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Prediction Model to Estimate the Zero Crossing Point for Faulted Waveforms

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Prediction Model to Estimate the Zero Crossing Point for Faulted Waveforms

THESIS

A thesis submitted in partial fulfillment of the
requirements for the degree of Master of Science in Electrical Engineering
in the College of Engineering
at the University of Kentucky

By

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Lexington, Kentucky

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ABSTRACT OF THESIS

In any power system, fault means abnormal flow of current. Insulation breakdown is the cause of fault generation. Different factors can cause the breakdown: Wires drifting together in the wind, Lightning ionizing air, wires with contacts of animals and plants, Salt spray or pollution on insulators. The common type of faults on a three phase system are single line-to-ground (SLG), Line-to-line faults (LL), double line-to-ground (DLG) faults, and balanced three phase faults. And these faults can be symmetrical (balanced) or Unsymmetrical (imbalanced).In this Study, a technique to predict the zero crossing point has been discussed and simulated. Zero crossing point prediction for reliable transmission and distribution plays a significant role. Electrical power control switching works in zero crossing point when a fault occurs. The precision of measuring zero crossing point for syncing power system control and instrumentation requires a thoughtful approach to minimize noise and external signals from the corrupted waveforms A faulted current waveform with estimated faulted phase/s, the technique is capable of identifying the time of zero crossing point. Proper Simulation has been organized on MATLAB R2012a.

KEYWORDS: Zero Crossing Point, System Protection, Reliable Power Transmission, Fault Minimization, Fault Protection

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Chapter 1 Introduction

If a question is asked what the most important factor is at this moment to live on this earth, "Electricity" will probably be the answer from most of the surveys. But, to be sensible or to be more technical the answer would be reliable power system. This thesis includes a topic where an important thing for reliable power system has been discussed which can be mentioned as zero crossing estimation technique. Before the discussion will go into deeper, brief words will be exchanged about history of electric power system, components of electric power system and the importance of zero crossing in power system.

1.1 History of Electric Power System

In the early days or one might say at the beginning era of electricity, power systems were so small and localized. Among all the accomplished systems "The Pearl Street Station" in New York City was the first that connected a 100 V generator that burned coal to power a few hundred lamps in the neighborhood; it was founded by Thomas Edison and his company which was established in 1882 [12]. At the very beginning the station provided power around 3,000 lamps for 59 customers. Soon, many similar complete, independent and isolated systems were built throughout the country. The power station was run by direct current and operated at a single designated voltage. At that time for long-distance transmission it was not possible to transform Direct Current (DC) easily to the required higher voltages to minimize power loss; so it was obvious to have the maximum economical distance around half-a-mile (800 m) between the generating station and end user load [13]. To solve the issue, a number of the AC equipment including generators and transformers were imported from Europe by George Westinghouse who was known an American entrepreneur and he hired engineers for experimenting with them for making a structure of commercial power system [18]. In July 1888, Westinghouse also purchased Nikola Tesla's US patents for a poly phase AC induction motor and hired Tesla for one year to be a consultant at the Westinghouse Electric & Manufacturing Company's Pittsburg labs to set up the AC motor [14,18].

From this lab, the first generator that used alternating current (AC) was built by William Stanley, Jr. Instead of flowing in one direction, its direction of electricity was backward and forward [18]. Alternating Current is being used almost exclusively worldwide today, but in the late 1800s it was hardly imaginable to use AC than DC. The major advantage of Alternating Current is that it is possible to transmit AC power as high voltage and convert it to low voltage to serve individual users using the step up and step down transformers [12]. From the late 1800s to ahead, a jumble of AC and DC grids popped up over the country, in direct competition with one another. By 1930s regulated electric utilities became well-demonstrated, providing all three major aspects of electricity- generation, transmission, and distribution.

1.2 Electric Power System Structure

An electric power system can be defined as an electrical network that supplies residential and industrial area with power - this total power system can be named as the grid and generally can be divided into three basic or major parts: generators, transmission line and distribution system. Generators usually supply/produce the power; Transmission system carries the power from the generating stations to the load centers and the distribution system that makes flow the power to nearby houses and industries. Bulks of these systems are established on three-phase AC power which is basically the standard for large-scale power transmission and distribution throughout the present world. To deliver from generation plant to distribution station step up transformers are used so that voltage always remain up above of 110kV or equal to that as the distance from generation plant to distribution substation may long enough. Otherwise, signal might be distorted. Due to this voltage range transmission loss for long transmission line is less. After transmitting to the distribution substation step down transformers are used for delivering to the end users.

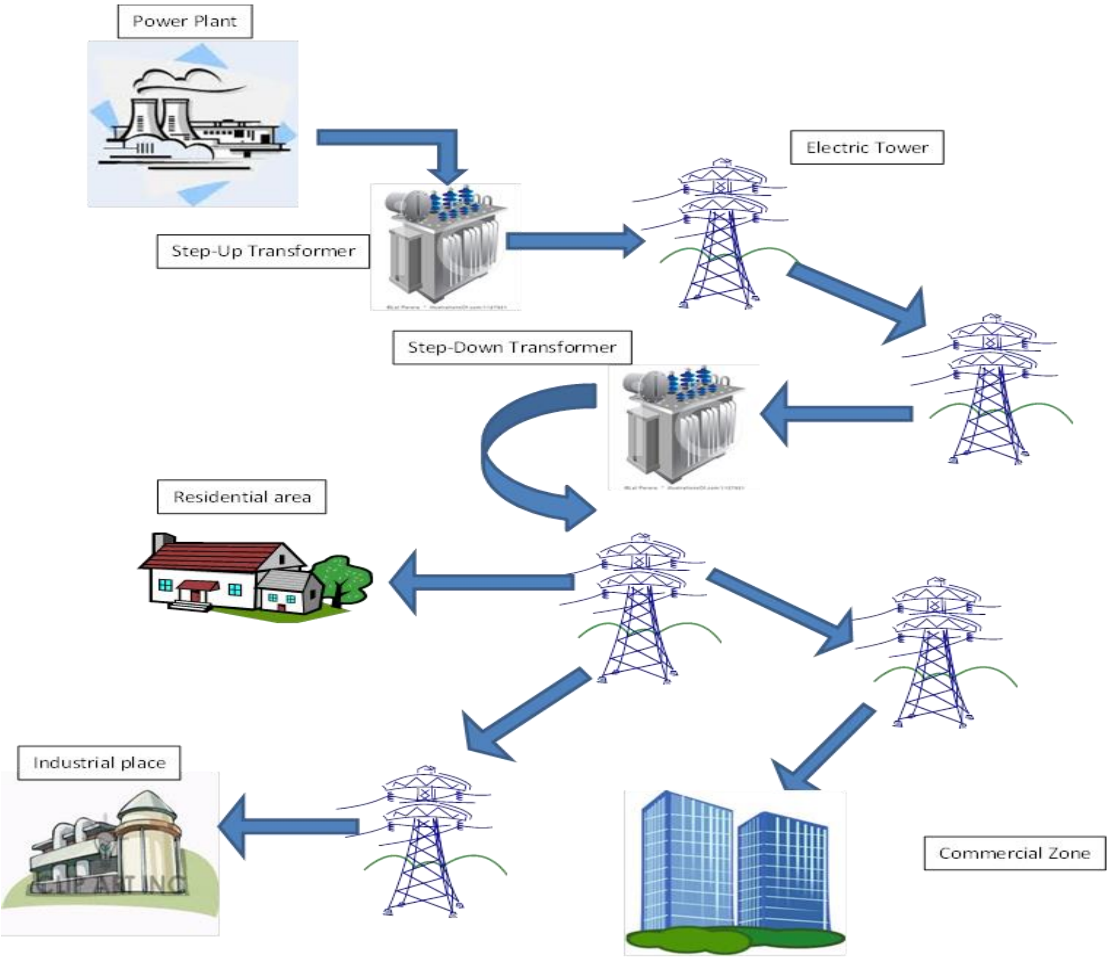

Figure 1: Electric Power System Components

1.3 Importance of Zero crossing point in power System

Due to the importance of zero crossing estimation point, we have to discuss about the importance of fault recognition first. In power system, fault infers the abnormality in electric current. In three-phase systems, a fault is usually engaged in one or more phases and ground, or may take place only between phases. In a "ground fault" or "earth fault", charge falls into the ground. In a poly phase system, all the phases may be affected by a fault equally which is a "symmetrical fault". If only some phases are affected, it becomes more complicated to analyze the ensuing "asymmetrical fault" due to not applicable of the simplifying assumption of equal current magnitude in all phases. Here the term "Symmetrical components" comes to solve this issue. Faults may also be caused by either short circuits to ground or between conductors or may be caused by broken conductors in one or more phases which are known as open circuit fault. In a nutshell, if we describe the fault in types there are two kinds: symmetrical faults and asymmetrical faults. The definition of symmetric and asymmetric fault has been given. Due to the nature of asymmetric fault it can be divided in three types: line to line, line to ground and double line to ground fault. When such fault occurs, equipment used for power system protection operates to separate the circuit of the fault. According to the tenure of fault types there are two types: Transient and persistent. Transient fault is a temporary fault in nature which usually occurs for a very short time and restores then. Faults in overhead lines are generally transient in nature too. Typical examples of Transient faults are: momentarily tree contacts, Bird and other animal contacts, lightning strike and conductor cashing. Persistent fault doesn't appear when the power is disconnected. This kind of fault happens due to mechanical damage in underground lines [16]. Among two types of faults we care mostly concerned about the asymmetric faults because of the system imbalanced, and this is not easy to resolve. The studies and detection of these faults are necessary to ensure that the reliability and stability of the power system. Protection system consists of instrumental transformers, relays and circuit breaker. If the secondary windings current of the instrument transformer exceeds the “pick value” then the relay contacts close automatically, this then causes the circuit breaker to open. So, Information retrieved from the instrumental transformers make the relay and circuit breaker work for power system protection.

In Mathematics, a "zero-crossing" is a point where the sign of a function changes (e.g. from +ve to -ve), due to the crossing of the axis (zero value) in the graph of that function. In alternating current (AC), the “zero crossing point” is the certain point at which there is no voltage posed. In a sine wave or other simple waveform, in each cycle this naturally occurs twice. Zero crossing point estimation is important in electrical power system control. When a fault occur in transmission line, Electric Arc happens, system gets unbalanced, and this fault needs to be minimized soon as the fault signal is consisted of voltage spike and electrical noise which may create damage of the equipment(Circuit Breaker/ Switch) depending on the fault. More specifically, if the arc is secondary it can hamper in Ultra High Voltage (UHV) transmission line. If the secondary arc is not put out quickly, the circuit breaker will be reclosed the trouble in arc light， which continues affecting the stability of system [17]. In Alternating Current, the significant point is to separate the circuit at zero crossing point next to the fault occurring point to lessen the damages. In system protection, if we can detect the zero crossing point next to the fault occurring point and synchronize the fault occurring time with the breaker, the circuit will be separated at the zero crossing point which can minimize the noise effect or damage in the transmission line. There are two types of noise: noise on power lines and noise emitted into open spaces caused by arc. The zero crossing function is effective against both types of noise. AC type Relays turns “ON” at the zero crossing point of the AC sinusoidal wave form, prevents high inrush currents when switching to inductive or capacitive loads.

Chapter 2 Literature review

The power system, as a whole, is a complex dynamic system. The voltage and current phasors are widely used for monitoring and control in a large interconnected power system. Voltage and current phasors in power system network undergo through a dynamic variation under system disturbances and faults. Fast prediction of rate of change of these phasors and acceleration of changes will find direct applications in system protection and stability prediction. A fault signal is consisted of fundamental and harmonic components that can be represented as sinusoidal function and DC decaying components, which is expressed as decaying exponential function [1,2], known as DC-Offset. To calculate the fault signal it is utmost important to get the values of the phasors. Discrete Fourier Transform, Walsh, Harr, Least Square or Kalman filtering have been used for phasor estimation [3-5], which usually suppress the dc offset too.

In paper [6], a modified dynamic phasor estimation method was proposed to estimate a phasor that changes magnitude according to time during the transient period. Envelope of a phasor was estimated by assuming an input signal inclusive of a Taylor series of fundamental frequency component and high frequency components. A decaying dc offset is a non-periodic signal and has a relatively wide range frequency spectrum with larger distribution at lower frequencies. Since conventional full-cycle DFT cannot effectively attenuate the lower frequency components, unwanted errors in forms of overshoot and decaying oscillations result in the magnitude and angle of the estimated phasor, paper [7] has described an extemporized DFT based phasor algorithm. This paper first presents a new method in which the decaying dc parameters are estimated by averaging the current signal over the power system cycle.

However, instead of the direct use of decaying dc magnitude and time constant, two interim variables are defined to reduce the amount of computation. Second, a new method is proposed to compensate the current phasor estimated by full-cycle DFT in the phasor domain using very few basic mathematical operations. Moreover, in this paper, the standard performance indices used in control theory and signal processing including rise time, settling time, and overshoot have been adopted to compare the proposed algorithm with the full-cycle DFT, cosine and mimic filter (a high-pass or band pass filter which can completely remove the decaying dc offset only when the time constant of the dc offset matches with the presumed one) DFT algorithms. An improved DFT method has been discussed in paper [1]. Using the odd and even samples of fault signal after DFT it has shown the calculation of amplitude and phase of harmonic and exponential component. Paper [8] presented a novel adaptive mimic filter (AMF) to eliminate the decaying DC component effect on phasor estimation. The key idea is to use an adaptive algorithm to obtain the decaying time constant of the signal; thereby the digital mimic filter parameters are readjusted. As a result, the decaying DC component may be completely filtered out. The proposed phasor estimation algorithm combines the AMF with the Full-Cycle discrete Fourier Transform (FCDFT) algorithm. Paper [9] is about calculating impedances from digitized voltages and currents sampled of a faulted waveform. This paper presents a novel algorithm which is based on the least error squares curve matching technique. The algorithm assumes that the input is consisted of a fundamental frequency component, a decaying d. c. and harmonics of defined order. The decay rate of the d. c. component is not assumed in advance because it is affected both by the resistance of the arc at the fault and the effective resistance of the system. The paper describes the least squares approach for developing a digital filter which explicitly takes account the decaying d. c. components in the system voltages and currents. The concept of pseudo-inverse which has been used in developing the algorithm is also presented.

Beside those, several methods of calculating zero crossing point next to the fault has been discussed in [10], these are: Pre-Detection Low Pass filtering, Post Processing Signal Conditioning, Simple Optical Isolated Semiconductor Devices, Zero-Crossing Detection by Interpolation, Comparator Circuits with Fixed Hysteresis, Comparator Output Frequency Filtering.

All those description are ended by calculating the magnitude of the dc offset and the peak value of the definite harmonic component. And here the next step of the formulation starts to calculate the time of zero crossing point.

Chapter 3 Theoretical Discussion for Zero Crossing Estimation

To solve the problem of zero crossing point estimation there are several points like components of faulted voltage/current wave, Faults in the transmission line and sequence components to solve the unbalanced system. Least square method has been used here to solve the zero crossing point [9].

3.1 Fault wave fundamentals

Fundamental and harmonic components in fault current/voltage can be represented as sinusoidal functions, and the decaying dc component is expressed as a decaying exponential function [1,9]. The equation is:

$$I(\mathrm{t}) = \mathrm{K}_1 e^{-t/\tau} + \sum_{n=1}^{N} K_{2n} Sin(\mathrm{n}\,\omega_0\,\mathrm{t} + \theta_n) \qquad (3.1)$$

Where, $I(t)$ is the instantaneous Current at time t;

τ is the time constant of the decaying d. c. component;

N is the highest order of the harmonic component present in the signal;

ω_0 is the fundamental frequency of the system;

K_1 is the magnitude of the d. c. offset of t=0;

K_{2n} is the peak value of the nth harmonic component and

θ_n is the phase angle of the nth harmonic component.

$e^{-t/\tau}$ can be expanded by Taylor series as below :

$$e^{-t/\tau} = 1 - t/\tau + \frac{1}{2!}\frac{t}{\tau^2} - \frac{1}{3!}\frac{t}{\tau^3} \qquad (3.2)$$

In real world, present in fault voltages and currents there are no even harmonics. Also, signal conditioning equipment which usually contains analog filters, blocks the higher order harmonics to reach the relay [9]. First three terms have been used and assumed here to expand the equation. The signal conditioning equipment will block the fifth and higher order harmonics in an effective manner and no even harmonics will be presented in the input, a current sampled at definite time t can be expressed by Equation 3.3 which is an abbreviated form of Equation 3.1.

$$I(t) = K_1 - K_1(\frac{t}{\tau}) + \frac{K_1}{2!}(\frac{t}{\tau^2}) + K_{21}Sin(\omega_0 t + \theta_1) + K_{23}Sin(3\omega_0 t + \theta_3) \tag{3.3}$$

Equation (3.4) is obtained by expanding $Sin(\omega_0 t + \theta_1)$ and, $Sin(3\omega_0 t + \theta_3)$ using trigonometric formulation of *Sin(A+B)* and substituting in Equation 3.3.

$$\begin{aligned} I(t) &= K_1 - K_1(\frac{t}{\tau}) + \frac{K_1}{2!}(\frac{t}{\tau^2}) + K_{21}(Sin\omega_0 t)(Cos\theta_1) + K_{21}(Sin\theta_1)(Cos\omega_0 t) \\ &+ K_{23}(Sin3\omega_0 t)(Cos\theta_3) + K_{23}(Cos3\omega_0 t)(Sin\theta_3) \end{aligned} \tag{3.4}$$

Now for calculating the unknowns and applying mathematical formulations we will organize equation (3.4) in a formation of matrix. Suppose,

$$X_1 = K_1 \tag{3.5}$$

$$X_2 = K_{21}(Cos\theta_1) \tag{3.6}$$

$$X_3 = K_{21}(Sin\theta_1) \tag{3.7}$$

$$X_4 = K_{23}(Cos\theta_3) \tag{3.8}$$

$$X_5 = K_{23}(Sin\theta_3) \tag{3.9}$$

$$X_6 = \frac{-K_1}{\tau} \tag{3.10}$$

$$X_7 = \frac{K_1}{2\tau^2} \tag{3.11}$$

and,

$$A_{11} = 1 \tag{3.12}$$

$$A_{12} = Sin(\omega_0 t) \tag{3.13}$$

$$A_{13} = Cos(\omega_0 t) \tag{3.14}$$

$$A_{14} = Sin(3\omega_0 t) \tag{3.15}$$

$$A_{15} = Cos(3\omega_0 t) \tag{3.16}$$

$$A_{16} = t \tag{3.17}$$

$$A_{16} = t^2 \tag{3.18}$$

So, now we can write equation (3.4) as

$$I(t) = A_{11}X_1 + A_{12}X_2 + A_{13}X_3 + A_{14}X_4 + A_{15}X_5 + A_{16}X_6 + A_{17}X_7 \tag{3.19}$$

Then depending on time, we can have the equations of I (t_1), I (t_2), I (t_3), and so on. And using these sample currents we can make a model of matrix to get the unknowns in X. The equation (3.19) can be written in a format of matrix as below:

$$\left[\underset{m\times 1}{I}\right] = \left[\underset{m\times 7}{A}\right]\left[\underset{7\times 1}{X}\right] \tag{3.20}$$

We can easily retrieve the fault currents what we will discuss in the later part of the theory and we know the values of matrix A, which can be predetermined and depends on the values of sampling rate and the time reference. Using the values we can get the solution of X which is consisted of the magnitude of the dc offset and magnitude & phase angle of the harmonic components. From the dimension of the matrix, we can easily see that we need at least seven current samples from the faulted signal. If we take seven values and it becomes a square matrix, then we can use inverse of the matrix. But, for more accuracy we may take some more values. And the matrix A becomes rectangular matrix. Here, we must have to use the mathematical constraints that we can't do inverse of a rectangular matrix. So, here comes the function of pseudo inverse, which can be referred as Moore –Penrose pseudo inverse. A common use of the Moore –Penrose pseudo-inverse (hereafter, just pseudo inverse) is to compute a best fit'(least squares) solution to a system of linear equations that lacks a unique solution. We usually represent pseudo-inverse by representing A^+.

So, the value of unknowns can be determined as follows:

$$[X]=[A]^T[I] \tag{3.21}$$

3.2 Sequence Components & Fault Types

Here comes the usage of current samples. Now, faulted current samples can be taken from any type of fault like single line to ground fault, Line to Line fault, double line to ground fault. To discuss about these faults, we must have to review the symmetrical component theories cause it is utmost useful to solve the unbalanced three phase circuits. For these theories Glover, Sharma & Overbye's book [11] and Grainger ,John J., Stevenson, William D.'s [16] can be referred. We will infer some parts of these books.

