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Bi-Objective Optimization of Kidney Exchanges

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Bi-Objective Optimization of Kidney Exchanges

THESIS

A thesis submitted in partial
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the degree of Master of Science in
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By
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Lexington, Kentucky

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ABSTRACT OF THESIS

Bi-Objective Optimization of Kidney Exchanges

Matching people to their preferences is an algorithmic topic with real world applications. One such application is the kidney exchange. The best "cure" for patients whose kidneys are failing is to replace it with a healthy one. Unfortunately, biological factors (e.g., blood type) constrain the number of possible replacements. Kidney exchanges seek to alleviate some of this pressure by allowing donors to give their kidney to a patient besides the one they most care about and in turn the donor for that patient gives her kidney to the patient that this first donor most cares about. Roth et al. first discussed the classic kidney exchange problem. Freedman et al. expanded upon this work by optimizing an additional objective in addition to maximal matching. In this work, I implement the traditional kidney exchange algorithm as well as expand upon more recent work by considering multi-objective optimization of the exchange. In addition I compare the use of 2-cycles to 3-cycles. I offer two hypotheses regarding the results of my implementation. I end with a summary and a discussion about potential future work.

KEYWORDS: algorithmic matching, multi-objective optimization, kidney exchanges, bi-objective optimization, top trading cycle and chain, algorithmic analysis

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Date: April 10, 2018

Bi-Objective Optimization of Kidney Exchanges

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With great love I dedicate this small thesis to my dad (the hardest working man I know), my mom (for her wisdom and guidance), my brother Lanny (for being the best brother), my sister Kristi (for being a wonderful addition), Charlie (for being the emperor), and my cousin XinXin (for teaching me about sumo and other things).

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TABLE OF CONTENTS

LIST OF FIGURES

Chapter 1 Introduction

Matching people according to their preferences is a well studied algorithmic problem. Classic problems in the field include stable marriage and housing preferences. In both of those cases, we are attempting to maximize the utility of each person. In the case of kidney exchanges, lives (not just utility) are literally on the line [22, 19].

From an algorithmic perspective, kidney exchanges present an interesting matching problem for at least three reasons. The first issue is the online nature of kidney exchanges. This simply means that when an organ becomes available is highly random and unknown. Kidneys outside the body can only last a maximum of 30 hours. In some instances, donors are flown to the hospital, which alleviates some of the problem.

The second issue is the simultaneity involved in the surgeries. In order to prevent reneging, all the people involved in the swap simultaneously undergo surgery. This means that assuming n pairs of people, there will be $2n$ operations.In both the academic literature and our work, cycles are usually capped at some small integer L, usually 2 or 3. One reason for capping the cycle is so that if a pair wanted the back out at the last minute fewer agents are affected [9, 2].

Finally, there is the issue of social welfare. For example it is generally considered not wise from a societal standpoint to give old kidneys to young people and young kidneys to old people [22]. Among other reasons, if the old kidney that is transplanted to a young person suddenly fails, the next kidney will be harder to graft in from a biological standpoint.

Alvin Roth first described the classic kidney exchange problem. He introduced the top trading cycle and chain mechanism [18]. Essentially, he added a “wait option” and allowed for the formation of chains in addition to the use of cycles to find maximal matchings. The use of cycles alone was established earlier by Gale and Shapley [10]. Dickerson, Conitzer et al. expanded on this work to optimize a second objective once the maximal matching has been determined [9]. In this work, I optimize over multiple objectives once the maximal matching has been determined. In both Freedman’s work and mine, we use only cycles to compute maximal matchings.

The rest of this thesis is structured as follows: Chapter 2 discusses the importance of kidneys and the classic kidney exchange problem. Both the direct exchange as well as the indirect kidney exchange will be discussed. This is followed by pseudo-code as well as a discussion of some results of the experiment. Chapter 3 focuses on bi-criteria optimization, what it is and how it relates to the classic kidney exchange problem. The two criteria under consideration are age and Euclidean distance. Chapter 4 discusses the implementation and results of the experiment. Chapter 5 summarizes and discusses potential avenues for future work.

Chapter 2 The Classical Kidney Exchange Problem

In this chapter, we note the biological importance of kidneys and then discuss the direct kidney exchange and the indirect kidney exchange. These concepts are combined algorithmically to form the "top trading cycle and chain." Implementation of TTCC and its results are briefly discussed.

I Biological Importance of Kidneys and Other Facts

Kidneys are vital to the human body as they are responsible for filtering out the toxins and waste of the body. Out of the over 121,000 people in the United States waiting for an organ, over 100,000 of them are waiting for a kidney [7]. Every person starts life with two kidneys but only needs one to live.

Due to various biological complications (such as blood types), people are not always eligible to give a kidney to the person to whom they most want to give [5, 16, 8]. Recent advances in medicine however allow this potential donor to give to another waiting patient while that patient's potential donor donates her kidney to this original donor's patient. Such a swap is called a kidney exchange. Kidney exchanges come in two basic types known as direct exchange and indirect exchange.

II Direct Kidney Exchange Algorithm

The direct kidney exchange is similar to the housing allocation problem. In the housing allocation problem there is a set of people, each of whom owns a house. Each person has a set of strict preferences for every house [14, 10]. Consider the following example:

Example:

Person 1 [1,2,3]
Person 2 [3,1,2]
Person 3 [1,3,2]

In the above illustration, person 1 prefers house 1 (e.g., his own house) the most and then houses of person 2 and 3. Person 2 prefers the house of person 3 and then the house of person 1 and finally her own house. Person 3 prefers the house of person 1, then his own house, then the house of person 2.

We say that matchings are <u>Pareto optimal</u> if no person can get a better house without another person getting a worse house. We say that matchings are <u>stable</u> in this context to mean that even if a person had an incentive to exchange for a better house, no other person would be available to exchange with her [20].

To find the optimal stable matching of person to house, Shapley introduced the top trading cycle algorithm. The top trading cycle proceeds in rounds. In each round the (remaining) people point to their most preferred (remaining) house. A person can point to their own house (e.g., self cycles are allowed.) Claim: In each round of pointing, there must exist a cycle.

Proof. Let there be n nodes. Each node must have an edge pointing to another node or to itself. In the case of a node pointing to itself the existence of a cycle is trivial as it is a self cycle. In the case of all nodes pointing to a different node, every node has an out-degree of one. As you traverse through the graph going through the finite set of nodes, eventually you must revisit an node and thus a cycle is formed. $\square$

Execute the trade of such a cycle or in the case of a self cycle, let that person remain in her current house. Remove the nodes in that cycle from consideration. Repeat this until every person is matched to a house. When this algorithm terminates there will be a Pareto optimal as well as stable set of matchings [12]. This algorithm has the property that the dominant strategy is being truthful in your preference ordering [12].

