)

To preserve continuity, the algebraic sum of the changes in flow at any junction must be zero.

Thus at any junction

\[
\Sigma dQ_i = 0
\]  

(30)

and from eqn. (29)

\[
\Sigma A_i dv_i = -2\Sigma \frac{Q_i}{D_i} dD_i
\]  

(31)

It is clear that by considering changes in flow at many junctions, equations involving \( A_i dv_i \) and \( dD_i \) can be obtained. It also follows from the continuity equations that one of these equations would be redundant. The system equations can be expressed in the following form:
Matrix coefficients are 1, 0, -1

\[
\begin{bmatrix}
\text{Matrix coefficients} \\
\text{are 1, 0, -1}
\end{bmatrix}
\begin{bmatrix}
A_1 dv_1 \\
A_2 dv_2 \\
A_3 dv_3 \\
\vdots \\
A_m dv_m
\end{bmatrix}
= \begin{bmatrix}
\text{Matrix coefficients} \\
\text{are } -\frac{2Q}{D} \\
or 0
\end{bmatrix}
\begin{bmatrix}
dD_1 \\
dD_2 \\
dD_3 \\
\vdots \\
dD_m
\end{bmatrix}
\] (32)

By considering first order derivatives of the loop equations, another set of equations involving \(A_i dv_i\) and \(dD_i\) can be obtained as follows:

\[
H_L = KQ^{1.8518}
\]

is differentiated to obtain

\[
dH_L = Q^{1.8518} \frac{dK}{dD} dD + 1.8518 KQ^{0.8518} \frac{dQ}{dD} dD
\]

\[
+ 1.8518 KQ^{0.8518} \frac{dQ}{dv} dv
\]

From (29) \(\frac{dQ}{dD} = \frac{2Q}{D}\) and \(\frac{dQ}{dv} = A\)

By substituting in (35)

\[
dH_L = Q^{1.8518} \frac{dK}{dD} dD + 1.8518 KQ^{0.8518} \frac{2Q}{D} dD
\]

\[
+ 1.8518 KQ^{0.8518} Adv
\]

From (26) \(K = \frac{4.77L(12)^{4.87}}{C^{1.8518}D^{4.87}}\)

By differentiation

\[
\frac{dK}{dD} = -\frac{4.87}{D^{5.87}} \frac{4.77L(12)^{4.87}}{C^{1.8518}} = -4.87 \frac{K}{D}
\]

(35)
Substitute (35) for \( \frac{dK}{dD} \) in (34), then

\[
dH_L = -4.87 \frac{KQ^{1.8518}}{D} \ dD + 3.7036 \frac{KQ}{D} \ dD + 1.8518 \ KQ^{0.8518} \ Adv
\]

(36)

Simplifying and combining terms give

\[
dH_L = -1.1664 \ \frac{KQ^{1.8518}}{D} \ dD + 1.8518 \ KQ^{0.8518} \ Adv
\]

(37)

These equations of the form \( \Sigma dH_L = 0 \) can be transformed into

\[
1.8518 \ \sum KQ^{0.8518} \ (Adv) = 1.1664 \ \sum KQ^{0.8518} \ dD
\]

(38)

or in matrix form:

\[
\begin{bmatrix}
\text{Matrix Coefficients} \\
\pm 1.8518 \ KQ^{0.8518} \\
\text{or } 0
\end{bmatrix}
\begin{bmatrix}
A_1 \ dv_1 \\
A_2 \ dv_2 \\
A_3 \ dv_3 \\
\vdots \\
A_m \ dv_m
\end{bmatrix}
= \begin{bmatrix}
\text{Matrix coefficients} \\
\pm 1.1664 \ \frac{KQ^{1.8518}}{D} \\
\text{or } 0
\end{bmatrix}
\begin{bmatrix}
dD_1 \\
dD_2 \\
dD_3 \\
\vdots \\
dD_m
\end{bmatrix}
\]

(39)

By combining all independent equations derived from the basic hydraulic relations, (32) and (39), the following simultaneous equations are obtained.

\[
\begin{bmatrix}
\text{Matrix Coefficients} \\
\pm 1 \text{ or } 0
\end{bmatrix}
\begin{bmatrix}
A_1 \ dv_1 \\
A_2 \ dv_2 \\
A_3 \ dv_3 \\
\vdots \\
A_m \ dv_m
\end{bmatrix}
= \begin{bmatrix}
\text{Matrix coefficients} \\
-2Q \frac{dQ}{D} \text{ or } 0 \\
\text{or } \pm 1.1664 \ \frac{KQ^{1.8518}}{D}
\end{bmatrix}
\begin{bmatrix}
dD_1 \\
dD_2 \\
dD_3 \\
\vdots \\
dD_m
\end{bmatrix}
\]

(40)
It is observed that, if \( m \) is the number of pipes in the network then the matrices on both sides of equations (40) are of order \( m \times m \).

The cost function, as given by eqn. (27) is expressed in terms of the diameter, \( D \), flow \( Q \), and Hazen-Williams' loss coefficients. \( Q \) is a function of \( D \) and \( v \), \( K \) is a function of \( D \). Thus the cost function can be expressed in terms of \( D \) and \( v \) and the first derivative of the cost function can be put in a form similar to (40).

The cost function previously defined can be written as

\[
R = \sum a_i D_i^{1.29} + \sum b_i K_i Q_i^{2.8518}
\]

where \( a_i \) and \( b_i \) are constants. Differentiation of \( R \) with respect to \( D_i \)'s and \( v_i \)'s gives

\[
dR = \left( 1.29 \sum a_i D_i^{0.29} + \sum b_i \frac{\delta K_i}{\delta D_i} Q_i^{2.8518} \right) dD_i
\]

\[
+ 2.8518 \sum b_i K_i Q_i^{1.8518} \frac{\delta Q_i}{\delta D_i} dD_i
\]

\[
+ 2.8518 \sum b_i K_i Q_i^{1.8518} \frac{\delta Q_i}{\delta v_i} dv_i
\]

Since \( \frac{\delta K_i}{\delta D_i} = -4.87 \frac{K_i}{D_i} \) (from 35) and from (29) \( \frac{\delta Q_i}{\delta D_i} = \frac{2Q_i}{D_i} \)

and \( \frac{\delta Q_i}{\delta v_i} = A_i \) the derivative becomes

\[
dR = 1.29 \sum a_i D_i^{0.29} dD_i + 0.8336 \sum b_i \frac{K_i Q_i}{D_i}^{2.8518} dD_i
\]

\[
+ 2.8518 \sum b_i K_i Q_i^{1.8518} (A_i dv_i)
\]

This can be expressed in matrix form as follows:
where \( e_1, e_2, e_3, \ldots, e_m \) are coefficients of \( A_1 dv_1, A_2 dv_2, A_3 dv_3, \ldots, A_m dv_m \) and \( f_1, f_2, f_3, \ldots, f_m \) are coefficients of \( dD_1, dD_2, dD_3, \ldots, dD_m \).

