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## Error Analysis of Stored Grain Inventory Determination

Aaron P. Turner

*University of Kentucky, aaron.turner@uky.edu*

Michael D. Montross

*University of Kentucky, michael.montross@uky.edu*

Joshua J. Jackson

*University of Kentucky, joshjackson@uky.edu*

Samuel G. McNeill

*University of Kentucky, sam.mcneill@uky.edu*

Mark E. Casada

*USDA Agricultural Research Service*

*See next page for additional authors*

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### Authors

Aaron P. Turner, Michael D. Montross, Joshua J. Jackson, Samuel G. McNeill, Mark E. Casada, Josephine M. Boac, Rumela Bhadra, Ronaldo G. Maghirang, and Sidney A. Thompson

# ERROR ANALYSIS OF STORED GRAIN INVENTORY DETERMINATION

A. P. Turner, M. D. Montross, J. J. Jackson, S. G. McNeill, M. E. Casada,  
J. M. Boac, R. Bhadra, R. G. Maghirang, S. A. Thompson

**ABSTRACT.** Estimation of the quantity of stored grain is important for crop insurance, financial statements, and inventory control. Traditionally, the height of grain has been measured using weighted tape measures, and the volume is subsequently computed using standard geometric shapes (cylinders and cones) along with visual correction of the grain surface. Field measurements by four trained USDA Farm Service Agency and crop insurance agents on older farm-sized bins (8.2 to 11.0 m, or 27 to 36 ft, in diameter) resulted in standard deviations between 0.02 and 0.30 m for the equivalent height when the grain surface was not level. The largest errors were observed with off-center surface profiles. When the grain surface inside the bins was manually leveled, the standard deviation of the equivalent height varied between 0.02 and 0.18 m. Error propagation analysis was performed to evaluate the error in measuring the volume of stored grain caused by the uncertainty associated with measuring the bin diameter and grain height as a function of the ratio of equivalent level grain height to bin diameter (EH/D). The errors were examined using an assumed range of uncertainties to explore how each factor contributed to the error in different scenarios. The uncertainty increased as the EH/D ratio decreased, especially in small-diameter bins with shallow grain heights where the volume bounded by the surface profile of the grain represented a large percentage of the total volume within the structure. Therefore, any errors in defining the surface profile resulted in large errors in the total estimated volume of grain in small-diameter bins. Conversely, for large-diameter bins with large grain heights, the surface profile represented a very small percentage of the total volume of grain. Consequently, any errors in defining the profile produced much smaller errors in the total grain volume. For accurate measurements, defined as a standard deviation of 1.2 cm (0.04 ft) in the diameter and 7.6 cm (0.25 ft) in the equivalent level height, the overall uncertainty in the volume measurement never exceeded 5% for smaller bins (<10 m in diameter) and decreased to less than 1% for larger bins (>10 m in diameter). A sensitivity analysis was performed on the three most common methods used to convert the measured volume to a quantity of grain. In each method, the quantity of grain stored in a bin is the product of the volume measurement and the pack factor. With all three methods, the sensitivity of the pack factor determination resulted in an error of less than 1% in the estimated total quantity of stored grain. The volume measurement accounted for the majority of the error in the estimation of bin inventory. As a result, accurate measurement of the bin volume is critical for determining the quantity of stored grain.

**Keywords.** Corn, Error propagation, Stored grain inventory, Technician variation, Wheat.

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The authors are **Aaron P. Turner, ASABE Member**, Engineer Associate, **Michael D. Montross, ASABE Member**, Professor, **Joshua J. Jackson, ASABE Member**, Post-Doctoral Scholar, and **Samuel G. McNeill, ASABE Member**, Associate Extension Professor, Department of Biosystems and Agricultural Engineering, University of Kentucky, Lexington, Kentucky; **Mark E. Casada, ASABE Member**, Research Agricultural Engineer, USDA-ARS Stored Product Insect and Engineering Research Unit (SPIERU), Manhattan, Kansas; **Josephine M. Boac, ASABE Member**, Research Associate, **Rumela Bhadra, ASABE Member**, Research Associate, and **Ronaldo G. Maghirang, ASABE Member**, Professor, Department of Biological and Agricultural Engineering, Kansas State University, Manhattan, Kansas; **Sidney A. Thompson, ASABE Member**, Professor, College of Engineering, University of Georgia, Athens, Georgia. **Corresponding author:** Michael Montross, 128 Barnhart Bldg., University of Kentucky, Lexington, KY 40546-0276; phone: 859-218-4319; e-mail: michael.montross@uky.edu.

Grain is bought and sold based on weight, with discounts or premiums based on the moisture content, test weight, and various other quality indicators associated with different grain types (i.e. dockage, foreign material, broken corn). Adjusters regularly need to measure the quantity of on-farm stored grain for insurance claims as well as collateral for loans. Additionally, grain processors, grain dealers, and elevators are subject to routine audits to ensure that enough grain is in storage to meet their physical and financial obligations. Therefore, maintaining accurate grain inventory is important for feed and commercial elevators, farms with storage facilities, and any other facilities associated with the grain trade.

Although grain is weighed at the time of sale, it is impractical to measure the mass once the grain is placed into storage. Another complicating factor is that the U.S. system trades grain based on bushels, which is the mass of grain divided by the standard test weight. Multiple methods have been employed to estimate the quantity of stored grain within grain bins, silos, piles, and warehouses (ASABE,

2010; IAAA, 1980; Thompson et al., 1987; USDA, 2011, n.d.). Each of these methods requires the operator to measure the volume of stored grain. The quantity of stored grain is subsequently estimated by multiplying the volume by a pack factor that is based on bin geometry (diameter) and test weight that provides the auditor with the bushels at the standard test weight for each grain type. Moisture content influences packing: the higher the moisture, the greater the packing (Thompson et al., 1987; USDA, n.d.). For wheat, the packing factor should be increased by 1% for every 1% in moisture above 11% (USDA, n.d.), although this is not typically done in practice. Grain packing is a complex issue because grain density *in situ* or packing within the bin is likely influenced by several additional factors, including the initial bulk density, grain depth, moisture content, type and level of dockage, friction properties of the grain, shape of the storage structure, type of grain, method of filling, kernel dimensions, kernel density, vibration due to machinery, and biological activity (insects and molds).

Several tools and methodologies have been developed to measure the volume of stored grain. These systems use a variety of sensors such as contact level indicators (tilt switches, pressure diaphragms, and rotary paddles) and non-contact level indicators (stereovision, radar, ultrasound, and lasers). Each system has advantages and disadvantages. Contact sensors provide an inexpensive method of measuring grain height at one point and are not influenced by dust. While proper placement of contact sensors can provide accurate point measurements, details of the overall surface condition are not provided. Non-contact sensors often have the ability to map the grain surface and provide an accurate grain height measurement. However, non-contact sensors are not commonly used because permanent mounting is required, they are relatively expensive, and dust can influence or impair their function.

The majority of measurements are still taken using a weighted fiberglass tape, and grain depth is often measured from the access door located in the bin roof near the wall, which provides only a single data point. The equivalent level height of the grain is then determined based on the depth measurement and a visual assessment of the surface profile.