III Indirect Kidney Exchange Algorithm

In addition to direct kidney exchanges we can also envision a wait list available to people who currently do not have an available donor. Donors that have a patient that they care about but cannot donate to, can give their kidneys to any patient on the waiting list and in exchange their preferred patient will be placed in a more favorable spot on the the waiting list. This concept of indirect exchange was introduced by Roth in the context of student housing. The mechanism is called, “I get your house, you get my turn” [18, 1]. The algorithm is described as follows.

Let there be four entities: new students, existing students, occupied houses, vacant houses. Let there be a strict ordering of housing preferences for each student. There is now a line of students each of whom has a set of housing preferences. Assign each new student his top choice and the second new student her top choice among remaining houses. As this line of student preferences proceeds there may be a new student who requests the house of an existing student. Before allocating the new student the house that currently has an existing student, move that current student to the top of the student preference line. Ask her what her current top housing preference is. One of three things can happen. She can prefer her current house, in which case, the new student must choose his next preferred house. She can prefer a vacant house, which will be assigned to her and her current house is then assigned to the new student. She can prefer the house of another existing student, in which case that student is now brought to the front of the student preferences line with the same three options available to him. As these assignments proceed, if a cycle develops, it will be between existing tenants. Each existing tenant will request the room of the tenant next in this cycle. Remove that cycle by executing the trade. This process of matching houses to students incentives existing students who already have a house to join the market because they are guaranteed to be at least as well off as not joining the student housing market and can possibly improve [18].

Consider the following illustration. Let there be three new students, 1 and 2 and 3. Let there be three existing students (4,5,6) that already live in houses 4, 5 and 6 respectively. Let there be three vacant houses labeled 1, 2 and 3. Let the following be the housing preferences of each student.

Example:

New Student 1 [4,1,2,6,5,3]
New Student 2 [2,3,5,6,1,4]
New Student 3 [1,2,3,4,5,6]
Current Student 4 [4,5,3,6,2,1]
Current Student 5 [6,5,1,3,4,2]
Current Student 6 [5,3,2,4,1,6]

According to our algorithm, new student 1, who is currently at the top of the student preferences line wants house 4. House 4, however is occupied by current student 4. The algorithm now moves her to the head of the line and considers her preferences. She likes her current house, and thus student 4 remains with house 4 and student 1 now points to his next most preferred house. Student 1 prefers house 1 and since that is a vacant house, gets it. Student 2 points to a vacant house as well and gets paired with it. Student 3 prefers house 1 the most, but that house was already assigned to student 1 and similarly for house 2, therefore student 3 is stuck with her third choice which is vacant house 3. Student 4 has already been taken care of by virtue of jumping the line thanks to student 1. Current student 5 prefers house 6 which is currently occupied. Current student 6 prefers house 5 and thus they engage in a mutually beneficial swap. The final student:house pairings are [1:1, 2:2, 3:3, 4:4, 5:6, 6:5].

IV Top Trading Cycle and Chains: A Combination of the Direct and Indirect Exchanges

We are now ready to combine both direct and indirect exchanges to solve the classical kidney exchange problem.

First note that based on the indirect exchange algorithm above, a natural analogy appears. The cadaveric kidneys are vacant houses, the people with no donors are the new students, the people with a loved one as a donor are the current students, and the people who are available to exchange kidneys are the occupied houses.

In this setup each person has a list of preferences including a new option called *wait*. Note that because there is now a wait option, there need not be a strict ordering of preferences over all available options [18].

Like the top trading cycle each (remaining) person points to their favorite (remaining) kidney in each round.

At this point one of two things can happen. A cycle can form, in which case a trade should be executed among the members of the cycle. Both the patients and kidneys should then be removed from future consideration. The other possibility is that a wait chain will be created. The patients in this wait chain[1] are pointing to their most preferred kidney. The wait chain ends with a patient who points to the wait option. This patient is called the head of the chain. All of the patients and their kidneys on the wait chain will be removed from future consideration as well. As soon as a kidney exogenously appears (e.g., an altruistic or cadaver donor) and satisfies the person at the head of the chain, then the rest of the people in that w-chain will get their kidney [18].

Consider the following illustration. Let there be four donor-patients pairs such that pair 1 is donor 1 and patient 1 and so on. There are no self cycles so only exchange cycles and wait chains can occur. Let the following be an example of the preferred kidneys of each donor by each patient.

Example:

Patient 1 [4,2,w]
Patient 2 [3,1,4]
Patient 3 [2,w]
Patient 4 [3,1,2]

In the first round patient 1 prefers the kidney of donor 4; patient 2 prefers the kidney of donor 3; patient 3 prefers the kidney of donor 2; and patient 4 prefers the kidney of donor 3. The cycle that is formed is between pairs 2 and 3.

In the second round patient 1 prefers to wait while patient 4 prefers the kidney of donor 1. Here we have a wait chain of $4 \rightarrow 1 \rightarrow w$. As soon as patient 1 gets a kidney exogenously (e.g., from a cadaver), person 4 will get a kidney from donor 1.

