SYNTHESIS OF SINGLE-HOLE VIBRATION WAVEFORMS FROM A MINING BLAST

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SYNTHESIS OF SINGLE-HOLE VIBRATION WAVEFORMS FROM A MINING BLAST

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Engineering at the University of Kentucky

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In mining engineering, blast-induced ground vibration has become one of the major concerns when production blasts are conducted, especially when the mining areas and the blast sites are near inhabited areas or infrastructure of interest. To comply with regulations, a vibration monitoring program should be developed for each mining operation. The vibration level, which is usually indicated by the peak particle velocity (PPV) of the vibration waveform, should fall below the maximum allowable values. Ideally, when blasting is near structures of interest (power towers, dams, houses, etc.), the vibration level (PPV) should be predicted prior to the actual production blasts. There are different techniques to predict the PPV, one in particular is the signature hole technique. This technique is based on signals and systems theory and uses a mathematical operation called convolution to assess the waveform of the production blast. This technique uses both the vibration waveform of an isolated hole and the timing function given by the timing used in the blast.

The signature hole technique requires an isolated single-hole waveform to create a prediction. Sometimes this information is difficult to acquire, as it requires the synthesis of a single-hole vibration waveform from a production blast vibration signal. The topic of ground vibrations from mining blasts, and more specifically the synthesis of a single-hole vibration waveform, has been studied by researchers in past decades, but without any concrete success. This lack of success may be partially due to the complexity and difficulty of modelling and calculation. However, this inverse methodology can be very meaningful if successfully applied in blasting engineering. It provides a convenient and economical way to obtain the single-hole vibration waveform and make the prediction of a production blast waveform easier.

This dissertation research involves the theories of deconvolution, linear superposition, and Fourier phases to recover single-hole vibration waveforms from a production waveform. Preliminary studies of deconvolution included spectral division deconvolution and Wiener filtering deconvolution. In addition to the adaptation of such methodologies to the blast vibrations problems, the effectiveness of the two deconvolution methods by the influence of delay interval and number of holes is also discussed. Additionally, a new statistical waveform synthesis method based on the theories of linear superposition, properties of
Fourier phase, and group delays was developed. The validation of the proposed methodology was also conducted through several field blasting tests.

Instead of synthesizing one normalized single-hole vibration waveform by deconvolution, the proposed statistical waveform synthesis methodology generates a different single-hole vibration waveform for each blast hole. This method is more effective and adaptable when synthesizing single-hole vibration waveforms. Recommendations for future work is also provided to improve the methodology and to study other inverse problems of blast vibrations.

KEYWORDS: Inverse problems of Blast vibrations, Single-hole vibration waveforms, Deconvolution, Statistical waveform synthesis, Electronic detonators
SYNTHESIS OF SINGLE-HOLE VIBRATION WAVEFORMS
FROM A MINING BLAST

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07/16/2018
For the glory of God.
To my beloved wife, Tuopu, who accompanied and encouraged me during the past five years of my PhD study, and whom I will keep in my heart forever.
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Chapter 1 INTRODUCTION

This chapter introduces the background and prediction of blast-induced ground vibrations, as it has been progressively researched in field of mining engineering. The signature hole method is reviewed and shared assumptions with the deconvolution methodology are highlighted in the research. The concepts associated with inverse problems bring a higher level of theoretical foundation to the dissertation research. The question of why the dissertation topic was chosen and its application potentials form the last section.

1.1. Background

There is a long history of explosive utilization and rock blasting, dating back to around the tenth century when black powder was invented in China. Originally, the rock was fragmented by instantaneous initiation of detonators and explosives. In 1895, fuse-type delay blasting caps were introduced by Smith (Dupont, 1980). People found that delay initiation could improve the performance of fragmentation. Vented delay initiators were introduced in the 1920s (Leslie, 1926, 1921; Wilhelm, 1926) followed by ventless delay initiators in the 1930s (Burrows and Noddin, 1938; Ellsworth, 1931; Hanley, 1938; Nash, 1935). Having an even greater impact, short delay (millisecond) detonators were brought into use in the 1940s (Leet, 1949; Scherrer, 1947). In the early stage, blast designs with delay initiations were mainly focused on rock fragmentation rather than its side effects including airblast and ground vibrations. This approach was acceptable at that time because mine blasting was mainly limited to remote areas. However, over time as the mining industry has developed, industrial blasting activities sometimes approached inhabited areas. This shift has made the side effects of blasting more hazardous to populated environments. As a result, ground vibration control has become a necessary part of blast designs.

To control the adverse effects from mining blasts, research has been conducted and regulations for blast vibrations have been established. Originally, criteria were focused on a single variable: peak particle velocity (PPV) (Duvall and Fogelson, 1962; Edwards and Northwood, 1960; Langefors, 1958; Northwood et al., 1963). However, later blasting practice proved that PPV alone was not effective enough to control or prevent building damage.
In 1974, the Bureau of Mines performed research and investigations to determine the relationship between blast damage, particle velocity, and frequency content, to develop a new blast vibration criterion. In the 1980s, after the publication of RI 8507 (Siskind et al., 1980), the new blast vibration damage criterion contained both PPV and frequency, referred to as the “Z-curve”. In addition, Germany, Sweden, and Switzerland also published similar criteria with both PPV and frequency content. (Anderson, 1993)

$$V = K \left( \frac{R}{\sqrt{W}} \right)^\beta$$  \hspace{1cm} (1)

where  \( R \) is the distance from the explosion,
\( W \) is maximum charge weight per delay,
\( K, \beta \) are quantities related to local ground conditions.

Other researchers adopted \( R/\sqrt{W} \) instead of \( R/\sqrt{W} \) (Drake and Little Jr, 1983; Villano and Charlie, 1993; Yun et al., 2007).

However, there are two disadvantages of such empirical laws: (1) The units of both sides of Equation (1) are not consistent according to dimensional analysis (Blair, 2004), which
makes empirical laws not fundamental equations for vibration prediction. (2) The scale distance equation is achieved by collecting a set of field test data and statistical regression. The accuracy of prediction depends on if the future blast vibration measurements have same or similar ground conditions to previous field tests.

When acquiring a large number of field measurements from ground vibrations is impractical, numerical simulation may be an alternative way of predicting ground vibrations. Different numerical methods have been proposed by researchers worldwide (Chafi et al., 2009; Saharan and Mitri, 2008; Schimizze et al., 2013; Toraño et al., 2006; Wu et al., 2004; Xu and Yan, 2006). However, it is too complicated to accurately model the physical process during a blast. So, most current numerical models make some simplifications of rock fragmentation and seismic wave propagation. In addition, running a large numerical model is still very time consuming and needs advanced professional knowledge, which usually makes those methods unpractical.

Another prediction method of blast vibrations is the utilization of artificial intelligence. This type of methodology require building an artificial neural network, which can then predict ground vibration levels according to blast parameters through a learning process (Khandelwal and Singh, 2006; Singh et al., 2004). This method in existing literature only predicts the PPV and dominant frequencies rather than the entire vibration waveform. Moreover, an artificial neural network requires certain learning cost, and if rock mass properties or geological conditions are not included in the learning process, the validity of prediction results may be uncertain. However, artificial intelligence still seems a promising method for ground vibration research in the future.

Compared to predicting only blast ground vibration levels (e.g. PPV), ground vibration waveform prediction is a more comprehensive method which depicts the vibration process of a particle during the entire blast. This method provides information including, but not limited to, PPVs and dominant frequencies. A popular waveform prediction method, and a main topic of this research, is called the signature hole method.

As previously mentioned, the first use of millisecond delay initiation was in the 1940s. During that period, Thoenen and Windes (1942) began to notice that certain appropriate delay times could be used to reduce ground vibrations. Due to the limited technology of
detonators at that time, the idea remained a theory. It was not until the 1980s that Anderson et al. (1983, 1985), Hinzen, et al. (1987), Crenwelge (1988) began to develop the basis of the signature hole method. Anderson (2008) made a comprehensive review of this method. The assumptions of signature hole method are summarized as below (Anderson, 2008):

1. All holes are detonated at the same location, so that the path traveled by the waves is identical. This assumption should be justified when the distance from seismograph to blast site is far larger than the dimension of the blast site.
2. All holes have the same explosive charge type and weight. In other words, the quantity of energy contained in explosives in each hole is the same.
3. All holes have the same explosive-rock interaction, so that the source pulse is the same. That is, the seismic energy transformed from total energy of each hole is the same.

The three assumptions ensure that each individual hole generates the same ground vibration waveform and together form the basis of the conventional signature hole method. In mathematics, the conventional signature hole method can be viewed as a convolution model.

Since its introduction, the signature hole method has played an important role in ground vibration prediction. However, the conventional signature hole method was restricted as a deterministic problem by its assumptions. In fact, due to the intrinsic randomness during the whole blast process (e.g. randomness in detonators, explosives, drilling and charging, rock damage, geology, wave propagation path etc.), the actual blast performance cannot be exactly as expected and cannot be consistent among each blast. Blair (1993) and Silva (2012) improved the signature hole method by including stochastic factors and Monte-Carlo method into the prediction of ground vibration, which gives more practical results. In their methodologies, the ground vibration waveform of each hole is varying. Therefore, the conventional signature hole method is no longer suitable to this condition, and the more general superposition concept was adopted.
1.2. Inverse problems

An inverse problem is a relative concept to a direct problem. Two problems are inverse to each other if the formulation of each of them requires full or partial knowledge of the other (Keller, 1976). There is no absolute difference between direct and inverse problems. If one of the two problems has been studied earlier or in more details, it is called the direct problem, where the other is the inverse problem. A reasonable understanding can be made upon time, space or causal orders which exist in any phenomenon, process, or physical system.

If the solution of some problems follows certain time, space or causal orders, they are called direct problems. On the contrary, inverse problems are trying to obtain the essence through the phenomena, or to find the causes through the effects. Specific to ground vibrations in mining blasts, direct and inverse problems may be schematically demonstrated in terms of system, input, and output in Figure 1-2.

![Figure 1-2 Direct and inverse problems of ground vibrations](image)

In Figure 1-2, the box in the center represents the system for ground vibrations. For blast ground vibrations, the system is the rock and soil medium between the blast event and the ground vibration measurement point. Seismic wave propagate through this system. Information about the system can be revealed through properties of rock mass, the geologic conditions of the ground medium, or an empirical Green’s function which reflects the
geologic conditions (Hutchings and Wu, 1990). Empirical Green’s functions are methods widely used in the research of earthquakes and they are generally referred to as the seismic measurements from a small earthquake event (Bour and Cara, 1997; Hartzell, 1978; Hutchings and Viegas, 2012; Koller et al., 1996). In mining blast engineering, a single-hole blast can also be viewed as an empirical Green’s function (Smith, 1993), which means it contains the geologic conditions of the ground medium to some extent. There will be a detailed description of the system from the point of view of signals and systems in subsequent chapters.

By receiving a stimulus or input, the system will generate a reaction or output. The input is the seismic energy emitted from a blast hole and the output is the measured ground vibrations at a certain location due to this input. If there are a series of blast holes, e.g. a series of inputs (input 1 to input n), the measured output will be a superposition of their corresponding outputs. For the case of multiple holes, the blast holes are usually initiated according to a certain timing sequence, and thus the information of inputs also includes the firing times of each blast holes.

According to Figure 1-2, ground vibration measurements and predictions belong to direct problems. When the blast design and some system information are known, ground vibrations at a certain distance can be predicted. The prediction methods include scaled distance law, numerical simulation, and signature hole method. But if information about the system or the input during the process of a blast or wave propagation is of interest, it becomes an inverse problem. Direct and inverse problems are usually complementary. For example, it is a direct problem to predict ground vibration waveforms of production blasts by signature hole method, and a single-hole vibration waveform is needed before prediction. On the contrary, the synthesis of a single-hole waveform conversely from the measurements of production blasts is actually solving an inverse problem. But compared to direct problems, inverse problems are usually more complicated, due to the fact that they are usually ill-posed problems.

Ill-posed problems are those which fail to satisfy any of the properties of well-posed problems defined by Hadamard (1902):

(1) The problem has a solution;
(2) The solution is unique;
(3) The solution is a continuous function of data.

Usually, a direct problem is a well-posed problem (Kirsch, 2011). But an inverse problem usually does not satisfy one or more of the three properties. This condition increases the difficulty to solve inverse problems. Therefore, some prior information or additional restrictions to problems are needed.

There are generally two types of inverse problems: inverse reconstruction problems and inverse identification problems (Neto and da Silva Neto, 2012). The purpose of an inverse reconstruction problem is to find the information of the input given the information of output. In contrast, the purpose of an inverse identification problems is to find the information about the system (e.g. single-hole vibration waveform), given information from the outputs. There are other inverse problems which involve both construction and identification problems. That is, information about both inputs and the system are unknown. This kind of inverse problems include blind deconvolution or blind source separation which is to separate the mixed measured signals into original unobserved signals (Shi, 2011). Those different types of inverse problems are also indicated in Figure 1-2.