Assume that a set of three-phase voltages designated V_a, V_b, and V_c is given. According to Fortescue, these phase voltages are resolved into the following three sets of sequence components:

1. Zero-sequence components, consisting of three phasors with equal magnitudes and with zero phase displacement, as shown in Figure 2(a)
2. Positive-sequence components, consisting of three phasors with equal magnitudes, $\pm 120^{o}$ phase displacement, and positive sequence, as in Figure 2(b)

3. Negative-sequence components, consisting of three phasors with equal magnitudes, ±120° phase displacement, and negative sequence, as in Figure 2(c)

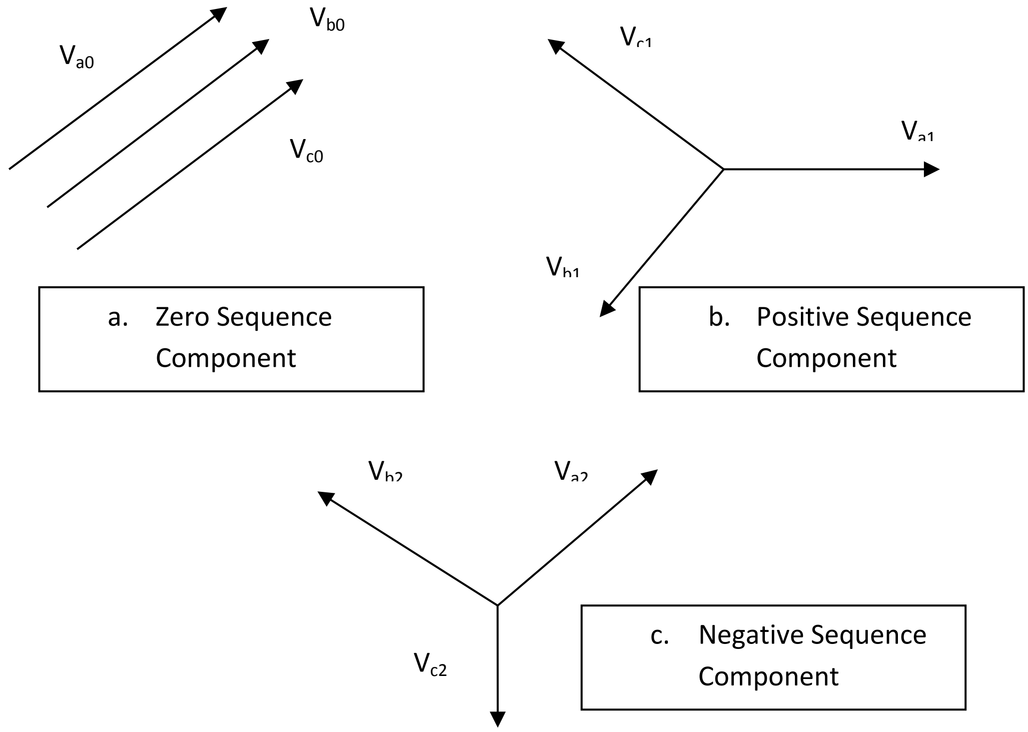

Figure 2: Resolving phase voltages/currents into three sets of sequence components [11]

An unbalanced three phase system can easily be transformed from *n* related phasors can be translated to n systems of balanced phasors by using this sequence network called symmetric component. Three voltage sequences of each phase are designated as V_{a0}, V_{b0}, and V_{c0} for zero Sequence, V_{a1}, V_{b1}, and V_{c1} for positive Sequence and V_{a2}, V_{b2}, and V_{c2} for negative Sequence. Relationship between phases and sequences can be described by the following matrix:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \tag{3.22}$$

Where, $a = 1\angle 120^0 = (-1/2) + \mathrm{j}(\sqrt{3}/2)$

By applying inverse of the above matrix, we can get as follows:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad (3.23)$$

Similarly, the relationship for current is as below:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \qquad (3.24)$$

and

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad (3.25)$$

Where, Three current sequences of each phase are designated as I_{a0}, I_{b0}, and I_{c0} for zero Sequence, I_{a1}, I_{b1}, and I_{c1} for positive Sequence and I_{a2}, I_{b2}, and I_{c2} for negative Sequence. Now, Different kinds of faults will be discussed. We will be using the theories from Granger and Stevenson's book [16] power system analysis and Glover, Sharma and Overbye's book [12] power system design and analysis for different fault type discussion.

3.2.1 Phase to ground Fault

Consider a single line-to-ground fault from phase *a* to ground at the general three-phase bus shown in Figure 3. In general, we include fault impedance Z_f. In the case of a bolted fault, Z_f =0, whereas for an arcing fault, Z_f is the arc impedance.

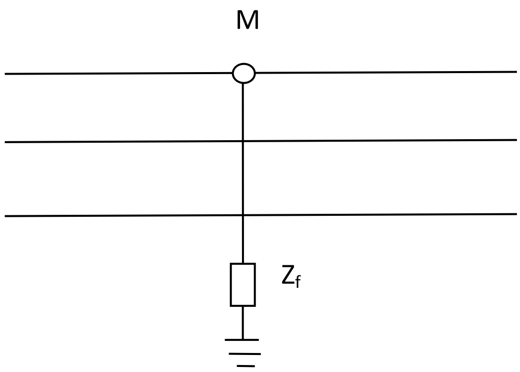

Figure 3: Phase a to ground Fault

In the case of a transmission-line insulator flashover, Z_f includes the total fault impedance between the line and ground, including the impedances of the arc and the transmission tower, as well as the tower footing if there are no neutral wires. The sequence network diagram is as below:

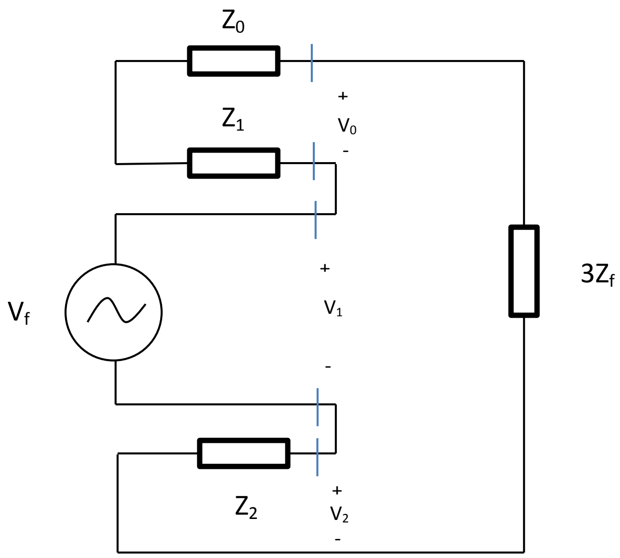

Figure 4: Interconnected sequence networks (Phase to ground faults) [11]

For the ease of equation mapping we will consider the fault from phase *a* to ground, whereas it can be arbitrarily chosen in any phase. The conditions from Figure 3 are expressed by the following equations:

$$I_b = 0 \tag{3.26}$$

$$I_c = 0 \tag{3.27}$$

$$V_{ag} = I_a Z_f \tag{3.28}$$

Now, from the symmetrical component equation (3.25), we can write,

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \tag{3.29}$$

And from equation (3.22) and equation (3.24) in equation (3.28), it is clear that

$$V_0 + V_1 + V_2 = \left(I_0 + I_1 + I_2\right) Z_f \tag{3.30}$$

From equation (3.29) and equation (3.30),

Fault condition in sequence domain, $I_0 = I_1 = I_2$, and $V_0 + V_1 + V_2 = 3Z_f I_1$

So, from the above equations and Figure 4 we can write,

$$I_0 = I_1 = I_2 = \frac{V_F}{Z_0 + Z_1 + Z_2 + 3Z_f} \tag{3.31}$$

And now from equation (3.31) it can be easily derived that,

$$I_a = I_0 + I_1 + I_2 = \frac{3V_F}{Z_0 + Z_1 + Z_2 + 3Z_f} \tag{3.32}$$

Finally,

$$V_{ag} = \frac{3V_F}{Z_0 + Z_1 + Z_2 + 3Z_f} Z_f \tag{3.33}$$

Now, we can have phase to ground voltage/current samples in different time instants. Using those voltage/current samples we can get the value of X from equation (3.21).

3.2.2 Phase to Phase Fault

For the ease of theory, we are considering a fault from phase b to phase c shown in Figure 5. Here, the fault impedance is Z_f between phase b to phase c. The conditions at bus M are,

$$I_a = 0 \tag{3.34}$$

$$I_c = -I_b \tag{3.35}$$

$$V_{bg} - V_{cg} = Z_f I_b \tag{3.36}$$

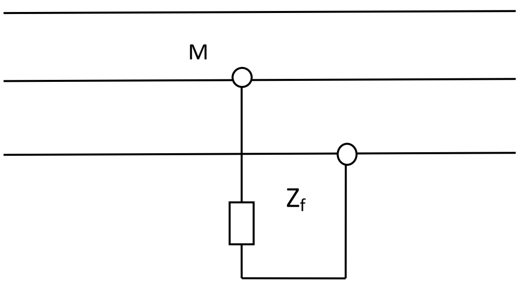

Figure 5: Phase b to Phase c Fault

Using the conditions, from equation 3.25, we can write-

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{3}\left(a - a^2\right) I_b \\ \frac{1}{3}\left(a^2 - a\right) I_b \end{bmatrix} \tag{3.37}$$

From equation 3.22 & 3.24 in equation 3.36, we can write,

$$\left(V_0 + a^2 V_1 + aV_2\right) - \left(V_0 + aV_1 + a^2 V_2\right) = Z_f\left(I_0 + a^2 I_1 + aI_2\right) \tag{3.38}$$

Equation (3.38) can be simplified as below,

$I_0 = 0$ and $I_2 = -I_1$, which can be referred as-

$$\left(a^2 - a\right)V_1 - \left(a^2 - a\right)V_2 = Z_f\left(a^2 - a\right)I_1 \;\; => \;\; V_1 - V_2 = Z_f I_1 \tag{3.39}$$

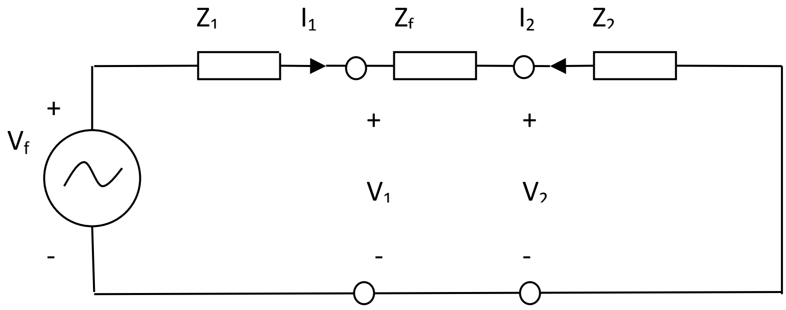

Figure 6: Interconnected Sequence network (Phase to Phase) [11]

From Figure 6 of interconnected sequence network, we can conclude:

$$I_1 = -I_2 = \frac{V_F}{Z_1 + Z_2 + Z_F}, I_0 = 0 \tag{3.40}$$

Transforming equation (3.40) in phase domain using equation (3.24) and using identity $\left(a^2 - a\right) = -j\sqrt{3}$, the fault current in phase b is

$$I_b = I_0 + a^2 I_1 + aI_2 = \left(a^2 - a\right) I_1 = -j\sqrt{3} I_1 = \frac{-j\sqrt{3}V_F}{Z_1 + Z_2 + Z_F} \quad (3.41),$$

$$I_a = 0 \quad (3.42),$$

and

$$I_c = I_0 + aI_1 + a^2 I_2 = \left(a - a^2\right) I_1 = -I_b \tag{3.43}$$

3.2.3 Double Phase to ground Fault

A double line to ground fault from phase b to phase c to ground through fault impedance Z_f has been shown in Figure 7. It is must to have the following conditions in case of phase b to phase c to ground type fault:

$$I_a = 0 \tag{3.44}$$

$$V_{cg} = V_{bg} \tag{3.45}$$

$$V_{bg} = Z_f \left(I_b + I_c \right) \tag{3.46}$$

Using equation (3.24) in equation (3.44) and using equation (3.22) in equation (3.46), we can write respectively-

$$I_0 + I_1 + I_2 = 0 \tag{3.47}$$

$$(V_0 + aV_1 + a^2V_2) = (V_0 + a^2V_1 + aV_2) \tag{3.48}$$

Simplifying that, we can conclude it with,

$$V_2 = V_1 \tag{3.49}$$

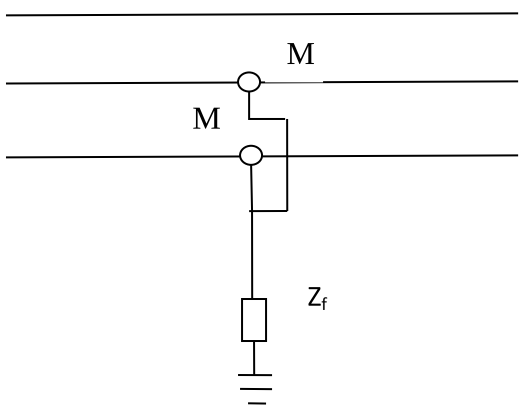

Figure 7: Phase b to C to Ground fault

Now plotting equation (3.22) & (3.24) in (3.46), we can write

$$(V_0 + a^2V_1 + aV_2) = Z_f \left(I_0 + a^2 I_1 + aI_2 + I_0 + aI_1 + a^2 I_2 \right) \tag{3.50}$$

Using the Identity $a^2 + a = -1$ and equation (3.50), it can be written as

$$\left(V_0 - V_1\right) = Z_f \left(2I_0 - I_1 - I_2\right) \tag{3.51}$$

Using equation (3.47), we can simplify (3.51) as below:

$$V_0 - V_1 = (3Z_f) I_0 \tag{3.52}$$

So, Equation (3.47), (3.49) and (3.52) summarize the fault conditions in sequence network.

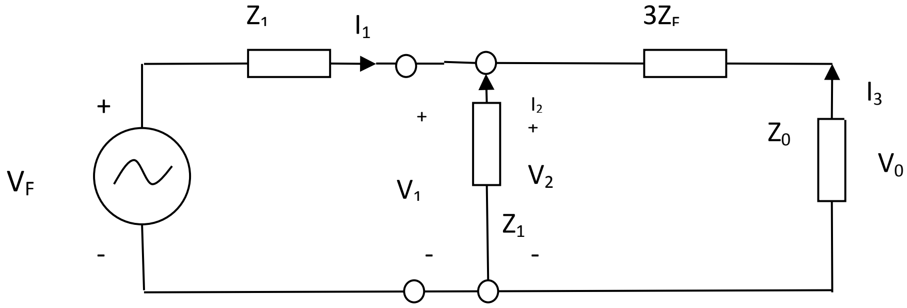

Figure 8: Interconnected Sequence network (Double Phase to ground) [11]

Zero, Positive and Negative sequence networks have been connected in parallel in the fault terminal in Figure 8 where $3Z_F$ has been connected in series with zero sequence. Sequence fault current can be retrieved from the circuit.

$$I_1 = \frac{V_F}{Z_1 + \left[Z_2 // \left(Z_0 + 3Z_f\right)\right]} = \frac{V_F}{Z_1 + \left[\dfrac{Z_2\left(Z_0 + 3Z_f\right)}{Z_0 + Z_2 + 3Z_f}\right]} \qquad (3.53),$$

$$I_2 = \left(-I_1\right)\left(\frac{Z_0 + 3Z_f}{Z_0 + Z_2 + 3Z_f}\right) \qquad (3.54)$$

and

$$I_0 = \left(-I_1\right)\left(\frac{Z_2}{Z_2 + Z_0 + 3Z_f}\right) \qquad (3.55)$$

Using equation (3.53), (3.54) & (3.55), we get the fault currents at phase domain.

3.3 Unknown Parameters calculation of Faulted Wave

Using equation (3.5) and (3.6), we can retrieve the value of $K_{21}, K_{23}, \theta_1, \theta_3$ and τ after having the following formulation:

$$K_{21} = \frac{X_2}{Cos\theta_1} = \frac{X_3}{Sin\theta_1}$$

$$=> \ Tan\theta_1 = \frac{X_3}{X_2}$$

$$=> \ \theta_1 = Tan^{-1}(\frac{X_3}{X_2}) \tag{3.56}$$

Similarly, we can have $\theta_3 = Tan^{-1}(\frac{X_5}{X_4})$ from equation (3.8) and (3.9) which can be written as below:

$$K_{23} = \frac{X_4}{Cos\theta_3} = \frac{X_5}{Sin\theta_3} \tag{3.57}$$

θ_1 & θ_3 can lead to calculate the value K_{21} & K_{23}.

To get the value of τ we can write from equation (3.10) and (3.11), $K_1 = -X_6\tau = 2X_7\tau^2$

$$=> \tau = -\frac{K_1}{X_6} \tag{3.58}$$

From the above equation (3.35) we can easily estimate the roots of τ

Now, after having all those values/unknowns of X we can write, I (t) = 0 to solve the function for getting the value of t at what instant the current will have a value of zero.

So, the equation that we need to solve is-

$$K_1 - K_1(\frac{t}{\tau}) + \frac{K_1}{2!}(\frac{t}{\tau^2}) + K_{21}Sin\left(\omega_0 t + \theta_1\right) + K_{23}Sin\left(3\omega_0 t + \theta_3\right) = 0 \tag{3.59}$$

The equation that has been stated above is a continuous function of t. from the mathematical definition of continuous function; we know continuous function is a function where a small change in the independent variable produces a small change in the output of the function. Here, depending on time (t) function changes. We have to solve, at which point of t the value of the function is zero. To solve this function we will use the computational method in mat lab.

Chapter 4 Evaluation Study

4.1 Introduction to Simulation Steps

In this section, we will talk about the implementation of the discussed theories. We have simulated an equivalent circuit that we discussed in the theoretical part. A 60 Hz power system model has been used for this study. The simulated power system consists of two generators at the two ends and a 177.1000-mile/285.13 km transmission line. Sampling frequency has been considered as 7680 Hz. Generator information and line parameters are presented in Table 1 and Table 2. Base values of 500kV and 100MVA are utilized in the per unit system. The simulated system has been portrayed in Figure 9.

Figure 9: Schematic Diagram of the System

Table 1: Source Parameters

Generator	Voltage (kV)	Positive-sequence Impedance (p.u)	Zero-sequence Impedance (p.u)
G_1	500	$(3.408+j9.033)\times10^{-3}$	$(3.059+j9.178)\times10^{-3}$
G_2	500	$(51.758+j2.944)\times10^{-3}$	$(.144+j3.023)\times10^{-3}$

Table 2: Transmission line parameters

Positive Sequence Impedance (Ω/mile)	Zero Sequence Impedance (Ω/mile)
$.4982- j\ 6.8156\times10^{5}$	$.06241+ j\ 2.1993\times10^{5}$

The voltage/current samples from this circuit will be used in unknown parameters calculation from the faulted wave equation. In this simulation, the zero crossing parameter is time (t) which we need to calculate. We need to calculate the time using the unknown parameters at which point function changes the sign. Here the function is a continuous function and we have used the "fzero" command to calculate the zero of the function.

4.2 Waveforms under Different Fault Conditions

Some typical voltage and/or current waveforms under different fault conditions and fault resistances are presented in this subsection using simulation through MATLAB script and Simulink. For each fault type, Using Fault resistance (Z_f) .01 ohms and ground resistance .01 ohms we will see the results of zero crossing point. The total simulation lasts for 0.167 seconds, and the fault occurs at 0.0333 second as the transition times were fixed there. We will also simulate the things with a different transitions time at .05 seconds.

4.2.1 Phase A to Ground Fault

The following Figure 10 & Figure 11 represent fault voltage and current waveforms respectively, under phase a to ground fault. From the simulation, we have seen that the fault has been occurred at .0333 second which is visible from the graphs. Here, the system gets unbalanced.

