ADAPTIVE SCHEDULING FOR OPERATING ROOM MANAGEMENT

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ADAPTIVE SCHEDULING FOR OPERATING ROOM MANAGEMENT

THESIS

A thesis submitted in partial fulfillment of the requirement for the degree of Master of Science in Mechanical Engineering in the College of Engineering at the University of Kentucky

By
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Lexington, Kentucky
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2017
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ABSTRACT OF THESIS

ADAPTIVE SCHEDULING FOR OPERATING ROOM MANAGEMENT

The perioperative process in hospitals can be modelled as a 3-stage no-wait flow shop. The utilization of OR units and the average waiting time of patients are related to makespan and total completion time, respectively. However, minimizations of makespan and total completion time are $NP$-hard and $NP$-complete. Consequently, achieving good effectiveness and efficiency is a challenge in no-wait flow shop scheduling. The average idle time (AIT) and current and future idle time (CFI) heuristics are proposed to minimize makespan and total completion time, respectively. To improve effectiveness, current idle times and future idle times are taken into consideration and the insertion and neighborhood exchanging techniques are used. To improve efficiency, an objective increment method is introduced and the number of iterations is determined to reduce the computation times. Compared with three best-known heuristics for each objective, AIT and CFI heuristics can achieve greater effectiveness in the same computational complexity based on a variety of benchmarks. Furthermore, AIT and CFI heuristics perform better on trade-off balancing compared with other two best-known heuristics. Moreover, using the CFI heuristic for operating room (OR) scheduling, the average patient flow times are decreased by 11.2% over historical ones at University of Kentucky Health Care.

KEYWORDS: Operating Room Scheduling, No-wait Flow Shop, Makespan and Total Completion Time, Trade-off Balancing, Heuristics.

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Chapter 1 Introduction

1.1 Background

Operating rooms (OR) are the most cost and revenue intensive areas in hospitals. In 2002, there were 36.5 million hospital stays in the United States, with an average length of stay of 4.5 days and an average cost of $10,400 per stay. About 21.8% of hospital stays in 2012 were surgical (Weiss and Elixhauser, 2014). In surgical procedures, ORs have been estimated to account for over 40% of total expenditure of a hospital (Denton et al., 2007). On the one hand, the operating rooms are one of most important resources, which has the largest cost and revenues (Ghazalbash et al, 2012). Mean hospital costs of surgical stays are $21,200 in 2012, which is 2.5 times the mean costs of $8,500 for medical stays and nearly 5 times the mean costs of $4,300 for maternal and neonatal stays (Moore et al, 2014). On the other hand, because of the aging population, there is a sharply increasing trend of demand for surgical services in recent years (Etzioni et al., 2003). There were around 51 million inpatient surgical procedures performed in the United States in 2010, according to the latest data from the Centers for Disease Control and Prevention (Hall and Owings, 2014).

Long waiting time, a large number of emergencies, and resource overload are coming along with this increasingly demand in healthcare systems (Meskens et al, 2013). Therefore, hospital managers are continuously looking for new methods to increase the utilization of OR units and to reduce the average waiting time of patients (Cardoen et al., 2010). From flow shop perspective, the utilization of an OR unit is defined as the workload divided by the completion time of the last patient, where workload is the sum of case times. The average waiting time is defined as the length of time that a patient stays in the
perioperative (periop) process, which consists of the preoperative (preop), intraoperative (intraop), and postoperative (postop) stages. Average waiting time can be represented by the average flow time in a flow shop, which is the total completion time divided by the number of patients. Minimization of maximum completion time or makespan can improve OR utilization, which directly reduces surgical overtime and its cost. Minimization of average waiting time can improve patient flow through the periop process, which improves patient throughput and generates more revenue.

1.2 Motivation

OR scheduling is extraordinarily complicated. The scheduling process must take different surgical specialties into consideration, each of which has different priorities, procedures, and case times. It also should consider different resources for specialties and surgical procedures. Resources include human resources (e.g., surgeons, anesthesiologists, nurses, and staff), equipment used in different periop stages (e.g., induction equipment, surgical instruments, and electro-medical equipment). In addition, the scheduling process must take into account the disturbances across the periop process, such as emergency cases, cancellations, or no-shows, variations in case times due to surgical complexities, post-anesthesia care units (PACU) boarding, and length of stay in the periop process (Bosse et al., 2013). Another consideration is about different preferences of stakeholders involved in scheduling processes (Glouberman and Mintzberg, 2001), which might possibly be in conflict.

Typically, the three stages of the periop process are shown in Figure 1.1 (Gupta, 2007). There are many operations in each stage. For example, the collection of patient information and the preparation for surgeries occur in the preop stage, surgeries occur in
ORs in the intraop stage, and PACU, intensive care units (ICU), or ward for recovery are in the postop stage. Each stage requires different resources to accommodate specific patient needs. For simplicity, we model the OR scheduling problem across the periop process as a 3-stage no-wait flow shop.

The three stages in the periop process are tightly coupled, because performance of one stage affects the performance of adjacent stages. This is also the characteristic of a 3-stage no-wait flow shop. For example, the delay of patients from a preop unit to ORs lowers OR utilization, especially for the first case (Roberts et al., 2015), which is the performance of stage 1 affects that of stage 2. Marcon and Dexter (2006) studied the impact of OR performance on PACU staffing, and found the performance of stage 2 affects that of stage 3. These two examples show how upstream stages affect downstream stages. Downstream stages can affect upstream stages as well. For example, when all postop beds are occupied, blocking occurs between the intraop and postop stages. Consequently, case times are extended because patients cannot be transferred out of ORs in the absence of recovery beds in PACU or ICU (Augusto et al., 2010; Wang et al., 2015). This scenario is accentuated with PACU boarding, which means patients stay in PACU overnight (Price et al., 2011). Therefore, the 3-stage no-wait flow shop is suitable to model the periop process, in which no waiting time between stages is allowed.

Figure 1.3: The perioperative process
To evaluate OR scheduling across the periop process, there are two main performance measures: OR utilizations and average waiting time. From flow shop perspective, \( C_{\text{max,2}} \) as the maximum completion time of the intraop stage affects OR utilization, because utilization equals to the workload divided by the working period, and the working period is equal to the completion time of the last job \( (C_{\text{max,2}}) \) minus the start time of the first job. \( \bar{C} = \sum C_{j,3}/n \) as the average completion time of the postop stage affects average waiting time, which is the total completion time in the postop stage \( (\sum C_{j,3}) \) divided by the number of patients \( (n) \) (Pinedo, 2014). There are several other objectives to evaluate the performance of the periop process. The objective of throughput is closely related to the average patient waiting time. According to the Little’s Law, the average inventory in a system equals the average cycle time (which includes waiting time and processing time) times the average throughput (Little, 1961). The objective of leveling resources is mainly to develop a schedule by smoothing resource occupancies without over usage. Leveling resources involves the utilization and average flow time in each of the three stages across the periop process. Therefore, maximum completion time and average completion time are most important performance measures, which define the utilizations and average waiting times.

ORs are the largest cost center and the greatest revenue source simultaneously for hospitals (Ghazalbash et al, 2012). OR scheduling affects the progression of surgical cases going through the periop process. Most hospitals use a three-phase block scheduling framework to plan this progression. OR planning is phase 1, focusing on long-term strategies, where resources and services are allocated to OR blocks. OR scheduling is phase 2, focusing on medium-term tactics, where a master surgical schedule (MSS) is generated.
The MSS generates the number of available surgical suites, operation hours, and OR block times for a type of services. Case sequencing is phase 3, focusing on short-term execution of MSS, where daily surgical cases are sequenced by operating rooms (Banditori et al., 2013, Cardoen et al., 2010).

Different sequencing methods can address stakeholders’ objectives differently. For example, the longest processing time (LPT) rule is recommended to improve OR utilization (Magerlein and Martin, 1978; Gupta and Denton, 2008). The shortest processing time (SPT) rule is recommended to reduce the number of case delays and to speed up patient flows across the periop process (Testi et al., 2007). Both approaches give rise to schedule slippage in that we cannot generate the best solutions of OR utilization and patient time simultaneously through LPT and SPT rules. Therefore, it’s of great interest and importance to balance different performance measures and to achieve adaptive scheduling and control.

1.3 The challenges in no-wait flow shop scheduling

Given the above complexities in OR scheduling across the periop process, we model it as min($C_{max}$ and $\sum C_j$) problems for a 3-machine no-wait flow shop. Therefore, we have the following challenges in no-wait flow shop scheduling.

For the convenience of describing scheduling problems in no-wait flow shop, we use $F_m |nwt| C_{max}$ to denote minimization of makespan and $F_m |nwt| \sum C_j$ to denote minimization of total completion time, where $F_m$ is for a flow shop problem with $m$ machines, $nwt$ for the constraint of no-wait, and $C_{max}$ for the objective to minimize maximum completion time and $\sum C_j$ for the objective of minimize total completion time (Graham et al., 1979).
No-wait flow shop scheduling problems are categorized as combinatorial optimization problems in which the feasible region is countable (Garey and Johnson, 2002). The complexity of different classes of problems in combinational optimization is shown in Figure 1.2. $F_m |nwt| C_{max}$ problems have been proved to be $NP$-hard when the number of machines is larger than or equal to three, and $F_m |nwt| \sum C_j$ problems are $NP$-complete when the number of machines is larger than or equal to two (Garey and Johnson, 2002; Röck, 1984). For an $NP$-complete or $NP$-hard problem, we cannot describe the problem by polynomials completely, or in other words, we cannot optimally solve the problem in a polynomial time. As a result of the $NP$-hardness or $NP$-completeness, it is extremely time consuming to find optimal solutions by using exact methods even for moderate-scale problems (Ding et al., 2014). Consequently, the complexity of these problems makes it difficult to optimally improve OR utilization and/or reduce average waiting time, although optimal solutions can be derived for 2-machine flow shop production to minimize $C_{max}$ (Johnson, 1954), and for 1-machine production to minimize $\sum C_j$. (Pinedo, 2014).

Figure 1.4: Decision making problems and complexity (Samarghandi, 2011)
Given $\bar{C} = \frac{\sum C_j}{n}$, we can see that the completion time of the last job $n$ ($C_{max}$) is included in $\sum C_j$. If $n$ is fixed, minimization of total completion time, $\min(\sum C_j)$, is the same as $\min(\bar{C})$. However, $\min(C_{max})$ does not necessarily mean $\min(\sum C_j)$, or vice versa, although $C_{max}$ is included in $\sum C_j$. These two scheduling objectives are inconsistent. In the investigation of OR scheduling methods (Li et al., 2014), the authors found these two common OR scheduling objectives were inconsistent. One objective of $\min(C_{max})$ is to minimize the maximum completion time for the last surgical case of the day. The second scheduling objective of $\min(\sum C_j)$ is to minimize the total completion time of an OR’s daily slate, which is analogous to minimizing the average completion time if the number of surgeries ($n$) is fixed, i.e., to $\min(\bar{C})$, where $j = 1, \ldots, n$. This inconsistency partially explains why delays can occur between any two perioperative stages -- improving utilization in any stage may reduce patient flow out of that stage, i.e., minimizing $C_{max}$ may maximize $\sum C_j$. Consequently, such inconsistency between $\min(C_{max})$ and $\min(\sum C_j)$ makes it more difficult to improve OR utilization and reduce average waiting time at the same time, which is a multi-objective optimization problem.

As OR management concerns evolve, on the one hand, the system changes over time and the relationship among system components creates inconsistencies in system performance (Davis et al., 2013; Beck et al., 2014; Pellegrino, 2015). Therefore, an OR scheduling process must adapt to changing relationships and facilitate OR management as concerns evolve. On the other hand, OR schedules are constructed as static timetables, with little ability to adapt to dynamic changes in demand (e.g., emergency cases or case time variation). With the time goes by, OR planning, scheduling, and control are interacted with each other. For example, if the performance at this time is not good as expected, it may
affect the schedulers to adjust scheduling at next time, or even affect the OR manager to change resource allocation at the next planning time. Consequently, the complexity of the interaction between OR planning, scheduling, and control makes it difficult to achieve adaptive scheduling in perioperative process.

1.4 The contribution

The contribution of our work comes from three aspects: (1) new methods to \( \min(C_{\text{max}}) \) and \( \min(\sum C_j) \) for no-wait flow shop respectively; (2) a trade-off balancing function to evaluate trade-off between \( C_{\text{max}} \) and \( \sum C_j \); (3) a validation of the CFI heuristic based on the historical data at University of Kentucky HealthCare (UKHC), along the time horizon.