By combining (40) and (42) the left hand side becomes a matrix of order \((m+1) \times (m+1)\) and the right hand one of \((m+1) \times m\).

It is also observed that "dR" only occurs once and that it is on the left hand side. Thus one of the columns on this side would have coefficients of

\[ 1, 0, 0, 0, \ldots, 0. \]

Thus the following sets of equations are obtained

\[
\begin{bmatrix}
1 & e_1 & e_2 & \ldots & e_m \\
A_1 dv_1 \\
A_2 dv_2 \\
\vdots \\
A_m dv_m
\end{bmatrix}
\begin{bmatrix}
dR \\
dD_1 \\
dD_2 \\
\vdots \\
dD_m
\end{bmatrix}
= 
\begin{bmatrix}
f_1 & f_2 & f_3 & \ldots & f_m \\
dD_1 \\
dD_2 \\
\vdots \\
dD_m
\end{bmatrix}
\]

\[ (m+1)(m+1) \quad (m+1)m \]

(43)
Before (43) can be used it is necessary to have values of $Q_i$ and $D_i$. Values of $D$ are assumed and values of $Q$ are calculated. The pressure at the junctions are now computed and, if the pressure constraints are broken, new values of $D$ are assumed and the process repeated until these constraints are satisfied.

In this initial stage, if new values of $D$ are required it has been found convenient to add or subtract a constant increment to each of the diameters. It is usual to add the increments, but if the pressure at any point is greater than $ABMAX$ the increment is subtracted.

After values of $D$ and $Q$ are obtained which are feasible solutions the partial derivatives with respect to diameters and velocities are computed and the matrix coefficients for (43) are not known. From these equations, by manipulating the matrices the following is obtained:

\[
\begin{bmatrix}
\begin{array}{c}
dR \\ A_1 \text{d}v_1 \\ A_2 \text{d}v_2 \\ A_3 \text{d}v_3 \\
\vdots \\
A_m \text{d}v_m
\end{array}
\end{bmatrix} = (m + 1)x m\text{ matrix}
\begin{bmatrix}
\begin{array}{c}
dD_1 \\ dD_2 \\ dD_3 \\
\vdots \\
dD_m
\end{array}
\end{bmatrix}
\]

Since

\[
dR = g_1 dD_1 + g_2 dD_2 + g_3 dD_3 + \ldots + g_m dD_m
\]

(where $g_1, g_2, \ldots, g_m$ are coefficients of the first row of matrix in (44), then

\[
\frac{dD_1}{g_1} = \frac{dD_2}{g_2} = \frac{dD_3}{g_3} = \ldots = \frac{dD_m}{g_m} = \rho
\]
To obtain an optimal value of \( p \), using the method of steepest ascent, it is necessary to solve the equation \( \frac{dR}{dp} = 0 \). This is a complex equation and calculating the value of \( p \) to satisfy it is very cumbersome and at best approximate. The value assigned to \( p \) is a critical factor in using the steepest ascent (descent) method. Too big a value of \( p \) oversteps the optimum, too small a value requires too many unnecessary calculations. This makes it difficult to find the optimum value of \( p \) and thus a form of direct search technique was developed to determine this value.

Of the coefficients, \( g_1, g_2, g_3, \ldots, g_m \), the one with the biggest absolute value is chosen, say \( g_k \), and then \( dD_k \) corresponding to \( g_k \) is given a value. The ratio \( dD_k / g_k \) is now known and from (45) all values of \( dD_1, dD_2, \ldots, dD_m \) are computed. These increments, \( dD_1, dD_2, \ldots, dD_m \) may be either positive or negative. If \( g_k \) is negative these increments are first of all algebraically added and if \( g_k \) is positive they are subtracted.

If there is a reduction in cost and no constraints are broken, then the process of algebraically adding or subtracting can be continued, or new partial derivatives computed and the entire procedure so far repeated. If there is no reduction in cost within the specified constraints then the increments are halved and the process of algebraically adding or subtracting is continued. If again there is no reduction, the procedure of halving increments and algebraically adding or subtracting is continued until there is such reduction or the tolerance level is reached. At this stage new matrix coefficients are computed and the whole process is repeated. Thus, for an initial increment size of two inches and tolerance level of half inch, the possible increments likely to be tried after the first are one inch and half-inch.

The process eventually converges to a situation where changes in cost are negligible. At this stage most of the diameters do not satisfy constraint number (vi) that is \( D_1 (d_1, d_2, d_3, \ldots, d_m) \) and the final step in the procedure is to impose this condition.

If \( D_1 \) is the diameter corresponding to \( g_i \) where \( g_i >
any other $g$ and $D_i$ lies between $d_p$ and $d_q$ -- where $d_p$ and $d_q$ are members of the class $(d_1, d_2, d_3, \ldots, d_m)$ and $D_i$ is not -- then $D_i$ is increased or decreased to $d_p$ or $d_q$ by making $dD_i = d_p - D_i$ or $dD_i = d_q - D_i$. The other $dD'$s are computed and all the diameters are altered proportionately. The cost is computed for both cases when $dD_i = d_p - D_i$ and $dD_i = d_q - D_i$. The cost which is cheaper is noted and the corresponding diameters are chosen.

From now on the diameter for the $i$-th pipe is fixed and $dD_i = 0$. The derivative of cost function with respect to $D_i$ will not be a term of equation (40), also any derivative with respect to $D_i$ that is involved in the matrices of (40). The process is repeated until all the diameters are now in an acceptable set.

If, in making incremental changes in diameters, constraint (v) is broken, $(D > \text{DIMIN})$ and if the diameter is not that of the pipe with the largest absolute partial derivative of cost function, the diameter of the pipe breaking this constraint will be assigned the value of DIMIN. The diameter increments for the other pipes will be unaffected. If the pipe breaking the diameter constraint is the one with the largest absolute partial derivative of cost function, then not only is this diameter assigned the value DIMIN, but the changes in the other diameters are proportionately adjusted. If a pressure constraint is broken the incremental changes are reduced proportionately until the pressure constraint is satisfied.

