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A Direct, Approximate Solution to the Modified Green-Ampt Infiltration Equation

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TECHNICAL NOTES:

A DIRECT, APPROXIMATE SOLUTION TO THE MODIFIED GREEN-AMPT INFILTRATION EQUATION

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ABSTRACT. *Accurately predicting the rainfall-runoff process is of vital importance for water quality models as well as for correct design of various types of hydraulic structures. This article presents a method of describing the cumulative infiltration process as an explicit function of time using an approximation to the modified Green-Ampt equation given by Mein and Larson (1971). The resulting equation is helpful in predicting cumulative infiltration and therefore infiltration capacity for computer simulation models. The proposed method takes about 50% less time than the usual iterative technique for the same degree of accuracy. The maximum error due to approximation was 1% and generally the error was much less, making this solution acceptable for most practical problems.* **Keywords.** *Infiltration, Numerical methods, Modeling.*

Predicting the rainfall-runoff process accurately is an intriguing problem that has been studied by researchers in hydrologic science for several decades. Rainfall may infiltrate, runoff, or evaporate, depending on the soil properties, rainfall intensity, rainfall duration and a number of other factors. For the continental United States, approximately 30% of the total precipitation becomes streamflow, which implies that nearly 70% of the annual precipitation infiltrates (Mein and Larson, 1971). It is important therefore for a watershed model to predict infiltration accurately.

Rainfall on a differential area (or simply a point) can be represented by its time distribution. Mein and Larson (1971) modified the Green-Ampt equation (Green and Ampt, 1911) to describe the resulting point infiltration process both prior to runoff and after runoff begins. This modified Green-Ampt equation describes the cumulative infiltration after runoff begins as an *implicit* function of time. In other words, cumulative infiltration cannot be found by a formal algebraic method of substitution. Instead, iterative techniques, numerical approximation procedures, or graphical aids are required.

Chu (1978) presented a graphical approach that gives a good approximation of the cumulative infiltration, but the method cannot be directly implemented within computer simulation models. Many researchers (Muñoz-Carpena et al., 1992; Chaubey et al., 1994; and others) have used iterative techniques to solve the modified Green-Ampt infiltration equation for use in water quality modeling. Chaubey et al. (1994) used an iterative solution to the modified Green-Ampt equation in which iteration continued until the difference between successive estimates of cumulative infiltration (F) was less than the stopping criteria (Ω) of 10^{-7} m. Such iterative techniques are not always acceptable due to the uncertainty of convergence and because of the time consumed in calculation. Li et al. (1976) presented an explicit solution based on the power series expansion of the logarithmic term in the equation. The maximum error observed in Li's approximation was 8%, and this error was observed when simulating conditions often encountered in practical problems.

The objective of this study was to develop an accurate approximation to the modified Green-Ampt equation to represent cumulative infiltration as an explicit function of time. Such an explicit equation would aid in predicting cumulative infiltration accurately and quickly within computer simulation models.

DEVELOPMENT

The modified Green-Ampt infiltration equation given by Mein and Larson (1971) for cumulative infiltration at any time t (greater than ponding time) after rainfall begins is:

$$\frac{K_s(t - t_p + t_s)}{MS_{av}} = \frac{F}{MS_{av}} - \ln\left(1 + \frac{F}{MS_{av}}\right) \qquad (1)$$

where

K_s = saturated hydraulic conductivity (m/s)
t = time since the rainfall begins (s)
t_p = time to ponding (s)
t_s = shift in time scale due to the effect of cumulative infiltration at the ponding time (s)
F = cumulative infiltration at any time $t > t_p$ (m)
MS_{av} = product of antecedent soil moisture deficit and capillary potential at the wetting front (m)

It is clear from this equation that cumulative infiltration is an implicit function of time, i.e., it cannot be found by the simple method of substitution. Chu (1978) presented a

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graphical method of solution in which F/MS_{av} was plotted versus a dimensionless parameter ϕ on a log-log scale (fig. 1), where:

$$\phi = \frac{K_s\left(t - t_p + t_s\right)}{MS_{av}} \qquad (2)$$

In practice, one would compute the value of ϕ, refer to the graph, visually estimate the corresponding value of F/MS_{av}, and then compute F. We envisioned a numerical analogy to this graphical process.