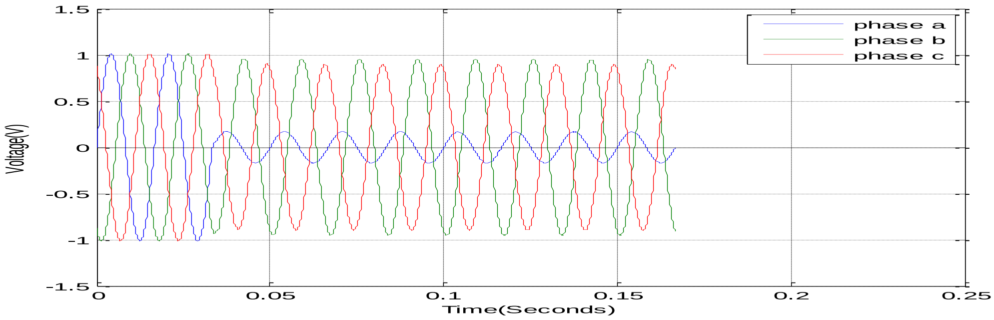

Figure 10: Three phase Ground to Fault Voltage Graph (AG Fault, Z_f = .01 ohm)

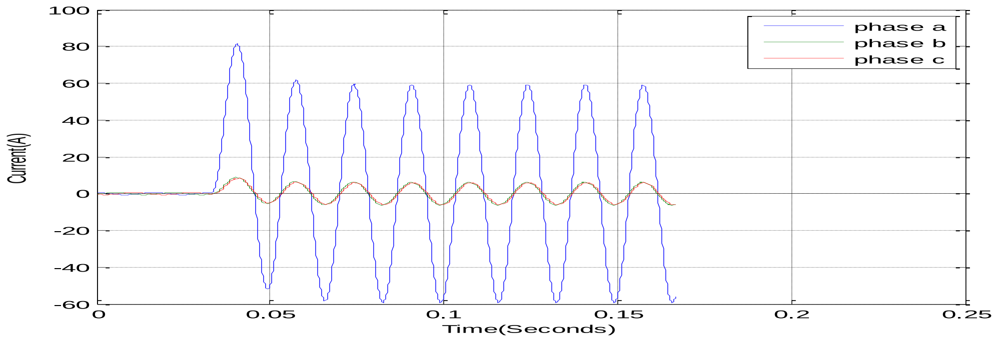

Figure 11: Three phase Ground to Fault Current Graph (AG Fault, Z_f = .01 ohm, Transition=.0333s)

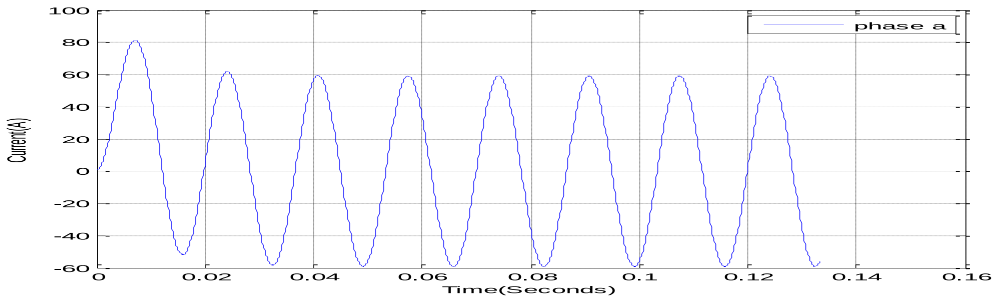

Figure 12: Phase a to Ground Fault Current Graph (Retrieved from Simulink)

Parameters from faulted wave equation, those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below:

Table 3: Faulted wave function parameters (Phase a to ground)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	57.8859 V
Peak value of the 1st harmonic component (K_{21})	60.209 V
Peak value of the 3rd harmonic component (K_{23})	-0.041713 V
Phase angle of the 1st harmonic component (θ_1)	-72.1633 ^{o}
Phase angle of the 3rd harmonic component (θ_3)	-51.1167 ^{o}
Time constant of the decaying d. c. component (τ)	0.0085469 s

By using the unknown parameters of faulted wave current those have been retrieved we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 13. This current wave symbolizes phase a to ground fault. This symbolizes the zero crossing point at .0123s. So, the zero crossing point for real transmission line would be at (.0333+.0123) s = .0456 s

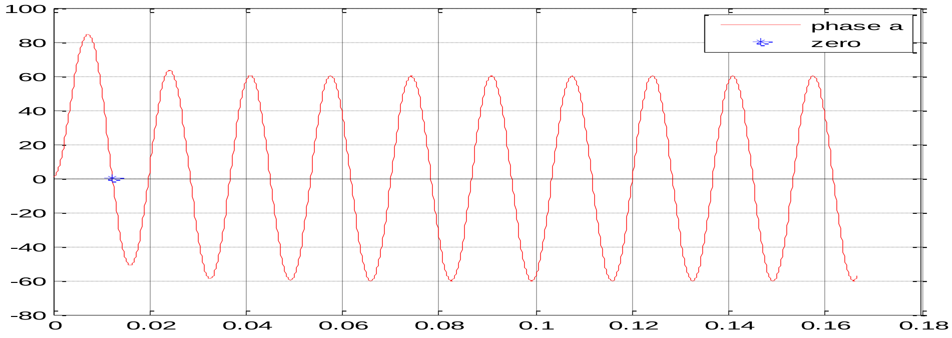

Figure 13: Phase a to ground fault zero crossing point

Now using different transition time we will see the zero crossing point. This transition time has been designated at .05 seconds.

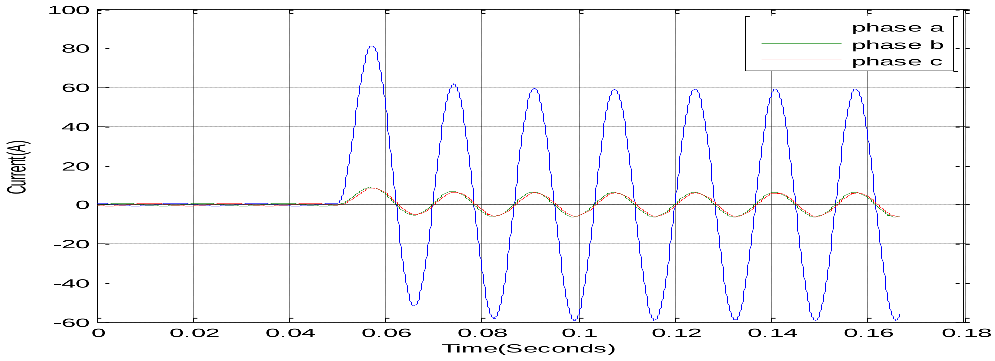

Figure 14: Three phase Ground to Fault Current Graph (AG Fault, Z_f = .01 ohm, Transition=.05s)

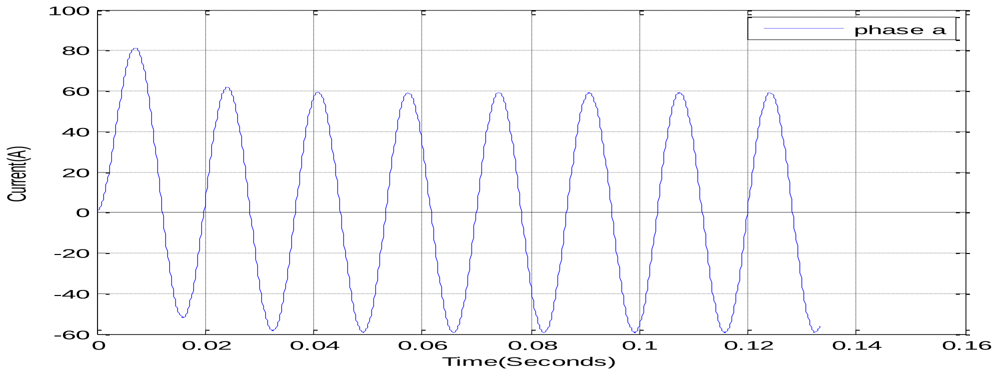

Figure 15: Phase a to Ground Fault Current Graph (Retrieved from Simulink)

Unknown parameters for Figure 15 are:

Table 4: Faulted wave function parameters (Phase a to ground)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	57.8759 V
Peak value of the 1st harmonic component (K_{21})	60.197 V
Peak value of the 3rd harmonic component(K_{23})	-0.044714 V
Phase angle of the 1st harmonic component (θ_1)	-72.1706 °
Phase angle of the 3rd harmonic component (θ_3)	-51.455 °
Time constant of the decaying d. c. component (τ)	0.0085557s

Now using these parameters, we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 16 where the zero crossing point shows at .0123s. That implies in the Simulink diagram zero crossing point would be at (.05+.0123) s=.0623s

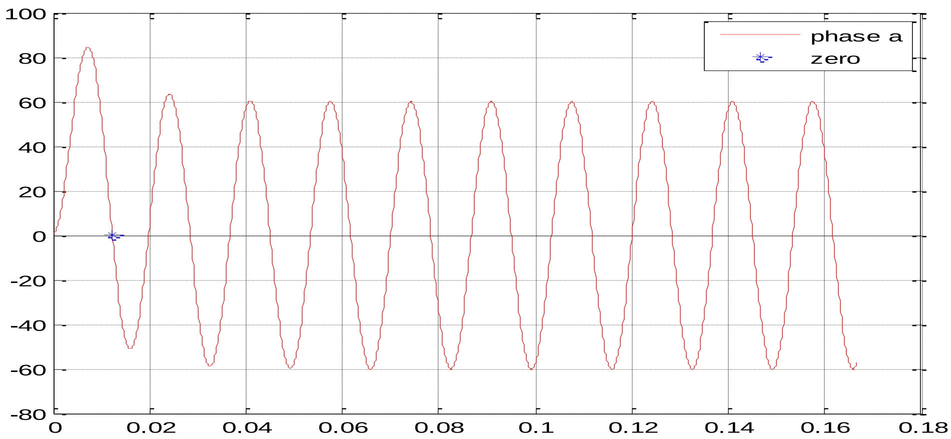

Figure 16: Phase *a* to ground fault zero crossing point

4.2.2 Phase B to Ground Fault

Alike phase a to ground, three phase voltage and current waveforms under phase b to ground fault have been depicted below in Figure 17 & Figure 18.Here the fault occurs in .0333s, which is the transition time.

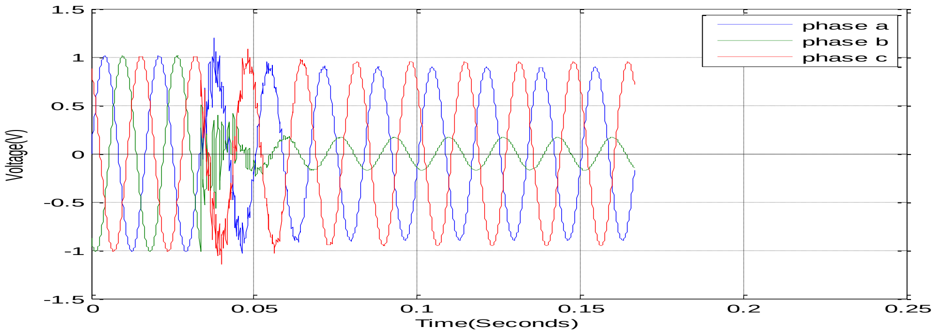

Figure 17: Three phase Ground to Fault Voltage Graph (BG Fault, $Z_f = .01$ ohm)

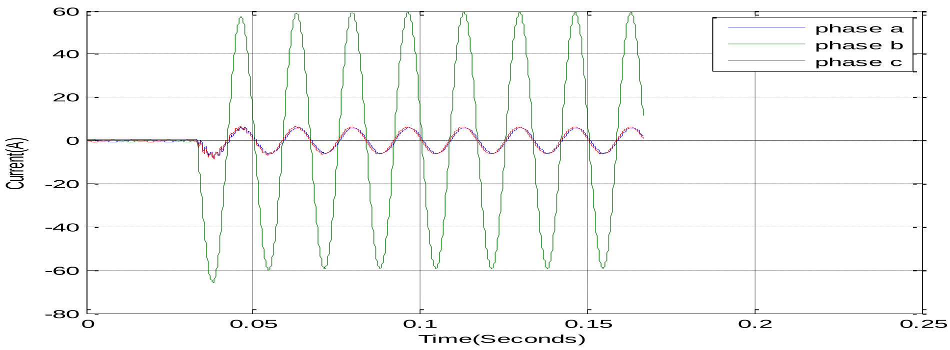

Figure 18: Three phase Ground to Fault Current Graph (BG Fault, $Z_f = .01$ ohm, Transition time=.0333s)

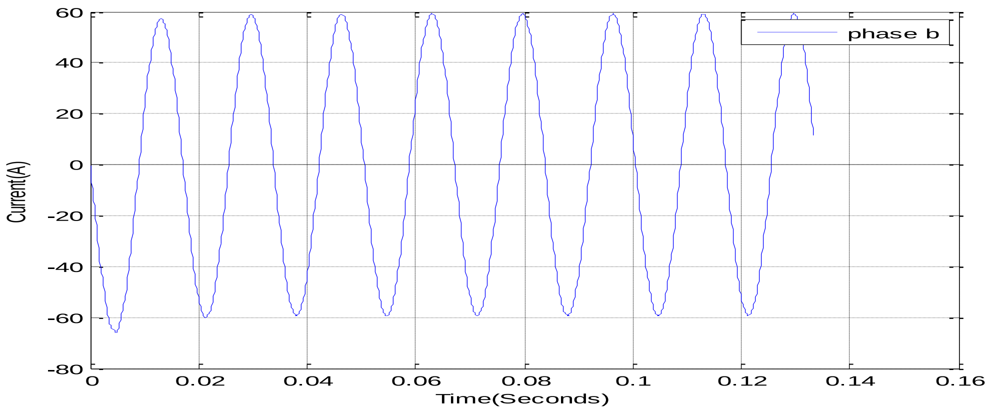

Figure 19: Phase b to Ground Fault Current Graph (Retrieved from Simulink)

Parameters from faulted wave equation, those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below after taking samples from Figure 19.

Table 5: Faulted wave function parameters (Phase b to ground)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	-13.2289 V
Peak value of the 1st harmonic component(K_{21})	-59.639 V
Peak value of the 3rd harmonic component (K_{23})	0.074298 V
Phase angle of the 1st harmonic component (θ_1)	-11.9721 º
Phase angle of the 3rd harmonic component (θ_3)	-8.3778 º
Time constant of the decaying d. c. component (τ)	0.0066779 s

By using the unknown parameters of faulted wave current those have been retrieved we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 20. This current wave symbolizes phase b to ground fault. Here the zero crossing time is .0090 s. So, the zero crossing time at the transmission line is (.0333+.0090) s = .0423 s

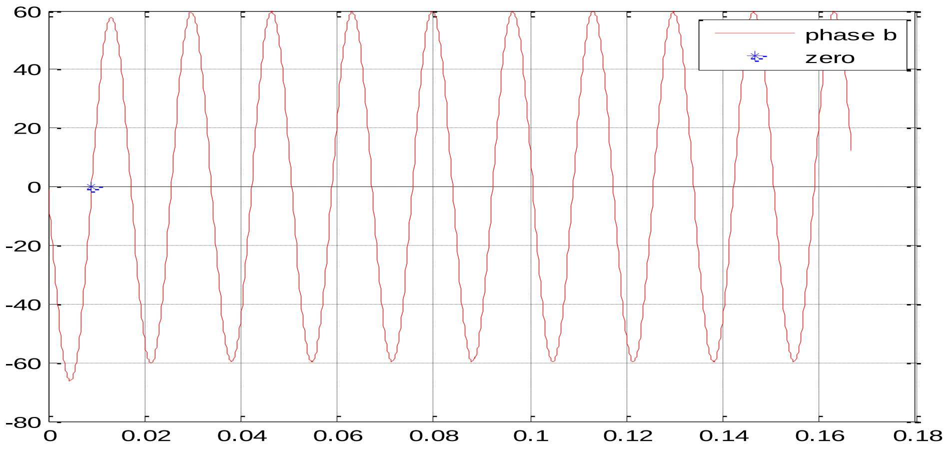

Figure 20: Phase b to ground fault zero crossing point

4.2.3 Phase C to Ground Fault

Similar to other phase to ground fault type, three phase voltage and current waveforms under phase c to ground fault have been depicted below in Figure 21 & Figure 22. Here the fault occurs in .0335s.

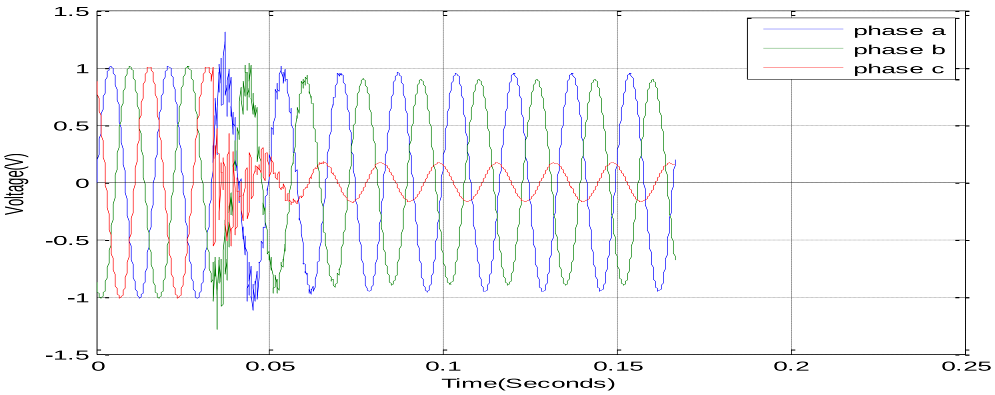

Figure 21: Three phase Ground to Fault Voltage Graph (CG Fault, $Z_f = .01$ ohm)

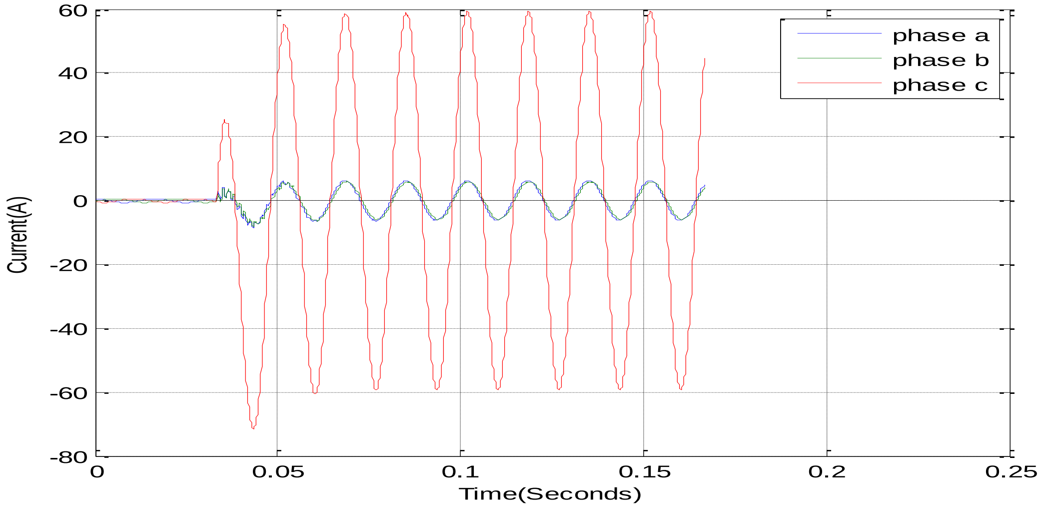

Figure 22: Three phase Ground to Fault Current Graph (CG Fault, $Z_f = .01$ ohm, Transition time=.0333s)

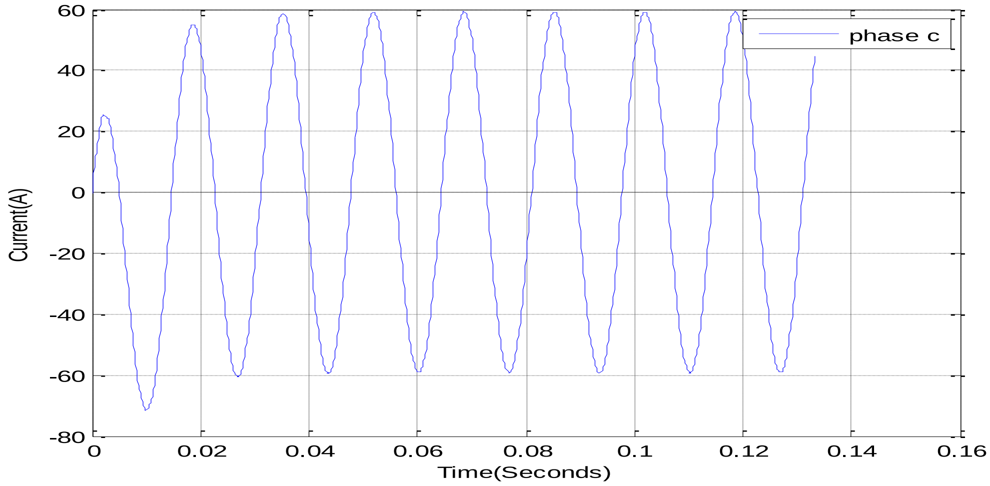

Figure 23: Phase c to Ground Fault Current Graph (Retrieved from Simulink)

Parameters from faulted wave equation, those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below after taking samples from Figure 23.

Table 6: Faulted wave function parameters (Phase c to ground)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	-44.616 V
Peak value of the 1st harmonic component (K_{21})	60.0983 V
Peak value of the 3rd harmonic component (K_{23})	-0.055446 V
Phase angle of the 1st harmonic component (θ_1)	48.3847 ^{o}
Phase angle of the 3rd harmonic component (θ_3)	27.5579 ^{o}
Time constant of the decaying d. c. component (τ)	0.0093423 s

By using the unknown parameters of faulted wave current those have been retrieved we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 24. This current wave symbolizes phase c to ground fault. Here, the zero crossing point is at .0049 s. So, at the transmission line zero crossing point would be at (.0333+.0049) s =.0382 s.