The method just outlined above is dependent on the shape of the cost function to obtain the optimum. If the function is convex then the global optimum is obtained. The pressure constraints, especially (iv) $(p_A \geq P_A)$ may have the effect of rendering the function non-convex, i.e., a "hole" in the feasible region. In this case it is advisable to ignore the pressure constraint in the initial stages of the process.

Computer Program - Using the method just described a computer
FLOWCHART FOR OPTIMUM DESIGN

READ DATA

COMPUTE FLOWS, PRESSURES AND COSTS

ADJUST DIA METERS

ARE CONSTRAINTS BROKEN?

NO

COMPUTE INCREMENTAL CHANGES IN DIAMETERS

COMPUTE COST DUE TO NEW DIAMETERS

HAS COST DECREASED?

NO

ARE INCREMENTS WITHIN TOLERANCE LEVEL?

NO

COMPUTE NEW INCREMENTAL CHANGES IN DIAMETERS

YES

ARE CONSTRAINTS BROKEN?

NO

STORE COST KEEP NEW DIAMETERS

FIGURE 1A
FLOWCHART FOR OPTIMUM DESIGN (CONT'D)

1. IS COST REDUCTION WITHIN TOLERANCE?
   - NO → 2
   - YES → ADJUST DIAMETERS AND FIX DIAMETER WITH LARGEST EFFECT ON COST

2. ARE ALL DIAMETERS FIXED?
   - NO
   - YES → PRINT RESULTS → END

FIGURE 1B
program was written for the optimum design of closed pipe networks. A flowchart depicting the logic of this program is given in Figure 1. A listing of the program is presented in Appendix I. The program is fairly involved and the reader is referred to Reference 5 for the details.

Example - A pipe network of nineteen pipes was analyzed by this program to determine the minimum cost design. Figure 2 shows the geometry of the network. The computer output for this example is presented on the following pages to illustrate the type of information output by the computer. For the given cost information and constraints it would have been highly unlikely that this configuration could have been determined without such an aid as this program. At the present time additional efforts are being made to refine and improve this computer program. When this is completed the program will be made available to potential users.
Figure 2  Hydraulic Network for Example
(to convert flowrate to m³/s multiply gpm by 6.309 x 10⁻⁵)
<table>
<thead>
<tr>
<th>Pipe Number</th>
<th>Diameter (in.)</th>
<th>Length (ft)</th>
<th>Roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.0</td>
<td>1500</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>8.0</td>
<td>1000</td>
<td>130</td>
</tr>
<tr>
<td>3</td>
<td>8.0</td>
<td>1200</td>
<td>120</td>
</tr>
<tr>
<td>4</td>
<td>8.0</td>
<td>2000</td>
<td>120</td>
</tr>
<tr>
<td>5</td>
<td>8.0</td>
<td>2800</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>8.0</td>
<td>1100</td>
<td>120</td>
</tr>
<tr>
<td>7</td>
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</tr>
<tr>
<td>8</td>
<td>8.0</td>
<td>2500</td>
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<td>100</td>
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<td>6.0</td>
<td>1300</td>
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<td>11</td>
<td>6.0</td>
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</tr>
<tr>
<td>12</td>
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</tr>
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</tr>
<tr>
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<td>1200</td>
<td>130</td>
</tr>
<tr>
<td>18</td>
<td>6.0</td>
<td>1800</td>
<td>120</td>
</tr>
<tr>
<td>19</td>
<td>6.0</td>
<td>1300</td>
<td>120</td>
</tr>
</tbody>
</table>

**TABLE I  PIPELINE DATA FOR EXAMPLE**
COMPUTER OUTPUT FOR EXAMPLE

MAXIMUM PRESSURE = 150.000 LBS PER SQ IN
MINIMUM PRESSURE = 30.000 LBS PER SQ IN
MINIMUM PRESSURE ALLOWED AT JUNCTION 9 IS 50.000 LBS PER SQ IN
SMALLEST DIAMETER ALLOWED = 6.000 INCHES
GREATEST INCREMENTAL CHANGE IN DIAMETER = 6.000 INCHES

WHEN THE LARGEST INCREMENTAL CHANGE IN DIAMETERS IS LESS THAN OR EQUAL TO 0.750 INCHES SUCH CHANGES ARE IGNORED

TOLERANCE ON FLOW = 15.000 GPM
LIFE OF PROJECT = 50.000 YEARS
RATE OF INTEREST = 5.000 PER ANNUM
COST OF ELECTRICITY = $0.01 PER KILOWATT-HOUR
TOLERANCE ON MONEY = 10.00
BUILD-UP FACTOR = 1.00
ENGINEERING NEWS RECORD INDEX = 877.

RESULTS OF OPTIMAL TRIAL

<table>
<thead>
<tr>
<th>PIPE NO</th>
<th>JUNCTION</th>
<th>LENGTH</th>
<th>ROUGHNESS</th>
<th>DIAMETER-INCHES</th>
<th>FLOW</th>
<th>PRESSURE AT JUNCTIONS</th>
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<td>130.</td>
<td>10.00 6.00</td>
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<td>120.000 123.172</td>
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</table>

**CAPITAL COST:** $12350.43

**OPTIMAL COST:** $13970.22

**CORE USAGE**

- OBJECT CODE: 31512 BYTES
- ARRAY AREA: 119800 BYTES
- TOTAL AREA AVAILABLE: 170080 BYTES

**DIAGNOSTICS**

- NUMBER OF ERRORS: 0
- NUMBER OF WARNINGS: 0
- NUMBER OF EXTENSIONS: 0

**COMPILE TIME:** 4.45 SEC

**EXECUTION TIME:** 716.00 SEC

**WATFIV - VERSION 1 LEVEL 3 MARCH 1971**

**DATE:** 72/344
CONCLUSIONS

The linear method developed as a result of this study for the hydraulic analyses of water distribution systems is proving to be a valuable aid to practicing engineers. This is supported by the large number of engineers who are presently using the program.

This method is also a useful tool in the minimum cost study of water distribution systems. The feasibility of developing a routine for minimum cost design also has been established through this study. A working program for closed loop systems has been developed.

A method for solving the hydraulics of water distribution systems on a standard analog computer also has been developed. This provides the basic tool for an analog-digital model for optimum network design. However, while this appears to be a very promising technique its practicality is limited. This is because most practicing engineers do not have access to the necessary analog-digital systems.