Note that the graph of F/MS_{av} versus ϕ (fig. 1) is very smooth. We approximated this curve using two or more polynomials of degree two, fitted individually for fairly large subranges of ϕ, over the entire range from 0.0001 to 17.0. A polynomial of degree two on a log-log scale can be rewritten using an equation of the form $y = \alpha(x)^{[\beta + \delta \ln(x)]}$, where α, β, and δ are empirical constants. Therefore, the model we used to represent F, fitted for different subranges of ϕ was of the form:

$$F = \alpha MS_{av}(\phi)^{[(\beta + \delta \ln(\phi)]}$$

Values of ϕ ranging from 0.0001 to 17.0 were obtained by substituting arbitrary values of F/MS_{av}, ranging from 0.01 to 20, into equation 1. Multiple linear regression analysis was performed in which the dependent variable was $\ln(F/MS_{av})$ and independent variables were $\ln(\phi)$ and $[\ln(\phi)]^2$, to fit the constants α, β, and δ for three subranges of ϕ. Several break-points between adjacent subranges of ϕ were tested and those that gave the best fit were chosen.

Results and Discussion

The fitted values of α, β, and δ for three subranges of ϕ are presented in table 1. The fit was excellent with $R^2 > 0.99$ for each subrange. The function is continuous at the intersection of two subranges (i.e., the function value is equal when approached from either the left or right of an intersection point). Figure 2 shows the actual and predicted values of cumulative infiltration for three arbitrary values of MS_{av} representing the range in MS_{av} for various soils given by Chu (1978). The maximum relative error was 1%, which was much less than the relative errors reported by Li et al. (1976) for the power series solution. Since ϕ is dimensionless, the parameter values in table 1 will apply for any consistent units for F and MS_{av} in equation 3. Values of MS_{av} and K_s, needed to use this method in an application for a specific site, can be estimated using procedures given by Bouwer (1966), Brooks and Corey (1966) and others.

Table 1. Fitted values of α, β, δ, and R^2 from the regression analyses

Ranges of ϕ	α	β	δ	R^2
0.0001 to 0.095	1.851	0.565	0.004	0.999
0.095 to 0.911	2.137	0.667	0.021	0.999
0.911 to 17.00	2.141	0.689	0.035	0.999

Comparison with Iterative Technique

Chaubey et al. (1994) used an iterative solution to equation 1 in which iteration continued until the difference between successive estimates of F was less than the stopping criterion, Ω, of 10^{-7} m. The selected value of Ω determined the accuracy of the estimates of F and the computer execution time required to perform the iterations. Computer execution time and accuracy of the estimates of F were tested by simulating an arbitrary scenario in which a steady rainfall occurred for 1.67 h on a Captina silt loam soil (a fine silty, mixed, mesic, Typic Fragiudult), with $K_s = 0.012$ m and $MS_{av} = 0.024$ m, planted in fescue grass (*Festuca arundinacea* Schreb.). Computer execution time (fig. 3) and accuracy (fig. 4) of the estimates of F were determined for several values of Ω. Accuracy was measured as the root mean squared error (RMSE) between the estimate of F at each time step (using the selected value of Ω) and the associated true value of F. The true value of F was defined as the value found using an arbitrarily small value of Ω (10^{-8}) which was assumed to produce negligible

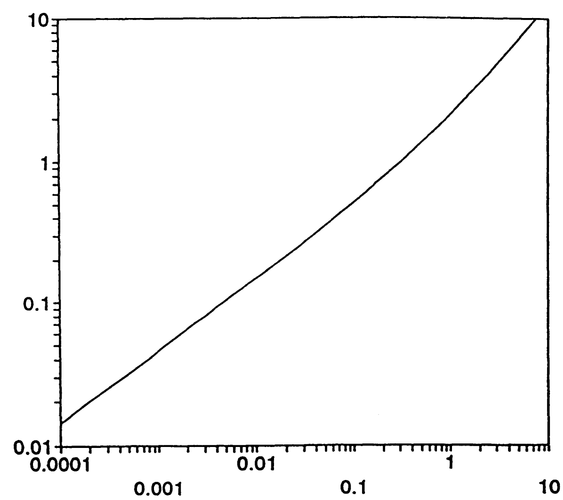

Figure 1–Graphical representation of Green-Ampt equation, from Chu et al. (1978).