Now, reviewing the ground vibration research methods previously, the inverse problem of scaled distance law method is to determine the local-ground-related parameters $K$ and $\beta$ in Equation (1) based on a set of collected data. So, it is an inverse identification problem and has been studied for a long time. For the methods of artificial neural network and numerical simulation, they both have potential to solve inverse reconstruction and identification problems. Unfortunately, there are still no literature of research on inverse problems of blast ground vibrations. The signature hole method involves information from both a timing sequence and a single-hole vibration waveform. So, its inverse problem also has the potential to conduct tasks of reconstruction or identification. If the measurements of a single hole blast and a production blast are known, and the goal is to estimate the initiation timing sequence, it is an inverse reconstruction problem. If the measurements of a production blast and the timing sequence are known, and the single hole waveform is desired, the problem becomes to an identification problem. If both the timing sequence and
single-hole waveform are unknown, it becomes to a mix of both inverse reconstruction and identification.

For the conventional signature hole method, deconvolution is the way to solve its inverse problem. In geophysical exploration area, the research of deconvolution has witnessed a long history (Arya and Holden, 1978). However, in the research of mining blast ground vibrations, little work has been done on inverse problems. There are already some attempts on deconvolution of blast ground vibrations, but the results are inconclusive (Balbas and Diaz-Villafranca, 2001; Hinzen, 1988) or detailed analysis is missing (Bernard, 2012, 2009; Crenwelge and Peterson, 1986; Tshibangu and Lefebvre, 2008; Yamamoto et al., 2001).

One thing to note is the inverse problem of the improved statistical signature hole method (Blair, 1993; Silva-Castro, 2012) may not be treated simply as deconvolution in that the direct problem is not simply a convolution model based on its assumptions. Its inverse problem will need analysis in detail in this dissertation research.

In recent years, Anderson (2013) did an interesting inverse problem of blast ground vibrations. He applied wavelet transform to ground vibration signals and obtained a time-frequency map. This map spread the time-domain signal into time-frequency domain, in which high frequency components revealed some information about the timing. His work could be viewed as the inverse reconstruction problems of blast ground vibrations, and that the time-frequency method also has the potential to conduct inverse identification problems.

1.3. Origin of the research and its significance

As reviewed in the sections above, one common characteristic of foregoing research is the attention put on direct problems for the purpose of ground vibration prediction, while few efforts were made on the inverse problems. The idea of this dissertation research comes from the inverse problems of the signature hole method.

The signature hole method has proved its effectiveness in practical applications by measuring a single-hole vibration waveform in advance. For a blast vibration to be predicted, the signature waveform must represent any geological changes. For this reason, it should be measured before each production blast. However, measuring a single-hole vibration waveform requires additional operations besides the regular blast designs. These
operations also have possibilities to interrupt the continuity and integrity of production blasts, and thus increase the costs. A compromising solution is to update the signature hole after several rounds of blasts.

Blast vibration monitoring has already become a regular part in the design and conduction of blasts. Generally, the recorded data are only used to obtain the vibration level (the PPV) to conform to regulations. This is a waste of the monitored signals which contain further useful information, and should be utilized adequately in a more important role for blast engineering.

Therefore, the core task throughout the whole dissertation research is to synthesize single-hole vibration waveforms from production blast vibration waveforms based on the utilization of electronic detonators. This dissertation research is solving inverse identification problems, and the achievements of it will hopefully provide a new perspective to blast engineering.
Chapter 2  DIRECT AND INVERSE PROBLEMS IN BLAST VIBRATIONS

This chapter describes the direct and inverse problems of blast vibrations in detail, inspired by the signature hole method.

2.1 Characteristics of blast vibrations

2.1.1 Characteristics of blast vibrations and its propagation

Blast-induced ground vibrations result from energy released from chemical explosives initiated in blast holes. In the detonation process, the solid mass of explosives converts into gaseous products. The change in the pressure of the gas occurring during the explosion generates a rapid change in the initial stress state of the surrounding rock which crushes the rock near the hole and displaces the rock into a muck pile. (Saharan et al., 2006). Beyond the hole, at some distance related to the initial hole diameter, the stress deforms the rock elastically and part of the energy released during the detonation travels as a stress wave or a seismic wave through the medium generating vibrations (Sally and Daemen, 1983).

When examining the energy balance, the energy initially stored in chemical explosives (ANFO, dynamite, Emulsions etc.) is equal to the sum of all the types of energy generated during the blasting process (Sanchidrián et al., 2007). The energy balance is expressed as

\[ E_E = E_F + E_S + E_K + E_{NM} \]  

(2)

Where \( E_E \) is explosive energy in the chemical materials;

\( E_F \) is energy used in fragmentation;

\( E_S \) is energy in seismic wave;

\( E_K \) is kinetic energy;

\( E_{NM} \) denotes not measurable energy including sound (airblast), heat, light, and other phenomena that occurs in the explosion.

The ground vibrations studied in this dissertation mainly come from the seismic energy, \( E_S \), which is released into the earth and generates seismic waves. Seismic waves can travel considerable distances (Frantti, 1963). In a production blast, holes are detonated at varying
times generating different pulses and ground vibrations. The interaction of the ground vibrations in a constructive or destructive manner produces a complex vibration pattern which is recorded at a specific location. The vibration waveform recorded in a blast event usually consists of three orthogonal components: radial, longitudinal, and vertical. Generally, the radial and longitudinal components are in a horizontal plane while the vertical component is perpendicular to the other two. This arrangement is merely due to the construction of the geophone.

The seismic waves can be divided into two variates: body waves and surface waves. The body waves propagate through the body of the rock and soil. The body waves can be further subdivided into compressive or sound-like waves (denoted as $P$) and distortional or shear waves (denoted as $S$). Explosions produce predominately body waves at small distances. These body waves propagate outward in a spherical manner until they intersect a boundary such as another rock layer, soil, or the ground surface. At this intersection, shear and surfaces waves are produced. The surface waves are transmitted along a surface (usually the upper ground surface). The most important surface wave is the Rayleigh wave (denoted as $R$). Rayleigh surface waves become important at larger transmission distances.

At small distances, all three wave types will arrive together, and it will be very hard to identify one from another. However, at larger distances, the more slowly moving shear and surface waves begin to separate from the compressive wave and allow identification (shown in Figure 2-1). In mining applications, most explosions are detonated as a series of smaller explosions which are delayed by milliseconds. Differences in travel paths and delay times result in the overlapping arrival of both wave fronts and wave types. This complicated arrival sequence has created difficulties in blasting seismology.
During a production blast event, the propagation of the generated seismic waves is influenced by the geological conditions between the blast site and the point under study (joints, faults, lithology, etc.). So, the body waves (P and S waves) and surface wave (R wave) propagate to the monitoring point as direct waves, reflections, and refractions, in different paths. Figure 2-2 shows a sketch of propagation model of a single-hole shot.

![Seismic wave propagation model](image)
In Figure 2-2 there are two monitoring points: A and B. Point A is near to the blast source and point B is relatively far from the blast source. The measured waveform at point A is relatively simple, and the surface wave and direct body wave have not separate yet. Whereas the waveform at point B is more complicated, and consists of direct waves, reflections from each layer interface, refactorions and surface waves. Multiple reflections are not included in this model.

2.1.2 Signals and system representation of blast vibrations

As defined before, based on the characteristics of blast vibrations above, the system can be viewed as the entity that wraps the site specific geological conditions (joints, faults, lithology, etc.) between the event site and the point under study, and the path of the vibration waves, including reflections and refactorions of waves propagating away from the event site. Figure 2-2 shows this concept.

By signals and systems theory, the input of the system is the explosive energy and the output is measured as a ground vibration signal at a recording point. The system is assumed as a causal linear time-invariant (LTI) system in this dissertation research. A LTI system can be expressed as convolution integral:

\[
y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau = x(t) * h(t)
\]  

(3)

A system is causal if its output depends only on past and present inputs to the system. In other words, the input should precede the output of the system. For a LTI system to be causal, its impulse response needs to meet the following relationship (Oppenheim, 1997):

\[
h(t) = 0 \quad for \ t < 0
\]  

(4)

A system is linear if it meets the superposition property (Oppenheim, 1997):

\[
\{x_1(t) + x_2(t)\} \rightarrow \{y_1(t) + y_2(t)\}, (additivity)
\]  

(5a)

and

\[
\alpha x_1(t) \rightarrow \alpha y_1(t), (homogeneity)
\]  

(5b)

Where \(x_1(t)\) and \(x_2(t)\) are inputs,
$y_1(t)$ and $y_2(t)$ are outputs.

The concept of superposition has been verified experimentally (Stump and Reinke, 1988) and adopted by signature hole method within blasting area (Anderson, 2008). It has also been suggested that it is possible for vibration waves from each blast hole to superpose in a non-linear way (Blair, 2008, 2004, 1990).

To understand the linearity or non-linearity better during a blast, it’s beneficial to briefly review the rock fracture process during a blast. The deformation of rock around a blast hole can be divided into four parts: (1) Explosion cavity zone; (2) Crushed zone; (3) Fractured zone; (4) Seismic zone (US Army Corps of Engineers, 1972). There is mainly a hydrodynamic process within the first zone. The second and third zones can be considered as transitional, non-linear deformation zones (Kutter and Fairhurst, 1971). The fracture phenomena range from severe crushing through plastic deformation to partial fracturing through shock waves, stress waves and gas pressures. The fourth zone is the seismic zone, where the stress is below the elastic limit of rock and no fragmentation occurs. In other words, the ground vibrations which are measured and analyzed in blast engineering usually exist within this area. The four zones are shown in Figure 2-3.

Figure 2-3 Fracture zones in a blast around a blast hole (US Army Corps of Engineers, 1972)

It is necessary to analyze the interaction between holes through inputs and outputs of a system and the superposition of waves. As shown in Figure 2-4, if the inputs are the
explosive charges in the holes (labelled a through f), the interaction between holes can be linear or non-linear depending on their spacing. If two holes are separated far away (for the scenario of hole 1 and hole 2), there is no interaction between them. Because of this, the output will be the linear superposition of ground vibrations from hole 1 and hole 2. When two holes are close enough, as with hole 3 and hole 4, their fracture zones began to affect the non-linear deformation of each other. If the two holes are delay fired, the prior fired hole will have a screening effect (an influencing factor) on the latter fired hole. When two holes are very close like hole 5 and hole 6, the non-linear effect between the two holes is so significant that the two holes can be viewed as one, as if they were initiated simultaneously. The three scenarios described above have been well illustrated by Blair (2004). Under this condition, linear superposition may not be suitable for the second and third scenarios and the system cannot be assumed as a LTI system.

Despite previous rationale, the system, taken as the seismic zone (all the rock mass) between the fractured zone of a hole and the measurement point, can be viewed as both linear and time-invariant. This approach is accurate if the ground vibration is measured within the seismic zone where there is no significant rock damage, and by assuming that the input that generates the vibrations happens in the first three zones, as classified above. This includes all the non-linear processes inside the area labelled a’ through f’. In a long-term timescale (millions to hundred million years), the earth is changing due to its internal and external forces. However, in a short timescale (hours to decades) the rock mass can be viewed as stable and invariant. Therefore, for a production blast, the rock medium can be viewed as LTI system under this condition.
2.2 Blast vibration’s direct and inverse problems

2.2.1 Direct problems in blast vibrations

In reflective seismology, the output can be modeled as a convolution of a wavelet with a reflectivity function plus noise. The wavelet here mainly refers to a simple waveform which is the response of the earth to a sharp seismic disturbance (explosion/detonation) (Ricker, 1940), and the reflection is a primary reflection. The reflectivity function can be considered as a Dirac (continuous-time) or Kronecker (discrete-time) comb with varying spacing and amplitudes. Other insignificant waves in the seismic reflection analysis can be taken as noise, such as: direct waves, refraction waves, surface waves, multiple reflections, and other random noise. Figure 2-2 also shows the convolution concept.

Blast vibrations are physically continuous-time signals when travelling through the earth medium, but the signals processed on a computer are sampled from the actual continuous signals. Because all the signals in this research are digital signals, choose discrete-time forms of equations in the following context.

The discrete-time convolution in Figure 2-2 can be expressed as:
\[ g[n] = x[n] + noise[n] = \sum_{\tau=0}^{n} r[\tau]w[n-\tau] + noise[n] \]  
\[ = r[n] * w[n] + noise[n] \]  

Where \( g[n] \), \( n = 0, 1, 2, ..., M \) is the ground vibration signal induced by a single hole, 
\( x[n] \) is noise free signal, 
\( w[n] \) is the wavelet, and 
\( noise[n] \) is the noise. 
\( r[n] \) is reflection function, and is written as 
\[ r[n] = \sum_{i=1}^{R} a_i \delta[k - t_i], n = 0,1,2, ..., t_R \]  

Where \( t_i \) is the time when reflection happened. 
\( a_i \) is the amplitude of reflectivity.