APPENDIX I

LISTING OF FORTRAN PROGRAM
FOR OPTIMUM DESIGN OF CLOSED LOOP SYSTEMS
IMPLICIT REAL*(A-H,O-Z)
INTEGER UNITS,TREES(50)
REAL*4 (T(50),L(50)),OABS,LIFE,DELIV(50),STAN(2500),SCAN(50)
DIMENSION (50,50),C(150,50),SAVE(2500),MAIN(2500)
INTRODUCED,OPRE(50),PRESS(50),HT(50),Y(50),PECED(50),G(50),DIA(50)
AND THE WWII,ocoder,SPACE,INDEX(50),D(50),D(50),INDEX(50),INDEX(50),INDEX(50),INDEX(50),INDEX(50)
Z ELDI(50),J(50),LOADS(50),ML(50),MT(50),JTMN(50),JTED(50),LZ(50)
1) MPPE(50)
READ (5,1000)UNITS,MXX,MXX
1020 FORMAT (16,51)
1700 FORMAT (1/8x, "RATE OF INTEREST", F6.3,1X," PER ANNUM")
1830 FORMAT (1/8x,"WHEN THE LARGEST INCREMENTAL CHANGE IN DIAMETERS IS"
1 Less than or equal to (F6.3,1X, "INCHES") /58X," SUCH CHANGES"
2 ARE IGNORED")
1 1930 FORMAT (1/8x, "MAXIMUM PRESSURE =", F6.3,1X," LBS PER SQ IN")
2010 FORMAT (1/8x, "MINIMUM PRESSURE =", F6.3,1X," LBS PER SQ IN")
2120 FORMAT (1/8x, "MINIMUM PRESSURE ALLOWED AT JUNCTION", I3,1X," IS",
1 F6.3,1X," LBS PER SQ IN")
2230 FORMAT (1/8x, "TOLERANCE ON FLOW =", F7.4,1X, "CFPS")
2340 FORMAT (1/8x, "TOLERANCE ON FLOW =", F7.4,1X, "CMOD")
2450 FORMAT (1/8x, "TOLERANCE ON PRESSURE =", F8.2,1X)
2420 FORMAT (1/8x, "GREATEST INCREMENTAL CHANGE IN DIAMETER =", F6.3,1X,
1 " INCHES")
2430 FORMAT (1/8x, "SMALLEST DIAMETER ALLOWED =", F6.3,1X," INCHES")
2440 FORMAT (1/8x, "LIFE OF PROJECT =", F6.1X," YEARS")
2450 FORMAT (1/8x, "COST OF ELECTRICITY =", F5.2,1X," PER KILOWATT-HOUR"
1 "")
2460 FORMAT (1/8x, "ENGINEERING NEWS INDEX =", F7.0)
1701 FORMAT (1/8x, "MULLED-UP FACTOR =", F6.3)
C IF UNITS=1, FLOW IS IN CMPS, IF UNITS=2, FLOW IS IN CMPS, IF UNITS=3 FLOW IS
C IN M C M. MAX=1 INDICATES THAT UNITS OF PRESSURE AT INLET ARE LBS/FIR**2,
C NOT "FOOT-HEAD OF WATER"
22 READ (5,1000) N,J,IP
23 READ (5,6000)ABMAX,ABMIN,CIMIN,D1,002,003,004
C N J= # OF JUNCTIONS,NP= # OF PIPES
24 READ (5,2000) (JBEGIN,JEND,J-D,JROUGHI,11111,=1,1,1)
25 TOTAL=0.0
26 TOTAL=0.0
27 WIND
28 WRITE(6,1110)
29 WRITE(6,1000)ABMAX
30 WRITE(6,2010)ABMIN
31 3000 FORMAT (1F10.4)
32 2090 FORMAT (1/8x, "COST OF ELECTRICITY =", F5.2,1X," PER KILOWATT-HOUR"
1 "")
33 WRITE(6,1110)
34 DO 11 I=1,NJ
35 READ (5,3000)PECED(I),DELEV(I),OPRE(I),HT(I)
C PECED(I)=1 Indicates node is an inlet, DEMAND(WHETHER INLET OR OUTLET)
C IS ALWAYS POSITIVE, DELV IS ELEVATION IN FEET, OPRE AND HT ARE IN LBS
C IN #2 UNITS, EXCEPT WHEN PECED=1 AND MAX
1 = 1 IN WHICH CASE OPRE
C MAY BE IN "FOOT-HEAD OF WATER" UNITS FOR THAT PARTICULAR INLET
37 IF (PECED(I)=1) TOTAL+TOTAL+DELEV(I)
38 IF (PECED(I)=1) TOTAL+TOTAL+DELEV(I)
39 IF (PECED(I)=1) TOTAL+TOTAL+DELEV(I)
40 IF (PECED(I)=1) TOTAL+TOTAL+DELEV(I)
41 IF (HT(I)=1) WRITE(6,2020)I,HT(I)
42 11 CONTINUE
43 IF (TOTAL=0.0) GO TO 100
44 WRITE (6,9100) TOTAL, TOTDEM
45 9100 FORMAT('TOTAL, TOTDEM = ', 1X, F10.2, 2X, *INFLOW')
46 STOP
47 100 DU 12 I=1,NP
48 SD(I)=0.0
49 INDEED(I)=DE
50 IF (DE(I).LT. DIM) DIM=DE
51 SCAN(I)=0.0
52 QI(I)=1.0
53 NPDE(I)=0
54 IF (I .GE. NJ) DEMAND(I)=0.0
55 ML(I)=0
56 IF (ML(I) .EQ. 0) MM=1
57 IF (ML(I) .EQ. 0) LL=1
58 101 READ (5,1000) LOOP(I,J), J=MM, LL
59 C LOOP(I,J) MAY BE POSITIVE OR NEGATIVE ACCORDING TO
60 C ASSUMED DIRECTION OF
61 C FLOW. IF FLUX FROM JUNCTION IS POSITIVE, INFLUX IS NEGATIVE.
62 C REGARD LAST
63 C JUNCTION AS REDUNDANT AND IGNORE IT.
64 C FROM I.EQ. NJ THRU I.EQ. NP
65 C LOOP(I,J) REFERS TO INDIVIDUAL COMPONENTS OF LOOP EQUATIONS
66 C SUM OF
67 C HEAD LOSSES = -0.0 WHILE FROM I.EQ. 1 THRU I.EQ. (NJ-1)
68 C LOOP(I,J) REFERS
69 C INDIVIDUAL COMPONENTS OF CONTINUITY EQUATIONS
70 GO TO 13 J=MM, LL
71 12 CONTINUE
72 IF (UNITS .EQ. 1) B=.1.
73 IF (UNITS .EQ. 2) B=.00228
74 IF (UNITS .EQ. 3) B=.15473
75 9000 FORMAT(9F10.4)
76 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
77 9000 FORMAT(9F10.5)
78 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
79 9000 FORMAT(9F10.5)