V Top Trading Cycle and Chain Pseudo Code

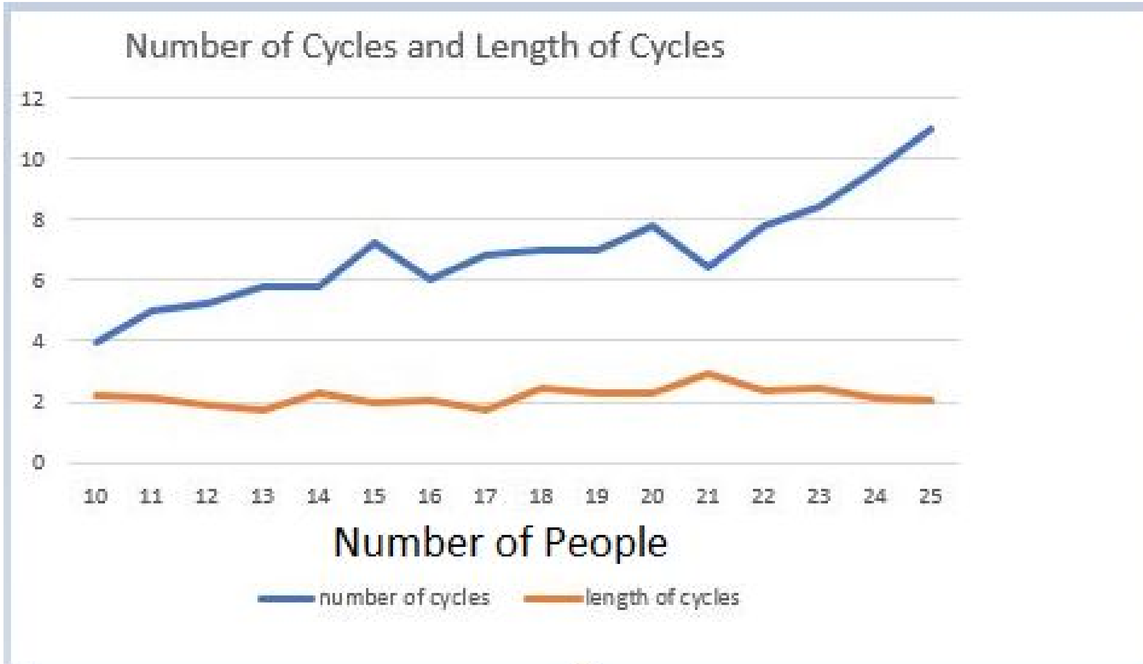
Algorithm 1 Top Trading Cycle and Chain

Initialize a random preference list for each person in the set
while Not every patient has a kidney **do**
  Let each undeclared person point to favorite kidney or 'w'
  Search through this graph to find the cycle or the wait chain
  Execute the trade in this cycle, or remove all people in the wait list chain
  Remove all such people and kidneys from the graph
  Continue the next round
  End once everyone either has a kidney or is part of some wait chain
end while

VI Experimental Setup and Results

The classic kidney exchange algorithm was implemented with two distinct components. The data generation was implemented in Python using the permutation function to generate random preference lists. Once the preference arrays were generated,

[1] "Waitlist", "Wait Chain" and w-chain are all interchangeable terms for the same thing.



[h]

Figure 2.1: The number and length of cycles

they were run through the TTCC program to find the matchings of patient to kidney. The experiment started off with 10 people and was incremented by 1 all the way up to 25 people. For each size of people, 5 trials were run and averaged.

The experiments analyzed the number of cycles and number of chains as well as the average number of people per cycle and average number of people per chain. Also the average preference of a kidney that the patient received was analyzed.

CPU time was run separately as it required a large number of people to see any effect. Running the experiment with just 25 people or fewer would often lead to 0 CPU seconds used. Therefore I also ran the experiment with 50, 100, 1000, 2000, 3000, 4000, and 5000 patients twice for each number of patients, and averaged the CPU time.

VII Discussion of the Top Trading Cycle and Chains Graphs

As the number of people increased, the number of cycles increased but the average length of each cycle stayed relatively steady. The opposite phenomenon occurred with chains. The number of chains averaged close to 1. There were a few times in the experiments when there was no chain and everyone was part of the kidney exchange cycle thus depressing the average a little bit.

The length of the chain had an upward trend as the number of people increased. The increase in the number of people also increased the number of rounds needed to terminate the algorithm. In turn, the increase in the number of rounds needed, seemed to decrease the average preference of the kidney received by the patient (i.e., the utility went down). As the number of people increased, the CPU time increased