First, we propose an initial sequence algorithm (ISA), based on which we propose an average idle time (AIT) heuristic to \( \min(C_{\text{max}}) \), and a current and future idle time (CFI) heuristic to \( \min(\sum C_j) \) for no-wait flow shop scheduling. In the ISA, we treat current idle time and future idle time differently by a lever concept introduced in Li et al (2011). Consequently, in the initial sequence, we assign higher weights to current idle times generated by jobs in the head of a sequence than those generated by jobs in the tail of the sequence. In both AIT and CFI heuristics, search techniques of insertion and neighborhood exchanging are used to further improve the solutions generated by the ISA. Based on a variety of benchmarks and randomly generated instances, our AIT and CFI heuristics perform better than other best-known existing heuristics for no-wait flow shop scheduling.

Second, we introduce a trade-off balancing (TOB) function for no-wait flow shop scheduling to evaluate trade-off between \( C_{\text{max}} \) and \( \sum C_j \). In the evaluation scheme, we assign
different preferences to each objective. The results show that the proposed heuristics perform better than the LS (Laha and Sapkal, 2014) and CH (Li et al., 2008) heuristics.

Third, we use statistical process control (SPC) and control charts to validate our CFI heuristic for operating room (OR) scheduling across the periop process in a healthcare system. The results indicate that potentially 3,000 additional patients could be served in a year if our CFI heuristic was applied for sequencing.

1.5 The structure of this thesis

The rest of the thesis is organized as follows:

The chapter 2 provides a thorough literature review, including current status of heuristics to minimize the $C_{\text{max}}$ and $\sum C_j$, current status of multi-objective optimizations in no-wait flow shop scheduling, and statistical process control in OR scheduling.

The chapter 3 gives the methodology of the proposed heuristics. The problem description of no-wait flow shop is provided first. Then the initial sequence algorithm (ISA) is proposed to generate the initial sequence. Based on ISA, the AIT and CFI heuristics are given in detail to minimize $C_{\text{max}}$ and $\sum C_j$, respectively. The increment objective method is introduced to reduce the computational complexity from $O(n)$ to $O(1)$ when using neighborhood exchanging technique to calculate the total completion time.

The chapter 4 provides the results of case study. First, the performance of AIT and CFI heuristics will be compared with other existing heuristics based on a variety of benchmarks and generated data. Second, by using trade-off balancing (TOB) function, based on the Taillard’s benchmarks, the performances of the trade-off between $C_{\text{max}}$ and $\sum C_j$ for each heuristic are compared. Last, based upon the historical data at the University of Kentucky HealthCare, the SPC charts with the CFI heuristic and UKHC are presented.
The chapter 5 presents the conclusions and directions for future research.
Chapter 2 Literature review

This chapter focuses on the literature review on three topics. First, heuristics currently for single objective optimization of no-wait flow shop scheduling are introduced, which are to minimize maximum completion time ($C_{\text{max}}$) and total completion time ($\sum C_j$), respectively. Second, heuristics currently for multi-objective optimizations of no-wait flow shop scheduling are presented. Finally, statistical process control (SPC) methods are introduced for operating room scheduling.

2.1 Heuristics for no-wait flow shop to minimize $C_{\text{max}}$ and $\sum C_j$

Minimizations of $C_{\text{max}}$ and $\sum C_j$ are $NP$-hard and $NP$-complete problems for no-wait flow shop production, and it is extremely time consuming to find optimal solutions using exact methods for such problems. There are mainly two ways to find near-optimal solutions by using heuristics and meta-heuristics (Ruiz and Maroto, 2005). The heuristics can be grouped into constructive heuristic and improvement heuristics. The constructive heuristics build a feasible schedule from scratch, such as the NEH heuristic (Nawaz et al., 1983), while the improvement heuristics improve the performance of feasible schedules by applying some search techniques, such as neighborhood exchanging (Dannenbring, 1997). Meta-heuristics start from an initial schedule constructed by constructive or improvement heuristics, and generate better performance by iterations until a stopping criterion is satisfied, such as the computation time, the number of iterations, etc. Typical examples of meta-heuristics are simulated annealing (Ogbru and Smith, 1990), tabu search (Moccellin, 1995), genetic algorithms (Murata et al., 1996). Compared to heuristics, meta-heuristics can generate better results in general, but have much higher computational complexities.
and take much longer computation time to solve even for moderately scaled instances. Consequently, with such a high computation burden, meta-heuristics are not commonly applied in industry where problem scales are changing from moderate to large. Therefore, heuristics with small computational efforts are proposed to minimize $C_{max}$ and $\sum C_j$ respectively in the thesis for adaptive scheduling.

2.1.1 Heuristics for no-wait flow shop to minimize $C_{max}$

Table 2.1 shows a summary of the heuristics for the $F_m \mid n\text{wt} \mid C_{max}$ problems reviewed in a chronological order.

<table>
<thead>
<tr>
<th>Year</th>
<th>Author(s)</th>
<th>Acronym</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976</td>
<td>Bonney and Gundry</td>
<td>BG</td>
<td>Based on a slope index</td>
</tr>
<tr>
<td>1980</td>
<td>King and Spachis</td>
<td>KS</td>
<td>Minimum covering level</td>
</tr>
<tr>
<td>1993</td>
<td>Gangadharan and Rajendran</td>
<td>GR</td>
<td>The jobs with increasing trends in processing time are processed ahead</td>
</tr>
<tr>
<td>1994</td>
<td>Rajendran</td>
<td>RAJ</td>
<td>Adjacent jobs match and put last job with short processing time</td>
</tr>
<tr>
<td>2008</td>
<td>Framinan and Nagano</td>
<td>FN</td>
<td>Based on Farthest Insertion Travelling Salesman Procedure</td>
</tr>
<tr>
<td>2008</td>
<td>Li et al.</td>
<td>CH</td>
<td>Based on job insertion and interchange techniques</td>
</tr>
<tr>
<td>2009</td>
<td>Laha and Chakraborty</td>
<td>LC</td>
<td>Based on job insertion considering two consecutive jobs as a block</td>
</tr>
<tr>
<td>2016</td>
<td>Ye et al.</td>
<td>ADT</td>
<td>Based on average departure time</td>
</tr>
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</table>

Bonney and Gundry (1976) and King and Spachis (1980) pioneered constructive heuristics for solving $F_m \mid n\text{wt} \mid C_{max}$ problems. Bonney and Gundry (1976) proposed a slope
index method to sequence jobs. King and Spachis (1980) proposed a minimum covering level (MCL) method for solving $F_m \mid nwt \mid C_{max}$ problems.

Gangadharan and Rajendran (1993) and Rajendran (1994) proposed two heuristics, named GR and RAJ, to solve the same problem. GR heuristic first sequences jobs in an increasing or decreasing trend of their times, then uses an insertion technique to improve the performance of the initial sequence. RAJ heuristic makes the adjacent jobs “match”, similar to the GR heuristic based on the increasing or decreasing trend of a job, as much as possible in order to minimize the inter-job delays and has the last job with short processing time. The computational results showed that GR and RAJ heuristics were superior to the heuristics proposed by BG heuristic (Bonney and Gundry, 1976) and KS heuristic (King and Spachis, 1980).

Framinan and Nagano (2008) proposed a new heuristic based on Farthest Insertion Travelling Salesman Procedure (FITSP) (Syslo, 1983), and compared this new heuristic with random ordering, descending sum of processing times, and RAJ initial sequence (Rajendran, 1994). The experimental results showed that the new heuristic performed better than other three heuristics.

Based on the objective increment method, Li et al. (2008) proposed a composite heuristic (CH) using job insertion and exchange techniques, and experimental results showed that the CH heuristic performed better than the GR (Gangadharan and Rajendran, 1993) and RAJ (Rajendran, 1994) heuristics and used the least CPU time for the same instances.

Laha and Chakraborty (2009) proposed a constructive heuristic (LC) to solve $F_m \mid nwt \mid C_{max}$ problems. The principle of job insertion in the LC heuristic is that every two
consecutive jobs are selected as a block from the initial sequence, which is to be inserted into a partial sequence, and each job in the block is inserted into each possible position of the partial sequence. Through each insertion, choose the best partial sequence with the smallest makespan and update the partial sequence until all jobs from the initial sequence have been inserted into the partial sequence. The computational results showed that the LC heuristic was significantly better than the GR (Gangadharan and Rajendran, 1993), RAJ (Rajendran, 1994) heuristics.

Ye et al. (2016) proposed an average departure time (ADT) heuristic to minimize $C_{\text{max}}$ for no-wait flow shop production. They first proposed the initial sequence based on the average of idle times, and then use group and insertion techniques to improve the initial solutions. The computational results showed that the ADT heuristic performed better than GR (Gangadharan and Rajendran, 1993), RAJ (Rajendran, 1994), and modified NEH (Nawaz et al., 1983) heuristics.

Overall, many researchers adopt the insertion and exchange techniques to improve the initial solutions for solving $F_m |n\text{wt}| C_{\text{max}}$ problems. However, the properties of no-wait flow shop scheduling still needs investigation to generate more effective and efficiency heuristics.

2.1.2 Heuristics for no-wait flow shop to minimize $\sum C_j$

Table 2.2 shows a summary of the heuristics for the $F_m |n\text{wt}| \sum C_j$ problems reviewed in a chronological order.
Table 2.2: The summary of heuristics to minimize $\sum C_j$

<table>
<thead>
<tr>
<th>Year</th>
<th>Author(s)</th>
<th>Acronym</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>Rajendran and Chaudhuri</td>
<td>RC</td>
<td>Based on preference relations</td>
</tr>
<tr>
<td>2000</td>
<td>Bertolissi</td>
<td>BER</td>
<td>Temporary flow time</td>
</tr>
<tr>
<td>2004</td>
<td>Aldowaisan and Allahverdi</td>
<td>PH1(p)</td>
<td>NEH insertion scheme and pair-wise exchange</td>
</tr>
<tr>
<td>2010</td>
<td>Framinan et al.</td>
<td>FNM</td>
<td>Insertion and exchange neighborhood techniques</td>
</tr>
<tr>
<td>2013</td>
<td>Gao et al.</td>
<td>IB</td>
<td>Improved Bertolissi heuristic</td>
</tr>
<tr>
<td>2013</td>
<td>Sapkal and Laha</td>
<td>SL</td>
<td>Priority on the bottleneck</td>
</tr>
<tr>
<td>2014</td>
<td>Laha et al.</td>
<td>PSI</td>
<td>Penalty-shift-insertion scheme</td>
</tr>
<tr>
<td>2014</td>
<td>Laha and Sapkal</td>
<td>LS</td>
<td>Based on average departure time</td>
</tr>
</tbody>
</table>

Rajendran and Chaudhuri (1990) proposed an RC heuristic based on the preference relations for the $F_m \mid nwt \mid \sum C_j$ problems. According to the results of computational experiments, their RC heuristic was more effective on the $F_m \mid nwt \mid \sum C_j$ problems than BG heuristic (Bonney and Gundry, 1976) and KS heuristic (King and Spachis, 1980).

Bertolissi (2000) proposed a BER heuristic to min ($\sum C_j$) based on an initial sequence and job insertion technique. The initial sequence is generated by comparing temporary flow times of each pair of jobs. Job insertion technique is applied to improve the performance by the initial sequence. The computational results showed that the BER heuristic performed better than RC heuristic (Rajendran and Chaudhuri, 1990) and BG heuristic (Bonney and Gundry, 1976).

Aldowaisan and Allahverdi (2004) proposed six improved heuristics by using three different search methods, first by the same insertion scheme as in the NEH heuristic (Nawaz et al., 1983), second by the same insertion technique as in Rajendran and Ziegler
(1997), and third by the adjacent pair-wise neighborhood exchanging method. The NEH heuristic is considered to be the best constructive heuristic to minimize makespan for permutation flow shop production (Kalczynski and Kamburowski, 2007). Among the six improved heuristics, the PH1(p) heuristic performed significantly better than the heuristic proposed by RC heuristic (Rajendran and Chaudhuri, 1990) and the genetic algorithm proposed by Chen et al (1996).

Framinan et al. (2010) proposed an FNM constructive heuristic to minimize total completion time based on insertion and exchange neighborhood techniques. The results of their case studies showed that the FNM heuristic performed better than the RC heuristic proposed by Rajendran and Chaudhuri (1990), the PH1(p) heuristic by Aldowaisan and Allahverdi (2004), BER heuristic by Bertolissi (2000), and the heuristic by Fink and Voß (2003).