The inverse problem of the convolution model in Figure 2-2 is the widely used deconvolution methods in geophysical studies, including predictive deconvolution (Robinson, 1954), homomorphic deconvolution (Ulrych, 1971), Kalman filter deconvolution (Bayless and Brigham, 1970; Crump, 1974), etc. Its purpose is to explore the subsurface structure of the earth in order to discover oil, gas, or mineral reserves by recovering the reflection function in Equation (7).

In mining engineering, a production blast is a full-scale blast compared to a single-hole blast and is usually ripple-fired. That is, the blast contains multiple holes which are detonated in a certain timing sequence. Each hole is an input to the system represented by an impulse and generates an output (a single-hole vibration waveform) described by Equation (6) at the recording location. For the conventional signature hole method, according to its assumptions, all the holes can be combined into an impulse train and form another convolution model. Figure 2-5 illustrates the concept of ripple-fired blast convolution. It can be expressed as:
\[ y[n] = \sum_{\tau=0}^{n} d[\tau] g[n - \tau] = d[n] * g[n], (n = 0, 1, 2, \ldots, N) \]  

(8)

Where \( g[n] \) is the single-hole vibration waveform, which is the response of the ground vibration system due to the detonation of a single hole.

\( d[n], (n = 0, 1, 2, \ldots, L) \) represents an impulse train which happens at the actual firing times. Each impulse represents a single-hole blast event which generates ground vibrations in a seismic zone. In the discrete time domain, \( d[n] \) can be written as a Kronecker comb function:

\[ d[n] = \sum_{i=0}^{D} b_i \delta[k - t_i], (n = 0, 1, \ldots, L) \]  

(9)

where \( t_i (i = 0, 1, \ldots, D) \) is the firing time of each charge with \( t_0 = 0 \), \( b_i \) is the amplitude coefficient of each impulse. For the signature hole method, \( b_i = 1 \).

Then, the impulse train becomes:

\[ d[n] = \sum_{i=0}^{D} \delta[n - t_i], (n = 0, 1, \ldots, L) \]  

(10)

Assuming that \( d[n] = (d_0, d_1, \cdots d_L) = (1, 0, \cdots 0, 1_{t_2}, 0, \cdots 0, 1_{t_D}); g[n] = (g_0, g_1, \cdots g_M) \), where; \( y[n] = (y_0, y_1, \cdots y_{L+M}) \). The matrix form of Equation (8) is expressed as
From Equation (11) it is possible to see that the output of the LTI system in Equation (8) can be expressed as a linear superposition of identical single-hole vibration signals delayed by the firing times $t_i = (t_1, t_2 \cdots t_D)$. This delayed superposition can be graphically illustrated in Figure 2-6.

Figure 2-5 Schematic diagram of conventional signature hole method in a mining blast
The convolution model of Equation (8) for the conventional signature hole method assumes identical single-hole vibrations waveform for every hole. However, each hole will generate a different vibration waveform due to several reasons: (1) The screening effect on a hole resulting from previously blasted holes. (2) Rock and soil medium is anisotropic due to in-situ factors including geology, joint rock mass systems, underground seepage, etc. This makes the wave propagation path between each hole and the measurement point vary spatially. (3) Random factors during the blast operation and process, such as: drilling, loading of holes with explosives, the detonation system and random noise.

If all the variabilities mentioned above are considered, the single-hole vibration waveforms in Figure 2-5 and Figure 2-6 should change hole to hole. The convolution equations in Equation (8) is not suitable under this condition. However, the superposition principle still holds. So, the problem can still be expressed graphically in Figure 2-7 and Figure 2-8. This methodology, including the hole-to-hole variability, is called the improved signature hole technique (Silva-Castro, 2012), and it also can be viewed as a statistical signature hole method.
2.2.2 Inverse problems of blast vibrations

Obviously, the process that all single-hole vibration waveforms superpose to production blast ground vibrations is a direct problem. The purpose of its inverse problem is to find the actual timing sequence or single hole vibration waveform(s). Throughout this dissertation, the issue of interest is to recover single-hole vibration waveforms with an assumption that the timing sequence is known. This assumption will be analyzed and discussed in the following chapter.

The aforementioned analysis already made clear that each blast hole produces a different ground vibration waveform. Therefore, for the convolutional model of the conventional signature hole method described by Equations (8) to (11) and Figure 2-5 to Figure 2-6, the
inverse problems are solved by a class of deconvolution methods, and the solution gives a normalized single-hole waveform which has averaged the variabilities among all single-hole blast vibrations. The concept is shown in Figure 2-9.

Figure 2-9 Inversion of convolutional signature hole method

For the statistical signature hole method depicted in Figure 2-7 and Figure 2-8, a statistical waveform synthesis methodology is expected to be developed, and the solution should give a series of statistically varying single-hole vibration waveforms. The concept is shown in Figure 2-10.

Figure 2-10 Inversion of statistical signature hole method

In addition, Figure 2-6 and Figure 2-8 reveal a relationship of waveform lengths among single hole vibrations, the comb function and the production blast vibrations. The length of the single hole vibration waveform, \( g[n] \) or \( g_i[n] \), the comb function, \( d[n] \), and the production blast vibration waveform, \( y[n] \) are denoted by \( L_g = M + 1 \), \( L_{comb} = L + 1 \) and \( L_y = N + 1 \), respectively. If the durations of \( g[n] \), \( d[n] \) and \( y[n] \) are denoted by \( l_g \), \( l_{comb} \), and \( l_y \), respectively, they are one sample point shorter than their corresponding lengths. That is,
According to Equation (11), the waveform lengths have the following relationship,

\[ L_y = L_g + L_{comb} - 1 = M + L + 1 = N + 1 \] (13)

So, there is an equation

\[ M + L = N \] (14)

Which is actually

\[ l_g + l_{comb} = l_y \] (15)

Therefore, it’s necessary to pay attention to the duration of synthetic single hole waveforms in practical operations. It cannot be longer than the production blast waveform. Ideally, there should be a difference equal to comb function duration between the synthetic single-hole waveform and the measured production blast waveform. So, if the synthesis result is too long, truncation is needed.

Finally, the purpose of this dissertation research is summarized in Equation (16).

\[
\begin{align*}
  d[n] & \ast g[n] = y[n] & \text{deconvolution with } d[n] \text{ known} & \Rightarrow \hat{g}[n] \\
g_1[n] \cdots g_i[n] \cdots g_D[n] & \Rightarrow y[n] & \text{statistically synthesize with } d[n] \text{ known} & \Rightarrow \hat{g}_i[n]
\end{align*}
\] (16)
Chapter 3 TIMING PERFORMANCE OF DETONATORS

In Equation (16), if \( y[k] \) is the only available data, the solution of the inverse problems should give results of both \( d[k] \) and \( g[k] \) (\( g_n[k] \)). That is, the solution is carried out by blind deconvolution or blind source separation, which is beyond the scope of this dissertation. With the assumption that \( d[k] \) is known, only \( g[k] \) (\( g_n[k] \)) is to be solved, and thus the obtainment of single-hole vibration waveforms from measured ground vibrations is reduced to a normal deconvolution or a practical statistical waveform synthesis methodology. Therefore, only stable and reliable timing performance of detonators can justify the assumption about \( d[k] \). In this chapter, the performance and characteristics of both electronic and pyrotechnic detonators will be analyzed in order to verify which type of detonator: electronic or pyrotechnic, can justify the assumption of \( d[k] \) as a known variable.

3.1 Scatter of Delay Time

High precision detonators were introduced in the 1960s and are very similar to the detonators used today. In general, there are two types of detonators: pyrotechnic and electronic detonators. In the pyrotechnic system, the delay is given by the length and burning rate of the chemical component. Therefore, varying the length of this element varies the time delay. In the electronic system, the time delay is controlled by an electronic circuit. Figure 3-1 shows the components of the two types of detonators and evolution of delay scatter.
It is very difficult to find detailed technical information about scatter in firing times for both initiation systems (pyro and electronic). Usually a manufacturer includes the nominal delay in the pyrotechnic type and the programmable firing time range in the electronics, without any other information about the scatter of the system.

The University of Kentucky Explosives Research Team (UKERT), through various studies, has tested pyrotechnic and electronic detonators with different delays. The delays used here were chosen based on a particular blasting application in a surface coal mine in West Virginia. The tests follow the testing setup proposed by Lusk et al. (Lusk et al., 2012). Table 1 shows the results.

Table 1 Summary of experimental results

<table>
<thead>
<tr>
<th>Detonator type</th>
<th>Nominal or programmed delay (ms)</th>
<th>Total detonators</th>
<th>Delay average (ms)</th>
<th>Standard deviation (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyrotechnic</td>
<td>25</td>
<td>59</td>
<td>27.751</td>
<td>0.765</td>
</tr>
<tr>
<td></td>
<td>100</td>
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<td>102.73</td>
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<td></td>
<td>700</td>
<td>59</td>
<td>715.71</td>
<td>6.195</td>
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<tr>
<td>Electronic</td>
<td>10</td>
<td>53</td>
<td>9.95</td>
<td>0.092</td>
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<td></td>
<td>25</td>
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</tr>
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<td></td>
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<td>20</td>
<td>675.33</td>
<td>0.417</td>
</tr>
<tr>
<td></td>
<td>700</td>
<td>20</td>
<td>700.22</td>
<td>0.342</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>43</td>
<td>1000.543</td>
<td>0.321</td>
</tr>
<tr>
<td></td>
<td>1400</td>
<td>20</td>
<td>1400.496</td>
<td>0.618</td>
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</table>

In this problem, the firing times of detonators are assumed to follow a normal distribution. The Shapiro-Wilk test (Shapiro and Wilk, 1965) was used to verify this assumption for the data of each delay time. The results are included in Figure 3-2.
For the Shapiro-Wilk test, the null hypothesis ($H_0$) is that the data is normally distributed. The chosen $\alpha$-level is 0.05. If the p-value is smaller than 0.05, the null hypothesis is rejected, and there is evidence that the data does not follow a normal distribution. As shown in Figure 3-2, the p-values of tests for pyrotechnic 100 ms, 700 ms, and electronic 675 ms are greater than 0.05, so the null hypothesis of the normal distribution cannot be rejected. However, for pyrotechnic 25 ms and electronic 700 ms, the p-values are smaller than 0.05. Thus, the null hypothesis for the two tests is rejected, and the collected data is concluded to not be normally distributed. The failure of the normality tests may be attributed to an insufficient sample size. However, for the problem under analysis, the normal distribution assumption is maintained. So, Figure 3-3 shows the distributions of delay times summarized in Table 1.
From Figure 3-3 it can be seen that the average values of delay time for electronic detonators are closer to their nominal delays compared to pyrotechnic detonators. So, it is a very accurate system. Also as observed in Figure 3-3, there is a positive relationship between the nominal delay time and the standard deviation for both systems. In other words, when the delay time increases, the standard deviation increases.

In order to find if such a trend is true for other detonator manufacturers besides those tested in UKERT, results from other sources were compiled in a database. The data of pyrotechnic detonator are included in Table 2, and those of electronic detonators is in Table 3.

Table 2 Scatters of delay times of pyrotechnic detonators

<table>
<thead>
<tr>
<th>Nominal delay (ms)</th>
<th>Standard deviation (ms)</th>
<th>Nominal delay (ms)</th>
<th>Standard deviation (ms)</th>
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<td>175</td>
<td>5.9</td>
<td>205</td>
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(Winzer et al., 1979)
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<td></td>
<td>50</td>
<td>5.3</td>
<td>75</td>
<td>1.4</td>
</tr>
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<td></td>
<td>75</td>
<td>11.8</td>
<td>100</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>5.1</td>
<td>125</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>6.2</td>
<td>150</td>
<td>5.7</td>
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<tr>
<td></td>
<td>150</td>
<td>5.7</td>
<td>175</td>
<td>7.8</td>
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<tr>
<td></td>
<td>175</td>
<td>7.5</td>
<td>200</td>
<td>5.8</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>7.5</td>
<td>250</td>
<td>3.9</td>
</tr>
</tbody>
</table>

**Set 2**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>12.5</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>12.5</td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>20.1</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>17.1</td>
<td></td>
</tr>
<tr>
<td>450</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>22.3</td>
<td></td>
</tr>
<tr>
<td>550</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>650</td>
<td>29.5</td>
<td></td>
</tr>
<tr>
<td>750</td>
<td>36.2</td>
<td></td>
</tr>
<tr>
<td>875</td>
<td>36.6</td>
<td></td>
</tr>
</tbody>
</table>
Table 3 Scatters of delay times of electronic detonators

<table>
<thead>
<tr>
<th>Nominal delay(ms)</th>
<th>Standard deviation(ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.092</td>
</tr>
<tr>
<td>25</td>
<td>0.117</td>
</tr>
<tr>
<td>100</td>
<td>0.104</td>
</tr>
<tr>
<td>Tests for this research</td>
<td>675</td>
</tr>
<tr>
<td>700</td>
<td>0.342</td>
</tr>
<tr>
<td>1000</td>
<td>0.321</td>
</tr>
<tr>
<td>1400</td>
<td>0.618</td>
</tr>
</tbody>
</table>

Information from Table 2 and Table 3 was plotted in Figure 3-4 for pyrotechnic and electronic initiation systems, respectively. In a previous paper published in 2015, (Li and Silva-Castro, 2015), the data were fitted by linear curves. By adding more data into Table 2 and Table 3, it shows that a second-order polynomial gives a higher value of $R^2$.