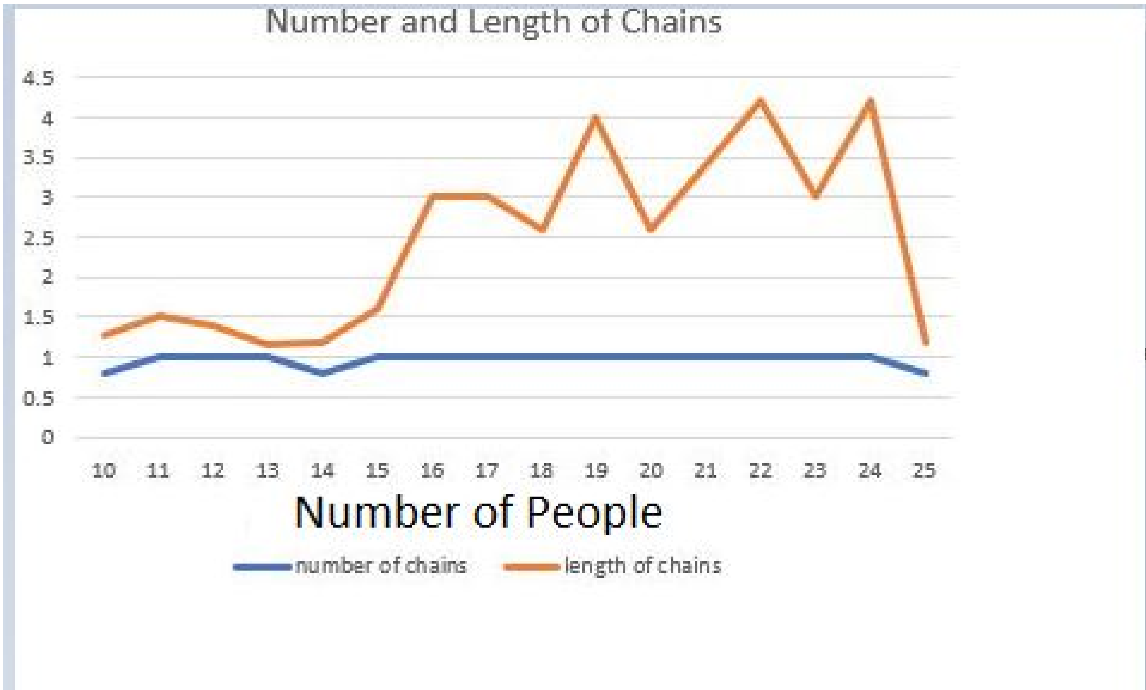


Figure 2.2: The number and length of chains

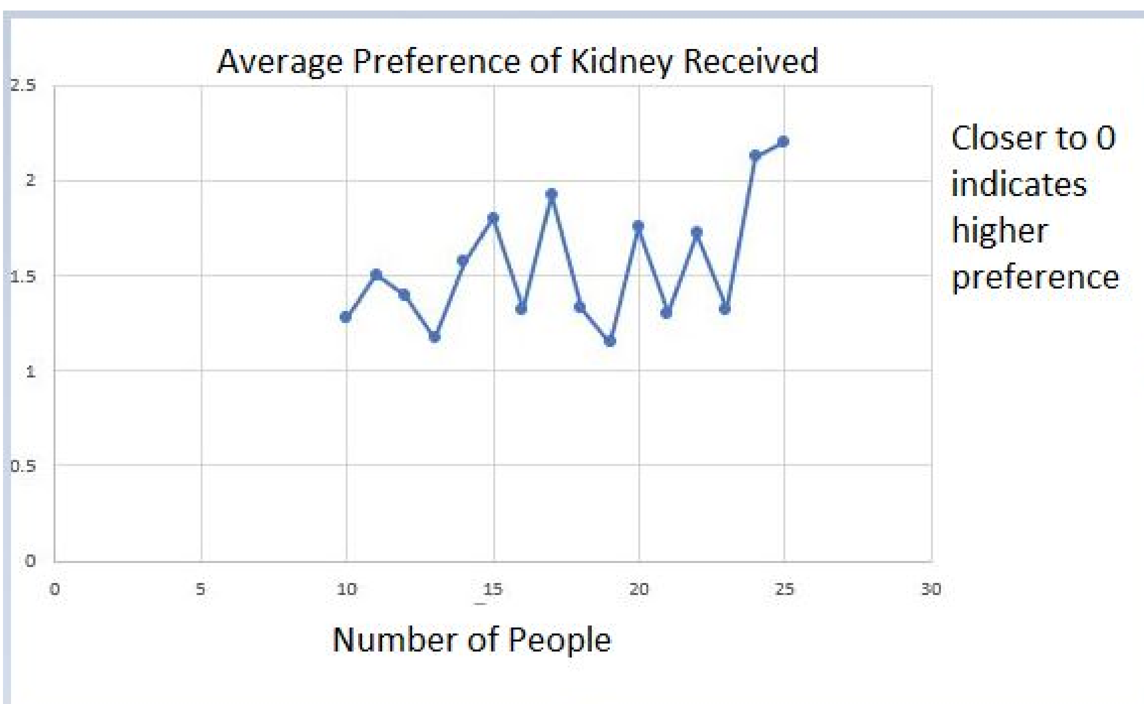


Figure 2.3: The average preference of the kidney received

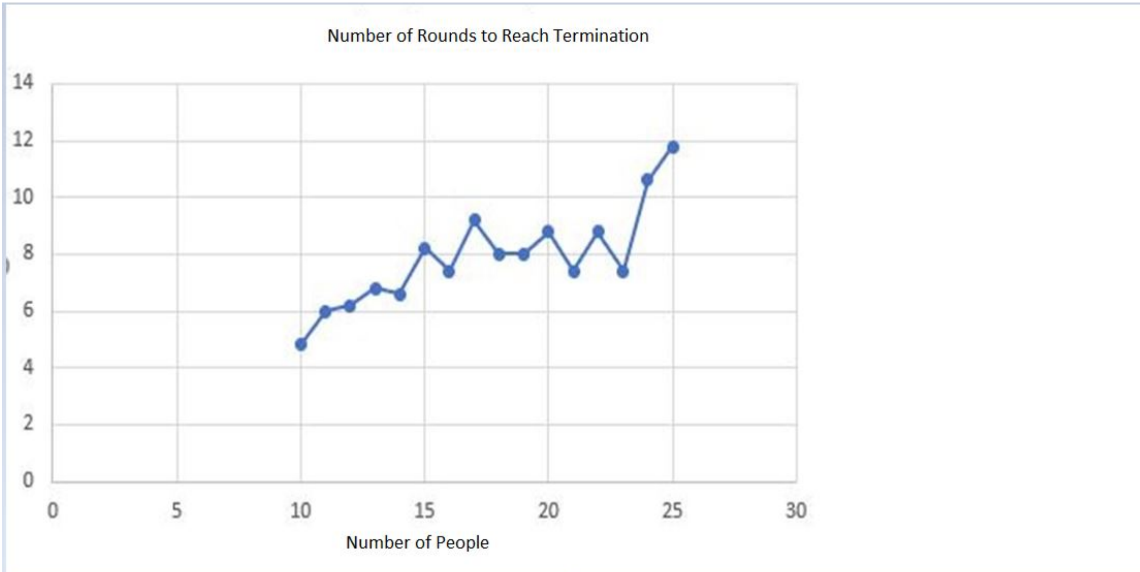


Figure 2.4: The average number of rounds needed before the algorithm terminated

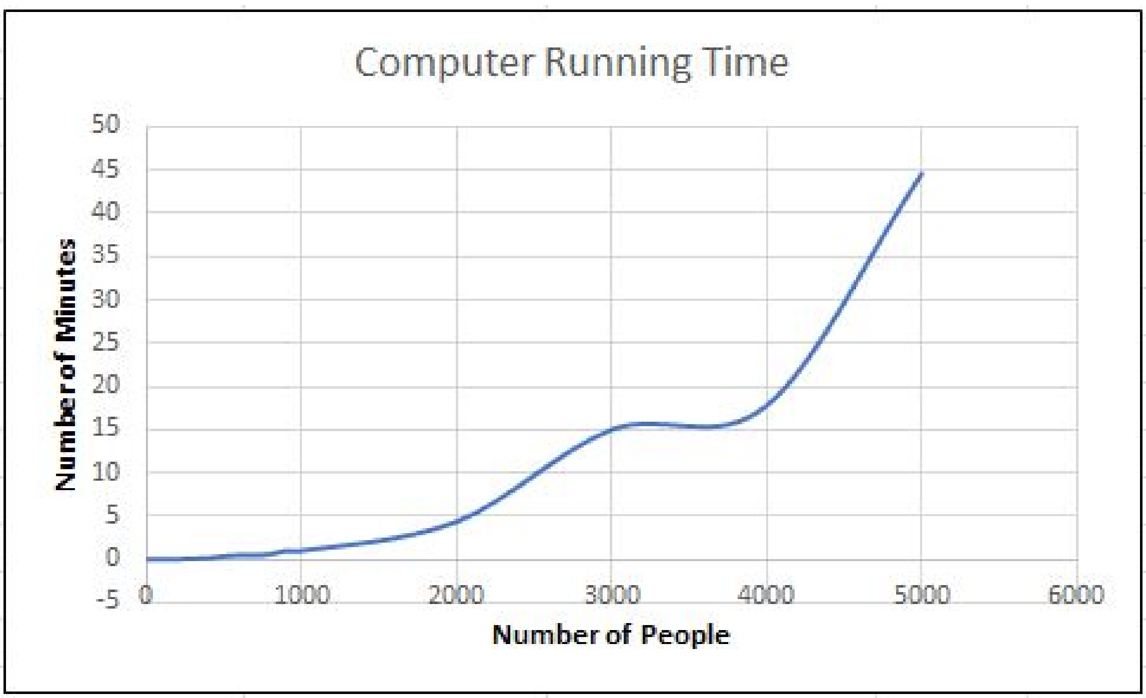


Figure 2.5: The CPU Runtime

in a quadratic manner. This is to be expected because the top trading cycle and chain algorithm is an $O(n^2)$ time algorithm [12].