Using the constructive procedure as in Laha and Chakraborty (2009), Gao et al. (2013) proposed two constructive heuristics, the improved standard deviation (ISD) heuristic and the improved Bertolissi (IB) heuristic, which were developed from the standard deviation heuristic by Gao et al. (2011) and the BER heuristic (Bertolissi, 2000), respectively. The results of their case studies showed that the IB heuristic performed better than the NEH (Nawaz et al., 1983) and BER heuristic (Bertolissi, 2000).

Sapkal and Laha (2013) proposed an efficient heuristic (SL heuristic) to minimize total flow time. The initial sequence is generated based on the assumption that the priority of a job in the initial sequence is given by the sum of processing times on the bottleneck machine. An insertion technique, the same as that in LC heuristic proposed by Laha and Chakraborty (2009), is applied to improve the performance of the initial sequence. The
results showed that the SL heuristic performed better than RC heuristic (Rajendran and Chaudhuri, 1990) and BER heuristic (Bertolissi, 2000).

Laha et al. (2014) proposed a penalty-shift-insertion (PSI) heuristic for $F_m | nwt | \sum C_j$ problems, and their computational experiments showed that the PSI heuristic was relatively more effective and efficient than other heuristics in the literature at the time.

Recently, Laha and Sapkal (2014) proposed an improved LS heuristic, and results showed that the LS heuristic performed better than the PH1(p) heuristic (Aldowaisan and Allahverdi, 2004) and the FNM heuristic (Framinan et al., 2010).

### 2.2 Heuristics for multi-objective optimization in no-wait flow shop

Although in the past decades, efforts have been made to obtain high-quality solutions with acceptable computation times by optimizing a single objective, multi-objective optimization is more reasonable for flow shop production scheduling in reality, because some objectives are inconsistent, such as $\min(C_{\text{max}})$ and $\min(\sum C_j)$ as indicated in Li et al., (2014). Allahverdi and Aldowaisan (2002) proposed a PAAH heuristic to minimize a weighted sum of makespan and total completion time based on insertion and exchange techniques. By their computational results, the PAAH heuristic performed better than existing heuristics for the single objective of $C_{\text{max}}$ and $\sum C_j$, such as RC heuristic (Rajendran and Chaudhuri, 1990), GR (Gangadharan and Rajendran, 1993), RAJ (Rajendran, 1994), and a genetic local search algorithm for multi-objective in flow shop (Ishibuchi and Murata, 1998).

Liao et al. (2008) proposed an evolutionary algorithm and Liu et.al (2008) proposed a new hybrid genetic algorithm. Both methods are for no-wait flow shop production to minimize both makespan and total flow time. A non-dominated sorting strategy and an
objective increment strategy are integrated into these two methods. Their experimental results showed that the proposed methods outperformed the PAAH heuristic (Allahverdi and Aldowaisan, 2002) and other heuristics.

There are several other multi-objectives for no-wait flow shop scheduling. Allahverdi and Aldowaisan (2004) proposed hybrid simulated annealing (SA) and genetic algorithm (GA) algorithms for the no-wait flow shop problem with makespan and maximum lateness criteria, and showed that the hybrid approach was efficient. Pan et.al (2008) proposed a novel particle swarm optimization algorithm for no-wait flow shop scheduling problems with makespan and maximum tardiness criteria. Jevadi et al. (2008) proposed a fuzzy multi-objective linear programming (FMOLP) model to minimize the weighted mean completion time and weighted earliness. This model provided a systematic framework that facilitated the fuzzy decision-making process until a satisfactory solution was obtained. Ruiz and Allahverdi (2009) proposed local search methods to minimize the weighted sum of makespan and maximum lateness. The local search methods are mainly based on the genetic algorithms and iterated greedy procedures. The computational results showed that their algorithms performed better than the PAAH heuristic (Allahverdi and Aldowaisan, 2002) and the heuristic proposed by Allahverdi and Aldowaisan (2004). Pan et al (2009) proposed a novel discrete differential evolution (DDE) algorithm to minimize makespan and maximum tardiness. The computational results showed that DDE algorithm performed better than the HDE algorithm (Qian et al., 2009) and IMMOGLS2 algorithm (Ishibuchi et al. 2003). Xie and Li (2012) proposed an evolved discrete harmony search (EDHS) algorithm to minimize makespan, total flow time, and maximum tardiness.
From the literature review above, we found that there is a very limited number of papers to address the multi-objective no-wait flow shop scheduling problem by using heuristics, while most researchers adopted meta-heuristics, such as SA, GA and local search, to solve this problem. We assign different weights to different objectives as introduced in the PAAH heuristic (Allahverdi and Aldowaisan, 2002), to evaluate the trade-off between $C_{\text{max}}$ and $\sum C_j$ based on our proposed and compared heuristics.

2.3 Statistical process control in operating room scheduling

Statistical process control (SPC) is a branch of statistics that combines a time series with historical data, generating good insights of scheduling in a more understandable way for decision makers. Conventional statistical analysis methods do account for natural variations without a time series. Therefore, it is a good way to use SPC and control charts to provide decision-makers to determine if changes in processes are making a real difference in outcomes.

The theory of statistical process control (SPC) was developed by Dr. Walter Shewhard (1931), and was popularized worldwide by Dr. W Edwards Deming (2000). The basic principles of SPC include (Benneyan et al., 2003):

- Individual measurement from any process will display a variation;
- If the data is from a stable common cause process, the variability is predictable within a knowable range that can be calculated from statistical model such as Gaussian, binomial, or Poisson distribution;
- If the data is from a special cause process, measured values will deviate in some observable way from these random distribution models;
• Assuming that the data are in control, we can establish the statistical control limits and test for data that deviate from predictions, providing statistical evidence of a change.

The control charts are the key tools of statistical process control (SPC). The control chart consists of two parts: one is the series of measurement plotted in the time order, and the other are three horizons lines, including center line (the mean line), the upper control limit (UCL) and lower control limit (LCL). Figure 2.1 shows an example of control chart of effective service rate of surgery type 1 (S1) with time series. To interpret the control chart in Figure 2.1, the series of measurements of effective service rate is plotted as the black line. The green line is the center line, we can obtain the mean service rate is 0.7708. Besides, there are two red lines to present the UCL (0.8231) and LCL (0.7185). The data between the UCL and LCL in the Figure 2.1 are considered as the common cause variation. However, there are three red points in the line falling outside the control limits. These data are indications of special cause variation, which means these data are out of control.

![Xbar Chart of S1](image)

Figure 2.1: The control chart of effective service rate of surgery type 1 (S1)
Where to draw the UCL and LCL is an important factor in the control charts. If the limits are too narrow, there is a high risk to have ‘type I error’. Type I error means we mistakenly consider some data as the special cause variations, which in fact they are common cause variations. If the limits are too wide, there is a high risk to have ‘type II error’. Type II error means we consider some data as common cause variations, which in fact they are special cause variations. It is recommended that the control limits are set as ±3 SD (standard deviation) for detecting a significant change while achieving a rational balance between two types of risks (Shewhart, 1931).

Above all, the statistical process control (SPC) and control charts are good tools to monitor the process and evaluate the performance, especially in the healthcare environment, such as flash sterilization rate, surgical site infections, etc. (Benneyan et al., 2003).
Chapter 3 Methodology

This chapter gives the methodology of our work. First, the problem description of no-wait flow shop is given in details. Second, the initial sequence algorithm (ISA) is illustrated based on the performance of current idle time and future idle time. Finally, the proposed AIT heuristic to min($C_{max}$) and the CFI heuristic to min($\Sigma C_j$) are presented respectively, of which an increment method is used to reduce the computational complexities. Moreover, a neighborhood exchanging technique is used in the CFI heuristic.

3.1 Problem description

The following notations are used in problem description and formulation.

- $\pi$: a sequence of $n$ jobs, $\pi = [J_1, J_2, \ldots, J_{j-1}, J_j, \ldots, J_n]$;
- $n$: the number of jobs;
- $m$: the number of machines;
- $p_{j,i}$: the processing time of job $j$ on machine $i$, where $j=1\ldots n$ and $i=1\ldots m$;
- $ST_{j,i}$: the start time of job $j$ on machine $i$;
- $CT_{j,i}$: the completion time of job $j$ on machine $i$;
- $d_{j-1,j}^i$: the potential distance between completion time of job $j-1$ and start time of job $j$ on machine $i$;
- $D_{j-1,j}$: the distance between two adjacent jobs’ completion times on the last machine.
The calculation of \( C_{\text{max}} \) and \( \sum C_j \) for no-wait flow shop production will be illustrated as follows. First, we assume the start time of job \( j \) on the first machine equals to the completion time of job \( j-1 \) on the last machine as shown in Figure 3.1(a) and Equation (3-1). Meanwhile, there is no waiting time on intermediate machines for each job. Accordingly, the start time of job \( j \) on machine \( i \) and the completion time of job \( j-1 \) on machine \( i \) in Figure 3.1(a) can be formulated by Equations (3-2) and (3-3).

Given initial conditions that \( CT_{0,m} = 0 \), \( p_{j,0} = 0 \), \( p_{0,i} = 0 \), \( \sum_{k=1}^{0} p_{j,k} = 0 \), and \( \sum_{k=m+1}^{m} p_{j,k} = 0 \),

\[
\begin{align*}
ST_{j,1} &= CT_{j-1,m} \quad \text{where } j=1,2,\ldots,n \quad (3-1) \\
ST_{j,i} &= ST_{j,1} + \sum_{k=1}^{i-1} p_{j,k} \quad \text{where } j=1,2,\ldots,n \text{ and } i=1,2,\ldots,m \quad (3-2) \\
CT_{j,i} &= CT_{j,m} - \sum_{k=i+1}^{m} p_{j,k} \quad \text{where } j=1,2,\ldots,n \text{ and } i=1,2,\ldots,m \quad (3-3)
\end{align*}
\]

Since the job is processed continuously on all machines, the start time of job \( j \) on machine \( i \) equals to its start time on the first machine plus the sum of its processing times on machines 1 to \( i-1 \) as shown in Equation (3-2). Similarly, the completion time of job \( j-1 \)
on machine \( i \) equals to its completion time on the last machine minus the sum of its processing times on machine \( i+1 \) to \( m \) as shown in Equation (3-3).

Consequently, the potential distances between the start time of job \( j \) and the completion time of job \( j-1 \) on machine \( i \) can be formulated by Equation (3-4).

\[
d^i_{j-1,j} = ST_{j,i} - CT_{j-1,i} = ST_{j,1} - CT_{j-1,m} + \sum_{k=1}^{i-1} p_{j,k} + \sum_{k=i+1}^{m} p_{j-1,k}
\]

\[
= \sum_{k=1}^{i-1} p_{j,k} + \sum_{k=i+1}^{m} p_{j-1,k}
\]

These potential distances between the start times of job \( j \) and the completion times of job \( j-1 \) can be reduced by shifting job \( j \) to the left with the amount of the minimum of these potential distances as in Figure 3.1(b), i.e., \( \min (d^i_{j-1,j}) \) for \( i = 1 \ldots m \). In other words, there exists at least one machine \( i \), where \( ST_{j,i} \) equals to \( CT_{j-1,i} \) (1 \( \leq \) \( i \) \( \leq \) \( m \)). Hence, the distance between jobs \( j \) and \( j-1 \) on the last machine \( D_{j-1,j} \) can be calculated by the total processing time of job \( j \) on all machines minus the minimum \( d^i_{j-1,j}(i = 1 \ldots m) \) as shown in Equation (3-5):

\[
D_{j-1,j} = \sum_{k=1}^{m} p_{j,k} - \min_{\{i\}} d^i_{j-1,j}
\]

\[
= \sum_{k=1}^{m} p_{j,k} - \min_{\{i\}} \left( \sum_{k=1}^{i-1} p_{j,k} + \sum_{k=i+1}^{m} p_{j-1,k} \right)
\]

\[
= \max_{\{i\}} \left( \sum_{k=i}^{m} p_{j,k} - \sum_{k=i+1}^{m} p_{j-1,k} \right)
\]
Therefore, the calculation of $C_{\text{max}}$ for a sequence $\pi = [J_1...J_n]$ can be transferred to the calculation of the total processing time of first job in the sequence and sum of $D_{j-1,j}$ ($j = 2,3,...,n$) as follows in Equation (3-6):

$$C_{\text{max}}(\pi) = \sum_{i=1}^{m} p_{\pi(1),i} + \sum_{j=2}^{n} D_{\pi(j-1),\pi(j)}$$

(3-6)

Similarly, the calculation of total completion time ($TCT$) for a sequence $\pi = [J_1...J_n]$ can be transferred to the calculation of the sum of the completion time of each job in the sequence as follows in Equation (3-7):

$$TCT(\pi) = \sum_{i=1}^{m} p_{\pi(1),i} + \sum_{j=2}^{n} \left( \sum_{i=1}^{m} p_{\pi(1),i} + \sum_{k=2}^{j} D_{\pi(k-1),\pi(k)} \right)$$

$$= n \sum_{i=1}^{m} p_{\pi(1),i} + \sum_{j=2}^{n} (n - j + 1) D_{\pi(j-1),\pi(j)}$$

(3-7)

3.2 Initial Sequence Algorithm (ISA)

We use the initial sequence algorithm (ISA) to construct an initial sequence, where we assign higher weights to current idle times generated by jobs in the head of the sequence than those generated by jobs in the tail of the sequence. The steps of ISA are as follows:

Step 1: Set the position index $k=1$, the set of sequenced jobs $S=\emptyset$ and the set of unsequenced jobs $U=\{\text{all jobs}\}$.