In Figure 3-4, it is evident that for both initiation systems, the standard deviation increases as the nominal delay time increases. The increase rate of scattering along with the nominal delay for pyrotechnic system gets larger with a large value of nominal delay. On the contrary, the scattering curve of the electronic system looks like a flat line compared to that of the pyrotechnic system. That is, the scatter of firing times for electronic initiation system keeps within a very low value range (0.1ms – 0.6ms) regardless of the programmed delay time. In other words, it is a very precise system, as expected.
3.2 Performance of Two Adjacent Holes

3.2.1 Description of particular situation under research

After analyzing the accuracy and precision of both pyrotechnic and electronic detonators, further analysis of the performance of two adjacent holes for both initiation systems can be done, as to observe the accuracy and precision of electronic detonators from a different perspective. For two adjacent holes, the key parameter is their delay interval. As mentioned before, based on a particular blasting application in a surface coal mine in West Virginia, it is common to use a delay time of 25 ms between explosive charges in a single hole in a production shot. This particular situation is explained in Figure 3-5.
At this particular mine, some problems in the production shot started to be evident when the timing for the blast was greater than 600ms (flyrock events and high vibration recordings). For that reason, the analysis included in this chapter was performed for timings of 675 and 700ms.

If an electronic system is used, the 25ms delay between charges can be reached just programming the detonators to 675ms for the upper charge and 700ms for the lower charge (Figure 3-6a). On the other hand, if the pyrotechnic system is used, because of products limitations the 25ms between charges can be obtained using two in-hole detonators of 700ms and one surface delay device of 25ms (Figure 3-6b).

Figure 3-5 Timing arrangement under analysis

Figure 3-6 Timing scenarios using electronic and pyrotechnic initiators

a. Arrangement for electronic initiation

b. Arrangement for pyrotechnic initiation
3.2.2 Analysis of the random variables of the problem

Before any analysis, it is necessary to define the independent random variables and their distributions for this particular problem (see Figure 3-7). In general, let $t_k$ denote the nominal firing time $k$, its mean and standard deviation are $\mu_k$ and $\sigma_k$, respectively. Also, $t_{k(max)}$ and $t_{k(min)}$ are the maximum and minimum value in the sample of detonator timing tests for the respective nominal firing time $k$. In the same way, the term $t_{k+1}$ can be defined as the next adjacent nominal firing time $k + 1$, its mean and standard deviation will be $\mu_{k+1}$ and $\sigma_{k+1}$. Similarly, $t_{k+1(max)}$ and $t_{k+1(min)}$ are the maximum and minimum value in the sample of m detonator for the nominal firing time $k + 1$. In this problem, it is assumed that the variables are independent and they follow a normal distribution.

![Figure 3-7 Variables definition](image)

As shown in the Figure 3-6, the objective of this analysis is to assess the performance of both initiation systems designed to achieve a 25 ms delay time between charges in the same hole.

The calculation of the 25 ms delay time is straightforward for the electronic system and can be done just using the results from nominal times of 675 and 700 ms. However, in the case of a pyrotechnic system, it is necessary to include the delay time given by the surface delay element. For pyrotechnic detonators, using the linear combination of the results for the nominal firing time of 700 ms and the results from the surface delay of 25 ms, the probabilistic parameters for a firing time of 725 ms was calculated. In this way, for $t_{725}$,
the average value was calculated as \( \mu_{725} = \mu_{700} + \mu_{25} \) and its standard deviation; \( \sigma_{725} = \sqrt{\sigma^2_{700} + \sigma^2_{25}} \). The statistics of the random variables are summarized in Table 4.

Table 4 Summary of random variables in this analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Electronic</th>
<th>Pyrotechnic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_k )</td>
<td>( t_{675} )</td>
<td>( t_{700} )</td>
</tr>
<tr>
<td>( \mu_k )</td>
<td>( \mu_{675} = 675.33 \text{ ms} )</td>
<td>( \mu_{700} = 715.710 \text{ ms} )</td>
</tr>
<tr>
<td>( \sigma_k )</td>
<td>( \sigma_{675} = 0.417 \text{ ms} )</td>
<td>( \sigma_{700} = 6.195 \text{ ms} )</td>
</tr>
<tr>
<td>( t_{k(\max)} )</td>
<td>( t_{675(\max)} = 675.90 \text{ ms} )</td>
<td>( t_{700(\max)} = 730.575 \text{ ms} )</td>
</tr>
<tr>
<td>( t_{k(\min)} )</td>
<td>( t_{675(\min)} = 674.580 \text{ ms} )</td>
<td>( t_{700(\min)} = 697.925 \text{ ms} )</td>
</tr>
<tr>
<td>( t_{k+1} )</td>
<td>( t_{700} )</td>
<td>( t_{725} )</td>
</tr>
<tr>
<td>( \mu_{k+1} )</td>
<td>( \mu_{700} = 700.220 \text{ ms} )</td>
<td>( \mu_{725} = 743.461 \text{ ms} )</td>
</tr>
<tr>
<td>( \sigma_{k+1} )</td>
<td>( \sigma_{700} = 0.3421 \text{ ms} )</td>
<td>( \sigma_{725} = 0.3421 \text{ ms} )</td>
</tr>
<tr>
<td>( t_{k+1(\max)} )</td>
<td>( t_{700(\max)} = 701.03 \text{ ms} )</td>
<td>( t_{725(\max)} = 759.879 \text{ ms} )</td>
</tr>
<tr>
<td>( t_{k+1(\min)} )</td>
<td>( t_{700(\min)} = 699.84 \text{ ms} )</td>
<td>( t_{725(\min)} = 724.080 \text{ ms} )</td>
</tr>
</tbody>
</table>

3.2.3 Success, crowding and overlapping concepts

To measure the performance of a delay time system in this problem, it is necessary to introduce the concepts of success, crowding, overlapping and off-design, in order to measure the performance of a delay time system. These concepts are related to the time window for which each classification in the delay time performance is valid. Figure 3-8 shows the mentioned concepts.

Figure 3-8 Success, crowding, overlapping and off-design concepts (Li and Silva-Castro, 2015)
Ideally, two detonators can fire exactly at designed firing times. However, both electronic and pyrotechnic initiation systems have scatter, it is necessary to introduce a definition of the success time window. At first sight, success can be defined as the detonation of the delayed charges within a time window (detonation of charges a and b in Figure 3-8). How wide the time window for success should be related to the standard adopted for the analysis.

The success range is defined as that of nominal delay interval ($\mu = 25ms$) plus/minus a certain value $b$, that is, $(\mu - b, \mu + b)$. The remaining problem is to determine the value of $b$.

In statistics, the coefficient of variation is used as a relative measure of the scatter of a random variable. Mathematically, the coefficient of variation is defined as (Harr, 1987)

$$CV(x) = \frac{\sigma(x)}{\mu(x)} \times 100\%$$

(17)

Where $x$ is a random variable,

$\mu(x)$ and $\sigma(x)$ are its mean and standard deviation respectively,

$CV(x)$ is coefficient of variation.

The coefficient of variation indicates the central tendency of a distribution. Higher coefficient of variation represents greater scatter. As a rule of thumb, low coefficients of variation are below 10% - 15%. A value of 30% is considered as moderate and greater than 30% is high. Here in this research, the coefficient of variation for a delay interval was limited to 5% as a strict standard. In other words, the standard deviation $\sigma(x)$ should be lower than $5\% \cdot \mu(x)$.

Based on this idea, let $b = 5\% \cdot \mu$, then the success range becomes to (23.75ms, 26.25ms). In other words, any delay interval in the production shot falling within this range is considered as a success.

Overlapping is defined as the event when two adjacent periods of detonators fire with a delay interval shorter than 8 ms (detonation of charges a and c in Figure 3-8) This assumption is based on the 8 ms rule specified in the federal regulations code (CFR, Title
30, § 816.67), which means that all charges firing within 8-millisecond interval will be considered as simultaneous detonation.

The range between overlapping and success can be defined as "crowding", which is a “grey” zone or transition region between overlapping and success. The probability of crowding means the possibility that the delay interval will be crowded in between the success and overlapping regions. Furthermore, the transition region of crowding between less crowded and crowded should include weights for different degrees of risk (e.g. high risk for the detonation of charges a and d in Figure 3-8). Here the term “risk” is related to practical problems (flyrock or high vibration) which may happen in production blasts. Finally, the time window above success is undesired. If a charge detonates beyond the desired delay time (detonation of charges a and e in Figure 3-8), it can generate problems for the next firing charge. This area was denoted as “off-design”. The boundaries for each zone can be summarized as (-∞←overlapping→8ms←crowding→23.75ms←success→26.25ms←off-design→∞). Moreover, they can be sorted in a descending order of severity: overlapping> crowding (high, medium, low)> off-design.

3.2.4 Reliability of single delay interval
There have been random variables $t_k$ and $t_{k+1}$. Take $t_n = t_{k+1} - t_k$ as a new variable. It’s the linear combination of $t_k$ and $t_{k+1}$. According to the theory of linear combination of normal independent random variables, $t_n$ also follows normal distribution, and its mean $\mu_n = \mu_{k+1} - \mu_k$ and standard deviation $\sigma_n = \sqrt{\sigma_k^2 + \sigma_{k+1}^2}$. Using linear combination theory, it is possible to calculate the normal distribution for the nominal delay interval of 25 ms for both systems (electronic and pyrotechnic).

Figure 3-9 shows the results.
To assess the timing performance of two adjacent holes, take the delay interval between the two holes as a variable and analyze its reliability which is the probability of success. Based on the definitions above, the reliability for a single delay interval can be calculated by Equation (18).

$$R(I_k) = P(23.75 \leq I_k \leq 26.25) = \int_{23.75}^{26.25} N(\mu_k, \sigma_k) dI_k$$  \hspace{1cm} (18)

Similarly, the probabilities of overlapping, crowding, and off-design can be computed. The results for both pyrotechnic and electronic initiations are summarized in Table 5.

<table>
<thead>
<tr>
<th>Component state</th>
<th>Probability Pyrotechnic</th>
<th>Probability Electronic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overlapping</td>
<td>0.0124</td>
<td>0.0000</td>
</tr>
<tr>
<td>Crowding</td>
<td>0.3121</td>
<td>0.0464</td>
</tr>
<tr>
<td>Success (Reliability)</td>
<td>0.1077</td>
<td>0.9046</td>
</tr>
<tr>
<td>Off-design</td>
<td>0.5678</td>
<td>0.0490</td>
</tr>
</tbody>
</table>

The analysis tells us some conclusions about the timing performance of detonators for a single delay interval (two adjacent holes):
• The scatter of delay time of electronic detonators is much smaller than that of pyrotechnic detonators.
• The test results of electronic detonators are both accurate and precise. This larger scatter for pyrotechnic detonators serves as a big factor inducing higher overlapping probability.

When the delay interval is 25ms, the overlapping probability for electronic detonators is 0, while that of pyrotechnic detonators is 0.0124. This case study is a robust proof of electronic detonator’s high accuracy and precision. Correspondingly, electronic detonators have an overwhelmingly higher chance of success (more than 8 times) than pyrotechnics.

3.3 Conclusion of this chapter
When electronic detonators are used, the actual firing times approximate the designed timing compared to pyrotechnic detonators. In other words, the comb function can be viewed as known when electronic detonators are used.
Chapter 4 PRELIMINARY ANALYSIS OF DECONVOLUTION

4.1 Introduction

Using the convolution equation in Equation (8), it is possible to explain the target of the deconvolution. In Equation (9), \( y(k) \) is the measured production blast ground vibration signal and the problem is to estimate \( g(k) \) from this measured production blast signal, assuming that \( d(k) \) is known. Defined in this way, the deconvolution procedure will result in a single normalized single-hole vibration waveform. This chapter presents the field data collected for the application of deconvolution techniques. It also shows the procedures and results when some of the deconvolution techniques explored for the blast vibration problems.

4.2 Test data description

The collected field data used in this dissertation contains five (5) different sets of ground vibration signals. They have different numbers of holes: 6, 27, 47, 62, and 83. The 6-hole data is from a test conducted at Guyan surface coal mine in West Virginia in 2011 (Silva-Castro, 2012). The other four data sets are from a different surface coal mine also in West Virginia. Every data set contains a production blast event and at least one single-hole vibration waveform. The sample rate used to collect the vibration waveform information was 1024 samples/sec (typical for mining applications), and this is the same for all sets of data.

For the 6-hole data, there were 8 blast holes and 3 signature holes in this test. The layout of the test is summarized in Table 6 and Figure 4-1.