The volume of grain can then be calculated. Commercial facilities have a bin inventory sheet with the effective depth and volume per unit depth (bu ft<sup>-1</sup>) predetermined. In this case, only the headspace between the eave and the grain surface must be measured. Hurburgh (2002) presented an example of an error analysis in the volumetric measurement and determination of stored grain in a bin. An estimated maximum error ( $\pm 2$  standard deviations) was assumed for the stretch in the tape (0.2%), level fill depth estimate (1.0%), test weight (2.0%), moisture (1.2%), and pack factor (1.0%) that resulted in an overall error of 2.7% in the grain inventory when the standard deviations were added in quadrature.

Unusual grain surface topography can also lead to complications when determining the equivalent level height of grain. Figure 1, which shows six potential grain profiles, demonstrates the difficulty an operator could face by relying on a single point measurement. In each case, an 18.3 m (60 ft) diameter bin was assumed filled with the same volume of (3524 m<sup>3</sup> or 100,000 bu) of grain, assuming an angle of repose of 25°. For the peaked surface condition the equivalent level height of grain is 13.4 m (sidewall grain height = 12 m, peaked cone height = 1.4 m). An error of as little as 0.1 m in height could result in an error of 0.75%, or 26.4 m<sup>3</sup> (750 bu). If an operator assumed a peaked grain surface and did not visually inspect the surface when measuring, the equivalent level height of grain and the estimated grain volume could vary significantly. The most disproportionate situation involves a side draw unloading system using only a single measurement taken at the sidewall, where the equivalent level height of grain could vary from 9.9 to 18.5 m. This would result in errors of -26.1% (underestimate) to +38.0% (overestimate) in the volume of grain in the bin. For the other four conditions, errors in the estimated volume would vary from +7.5% for the partially inverted surface condition to +20.8% for the inverted cone condition. These errors represent the extremes. In practice, inspectors make visual adjustments to reduce these errors, but figure 1 illustrates the potential magnitude of measurement errors when accounting for grain surface topography.

The objective of this study was to evaluate the source and

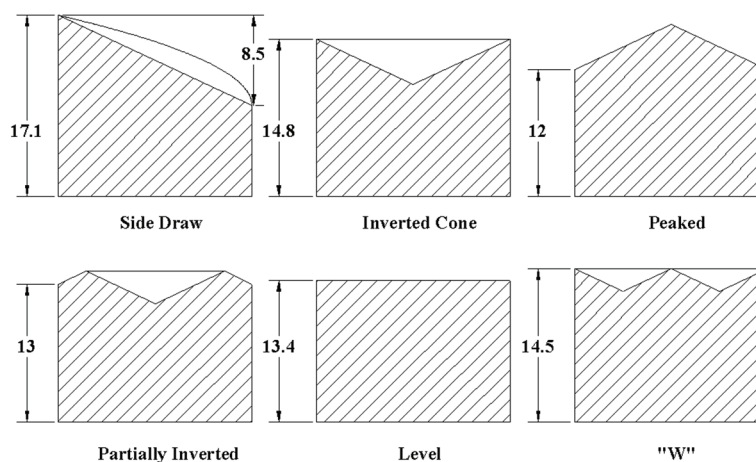


Figure 1. Variation of grain height (m) in an 18.3 m diameter bin filled to a constant volume of 3524 m<sup>3</sup> with typical grain surface conditions experienced during storage.

magnitude of errors associated with stored grain inventory estimation. This included an error analysis associated with the volume measurement, a sensitivity analysis on the prediction of the packing factor, and the overall impact on the measurement of stored grain inventory.

### VOLUME MEASUREMENT TECHNIQUES

The volume of grain in a bin is determined based on the bin diameter and the equivalent level height of grain. The methods used to account for the surface condition of the grain (fig. 1) is somewhat subjective and can be influenced by operator experience, visibility of the grain surface from the top access port, lighting, dust, and other factors.

A number of tools can be employed to better estimate the volume of grain in a bin. Standard angles of repose can be found in published literature for each grain type (Brooker et al., 1992; Eurocode, 2006). These can be used if the grain is fully peaked or fully inverted. Commercial facilities often have a measurement location in the center of the bin to determine the height of the cone in conjunction with a measurement point at the eave or sidewall. In other bins, the grain height is measured at a location 1/6 of the diameter from the wall. This location is commonly used because grain surfaces that are coned downward or coned upward have an equal volume of grain in the cone above and below this location, which would result in the equivalent level height of the grain.

Another method used in inventorying grain involves breaking the structure into easily measured geometries. This method works well for simple geometries but can become difficult when unusual grain surfaces are encountered. The state of Michigan switched from conventional audits for grain dealers to using the ExamHand self-inventory software (MDARD, 2012; Miller, 2010). This software uses measurements from key points on the grain surface to estimate the total volume of grain in the bin.

ASABE Standard EP413.2 (ASABE, 2010) provides guidelines for determining the capacity of grain bins and can be used to estimate the quantity of grain in storage when the grain is either peaked or leveled. This procedure uses the angle of repose of the grain to measure the peaked volume. Equation 1 determines the volume of grain in a full bin based on the bin diameter, angle of repose, and eave height (height of the bin wall above the floor). This equation works well for the ideal situation of simple conical and cylindrical shapes:

$$V_p = \frac{\pi D^2}{4} \times EH + \left( \frac{\pi D^2}{4} \right) \left( \frac{\frac{D}{2} \times \tan \alpha}{3} \right) \quad (1)$$

where

- $V_p$  = peaked volumetric capacity of the bin, m<sup>3</sup> (ft<sup>3</sup>)
- $D$  = bin diameter, m (ft)
- $EH$  = eave height, m (ft)
- $\alpha$  = angle of repose (degrees).

However, as shown in figure 1, bins often have grain profiles that are partially peaked or partially inverted, off-center cones, or side draws, to mention just a few. Most auditors

attempt to adjust for irregularities by visual inspection; however, as discussed later, this visual correction introduces variation among auditors and impacts the inventory estimate.

### PACKING ESTIMATION

#### Using Janssen's Equation

Janssen's equation (Janssen, 1895) can be used to predict the pressures within grain storage structures. The classic theory provided by Janssen (1895) for predicting pressure in grain bins, given in differential form, is:

$$\frac{dP}{dy} = gD(P) - \frac{kP\mu}{R} \quad (2)$$

where

- $P$  = vertical overburden pressure within the bin (kPa)
- $y$  = grain depth (m)
- $g$  = gravitational acceleration constant (kN m<sup>-3</sup>)
- $D(P)$  = bulk density within the bin as a function of pressure (kg m<sup>-3</sup>)
- $k$  = lateral to vertical pressure ratio (dimensionless)
- $\mu$  = coefficient of friction of grain on bin wall (dimensionless)
- $R$  = hydraulic radius (m).

Numerous studies involving grains have shown that the vertical overburden pressure ( $P$ ) caused by the cumulative weight of the overbearing material in a storage structure causes the stored material to compress, which results in an increased bulk density. This increase in bulk density caused by the overburden pressure is commonly referred to as packing and is a primary concern when estimating the amount of grain in storage. Previous research (McNeill et al., 2004; Thompson et al., 1987; Thompson and Ross, 1983) has shown that the differential form of Janssen's equation (eq. 2), assuming a variable bulk density, can be used to estimate the density increase, packing, and total quantity of stored grain in a bin. In these studies, the bulk density ( $D$ ) was assumed to vary with respect to overburden pressure, type of grain, and moisture content. An inventory tool (WPACKING) based on this premise has been proposed and evaluated in several works (Bhadra et al., 2015; Boac et al., 2015; Thompson et al., 1987, 1991).