Chapter 3 Bi-Objective Optimization of Kidney Exchanges

I Bi-objective Optimization

In the real world, we often care about more than just the optimization of one objective. For example a stock broker wants to maximize returns while minimizing risk and the driver wants to maximize scenic beauty while minimizing drive time [6]. Often these objectives are in conflict with one another and we are not sure what the relative weights of each objective are [21, 4, 13]. Indeed, if we were sure of the importance placed on each objective, we could transform the problem back into a single objective optimization problem [6].

In the multi-objective optimization case there is no single best solution [4]. Instead we are trying to find the "most preferred" outcome. "Most preferred" gives rise to the idea of non-dominated point. This is the point that cannot be improved upon in one objective without sacrificing in the other. A non-dominated point is called an efficient point and the Pareto set is composed of these efficient points. Note that there can be non-dominated points that are not in the Pareto set [4].

In the case of kidney exchanges we have the following objectives:

- Maximize the number of matchings in the kidney pair pool. This is the main objective.
- Minimize the total of pairwise age differences of the matchings. As an example, consider a 2-cycle in which pair one had a donor-age of 50 and a patient-age of 42. The second pair has a donor-age of 61 and a patient-age of 33. The total age difference would be $(50 - 33) + (61 - 42) = 36$.[1]
- Minimize the total geographic distance between the donating pairs. As an example, consider a 3-cycle in which the first donor-patient pair with a location of (20,40), the second donor-patient pair has a location of (43, 34), and the third has a location of (27, 72). The total distance would be the Euclidean distance of pairs 1 and 2 (23.7) plus Euclidean distance of pairs 2 and 3 (41.2) plus Euclidean distance of pairs 1 and 3 (32.7) for a total distance of 97.6.

In accordance with the literature [9] we make each edge used in a matching have a weight of one and the utility function is simply the summation of all the edges multiplied by a large number n. In our case the n was chosen to be 500. By multiplying this objective by such a large number, we are ensuring that this objective takes precedence over the two minimization objectives.

[1] Note that technically we are using the absolute value of the differences. To see why, consider a 2-cycle in which the first pair has a donor of age 40 and a patient of age 50. The other pair has has a donor of age 50 and a patient of age 40. To strictly use differences, we would have one difference of $(40 - 50) = -10$ and another difference of $50 - 40 = 10$. This would imply the summation of the exchange has a total age difference of 0, which is incorrect, it should be 20.

We then penalize the utility of the maximal match by subtracting it from the sum of the total age difference times $weight1$ (e.g., $\alpha \cdot \sum age\_differences$) as well as the sum of the total distance times $weight2$ (e.g., $\beta \cdot \sum distance$). The weights have an additional constraint: $(\alpha + \beta) = 1$. The two weights are used to show the importance we place on each of the two minimization objectives once the maximal matching has occurred.

- U= (500$\cdot \sum \max match$) - ($\alpha \cdot \sum age\_differences$) - ($\beta \cdot \sum distance$)
- $\alpha + \beta = 1$

II Experimental Setup

The experiment was implemented using Notepad++ and run via Windows PowerShell using Anaconda and Python 3.6.2. The main library used was GurobiPy and the starter code for finding the maximal matches came from the Gurobi kidney exchange example online [11].

First random kidney pairs were generated that contained blood type, location, and age. The blood type percentages were given by LiveScience [5], while the ages were uniformly randomized between 30 and 70. [17]. This corresponds to the 2017 US census breakdown of the age groups.[2] The ages were randomized by the following equation:

$$Age_{normalized} = \frac{Age_{actual} - 30}{40}.$$

Finally the y-axis coordinates were randomized between 24 and 49 while the x-axis coordinates were randomized between 66 and 124.[3] These numbers came from the most extreme points of the continental United States as noted by the World Almanac [3].

Once the kidney pairs were generated, we created three matrices. The first matrix is used to check whether a donor from one pair can donate kidneys to a patient from another pair and vice versa. This is done by creating directional edges in the graph.

The second matrix keeps track of the age differential between the donor of one pair and the patient of another. The third matrix keeps track of the distance between any two pairs. The distance was calculated by calling the numpy Euclidean distance function. The distance was then normalized by copying the distance matrix into another matrix called normalized distance. Using numpy, I got the maximum distance and the minimum distance for the instance. Then to normalize, I use the following equation:

$$Distance_{normalized} = \frac{Distance_{actual} - Distance_{min}}{Distance_{max} - Distance_{min}}.$$

The results display total utility as a function of the two weights.

[2] Age 30–39: 13 percent; 40–49: 13 percent; 50–59: 13 percent; 60–69: 11 percent

[3] The northernmost point is Northwest Angle, MN at 49° N. The southernmost point is Key West, FL at 24° N. The westernmost point is Bodelteh Islands, WA at 124° W. The easternmost point is West Quoddy Head, ME at 66° W.

III First Set of Experimental Results

The experiment consisted of 100 kidney pairs. Multiple instances of the experiment were run with the same pattern recurring. The x-axis represents the relative weighting of the two minimization objectives while the y-axis represents the total utility.

As the weighting increases in favor of distance and decreases in age, the total utility is decreasing fairly steadily and looks linear. The following is my hypotheses as to why this might be the case.

Hypothesis: The expected value of minimization of age is bigger than the expected value of minimization of distance.

Let us consider the 2-cycle case. The expected value of age is $\frac{1}{40} \cdot 0 + \frac{1}{40} \cdot 1 + ... \frac{1}{40} \cdot 40$ for an expected value of 20.5.