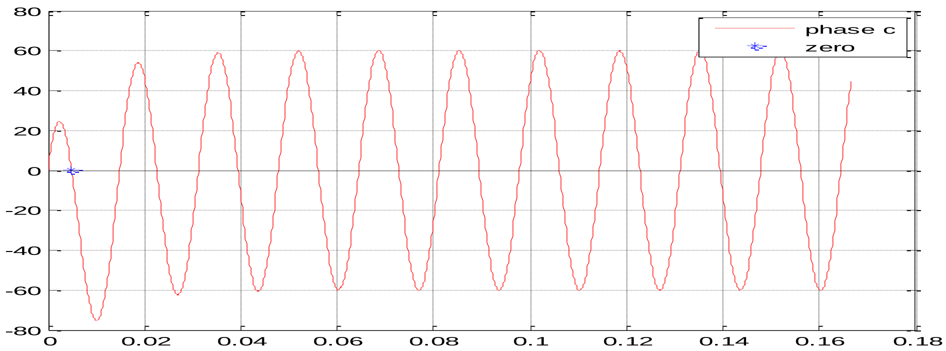

Figure 24: Phase c to ground fault zero crossing point

4.2.4 Phase A to Phase B Fault

Using phase a to phase b fault, derived three phase voltage and current curve has been portrayed in Figure 25 & Figure 26, respectively. Here, from the simulation we got that the fault occurred at .0336 second. The curves also represent that the fault time is near .03 second.

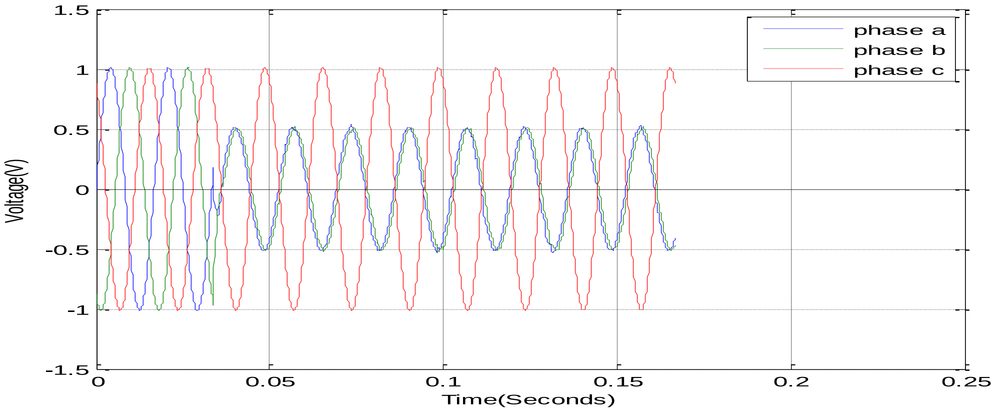

Figure 25: Three phase a to phase b fault Voltage Graph (AB Fault, Z_f = .01 ohm)

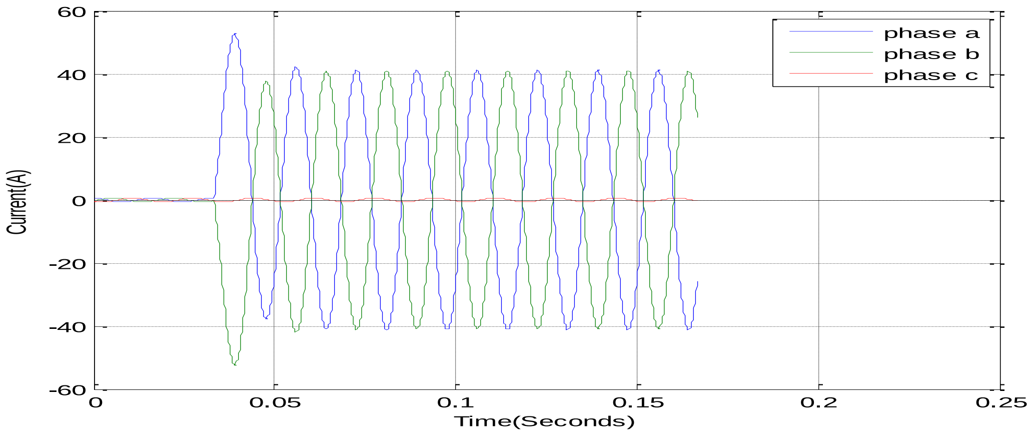

Figure 26: Three phase a to phase b fault current Graph (AB Fault, Z_f = .01 ohm, Transition time=.0333s)

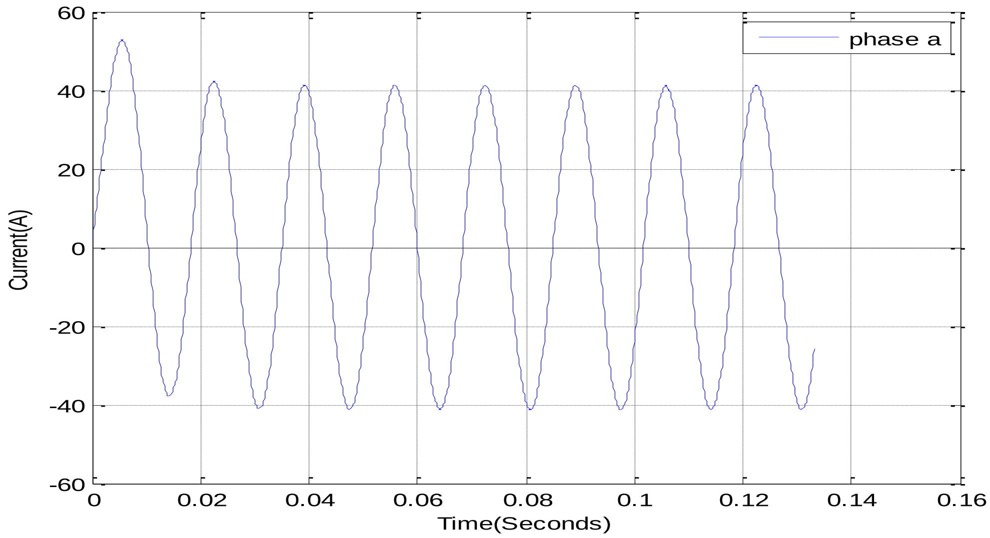

Figure 27: Phase a to phase b type fault (Phase a, retrieved from simulink)

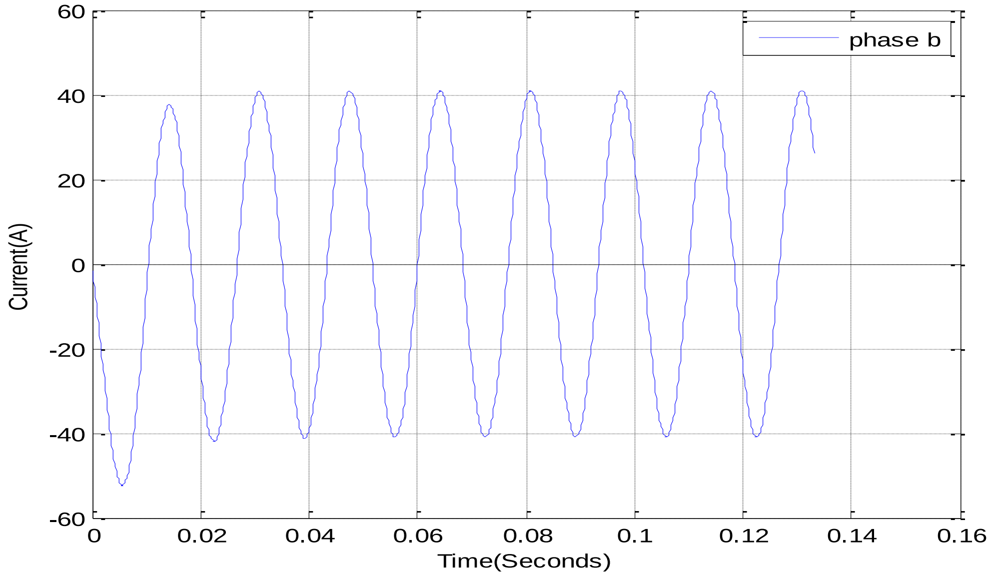

Figure 28: Phase a to phase b type fault (Phase b, retrieved from simulink)

Parameters from faulted wave equation of phase a (Figure 27) and phase b (Figure 28), those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below:

Table 7: Faulted wave function parameters (Phase a)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	26.794 V
Peak value of the 1st harmonic component(K_{21})	41.2787 V
Peak value of the 3rd harmonic component(K_{23})	0.029954 V
Phase angle of the 1st harmonic component(θ_1)	-37.1705 ^{o}
Phase angle of the 3rd harmonic component(θ_3)	84.292 ^{o}
Time constant of the decaying d. c. component (τ)	0.0078781 s

Table 8: Faulted wave function parameters (Phase b)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	-26.7939 V
Peak value of the 1st harmonic component(K_{21})	-41.0677 V
Peak value of the 3rd harmonic component(K_{23})	-0.029964 V
Phase angle of the 1st harmonic component(θ_1)	-38.0023 ^{o}
Phase angle of the 3rd harmonic component(θ_3)	84.2214 ^{o}
Time constant of the decaying d. c. component (τ)	0.0078782 s

By using the unknown parameters of faulted wave current of phase a, those have been retrieved we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 29. This current wave symbolizes phase a current in phase a to phase b type fault.

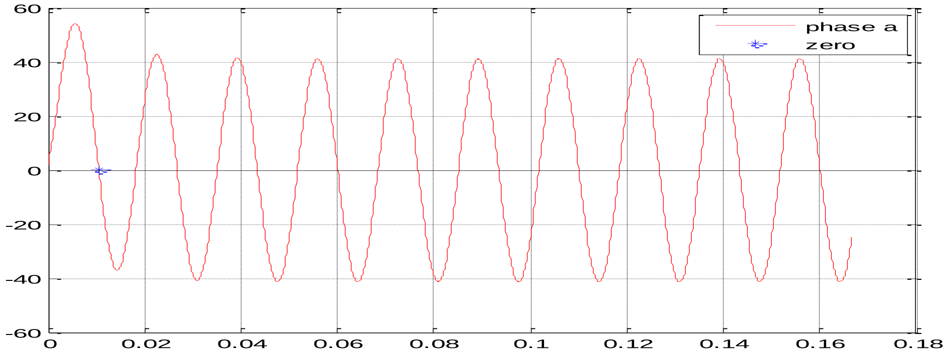

Figure 29: Phase a to Phase b fault zero crossing point (Phase a)

By using the unknown parameters of faulted wave current of phase b, those have been retrieved we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 30. This current wave symbolizes phase b current in phase a to phase b type fault.

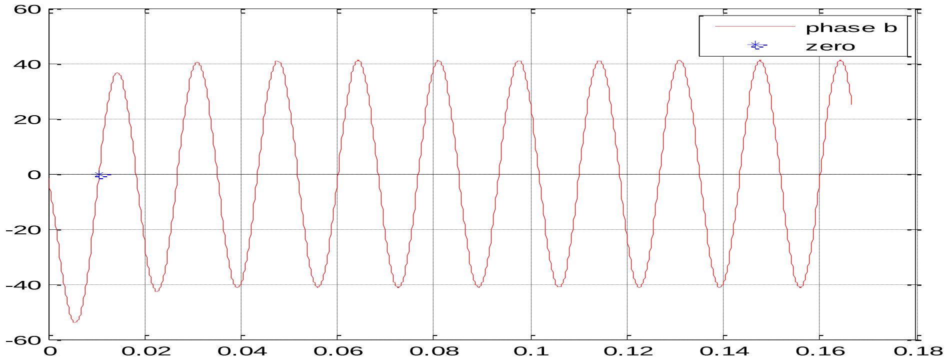

Figure 30: Phase a to Phase b fault zero crossing point (Phase b)

From the theory of phase a to phase b type fault we know, $I_a = -I_b$ which can be clearly observed in Figure 29 & Figure 30. So, it is clearly verified that the method worked well properly. We also found that both have the same zero crossing point after the fault occurrence. It's .0105 second. Now, It can be clearly mentioned that the zero crossing point at the transmission line that has been used in the simulation is (.0333+.0105) s =.0438 s. The joint graph of phase a and phase b under the phase a to phase b type fault, will make the observation much more clear which has been presented below:

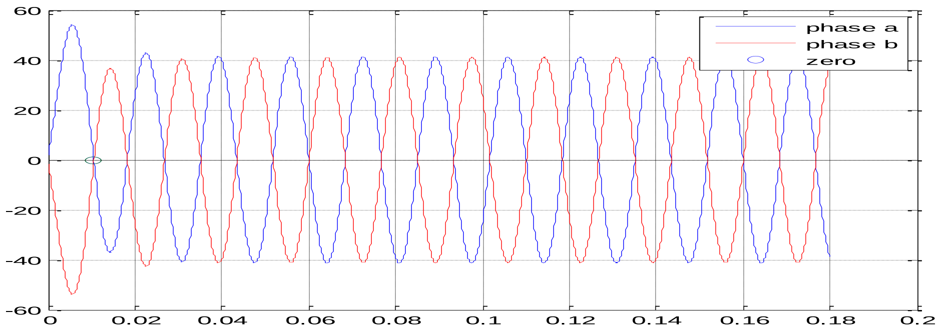

Figure 31: Phase a to Phase b fault zero crossing point (Phase a and phase b)

4.2.5 Phase B to Phase C Fault

Similar to phase a to phase b, derived three phase voltage and current curve has been portrayed in Figure 31 & Figure 32, respectively. Here, from the simulation we got that the fault occurred at .0333 second. The curves also represent that the fault time is near .03 second.

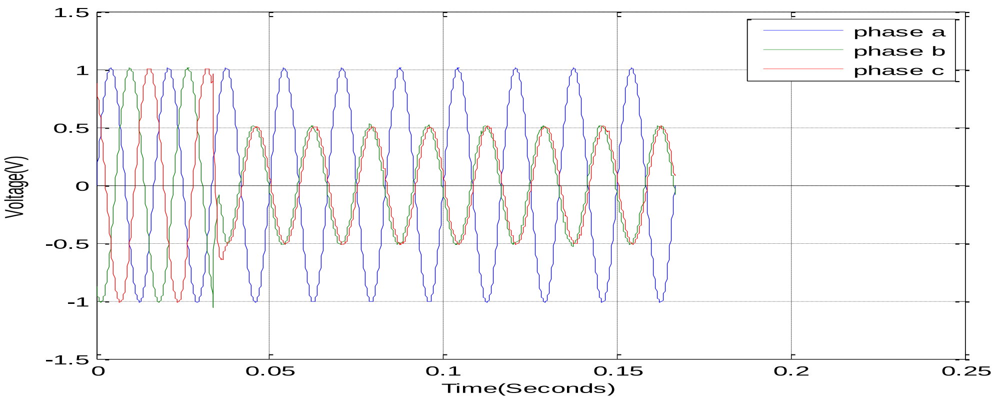

Figure 32: Three phase b to phase c fault Voltage Graph (BC Fault, Z_f = .01 ohm)

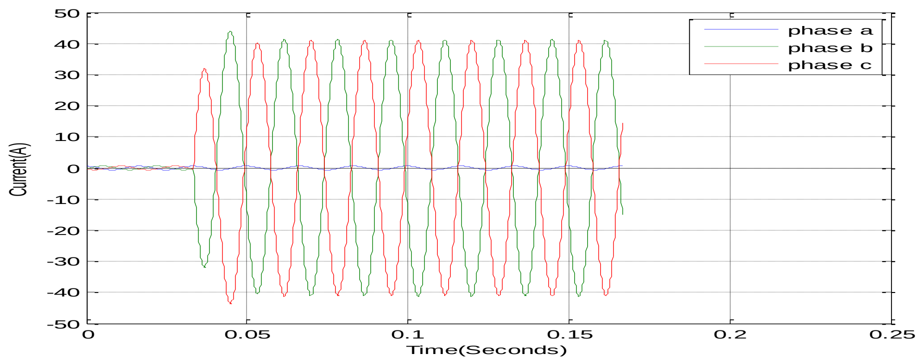

Figure 33: Three phase b to phase c fault current Graph (BC Fault, Z_f = .01 ohm)

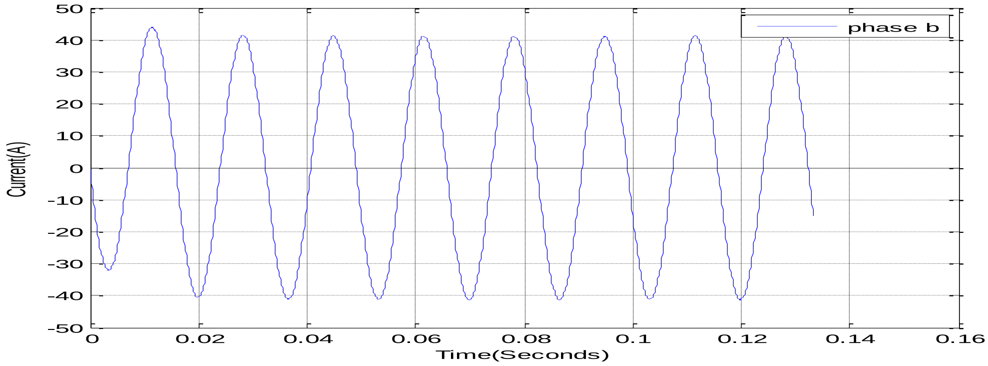

Figure 34: Phase b to phase c type fault (Phase b, retrieved from simulink)

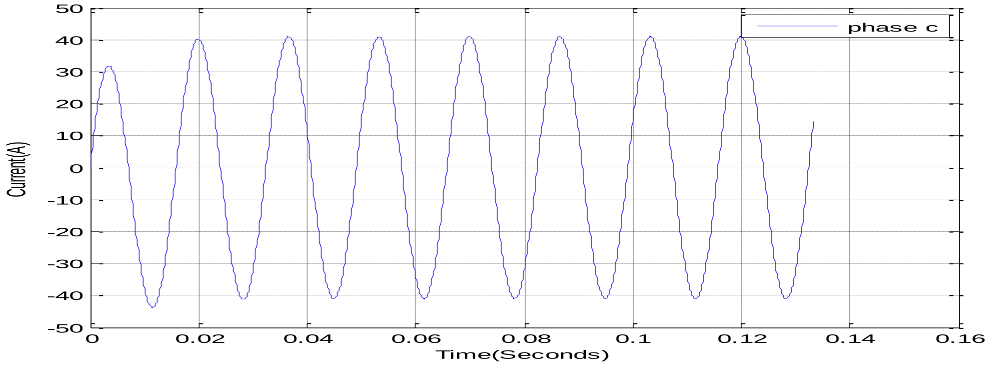

Figure 35: Phase b to phase c type fault (Phase c, retrieved from simulink)

Parameters from faulted wave equation of phase b (Figure 34) and phase c (Figure 35), those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below in Table 9 & Table 10:

Table 9: Faulted wave function parameters (Phase b)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	14.814 V
Peak value of the 1st harmonic component(K_{21})	-41.4267 V
Peak value of the 3rd harmonic component(K_{23})	-0.066072 V
Phase angle of the 1st harmonic component(θ_1)	21.9242 ^{o}
Phase angle of the 3rd harmonic component(θ_3)	12.5931 ^{o}
Time constant of the decaying d. c. component (τ)	0.0075692 s

Table 10: Faulted wave function parameters (Phase c)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	-14.8146 V
Peak value of the 1st harmonic component(K_{21})	41.196 V
Peak value of the 3rd harmonic component(K_{23})	0.066105 V
Phase angle of the 1st harmonic component (θ_1)	21.1058 ^{o}
Phase angle of the 3rd harmonic component(θ_3)	12.5776 ^{o}
Time constant of the decaying d. c. component (τ)	0.0075686 s

By using the unknown parameters of faulted wave current of phase b and phase c, those have been retrieved we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 36 & Figure 37. This current wave symbolizes phase b and phase c current under phase b to phase c type fault.