#### FSA and RMA Combined Test Weight and Pack Factor Adjustments

In the U.S., two methods are commonly used to estimate the amount of packing in bins. These two methods are administered by the USDA Farm Service Agency (FSA) and Risk Management Administration (RMA). Both techniques use empirical tables that assume packing is a function of test weight and bin diameter or cross-sectional area of the structure (USDA, 2011, 2012b, n.d.). The FSA method also allows the pack factor to be corrected for site-specific variables that impact packing (spreaders, vibrating machinery, moisture content, etc.). Further details and the application of these methods can be found in Bhadra et al. (2015) and Boac et al. (2015).

### GRAIN QUALITY FACTORS

Test weight impacts the combined test weight and pack

correction factor with the FSA and RMA methods. Test weight also impacts the packing prediction when using the WPACKING method and equation 2, as the bulk density at the top grain surface is assumed equal to the test weight. Moisture influences the compressibility relationship in Janssen's equation, and the FSA packing method includes an additional adjustment factor for conditions other than normal, which takes into account grain stored at elevated moisture. Moisture content and test weight are typically measured using electronic moisture meters specified by the USDA Grain Inspection, Packers, and Stockyards Administration (USDA, 2012a). Some stored grain facilities, and all official grades, use the Winchester cup method for measuring the test weight (FGIS, 2013).

For wheat, the tolerance for moisture meters is 0.04 of the percent moisture content, with a minimum tolerance of 0.7% in moisture content, while the tolerance for test weight is  $6 \text{ kg m}^{-3}$  ( $0.5 \text{ lb bu}^{-1}$ ) (NIST, 2015). Laux et al. (2015) evaluated the change in the standard deviation of moisture and test weight measurements from elevators before and after adoption of quality management systems. Those researchers found a standard deviation mean difference of approximately  $5.4$  to  $6.9 \text{ kg m}^{-3}$  ( $0.43$  to  $0.55 \text{ lb bu}^{-1}$ ) in the test weight measurement and  $0.19$  to  $0.36$  percentage points in the moisture measurement depending on the type of certification the facility had. This provides estimates of errors found in practice related to grain quality parameters that could influence grain packing.

## MATERIALS AND METHODS

### OBSERVED VARIATION IN FIELD MEASUREMENTS

Two experiments were used to evaluate potential errors with existing measurement methods. First, the errors associated with using a weighted tape measure and laser distance meter were evaluated in an indoor, lighted stairwell by personnel who were measuring grain bins for the project. Two weighted fiberglass tape measures (with gradations of  $0.3 \text{ cm}$  or  $0.01 \text{ ft}$ ) and two laser distance meters (Disto D8, Leica Geosystems, Norcross, Ga.) with a reported accuracy of  $1.0 \text{ mm}$  were used at three heights ( $5.3$ ,  $9.4$ , and  $13.5 \text{ m}$ ). Heights were measured relative to the ground and relative to a container filled level with corn to simulate the minimum expected error in the height measurement. In all cases, measurements were referenced to the top railing. These experiments allowed potential errors due to stretch in the tape measure and due to the weighted end of the tape sinking into the grain.

A second set of experiments was conducted in bins located on the C. Oran Little Research Center (Versailles, Ky.) of the University of Kentucky following the 2009 corn harvest. In these experiments, the inventory of five different bins was independently taken by four crop insurance and RMA auditors using standard RMA procedures and subsequently compared. The bin diameter and eave height (distance between top of the bin sidewall to the bin floor) of each bin are summarized in table 1. All bins were corrugated steel construction and ranged from  $7.3$  to  $11 \text{ m}$  ( $24$  to  $36 \text{ ft}$ ) in diameter. Bin measurements (eave height, diameter, plenum

**Table 1. Bin diameter and eave height.**

Bin	Diameter, m (ft)	Eave Height, m (ft)
1	11.0 (36)	9.4 (30.8)
2	8.2 (27)	6.1 (20.1)
3	8.2 (27)	7.4 (24.4)
4	7.3 (24)	7.4 (24.4)
5	8.2 (27)	7.4 (24.4)

height, and effective depth) were taken by the authors prior to filling, and all measurements were referenced to the bottom lip of the bin access. All of the bins had full aeration floors, and the plenum height was estimated based on the bolt position on the outside of the bin. The bins were in varying states of fill, representative of the loading conditions encountered in actual bins. Four of the bins were filled with corn, and one was filled with soft red winter wheat. The majority of the corn used in these experiments was previously dried to approximately  $14\%$  using a high-temperature dryer. The bins had distinctly different grain surface profiles and equivalent level grain height to diameter ratios ( $EH/D$ ). Bin 1 had been partially unloaded from an off-center well and contained an inverted off-center cone. After initial unloading, additional grain had been added to the top surface of the grain. Bin 2 was approximately full, and the grain surface profile beyond the manhole was difficult to see; all auditors assumed that the bin was symmetrically filled. Bin 3 was almost empty, but the grain surface profile was not symmetrical. Bin 4 was nearly empty and appeared to have been unloaded from an off-center well. Bin 5 had a typical symmetrical inverted cone formed by unloading from a center well.

The characteristic bin dimensions, such as circumference, eave height, and false floor height, were measured by each auditor using a fiberglass tape measure. Equivalent grain height was measured using a weighted fiberglass tape measure and adjusted according to standard practices by each auditor. The volume of grain per foot ( $\text{bu ft}^{-1}$ ) was determined using the cross-sectional area based on the measured bin circumference. The total grain volume was then determined by multiplying the volume of grain per foot by the equivalent height of grain. Once the volume was estimated, the amount of packing was estimated (USDA, 2011) and applied to the volume measurement to determine the total quantity of grain stored. The results of the inventory were reported using a USDA Commodity Credit Corporation farm storage loan worksheet (document CCC-677-1). After initial measurements, the bins were manually leveled and measured again. Before and after inventory estimates were compared for significance using paired t-tests.

### ERROR PROPAGATION IN VOLUME MEASUREMENT

Error propagation is commonly used to determine the level of cumulative uncertainty in a measurement (Dally et al., 1984; Doebelin, 2004; NIST, 2012). Inventorying grain requires measurement of multiple variables to calculate an overall desired dependent variable. The uncertainty in the calculated value can be determined from variance summation (Dally et al., 1984; NIST, 2012). To determine the volume of grain in a bin, the bin diameter and equivalent level grain height need to be measured. The volume can then be determined using equation 3:

$$V = \frac{\pi D^2 \times EH}{4} \quad (3)$$

where

$V$  = volume of stored grain, m<sup>3</sup> (ft<sup>3</sup>)

$D$  = diameter of bin, m (ft)

$EH$  = equivalent level height of grain, m (ft).

In equation 3, the equivalent level height represents the height of a level cylinder after the headspace, surface topography, and plenum height have been taken into account. The total uncertainty in the volume measurement is a function of the error associated with each independent variable and can be determined from:

$$\delta V = \sqrt{\left(\frac{\partial V}{\partial D}\right)^2 \sigma_D^2 + \left(\frac{\partial V}{\partial H}\right)^2 \sigma_H^2} \quad (4)$$

where  $\sigma_D$  and  $\sigma_H$  are the standard deviations of the error in the diameter and equivalent level height measurements, respectively.