Finding the expected value of distance was tricker but recall that we transformed the continental United States into a rectangle with width of 58 and height of 25 and diagonal of approximately 63. The formula for finding the expected value of a rectangle [15] is:

$$\frac{1}{15}\left(\frac{w^3}{h^2} + \frac{h^3}{w^2} + d\left(3 - \frac{w^2}{h^2} - \frac{h^2}{w^2}\right) + \frac{5}{2}\left(\left(\frac{h^2}{w}\right)\left(log\frac{w+d}{h}\right) + \left(\frac{w^2}{h}\right)\left(log\frac{h+d}{w}\right)\right)\right).$$

After plugging in the numbers, we have an expected value for distance of 15.6. This number is smaller than the expected value of age which is 20.5.

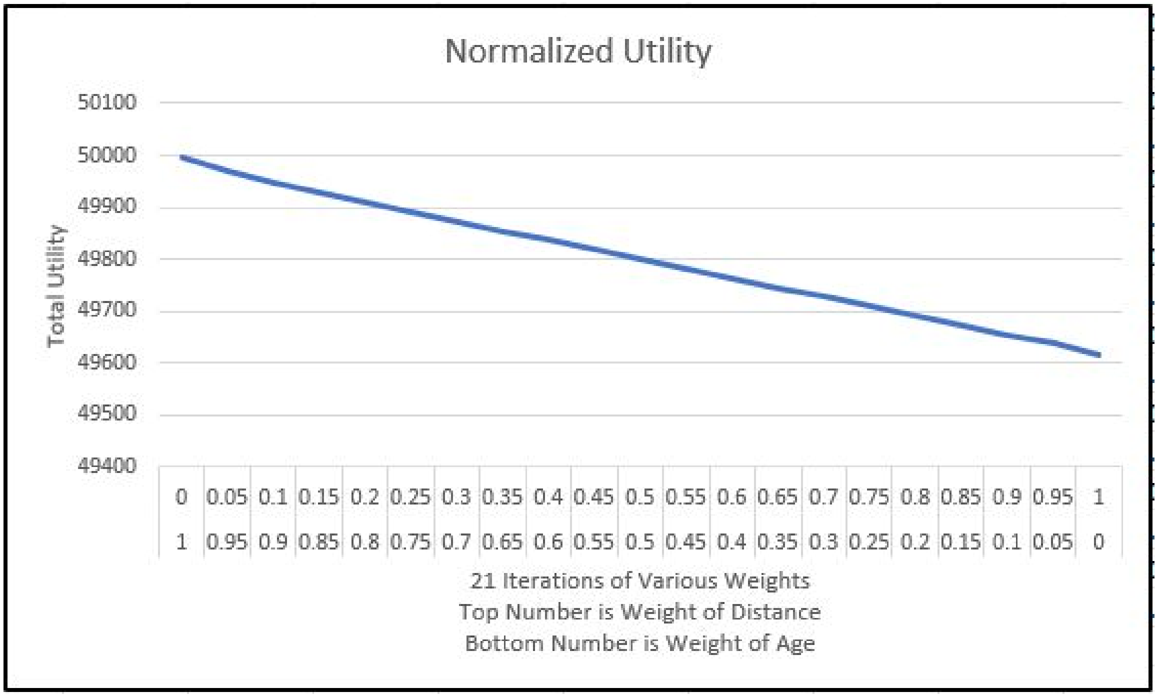
IV Second Set of Experimental Results

I also looked at the number and percentages of 2-cycles used versus 3-cycles. As I ran many more instances of Pareto front generation I found that the program favored usage of 2-cycles, without exception and often dramatically. I also found that after a few iterations, without fail, the number of total cycles used, 2-cycles used and 3-cycles did not change.

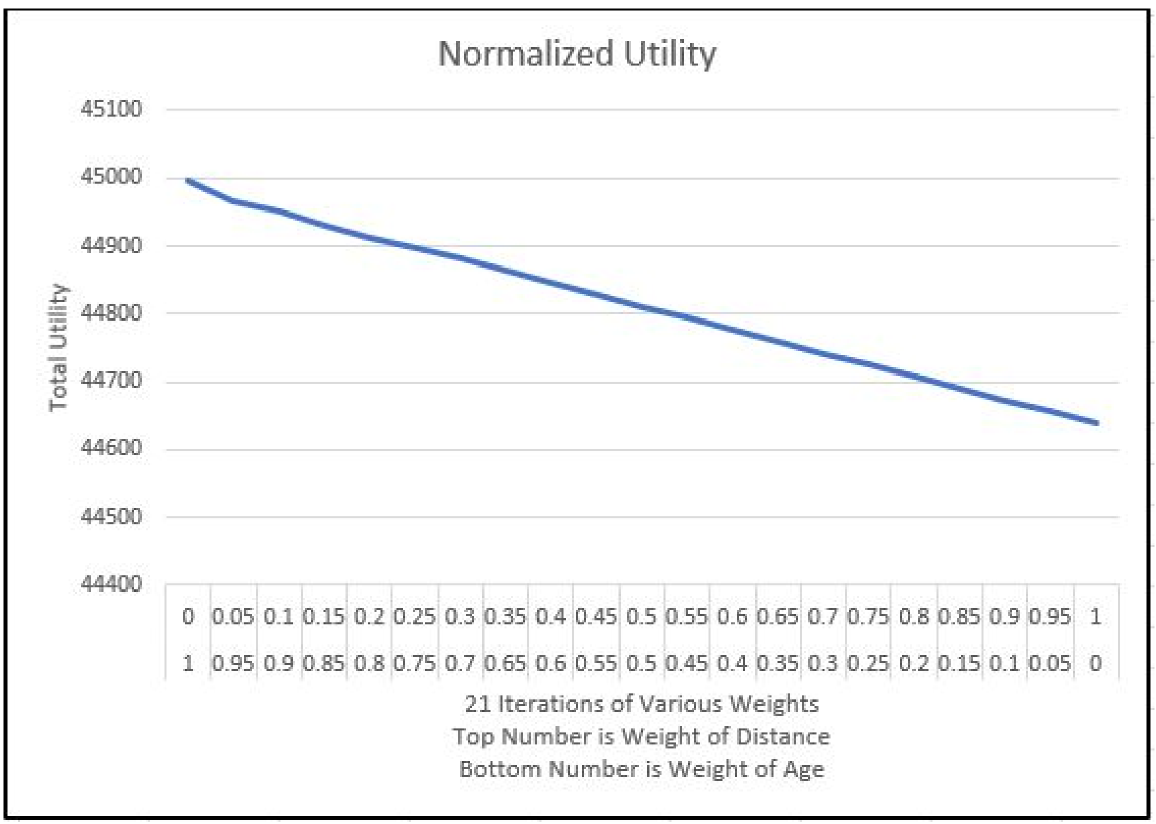
Dominant use of 2-cycles is further evidenced by the fact that as the iterations are being computed, the weights being placed on each objective are slowly shifting by small increments, yet the overwhelming usage of 2-cycles remains constant.