80 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
81 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
82 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
83 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
84 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
85 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
86 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
87 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
88 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
89 READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
90 N=1
91 INN=N
92 141 READ(5,1100) (014(I), I=INN)
93 N=N+1
94 INN=N+1
95 1100 FORMAT(10F5.2)
96 IF 16 OR A MULTIPLE OF 16, THEN A BLANK CARD MUST BE INSERTED
IF (DIA(INN=16).NE. 0.0) GO TO 141
147 N*0
98 DO 44 [1.1N1
99 IF (DIA(InN.0.0) N*IN+1
100 IF (DIA(INN).EQ. 0.0) GO TO 223
101 44 CONTINUE
102 223 INN=N1
103 N=0
104 ATD=0.01
105 LPL=0
106 CHANGE=0.0
107 EXTRA=0.0
108 CT2=0.0
109 KXX=0
110 CT=0.0
111 KX=0
112 M=1
113 LK=16
114 IF (MXEq. 0.0) GO TO 118
115 609 READ (3,1000) (MN(I),I=M,LX)
116 LX=LX+16
117 M=M+1
118 IF (LX<16) (LT. MX) GO TO 609
119 DO 374 14.1=MX
120 LM=MN(I)
121 674 HL ine(LR)=1
122 118 DO 14 J=1,1N1
123 IF (D(I).LT. DIMIN) SCAN[I]=D(I)
124 IF (D(I).LT. DIMIN D(I)=DIMIN
125 K(I)=4.*7712**4.87*H(I)*DABS(Q(I)**1.0518/(ROUGH**1)**1.0518*
126 1DDE**4.87)
127 IF (UNITS EQ. 23 K(I)=0.00012/K(I)
128 IF (UNITS EQ. 33 K(I)=2.24416*K(I)
129 DO 14 J=1,1N1
130 16 A(I,J)=0.0
131 602 LL=NJ=1
132 131 DO 16 J=1,LL
133 132 DO 133 J=LJ,LK
134 LNK=LOOP(I,J)
135 LM=ABS(LN)
136 IF (LM.LT. 0.0) A(I,J)=LM+1.0
137 IF (LM.GT. 0.0) A(I,J)=LM-1.0
138 16 CONTINUE
139 102 DO 17 J=1,NP
140 140 LXX=MN(I)
141 DO 141 J=1,1LX
142 LNK=LOOP(I,J)
143 LM=ABS(LN)
144 A(I,J)=LM
145 IF (LJ.LT. 1) A(I,J)=A(I,J-L)
146 17 CONTINUE
147 DO 18 J=1,NP
148 DO 18 J=1,NP
149 18 SAVE(I) =A(I,J)
150 CALL AINV (SAVE,NP,DOOD,LJ,MN,NP,NP)
151 CALL GMPP (SAVE,DPMM,Y,NP,NP,NP,NNP,NP,1/NP,NP,1/NP)
152 DO 19 J=1,NP
153 IF (KX.EQ. 0) Y(J)=Y(I)
154 IF (KX.NE. 0) Y(J)=Y(I)+Y(I)/2.
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156  K(I)=.77*2.5*10(I)*I(RUGH(I))**.8518/(ROUGH(I)**.1*.8518)
157  IF (UNITS .EQ. 2) K(I)=.000125*K(I)
158  IF (UNITS .EQ. 3) K(I)=.2561*K(I)
159  19 CONTINUE
160  IF (KK .EQ. 1) GO TO 102
161  C=0.0
162  DO 20 I=1,4
163  DIFF=KANSIG(I)-Y(I)
164  IF (KK .EQ. 5) GO TO 807
165  IF (DIFF .GT. 0.0) GO TO 102
166  CT=CT+KMK(I)
167  807 CT=CT+.555*G(M)**.12*K(I)/1877.*BUILD+LAC
168  CT=CT+.75/550.*K(I)*.365.*CNSTK(I)**2*2.624
169  20 CONTINUE
170  CT=CT
171  CT=CT
172  652 KK=0
173  129 NL=J-1
174  134 LL=0
175  176 DO 21 L=1, J
176  177 DO 22 K=1, J
178  22 A(I,J)=0.0
179  179 LM=JAMS(TREE(I))
180  180 LN=JGTH(1)
181  181 LD=JENDLM
182  182 IF (LN .EQ. VK) GO TO 113
183  183 IF (LD .EQ. VK) GO TO 113
184  184 IF (PECEDILN) .EQ. 1.0 THEN NL=NL+1
185  185 IF (PECEDILN) .EQ. 1.0 THEN LN=LN+1
186  186 IF (PECEDILD) .EQ. 1.0 THEN K=K+1
187  187 IF (PECEDILD) .EQ. 1.0 THEN L=L+1
188  188 IF (PECEDILN) .NE. 1.0 AND PECEDILD .NE. 1.0 THEN GO TO 113
189  189 IF (NL+K-L,L) .GT. NJ THEN GO TO S01
190  190 DO 23 J=1, NJ
191  23 A(K,L,J)=0.0
192  113 IF (PECEDILN) .EQ. 1.0 THEN NL+K-L,J=L
193  193 IF (PECEDILD) .EQ. 1.0 THEN NL+K-L,J=L
194  194 IF (PECEDILN) .EQ. 1.0 AND MAX .EQ. 13 THEN PRED(R+K+L,J)=PRELM
195  195 IF (PECEDILD) .EQ. 1.0 AND MAX .EQ. 13 THEN PRED(R+K+L,J)=PRELM
196  196 IF (PECEDILN) .EQ. 1.0 AND MAX .EQ. 11 THEN PRED(R+K+L,J)=PRELM
197  197 IF (PECEDILD) .EQ. 1.0 AND MAX .EQ. 11 THEN PRED(R+K+L,J)=PRELM
198  198 IF (NL+K+L,J) .GT. NJ THEN A(K,L,J)
199  199 IF (NL+K+L,J) .GT. NJ THEN A(K,L,J)
200  200 IF (NL+K+L,J) .GT. NJ THEN A(K,L,J)
201  201 IF (NL+K+L,J) .GT. NJ THEN A(K,L,J)
202  202 PRESS(I)=KLM*U(LM)
203  203 IF (TREE(I) .LT. 0) PRESS(I)=PRESS(I)