The effect of the uncertainty in each variable on the total volume was examined using three estimates of the standard deviation of the equivalent level height measurement and two estimates of the standard deviation of the bin diameter. These values were intended to represent possible scenarios that would be encountered in the field. The values were based on field measurements and engineering judgement.

The bin diameters analyzed varied between 3.0 and 32.0 m (10 and 105 ft) with  $EH/D$  of 0.25, 0.5, 1.0, 1.5, and 6.0. The heights used in the  $EH/D$  ratio were the equivalent level grain height. For example, a 30 m diameter bin filled to an  $EH/D$  ratio of 0.5 would have an equivalent level grain height of 15 m. An  $EH/D$  ratio of 0.25 would correspond to a bin filled with a very shallow grain depth, while an  $EH/D$  ratio of 6.0 would correspond to a tall slender structure with a very deep grain depth, such as a concrete silo. Typical  $EH/D$  ratios for steel bins are between 0.5 and 1.5. Concrete bins frequently have ratios over 2.0 and can approach 6.0 in very deep small-diameter bins. In this analysis,  $EH/D$  ratio always refers to equivalent level grain height and not the actual eave height and bin diameter. A bin with a diameter of 20 m and an eave height of 20 m would have an  $EH/D$  ratio of 0.25 if the bin was filled to a depth of 5 m.

#### SENSITIVITY ANALYSIS IN PACKING PREDICTION AND ESTIMATION OF STORED GRAIN

A sensitivity analysis was performed to examine the implication of uncertainty in the parameters used to determine the amount of packing and subsequently the quantity of stored grain at the standard test weight. The percentage change was determined for both the predicted pack factor and the quantity of stored grain. The analysis was conducted using the standard FSA and RMA estimation methods and the WPACKING technique (Thompson et al., 1987).

#### FSA and RMA Combined Test Weight and Pack Factor

The FSA method uses a combined test weight and pack factor adjustment in conjunction with the standard test weight to estimate stored grain. The impact of an error in test

weight measurement on the total quantity of grain predicted using this method was evaluated by comparing the predicted quantity at the standard test weight with the quantity predicted when a simulated error was introduced. For both cases, the predicted quantity of grain was determined using the procedures and tables in the warehouse examiner's guide (USDA, n.d.). The pack index for wheat was determined based on cross-sectional area and geometry. This value was then adjusted based on the test weight to produce the combined test weight and pack factor. The resulting percentage error was determined for bin diameters of 4.6, 9.1, 18.3, and 27.4 m (15, 30, 60, and 90 ft). The pack index values were based on the bin diameters taken from tables in the warehouse examiners guide (USDA, n.d.); thus, they were the same for both the base case and the case with simulated error.

The RMA procedure for determining the test weight and packing adjustment is similar to the FSA method. An empirical pack factor table based on the bin diameter, grain type, and test weight (USDA, 2011, 2012b) provided pack factors for seven types of grain. The adjusted pack factor was multiplied by the measured volume of grain in the bin to determine the actual inventory.

Two errors were simulated in the combined test weight and pack factor. Wheat with a standard test weight of 772 kg m<sup>-3</sup> (60 lb bu<sup>-1</sup>) was used as the base case, and a test weight error of +6 kg m<sup>-3</sup> (0.5 lb bu<sup>-1</sup>), i.e., an error of 0.8%, was assumed based on Laux et al. (2015). An error of 20% in the combined test weight and pack factor was also assumed.

#### WPACKING Method Using Differential Form of Janssen's Equation

To determine the quantity of grain stored in a bin, the WPACKING method (Thompson et al., 1987) uses equation 2 to predict the overburden pressure within the bin. The ordinary differential equation (ODE45) solver in MATLAB (ver. R2014B, The MathWorks, Inc., Natick, Mass.) was used with the appropriate values for  $\mu$  and  $k$  to predict the overburden pressures (ASABE, 2010). The corresponding variation in bulk density at each pressure level was determined as a function of test weight, moisture content, and pressure based on the compressibility equation for hard red winter wheat (HRWW) (Turner et al., 2016). The mass of grain in each layer was then calculated using the trapezoidal integration function (TRAPZ) in MATLAB. The quantity of stored grain (bu) was found by dividing the mass by the standard test weight.

The predicted packing and estimated quantity of HRWW were calculated using the differential form of Janssen's equation for bins with diameters of 9.1, 18.3, and 27.4 m (30, 60, and 90 ft) and  $EH/D$  ratios ranging from 0.33 to 3.0. Janssen's equation has three parameters that impact predicted mass:  $\mu$ ,  $k$ , and the compressibility relationship, which is dependent on the grain type, test weight, and moisture content. To explore their impacts, various combinations of assumed errors in these parameters were evaluated. Standard values of  $\mu$ ,  $k$ , test weight, and moisture content (0.6, 0.5, 772 kg m<sup>-3</sup>, and 10%, respectively) were initially selected as baselines for comparison. The change in estimated bin in-

ventory was then calculated relative to the baseline prediction. Five scenarios were examined based on the standard values of the parameters and the standard deviations of their estimate:  $\mu + 0.1$ ;  $k + 0.05$ ;  $\mu + 0.1$  and  $k + 0.05$ ; test weight  $+ 6 \text{ kg m}^{-3}$ ; and moisture content  $+ 0.25\%$ . The compressibility relationship for HRWW was assumed to manifest an error up to 10%. The errors for the pack factor were reported as absolute errors.

## RESULTS AND DISCUSSION

### IDEAL HEIGHT MEASUREMENT

The standard deviation and coefficient of variation for the height measurement at the three heights evaluated in a stairwell are shown in table 2. These tests were conducted in a well-lit stairwell with a level grain surface, no obstruction from ladders, or other complicating factors that would lead to higher errors. There was no difference in the coefficient of variation (CV) between the two targets (concrete floor or container filled with corn), but there were differences between the laser and tape measurements. The two measurement devices were not significantly different at any level for heights measured relative to the level ground; thus, only measurements relative to the grain are shown in table 2. The average heights measured by the laser distance meter and the fiberglass tape were significantly different (95% confidence interval) at the low and medium height levels. However, at the largest height, the measurements were not significantly different. The lack of significant difference between the laser and tape at all levels relative to the ground and at the highest level measured above the grain indicate that tape measure stretch was not a factor. The difference seen at the lower heights measured above the grain indicates that the grain/tape interface has some impact on the height measurement. These effects could be exacerbated in practice where obstructions and limited visibility make it difficult to determine when the tape is in contact with the grain. The average

**Table 2. Average, standard deviation, and coefficient of variation in the measurement of height above corn under ideal conditions using a laser distance meter and a weighted fiberglass tape ( $n = 4$ ).**

Measurement Method	Average Height (m)	$\sigma$ (cm)	CV (%)
Laser meter	5.35	0.8	0.16
	9.43	0.9	0.10
	13.53	1.1	0.08
Fiberglass tape	5.31	1.4	0.26
	9.40	4.6	0.49
	13.51	3.3	0.24

CV was 0.11% with the laser meter and 0.33% with the fiberglass tape. Despite the significant difference at some heights, the differences between measurement methods and the standard deviations relative to the height measured were so small that there would be little practical difference between the laser and the tape.