Step 2: Select the $j$th job (denoted as $J_{[j]}$) in $U$ ($j=1,...,n-k+1$), place it into the position $k$ in $S$, and calculate the average processing time ($APT_i$) of all jobs in $U$ except the selected $J_{[j]}$ on each machine. Set up an artificial job, and its processing time on each machine equals to $APT_i$ (Liu and Reeves, 2001; Li and Freiheit, 2016). Append this artificial job to $J_{[j]}$, that is the artificial job
is located on the \((k+1)\)th position in \(S\).

**Step 3:** Calculate the idle time between \(J_{[k]}\) and the \((k-1)\)th job in \(S\), which is considered as the current idle time \(CI(j) = \sum_{i=1}^{m} (C_{j,i} - p_{j,i} - C_{k-1,i})\), where \(C_{0,i} = 0 \quad \forall \ i\). Calculate the idle time between \(J_{[k]}\) and the artificial job, which is considered as the future idle time \(FI(j) = \sum_{i=1}^{m} (C_{k+1,i} - APT_i - C_{k,i})\). The index function \(f(j) = (n-k) \ast CI(j) + FI(j)\) is computed. For \(j=1, \ldots, n-k+1\), each job in \(U\) has its own index function value, and we remove the job which has the minimum value of \(f(j)\) from \(U\) and put it into the \(k\)th position in \(S\). Set \(k=k+1\).

**Step 4:** If \(k < n\), go to Step 2, otherwise, append the last one job in \(U\) to the last position in \(S\), and output \(S\) as the initial sequence \(\pi_0\).

To illustrate main procedures of the ISA heuristic in detail, we take a 6-job 5-machine instance as an example, which is the same as in Li et al. (2008). The main steps are listed as follows, and the processing time of each job on each machine can be found in Table 3.1.

1) Set \(S=\emptyset\) and the \(U=\{J_1, J_2, J_3, J_4, J_5, J_6\}\).

2) Consider \(J_1\) in the 1st position of \(S\), and the average processing times of \(J_2, J_3, J_4, J_5\) and \(J_6\) on each machine are computed as \(AP_{T_1}=[46.4, 53.8, 57, 42.4, 38.4]\), which equal to the processing times of an artificial job. Append this artificial job to \(J_1\), and we can obtain the current idle time of 150, and future idle time of 262.4. The index function value for \(J_1\), namely \(f(1)\), is 1012.4. We can consider \(J_2\) in the 1st position of \(S\) and obtain \(f(2)=1596.8\). Similarly, we can obtain \(f(3)=1430.8, f(4)=1146,\)
\( f(5) = 945.2 \), and \( f(6) = 822.8 \). Hence, we remove \( J_6 \) that has the minimum \( f \) value from \( U \) and put it into the 1st position of \( S \).

3) For the 2nd position in \( S \), we can do the similar procedure as in Step 2 in ISA, and obtain the index function values \( f \) for each job in \( U \), which are \( f = [648.25, 1452.8, 991.5, 960.75, 379.5] \). Hence we remove \( J_5 \) from \( U \) and put it into the 2nd position of \( S \). Similarly, we generate the initial sequence \( \pi_0 \) as \{\( J_6, J_5, J_1, J_4, J_3, J_2 \)\}.

### Table 3.1: Processing times of a 6-job 5-machine instance

<table>
<thead>
<tr>
<th></th>
<th>( M_1 )</th>
<th>( M_2 )</th>
<th>( M_3 )</th>
<th>( M_4 )</th>
<th>( M_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_1 )</td>
<td>72</td>
<td>68</td>
<td>9</td>
<td>1</td>
<td>48</td>
</tr>
<tr>
<td>( J_2 )</td>
<td>83</td>
<td>83</td>
<td>31</td>
<td>66</td>
<td>90</td>
</tr>
<tr>
<td>( J_3 )</td>
<td>11</td>
<td>90</td>
<td>74</td>
<td>72</td>
<td>36</td>
</tr>
<tr>
<td>( J_4 )</td>
<td>89</td>
<td>7</td>
<td>57</td>
<td>37</td>
<td>31</td>
</tr>
<tr>
<td>( J_5 )</td>
<td>44</td>
<td>62</td>
<td>41</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td>( J_6 )</td>
<td>5</td>
<td>27</td>
<td>82</td>
<td>24</td>
<td>13</td>
</tr>
</tbody>
</table>

### 3.3 AIT heuristic to \( \min(C_{\text{max}}) \)

The AIT heuristic consists of three phases: phase 1 for initial sequence generation, phase 2 for the insertion and neighborhood exchanging, and phase 3 for iteration improvement. In phase 1, we take both current idle times and future idle times into consideration to generate the initial sequence based on the ISA as in Section 3.2. In phase 2, we apply the insertion and neighborhood exchanging techniques to improve solutions. In phase 3, we use iterations to further improve solutions.

The techniques of insertion and neighborhood exchanging are used to improve solutions found by the ISA. In addition, when using insertion and neighborhood exchanging techniques, an objective increment method (Li et al., 2008) is used to calculate
the increment of makespan ($\Delta C_{\text{max}}$), reducing the computational complexity of calculating $C_{\text{max}}$ from $O(n)$ to $O(1)$.

Assume there are four jobs to be scheduled and the distance matrix $D_{4 \times 4}$ has been calculated in advance. For a temporary sequence $\pi_1=[J_1,J_2,J_3]$, $C_{\text{max}}(\pi_1) = \sum_{i=1}^{m} p_{\pi(1),i} + D_{\pi(1),\pi(2)} + D_{\pi(2),\pi(3)}$ using Equation (3-6). Assume $J_4$ will be inserted into the second position of $\pi_1$ and the sequence will be updated as $\pi_2=[J_1,J_4,J_2,J_3]$. The objective increment is $\Delta C_{\text{max}} = D_{1,4}+D_{4,2}-D_{1,2}$. Then $C_{\text{max}}(\pi_2) = C_{\text{max}}(\pi_1) + \Delta C_{\text{max}}$. In addition, if $J_4$ and $J_2$ are exchanged in $\pi_2$ and the sequence is updated as $\pi_3=[J_1,J_2,J_4,J_3]$, the objective increment is $\Delta C_{\text{max}} = D_{1,2}+D_{2,4}+D_{4,3}-D_{1,4}-D_{4,2}-D_{2,3}$. Then $C_{\text{max}}(\pi_3) = C_{\text{max}}(\pi_2) + \Delta C_{\text{max}}$. Therefore, by the objective increment method, the makespan can be calculated without computing makespan for the whole sequence.

The steps for the AIT heuristic are as follows:

Step 1: Compute the distance matrix $D_{n \times n}$ and obtain the initial sequence $\pi_0$ using ISA. Let $C_{\text{max}}$ be the makespan of $\pi_0$. Set the current best makespan $C_{\text{max}}=C_{\text{max}0}$, the current best sequence $\pi_b=\pi_0$, and the number of iterations $r$ changes from 1 to 5 (the experiment shows that when $r$ exceeds 5, there is little improvement of solutions) for Steps 2 to 6.

Step 2: Select first two jobs from $\pi_b$, and choose the partial sequence with a smaller $C_{\text{max}}$. Set $k=3$.

Step 3: Select $k$th job in $\pi_b$ and insert it in all possible positions of the current partial sequence. Calculate $\Delta C_{\text{max}}$ for all resultant temporary sequences. The temporary sequence whose job position has the minimum $\Delta C_{\text{max}}$ is selected as the current sequence. Next, exchange the position of each job in the
current sequence with that of the rest jobs. Among sequences generated by neighborhood exchanging, if one sequence yields the smallest negative $\Delta C_{\text{max}}$, set this sequence as the current sequence, otherwise, keep the current one. $k = k + 1$.

Step 4: Repeat Step 3 until all jobs are scheduled, and set the current sequence as $\pi_r$ with $C_{\text{max}}$.

Step 5: If $C_{\text{max}} < C_{\text{maxb}}$, set $C_{\text{maxb}} = C_{\text{max}}$ and $\pi_b = \pi_r$.

Step 6: For $j = 1$ to $n-1$, insert the $j$th job in $\pi_r$ into $n-j$ possible positions in the forward direction. If these sequences generate a lower $C_{\text{max}}$ than $C_{\text{maxb}}$, then update $\pi_b$ and $C_{\text{maxb}}$.

Step 7: Update $r = r + 1$. If $r \leq 5$, return to Step 2; otherwise, go to Step 8.

Step 8: Output the final $\pi_b$ and $C_{\text{maxb}}$.

The computational complexity is $O(mn^2)$ for Step 1, which computes $D_{n\times n}$ by Eq. (3-5) and generates the initial sequence, $O(n^3)$ for Step 3 and 4, which results from the insertion and neighborhood exchanging techniques using the objective increment method, and $O(n^2)$ for Step 6, which generates $n(n-1)/2$ sequences and calculates corresponding makespan with $O(1)$ Therefore, the computational complexity of the AIT heuristic is $O(n^3 + mn^2)$, which is the same as that of the LC, ADT and CH heuristics. An example, which is the same as the example in Table 3.1, is given below to illustrate the main steps of the AIT heuristic.

1) The distance matrix $D_{n\times n}$ calculated by Equation (3-5) is shown in Table 3.2. From the ISA, we obtain the initial sequence $\pi_0 = \{J_6, J_5, J_1, J_4, J_3, J_2\}$ and $C_{\text{max0}} = 616$. Set $C_{\text{maxb}} = 616$, $\pi_b = \{J_6, J_5, J_1, J_4, J_3, J_2\}$, and $r = 1$. 

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2) The sequence from Step 2 to 6 in the AIT heuristic is \( \pi_1 = \{J_6, J_3, J_2, J_5, J_1, J_4\} \) and \( C_{max1} = 601 \). Update \( C_{maxb} = 601 \), \( \pi_b = \{J_6, J_5, J_1, J_4, J_3, J_2\} \).

3) The sequence from second iteration \((r=2)\) is \( \pi_2 = \{J_3, J_2, J_4, J_6, J_5, J_1\} \) and \( C_{max2} = 584 \). Update \( C_{maxb} = 584 \), \( \pi_b = \{J_3, J_2, J_4, J_6, J_5, J_1\} \).

4) The sequence from third iteration \((r=3)\) is \( \pi_3 = \{J_6, J_3, J_2, J_4, J_5, J_1\} \) and \( C_{max3} = 565 \). Update \( C_{maxb} = 565 \), \( \pi_b = \{J_6, J_3, J_2, J_4, J_5, J_1\} \).

5) In the iterations from 4 to 5, the sequence and makespan remain unchanged.

Therefore, the final sequence is \( \{J_6, J_3, J_2, J_4, J_5, J_1\} \) with makespan 565.