204  204 IF (NL+K+L,J) .GT. NJ THEN Z(I)
205  205 IF (PECEDILN) .EQ. 1.0 THEN PRESS(I)=PRESS(I)+PRESS(N+K+L)
206  206 IF (PECEDILD) .EQ. 1.0 THEN PRESS(I)=PRESS(I)+PRESS(N+K+L)
207  207 21 CONTINUE
208  21 DO 24 J=1, NJ
209  24 SAVE(I)=A(I,J)
```
CALL MVW (SAVE,NJ,DDD,LL,MM,NJ,NJ)
DO 25 1=1,NJ
OPRE(I)=Y(I-1)-DELEV(I)*62.4/144.
IF (KK .NE. 0) GO TO 104
IF (OPRE(1) .GT. ABMAX) GO TO 105
IF (OPRE(I) .LT. ABMIN) GO TO 105
IF (OPRE(I) .LT. H(I) .AND. H(I) .NE. 0.0) GO TO 105
GO TO 25
105 IF (DUIL .LE. 0.02) GO TO 673
DO 26 J=1,NP
D(I+J,J+1)=0.0
IF (OPRE(I) .GT. ABMAX) GO TO 106
IF (OPRE(I) .LT. ABMIN) GO TO 106
IF (OPRE(I) .LT. H(I) .AND. H(I) .NE. 0.0) GO TO 106
GO TO 25
106 IF (DIFF=ABMAX(1)-DELEV(I)) GO TO 107
IF (DIFF .LE. 0.0001) .AND. CT4 .NE. 0.01 GO TO 108
DIFF=ABS(DELEV(I)-EXTRA)
IF (DIFF .LE. 0.0001) DELEV(I)=EXTRA
IF (DELEV(I) .NE. EXTRA) GO TO 671
DIFF=ABS(DELEV(I)-0.0)
IF (DIFF .LE. 0.0001) GO TO 671
661 DO 271 J=1,NP
D(I+J,J+1)=DELEV(I)+DELEV(I)
GO TO 118
271 CONTINUE
NPipe(1)=1
CHANGE=0.0
EXTRA=0.0
KX=0
GO TO 118
208 DO 209 J=1,NP
S(I,J)=0.0
GO TO 118
209 S(I,J)=DELEV(I)+DELEV(I)
NPipe(1)=1
CHANGE=0.0
EXTRA=0.0
KX=0
GO TO 118
671 CT4=0.0
GO TO 205
25 CONTINUE
DIFF=ABS(EXTRA-0.01)
IF (DIFF .LE. 0.0001) EXTRA=0.0
IF (EXTRA .EQ. 0.01) GO TO 669
DIFF=ABS(DELEV(I)-EXTRA)
IF (DIFF .LE. 0.0001) DELEV(I)=EXTRA
IF (DELEV(I) .EQ. EXTRA) NPipe(1)=1
689 AXX=1
IF (CT4 .EQ. 0.01) GO TO 694
690 IF (CT1 .NE. 0.0 .AND. CT1 .GT. CT4) CT3=CT4
IF (CT1 .NE. 0.0 .AND. CT1 .LT. CT4) CT4=CT1
694 IF (CT1 .EQ. 0.01) GO TO 646
IF (CT3 .GT. CT4 .AND. 0.01 .LE. 0.02) CT3=CT4
273 046 IF (KK ,EQ. 11) GO TO 139
274 IF (CT3 .GT. CT4 .AND. CT4 .NE. 0.01) CT3=CT4
275 IF (CT3 .NE. CT4) GO TO 139
276 IF (D01 .NE. A01) D01=D01/2.
277 DIFF=OABS(CT2-CT3)
278 IF (DIFF .LE. 001 .AND. D01 .LE. 002) GO TO 673
279 IF (D01 .LE. D02) GO TO 139
280 139 NNN=NPI+1
281 KKXK=0
282 KX=1
283 D01=A01
284 EXTR=0.0
285 CHANGE=0.0
286 CT1=0.0
287 LPL=0
288 CT2=CT3
289 NNM=0
290 N=1
291 DD 34 J=1, NM
292 IF (1 .GT. NM) GO TO 666
293 S01=0.0
294 IF (NPPIPE(J) .EQ. 1) NNM=NNN+1
295 IF (NPPIPE(J) .EQ. 11) NNM=1
296 NPP=NPI+1
297 IF (NNN .GT. NPP/2) D01=A01/2.
298 IF (D01 .LE. D02) D01=A01
299 IF (NNN .LT. NP) GO TO 137
300 666 DD 35 J=1, NM
301 A1(J,J)=0.0
302 C1(J,J)=0.0
303 35 CONTINUE
304 34 CONTINUE
305 A1(J,J)=1.0
306 DD 32 L=1, NM
307 DD 33 J=1, NM
308 IF (1 .NE. 11) GO TO 121
309 IF (J .EQ. 11) GO TO 142
310 A1(J,J)=746/5590.24*365.9*CENTS*2.8518*K(J-1)*Q1(J-1)*B*62.4
311 1*=-1.1
312 IF (J .EQ. NM) GO TO 33
313 142 C1(J,J)=.35391.29*Q(J)*.29*Q(J)/1877.*BUILD1+1.71*365
314 1*=-CENTS*8336*Q(J)*Q01(J)*Q(J)/2435.*02
315 IF (NPPIPE(J) .EQ. 11) C1(J,J)=0.0
316 GO TO 33
317 121 IF (J .EQ. 1) GO TO 33
318 LN1=AUS(L00P(I-1)-1).J(J++)
319 IF (L00P(I-1).J(J++)<>120.32.122
320 120 IF (1 .LE. NJJ A1(I,J(J++)<>1.0
321 IF (1 .LE. NJJ C1(I,J++)<>2.*QLN/DILN)
322 IF (NPPIPE(LN) .EQ. 11) C1(LN)=0.0
323 IF (1 .GT. NJJ A1(I,J(J++)<>1.8518*QLN)
324 IF (QLN .LT. 0.0) A1(I,J(J++)<>A1(I,J++)
325 IF (1 .GT. NJJ C1(LN)=1.1664*QLN*QLN/DILN)
326 GO TO 33
327 122 IF (1 .LE. NJJ A1(I,J(J++)<>1.0
328 IF (1 .LE. NJJ C1(J,J++)<>2.*QLN/DILN)
329 IF (NPPIPE(LN) .EQ. 11) C1(LN)=0.0
330 IF (1 .GT. NJJ A1(I,J(J++)<>1.0518*QLN)
331 IF (QLN .LT. 0.0) A1(I,J(J++)<>A1(I,J++)
IF (NPPIPE(LN) .EQ. 1) CL(LN)=0.0
333 CONTINUE
334 DO 50 J=1,NM
335 DO 40 J=1,NM
336 IF (CL(J,NM) .GT. 0.0) GO TO 60
337 CG=J=1-1)NM
338 SAVE(IP)=A(J+1)
339 IF (IP .EQ. NM) GO TO 60
340 CG=J=1-1)NM
341 CHAIN(IP)=CI(J+1)
342 SCAN(1)=0.0
343 CONTINUE
344 CALL MINVE(SAVE,NM,00,LL,NN,NM,NM)
345 CALL GMPRO(SAVE,CHAIN,STAN,NM,NM,NP,NM*np,NM*NPI)
346 N=1
347 DO 30 J=1,NP
348 IF (NPPIPE(N) .EQ. 1) N=N+1
349 NT=1+J(NM)NM
350 NX=1+J(NM)NM
351 IF (DABS(STAN(NK)) .GT. DABS(STAN(NJ))*NM)
352 NT=1+J(NM)NM
353 CI(LJ)=STAN(NK)
354 CONTINUE
355 CONTINUE