In an actual bin, a minimum of two measurements are typically required to calculate grain height: the eave height of the bin and the void space above the grain. If the bin contains a plenum, a third measurement of plenum height is required. Thus, the uncertainty value applicable to error propagation would be determined from equation 5 using the standard deviations in the eave height ( $\sigma_{ev}$ ), in the void space height ( $\sigma_{vs}$ ), and in the plenum height ( $\sigma_{pl}$ ). During addition and subtraction, uncertainties add in quadrature (Doebelin, 2004):

$$\delta H = \sqrt{\sigma_{ev}^2 + \sigma_{vs}^2 + \sigma_{pl}^2} \quad (5)$$

Equation 5 was used to calculate the uncertainty in determining the equivalent level height in a grain bin. Based on the data in table 2, a standard deviation of 4 cm in eave height and void space measurement and a standard deviation of 1.5 cm in plenum height were assumed. This resulted in an overall height uncertainty of approximately 6 cm. In practical terms, this corresponds to an uncertainty in the volume measurement ranging from 0.2% to 1.2% for bins with eave heights between 5 and 30 m. This represents the uncertainty in measuring a flat level surface and is the minimum level of uncertainty, before any correction for surface topography or other factors are considered.

### VOLUME MEASUREMENT IN TYPICAL BINS

To examine the uncertainty associated with the equivalent level height, five older farm bins (table 1) were measured in their original condition and after manual leveling. Descriptive statistics for the mean volume, standard deviation of the volume ( $\sigma_v$ ), and standard deviation of the equivalent level height ( $\sigma_H$ ) for unleveled and leveled bins are shown in table 3.

The standard deviation for the eave height in the unleveled bins varied between 0.02 and 0.30 m (0.05 and 0.99 ft). The height from the bin floor to the eave was given, so these values represent the uncertainty associated with measuring the headspace and accounting for surface conditions. This resulted in a standard deviation between 0.3 and 17.3 m<sup>3</sup> for the volume and a coefficient of variation that

**Table 3. Surface conditions and descriptive statistics of mean grain volume, standard deviation of volume ( $\sigma_v$ ), and standard deviation in level height ( $\sigma_H$ ) determined by four FSA and crop insurance adjusters in unleveled and leveled bins when using weighted fiberglass tape ( $n = 4$ ).**

Bin	EH/D Ratio	Surface Condition	Unleveled Bin			Leveled Bin			Percent Change in Volume Estimate (%)
			Mean Volume (m <sup>3</sup> )	$\sigma_v$ (m <sup>3</sup> )	$\sigma_H$ (m)	Mean Volume (m <sup>3</sup> )	$\sigma_v$ (m <sup>3</sup> )	$\sigma_H$ (m)	
1	0.82	Inverted, off center	842.3	17.3	0.18	864.7	17.5	0.18	2.6 <sup>[a]</sup>
2	0.73	Almost full, level	322.0	4.6	0.02	322.0	4.6	0.02	-
3	0.11	Almost empty, off center	50.4	2.6	0.30	44.4	1.1	0.02	-11.8 <sup>[a]</sup>
4	0.16	Almost empty, off center	52.8	6.3	0.15	50.3	1.2	0.03	-4.8
5	0.49	Inverted, centered	210.6	0.3	0.05	218.1	0.9	0.02	3.5 <sup>[a]</sup>
Operator influence on total inventory <sup>[b]</sup>			1478.1	30.3		1499.4	24.0		1.6 <sup>[a]</sup>

<sup>[a]</sup> The unleveled and leveled measurements are significantly different based on a t-test ( $p < 0.05$ ).

<sup>[b]</sup> Sum of the volumes measured by each auditor and standard deviation calculated from the total volume from each auditor.



ranged between 0.13% and 11.94% for the five bins. Bin 3, which had the smallest quantity of stored corn and the most complex surface geometry, also had the highest uncertainty. The largest coefficients of variation were observed for those bins with the most non-uniform grain surface profiles, reinforcing the argument that these conditions lead to increased uncertainty.

For unleveled bins, each of the auditors summed their individual bin measurements to find the total inventory. The average total inventory was 1478.1 m<sup>3</sup>, with a standard deviation of 30.3 m<sup>3</sup> and a coefficient of variation of 2.05% that represented the variation between auditors for determining total inventory. The range in inventory predicted by the auditors varied between 1433.9 and 1500.9 m<sup>3</sup> for unleveled bins. Assuming a grain price of \$120 t<sup>-1</sup> (~\$3.00 bu<sup>-1</sup>), the value of the inventory in storage could range from \$132,850 to \$139,050. This range represents approximately 5% of the economic value of the grain stored in these older farm bins.

Bin 2 was initially level, and repeated measurements gave the same average volume. Of the four remaining bins, manual leveling resulted in significantly different volume estimates for bins 1, 3, and 5, with bin 3 having the largest percent change in estimated volume. This shows that surface conditions can impact the precision of volume estimates and illustrates the need to better estimate the grain surface. Based on the total inventory estimates from the individual auditors, manual leveling resulted in a significantly different total volume, with a mean of 1499.4 m<sup>3</sup>, a standard deviation of 24.0 m<sup>3</sup>, and a range of 1463.5 to 1513.1 m<sup>3</sup>.

As seen in figure 1, the total volume of grain in a round bin can be broken into two parts: a cylinder and the volume of the irregular profile of the grain surface. While calculating the volume of the cylinder is straightforward, calculating the volume of the irregular profile is considerably more difficult. For bins with large  $EH/D$  ratios, the volume of grain within the irregular profile would normally be only a small percentage of the total volume of grain in the bin. Therefore, any errors in estimating the irregular volume represent a small overall error in the inventory. By comparison, for bins with small  $EH/D$  ratios, the volume of grain within the irregular profile is a larger portion of the total volume, and any error in estimating the second volume translates to a larger inventory error.

#### ASSUMED ERROR VALUES IN EQUIVALENT HEIGHT AND DIAMETER

The equivalent level height measurement is subjective in nature and depends on the surface topography, making the standard deviation difficult to quantify for use in an error analysis. However, based on tests conducted in a well-lit stairwell, a standard deviation of 7.6 cm (0.25 ft) was assumed for the error in height measurement for the most accurate case. Based on measurements in bin 3 (standard deviation of 30 cm), a typical error was assumed to be 30 cm (1 ft). A maximum error of 53.3 cm (1.75 ft) was also assumed. While the maximum error appears high and should be lower in practice, large-diameter bins with complex surfaces due to side draws, eccentric loading/unloading, and poor visibility could result in large errors. Inventory measurements on large farm bins in varying states of unloading

could result in such large errors. At commercial facilities, inventory is often determined during annual weigh ups in which bins are consolidated and large errors in the height measurement would not be expected.

Similarly, the error in bin diameter was assumed to be between 1.2 and 4.6 cm (0.04 and 0.15 ft). A number of factors influence the magnitude of this error. For example, some manufacturers of corrugated steel bins (Westeel, 2010) specify that the bin should not be more than 3.8 cm (0.12 ft) out of round. Corrugated steel bins are typically sold in “standard” or “nominal” diameters. However, confusion can arise because some manufacturers use SI units while others use inch/pound units (Tom Gettings, GSI, personal communication, 23 August 2013). Most U.S. bin manufacturers, farmers, elevator managers, and auditors use the nominal diameter in feet. However, bins manufactured using inch/pound units with a nominal diameter of 90 ft would have an actual diameter of 89.5 ft, while a bin with a nominal diameter of 90 ft manufactured using SI units would have an actual diameter of 27 m (88.6 ft). Within the grain industry, both bins would most likely be described as a 90 ft diameter. An auditor could be tempted to calculate the volume using a bin’s nominal diameter because measuring the diameter is often difficult due to obstructions at the foundation. In addition, external vertical wall stiffeners are frequently 7.6 cm (3 in.) in depth or greater, which would also make diameter measurement difficult.