<table>
<thead>
<tr>
<th></th>
<th>( J_1 )</th>
<th>( J_2 )</th>
<th>( J_3 )</th>
<th>( J_4 )</th>
<th>( J_5 )</th>
<th>( J_6 )</th>
</tr>
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<td>151</td>
<td>-</td>
<td>50</td>
<td>51</td>
</tr>
<tr>
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<td>84</td>
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<tr>
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<td>207</td>
<td>153</td>
<td>88</td>
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</tr>
</tbody>
</table>

### 3.4 CFI heuristic to \( \min(\Sigma C_j) \)

The CFI heuristic consists of three phases: phase 1 for initial sequence generation, phase 2 for the insertion and neighborhood exchanging, and phase 3 for iteration improvement. To improve effectiveness of the CFI heuristic, we take both current idle times and future idle times into consideration to generate the initial sequence by the ISA as in Section 3.2, and apply the insertion and neighborhood exchanging techniques. To improve efficiency of the CFI heuristic, we introduce an objective increment method to calculate TCT while applying neighborhood exchanging. In addition, we determine the number of iterations as 6 rather than 10 as used in the PH1(p) (Aldowaisan and Allahverdi,
2004) and LS (Laha and Sapkal, 2014) heuristics. Six iterations reduce the computation time and maintain effectiveness of the CFI heuristic. The steps of the CFI heuristic are as follows:

Step 1: Compute the distance matrix $D_{n \times n}$ and obtain the initial sequence $\pi_0$ using ISA. Let $\text{TCT}_0$ be the total completion time of $\pi_0$. Set the current best total completion time $\text{TCT}_b=\text{TCT}_0$, the current best sequence $\pi_b=\pi_0$, and the number of iterations $r$ from 1 to 6 for Steps 2 to 6.

Step 2: Select first two jobs from $\pi_b$, and choose the partial sequence with a smaller TCT.

Step 3: First, apply the NEH insertion technique (Nawaz et al., 1983) to the obtained partial sequences, select the best partial sequence with minimum TCT as current sequence. Next, exchange the position of each job in the current sequence with that of the rest jobs. Among sequences generated by interchanging, the objective increment method is used to calculate $\Delta \text{TCT}$. If one sequence yields the smallest negative $\Delta \text{TCT}$, set this sequence as the current sequence, otherwise, keep the current one.

Step 4: Repeat Step 3 until all jobs are scheduled, and set the current sequence as $\pi_r$ with $\text{TCT}_r$.

Step 5: If $\text{TCT}_r<\text{TCT}_b$, set $\text{TCT}_b=\text{TCT}_r$ and $\pi_b=\pi_r$.

Step 6: For $j=1$ to $n-1$, insert the $j$th job in $\pi_r$ into $n-j$ possible positions in the forward direction. If these sequences generate a lower TCT than $\text{TCT}_b$, then update $\pi_b$ and $\text{TCT}_b$. 
Step 7: Update \( r = r + 1 \). If \( r \leq 6 \), return to Step 2; otherwise, go to Step 8. (Note: the condition \( r \leq 6 \) is concluded from a case study.)

Step 8: Output the final \( \pi_b \) and \( TCT_b \).

While using neighborhood exchanging technique in Step 3, an objective increment method is introduced to calculate TCT. For example, there are five jobs scheduled and the distance matrix \( D_{5 \times 5} \) is computed. For the sequence \( \pi = \{J_1, J_2, J_3, J_4, J_5\} \),

\[
TCT_{\pi} = 5 \sum_{i=1}^{m} p_{\pi(1),i} + \sum_{j=2}^{5} (n - j + 1)D_{\pi(j-1),\pi(j)}
\]

using Equation (3-7). Let \( J_1 \) and \( J_2 \) be exchanged, and update the sequence as \( \pi' = \{J_2, J_1, J_3, J_4, J_5\} \). The objective increment is

\[
\Delta TCT = 5(\sum_{i=1}^{m} p_{2,i} - \sum_{i=1}^{m} p_{1,i}) + 4(D_{2,1}-D_{1,2}) + 3(D_{1,3}-D_{2,3}).
\]

Therefore, using the objective increment method, the TCT can be calculated without computing TCT for the whole sequence. The details of the objective increment method are provided in section 3.5.

The main computational burden of the CFI heuristic is determined by the NEH insertion and neighborhood exchanging techniques in Step 3. The computational complexity for the NEH insertion is \( O(n^3) \) including calculating TCT with \( O(n) \) when selecting the best insertion position. The computational complexity for neighborhood exchanging technique is also \( O(n^3) \) including calculating TCT with \( O(1) \) when selecting the best exchanged pair. Therefore, the overall computational complexity of the CFI heuristic is \( O(n^3) \), which is the same as that of the PH1(p) and LS heuristics, and less than that of the FNM heuristic.

To illustrate the main steps of the CFI heuristic, we provide a 5-job 4-machine instance as shown in Table 3.3, which is the same as in Bertolissi (2000).
**Initial sequence algorithm**

Step 1: \( S = \emptyset \) and the \( U = \{ J_1, J_2, J_3, J_4, J_5 \} \).

Step 2: Consider \( J_1 \) in the 1st position of \( S \), the processing times of \( J_2, J_3, J_4, \) and \( J_5 \) on each machine are the average processing times. \( APT_i = [18, 15.25, 13.75, 15.5] \). Append this artificial job to \( J_1 \), and we can obtain the current idle time of 48, and future idle time of 13.25. The index function value for \( J_1 \), namely \( f(1) \), is 205.25. We can consider \( J_2 \) in the 1st position of \( S \) and obtain \( f(2) = 211 \). Similarly, we can obtain \( f(3) = 199.25, f(4) = 222.25 \), and \( f(5) = 230.25 \). Hence, we remove \( J_3 \) that has the minimum \( f \) value from \( U \) and put it into the 1st position of \( S \).

Step 3: For the 2nd position in \( S \), we can do the similar procedure as in Step 2, and obtain the index function values \( f \) for each job in \( U \), which are \( f = [155, 53.33, 153, 100.33] \). Hence we remove \( J_2 \) from \( U \) and put it into the 2nd position of \( S \). Similarly, we generate the initial sequence \( \pi_0 \) as \( \{ J_3, J_2, J_1, J_5, J_4 \} \).

**CFI heuristic**

The distance matrix \( D_{n \times n} \) calculated by Equation (3-5) is shown in Table 3.4. From the ISA, we obtain initial sequence \( \pi_0 = \{ J_3, J_2, J_1, J_5, J_4 \} \) and \( TCT_0 = 501 \). Set \( TCT_b = 501 \), \( \pi_b = \{ J_3, J_2, J_1, J_5, J_4 \} \), and \( r = 1 \).
We can select first two jobs, $J_3$ and $J_2$, from the initial sequence, and obtain a TCT of 129 for a partial sequence $\{J_3, J_2\}$. Exchange the two jobs and obtain a TCT of 130 for a partial sequence $\{J_2, J_3\}$. Hence, we fix the relative positions of two jobs as a partial sequence of $\{J_3, J_2\}$.

Inserting $J_1$ from the initial sequence to each possible position of the partial sequence $\{J_3, J_2\}$, we can have the following partial sequences, $\{J_1, J_3, J_2\}$, $\{J_3, J_1, J_2\}$ and $\{J_3, J_2, J_1\}$ with partial TCTs of 228, 250, 229, respectively. Hence, we choose the partial sequence of $\{J_1, J_3, J_2\}$ as the current sequence with the minimum partial TCT of 228. The neighborhood exchanging method is applied, and the following partial sequences are examined, $\{J_3, J_1, J_2\}$, $\{J_2, J_3, J_1\}$ and $\{J_1, J_2, J_3\}$ with $\Delta TCTs$ of 22, 10 and 13, respectively. None of these values is lower than 0, therefore, the current sequence remains as $\{J_1, J_3, J_2\}$.

Insert $J_5$ from the initial sequence to each possible position of the current sequence, and the following partial sequences are examined: $\{J_5, J_1, J_3, J_2\}$, $\{J_1, J_5, J_3, J_2\}$, $\{J_1, J_3, J_5, J_2\}$ and $\{J_1, J_3, J_2, J_5\}$ with partial TCTs of 376, 376, 372, and 359, respectively. Hence, we choose $\{J_1, J_3, J_2, J_5\}$ as the current sequence with the minimum partial TCT of 359. The neighborhood exchanging method is applied, and the following partial sequences are examined: $\{J_3, J_1, J_2, J_5\}$, $\{J_2, J_3, J_1, J_5\}$, $\{J_5, J_3, J_2, J_1\}$, $\{J_1, J_2, J_3, J_5\}$, $\{J_1, J_5, J_2, J_3\}$ and

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<td>11</td>
<td>15</td>
<td>14</td>
<td>-</td>
</tr>
</tbody>
</table>
{J_1, J_3, J_5, J_2} with ΔTCTs of 36, 16, 36, 20, 18, and 13, respectively. None of these values is lower than 0, therefore, the current sequence remains as \{J_1, J_3, J_2, J_5\}.

Insert \( J_4 \) from the initial sequence to each possible position of the current sequence and the following candidates are tried: \{J_4, J_1, J_3, J_2, J_5\}, \{J_1, J_4, J_3, J_2, J_5\}, \{J_1, J_3, J_4, J_2, J_5\}, \{J_1, J_3, J_2, J_4, J_5\} and \{J_1, J_3, J_2, J_5, J_4\} with TCTs of 526, 532, 542, 503 and 504. Hence, we choose \{J_1, J_3, J_2, J_4, J_5\} as the current sequence with minimum TCT of 503.

The neighborhood exchanging method is applied and the following partial sequences are examined: \{J_3, J_1, J_2, J_4, J_5\}, \{J_3, J_1, J_2, J_5, J_4\}, \{J_1, J_3, J_2, J_4, J_5\}, \{J_1, J_3, J_2, J_5, J_4\} and \{J_1, J_3, J_2, J_4, J_5\} with TCTs of 50, 32, 22, 54, 37, 46, 34, 39, 14 and 1. None of these values is lower than 0, therefore, the current sequence remains as \{J_1, J_3, J_2, J_4, J_5\} with TCT 503.

After using insertion and neighborhood interchanging methods, we obtain \( \pi_1 = \{J_1, J_3, J_2, J_4, J_5\} \) and TCT\(_1\)=503. Since TCT\(_1\) is larger than TCT\(_b\), the \( \pi_b \) remains unaltered with TCT\(_b\) 501. For \( j=1 \) to 4, insert \( j \)th job into each possible position of \( \pi_1 \) in the forward direction and get the following sequences: \{J_3, J_1, J_2, J_4, J_5\}, \{J_3, J_1, J_2, J_5, J_4\}, \{J_1, J_3, J_2, J_4, J_5\}, \{J_1, J_3, J_2, J_5, J_4\}, \{J_1, J_3, J_4, J_5, J_2\}, \{J_1, J_3, J_4, J_5, J_2\}, \{J_1, J_3, J_4, J_5, J_2\} and \{J_1, J_3, J_4, J_5, J_2\} with TCTs of 553, 510, 514, 510, 540, 541, 540, 542, 531, and 504. None of these values is lower than TCT\(_b\), the \( \pi_b \) remains unaltered with TCT\(_b\) 501 and is used for further process till \( r=6 \) iterations are completed. Hence, the final sequence is \{J_3, J_2, J_1, J_5, J_4\} with TCT 501.