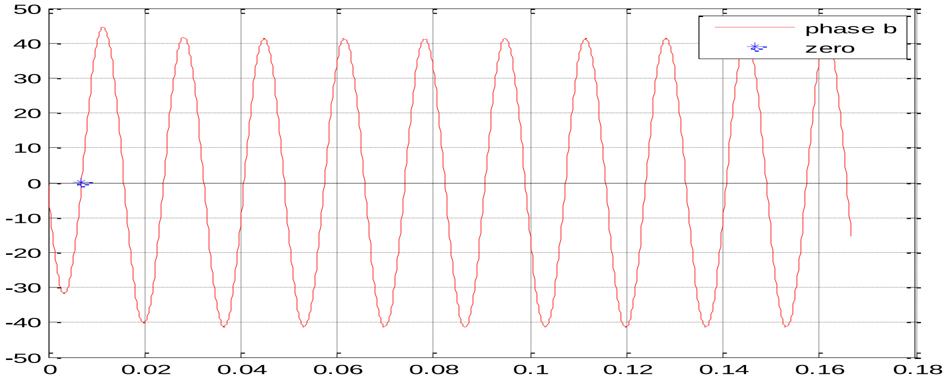

Figure 36: Phase b to Phase c fault zero crossing point (Phase b)

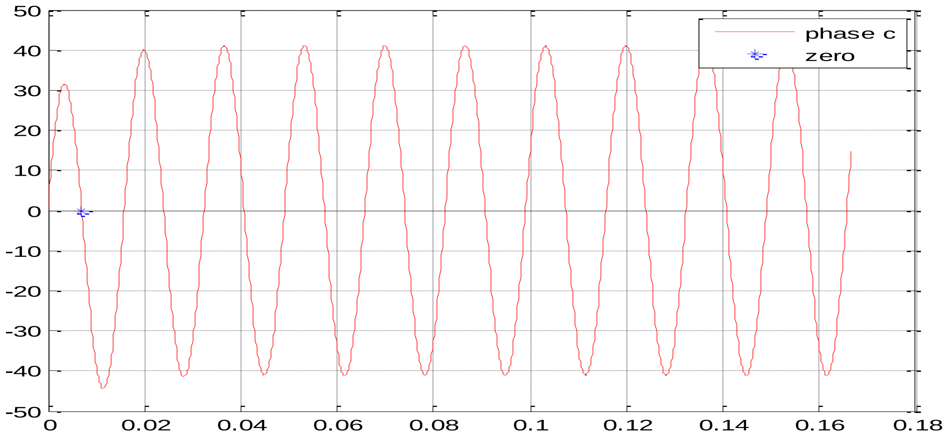

Figure 37: Phase b to Phase c fault zero crossing point (Phase c)

From the theory of phase b to phase c type fault we know, $I_b = -I_c$ which can be clearly observed in Figure 36 & Figure 37. So, it is clearly verified that the method worked properly. We found that both have the same zero crossing point after the fault occurrence. It's .0070 second. So, the transmission line zero crossing time after the fault would be (.0333+.0070) s = .0403 s. The joint graph of phase b and phase c under the phase b to phase c type fault, will make the observation much more clear which has been presented below:

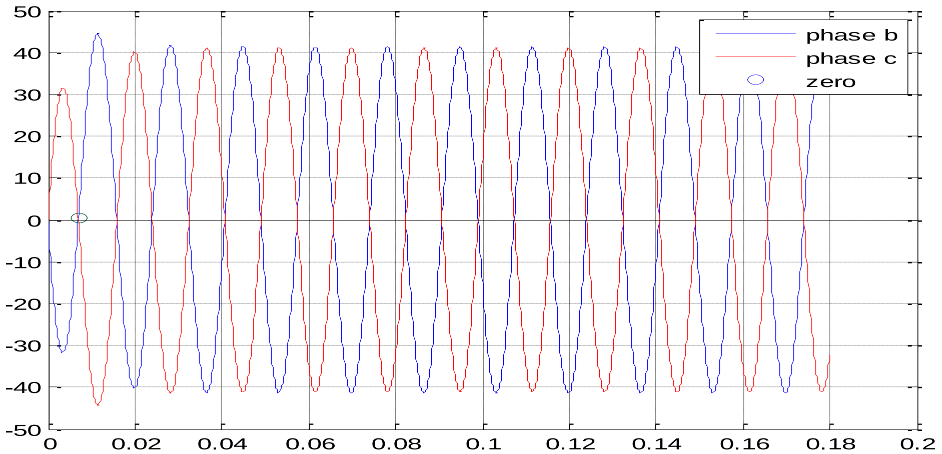

Figure 38: Phase b to Phase c fault zero crossing point (Phase b and phase c)

4.2.6 Phase A to Phase C Fault

Similar to other phase to phase fault type, derived three phase voltage and current curve has been portrayed in Figure 39 & Figure 40 respectively under phase a to phase c type fault. Here, from the simulation we got that the fault occurred at .0333 second. The curves also represent that the fault time is near .03 second as other curves.

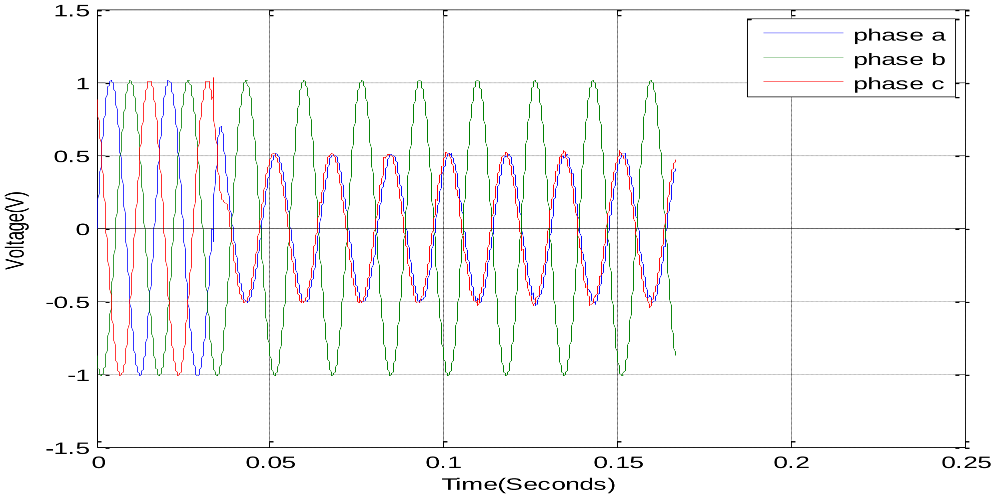

Figure 39: Three phase a to phase c fault Voltage Graph (AC Fault, Z_f = .01 ohm)

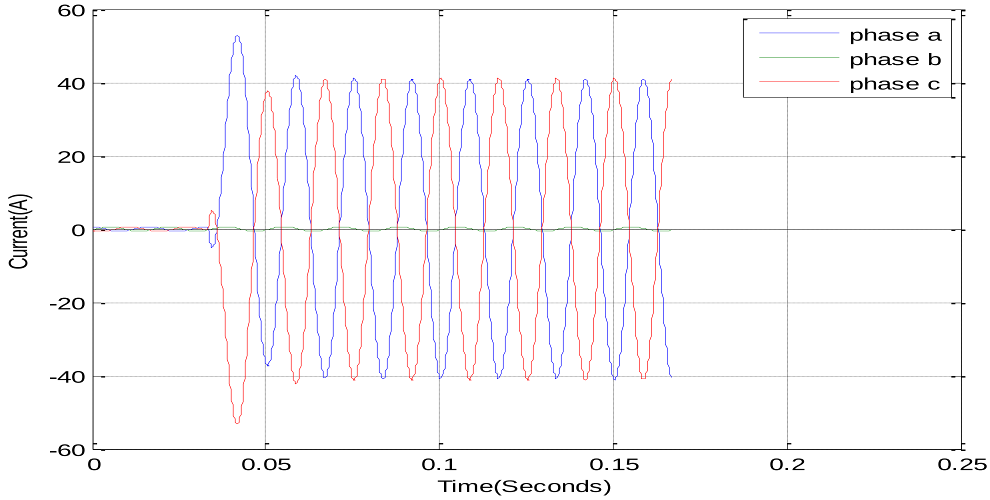

Figure 40: Three phase a to phase c fault Current Graph (AC Fault, Z_f = .01 ohm)

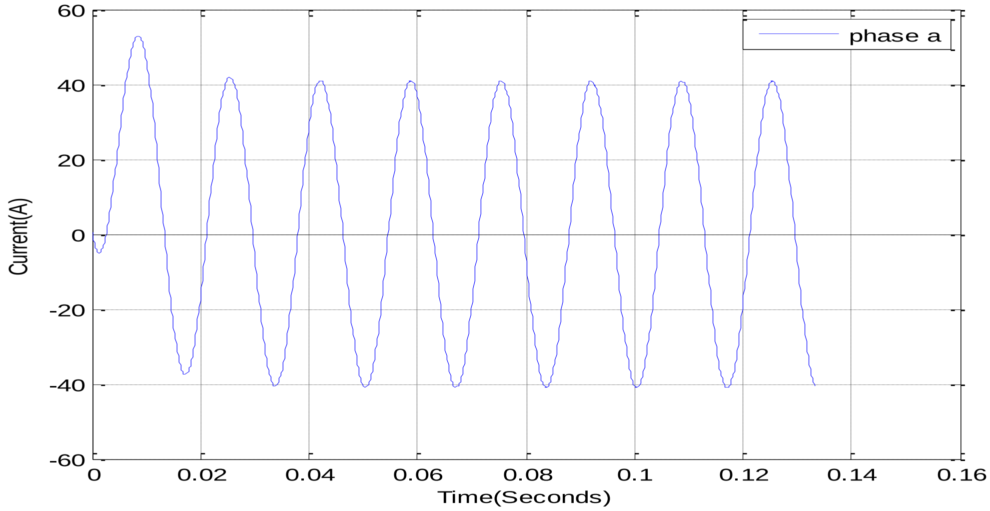

Figure 41: Phase a to phase c type fault (Phase a, retrieved from simulink)

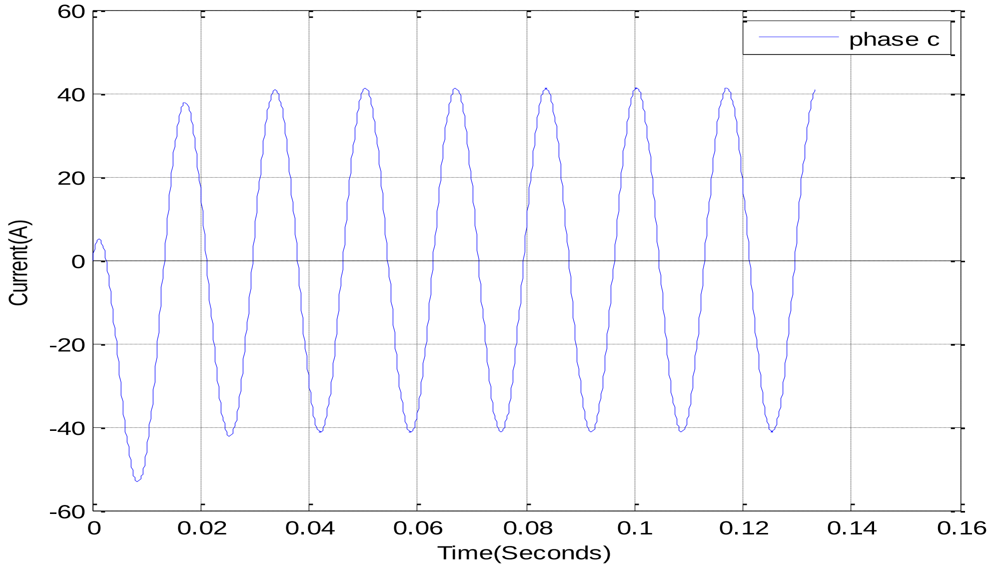

Figure 42: Phase a to phase c type fault (Phase c, retrieved from simulink)

Parameters from faulted wave equation of phase a (Figure 41) and phase c (Figure 42), those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below in Table 11 and Table 12.

Table 11: Faulted wave function parameters (Phase a)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	42.2008 V
Peak value of the 1st harmonic component(K_{21})	-42.4196 V
Peak value of the 3rd harmonic component (K_{23})	-0.072562 V
Phase angle of the 1st harmonic component (θ_1)	80.082 ^{o}
Phase angle of the 3rd harmonic component (θ_3)	-18.3045 ^{o}
Time constant of the decaying d. c. component (τ)	0.0077117 s

Table 12: Faulted wave function parameters (Phase c)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	-42.2016 V
Peak value of the 1st harmonic component (K_{21})	42.6402 V
Peak value of the 3rd harmonic component (K_{23})	0.072608 V
Phase angle of the 1st harmonic component (θ_1)	80.8835 ^{o}
Phase angle of the 3rd harmonic component (θ_3)	-18.2589^o
Time constant of the decaying d. c. component (τ)	0.0077112 s

By using the unknown parameters of faulted wave current of phase a and phase c, those have been retrieved, we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 43 & Figure 44. This current wave symbolizes phase a and phase c current under phase a to phase c type fault.

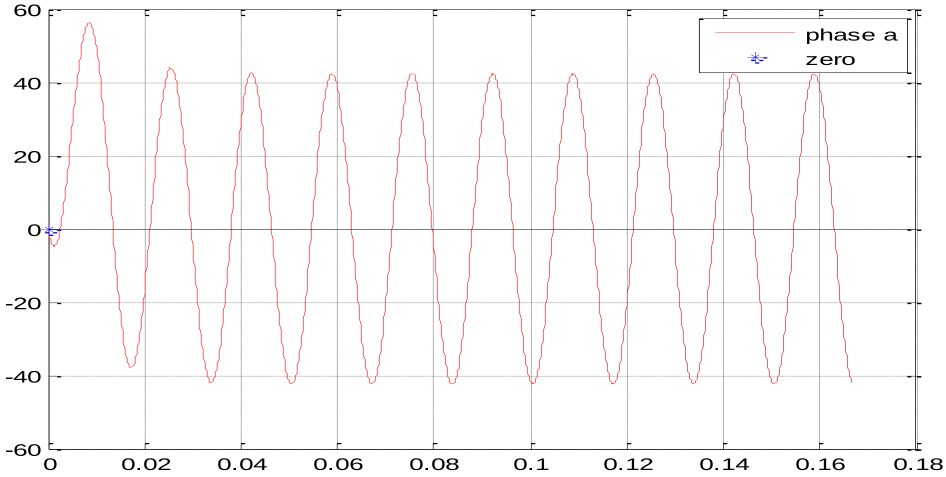

Figure 43: Phase a to Phase c fault zero crossing point (Phase a)

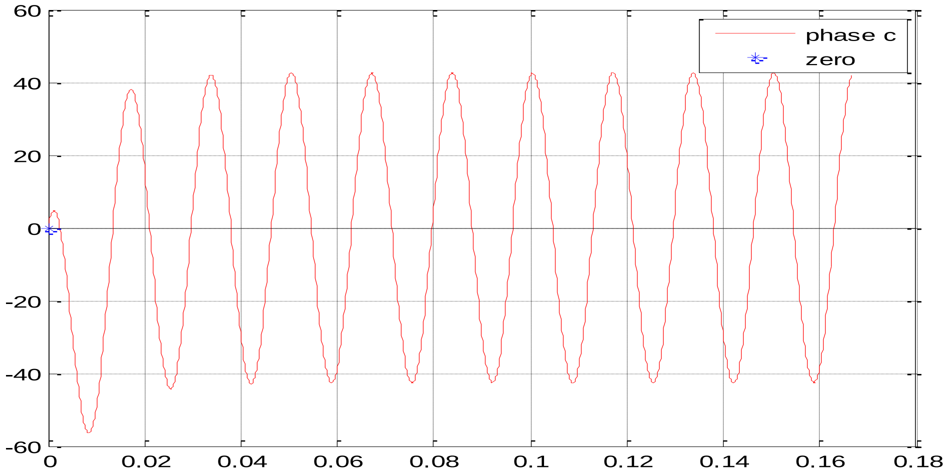

Figure 44: Phase a to Phase c fault zero crossing point (Phase c)

From the theory of phase a to phase c type fault we know, $I_a = -I_c$ which can be clearly observed in Figure 43 and Figure 44. So, it is clearly verified that the method worked well properly like other phase to phase fault type. Here, we found that both have very close zero crossing point after the fault occurrence. It's 5.3890×10^{-05} second for phase a and 1.5267×10^{-05} second for phase c. The joint graph of phase a and phase c under the phase a to phase c type fault, will make the observation much more clear which has been presented below:

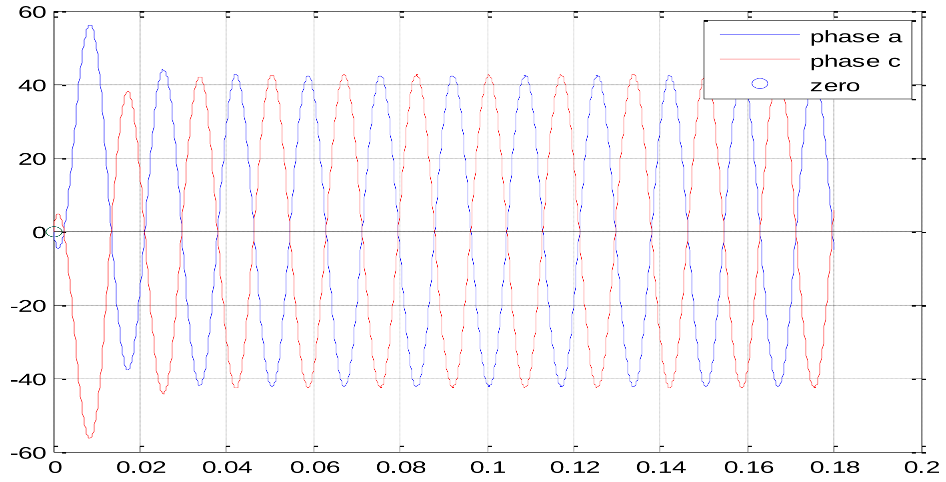

Figure 45: Phase b to Phase c fault zero crossing point (Phase b and phase c)

4.2.7 Phase A to Phase B to Ground Fault

Here, phase to phase to ground type faults and their zero crossing will be checked. Phase a to phase b to ground is taken into account in this section. As other fault types, in the simulation, we have set the fault time at .0333 s. Phase voltage and current has been depicted in the following diagrams which shows that, the fault occurred near .03 second.

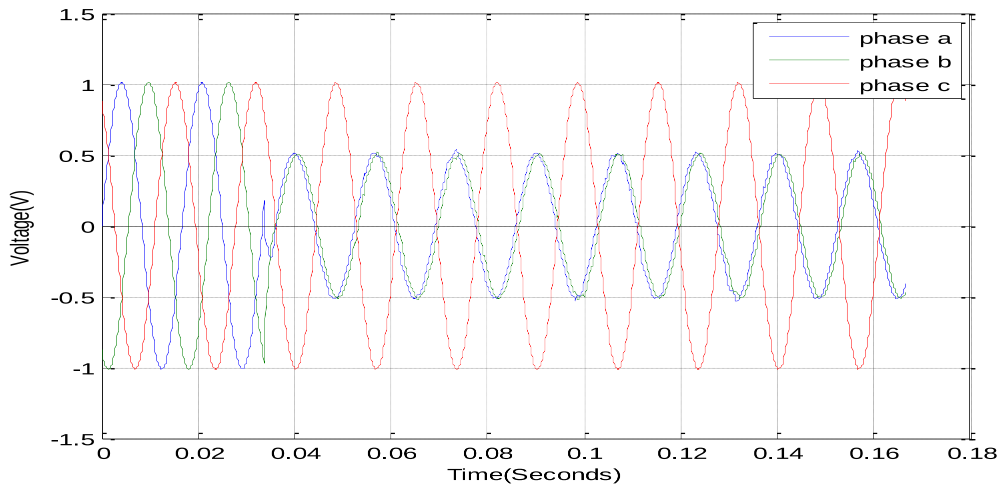

Figure 46: Three phase a to phase b to ground fault Voltage Graph (AC Fault, $Z_f = .01$ ohm)

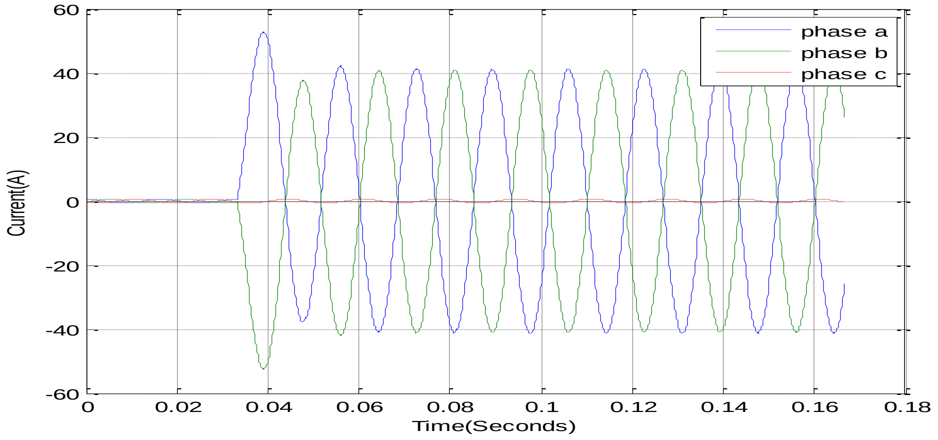

Figure 47: Three phase a to phase b to ground fault Current Graph (AC Fault, $Z_f = .01$ ohm)