#### POTENTIAL ERRORS IN VOLUME MEASUREMENT

Figure 2 shows an error analysis for a range of  $EH/D$  ratios as a function of bin diameter. The magnitude of the error in grain volume was based on equation 4 and assumed errors in both the bin diameter and grain height. Figure 2 shows six scenarios based on the assumed range of standard deviations for each variable: (a) best-case diameter and height measurement (errors of 1.2 cm in diameter and 7.6 cm in height), (b) worst-case diameter and best-case height (errors of 4.6 cm in diameter and 7.6 cm in height), (c) best-case diameter and average-case height (errors of 1.2 cm in diameter and 30 cm in height), (d) worst-case diameter and average-case height (errors of 4.6 cm in diameter and 30 cm in height), (e) best-case diameter and worst-case height (errors of 1.2 cm in diameter and 53.3 cm in height), and (f) worst-case diameter and worst-case height (errors of 4.6 cm in diameter and 53.3 cm in height).

The magnitude of the error in grain volume was expressed for bins with height-to-diameter ( $EH/D$ ) ratios between 0.25 and 6.0, which accounts for shallow grain depths ( $EH/D = 0.25$ ) and tall bins, such as those found in concrete silos ( $EH/D = 6.0$ ). In all six scenarios, the error decreased with increases in both bin diameter and  $EH/D$  ratio. Large errors in the height of grain (scenarios e and f) had a greater effect than the error in bin diameter on the magnitude of the error. In shallow bins ( $EH/D = 0.25$ ), the surface profile had a larger relative impact on the grain volume, and any errors made in estimating the profile can have an appreciable effect on the grain volume. However, as bin diameter increased, the error decreased to levels below 10% for all scenarios. In scenario a, the best-case diameter and height measurement

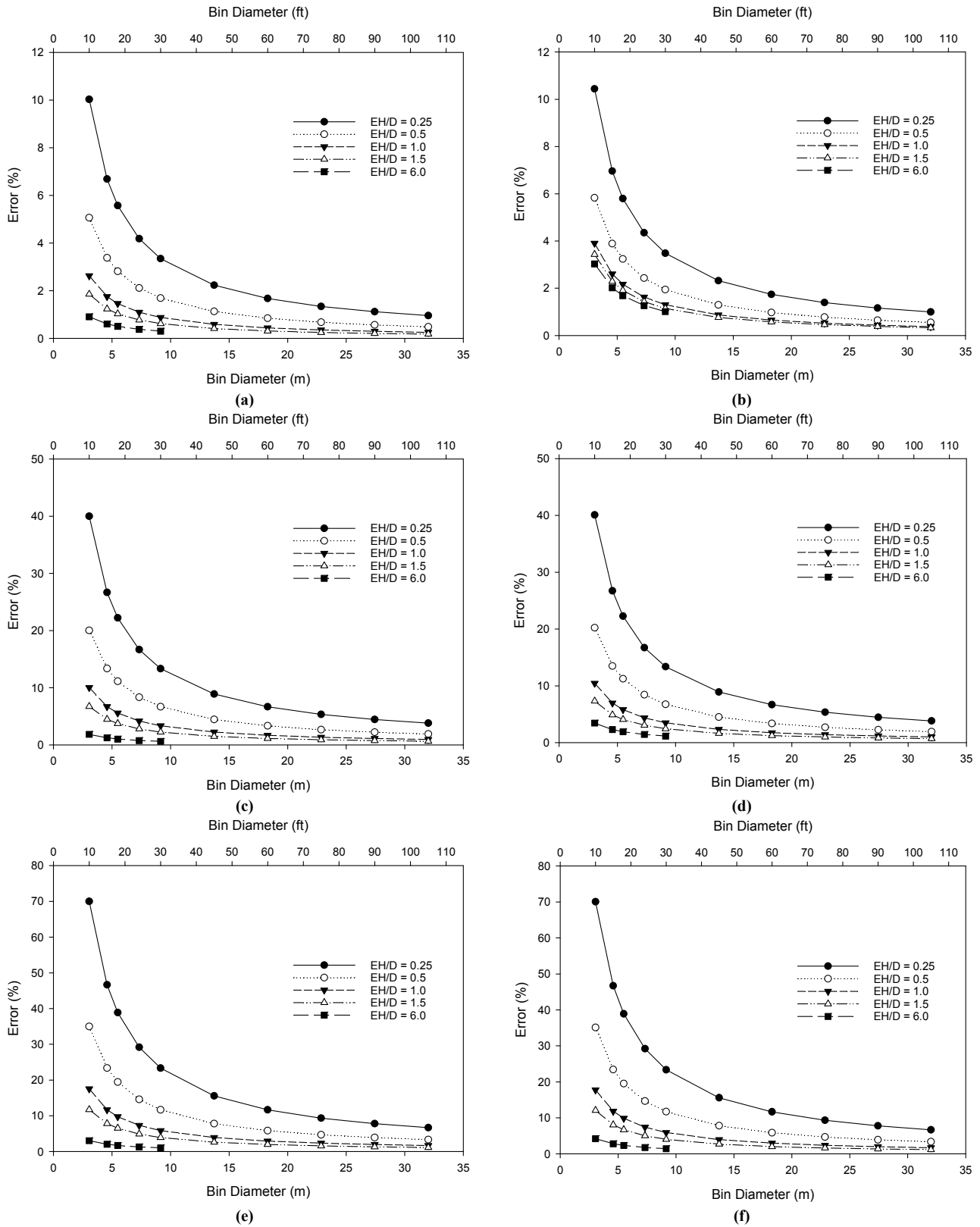


Figure 2. Total error in bin volume measurement as a function of bin diameter over a range of equivalent level grain height-to-diameter ratios ( $EH/D$ ) with standard deviations of (a) 1.2 cm in diameter and 7.6 cm in height (0.04 and 0.25 ft), (b) 4.6 cm in diameter and 7.6 cm in height (0.15 and 0.25 ft), (c) 1.2 cm in diameter and 30 cm in height (0.04 and 1.0 ft), (d) 4.6 cm in diameter and 30 cm in height (0.15 and 1.0 ft), (e) 1.2 cm in diameter and 53.3 cm in height (0.04 and 1.75 ft), and (f) 4.6 cm in diameter and 53.3 cm in height (0.15 and 1.75 ft).

(fig. 2a), small-diameter bins with  $EH/D \leq 0.5$  had an overall error of less than 5%, which decreased to less than 2% for

all  $EH/D$  ratios in large-diameter bins ( $\geq 15$  m). For scenario e, the best-case diameter and worst-case height (fig. 2e), the

error increased noticeably with small bin diameters and shallow grain depths but still decreased to below 10% at large bin diameters.