356 SCAN(N)+0.0
357 IF (CL(N,N) .GT. 0.0) AND. KXXX .EQ. 0.0) LOOK=1
358 IF (CL(N,N) .LT. 0.0) AND. KXXX .EQ. 0.0) LOOK=0
359 IF (CJL) .GT. 0.0) GO TO 129
360 CALL 672 J=1,INM
361 DIFF=DABS(DEN(DIAN(IJ)))
362 IF (DIFF .LE. 0.0001) DJ=DAJ(IJ)
363 IF (DAJ(IJ) .LT. D(NJ)) GO TO 672
364 IF (KX .EQ. 0.0) DELD(NJ)+DIJ(IJ)-D)
365 IF (D(NJ) .EQ. 0.0) DIJ(IJ) NPPIPE(N)=1
366 IF (D(NJ) .EQ. 0.0) DIJ(IJ) GO TO 118
367 IF (N) (N.IE. 0.0) DELD(NJ)+DIM-D(NJ)
368 IF (D(NJ) .GT. DIM) DELD(NJ)+DIJ(IJ)-D)
369 IF (N=0) DIJ(IJ) DELD(NJ)+DIJ(IJ)-D)
370 EXTRAS=DIJ(IJ)+D)
371 CHANGE=DELD(NJ)
372 LOOK=3
373 IF (K(N,J) .EQ. 0.0) KK=1
374 GO TO 668
375 CALL 672 K=1,NNN
376 IF (CJL) .GT. 0.0) DELD(NJ)=DD
377 NPPNP=1
378 CONTINUE
379 DIFF=DABS(CHANGE-O.0)
380 IF (DIFF .LE. 0.0001) CHANGE=O.0
381 DIFF=DABS(CL(N,N) .O.0)
382 IF (DIFF .LE. 0.0001) CI(N,N)=O.0
383 DD 37 1=1,INP
384 IF (NPPIPE(N) .EQ. 1) NNN=NNN+1
385 IF (C(N,N) .EQ. 0.0) ODL+002
386 IF (C(N,N) .EQ. 0.0) GO TO 654
387 DELD(NJ)+DEL(NJ)+CI(N,N)
388 IF (LOOK .EQ. 1) DELD(NJ)+DEL(NJ)
389 IF (SCAN(1) .EQ. 0.0) DIJ(I)=DEL(1)
390 =0
391 IF (SCAN(1) .NE. 0.0) AND. D(l) .EQ. DIMIN) M=1
392 IF (D(l) .EQ. DIMIN) AND. SCAN(1) .NE. 0.0) D(l)=SCAN(l)+DEL(l)
393 CONTINUE
394 CALL 672 1=1,NNN
393 IF IM .EQ. 11 GO TO 204
394 IF (SCANII .NE. 0.0) AND. (DI11 .NE. DIMIN) GO TO 37
395 801 IF (DI11 .GE. DIMIN GO TO 37
396 LPL=I
397 IF (LPL .NE. NI) SCANII=DI11
398 IF (LPL .NE. NI) GO TO 37
399 DILPL=DILPL-DELDIJ
400 DELDIJ=0.0111-DILPL
401 DD G1,41,LP,1
402 IF (JLT. LPL) (JII+DI11) DELDIJ
403 DDI11-37+37+DELDIJ
404 DILPL=0.0111
405 670 CONTINUE
406 IF (DELDII .EQ. 0.0) DD G1,0 DD 2
407 IF (DELDII .EQ. 0.0) GO TO 651
408 GO TO 127
409 37 CONTINUE
410 GO TO 127
411 127 KKK=0
412 00 78 =I=1,LP,1
413 IF (DII .LT. DIMIN) SCANII=DIII
414 IF (DII .LT. DIMIN) DII=DIMIN
415 KII=5.777*1.877+11*(OABSQIIII)**0.8518/ROUGHII**1.8518
416 IF (UNITS .EQ. 2) KII=-0000121KII
417 IF (UNITS .EQ. 3) KII=224416KII
418 IF (UNITS .EQ. 4) KII=444416KII
419 IF (UNITS .EQ. 5) KII=666616KII
420 DD G1,41,LP,1
421 70 ALL(J)=0.0
422 DO 79 =1,LP,1
423 LXX=NII(J)
424 DD 65 =J=1,LXK
425 LII=IABSLII(J)
426 ALL(L)=LII
427 IF (LII .LT. 0.0) AII,LII=0.0
428 05 CONTINUE
429 79 CONTINUE
430 144 DD 82 =J,N,LP,1
431 LII=LII(J)
432 DO 83 =J=1,LP,1
433 LII=IABSLII(J)
434 ALL(LM)=LII
435 IF (LII .LT. 0.0) AII,LII=0.0
436 83 CONTINUE
437 82 CONTINUE
438 DD G1,1=1,LP,1
439 DD G1,1=1,LP,1
440 1=I=J+1,LP,1
441 64 SAVE(GII)=ALL(I)
442 CALL MIVY (SAVE,NP,DD,LL,NN,NP,NP)
443 CALL GMPOD (SAVE,DIMNO,Y,NP,DP,NN,NP,NP,1,1,1,1)
444 DD 85 =1,LP,1
445 84 IF (KII .EQ. 0) QII=1
446 IF (KII .EQ. 0) QII=1
447 KII=5.777*1.877+11*(OABSQIIII)**0.8518/ROUGHII**1.8518
448 IF (UNITS .EQ. 2) KII=-0000121KII
449 IF (UNITS .EQ. 3) KII=224416KII
450 IF (UNITS .EQ. 4) KII=444416KII
451 IF (UNITS .EQ. 5) KII=666616KII
452 85 CONTINUE
453 KKK=KII+1
454
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513 IF (D11 .EQ. 0.0) GO TO 203
514 IF (M .EQ. 1.0) GO TO 203
515 IF (SCAL(I) .NE. 0.0) GO TO 203
516 CONTINUE
517 D01 = D01/2.
518 IF (X .EQ. 1.0) GO TO 621
519 DIFF = DIFF*(C12-C13)
520 IF (D01 .LE. D02 .AND. DIFF .LE. 0.0) GO TO 673
521 IF (D01 .LE. D02) GO TO 118
522 IF (DIFF/ABS(EXTRA-E0) .LE. 0.0) GO TO 624
523 IF (EXTRA .NE. 0.0) GO TO 627
524 IF (X .EQ. 2.0) GO TO 139
525 GO TO 123
526 IF (CT-A.0) GO TO 137
527 120 FORMAT (/40X, 'RESULTS OF OPTIMAL TRIAL')
528 WRITE (6,1200)
529 1200 FORMAT (/18X, 'CAPITAL COST ',F8.2,1X)
530 1201 FORMAT (/18X, 'OPTIMAL COST ',F8.2,1X)
531 1202 FORMAT (/18X, 'RESULTS OF OPTIMAL TRIAL')
532 WRITE (6,1200)
533 WRITE (6,1201)
534 1204 FORMAT (/10X, 'A = INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY RESULTANT INVERSE')
535 CALL MINV(A,N,O,L,MINV)
536 1205 FORMAT (/10X, 'RESULTANT INVERSE')
537 CALL MINV(A,N,O,L,MINV)
538 RETURN
539 END
```