### 3.5 Objective increment method to calculate total completion time (TCT)

Assume there is a sequence \( \pi=\{J_1, J_2, \ldots, J_{j-1}, J_j, \ldots, J_n\} \), and the corresponding TCT is TCT\(_\pi\). When \( \pi(k) \) and \( \pi(j) \) (0<\( k < j \leq n \)) in \( \pi \) are exchanged, the new sequence \( \pi' \) is generated,
the difference of TCT between $\pi'$ and $\pi$, i.e., $\Delta TCT(k, j)$, can be calculated by one of the following conditions:

Condition 1: $k=1$ and $j=2$

$$\Delta TCT(k, j) = n \left( \sum_{i=1}^{m} p_{\pi'(k), i} - \sum_{i=1}^{m} p_{\pi(k), i} \right) + (n - 1) \left( D_{\pi'(k), \pi'(j)} - D_{\pi(k), \pi(j)} \right)$$

$$+ (n - 2) \left( D_{\pi'(j), \pi'(j+1)} - D_{\pi(j), \pi(j+1)} \right)$$

Condition 2: $k=1$ and $j=3, \ldots, n-1$

$$\Delta TCT(k, j) = n \left( \sum_{i=1}^{m} p_{\pi'(k), i} - \sum_{i=1}^{m} p_{\pi(k), i} \right)$$

$$+ (n - 1) \left( D_{\pi'(k), \pi'(k+1)} - D_{\pi(k), \pi(k+1)} \right)$$

$$+ (n - j + 1) \left( D_{\pi'(j-1), \pi'(j)} - D_{\pi(j-1), \pi(j)} \right) + (n - j) \left( D_{\pi'(j), \pi'(j+1)} - D_{\pi(j), \pi(j+1)} \right)$$

Condition 3: $k=1$ and $j=n$

$$\Delta TCT(k, j) = n \left( \sum_{i=1}^{m} p_{\pi'(k), i} - \sum_{i=1}^{m} p_{\pi(k), i} \right)$$

$$+ (n - 1) \left( D_{\pi'(k), \pi'(k+1)} - D_{\pi(k), \pi(k+1)} \right) + D_{\pi'(j-1), \pi'(j)}$$

$$- D_{\pi(j-1), \pi(j)}$$

Condition 4: $k=2, 3, \ldots, n-2$ and $j=k+1$

$$\Delta TCT(k, j) = (n - k + 1) \left( D_{\pi'(k-1), \pi'(k)} - D_{\pi(k-1), \pi(k)} \right)$$

$$+ (n - k) \left( D_{\pi'(k), \pi'(j)} - D_{\pi(k), \pi(j)} \right)$$

$$+ (n - j) \left( D_{\pi'(j), \pi'(j+1)} - D_{\pi(j), \pi(j+1)} \right)$$

Condition 5: $k=2, 3, \ldots, n-3$ and $j=k+2, \ldots, n-1$
\[ \Delta TCT(k, j) = (n - k + 1) \left( D_{\pi'_{(k-1), \pi'(k)}} - D_{\pi(k-1), \pi(k)} \right) \]
\[ + (n - k) \left( D_{\pi'(k), \pi'(k+1)} - D_{\pi(k), \pi(k+1)} \right) \]
\[ + (n - j + 1) \left( D_{\pi'(j-1), \pi'(j)} - D_{\pi(j-1), \pi(j)} \right) \]
\[ + (n - j) \left( D_{\pi'(j), \pi'(j+1)} - D_{\pi(j), \pi(j+1)} \right) \]

Condition 6: \( k=2,3,\ldots,n-2 \) and \( j=n \)

\[ \Delta TCT(k, j) = (n - k + 1) \left( D_{\pi'_{(k-1), \pi'(k)}} - D_{\pi(k-1), \pi(k)} \right) \]
\[ + (n - k) \left( D_{\pi'(k), \pi'(k+1)} - D_{\pi(k), \pi(k+1)} \right) + D_{\pi'(j-1), \pi'(j)} \]

\[ - D_{\pi(k-1), \pi(k)} \]

Condition 7: \( k=n-1 \) and \( j=n \)

\[ \Delta TCT(k, j) = 2 \left( D_{\pi'_{(k-1), \pi'(k)}} - D_{\pi(k-1), \pi(k)} \right) + D_{\pi'(k), \pi'(j)} - D_{\pi(k), \pi(j)} \]

Hence, the TCT of new sequence \( \pi' \) can be calculated by the following equation:

\[ TCT_{\pi'} = TCT_{\pi} + \Delta TCT \]

Therefore, the calculation of TCT for the new sequence can be reduced from \( O(n) \) to \( O(1) \).
Chapter 4 Case study

To test the performance of our proposed heuristics on $F_m |nwt| C_{\text{max}}$ and $F_m |nwt| \sum C_j$ problems, we have done a series of case studies, and the results are presented in this chapter. First, we provide the schemes to generate the data and evaluate the heuristic performance. Second, for each single objective, the computational results of all compared heuristics are given from the perspectives of effectiveness and efficiency. Third, we introduce trade-off balancing function to evaluate of trade-off between min($C_{\text{max}}$) and min($\sum C_j$) for each heuristic. Finally, statistical process control (SPC) is used to evaluate the performance of the CFI heuristic along time horizon based on University of Kentucky HealthCare historical data.

4.1 Schemes to carry out case studies

First, to evaluate the performance of the AIT heuristic on solving $F_m |nwt| C_{\text{max}}$ problems, we compare the AIT heuristic with the LC (Laha and Chakraborty, 2009), ADT (Ye et al., 2016) and CH (Li et al., 2008) heuristics. Second, to evaluate the performance of the CFI heuristic on solving $F_m |nwt| \sum C_j$ problems, we compare the CFI heuristic with the PH1(p) (Aldowaisan and Allahverdi, 2004), the FNM (Framinan et al., 2010), and LS (Laha and Sapkal, 2014) heuristics.

For effectiveness, we use the average relative percentage deviation (ARPD), maximum percentage deviation (MPD), and percentage of the best solutions (PBS) to evaluate each heuristic based on both small-scale and large-scale instances. Analysis of variance (ANOVA) techniques and paired $t$-tests are used to statistically verify the improvement on effectiveness based on large-scale instances. To evaluate efficiency, we
use the computation times based only on large-scale instances, as the computation times for the small-scale instances are negligible.

For small-scale instances, the number of jobs is 5, 6, 7, or 8, and the number of machines is 5, 10, 15, 20, or 25. Thus, there are 20 combinations. For each combination, 30 instances are generated randomly, and the processing times for each instance are integers, following a uniform distribution in [1, 99]. In total, there are 600 instances in small-scale.

For large-scale instances, Taillard’s benchmarks (Taillard, 1994) are classic and commonly used to test the performance of heuristics for flow shop scheduling. Taillard’s benchmarks consist of 120 instances in 12 combinations, with 10 instance for each combination, where the number of jobs is 20, 50, 100, 200 or 500, and the number of machines is 5, 10 or 20.

For $F_m | nwt | C_{\text{max}}$ problems, three criteria are used to evaluate the effectiveness of each heuristic (Ye et al., 2016; Li et al., 2008):

(1) Average relative percent deviation (ARPD):

\[
ARPD = \frac{1}{N} \sum_{i=1}^{N} \frac{M_i(H) - \text{Best}_{\text{known}_i}}{\text{Best}_{\text{known}_i}} \times 100
\]  

(4-1)

(2) Maximum percent deviation (MPD):

\[
MPD = \max_{i=1,\ldots,N} \left( \frac{M_i(H) - \text{Best}_{\text{known}_i}}{\text{Best}_{\text{known}_i}} \right) \times 100
\]  

(4-2)

where, $M_i(H)$ is the makespan obtained by heuristic $H$ for an instance $i$ in a combination. $N$ is the number of instances for each combination. $N$ is 30 for small-scale instances but is 10 for large-scale instances. $\text{Best}_{\text{known}_i}$ is the optimal
solution for small-scale instances by using exhaustive enumeration. However, for large-scale instances, $Best_{known_i}$ is the best makespan among all four heuristics.

(3) Percentage of the best solutions (PBS)

PBS is the percentage of instances for which a heuristic achieves the best performance among the four heuristics. The row total for PBS does not necessarily sum to 100% since some heuristics may tie on the best performance for some instances.

Similarly, for $F_m \mid nwt \parallel \sum C_j$ problems, three similar criteria are used to evaluate effectiveness of each heuristic:

1. Average relative percent deviation (ARPD):

$$ARPD = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{TCT_i(H) - Best_{known_i}}{Best_{known_i}} \right) \times 100$$

(4-3)

2. Maximum percent deviation (MPD):

$$MPD = \max_{i=1,\ldots,N} \left( \frac{TCT_i(H) - Best_{known_i}}{Best_{known_i}} \right) \times 100$$

(4-4)

$TCT_i(H)$ is the total completion time obtained by heuristic $H$ for an instance $i$ in a combination. $N$ is the number of instances for each combination. $N$ is 30 for small-scale instances but is 10 for large-scale instances. $Best_{known_i}$ is the optimal solution for small-scale instances by using the exhaustive enumeration method. However, for large-scale instances, the best known solutions are from Qi et al. (2016), who proposed the best known upper bounds for $F_m \mid nwt \parallel \sum C_j$ problems based on Taillard’s benchmarks.

(3) Percentage of the best solutions (PBS)
PBS is the percentage of instances for which a heuristic achieves the best performance among the four heuristics. The row total for PBS does not necessarily sum to 100% since some heuristics may tie on the best performance for some instances.

4.2 Results of case study on $F_m | b_{\text{nt}} | C_{\text{max}}$ problems

4.2.1 Small-scale instances

For small-scale instances, the results are shown in Table 4.1. The AIT heuristic achieves the best performance on ARPD of 0.23%, on MPD of 5.14%, and on PBS of 82.17%.

As shown in Table 4.1 for small-scale instances, with respect to ARPD, the CH heuristic achieves an average of 0.30%, better than 1.28% of the LC heuristic and 0.62% of the ADT heuristic. However, the AIT heuristic achieves the smallest average of 0.23%, which has a 23.3% improvement over the CH heuristic. With regard to MPD, the CH heuristic achieves 5.18%, smaller than 11.42% of the LC heuristic and 7.00% of the ADT heuristic. The AIT heuristic also achieves the smallest MPD of 5.14%. With respect to PBS, the CH heuristic achieves 76.50%, better than 63.33% and 48.5% of the ADT and LC heuristics, respectively. However, the AIT heuristic achieves the best PBS of 82.17%. Overall, the AIT heuristic achieves the best performances in ARPD, MPD and PBS in small-scale instances, compared with the LC, ADT, and CH heuristics.
Table 4.1: Average Relative and Maximum percent deviations (ARPD & MPD) for small-scale instances to \( \min(C_{\text{max}}) \) (%)

<table>
<thead>
<tr>
<th>Size</th>
<th>LC</th>
<th>ADT</th>
<th>CH</th>
<th>AIT</th>
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<tr>
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<td>( m )</td>
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<td>MPD</td>
<td>ARPD</td>
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<tr>
<td>5</td>
<td>5</td>
<td>0.75</td>
<td>11.42</td>
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<td>76.50</td>
<td>82.17</td>
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</tbody>
</table>
4.2.2 Large-scale instances

For large-scale instances, the results are shown in Table 4.2. The AIT heuristic achieves the best performance on ARPD of 0.23% and on PBS of 65.83%, but not on MPD of 2.95%.

Table 4.2: Average Relative and Maximum percent deviations (ARPD & MPD) for large-scale instances to min($C_{max}$) (%)

<table>
<thead>
<tr>
<th>Size</th>
<th>LC</th>
<th>ADT</th>
<th>CH</th>
<th>AIT</th>
</tr>
</thead>
<tbody>
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<td>MPD</td>
<td>ARPD</td>
<td>MPD</td>
</tr>
<tr>
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<td>2.21</td>
<td>5.38</td>
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<td>1.33</td>
<td>2.12</td>
</tr>
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<td>2.85</td>
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<td>4.29</td>
<td>5.20</td>
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<tr>
<td>10</td>
<td>2.91</td>
<td>4.20</td>
<td>2.00</td>
<td>2.77</td>
</tr>
<tr>
<td>20</td>
<td>2.38</td>
<td>3.77</td>
<td>1.32</td>
<td>1.91</td>
</tr>
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<td>3.05</td>
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<tr>
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</tr>
<tr>
<td>500</td>
<td>2.67</td>
<td>3.39</td>
<td>1.89</td>
<td>2.53</td>
</tr>
<tr>
<td>All instances</td>
<td>2.63</td>
<td>6.46</td>
<td>2.14</td>
<td>5.38</td>
</tr>
</tbody>
</table>

As shown in Table 4.2, the LC heuristic achieves ARPD of 2.63%, MPD of 6.46% and PBS of 0.83%. The ADT heuristic achieves ARPD of 2.14%, MPD of 5.38% and PBS of 5.83%, better than the LC heuristic. The CH heuristic obtains ARPD of 0.64%, MPD of 2.32%, and PBS of 29.17%, and performs the best among the LC, ADT, and CH heuristics. The AIT heuristic achieves the smallest ARPD of 0.23%, 64% improvement over the CH heuristic. Besides, the AIT heuristic obtains the largest PBS of 65.83% among all four
heuristics, although the MPD of the AIT heuristic is 2.95%, close to 2.32% of the CH heuristic, and smaller than those of the LC and ADT heuristics, which are 6.46% and 5.38%, respectively.

ARPDs of heuristics for large-scale instances are used to plot the trend of deviations as the number of jobs or machines increases, as shown in Figure 4.1.

Figure 4.1: Deviations of $C_{\text{max}}$ as the number of jobs or machines increases

(a) Deviation of $C_{\text{max}}$ by no. of jobs (%);

(b) Deviation of $C_{\text{max}}$ by no. of machines (%)

Figure 4.1: Deviations of $C_{\text{max}}$ as the number of jobs or machines increases
As the number of jobs increases from 20 to 500, Figure 4.1(a) shows that the deviation of the AIT heuristic is smaller than those of other three heuristics. The trends of the LC and ADT heuristics are relatively flat as the number of jobs increases. However, the trend of the CH heuristic is going up when the number of jobs is larger than 100, whereas the trend of the AIT heuristic is continuously going downwards as the number of jobs increases.