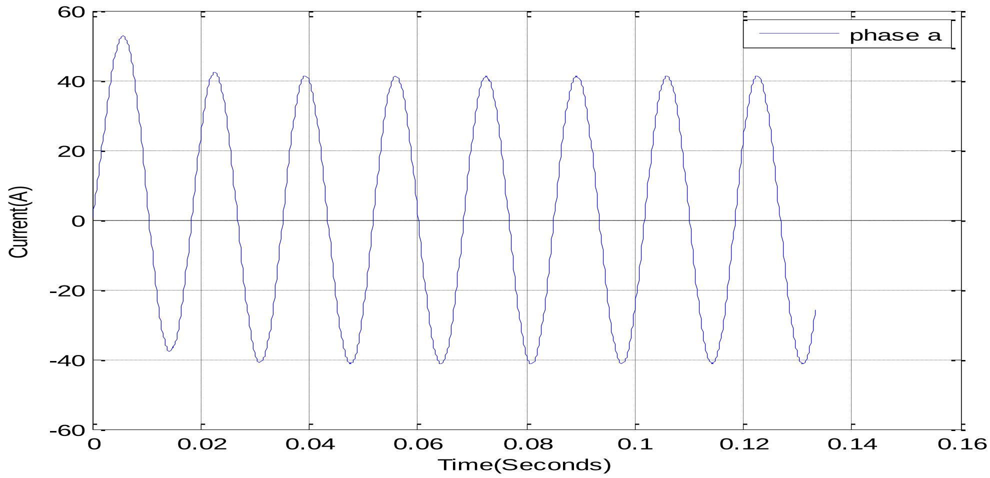

Figure 48: Phase a to phase b to ground type fault (Phase a, retrieved from simulink)

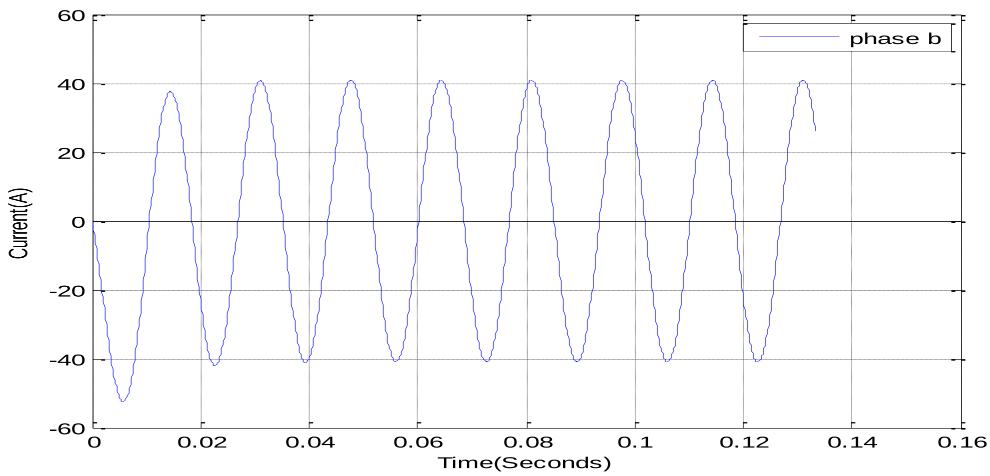

Figure 49: Phase a to phase b to ground type fault (Phase b, retrieved from simulink)

Parameters from faulted wave equation of phase a (Figure 48) and phase b (Figure 49), those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below:

Table 13: Faulted wave function parameters (Phase a)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	27.1777 V
Peak value of the 1st harmonic component (K_{21})	41.243 V
Peak value of the 3rd harmonic component (K_{23})	0.030824 V
Phase angle of the 1st harmonic component (θ_1)	-39.9608 ^{o}
Phase angle of the 3rd harmonic component (θ_3)	68.9803^o
Time constant of the decaying d. c. component (τ)	0.0079793 s

Table 14: Faulted wave function parameters (Phase b)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	-27.1764 V
Peak value of the 1st harmonic component (K_{21})	-41.0311 V
Peak value of the 3rd harmonic component (K_{23})	-0.03087 V
Phase angle of the 1st harmonic component (θ_1)	-40.7928^o
Phase angle of the 3rd harmonic component (θ_3)	68.807^o
Time constant of the decaying d. c. component (τ)	0.0079803 s

By using the unknown parameters of faulted wave current of phase a and phase b, those have been retrieved, we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 50 and Figure 51. This current wave symbolizes phase a and phase b current under phase a to phase b to ground type fault. We found that both have the same zero crossing point after the fault occurrence. It's .0106 second. So, the transmission line zero crossing time after the fault would be (.0333+.0106) s = .0439 s.

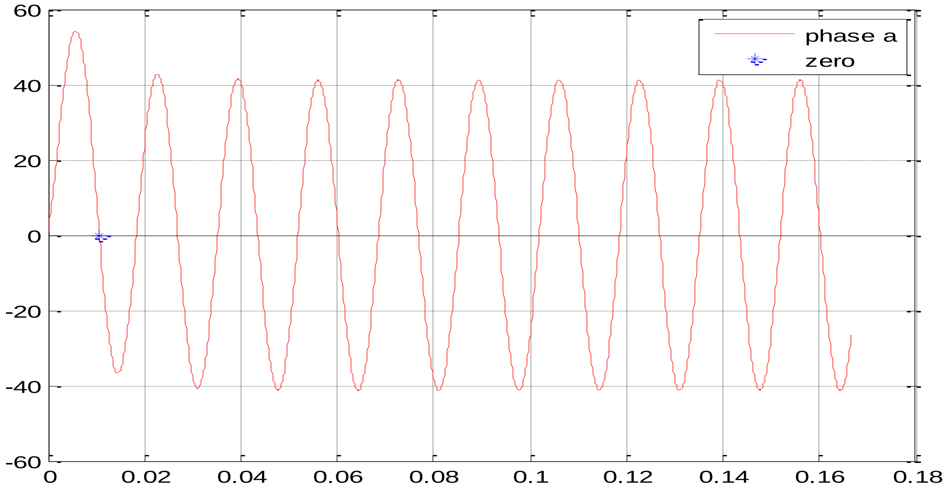

Figure 50: Phase a to Phase b to ground fault zero crossing point (Phase a)

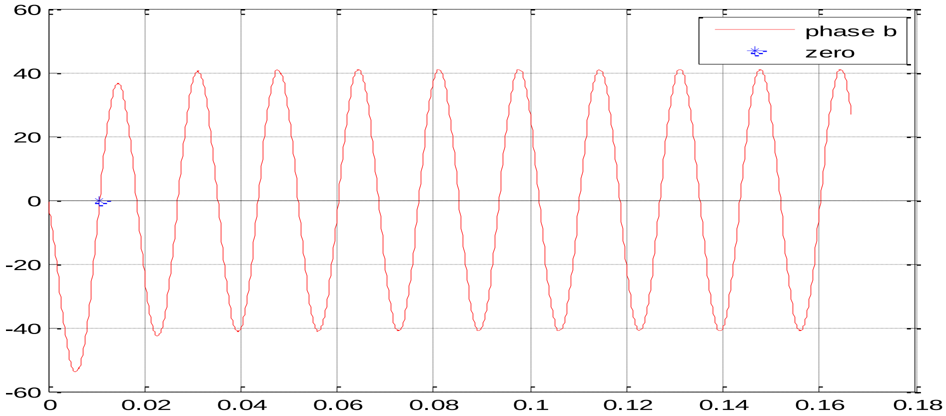

Figure 51: Phase a to Phase b to ground fault zero crossing point (Phase b)

If we join the zero crossing of phase a to phase b to ground, Figure 52 will represent that.

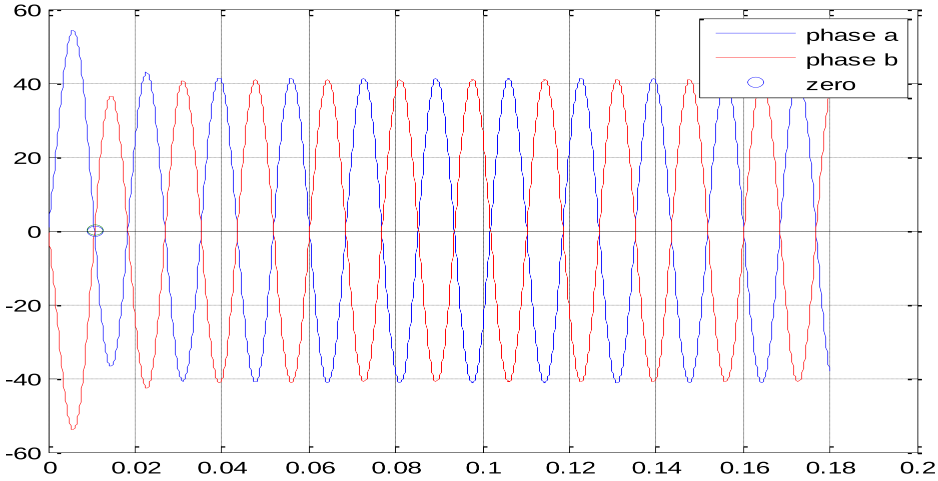

Figure 52: Phase a to Phase b to ground fault zero crossing point (Phase a and phase b)

4.2.8 Phase B to Phase C to Ground Fault

Similar to phase a to phase b to ground fault type, we have done simulation for phase b to phase c to ground fault. As other fault types, in the simulation, we have set the fault time at .0333 s. Phase voltage and current has been depicted in the following diagrams which shows that, the fault occurred near .03 second.

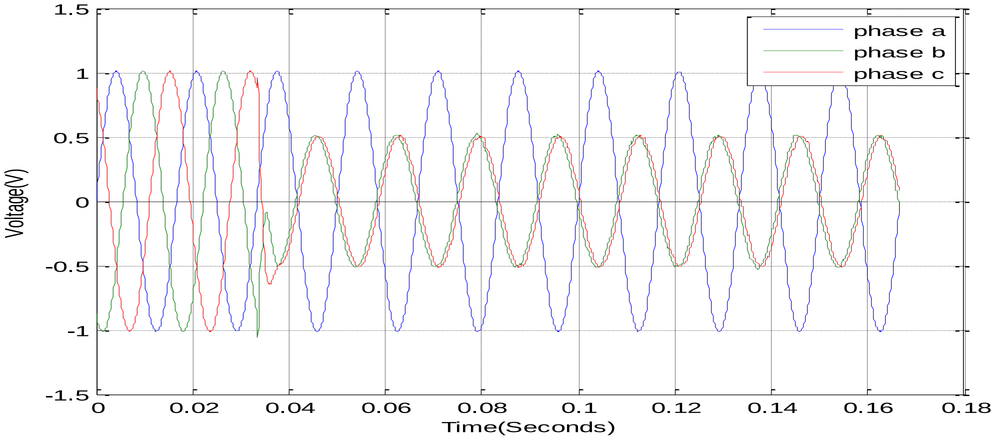

Figure 53: Three phase b to phase c to ground fault Voltage Graph (AC Fault, $Z_f = .01$ ohm)

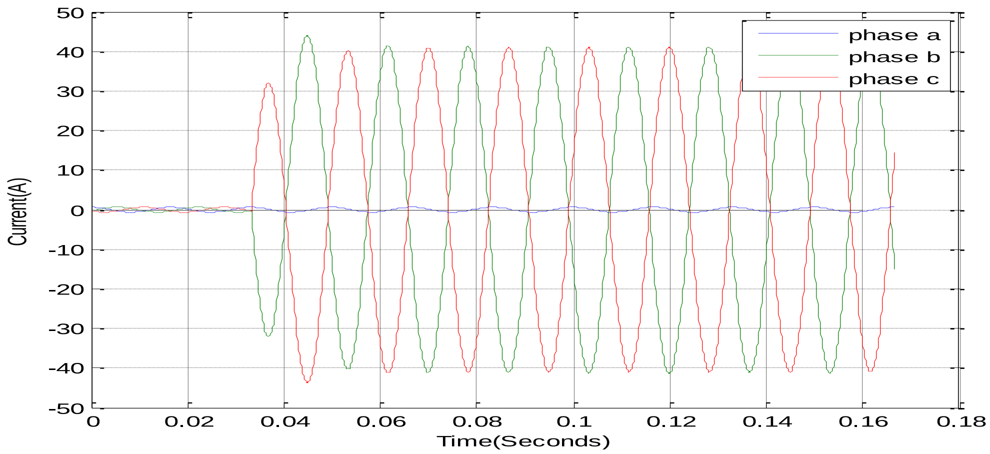

Figure 54: Three phase b to phase c to ground fault Current Graph (AC Fault, $Z_f = .01$ ohm)

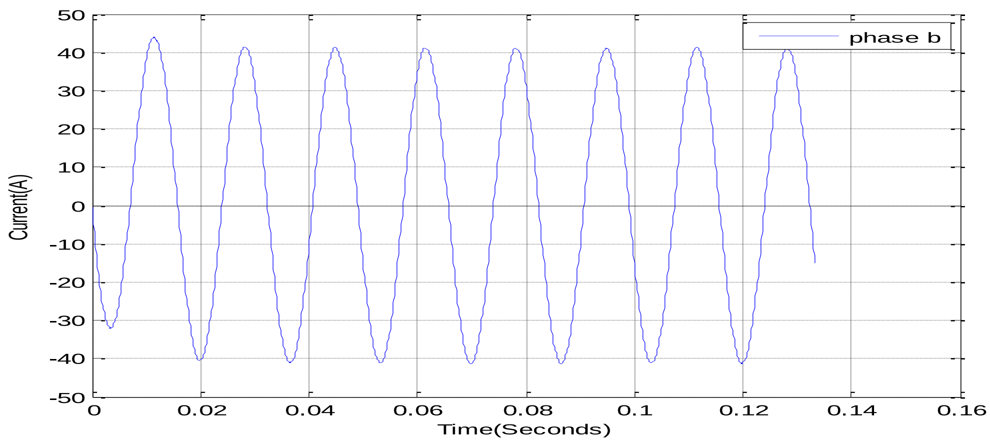

Figure 55: Phase b to phase c to ground type fault (Phase b, retrieved from simulink)

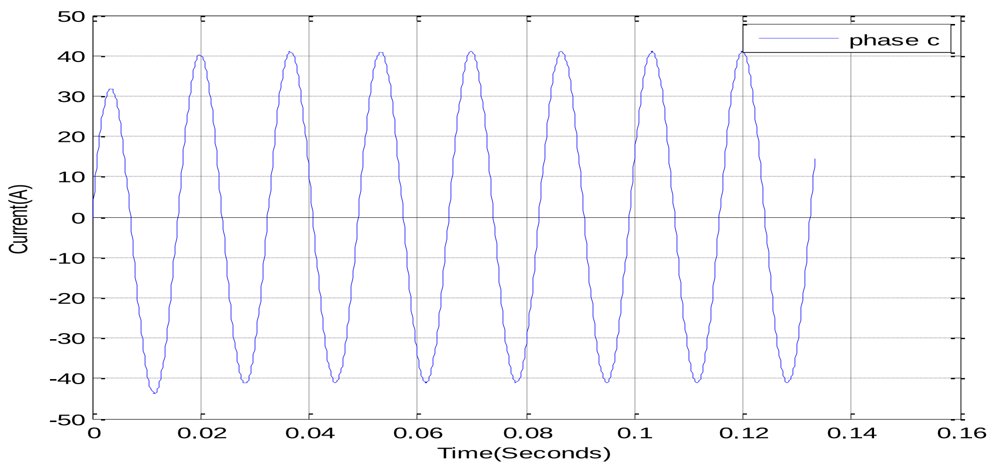

Figure 56: Phase b to phase c to ground type fault (Phase c, retrieved from simulink)

Parameters from faulted wave equation of phase b (Figure 55) and phase c (Figure 56), those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below in Table 15 & Table 16:

Table 15: Faulted wave function parameters (Phase b)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	14.814 V
Peak value of the 1st harmonic component (K_{21})	-41.4267 V
Peak value of the 3rd harmonic component (K_{23})	-0.066072 V
Phase angle of the 1st harmonic component (θ_1)	21.9242^o
Phase angle of the 3rd harmonic component (θ_3)	12.5931^o
Time constant of the decaying d. c. component (τ)	0.0075692 s

Table 16: Faulted wave function parameters (Phase c)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	-14.8146 V
Peak value of the 1st harmonic component (K_{21})	41.196 V
Peak value of the 3rd harmonic component (K_{23})	0.066105 V
Phase angle of the 1st harmonic component (θ_1)	21.1058^o
Phase angle of the 3rd harmonic component (θ_3)	12.5776^o
Time constant of the decaying d. c. component (τ)	0.0075686s

By using the unknown parameters of faulted wave current of phase b and phase c, those have been retrieved; we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 57 & Figure 58. This current wave symbolizes phase b and phase c current under phase b to phase c to ground type fault. We found that both have the same zero crossing point after the fault occurrence. It's .0070 second. So, the transmission line zero crossing time after the fault would be (.0333+.0070) s = .0403 s.

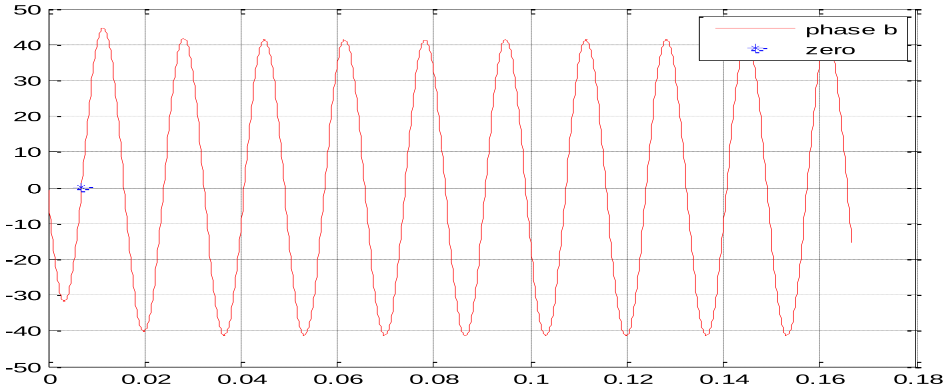

Figure 57: Phase b to Phase c to ground fault zero crossing point (Phase b)

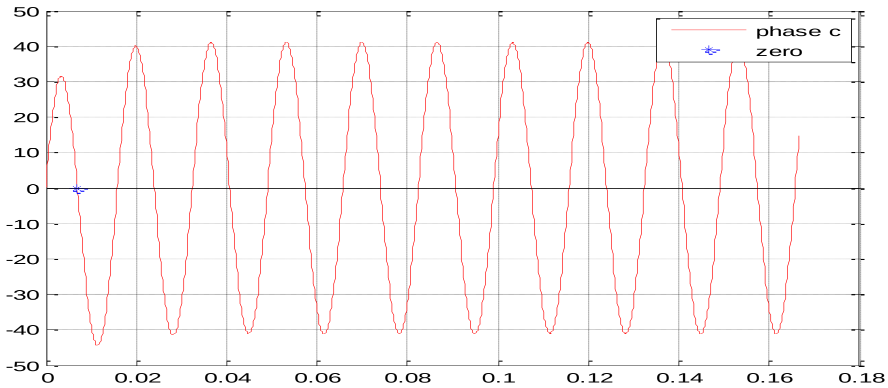

Figure 58: Phase b to Phase c to ground fault zero crossing point (Phase c)

If we join the zero crossing of phase b to phase c to ground, Figure 59 will represent that.

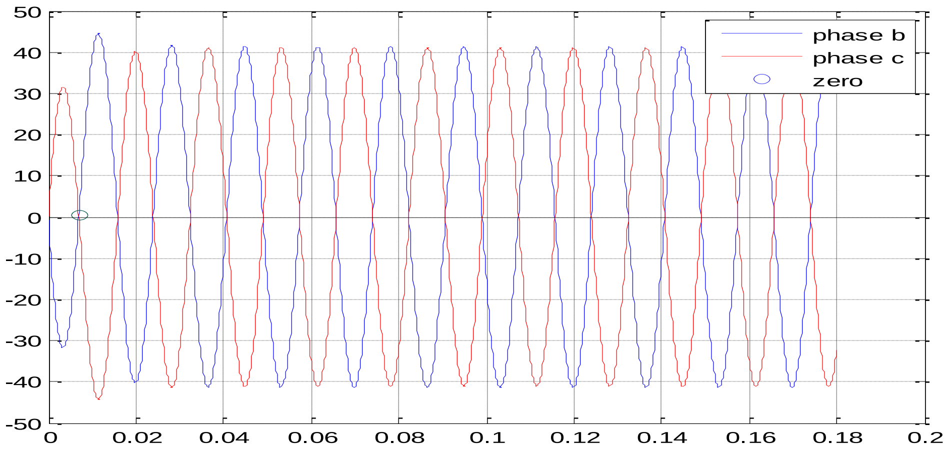

Figure 59: Phase b to Phase c to ground fault zero crossing point (Phase b and phase c)

4.2.9 Phase A to Phase C to Ground Fault

Similar phase to phase to ground fault type, we have done simulation for phase a to phase c to ground fault. As other fault types, in the simulation, we have set the fault time at .0333 s. Phase voltage and current has been depicted in the following diagrams which shows that, the fault occurred near .03 second.