<table>
<thead>
<tr>
<th>Holes</th>
<th>Hole diameter (in)</th>
<th>7.875</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Hole depth (ft)</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>Number of holes</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Spacing (ft)</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Burden (ft)</td>
<td>17</td>
</tr>
<tr>
<td>Timing</td>
<td>Detonator</td>
<td>Electronic</td>
</tr>
<tr>
<td></td>
<td>Delay interval of blast holes (ms)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Intervals for signature holes (ms)</td>
<td>1490 - 3000</td>
</tr>
</tbody>
</table>
In Figure 4-1, it is shown that the first six holes are initiated by delay interval of 5 ms, then followed by two signature holes in 3000 ms and 6000 ms. Two additional blast holes and one signature hole are detonated after the first two signature holes. The ground vibrations were recorded by a seismograph located 210 m (689 ft) from the blast site. The complete record of the whole blast is included in Figure 4-2.

Figure 4-2 The 6-hole seismograph record (Silva-Castro, 2012)
For the 27-hole blast test, the geometry and layout are shown in Table 7 and Figure 4-3, respectively.

### Table 7 Configuration of the 27-hole blast test

<table>
<thead>
<tr>
<th></th>
<th>Hole diameter (in)</th>
<th>Hole depth (ft)</th>
<th>Number of holes</th>
<th>Spacing (ft)</th>
<th>Burden (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Timing</td>
<td>Detonator</td>
<td>Electronic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Delay interval of blast holes (ms)</td>
<td>20</td>
<td>Intervals for signature holes (ms)</td>
<td>2000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Figure 4-3, a delay interval of 20 ms is shown for the whole blast, then followed by a signature hole in 2000 ms (the final blast hole). The ground vibration waveform was recorded by a seismograph located 186.75 m (612.69 ft) from the blast site. The complete record of the whole blast is included in Figure 4-4.
Figure 4-4 The 27-hole seismograph record

For the 47-hole blast test, the configuration and layout are shown in Table 8 and Figure 4-5, respectively.

Table 8 Configuration of the 47-hole blast test

<table>
<thead>
<tr>
<th>Holes</th>
<th>Hole diameter (in)</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole depth (ft)</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>Number of holes</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>Spacing (ft)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Burden (ft)</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Timing

<table>
<thead>
<tr>
<th>Detonator</th>
<th>Electronic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay interval of blast holes (ms)</td>
<td>20</td>
</tr>
<tr>
<td>Intervals for signature holes (ms)</td>
<td>2920</td>
</tr>
</tbody>
</table>

Particle velocity (in/s)

Time (s)

-0.05 0 0.05 0.5 1 1.5 2 2.5 3
In Figure 4-5, the delay interval of 20 ms is shown, then followed by a signature hole in 2920 ms (the last blast hole). The ground vibration waveform was recorded by a seismograph located 150 m (492.38 ft) southwest from the blast site. The complete record of the whole blast is included in Figure 4-6.
Figure 4-6 The 47-hole seismograph record

For the 62-hole blast test, the configuration and layout are shown in Table 9 and Figure 4-7.

Table 9 Configuration of the 62-hole blast test

<table>
<thead>
<tr>
<th>Holes</th>
<th>Hole diameter (in)</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hole depth (ft)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Number of holes</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>Spacing (ft)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Burden (ft)</td>
<td>-</td>
</tr>
<tr>
<td>Timing</td>
<td>Detonator</td>
<td>Electronic</td>
</tr>
<tr>
<td></td>
<td>Delay interval of blast holes (ms)</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Intervals for signature holes (ms)</td>
<td>3240</td>
</tr>
</tbody>
</table>
In Figure 4-7, the delay interval of 20 ms is shown, then followed by a signature hole in 3240 ms. The ground vibration waveform was recorded by a seismograph located 120.37 m (394.91 ft) from the blast site. The complete record of the whole blast is included in Figure 4-8.
Finally, for the 83-hole blast test, the configuration and layout are shown in Table 10 and Figure 4-9.

Table 10 Configuration of the 83-hole blast test

<table>
<thead>
<tr>
<th>Holes</th>
<th>Hole diameter (in)</th>
<th>Hole depth (ft)</th>
<th>Number of holes</th>
<th>Spacing (ft)</th>
<th>Burden (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>83</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Timing

<table>
<thead>
<tr>
<th>Detonator</th>
<th>Delay interval of blast holes (ms)</th>
<th>Intervals for signature holes (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic</td>
<td>20</td>
<td>3140</td>
</tr>
</tbody>
</table>
In Figure 4-9, the delay interval of 20 ms is shown, then followed by a signature hole in 3140 ms. The ground vibration waveform was recorded by a seismograph located 54.56 m (179 ft) from the blast site. The complete record of the whole blast is included in Figure 4-10.
Figure 4-10 The 83-hole seismograph record

All the data was collected with seismographs typically used to monitoring blast vibrations from mines. The application of various deconvolution techniques are included.

4.3 Spectral Division

4.3.1 Fourier series and Fourier transform

From the study of signals and systems, it is accepted that a time domain signal can also be represented by a frequency domain signal. For a periodic signal, a Fourier series can be used. The Fourier series for a periodic function \( x(t) \) with a period of \( T \) can be represented as (Oppenheim, 1997; Yang, 2009)

\[
x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + \sum_{k=1}^{\infty} b_k \sin k\omega_0 t
\]

Where \( \omega_0 = 2\pi/T \);

\[
a_0 = \frac{1}{T} \int_0^T x(t) dt;
\]

\[
a_k = \frac{2}{T} \int_0^T x(t) \cos k\omega_0 t dt;
\]

\[
b_k = \frac{2}{T} \int_0^T x(t) \sin k\omega_0 t dt.
\]
Equation (19) can also be written as magnitude and phase form:

\[ x(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k) \]  \hspace{1cm} (20)

Where \( A_k = \sqrt{a_k^2 + b_k^2} \);

\[ \theta_k = \tan^{-1}(-b_k/a_k). \]

While for an aperiodic signal, Fourier transform is used (Oppenheim, 1997):

\[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t} d\omega \]

\[ X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \]  \hspace{1cm} (21)

Equation (21) is referred to as the Fourier transform pair. When conducting Fourier transform on a computer, usually Fast Fourier Transform is used instead of the continuous time equation.

4.3.2 Convolution from time domain to frequency domain

By transforming a signal from time domain to frequency domain through Fourier transform, it is possible to conduct a deterministic spectral division deconvolution.

Apply Equation (9) into Equation (8),

\[ y[n] = d[n] \ast g[n] = \sum_{i=1}^{D} a_i \delta[n - t_i] \ast g[n] \]  \hspace{1cm} (22)

Taking the discrete Fourier transform to Equation (22),

\[ Y(f) = D(f) \cdot G(f) \]  \hspace{1cm} (23)

If \( d[n] \) is known, the discrete Fourier transform of a single-hole vibration waveform can be estimated by

\[ G(f) = \frac{Y(f)}{D(f)} = \frac{Y(f)}{\sum_{i=1}^{L} a_i e^{-j2\pi f \cdot t_i}} \]  \hspace{1cm} (24)
The actual firing time, $t_i$, can be measured by a special measurement setup (Birch et al., 2011; Hinzen et al., 1987). As reviewed in chapter 3, if electronic detonators are used in the blast, the actual firing times deviate from the nominal delay by only a small error, so the designed timing sequence in this situation may be approximately used as the actual firing times. For simplicity, assume the amplitude $a_i$ as one (1).

Taking the inverse discrete Fourier transform of $G(f)$, the single-hole vibration waveform $g[n]$ can be estimated. Note that the deconvolution result has the same length to $G(f)$ in the frequency domain. As long as the actual duration of $g[n]$ after attenuation approximately meets the relationship in Equation (15), the deconvolution result is acceptable.

4.3.3 Effective range of magnitude spectrum

Magnitude is the more commonly used constituent part of Fourier transform, so it is necessary to first investigate magnitude spectrum a little bit more in detail.

![Graphs showing magnitude spectrum and cumulative integral](image)

**a. Full range magnitude**

**b. Truncated magnitude and $f_{lim}$**

Figure 4-11 Typical magnitude spectrum of transverse component from a production blast (No. of delays: 78; Delay interval: 10ms)

Figure 4-11 shows a typical magnitude spectrum of a production blast by Fourier transform. It is observed that the magnitude tends to diminishes to zero beyond a certain frequency. That is, the frequency components higher than this certain frequency scarcely contribute to the ground vibrations. So, focus is on the main part under this frequency for a specific blast. This certain frequency value may be called “effective frequency limit” ($f_{lim}$). Different blasts may have various values of $f_{lim}$. It is hard to uniquely determine this value. Instead,
it can be selected within a range as long as it covers all the dominant frequency zones. For example, in the magnitude spectrum in Figure 4-11, the effective frequency limit can be a frequency between 55Hz and 80Hz. However, a quantitative method is needed to select a value within this range. After a series of trials, the effective frequency limit can be determined as the frequency corresponding to 0.98 quantile of magnitude’s cumulative integral with respect to frequency. For the magnitude shown in Figure 4-11, \( f_{lim} \) is determined as 59.5 Hz.

Note that the magnitude spectrum is truncated at 100 Hz to eliminate any influence from higher frequency noise before determining \( f_{lim} \). The truncation frequency can be different depending on the energy distribution of the magnitude spectrum so that it is beyond the significant part of the magnitude spectrum.

4.3.4 Illustration of computation

4.3.4.1 Instability of division

In this analysis, the production blast is the 6-hole blast. The radial components of the production blast waveform and three single-hole vibration waveforms were used to demonstrate the methodology procedure. However, it can be applied to any other number of holes and component. Figure 4-12 shows the details of the vibration waveforms for the radial components.
To perform spectrum division, time-domain signals must be transformed into Fourier spectrums in the frequency domain. A Fast Fourier transform is used to perform this task. During the frequency-domain operation, there are two issues of importance: 1) instability due to zeros in the denominator and 2) undesirable content in the higher frequency range.

The comb function $d[n]$ only contains 6 delta functions and is shown in Figure 4-13a. Compared to the measured ground vibration signal of the production blast $y[n]$, the comb function is much shorter than the six-hole waveform (only 25 ms compared to 1000 ms). Spectral division requires the Fast Fourier transform of the production blast and the comb function have the same size. The FFT size is selected as the next highest power of 2 of the length of $y[n]$. So, the comb function has to be padded with zeros to the same length as the discrete Fourier transform. The magnitude spectrum of the comb function is shown in Figure 4-13b.
From Figure 4-13b, a zero point at 512 Hz can be found in the $D(f)$ after padding zeros in $d[n]$. Because $D(f)$ is in the denominator of Equation (24), it is possible to make the solution unstable and obtain results with infinite values (zero divisions).

However, reviewing the transform results, the number of zero points is finite. So, other values can be used to substitute the zeros and keep the continuity of the curve at the same time. One method is to replace the zero point by the previous or following point, depending on which one has a smaller magnitude. If the two adjacent points are of the same magnitudes, the prior one is used as the substitute. Note that the substitution is for the complex numbers, not simply for the magnitudes. Figure 4-14 shows the detailed local magnitude spectrum of comb function and substitution of a zero point. The values in the dash line boxes are those used for substitution of the zero point.

Figure 4-13 Comb function and its amplitude spectrum
4.3.4.2 Operations of spectral division

After fixing the zero point problem, it is possible to calculate Equation (24). The FFT operation can be applied to the six-hole waveform, $y[n]$, and get $Y(f)$. By applying $Y(f)$ and the modified $D(f)$ into Equation (24), then the spectral division can be performed to get the result $G(f)$. The magnitude spectra of the denominator, numerator and quotient of Equation (24) are shown in Figure 4-15.
Figure 4-15 Graphical illustration of the spectral division methodology

In Figure 4-15, the main frequency zone of $Y(f)$ in the numerator is concentrated below 30 Hz which is followed by a relatively flat tail between 30 Hz and 50 Hz, then the spectrum of $Y(f)$ becomes almost close to zero after 50 Hz. Compared to the spectrum of $Y(f)$, the spectrum of $G(f)$ in the quotient is also expected to have a similar but flatter shape within the same main frequency zone below 30 Hz. However, there are multiple large peaks which are abnormal contents among the spectrum of $G(f)$, which makes the expected contents look like minor parts by contrast. This is caused by the extremely small values in the denominator, even if the spectrum of $Y(f)$ is very flat after 30 Hz. The correspondence of large peaks and extreme small values are indicated by the dashed lines in Figure 4-15.