Hypothesis: The (sometimes overwhelming) preference for 2-cycles is a consequence of the utility function.

Recall that in our utility function, there is penalty for the distance and the age. In a 3-cycle, you must add an additional age and an additional distance versus a 2-cycle. This extra penalty is enough to make the preference for 2-cycles fairly strong.

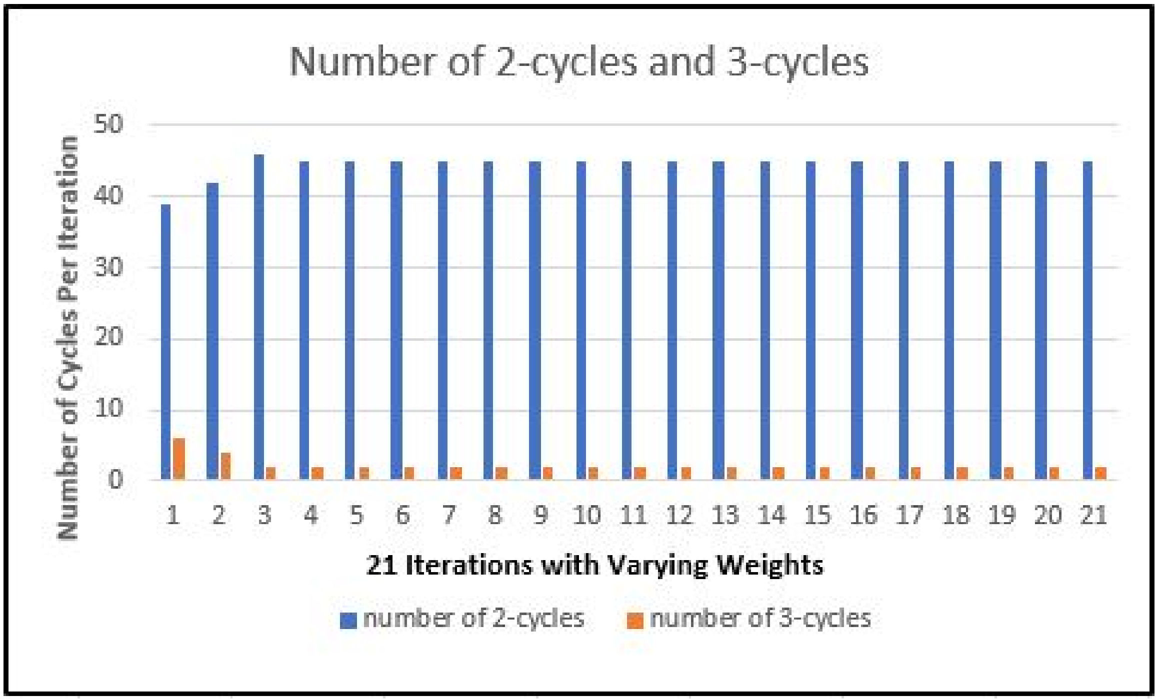


(a) Pareto Front 1

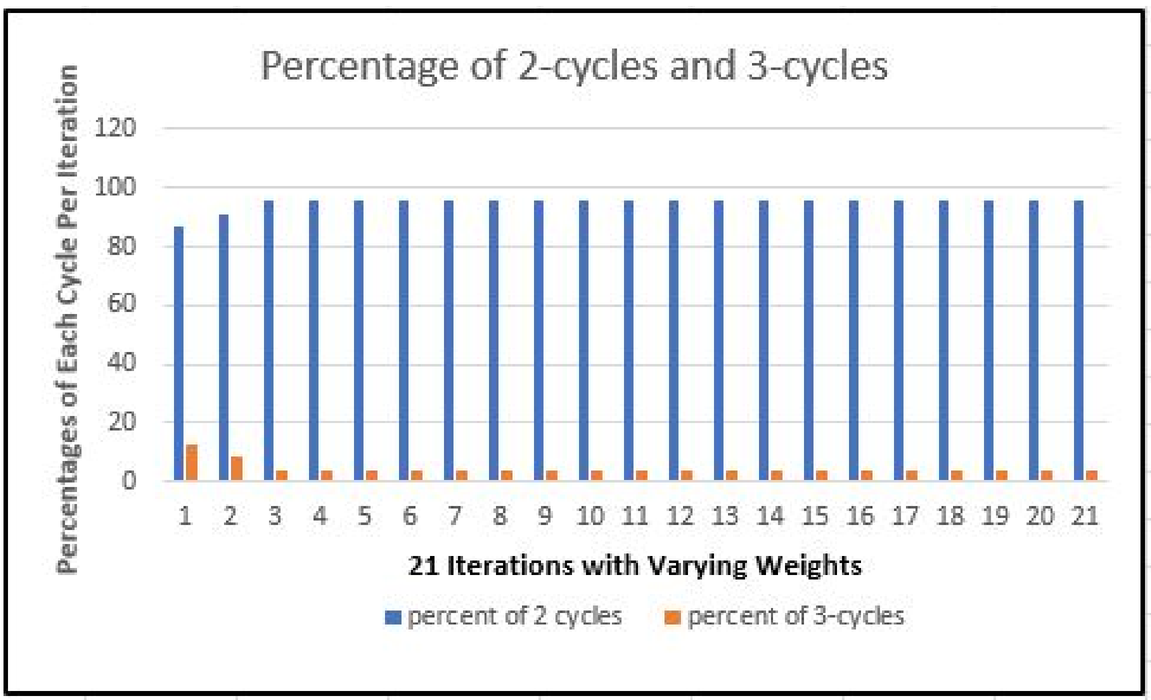


(b) Pareto Front 2

Figure 3.1: Two Instances of the Bi-Objective Kidney Exchange

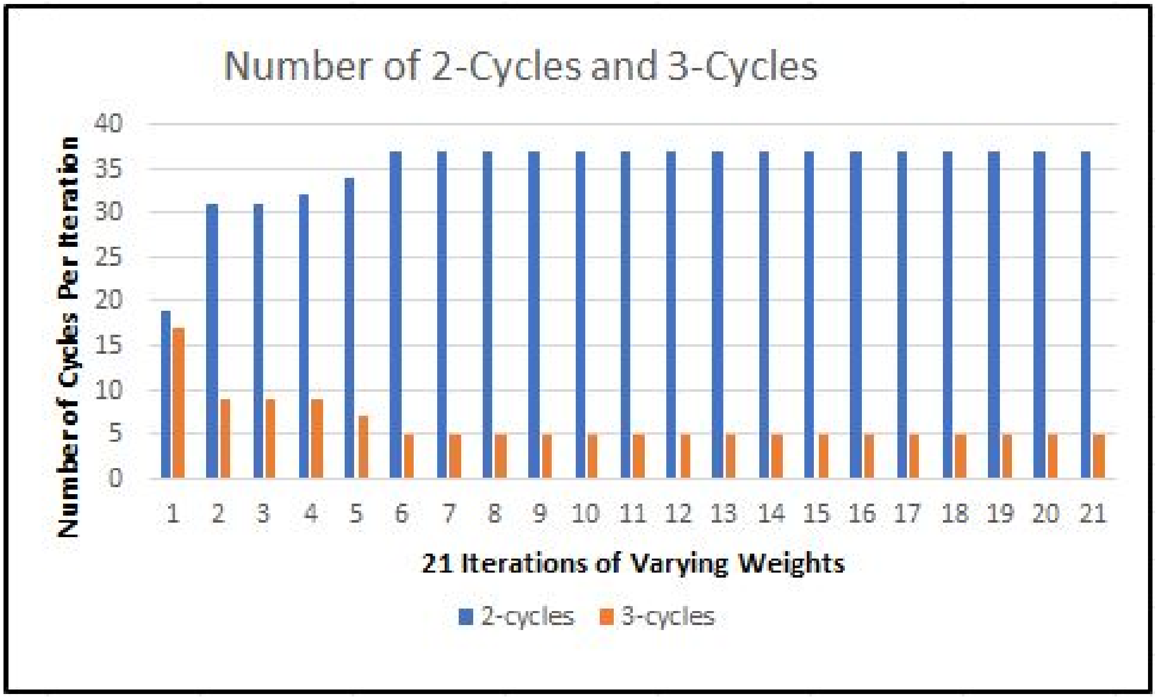


(a) Second Question 1

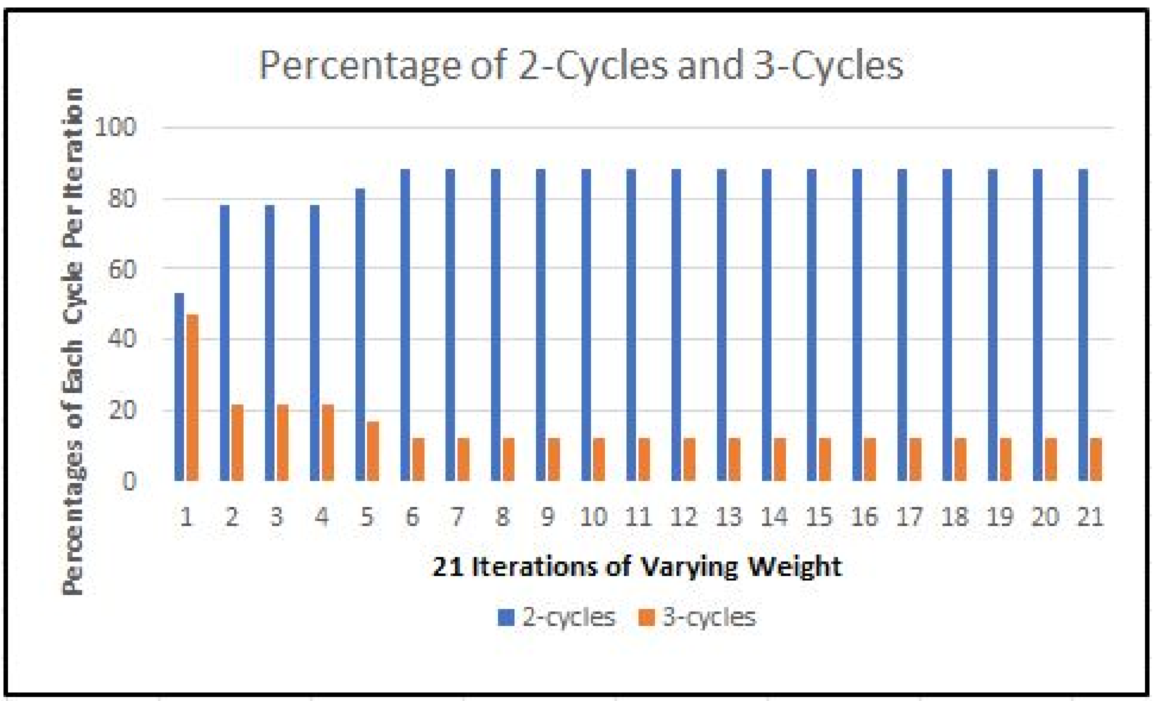


(b) Second Question 2

Figure 3.2: One set of 2-Cycle versus 3-Cycle Graphs



(a) Second Example 1



(b) Second Example 2

Figure 3.3: Second set of 2-Cycle versus 3-Cycle Graphs

Chapter 4 Summary and Future Work

Matching people to preferences has real world applications, none more so than kidney exchanges. I have reviewed classical kidney exchange problem with results from my implementations. I then looked at the kidney exchange problem as a bi-objective optimization problem. The two objectives were minimization of age and distance, once the maximal matching occurred. Both of these objectives arose from real world concerns related to kidney exchange. By using 2- and 3-cycle kidney matchings and transforming the dual objective problem into a sequence of single objective maximization problems I then generated elements of the Pareto front. I hypothesized that the greater expected value of minimizing age explains the phenomena and was right.

I also showed that 2-cycles are much more preferred than 3-cycles and I hypothesize that this is due to the utility function used.

While the results of the work show promise in multi-objective optimization of the kidney exchange problem, it is just a first step.

The next steps could include trying to scale up the optimiztion as well as trying to generate ϵ-covers that uniformly approximate the Pareto front rather than sampling it. There are also a couple outstanding questions still to be resolved.

1. Must the normalized Pareto be decreasing linearly or was that a consequence of how we set up the problem?

2. After a few iterations, the number of 2-cycles used settled on a number and percentage and never deviated afterwards. What explains this phenomenon?

If this thesis whets interest in the topic of kidney exchanges with multi-criteria optimization, it will have served its purpose.

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