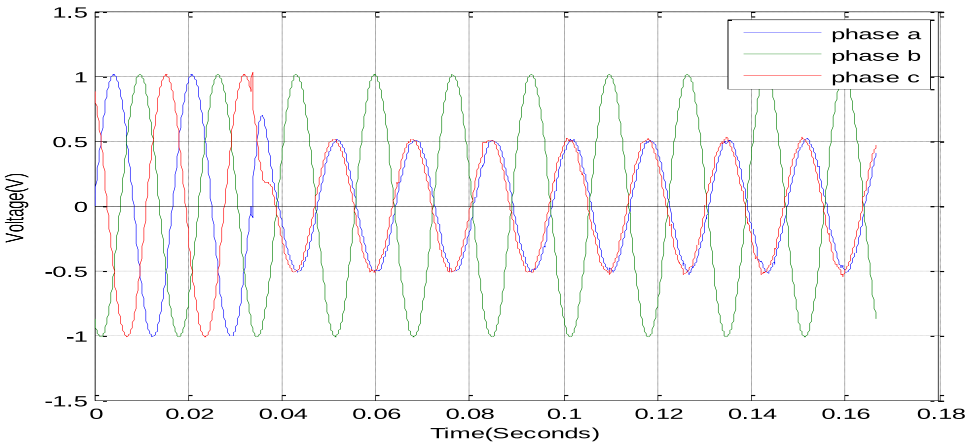

Figure 60: Three phase b to phase c to ground fault Voltage Graph (ACG Fault, Z_f = .01 ohm)

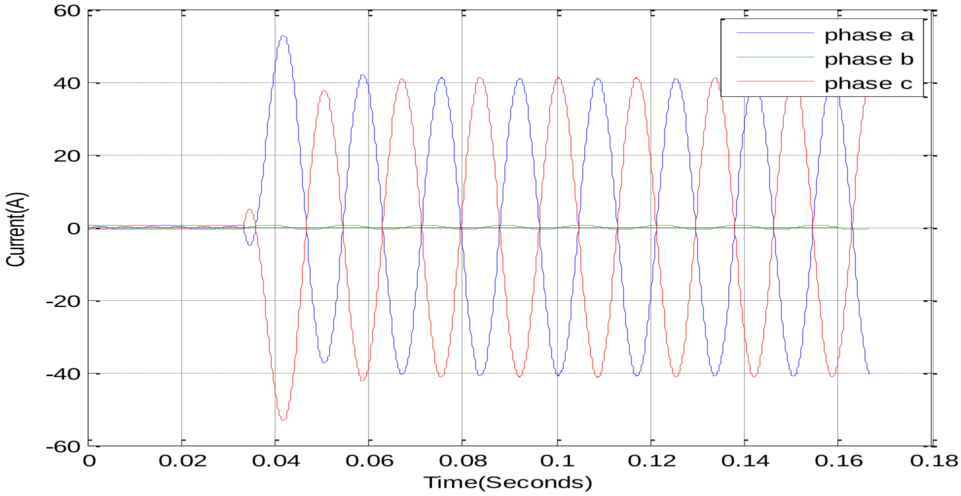

Figure 61: Three Phase a to phase c to ground fault Current Graph (ACG Fault, Z_f = .01 ohm)

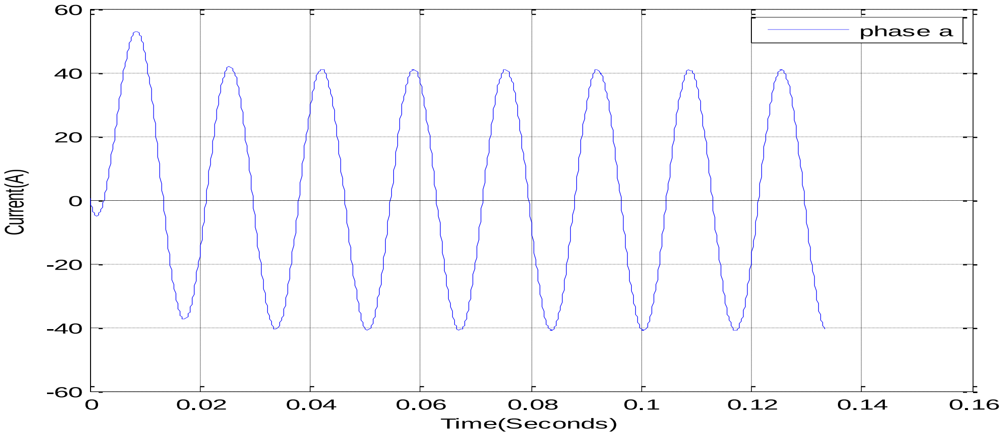

Figure 62: Phase a to phase c to ground type fault current graph (Phase a, retrieved from simulink)

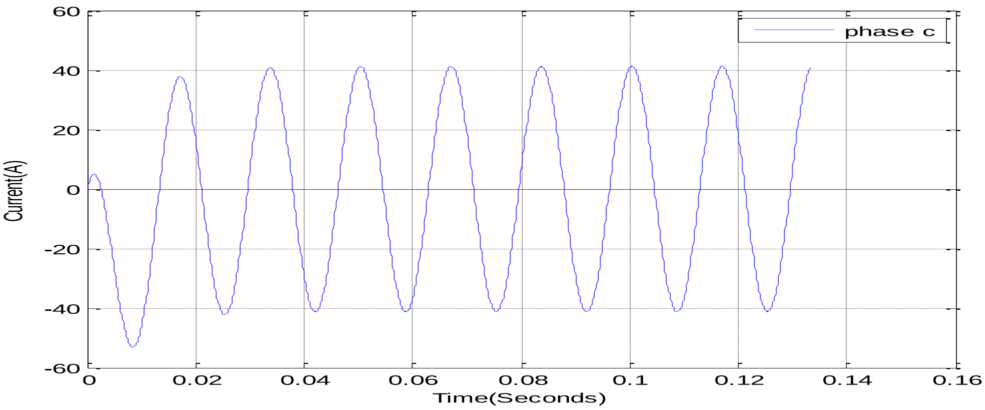

Figure 63: Phase a to phase c to ground type fault current graph (Phase c, retrieved from simulink)

Parameters from faulted wave equation of phase a (Figure 62) and phase c (Figure 63), those will be used for calculating zero crossing point time calculation right after the fault occurrence has been specified below:

Table 17: Faulted wave function parameters (Phase a)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	42.2008 V
Peak value of the 1st harmonic component (K_{21})	-42.4196 V
Peak value of the 3rd harmonic component (K_{23})	-0.072562 V
Phase angle of the 1st harmonic component (θ_1)	80.082^o
Phase angle of the 3rd harmonic component (θ_3)	-18.3045^o
Time constant of the decaying d. c. component (τ)	0.0077117 s

Table 18: Faulted wave function parameters (Phase c)

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	-42.2016 V
Peak value of the 1st harmonic component (K_{21})	42.6402 V
Peak value of the 3rd harmonic component (K_{23})	0.072608 V
Phase angle of the 1st harmonic component (θ_1)	80.8835^o
Phase angle of the 3rd harmonic component (θ_3)	-18.2589^o
Time constant of the decaying d. c. component (τ)	0.0077112 s

By using the unknown parameters of faulted wave current of phase a and phase c, those have been retrieved, we can calculate the zero crossing time next to the faulted point which has been depicted in Figure 64 & Figure 65. This current wave symbolizes phase a and phase c current under phase a to phase c to ground type fault. Here, we found that both have very close zero crossing point after the fault occurrence. It's 5.3893×10^{-05} second for phase a and 1.5257×10^{-05} second for phase c.

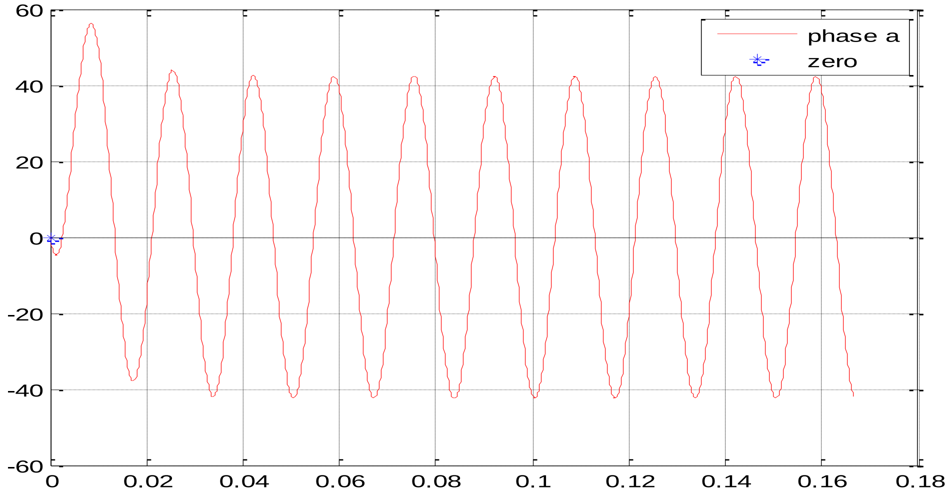

Figure 64: Phase a to Phase c to ground fault zero crossing point (Phase a)

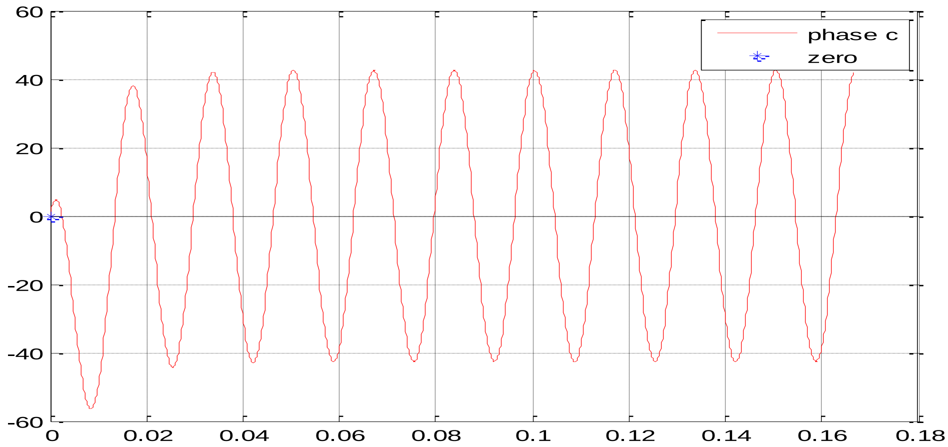

Figure 65: Phase a to Phase c to ground fault zero crossing point (Phase c)

If we join the zero crossing of phase a to phase c to ground, figure 66 will represent that.

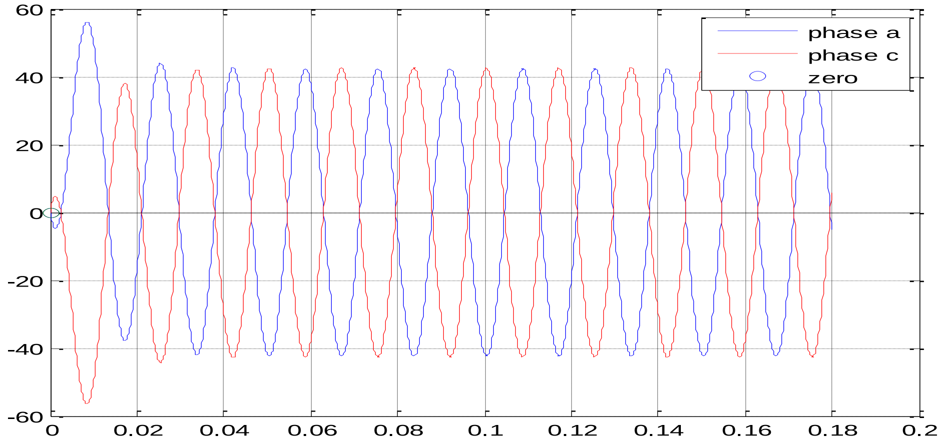

Figure 66: Phase a to Phase c to ground fault zero crossing point (Phase a and phase c)

4.3 Second zero crossing point

In any case, if we miss the zero crossing point next to the fault occurring point, we must have to separate the circuit at next most zero crossing point. The faulted wave signal is a sinusoidal signal. So, the distance it crosses from the origin to meet the first zero crossing, if we add the same distance it will cross for the second zero crossing. The following graph shows the second zero crossing point. Here, we have used the data table and curve from phase a to ground fault.

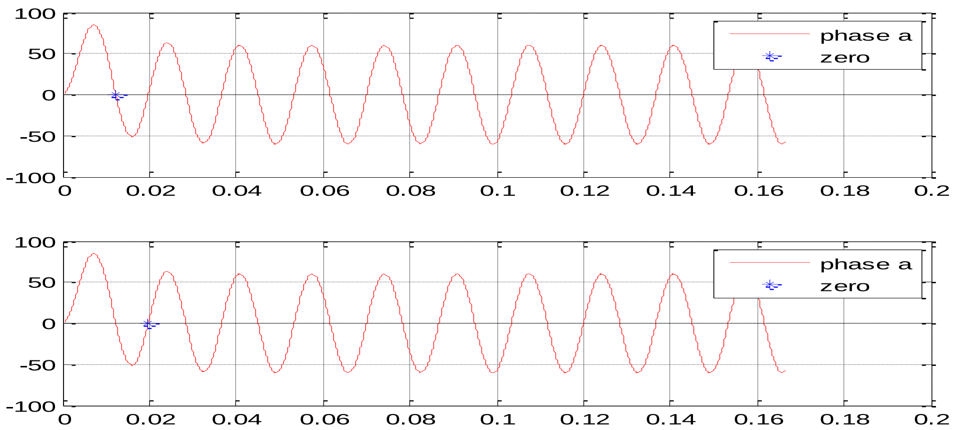

Figure 67: Two zero crossing point in a sequence (Phase a to ground fault)

4.4 Separation of the circuit at Zero Crossing

In this section, we have used balanced three phase fault to determine the zero crossing point and how it works in using of breaker in the transmission line. Figure 68 represents a current graph of balanced three phase fault.

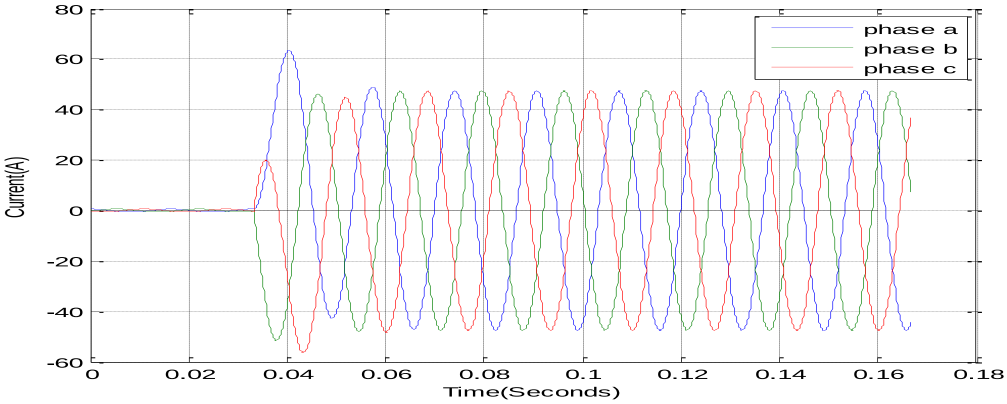

Figure 68: Current Graph in Three Phase balanced fault

If we integrate a three phase breaker and synchronize the fault time with the breaker time, it will be tripped the next zero crossing point after the fault occurring point. The above figure depicts that the fault occurred near .03 seconds. Based on the zero crossing point time of each phase we can select which phase to open first in the breaker.

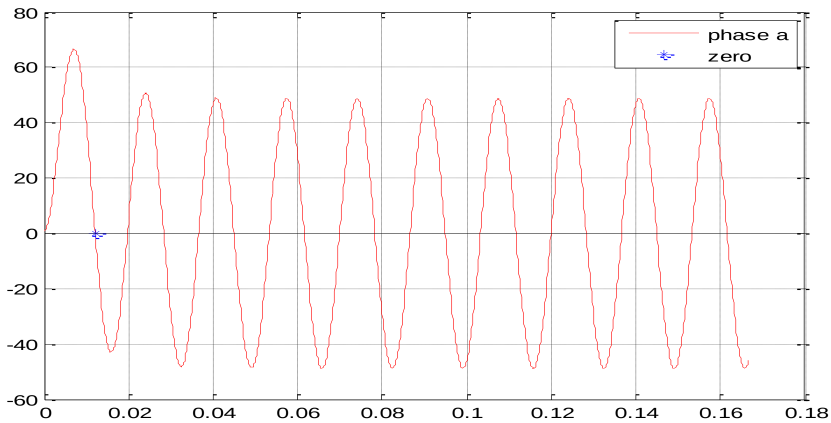

Figure 69: Balanced Fault (Phase A, Zero Crossing Time= .0121 s)

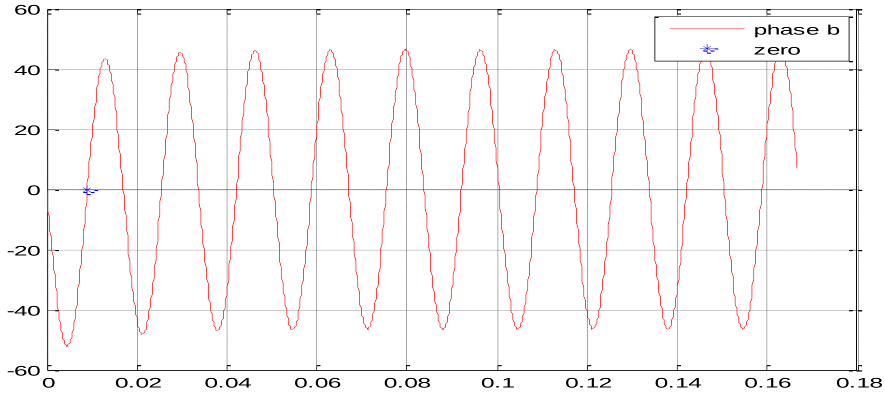

Figure 70: Balanced Fault (Phase B, Zero Crossing Time=0.0090 s)

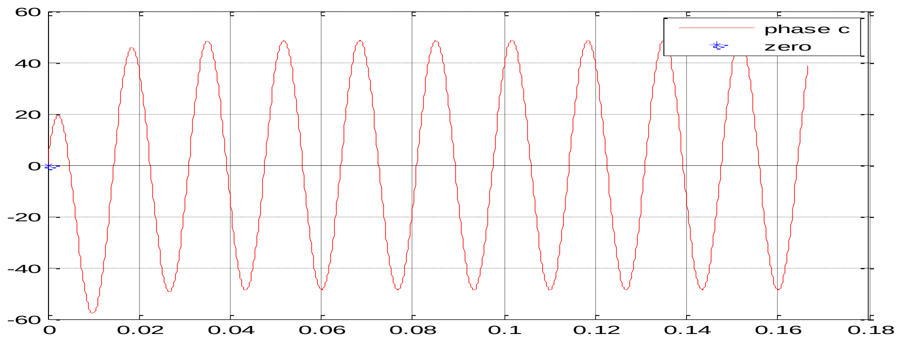

Figure 71: Balanced Fault (Phase C, Zero Crossing Time= 4.1554e-06 s)

From the above figures, It is clearly visible that Phase C has the first Zero Crossing after the fault. So, for phase C breaker will be opened 1st and the other two phases will not remain same after opening phase C breaker. Comparison of Figure 72 with Figure 68 reveals that.

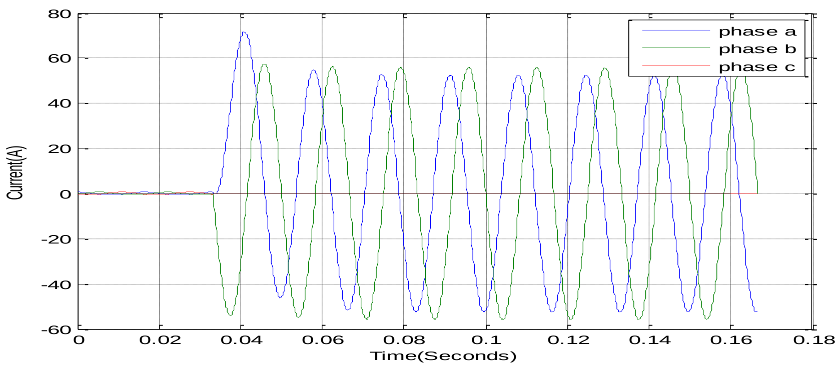

Figure 72: Balanced Fault (Three Phase, After Opening Breaker for Phase C)

Now, whichever has the first zero crossing time in between Phase A and Phase B, will be tripped.