Figure 4.1(b) plots the trend of ARPDs against the number of machines, ranging from 5 to 20 machines. The trend obtained by the ADT heuristic continuously goes downwards as the number of machines increases. However, the trends obtained by the LC, CH and AIT heuristics are relatively flat, when the number of machines increases from 5 to 20. Among four heuristics, the deviation of the AIT heuristic is the smallest.

To verify the effectiveness of the AIT heuristic, two statistical analyses are conducted based on ARPDs for large-scale instances. First, the analysis of variance (ANOVA) is used to test the difference among the ARPDs of the LC, ADT, CH and AIT heuristics. As shown in Table 4.3, the results show that the difference of ARPDs among four heuristics is statistically significant at 95% confidence interval with \( p \)-value=0.000.

<table>
<thead>
<tr>
<th>Source</th>
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<th>SS</th>
<th>MS</th>
<th>P</th>
</tr>
</thead>
<tbody>
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<td>160.782</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
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<td>437</td>
<td>0.918</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>479</td>
<td>919.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Second, paired \( t \)-tests on the ARPD are performed to validate whether there are significant differences among the LC, ADT, CH and AIT heuristics. As shown in Table 4.4, the estimates for mean differences between the AIT heuristic and other heuristics are all
smaller than 0 with \( p \)-value=0.000, and the 95% confidence intervals for mean differences fall into the negative interval, indicating that the ARPD of the AIT heuristic is significantly smaller than those of other three heuristics at confidence level \( \alpha=0.05 \).

Table 4.4: Paired \( t \)-tests results to \( \min(C_{\text{max}}) \) \((\alpha=0.05)\)

<table>
<thead>
<tr>
<th></th>
<th>AIT vs.</th>
<th>LC</th>
<th>ADT</th>
<th>CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Estimate for mean difference</td>
<td>-2.407</td>
<td>-1.911</td>
<td>-0.414</td>
<td></td>
</tr>
<tr>
<td>95% CI for mean difference</td>
<td>(-2.618, -2.196)</td>
<td>(-2.183, -1.639)</td>
<td>(-0.574, -0.254)</td>
<td></td>
</tr>
</tbody>
</table>

We use the amount of CPU time to measure the efficiency of each heuristic. All four heuristics are programmed in Matlab and run on a Dell Precision T1700 with Intel Core i5-4590 CPUs of 3.3 GHz. For 500-job 20-machine instances, we have run one case for 100 times and obtain the average CPU time, which is 38.31 seconds for the AIT heuristic, and 21.97, 4.98, and 5.27 seconds for the CH, ADT, and LC heuristics, respectively. The performance of the AIT heuristic on deviation justifies the additional computation time, although the AIT heuristic uses the insertion and neighborhood exchanging techniques and generates more sequences than the other three heuristics.

Overall, the AIT heuristic outperforms other heuristics on ARPD, MPD and PBS criteria. Moreover, the deviation of makespan for the AIT heuristic has a decreasing trend as the number of jobs or machines increases. Since all four heuristics have the same computational complexity, the AIT heuristic statistically performs better than the other three heuristics for no-wait flow shop production to \( \min(C_{\text{max}}) \).

4.3 Results of case study on \( F_m |nwt| \sum C_j \) problems

4.3.1 Small-scale instances
For small-scale instances, the results are shown in Table 4.5. The CFI heuristic achieves the best performance on ARPD of 0.06%, on MPD of 2.98%, and on PBS of 88%.

Table 4.5: Average relative and maximum percent deviation (ARPD & MPD) for small-scale instances to \( \min(\sum C_j) \) (%)

<table>
<thead>
<tr>
<th>Size</th>
<th>PH1(p)</th>
<th>FNM</th>
<th>LS</th>
<th>CFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>m</td>
<td>ARPD</td>
<td>MPD</td>
<td>ARPD</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.41</td>
<td>5.94</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.13</td>
<td>1.36</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.27</td>
<td>2.77</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.07</td>
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</tr>
<tr>
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<td>25</td>
<td>0.06</td>
<td>1.31</td>
<td>0.01</td>
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<td>5</td>
<td>0.46</td>
<td>3.93</td>
<td>0.24</td>
</tr>
<tr>
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<td>0.38</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.28</td>
<td>4.11</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.40</td>
<td>3.09</td>
<td>0.16</td>
</tr>
<tr>
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<td>25</td>
<td>0.16</td>
<td>1.69</td>
<td>0.05</td>
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<td>7</td>
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<td>0.51</td>
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<td>0.30</td>
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<td>0.16</td>
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<td>0.09</td>
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<tr>
<td>PBS</td>
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<td>73</td>
<td>75</td>
<td>88</td>
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As shown in Table 4.5 for small-scale instances, with respect to ARPD, the LS heuristic achieves 0.21%, better than the PH1(p) heuristic of 0.37% and FNM heuristic of 0.23%. The CFI heuristic achieves the smallest ARPD of 0.06% from the optimal. With regard to MPD, the LS heuristic achieves 4.11%, smaller than the PH1(p) of 5.99% and FNM heuristic of 5.61%. The CFI heuristic also achieves the smallest MPD of 2.98%. With respect to PBS, the LS and FNM heuristics are very close, 75% and 73%, respectively, better than the PH1(p) heuristic of 65%. The CFI heuristic reaches 88% of the best solutions, 17% improvement over the LS heuristic.

4.3.2 Large-scale instances

For large-scale instances, the results are shown in Table 4.6. The CFI heuristic achieves the best performance on ARPD of 2.08% and on PBS of 53%, but not on MPD of 7.05%.

As shown in Table 4.6, the PH1(p) heuristic achieves ARPD of 3.47%, MPD of 7.15% and PBS of 7%. The FNM heuristic achieves ARPD of 2.74%, MPD of 5.33% and PBS of 12%, better than the PH1(p) heuristic. The LS heuristic obtains ARPD of 2.33%, MPD of 4.91%, and PBS of 30%, better than the PH1(p) and FNM heuristics on effectiveness. The CFI heuristic achieves the smallest ARPD of 2.08% and the largest PBS of 53% among all four heuristics, although the MPD of the CFI heuristic is not as good as the FNM and LS heuristics.
ARPDs of heuristics for large-scale instances are used to plot the trend of deviations as the number of jobs or machines increases, as shown in Figure 4.2.

As the number of jobs increases from 20 to 500, Figure 4.2 (a) shows that the deviations of all four heuristics are very close when the number of jobs is 20, and then increase when the number of jobs changes from 20 to 100. In addition, as the number of jobs increases from 100 to 500, the deviation of the CFI heuristic drops the fastest compared with those of other three heuristics.

Figure 4.2 (b) plots the trend of ARPDs against the number of machines, ranging from 5 to 20 machines. The trends obtained by the PH1(p) and LS heuristics go downwards

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<tr>
<th>Size</th>
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<th>MPD</th>
<th>LS ARPD</th>
<th>MPD</th>
<th>CFI ARPD</th>
<th>MPD</th>
</tr>
</thead>
<tbody>
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<td>n</td>
<td>m ARPD</td>
<td>MPD</td>
<td>n ARPD</td>
<td>MPD</td>
<td>n ARPD</td>
<td>MPD</td>
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<td>0.98</td>
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<tr>
<td>All instances</td>
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<td>53</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

ARPDs of heuristics for large-scale instances are used to plot the trend of deviations as the number of jobs or machines increases, as shown in Figure 4.2.
as the number of machines increases from 5 to 15, whereas the deviations of both the FNM and CFI heuristics go up. However, when the number of machines increases from 10 to 20, the deviation of the CFI heuristic drops faster than those of other heuristics and reaches to the lowest point of deviations.

Figure 4.2: Deviations of TCT as the number of jobs or machines increases
To verify effectiveness of the CFI heuristic, two statistical analyses are conducted based on the large-scale instances. Firstly, the analysis of variance (ANOVA) is used to test whether the ARPDs of the PH1(p), FNM, LS and CFI heuristics are the same or whether some ARPDs are different. The ANOVA results from Table 4.7 show that the difference among heuristics is statistically significant with \( p\)-value=0.000.

Table 4.7: ANOVA results to \( \min(\sum C_j) \) (95% Confidence Interval)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heuristics</td>
<td>3</td>
<td>134.27</td>
<td>44.76</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>476</td>
<td>890.40</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>479</td>
<td>1024.68</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Secondly, paired \( t \)-tests on the ARPD are performed to validate whether or not there are significant differences among the PH1(p), FNM, LS and CFI heuristics. Table 4.8 shows the summarized results of the paired \( t \)-test for confidence level \( \alpha=0.05 \). As Table 4.8 indicates, the CFI heuristic significantly outperforms the other three heuristics.

Table 4.8: Paired \( t \)-test results to \( \min(\sum C_j) \) (\( \alpha=0.05 \))

<table>
<thead>
<tr>
<th>CFI vs.</th>
<th>PH1(p)</th>
<th>FNM</th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.012</td>
</tr>
<tr>
<td>Estimate for mean difference</td>
<td>(-1.397)</td>
<td>(-0.664)</td>
<td>(-0.2517)</td>
</tr>
<tr>
<td>95% CI for mean difference</td>
<td>((-1.659,-1.135))</td>
<td>((-0.886,-0.442))</td>
<td>((-0.4461,-0.0573))</td>
</tr>
</tbody>
</table>

Computation times are used to evaluate efficiency of each heuristic. All four heuristics are programmed in Matlab and run on a Dell Precision T1700 with Intel Core i5-4590 CPUs of 3.3 GHz.
In order to improve efficiency of the CFI heuristic, we determine the number of iterations $r$ by changing $r$ from 1 to 16 based on large-scale instances. We calculate the deviations from upper bound for each value of $r$. Figure 4.3 indicates that when the value of $r$ is larger than or equal to 6, the deviation asymptotically reaches to the same level of 2.08%, which means our CFI heuristic generates all possible sequences in 6 iterations. Therefore, we set the number of iterations $r$ as 6.

The average computation time (in seconds) required for large-scale instances by each heuristic is given in Table 4.9. On average, the CFI heuristic uses less CPU time than the LS heuristic, but more CPU time than the PH1(p) and FNM heuristics. Although the computational complexity of the FNM heuristic is $O(n^4)$, higher than that of the LS and CFI heuristics, respectively, the FNM heuristic takes less computation time, because there are 10 iterations in the LS heuristic and 6 in our CFI heuristic. Overall, the CFI heuristic achieves the best effectiveness in less computation time among three popular heuristics.
Table 4.9: CPU times of four heuristics for large-scale instances to \( \min(\sum C_j) \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( m )</th>
<th>PH1(p)</th>
<th>FNM</th>
<th>LS</th>
<th>CFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>0.00</td>
<td>0.02</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.00</td>
<td>0.01</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.00</td>
<td>0.01</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td>0.02</td>
<td>0.10</td>
<td>0.71</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.02</td>
<td>0.10</td>
<td>0.72</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.02</td>
<td>0.10</td>
<td>0.72</td>
<td>0.46</td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>0.09</td>
<td>0.80</td>
<td>4.10</td>
<td>2.66</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.10</td>
<td>0.83</td>
<td>4.23</td>
<td>2.78</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.12</td>
<td>0.86</td>
<td>4.17</td>
<td>2.81</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
<td>0.69</td>
<td>9.24</td>
<td>28.78</td>
<td>19.26</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.77</td>
<td>9.38</td>
<td>28.65</td>
<td>19.51</td>
</tr>
<tr>
<td>500</td>
<td>20</td>
<td>10.64</td>
<td>283.49</td>
<td>467.04</td>
<td>319.58</td>
</tr>
</tbody>
</table>

(Units: seconds)

4.4 Trade-off balancing

For bi-objective optimization of \( C_{\text{max}} \) and \( \sum C_j \), we introduce the trade-off balancing function to evaluate the performance of each heuristic. The trade-off balancing function is shown in Equation (4-5) as follows:

\[
TOB(\alpha) = \alpha C_{\text{max}}^N + (1 - \alpha) TCT^N
\]  

(4-5)

Where the \( \alpha \) is the preference or weight for the objective of \( C_{\text{max}} \), \( C_{\text{max}}^N \) is the normalized value of \( C_{\text{max}} \) and \( TCT^N \) is the normalized value of TCT. We compare our AIT heuristic and CFI heuristic with LS and CH heuristics based on large-scale instances. We have chosen \( \alpha \) to be 0, 0.25, 0.5, 0.75, and 1. If \( \alpha \) is less than 0.5, we give more preference to TCT and give more preference to \( C_{\text{max}} \) when \( \alpha \) is greater than 0.5. The following Figure 4.4 shows how the performances of each heuristic behave given different values of \( \alpha \).
(a) The trade-off balancing when $\alpha = 0$

(b) The trade-off balancing when $\alpha = 0.25$
(c) The trade-off balancing when $\alpha = 0.5$

(d) The trade-off balancing when $\alpha = 0.75$
(e) The trade-off balancing when $\alpha = 1$

Figure 4.4: The performances of each heuristic given different values of $\alpha$

From Figure 4.4, given different values of $\alpha$, we can see that both AIT and CFI heuristics obtain the best performance. When the preference is given more to $C_{max}$, the AIT heuristic performs the best. When the preference is given more to the $TCT$, the CFI heuristic performs the best. Besides, the trends obtained by the AIT and CFI heuristics go downwards as the number of jobs increases from 20 to 500, whereas the trends of both the LS and CH heuristics go up. In addition, when the number of job is 500, the performances of AIT and CFI heuristic almost converge together, which indicates that the initial sequence algorithm (ISA) plays an important role to generate the final solutions, since both heuristics use the same ISA to generate initial sequence. Overall, our proposed heuristics have better performances to minimize the trade-off than the other two heuristics.