The spectral division quotient results for all directions and all the five (5) case studies under analysis are included in Figure 4-16 through Figure 4-20. For a better observation, only the
frequency range of 0 to 100 Hz is shown, and the effective frequency limit ($f_{lim}$) is also included.

\[ f_{lim} = 40.25 \text{Hz} \]
Figure 4-16 Spectral division results of 6-hole blast

\[ f_{\text{lim}} = 50.5 \text{Hz} \]

\[ f_{\text{lim}} = 55 \text{Hz} \]
$f_{lm} = 85.5 \text{ Hz}$
Figure 4-17 Spectral division results of 27-hole blast
\[ f_{lm} = 64.5 \text{ Hz} \]
Figure 4-18 Spectral division results of 47-hole blast
Figure 4-19 Spectral division results of 62-hole blast
Magnitude of complete blast, 83 holes, radial

Magnitude of comb, 83 holes

Magnitude of synthetic signature, 83 holes, radial

Magnitude of measured signature, 83 holes, radial

\[ f_{lim} = 53.75 \text{Hz} \]
Figure 4-20 Spectral division results of 83-hole blast

$\text{lim}_f = 49 \text{Hz}$

$\text{lim}_f = 56.5 \text{Hz}$
4.3.4.3 Treatment of the unwanted contents after the spectral division

When deconvolution is applied to mining engineering problems, the only data available are the production blast vibration signals and the comb function. The five case studies’ data included in this document have at least one signature, though this is not always the case. After analysis of the previous results, it is observed that the effective frequency limit of the production blast waveform can also be used as the frequency limit of the single-hole vibration waveform. This assumption can be proved by observing the top and bottom figures from Figure 4-16 to Figure 4-20.

From all the figures, it is shown that the large peaks concentrate below the effective frequency limit ($f_{lim}$) within the range of 0Hz to 100Hz. In general, as the number of holes and delay interval increase, there are more condensed peaks which make the supposed magnitude spectrum of synthetic single-hole vibration waveform out of recognition. The use of filters in the analysis of signals is a customary methodology to deal with the unwanted frequency contents introduced by the peaks. The useful frequency contents of the production blast and the single-hole vibration waveform are concentrated within low frequency range. So, a low-pass filter is preferable to remove the peaks. However, the effectiveness of the low-pass filter depends on the locations of the peaks.

For the 6-hole blast, there is only one peak below $f_{lim}$ for each of the three components. Even if the peak is below the effective frequency limit, it is still beyond the dominant frequency zones of the production blast. So, if a low-pass filter is applied to remove the peak, the main part of the magnitude spectrum can still be retained. An IIR or FIR filter can be designed by using the function “designfilt” in MATLAB®. As an example, an IIR filter is generated by the following script and the filter is visualized in Figure 4-21:

```matlab
lpfilt = designfilt('lowpassiir','FilterOrder',10,...
    'PassbandFrequency',30,'PassbandRipple',0.2,...
    'SampleRate',1024);
```
However, starting from 27 holes with 20ms delay interval, the large peaks appear among the supposed dominant frequency zones, which makes the magnitude spectrum of the synthetic single-hole vibration waveform an abnormal sharp fluctuation with much higher magnitudes than that of the production blast. Under this condition, if a low-pass filter is applied to remove all the large peaks in the synthetic spectrum, all the useful frequency contents will also be removed. Therefore, spectral division deconvolution is not suitable for the situation of large delay interval or a large number of holes. In the next section, the reason why spectral division deconvolution has such limitations will be explained.

4.3.4.4 Obtaining the synthetic single-hole vibration waveforms

While the spectral division deconvolution does not work well for a extensive number of holes or large delay intervals, the methodology works well for a small number of holes and a small delay interval. For these reasons, it will be applied to the 6-hole signals. By taking inverse discrete Fourier transform to the spectral division result, $G(f)$, one can get its time domain waveform. After applying the designed filter to this waveform, the synthetic single-hole vibration waveform $g[n]$ can be estimated. To verify the accuracy of the results, the measured single-hole vibration waveforms are used to compare with the synthetic single-hole vibration waveform in both the time domain and the frequency domain for all three direction components.

Cross-correlation coefficient is used to compare the similarity between waveforms in time domain. A general form of cross-correlation coefficient is defined below when $x$, $y$ and $l$ are two waveforms to be compared and the time lag, respectively (Alessio, 2015):
Equation (25) shows the cross-correlation is a function of time lag, \( l \), and its value lies between -1 and 1. The higher its absolute value is, the more similar the two waveforms are. A negative value means an upsidedown similarity between two signals. Usually, a cross-correlation coefficient with maximum absolute value is selected. If the time lag corresponding to the maximum absolute value is positive, it means \( y \) moves right or \( x \) moves left. On the contrary, a negative time lag means \( y \) moves left or \( x \) moves right.

The deconvolution results are included in Figure 4-22 and Figure 4-23. In Figure 4-22, the cross-correlation coefficients between the synthetic single-hole vibration waveforms and the measured single-hole vibration waveforms are between 0.4 and 0.7. The cross-correlation coefficient values are not very large (a value of one (1) is given when the signals are the same), but the results can still be considered acceptable for blast ground vibration applications, under the assumption that there are many random variables in a blast operation process and the earth medium (rock mass) is very heterogeneous.

The acceptance of low cross-correlation values among measured signatures and recovered signatures using spectral division is confirmed when the recovered signatures are used to reconstruct the production waveform. The cross-correlation between the synthetic and the measured production blast waveforms is much higher, and it can be as high as a value of 0.974. This shows that a good result of synthetic production blast waveform does not necessarily require a high cross-correlation value among single-hole vibration waveforms. It can be seen that the envelope of the synthetic single-hole vibration waveform generally matches the measured single-hole vibration waveforms. This concept also coincides with the statistical concept to solve blast vibration problems proposed in the improved methodology of signature hole technique developed by Jhon Silva (Silva-Castro and Lusk, 2012).

Another significant finding in Figure 4-22 is that the amplitude of the synthetic single-hole vibration waveform is higher than the measured ones. The reason may be the confinement condition of each hole in a production blast is different to that of a signature hole. So, sometimes a measured single-hole vibration waveform also needs calibration by
amplification or reduction to produce a reasonable production blast waveform (Silva-Castro, 2017). The synthetic single-hole vibration waveform can be viewed as a calibrated or normalized single-hole vibration waveform, which can be proved by the high correlation between the synthetic and measured production blast waveforms in Figure 4-23.

The lags in Figure 4-22 and Figure 4-23 are the time lag \( l \) values corresponding to the maximum value of cross-correlation coefficients when comparing the synthetic single-hole or six-hole vibration waveforms to the measured waveforms by Equation (25). The main causes include the time shift of a low-pass filter and the dissimilarities between waveforms due to various random factors.

Finally, it is observed that the synthetic single-hole vibration waveforms have more energy within higher frequency range compared to measured single-hole vibration waveforms. This difference is caused by the decreasing magnitude curve of comb function under 30 Hz, as shown in Figure 4-16.
Figure 4-22 Comparison between synthetic and measured single-hole vibration waveforms
Figure 4-23 Comparison between synthetic and measured production blast vibrations
4.3.5 Influence of delay interval and number of holes in the spectral division procedure

The case study in the previous section only covered 6 holes and 5ms delay interval. It is still necessary to check if the proposed spectral division methodology works for higher values of delay intervals and larger numbers of holes.

As indicated in the analysis in section 4.3.4.2, the key of spectral division deconvolution is the absence of zero values in the denominator in Equation (24). In the case study above, successfully conducting the spectral division deconvolution allows the denominator to keep relatively large values (different from zero) within the frequency range where the numerator has significant frequency contents (or effective frequency contents). The Fourier spectrum of an impulse train looks like the trajectory of a bouncing ball with crests and valleys. The first valley is a value close to zero, and it has been observed that the first valley should be located after the effective frequency content of the production shot (the numerator), to avoid a large abnormal quotient and make the deconvolution successful. This section will analyze how delay intervals and the numbers of delays influences the location of the first valley in the Fourier spectrum of the impulse train (or comb function).

Two methods in this document were used to analyze the Fourier magnitude spectrum of a comb function: (1) enlarging the delay interval while keeping the number of holes the same; (2) keep the delay interval fixed while increasing the number of holes. For the first scenario, the delay intervals are chosen as 5ms, 10ms, 15ms, 20ms, 25ms, and the number of holes is fixed as 6. For the second scenario, the delay interval is fixed as 5ms, and numbers of holes are: 6, 10, 20, 30 and 40.

The results are shown in Figure 4-24 and Figure 4-25. From the two figures it is shown that either increasing delay intervals or increasing numbers of holes can make the location of the first valley change to lower frequency values. In mining blasts, it is common to choose a delay interval as 20 or 25ms, and the number of holes in a blast easily exceeds 40. Therefore, the location of the first valley in the spectrum of the comb function will generally be under 30Hz. As previously mentioned, the spectral division method requires that the Fast Fourier transform of the production blast and the comb function have the same size. When only the range of frequencies below the first valley of the comb function is used
to recover the synthetic signature, it is necessary to filter all other frequencies above the frequency value of the first valley in the spectral division results. If a low-pass filter is applied to the spectral division results and the production waveform has useful high frequency contents above the first valley of the comb function’s Fourier magnitude spectrum, the synthetic single-hole waveform will not be accurate.

Figure 4-24 Influence of delay interval on magnitude spectrum of a comb function

Figure 4-25 Influence of number of holes on magnitude spectrum of a comb function
4.3.6 Discussion of the applicability of spectral division deconvolution for blast vibration problems

As analyzed in the previous section, in most cases the “useful” frequency content of the comb function is low compared to the frequency content of the production blast waveform. In the comb function for the blast events of 27, 47, 62 and 83 holes, there are several zero points in the denominator that produce large values in the operation. That situation makes the results unusable. Therefore, spectral division deconvolution does not work for production shots with a frequency content higher than 30 Hz, or production shots with a larger number of holes or long delay times.

In Equations (6) and (8), the convolution is usually used to describe the input-output relationship by an impulse response. In addition to the limitations in the frequency of the spectral division method previously explained, there is an inconvenience with the comb function. All the amplitudes of the delta functions used in the comb function are equal to one. If all the amplitudes are equal to one, the comb function can only be used to represent the occurrence of detonation and the firing time of each blast hole, and cannot represent the relative energy released to the rock mass by each detonation. To represent the relative energy of each detonation, the delta functions in the comb function should have different weights. This concept of weights is similar to the application reported by Yuill’s study (Yuill, 2003). However, even if different weights are assigned to the delta functions, they can only amplify, compress, or flip vertically the single-hole vibration waveforms. The phases of the waveforms will still remain the same. Moreover, the determination of the weight for each delta function in the case of a production shot is almost impossible to assess and is beyond the objective of this dissertation. Because of frequency limitations and unknowns in the comb function, the spectral division deconvolution in this dissertation is considered as a direct methodology, and proves it is not possible to simply compute the synthetic single-hole vibration waveforms in this direct way. It is necessary to find an alternative method to overcome the drawbacks of the comb function’s Fourier magnitude spectrum and unknowns. In next section, an estimation methodology using a least-square approach is investigated.
4.4 Wiener filtering applied to blast vibration problems

4.4.1 Introduction of Wiener filtering deconvolution

The limitation of the direct spectral division method in the frequency domain makes it unstable and a consistent choice to conduct deconvolution for most of the production shots. However, it is possible to do analysis in the time domain to avoid the instability in the frequency domain of the direct division. This option involves getting an optimum estimation of a single-hole vibration waveform by designing an inverse filter. This filter is the so-called Wiener filter, and the deconvolution method is thus called Wiener filtering deconvolution (Lee, 1961; Robinson and Treitel, 1967; Wiener, 1949). The essence of Wiener filtering is to design an inverse filter, using the least-squares approach, which can transform an input signal waveform into an output, which is as similar to the desired output as possible. Thus this method is also called a wave shaping filter. If the desired output is an impulse or a spike, the filter is also called a spike filter. Spike filters are often used in geophysics problems (Robinson, 1978).

For the problem in this dissertation research, the input is a comb function defined by the designed timing sequence, and the desired output is a single spike.

In Equation (8), \( y[n] \) is known as the measured production blast vibration waveform. The variable \( d[n] \) is also known, provided that electronic detonators are used, which is shown in Equation (10). As mentioned above, the core task of Wiener filtering deconvolution is to design a Wiener filter, \( f[n] \), which can compress the Kronecker comb function \( d[n] \) into a spike (as much as possible), denoted \( b[n] \). Intuitively, the spike should be at time zero. For a better filter performance and smaller sum of least-squared error, it is usually useful to added a time lag to the desired spike (Claerbout and Robinson, 1964). This operation can be expressed as:

\[
d[n] \ast f[n] = c[n] \quad \text{approximate to} \quad b[n]
\]

\[
b[n] = \delta[n - k], k \text{ is a nonnegative integer}
\]

Combining Equation (26) and Equation (8),
\[ y[n] * f[n] = d[n] * f[n] * g[n] = c[n] * g[n] \]
\[ \approx g[n - k] \quad \text{approximate to} \quad \delta[n - k] * g[n] = g[n - k] \quad (27) \]

Equation (27) shows that by applying the Wiener filter to the measured production blast vibration waveform, it is possible to synthesize a single-hole vibration waveform (probably with a time lag) to approximate the actual single hole vibrations. The detailed analysis using this technique for blast ground vibrations will be presented in the next section.