The trends of the error analysis matched the trends of the tests conducted in the five farm bins. In general, the errors increased as the total measured volume of grain in the bin decreased, and bins with the smallest *EH/D* ratios had the highest coefficients of variation (table 3). This trend was expected, as the surface represented an increasing proportion of the total volume. As bin diameter increases and the surface condition becomes less uniform, very large errors in equivalent grain height could be possible. However, the uncertainty in the bin volume decreased to below 10% in bins with diameters greater than 5.5 m (18 ft) filled to an *EH/D* ratio greater than 1.0 in the worst-case scenario.

Commercial facilities often mitigate the errors associated with measuring inventory by consolidating grain before an inspection. This reduces the number of bins that must be measured and results in grain surface conditions that are easily accounted for (surcharge cone). In a farm setting, this practice of consolidating grain is uncommon. Moreover, inventory in a farm setting is often complicated by the need to determine bin characteristics, such as effective depth, with grain in the bin. With these points in mind, larger errors can be expected in a farm setting.

#### POTENTIAL PACK FACTOR ESTIMATION ERRORS

##### *Sensitivity Analysis of FSA and RMA Methods*

After the volume is measured, pack factors are determined based primarily on bin diameter, grain type, and test weight for the FSA and RMA methods. Simulated errors in the test weight and pack factor were investigated for the FSA method (table 4) and the RMA method (table 5) with four bin diameters and an assumed grain height equal to the bin diameter. The pack factor for bins with diameters of 4.6, 9.1, 18.3, and 27.4 m (15, 30, 60, 90 ft) were 1.055, 1.090, 1.100, and 1.100, respectively, using the FSA method. An error of 6 kg m<sup>-3</sup> (0.5 lb bu<sup>-1</sup>) in the test weight changed the pack factors to 1.062, 1.098, 1.108, and 1.108 for bin diameters of 4.6, 9.1, 18.3, and 27.4 m, respectively. This resulted in a change of 0.7% in the estimated inventory. An error of 20%

in the packing percentage resulted in an inventory change between 1.0% and 1.8% based on bin diameter. The small change in the inventory due to the 20% error in the pack factor was partially due to the definition of the pack factor. A pack factor of 1.10 implies that the bin has 10% packing, or that the quantity of wheat was increased by 10% in the bin. A 20% error in packing changes the packing in the bin from 10% to 12% but changes the pack factor only from 1.100 to 1.120.

Similar trends were observed due to the simulated error in the test weight and pack factor with the RMA method (table 5). An error of 6 kg m<sup>-3</sup> (0.5 lb bu<sup>-1</sup>) in the test weight resulted in a change of 0.7% to 0.8% in the inventory, similar to the FSA method. An error of 20% in the packing resulted in a change of 0.7% and 1.2% for the 4.6 and 9.1 m diameter bins, which was less than the FSA method. For larger bins (18.3 and 27.4 m diameter), the change in inventory due to an error in packing was 2.5% and was slightly greater than the FSA method.

Both the FSA and RMA methods use tabulated data, which partially explains the behavior of the errors as a function of bin diameter. For all cases, the grain height was assumed equal to the bin diameter. Larger variations in the total quantity of grain were observed using the RMA method. A simulated error of 6 kg m<sup>-3</sup> (0.5 lb bu<sup>-1</sup>) with a test weight of 772 kg m<sup>-3</sup> (60 lb bu<sup>-1</sup>) led to a 0.7% change in the total quantity of grain depending on the bin diameter. An error of 20% in the combined test weight and pack factor resulted in an error in the total quantity of grain between 0.7% and 2.5%. Based only on errors in the test weight and pack factor, Hurburgh (2002) found an error of 1.7%, which compares well to this study.

For practical applications, the FSA and RMA estimates provided similar sensitivities to an assumed error in the test weight and packing percentage. The magnitude of the change in the total predicted quantity of grain due to test weight was slightly less than the 0.8% (6 kg m<sup>-3</sup> out of 772 kg m<sup>-3</sup>) error associated with the test weight measurement. A 20% change in the packing percentage resulted in changes of 0.7% to 2.5% in the total inventory for both the FSA and RMA methods. These values were determined for

**Table 4. Pack factor using the FSA method for varying bin diameters for wheat with a test weight of 772 kg m<sup>-3</sup> (60 lb bu<sup>-1</sup>), pack factor due to an error of 6 kg m<sup>-3</sup> (0.5 lb bu<sup>-1</sup>) in the test weight, pack factor including a 20% error, and the absolute percent sensitivity in the inventory.**

Bin Diameter, m (ft)	Pack Factor	Error Due to Test Weight		Error Due to Pack Factor	
		Pack Factor with Test Weight Error	Inventory Sensitivity (%)	Pack Factor with 20% Error	Inventory Sensitivity (%)
4.6 (15)	1.055	1.062	0.7	1.066	1.0
9.1 (30)	1.090	1.098	0.7	1.108	1.7
18.3 (60)	1.100	1.108	0.7	1.120	1.8
27.4 (90)	1.100	1.108	0.7	1.120	1.8

**Table 5. Pack factor using the RMA method for varying bin diameters for wheat with a test weight of 772 kg m<sup>-3</sup> (60 lb bu<sup>-1</sup>), the pack factor due to an error of 6 kg m<sup>-3</sup> (0.5 lb bu<sup>-1</sup>) in the test weight, pack factor including a 20% error, and the absolute percent sensitivity in the inventory.**

Bin Diameter, m (ft)	Pack Factor	Error Due to Test Weight		Error Due to Pack Factor	
		Pack Factor with Test Weight Error	Inventory Sensitivity (%)	Pack Factor with 20% Error	Inventory Sensitivity (%)
4.6 (15)	1.035	1.042	0.7	1.042	0.7
9.1 (30)	1.065	1.072	0.7	1.078	1.2
18.3 (60)	1.141	1.150	0.8	1.169	2.5
27.4 (90)	1.141	1.150	0.8	1.169	2.5

wheat, but similar trends could be expected for other crops.

In practice, moisture content is not considered by RMA or FSA auditors, despite the fact that the FSA manual (USDA, n.d.) defines the base moisture (11%) for crops such as wheat. For wheat, 1% should technically be added to the pack percent for each percent increase in moisture above 11% (USDA, n.d.). Based on the 20% change in the packing evaluated in table 4, changes in packing due to moisture with the FSA method would be negligible.

#### ***Sensitivity of Input Parameters to Janssen’s Equation Required for WPACKING***

WPACKING uses the differential form of Janssen’s equation to determine the vertical pressure and the bulk density in bins. Variations in the friction factor and pressure ratio were examined to determine their impact on the mass of grain estimated by Janssen’s equation. The quantity of grain stored in corrugated steel bins ranging from 9.1 to 27.4 m in diameter with height-to-diameter ratios of 0.33 to 3 was simulated. The sensitivity of predicted inventory to changes in  $\mu$  and  $k$  were small; thus, only the worst-case scenario is discussed, in which  $\mu$  and  $k$  were both varied by one standard deviation and created a maximum difference. The maximum change in the estimated inventory was 0.2% for the smallest diameter bin at a grain height of 27.4 m and decreased to 0.0% for a large-diameter bin at a grain height of 9.1 m.

In Janssen’s equation, the friction factor ( $\mu$ ) and pressure ratio ( $k$ ) are used in bin design to produce conservative bin pressures and are not necessarily representative of actual values that exist within the grain mass. There are a large number of sources for  $\mu$  and  $k$  (ASABE, 2010, 2011; Eurocode, 2006), but the specific choice of the parameters will likely not meaningfully change the estimated inventory based on the analysis in this study.