**SUBROUTINE MINV**

**PURPOSE**

**INVERT A MATRIX**

**USAGE**

CALL MINV(A,N,O,L,MINV)

**DESCRIPTION OF PARAMETERS**

A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY RESULTANT INVERSE.

N - ORDER OF MATRIX A

O - RESULTANT DETERMINANT

L - WORK VECTOR OF LENGTH N

M - WORK VECTOR OF LENGTH N

**REMARKS**

MATRIX A MUST BE A GENERAL MATRIX

MINV 10

MINV 20

MINV 30

MINV 40

MINV 50

MINV 60

MINV 70

MINV 80

MINV 90

MINV 100

MINV 110

MINV 120

MINV 130

MINV 140

MINV 150

MINV 160

MINV 170

MINV 180

MINV 190

MINV 200

MINV 210
574  
575  K=I+1
576  HULD=Ali+J
577  J=I+K+J
578  10  AI(J) = HULD

C  
C      INTERCHANGE COLUMNS
C  
579  35  I=K(J)
580  IF(I-K) = 5, 45, 38
581  38  J=NP+1-J
582  DO  40  J=1-N
583  J=H(J)+J
584  40  AI(J)=HOLD

C  
C      DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS CONTAINED IN BIGAI)
C  
585  45  IF(I+1=J)  45, 46, 48
586  46  D=0.0
587  RETURN
588  48  DO  55  I=1-N
589  IF(I-K) = 50, 55, 50
590  50  I=H(I)+1
591  AI(I)=AI(I)+BIGAI
592  55  CONTINUE

C  
C      REDUCE MATRIX
C  
593  56  GO TO 1-N
594  57  I=K+1
595  58  HOLD=AI(I)
596  59  J=1-I
600  60  GO  55  J=1-N
601  61  I=I+J
602  62  IF(I-K) = 50, 55, 60
603  63  H=H(J)+K
604  64  DO  70  J=1-K
605  70  AI(J)=HOLD+AI(K)+AI(I)
606  75  CONTINUE
607  76  CONTINUE

C  
C      DIVIDE ROW BY PIVOT
C  
608  77  KJ=K-N
609  DO  75  J=1-N
610  78  KJ=K+J
611  79  IF(J-K) = 70, 75, 70
612  80  AI(KJ)=AI(KJ)+BIGA
613  85  CONTINUE

C  
C      PRODUCT OF PIVOTS
C  
614  86  D=0*BIGA

C  
C      REPLACE PIVOT BY RECIPROCAL
C  
615  87  AI(KJ)=1.0/BIGA
80 CONTINUE

FINAL ROW AND COLUMN INTERCHANGE

K=IN
100 IF(K) 150,150,105
105 L=L(K)
J=[F(I-K) 100,120,108
108 JK=IK+J
110 HOLD(LJK)
J=J+J
115 ALKI=AIJ
120 AIKI=AIJ
125 IF(J-K) 100,100,125
130 DO 130 I=1,N
135 K=IK+H
140 HOLD=LKI
145 J=J-K+J
150 GO TO 100
155 RETURN

END

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SUBROUTINE GMPRO

PURPOSE
MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL
MATRIX

USAGE
CALL GMPRO(A,B,R,N,M,L)

DESCRIPTION OF PARAMETERS
A - NAME OF FIRST INPUT MATRIX
B - NAME OF SECOND INPUT MATRIX
R - NAME OF OUTPUT MATRIX
N - NUMBER OF ROWS IN A AND ROWS IN B
M - NUMBER OF COLUMNS IN A AND COLUMNS IN B

REMARKS
ALL MATRICES MUST BE STORED AS GENERAL MATRICES
MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A
MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B
NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWS OF MATRIX B
SUBPROGRAMS REQUIRED
NONE

METHOD
THE N BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320

GMPR 290
AND THE RESULT IS STORED IN THE N BY L MATRIX R.

SUBROUTINE GMPRD (A,B,R,N,M,L,NTM,MTL,NTL)

DIMENSION A(NTM), B(MTL), R(NTL)

DOUBLE PRECISION A,B,R

IR=0
IK=1
DO 10 K=1,L
IK=IK+M
DO 10 J=1,N
IR=IR+1
JI=J-J-1
IK=IK
R(IR)=0
DO 10 I=1,M
JI=J-J+1
IM=IM+1
10 R(IR)=R(IR)+A(JI)*B(I0)
RETURN
END