### 4.4.2 Derivation of Wiener filter for blast vibration problems

As mentioned above, the Wiener filter is also known as the “least squares” filter or the “optimum least squares” filter. The basic idea behind the Wiener filter is illustrated in Figure 4-26.

\[ f[n] (n = 0,1,...,K) \quad (28) \]

which can transform the input signal \( d[n] \) into an actual output

\[ c[n] = \sum_{s=0}^{K} f[s]d[n - s], (n = 0,1,...,K + L) \quad (29) \]

In Figure 4–26a, the desired output is a delta function. The calculated output is also expected to be a delta function that is calculated using the known impulse train information \( d[n] \) and one assumed function called a Wiener filter \( f[n] \). Once the desired and calculated outputs are compared using a least squares procedure, a Wiener filter will result in the function, \( I[n] \), that produces the minimum error in Figure 4-26b. The Wiener filter is expressed in the following form:

Figure 4-26 Winer filter elements (adapted from Treitel and Robinson, 1966)
Moreover, the actual output should be as close as possible to the desired output. This is achieved using the least-square technique.

\[ b[n] = \delta[n - k] \quad (n = 0,1, ..., K + L, 0 \leq k \leq K + L) \quad (30) \]

Next, the derivation of the Wiener filter for its application in the blast vibration problem is presented. The detailed theories and analysis about Wiener filtering can be found in existing literature (Robinson, 1954; Robinson and Treitel, 1967; Treitel and Robinson, 1966).

According to the least-square theory, the sum of squared errors between \( b[n] \) and \( c[n] \)

\[ I = \sum_{n=0}^{K+L} (b[n] - c[n])^2 \quad (31) \]

should be minimum. Applying Equation (29) into (31),

\[ I = \sum_{n=0}^{K+L} \left( b[n] - \sum_{s=0}^{K} f[s]d[n - s] \right)^2 \quad (32) \]

The value of \( I \) is minimum when the partial derivatives with respect to each \( f[n] \) is equal to zero. This condition is expressed as

\[
\frac{\partial I}{\partial f[t]} = \sum_{n=0}^{K+L} 2 \left( b[n] - \sum_{s=0}^{K} f[s]d[n - s] \right)(-d[n - t]) = 0, t = 0,1, ..., K
\]

(33)

This yields another equation

\[
\sum_{s=0}^{K} f[s] \sum_{n=0}^{K+L} d[n - s]d[n - t] = \sum_{n=0}^{K+L} b[n]d[n - t], t = 0,1, ..., K
\]

(34)

Completing substitutions, it is founded that

\[
\sum_{n=0}^{K+L} d[n - s]d[n - t] = r_{dd}[t - s], \quad (35)
\]

which is the autocorrelation of input \( d[n] \), and
\[ \sum_{n=0}^{K+L} b[n] d[n-t] = r_{bd}[t], \quad t = 0,1,2,\ldots, K \] (36)

which is the cross-correlation of desired output \( b[n] \) with the input \( d[n] \). Combining Equation (34), (35), (36),

\[ \sum_{s=0}^{K} f[s] r_{dd}[t-s] = r_{bd}[t], \quad t = 0,1,\ldots, K \] (37)

Equation (37) represents a set of \( N+1 \) equations. The matrix form is given by:

\[
\begin{bmatrix}
  r_{dd}[0] & r_{dd}[1] & \cdots & r_{dd}[N] \\
  r_{dd}[1] & r_{dd}[0] & \cdots & r_{dd}[N-1] \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{dd}[N] & r_{dd}[N-1] & \cdots & r_{dd}[0]
\end{bmatrix}
\begin{bmatrix}
  f[0] \\
  f[1] \\
  \vdots \\
  f[N]
\end{bmatrix}
= \begin{bmatrix}
  r_{bd}[0] \\
  r_{bd}[1] \\
  \vdots \\
  r_{bd}[N]
\end{bmatrix}
\] (38)

Equation (38) is called Wiener-Hopf equation, and by solving this equation, it is possible to obtain the Wiener filter coefficients \( f[n] \).

After obtaining a solution, a filter performance parameter \( P \) (Robinson and Treitel, 1967; Treitel and Robinson, 1966) can measure the performance of the calculated Wiener filter \( f[n] \). The filter performance parameter is giving by:

\[ P = \sum_{s=0}^{N} f[s] \frac{r_{bd}[s]}{r_{dd}[0]}, \quad (0 \leq P \leq 1) \] (39)

In Equation (39), \( P=0 \) means no agreement between the actual output \( c[n] \) and the desired output \( b[n] \); on the contrary, \( P=1 \) indicates an ideal performance of the derived filter \( f[n] \) which results in a perfect match between \( c[n] \) and \( b[n] \).

In theory, if the filter duration is infinite, a \( P \) value of 1 can be exactly achieved. However, the problem under study is a digital filter of finite duration. So, its value can only increase as much as possible towards one (1) by using a longer filter and delaying the desired spike.

4.4.3 Application of Wiener filter on blast vibrations to synthesize a single hole waveform

By using the calculated Wiener filter and the measured production blast vibration waveform in Equation (27), one can synthesize a single-hole waveform, \( \hat{g}[n] \), which is an estimation of the real single-hole vibration waveform \( g[n] \). Note that the desired synthetic
single-hole vibration waveform will be shorter than the deconvolution result in Equation (27). Recall from Equations (12) to (15), that the durations of the production blast, comb function, and the synthetic single hole waveform are $N$, $L$, and $M$, respectively. The desired duration of the synthetic single hole waveform is $M=N-L$ according to Equation (14) and (15). In addition, the duration of the estimated Wiener filter is $K$ in Equation (28). So, the duration of the deconvolution result in Equation (27) is $N+K$. This analysis shows that truncation is necessary to obtain the desired synthetic single hole waveform. Considering the spike location $k$, the starting point of truncation is the location of the spike lag $k$ in Equation (26) and (27), and the end point will be $k+M$. The truncation required in the calculations is illustrated in the following figure.

![Figure 4-27 Illustration of truncation](image)

By applying $\hat{g}[n]$ in Equation (8), the production blast vibration waveform, $\hat{y}[n]$, can be synthesized. By comparing $\hat{g}[n]$ and $\hat{y}[n]$ with the measured data, it can be known if the deconvolution methodology and results are satisfactory. Signals will be compared in both time domain and frequency domain.

4.4.4 Illustration of the computation for blast vibrations

To comprehensively observe the effectiveness of Wiener filtering deconvolution in mining blasts, the procedure will be conducted in two scenarios: (1) Zero lag of desired spike ($k=0$) and a Wiener filter with the same duration of the comb function; (2) A lagged spike ($k>0$) and a Wiener filter with a duration corresponding to a filter performance parameter value of 0.9. The purpose of computing the two scenarios is to illustrate how the spike lag and the filter length influence the deconvolution results.
The data used in this case study also come from the radial component of 6-hole blast tests described in Table 6, Figure 4-1, Figure 4-2 and Figure 4-12. Note that the six-hole waveform is the variable $y[n]$ in Equation (8). Also, the synthetic single hole waveform $\hat{y}[n]$ is compared against the measured single-hole vibration waveforms 1, 2 and 3, which are represented as $g_1[n], g_2[n]$ and $g_3[n]$. Once the synthetic single hole waveform is estimated, the convolution equation (Equation (8)) is used to synthesize the six-hole ground vibration waveform $\hat{y}[n]$. A comparison between $y[n]$ and $\hat{y}[n]$ will show the effectiveness of the deconvolution methodology presented in this case study. The length of $y[n]$ in Figure 4-12 is set as $N+1 = 1024$ sample points.

4.4.4.1 Wiener filter with an undelayed spike

The first step is to determine the specific impulse train $d[n]$ as the input of the filter. The desired output of the filter is $b[n]$. According to the previous description of the test data, the sample rate is 1024, which means the sampling period is rounded as $9.765625 \times 10^{-4}$ s. The firing times $t_i$, according to the design in Figure 4-1, is determined as

$$t_i = (0ms, 5ms, 10ms, 15ms, 20ms, 25ms)$$

In this case, $M$ is equal to 26 in Equation (10), and $d[n]$ can be expressed as a $27 \times 1$ vector

$$d[n] = [1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1]^T$$

where $T$ represents transpose of a vector. This comb function, $d[n]$, is also shown in Figure 4-13a. The length of the filter in Equation (28) is first set as equal to the length of $d[n]$, that is, $K = L = 26$. So, $f[n]$ can also be expressed as a $27 \times 1$ vector. According to Equation (29), the actual output $c[n]$ is the convolution of $d[n]$ with $f[n]$ and is a $53 \times 1$ vector.

With no lag on the desired spike in the initial analysis, the value of $k$ for the desired output $b[n]$ in Equation (30) is equal to 0. The $b[n]$ vector has the same length of $c[n]$.

$$b[n] = \begin{cases} 1, & n = 0 \\ 0, & 1 \leq n \leq 52 \end{cases}$$

Then, using Equations (35) and (36), the autocorrelation sequence of input, $r_{dd}[t - s]$, and the cross-correlation sequence between input and desired output, $r_{bd}[t]$, can be calculated. Substituting $r_{dd}[t - s]$ and $r_{bd}[t]$ into Equation (38), the solution of those equations is the Wiener filter, $f[n]$. To solve Equation (38), either the Levinson recursive algorithm
(Levinson, 1947; Treitel and Robinson, 1966) or matrix operations can be used (with the same results). The estimated Wiener filter is shown in Figure 4-28.

According to Equation (27), applying the estimated filter, $f[n]$, to the measured signal, $y[n]$, one can estimate $\hat{g}[n]$, and reconstruct the six-hole signal by convolving $\hat{g}[n]$ with $d[n]$. Note that there is an operation of truncating $\hat{g}[n]$ from the deconvolution result. The duration of $\hat{g}[n]$ is $M = N-L = 1024-27 = 997$, which is labelled by a reference line in Figure 4-29.

The comparison of the single-hole vibration waveforms is shown in Figure 4-30.
In the left column of Figure 4-30, the extracted single-hole vibration waveform ($\hat{v}[n]$) was plotted against the measured single-hole vibration waveforms. Their cross-correlation coefficients as well as the lag difference are also indicated in these figures. Here, the lag for cross-correlation coefficient is different than the lag of spike. The lag for cross-correlation coefficient represents the relative location of two signals corresponding to the maximum absolute value.

In the right column of Figure 4-30, the frequency contents of synthetic single-hole vibration waveform and measured single-hole vibration waveforms are compared. The main frequency of the synthetic single-hole vibration waveform is 18.0 Hz, while those of the measured single-hole vibration waveforms are 6.5 Hz, 7.0 Hz and 14.0 Hz for single-hole vibration waveform 1, 2 and 3, respectively.

After comparing single-hole vibration waveforms, the similarity between the synthetic and measured 6-hole waveforms in included in Figure 4-31.
Figure 4-31 Comparison of the 6-hole waveform in time and frequency domain

In the left side of Figure 4-31, both measured 6-hole waveform $y[n]$ and synthetic 6-hole waveform $\hat{y}[n]$ are presented. Their waveforms coincide in shape with a high cross-correlation coefficient of 0.982, but the maximum amplitude of the synthetic waveform is lower than that of the measured waveform. In the right side of Figure 4-31, the two Fourier spectra generally have a good similarity, but the magnitude of the synthetic waveform is significantly lower than that of the measured waveform.

The mismatching between the synthetic and measured signals appears only in amplitude. That means when the filter performance parameter is lower ($P=0.36$), there will be less energy reserved in the synthetic waveforms. If it is assumed that $y[n] = a \cdot \hat{y}[n]$, then the following equation according to the associative property of linear convolution is given:

$$y[n] = a \cdot \hat{y}[n] = a \cdot \hat{g}[n] \ast d[n] = (a \cdot \hat{g}[n]) \ast d[n]$$  \hspace{1cm} (41)

Equation (41) shows that if the difference between a synthetic production blast waveform and the measured waveform is just a scalar, the synthetic single hole waveform can be corrected by multiplying that scalar. Therefore, the results in Figure 4-30 and Figure 4-31 are acceptable.

4.4.4.2 Wiener filter with a lagged spike

As mentioned previously, increasing the length of the filter or delaying the desired spike can improve the filter’s performance parameter. Different combinations of filter length and spike lag result in different values of filter performance parameter ($P$). To avoid massive computation, a performance parameter of 0.9 with the shortest filter length is selected. To find the shortest filter length and its corresponding spike lag, it is possible to plot a contour of performance parameter values by varying the filter lengths and the spike lags. The result
is shown in Figure 4-32, and the shortest filter length corresponding to a \( P \) value of 0.9 is given by \( K+1=74 \) with a spike lag of \( k=61 \). This spike lag is taken as the optimum lag \( k_{\text{opt}} \) and it is possible to find its corresponding optimum-lag Wiener filter, \( f_{\text{opt}}[n] \). As a result, Figure 4-33 shows the optimum filter and its output from the comb function. It is shown that the spike has been moved to the position of \( k=61 \) (Figure 4-33b), and the output is more like a single spike. As a result of the spike lag, the synthetic single-hole waveform is also delayed and is contained within the convolution result of the filter \( f_{\text{opt}} \) and the 6-hole waveform. Thus, truncation from \( k=61 \) to \( k+M = 61+997 = 1058 \) is required to extract the waveform from the convolution result.