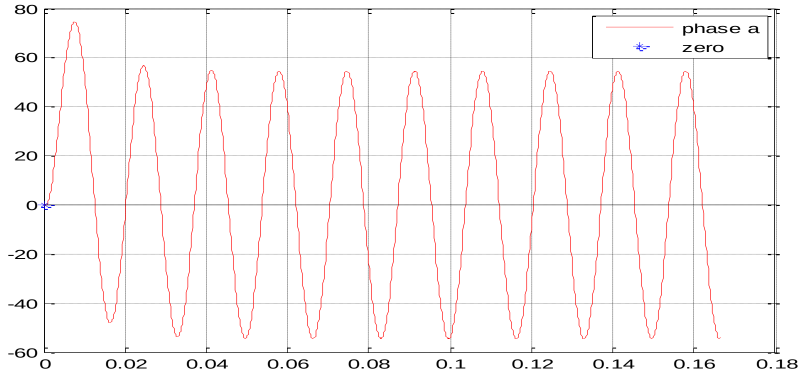

Figure 73: Balanced Fault (Phase A, after Opening Breaker for Phase C, Zero Crossing Time=1.8403e-04)

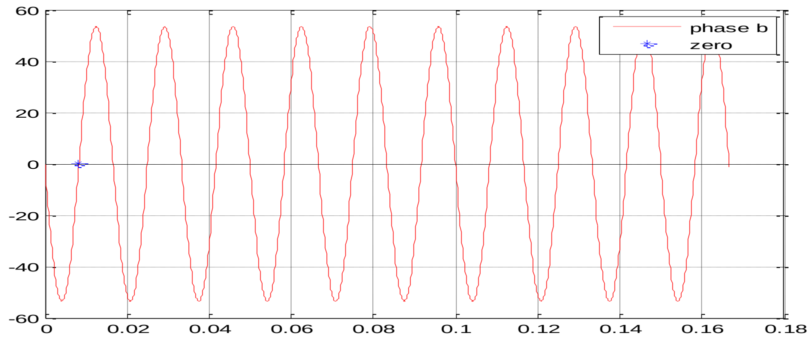

Figure 74: Balanced Fault (Phase B, After Opening Breaker for Phase C, Zero Crossing Time=0.0083)

From Figure 73 and Figure 74, we can come to a decision to trip Phase A first. After opening breaker for Phase A there will be some change in phase B current profile.

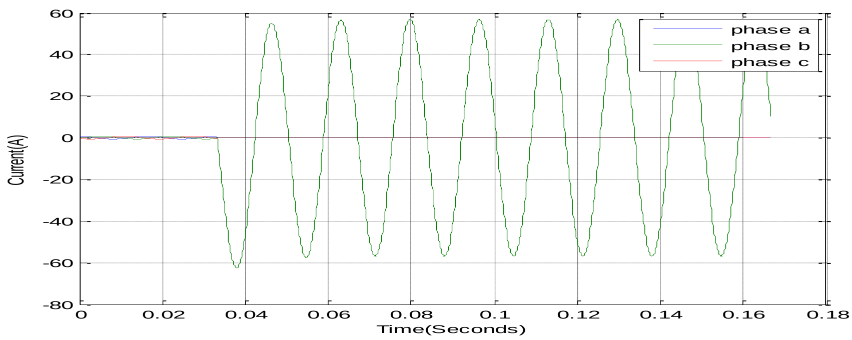

Figure 75: Balanced Fault (Three Phase, After Opening Breaker for Phase C & Phase A)

Figure 76 depicts the zero crossing for Phase B.

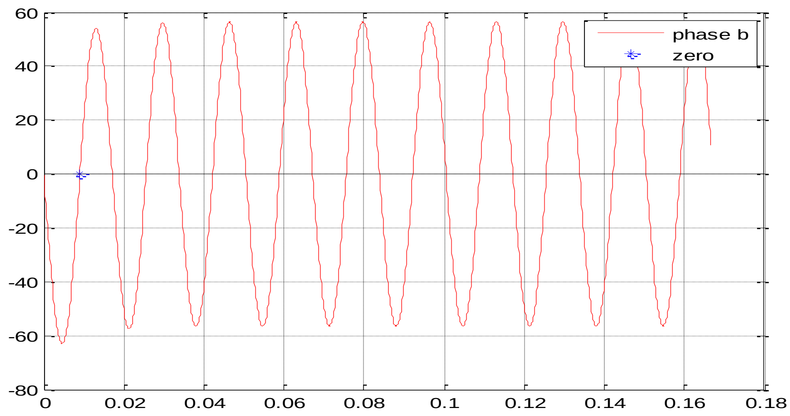

Figure 76: Balanced Fault (Phase B, After Opening Breaker for Phase C & Phase A, Zero Crossing Time=0.0090)

After opening Phase B, the total current profile curve after the fault occurring point will be represented by,

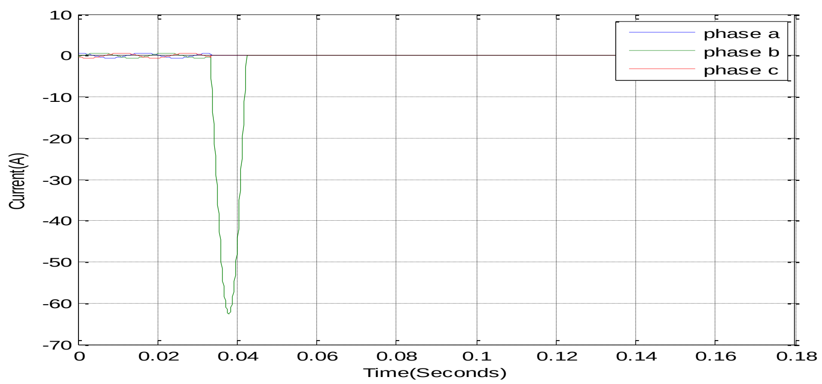

Figure 77: Balanced Fault (Three Phase, After Opening Breaker for Phase C , Phase A and Phase B sequentially)

4.5 Verification of the method with typical values

The method we proposed has been verified by using some typical values for unknown parameters for zero crossing estimation point which are mentioned in table 12.

Table 19: Typical parameters for unknown values

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	100 V
Peak value of the 1st harmonic component (K_{21})	$150\sqrt{2}$ V
Peak value of the 3rd harmonic component (K_{23})	$10\sqrt{2}$ V
Phase angle of the 1st harmonic component (θ_1)	10^o
Phase angle of the 3rd harmonic component (θ_3)	30^o
Time constant of the decaying d. c. component (τ)	.1000 s

These parameters can be placed in the faulted wave equation as following:

$$I(t) = 100 - 100(\frac{t}{\tau}) + \frac{100}{2!}(\frac{t}{\tau^2}) + 150\sqrt{2}Sin(\omega_0 t + 10) + 10\sqrt{2}Sin(3\omega_0 t + 30) \qquad (4.5.1)$$

Here, the fundamental frequency is 60 Hz. Using matlab the value of t has been gained here, which is 0.0089 s. We assumed current sample zero in a particular time instant to get the value of t at zero crossing. We can verify this solution by using the back calculation from the samples of the graph, that we retrieved using the above information. The graph has been depicted below:

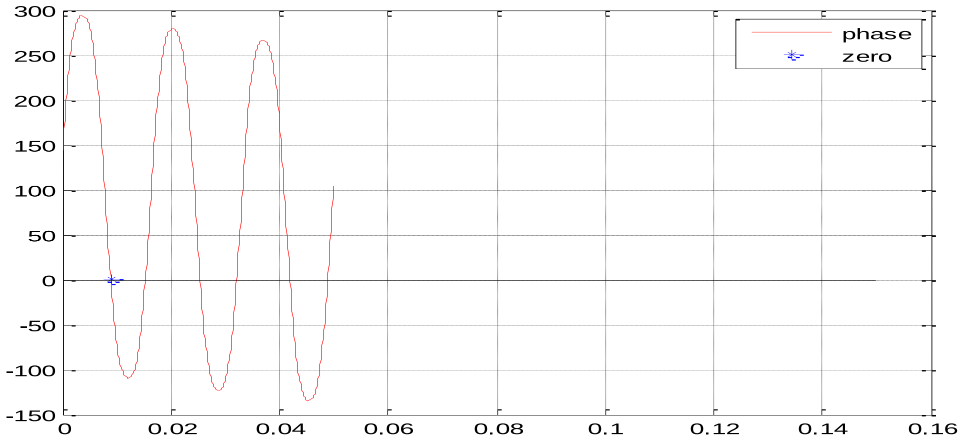

Figure 78: Faulted wave using the typical parameters

Now using time and current samples from the graph above, using equation 3.21 we can calculate the unknown parameters for zero crossing time calculation next to the fault occurrence. These have been stated in the following table:

Table 20: Unknown parameters from the back calculation

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	100.002 V
Peak value of the 1st harmonic component (K_{21})	212.1325 V
Peak value of the 3rd harmonic component (K_{23})	14.1421 V
Phase angle of the 1st harmonic component (θ_1)	10^{o}
Phase angle of the 3rd harmonic component (θ_3)	30^{o}
Time constant of the decaying d. c. component (τ)	0.09997s

Placing these values in the faulted wave function we can represent a curve which is pictured in Figure 79.

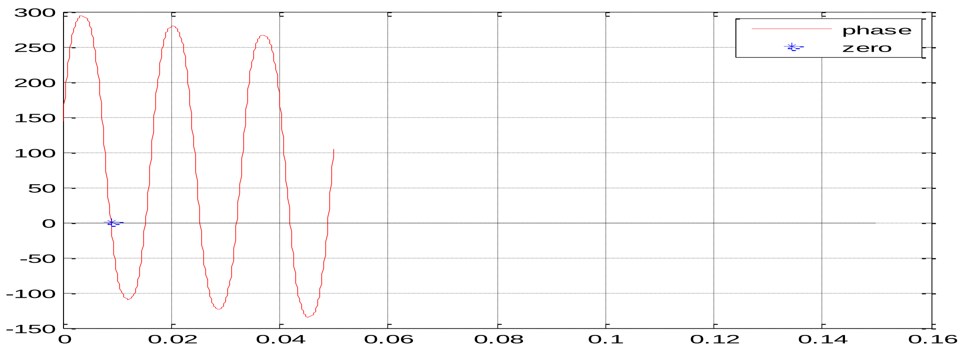

Figure 79: Faulted wave using the parameters from back calculation

Pattern of the graph is same as Figure 78. Here, the zero crossing time from the back calculation has been found in .0089 second. This can be clarified as accurate as the original. So, the verification method was correct in accordance to the procedure.

We can use some more typical values to get ensured of the method of this back calculation. Such typical values for unknown parameters have been stated in the following table.

Table 21: Typical parameters for unknown values

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	140 V
Peak value of the 1st harmonic component (K_{21})	$200\sqrt{2}$ V
Peak value of the 3rd harmonic component (K_{23})	$20\sqrt{2}$ V
Phase angle of the 1st harmonic component (θ_1)	12^o
Phase angle of the 3rd harmonic component (θ_3)	36^o
Time constant of the decaying d. c. component (τ)	.2000 s

Again if we place the parameters in the equation we will get something as below:

$$I(t) = 140 - 140(\frac{t}{\tau}) + \frac{140}{2!}(\frac{t}{\tau^2}) + 200\sqrt{2}Sin(\omega_0 t + 12) + 20\sqrt{2}Sin(3\omega_0 t + 36) \qquad (4.5.2)$$

Using the same procedure as the previous one and assuming fundamental frequency of 60 Hz and current sample zero at a particular time instant, we can get the value of t = 0.0088s. The graph has been depicted in Figure 80 .

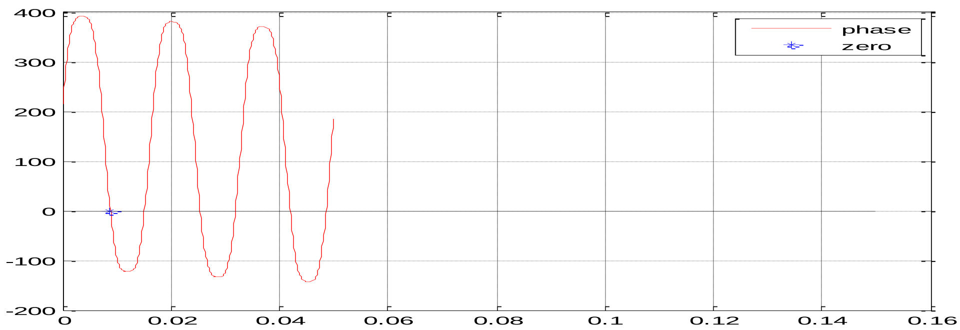

Figure 80: Faulted wave using the typical parameters

Now using time and current samples from the graph above, using equation 21 we can calculate the unknown parameters for zero crossing time calculation next to the fault occurrence. These have been stated in the following table:

Table 22: Unknown parameters from the back calculation

Parameter	Value
Magnitude of the d. c. offset of t=0 (K_1)	140.0004 V
Peak value of the 1st harmonic component (K_{21})	282.8428 V
Peak value of the 3rd harmonic component (K_{23})	28.2843 V
Phase angle of the 1st harmonic component (θ_1)	12^o
Phase angle of the 3rd harmonic component (θ_3)	36^o
Time constant of the decaying d. c. component (τ)	0.19998 s

We can represent a curve in the faulted wave function using these values which is pictured in Figure 81.

Alike the 1st example, Figure 80 and Figure 81 also looks alike and here in Figure 81, the zero crossing time is .0088 s which differs slightly from Figure 81.

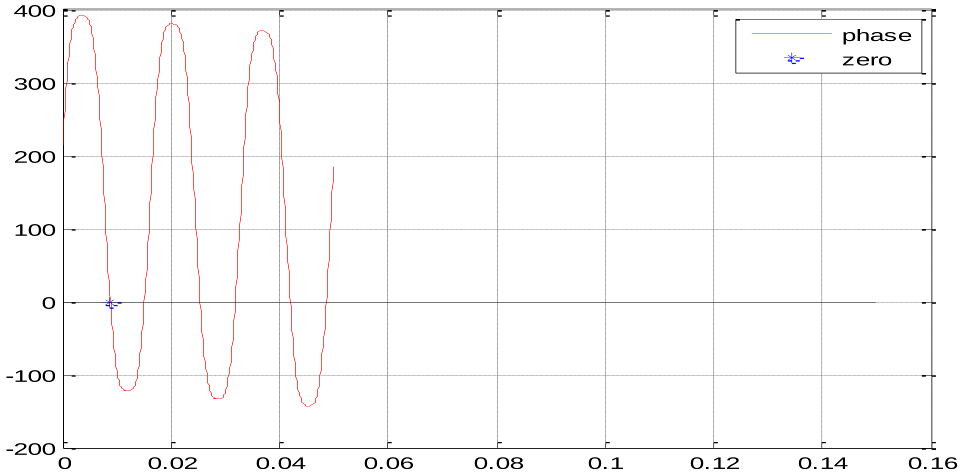

Figure 81: Faulted wave using the parameters from back calculation

So, from the verification we can come to a decision that the methodology we have used throughout the evaluation study, it's a working method.

Chapter 5 Conclusion

When a fault occurs, the first priority is to minimize the fault as soon as possible to lessen the damage. Proper Estimation of Zero Crossing point is utmost important for proper relaying or controlling of power system. A technique based on least square method has been discussed to determine the parameters to calculate the zero crossing point in this thesis. Every possible faults in Line to Line (L-L), Line to Ground (L-G) and Double Line to Ground (DL-G) has been used in the simulation to fulfill the study and the method has been verified by typical parameters and breaker whether it puts off at the zero crossing and separates the circuit or not. Simulation studies have demonstrated that the procedure is quite accurate.

APPENDIX

- Taylor Series: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ for all x (A.1)

- Trigonometric rule: $Sin(A+B) = SinACosB + CosASinB$ (A.2)

- In any matrix multiplication generally AX is not equal to XA because they might have different dimensions. (A.3)

- In transpose matrix, $(AB)^T = A^T B^T$ (A.4)

- Consider m linear equation n unknowns in least square method where m>n

$$\underset{m\times n}{[A]}\underset{n\times 1}{[X]} = \underset{m\times 1}{[Y]} \qquad \text{(A.5)}$$

- If A is full column rank, meaning rank(A) $\equiv n \leq m$, that is, $A^T A$ is not singular, then A^+ is a left inverse of A, in the sense that $A^T A = I_n$. We have the closed-form expression

$$A^+ = (A^T A)^{-1} A^T \qquad \text{(A.6)}$$

- If A is full row rank, meaning rank(A)$\equiv m \leq n$,, that is, AA^T is not singular, then A^+ is a right inverse of A, in the sense that . We have the closed-form expression

$$A^+ = A^T (AA^T)^{-1} \qquad \text{(A.7)}$$

- If A is square, invertible, then its inverse is $A^+ = A^{-1}$ (A.8)
- Neutral Current, $I_n = I_a + I_b + I_c$ (A.9)
- Neutral Current, $I_n = 3I_0$ (A.10)
- In a balanced Y connected system $I_n = 0$ (A.11)

REFERENCES

1. Rahmati, A.; Adhami, R., "An Accurate Filtering Technique to Mitigate Transient Decaying DC Offset," *Power Delivery, IEEE Transactions on* , vol.29, no.2, pp.966-968, April 2014.
2. Y. Guo, M. Kezunovic, and D. Chen, “Simplified algorithms for removal of the effect of exponentially decaying DC-offset on the Fourier algorithm,” IEEE Trans. Power Del., vol. 18, no. 3, pp. 711–717, Jul.2003.
3. C. Gasquet and P. Witimski, Fourier analysis and applictions. Springer, New York, 1998.
4. T. Lobos, T. Kozina and H.-J. Koglin, Power system harmonics estimation using linear least squares method and SVD, IEEE Proceedings on Generation, Transmission and Distribution, vol. 148, no.6, pp. 567-572, 2001.
5. A. Routray, A. K. Pradhan and K. P. Rao, A novel Kalman filter for frequency estimation of distorted signals in power systems, IEEE Transactions on Instrumentation and Measurement, vol. 51, no. 3, pp. 469-479, 2002.
6. Young-Jin Kwon; Wan-Seok Song; Sang-hee Kang; Dong-Gyu Lee, "A phasor estimation algorithm for a decaying AC component," Modern Electric Power Systems (MEPS), 2010 Proceedings of the International Symposium ", vol., no., pp.1-6, 20-22 Sept. 2010
7. Dadash Zadeh, M.R.; Zhiying Zhang, "A New DFT-Based Current Phasor Estimation for Numerical Protective Relaying," Power Delivery, IEEE Transactions on , vol.28, no.4, pp.2172-2179, Oct. 2013
8. Kleber M. Silva, Bernard F. Kusel "Decaying dc component effect elimination on phasor estimation using an adaptive filtering algorithm," Department of Electrical Engineering, University of Brasilia.
9. Sachdev, M.S., Baribeau, M. A., "A New Algorithm for digital Impedance Relays," Power Aparatus and Systems, IEEE Transactions on ,Vol.PAS-98, no.6,pp.2232-2240,Nov.1979.
10. Ankita Gupta et al. "An Efficient Approach to Zero Crossing Detection Based On Opto-Coupler " Int. Journal of Engineering Research and Applications , ISSN : 2248-9622, Vol. 3, Issue 5, Sep-Oct 2013, pp.834-838
11. Glover,J.D, Sarma,M.S and Overbye,T.J (2012) Power System analysis and design, US:Cengage Learning
12. "The electricity grid: A history",Burn an energy journal.Retrived 2014-03-25.
13. "Bulk Electricity Grid Beginnings" (Press release). New York Independent System Operator. Retrieved 2014-03-10.
14. John W. Klooster, Icons of Invention: the Makers of the Modern World from Gutenberg to Gates, pages 305
15. Barron: How does the power system work? Part 1. June 29,2011. [Basic Structure of the Electric System]
    Retrieve from http://www.the-minuteman.org/2011/06/29/how-does-the-power-system-work-part-1/
16. Grainger ,John J., Stevenson, William D. (2005) Power System Analysis,US:Tata McGraw-Hill
17. He Baina; Lin Xin; Xu Jianyuan, "The Analysis of Secondary Arc Extinction Characteristics on UHV Transmission Lines," High Voltage Engineering and Application, 2008. ICHVE 2008. International Conference on , vol., no., pp.516,519, 9-12 Nov. 2008
18. “George Westinghouse”, IEEE Global History Network.

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Publications

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- Husnain-Al-Bustam, Md. Shakawat Hossan, “ Design Development of an 8bit 8051Architecture Microcontroller Learning Kit”, ISSN No. 0976-5697, Volume 2, No. 6, Nov-Dec 2011, International Journal of Advanced Research in Computer Science.