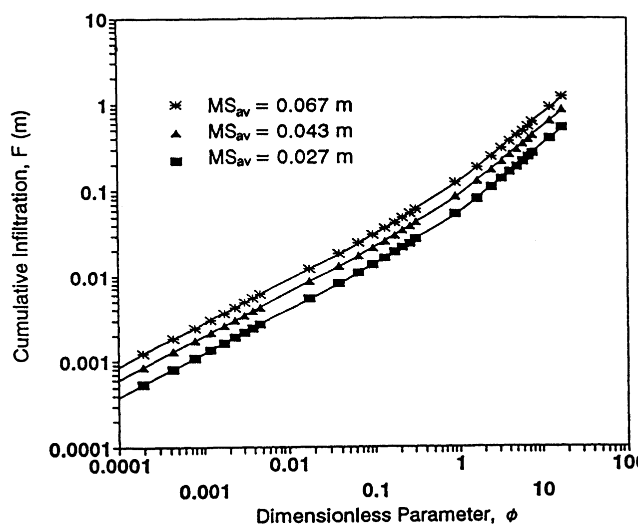

Figure 2–Cumulative infiltration (F) vs. ϕ for three values of MS_{av}. Solid lines represent the predicted value from equation 3 and the symbols represent actual values from the modified Green-Ampt equation (eq. 1).

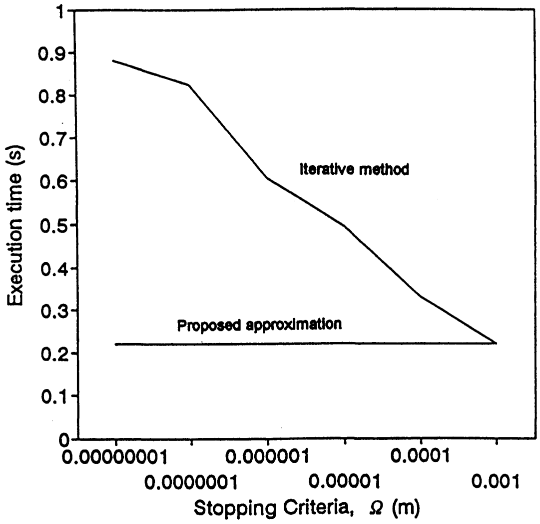

Figure 3–Execution time vs. Ω for the iterative method and the proposed method (eq. 3).

bias. Execution time was measured as the total elapsed computer execution time required to perform the 100 time steps (of 0.0167 h each) in the simulation. Also shown in figures 3 and 4 are the corresponding execution time and RMSE when F was computed using the proposed approximation (eq. 3).

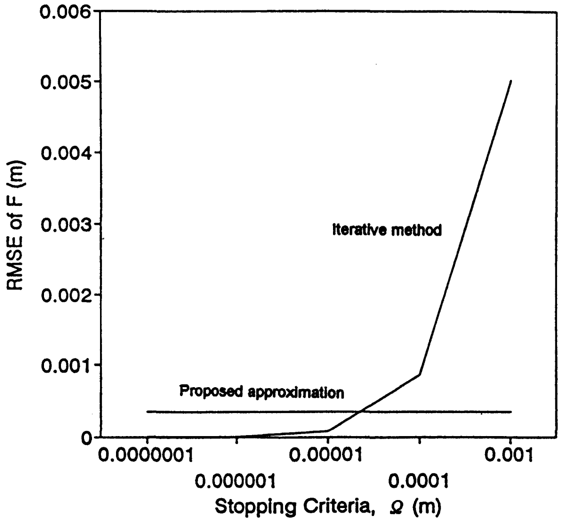

Figure 4–The RMSE of F (m) vs. Ω for the iterative method and the proposed approximation (eq. 3).

Note in figure 4 that the iterative method required $\Omega = 10^{-5}$ or less to exceed the accuracy of the proposed approximation. With this level of stopping criteria, the proposed method required much less (approximately 50% less) execution time than the iterative method (fig. 3). Although the execution time for this illustration was less than 1 s for both methods, the speed advantage of the direct approximation may become important in applications which include simulating multiple scenarios, with fine resolution in time and space, resulting in many repetitions of the calculation of F.

Summary

Equation 3 describes the cumulative infiltration as an explicit function of time, as opposed to the implicit form of the modified Green-Ampt equation given by Mein and Larson (1971). The proposed solution was more accurate than the power series solution and required much less computer execution time than the iterative technique. This direct solution for F will be helpful in predicting point cumulative infiltration, infiltration capacity, and watershed runoff accurately. Such prediction is a necessary component of water quality models and is also vital for the correct design of various types of hydraulic structures.

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