4.5 Case study on UKHC historical data

To validate our CFI heuristic for operating room (OR) scheduling across the periop process in a healthcare system, we carry out a case study based on historical OR data from
University of Kentucky HealthCare (UKHC), in which the first come first serve (FCFS) rule is used for OR scheduling, especially for emergencies.

The historical data set obtained from UKHC consists of almost 30,000 cases in 365 consecutive days from 2013 to 2014. UKHC schedules operating rooms on weekdays, and opens emergency rooms on weekends and holidays, thus the number of cases on weekends and holidays is much less than that on weekdays. Therefore, removing data on weekends and holidays, we have more than 27,000 cases in 50 weeks with 5 days a week, i.e., in 250 days. First, we compare the sequences of the PH1(p), FNM, LS, and CFI heuristics with the UKHC ones based on average patient flow time (APFT), which equals to the total completion time divided by the number of patients served in a day. Second, we use statistical process control (SPC) techniques to compare the process capability based on APFTs generated by our CFI heuristic and the actual UKHC data.

Table 4.10 shows the APFTs and standard deviations for four heuristics and the actual UKHC data. As shown in Table 4.10, our CFI heuristic can achieve the smallest APFT with the smallest standard deviation.

<table>
<thead>
<tr>
<th></th>
<th>PH1(p)</th>
<th>FNM</th>
<th>LS</th>
<th>CFI</th>
<th>UKHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>APFT</td>
<td>545.19</td>
<td>544.83</td>
<td>544.91</td>
<td><strong>544.74</strong></td>
<td>613.09</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>40.91</td>
<td>40.85</td>
<td>40.88</td>
<td><strong>40.82</strong></td>
<td>56.20</td>
</tr>
</tbody>
</table>

Using SPC techniques, we generate process capabilities for both CFI and the UKHC data as shown in Figure 4.5. The user-defined lower specification limit (LSL) and upper specification limit (USL) are set as 400 and 700 minutes, respectively, according to the historical data from UKHC. The process capabilities \( c_p \) and \( c_{pk} \) are defined as \( c_p = \frac{USL-LSL}{6\sigma} \) and \( c_{pk} = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right) \), where \( \mu \) is the average patient flow time and \( \sigma \) is...
the standard deviation for the process performance (Montgomery, 2007). Process
capability $c_p$ indicates if the outcomes of a process are within the control limits. With the
fixed range of specification limits, which is USL − LSL, the larger the $c_p$, the less the
variation in process, which is $6\sigma$. Process capability index $c_{pk}$ indicates if the outcomes are
centered around the average performance. The larger the $c_{pk}$, the less likely that the
outcomes will fall outside the limits, LSL or USL. As shown in Figure 4.5, the $c_p$ is 1.21
for our CFI heuristic and 0.90 for the UKHC data, and the $c_{pk}$ is 1.17 for our CFI heuristic
and 0.52 for the UKHC data. Obviously, the APFTs generated by our CFI heuristic are
more centered within the specification limits and with less variation, compared to those
from historical UKHC data.
Moreover, we generate the Xbar-R charts based on APFTs in 250 days as shown in Figure 4.6. The APFT is 544.7 minutes for our CFI heuristic, and 613.1 minutes for the data from UKHC. The improvement in average patient flow times can be calculated by $(613.1 - 544.7)/613.1 = 11.2\%$. The range of variation on our CFI heuristic is 205.5 minutes, less than that of 275.4 minutes for the UKHC data.
Figure 4.6: X-bar&R charts of average patient flow times

These results from X-bar&R charts support those of $c_p$ and $c_{pk}$, and the 11.2% improvement in average patient flow time indicates that potentially 3,000 additional patients could be served in a year if our CFI heuristic was applied for sequencing. However, in practice, OR scheduling and control is affected by many other factors in addition to
sequencing, such as emergencies, the availability of patients in the waiting list, surgical staff, and equipment, etc. These realities are reflected in the UKHC data.

Based on the above case studies, we have test the effectiveness, efficiency and robustness of our AIT and CFI heuristics. Compared with the best known heuristics for $F_{m|nwt|C_{max}}$ and $F_{m|nwt|\Sigma C_j}$ problems, such as LC, ADT and CH heuristics for $C_{max}$ and PH1(p), FNM and LS heuristics for $\Sigma C_j$, our heuristics outperform them significantly. In addition, for bi-objective optimization, our heuristics perform better than others based on the trade-off balancing function. Last, compared with actual performance at UKHC, our CFI heuristic can potentially serve 3,000 additional patients.
Chapter 5 Conclusion and future work

5.1 Concluding remarks

ORs are the largest cost center and the greatest revenue source simultaneously for hospitals (Ghazalbash et al., 2012). To achieve high utilization of OR units and reduce average waiting times are two main objectives in the 3-stage periop process. In the 3-stage periop process, the performance of one stage affects the performances of adjacent stages, which is also characteristic of a three-stage no-wait flow shop. The upstream stages affect downstream stages to influence the utilization and patient flow, and downstream stages can affect upstream stages as well, such as blocking and PACU boarding. Therefore, the 3-stage no-wait flow shop is suitable to model the periop process, in which no waiting time between stages is allowed.

No-wait flow shop production is common in industry, where no waiting time is allowed between intermediate operations. Minimization of makespan ($C_{max}$), which relates to utilizations, for no-wait flow shop production has been proven to be $NP$-hard and minimization of total completion time ($\sum C_j$), which relates to average waiting time, has been proven to be $NP$-complete. It is extremely time consuming to find optimal solutions using exact methods for such problems. Therefore, heuristics are widely used to find near optimal solutions for production scheduling in manufacturing. To min($C_{max}$), the LC, ADT, and CH heuristics are three typical ones developed recently (Laha and Chakraborty, 2009, Ye et al., 2016; Li et al., 2008), and to min($\sum C_j$), the PH1(p), FNM, and LS heuristics are three typical ones in the literature (Aldowaisan and Allahverdi, 2004; Framinan et al., 2010; Laha et al., 2014). These heuristics can obtain good solutions in a reasonable time, even for large-scale instances. However, there are some shortcomings of their heuristics, such
as high computational complexity in the FNM heuristic, lack of large-scale computational experiment in the LS heuristic.

To enhance effectiveness and efficiency beyond these heuristics, we propose the AIT heuristic to min($C_{max}$) and the CFI heuristic to min($\sum C_j$) for no-wait flow shop production. To improve effectiveness, we first take the current idle times and future idle times into consideration, proposing an initial sequence algorithm, and then use the insertion and neighborhood exchanging methods to further improve the solutions.

To increase efficiency, we first introduce an objective increment method to reduce the computational complexity from $O(n)$ to $O(1)$ in calculating total completion time, and then set the number of iterations in our AIT and CFI heuristic to further reduce the computation times.

Compared with the LC, ADT and CH heuristics for the $F_m |nwt| C_{max}$ problems, based on 600 small-scale instances, our AIT heuristic achieves the best performance on average relative percentage deviation (ARPD) of 0.23%, maximum percentage deviation (MPD) of 5.14%, and the percentage of the best solutions (PBS) of 82.17%. Based on large-scale instances in Taillard’s benchmarks, our AIT heuristic achieves the best performance on ARPD of 0.23% and PBS of 65.83%, although not on the MPD of 2.95%.

Compared with the PH1(p), FNM and LS heuristics for the $F_m |nwt| \sum C_j$ problems, based on 600 small-scale instances, our CFI heuristic achieves the best performance on ARPD of 0.06%, MPD of 2.98%, and PBS of 88%. Based on large-scale instances in Taillard’s benchmarks, our CFI heuristic achieves the best performance on ARPD of 2.08% and PBS of 53%, although not on the MPD of 7.05%. In addition, on average, the CPU
time of our CFI heuristic is 30.68 seconds, based on Taillard’s benchmarks, less than the 44.95 seconds of the LS heuristic.

In the trade-off balancing, our AIT and CFI heuristics outperform the LS and CH heuristics. When the preference is given more to $C_{\text{max}}$, the AIT heuristic performs the best. When the preference is given more to $\sum C_j$, the CFI heuristic performs the best. Besides, the trends obtained by the AIT and CFI heuristics go downwards as the number of jobs increases from 20 to 500, whereas the trends of both the LS and CH heuristics go up.

In a case study using historical data from UKHC, we found our CFI heuristic can achieve 11.2% improvement in average patient flow times over UKHC’s performance, and the average patient flow times generated by our CFI heuristic are under better process control with less variation. This means additional patients can potentially be served and there is a greater control of OR management across the peri-operative process.

Overall, our AIT and CFI heuristics can achieve good effectiveness and efficiency for no-wait flow shop scheduling and operating room scheduling.

5.2 Future work

Variation in processing times is a common disturbance to flow shop production in manufacturing or healthcare systems, which consequently generates an uneven workflow on the production lines. Cao, Patterson, and Bai (2005) have concluded that the main source of variations in processing time is the variation in actual processing times from their expected values. The difficulty to handle the variation in processing time is that we do not exactly know how the current variations and performance will impact on the future overall performance. One topic of our future work is adaptive production control by resequencing
jobs for this type of disturbances that the overall system performance can be under control in an acceptable range.

Besides variations in processing time, there are trade-offs between different objectives. Li et al. (2014) proved that the objectives of \( \min(C_{\text{max}}) \) and \( \min(\sum C_j) \) are not consistent in traditional permutation flow shop scheduling problems. The limitation of current work focuses on the evaluation scheme of trade-off for each single-objective heuristic, not addressing the root causes of trade-offs in no-wait flow shop production scheduling. The root cause of trade-off in flow shop scheduling is unbalanced cycle time at different stages in production lines. Therefore, another topic of our future work is to establish effective and efficient heuristics for multi-objective optimization in no-wait flow shop based on balancing cycle times at each stage.

The third topic of our future work is to apply these heuristics to the hybrid no-wait flow shop, in which there exist parallel machines at each stage. Based on the second future work, we want to extend 3-machine flow shop to hybrid no-wait flow shop, which relates to the resource allocation at each stage. On the one hand, from the second topic of future work, we can find how sequencing changes the cycle time in no-wait flow shop scheduling. On the other hand, different schemes of resource allocation affect the cycle time of each machine at each stage as well. Therefore, it is important and meaningful to study the following three questions: (1) given the fixed resources, how sequencing affects cycle time? (2) Given the fixed sequencing method, how can we obtain better performances by adjusting the resource allocation scheme? (3) How do the resource allocation scheme and sequencing interact with each other to affect the performance of the systems?
Overall, the future work will focus on achieving adaptive scheduling in operating rooms scheduling when (1) disturbances occur in systems; (2) optimizing different inconsistent objectives, and (3) resource allocation and sequencing interact with each other.
References:


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PUBLICATIONS AND PRESENTATIONS

Publications in referred international journals

Publications in referred international conferences
2. Abedini, A., W. Li, H. Ye. 2017. An optimization model for operating room scheduling to reduce blocking across the perioperative process. 45th North American Manufacturing Research Conference. (Accepted)

Presentations