In WPACKING, the compressibility equation, which is used to predict grain pressures, is a function of the moisture content of the grain. The predicted packing with a 0.25 percentage point error in moisture measurement was evaluated. The maximum error introduced by changes in moisture content were of little consequence, with the maximum change in the estimated inventory determined to be 0.1% for an  $EH/D$  of 1.0 and a grain height of 27.4 m.

The change in the predicted inventory based on a simulated 6 kg m<sup>-3</sup> error in test weight was 0.8% over the range of bin diameters and  $EH/D$  ratios examined. This is consistent with the results of the FSA and RMA methods and further illustrates that test weight errors translate directly to errors in predicted inventory (Laux et al., 2015). When solving Janssen’s equation, a large potential source of error is likely the assumption that the test weight, which is an uncompressed bulk density, is the bulk density of the grain at the top surface of the bin. Chang et al. (1983) studied the effect of grain spreaders on the bulk density of wheat, corn, and grain sorghum in a 6.4 m diameter bin and showed that the bulk density of grain was 9% to 12% higher when a grain spreader was used compared to no spreader. Similarly, Stephens and Foster (1976) demonstrated increased bulk density (766 to 871 kg m<sup>-3</sup>) in corn loaded into a bin using a spreader (a 13.7% increase).

The sensitivity of Janssen’s equation to the input variables was relatively minor. A 10% error in the compressibility equation translated to an error of approximately 0.1% in the total inventory. The coefficient of friction, lateral-to-vertical pressure ratio, moisture content, test weight, and compressibility equation each resulted in absolute overall errors of less than 1%. Based on WPACKING and the sensitivities discussed, the maximum absolute error varied from 1.0% to 1.2% over the range of bins and  $EH/D$  ratios examined.

#### ***Overall Impact of Volume and Pack Factor Errors***

A sensitivity analysis was performed to evaluate the overall uncertainty in the estimated bin inventory assuming errors in both the volume measurement and pack factor. Based on figure 2, errors of 0%, 2%, 5%, and 10% were assumed in the volume measurement. Errors of 0%, 5%, 10%, and 20% were assumed in the pack factor. Pack factors were obtained from WPACKING for three bin diameters (9.1, 18.2, and 27.4 m) filled to three grain heights (9.1, 18.2, and 27.4 m). Recommended values of 0.6 for  $\mu$  and 0.5 for  $k$  for hard red winter wheat in corrugated steel bins (ASABE, 2010) and the compressibility equation from Turner et al. (2016) were used in Janssen’s equation. Table 6 summarizes the potential range of uncertainty in the bin inventory measurement for each of the nine combinations of bin diameter and grain height. With an error of 2% in the volume and an error of 5% in the pack factor, the absolute uncertainty in the bin inventory was 2.2%. A similar error of 2.7% was found by Hurburgh (2002). The smallest uncertainty was found for a bin 9.1 m in diameter and a grain height of 9.1 m. The largest uncertainty was found for a bin 27.4 m diameter and a grain height of 27.4 m. However, only small variations were caused by bin size and grain height within the calculated uncertainties for a given set of errors.

Table 6 demonstrates how errors in the volume and pack factor contribute to the uncertainty in the bin capacity estimation. Errors in volume had a much greater impact than errors in pack factor. For example, a bin with a diameter of 27.4 m (90 ft) and a grain height of 27.4 m (90 ft) would have an uncompacted volume of approximately 16,200 m<sup>3</sup> (460,000 bu). A 10% error in the volume measurement would be 1620 m<sup>3</sup> (46,000 bu). However, if the bin had a pack factor of 1.05, meaning there was 5% packing in the bin, and the error in packing was 10%, then the net error introduced from the pack factor would be only 1% of the calculated capacity.

**Table 6. Percent uncertainty in estimated bin capacity based on the WPACKING model with standard inputs with errors of 0%, 2%, 5%, and 10% in the bin volume and errors of 0%, 5%, 10%, and 20% in the pack factor for bins ranging from 9.1 to 27.4 m in diameter and from 9.1 to 27.4 m in height.<sup>[a]</sup>**

Error in Bin Volume	Error in Pack Factor			
	0%	5%	10%	20%
0%	0	0.2	0.4 to 0.7	0.8 to 1.2
2%	2	2.2	2.5 to 2.7	2.9 to 3.2
5%	5	5.2	5.6 to 5.7	5.9 to 6.2
10%	10	10.2	10.6 to 10.8	10.9 to 11.3

<sup>[a]</sup> Standard inputs are corrugated steel bin with  $\mu = 0.6$ ,  $k = 0.5$ , moisture content = 10% w.b., test weight = 772 kg m<sup>-3</sup>, and hard red winter wheat compressibility equation from Turner et al. (2016).

## SUMMARY AND CONCLUSIONS

Error propagation analysis was performed to evaluate the effects that errors in measuring the volume, test weight and pack factors of grain had on inventory estimates. These are complex issues that are not easily evaluated using error propagation; thus, simulation was used to aid in the examination of their impact on the total inventory estimate. Based on engineering estimates of the errors in the bin diameter and height measurement, the overall error in volume was simulated for a series of bin configurations. Accurate height measurements were found to be critical to predicting grain volume. Errors decreased with increases in both the bin diameter and  $EH/D$  ratio. Large errors in equivalent level grain height estimation had a greater effect than an error in bin diameter on the magnitude of the error in volume. In shallow bins ( $EH/D = 0.25$ ), the surface profile had a larger relative impact on the grain volume, and any errors in describing the profile can impact grain volume calculations. However, as the bin diameter increased, the error decreased exponentially to levels between 2% and 10% for all scenarios.

To evaluate existing protocols, four FSA and crop insurance auditors measured typical farm bins with unleveled grain surfaces. The standard deviation of the volume measurements for the bins varied between 0.3 and 17.3 m<sup>3</sup>. The highest errors were observed in bins with the smallest volume, which confirmed the error propagation analysis. When the bins were manually leveled, the standard deviation of the volume measurement remained similar, between 0.9 and 17.5 m<sup>3</sup>. Leveling the bins resulted in significantly different volume estimates in three of the four bins that were leveled, as well as for the total volume estimate. This indicated that, in addition to accurate height measurements, care must be taken when accounting for surface conditions, especially with non-uniform surfaces and small  $EH/D$  ratios.

The uncertainty associated with the WPACKING method using Janssen's equation to convert the measured volume to a predicted quantity of stored grain was also examined. For each of the major inputs to WPACKING (coefficient of friction, lateral-to-vertical pressure ratio, moisture content, test weight, and the compressibility equation), the maximum uncertainty caused by each variable was less than 1% of the total predicted grain inventory. Uncertainties of similar magnitude were observed with the FSA and RMA methods, where a 0.8% change in test weight resulted in an error of less than 1% in the predicted grain inventory regardless of bin size. A 10% error in the packing percentage resulted in an overall error of 0.7% to 2.5% in the predicted inventory depending on the bin diameter.

Bin inventory is the product of the volume measurement and the pack factor estimation. The primary source of error in bin inventory estimation was the volume measurement. Over a range of typical bin diameters and heights, a 5% error in the volume and pack factor resulted in an overall error of approximately 5.2% in the grain inventory. Over 95% of this error (5 percentage points) was caused by errors in the volume measurement, while the remaining 5% (0.2 percentage points) resulted from errors in the pack factor.

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