Figure 4-32 Contour of filter performance parameter \( (P) \) for 6-hole comb function

![Figure 4-32 Contour of filter performance parameter (P) for 6-hole comb function](image)

Figure 4-33 Optimum-lag filter and its actual output

![Figure 4-33 Optimum-lag filter and its actual output](image)

a. Optimum-lag Wiener filter \( f_{\text{opt}}[n] \)  

b. Actual output with optimum-lag spike
Now this optimum Wiener filter can be applied to the measured 6-hole waveform again to obtain another synthetic single-hole vibration waveform. The truncation is shown in Figure 4-36. The comparison of Figure 4-29 and Figure 4-34 show that there is no large difference between the truncated part and the deconvolution result. The synthetic single-hole vibration waveform can then be compared to the measured single-hole vibration waveform in time domain and frequency domain again.

Figure 4-34 Truncation of single hole vibrations with lagged spike

Figure 4-35 Compare synthetic single-hole vibration waveform with optimum lag with measured single-hole vibration waveforms
In the left column of Figure 4-35, it is observed that the amplitude of the beginning part of the synthetic single-hole vibration waveform is much higher than that of measured single-hole vibration waveforms. The cross-correlation coefficients of single-hole vibration waveform 1 and 2 have increased to 0.505 and 0.486, while the cross-correlation of single-hole vibration waveform 3 decrease slightly to 0.581.

The synthetic single-hole vibration waveform with optimum filter is also compared with the measured single-hole vibration waveforms in the frequency domain in the right column of Figure 4-35. It can be seen that the frequency content distribution match better. The dominant frequency of synthetic single-hole vibration waveform (12.75Hz) is now closer to the dominant frequency of the measured single-hole vibration waveforms (6.5Hz, 6.75Hz, 13.75Hz).

Figure 4-36 Comparison of the 6-hole waveform with optimum lag in the frequency domain

In Figure 4-36, both measured 6-hole signal $y[n]$ and synthetic 6-hole signal $\tilde{y}[n]$ are presented. Their waveforms coincide in shape with a high cross-correlation coefficient of 0.987, and the amplitude of the synthetic 6-hole signal also matches the measured signal much better. The two Fourier spectra almost overlap with each other, which means the synthetic single-hole vibration waveform can do the reconstruction very well. Both zero-lag filter with scalar correction and optimum-lag filter can give good deconvolution results. However, it seems that the optimum filter can result in a better magnitude spectrum distribution of synthetic single hole waveform compared to the measured data.
4.4.4.3  **Deconvolution results of the other two directions of vibrations**

In the previous section, the Wiener filtering deconvolution works very well for the radial components of the 6-hole blast vibration data. Next, the proposed methodology will be also applied to the transverse and vertical direction components to check if it also works well for the other two directions. The results of both zero-lag filter with a scalar correction and optimum-lag filter are presented in Figure 4-37 through Figure 4-44.

Figure 4-37 Single-hole vibration waveform comparison with zero-lag filter for 6-hole waveform in transverse
Figure 4-38 Comparison of 6-hole waveform with zero-lag filter in transverse direction

Figure 4-39 Single-hole vibration waveform comparison with optimum filter for 6-hole waveform in transverse direction
Figure 4-40 Comparison of 6-hole waveform with optimum filter in transverse direction

Figure 4-41 Single-hole vibration waveform comparison with zero-lag filter for 6-hole waveform in transverse direction
Figure 4-42 Comparison of 6-hole waveform with zero-lag filter in transverse direction

Figure 4-43 Single-hole vibration waveform comparison with optimum filter for 6-hole waveform in vertical direction

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Figure 4-44 Comparison of 6-hole waveform with optimum filter in vertical direction

The Wiener filtering deconvolution works very well for the case study of 6-hole blast as shown in Figure 4-30 through Figure 4-44. Generally, higher P values can improve the performance of Wiener filters. Even if the synthetic single-hole vibration waveforms estimated by both zero-lag filter and optimum filter can reconstruct the 6-hole waveform very well, the frequency distribution the scenario of optimum filter coincides with the measured data better.

It is already known that spectral division deconvolution does not work well for the cases of longer delay interval and a large numbers of holes. Determining if Wiener filtering is capable of dealing with those situations is the problem analyzed next.

4.4.5 Influence of delay interval and number of holes when using Wiener filter

In the previous section, the case study of 6-hole blast was carried out. The delay interval for the 6-hole blast is 5ms. Now, the proposed methodology will be applied to the other four cases to check if this methodology still works for greater delay interval and numbers of holes. The delay interval is increased to 20ms, and the numbers of holes are 27, 47, 62 and 83, respectively. The test data are already shown in Figure 4-4, Figure 4-6, Figure 4-8 and Figure 4-10. For each blast case, a P value contour is plotted to select the combination of filter length and spike location corresponding to a P value of 0.9. Similarly, the deconvolution results from both zero-lag filter with scalar correction and optimum-lag filter are presented in Figure 4-45 through Figure 4-60. In each figure except the contours, the left column is the results from zero-lag filter and the right column contains the results from optimum filter.
4.4.5.1 Analysis for 27-hole blast

Figure 4.45 shows that for the 27 blast holes, and the desired filter performance parameter value is 0.9, the combination of filter length and spike location required is $K+1=1781$ and $k=1686$.

Figure 4-45 Contour of filter performance parameter ($P$) for 27-hole comb function

The synthetic single hole and production blast waveforms are shown from Figure 4-46 to Figure 4-48. A bandpass filter with a passband of 1Hz to 40Hz is used to remove the DC component and high-frequency noise.

![Graph showing filter performance parameter contours](image-url)
Figure 4-46 Deconvolution results with optimum filter for 27-hole waveform in radial direction
Figure 4-47 Deconvolution results for 27-hole waveform in transverse direction
Figure 4-48 Deconvolution results for 27-hole waveform in vertical direction.

- **Truncation, zero lag**
  - Time (s) vs. Particle velocity (in/s)
  - Synthetic results compared with measured data, corrected for lag.

- **Truncation, optm lag**
  - Time (s) vs. Particle velocity (in/s)
  - Synthetic results compared with measured data, corrected for optimal lag.

- **Frequency (Hz) vs. Magnitude**
  - Synthetic and measured results for single and 27-hole configurations, including corrected data.

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4.4.5.2 Analysis for the 47-hole blast

As shown in Figure 4.49, corresponding to a filter performance parameter of 0.9, the combination of filter length and spike location is $K+1=2938$ and $k=1933$.

Figure 4-49 Contour of filter performance parameter ($P$) for 47-hole comb function

The synthetic single hole and production blast waveforms are shown from Figure 4-50 to Figure 4-52. A bandpass filter with a passband of 1Hz to 40Hz is also used to remove the DC component and high-frequency noise.
Figure 4-50 Deconvolution results for 47-hole waveform in radial direction
Figure 4-51 Deconvolution results for 47-hole waveform in transverse direction
Figure 4-52 Deconvolution results for 47-hole waveform in vertical direction
4.4.5.3 Analysis for 62-hole blast

Corresponding to a filter performance parameter of 0.9, the combination of filter length and spike location is \( K+1=4366 \) and \( k=3052 \) as shown in Figure 4.53.

Figure 4-53 Contour of filter performance parameter \((P)\) for 62-hole comb function

The synthetic single-hole vibration waveforms and production blast waveforms are shown from Figure 4-54 to Figure 4-56. A bandpass filter with a passband of 1Hz to 40Hz is also used to remove the DC component and high-frequency noise.
Figure 4-54 Deconvolution results for 62-hole waveform in radial direction
Figure 4-55 Deconvolution results for 62-hole waveform in transverse direction
Figure 4.56 Deconvolution results for 62-hole waveform in vertical direction

4.4.5.4 Analysis for 83-hole blast

Figure 4.57 shows the optimum length and lag corresponding to a filter performance parameter of 0.9, in this case the location is $K+1=6374$ and $k=3950$. 
Figure 4-57 Contour of filter performance parameter ($P$) for 83-hole comb function

The synthetic single-hole vibration waveforms and production blast waveforms are shown from Figure 4-58 to Figure 4-60. A bandpass filter with a passband of 1Hz to 40Hz is also used to remove the DC component and high-frequency noise.
Figure 4-58 Deconvolution results for 83-hole waveform in radial direction
Figure 4-59 Deconvolution results for 83-hole waveform in transverse direction
Figure 4-60 Deconvolution results for 83-hole waveform in vertical direction
From the results above, it is observed that as the delay interval and number of holes increase, the production blast vibration waveforms become more complex. In consequence, the performance of the Wiener filters deteriorates significantly. The cross-correlation between the synthetic and measured single hole waveforms is still low. The cross-correlation of synthetic and measured 6-hole blast waveform can reach almost 1, while the cross-correlations for the complex cases have reduced to values between 0.4 and 0.8. It’s easy to observe that the synthetic waveforms don’t have a desired envelope with respect to the rising part, body part and decaying tail part.

In addition to the reduction of waveform similarity, the waveform amplitudes also do not match very well. Moreover, the results of the zero-lag filter with a scalar correction are even better than those results of the optimum filter in this respect. A scalar correction was only used for zero lag filter results to compare with the results from the optimum filter with a performance parameter (P) value of 0.9. It is observed that the scalar corrected synthetic production blast waveforms match the measured data quite well in the first half part concerning amplitude, but without decaying envelope in the latter part. Compared to the scalar corrected results, the deconvolution results from the optimum filter have lower particle velocities in the tail part, but its amplitude is usually lower than the measured data.

Also, the frequency magnitude spectra of the synthetic waveforms do not coincide with those of the measured data very well with respect to frequency content distribution. The synthetic waveforms tend to contain more contents in higher frequencies. The frequency spectra of the results of the zero-lag filter with scalar correction have a better coincidence of frequency distribution, but they usually have higher magnitudes than the measured data. On the contrary, the results of the optimum filter have lower magnitudes compared to the measured data, especially within the lower frequency range.

In conclusion, the Wiener filtering deconvolution does not work well for complex blasts with more holes.

4.4.6 Discussion about Wiener filtering deconvolution
The Wiener filtering deconvolution still works very well for the case of 6-hole blast, and can also be operated in the cases of longer delay interval and a large number of holes, but
the results are not satisfactory. So, this section will give more observations and discussion about the Wiener filtering deconvolution methodology.

(1) The scalar correction of synthetic single hole vibration waveform only works well when the synthetic production blast waveform has high similarity with the measured production blast. That is, the cross-correlation coefficient between the synthetic and measured production blast should be greater than 0.9, as in the case study of the 6-hole blast. This is the application condition of scalar correction in Equation (41).

(2) The secret of the unsatisfactory synthetic single hole waveforms partly hides in the truncation during the waveform synthesizing process. For the 6-hole blast case, neither the filter nor the comb function is long, and the optimum spike lag is also a small value compared to the deconvolution output. Thus, the truncation of the synthetic single hole waveform will occupy most part of the deconvolution output according to the illustration in Figure 4-27, and the truncation gives the correct results.

However, as the delay interval and number of holes increase, the comb function become much longer for the case of 27-hole, 47-hole, 62-hole and 83-hole blasts. This further results in a longer length of Wiener filter and a large value of optimum spike lag. The desired synthetic single-hole waveform becomes very short compared to the deconvolution output. So, truncation under this kind of situation introduces more inaccuracy about waveform envelope.

As it is known, a blast vibration waveform should contain a rising part and a delaying part which reflects attenuation effects. It is observed that the deconvolution output contains a rising part at the beginning and a decaying part at the tail. Unfortunately, they are not included when truncating the desired single-hole waveform from the deconvolution output. So, the truncated synthetic single-hole waveform does not look similar to the measured waveform, additionally its tail part may not decay like attenuation and may have higher amplitude than the front part.

(3) Indeed the optimum Wiener filter ($P=0.9$) can compress the long comb function into a spike very well, as seen clearly in Figure 4-61. But the corresponding Wiener filters cannot synthesize a single-hole waveform well as expected. This fact disputes the rationality of
the convolution assumption of traditional signature-hole method, which is demonstrated in Equation (8), Figure 2-5 and Figure 2-6. Also, recall the discussion in spectral division deconvolution, the fail of Wiener filtering for complex blast cases may be due to the fact that the comb function is not the actual input to the ground system.

Figure 4-61 Outputs of comb functions by optimum Wiener filter

(4) Another phenomenon observed from the successful 6-hole blast case is that the cross-correlation coefficients between synthetic single-hole vibration waveforms and measured
single-hole vibration waveforms is not very high, but the synthetic 6-hole waveforms did have high cross-correlation coefficients. By contrast, the deconvolution results for the other four case studies do not have high cross-correlation coefficients on either synthetic single-hole waveform or synthetic production blast waveform. However, the scalar corrected zero-lag filter outputs match well on amplitudes with the measured data, excluding the tail part. If the synthetic production blast waveforms have good decaying parts, the synthetic results will look better regardless of cross-correlation.

All these observations and discussion above lead to several inferences:

(1) One should not be restricted by cross-correlations when synthesizing single-hole vibration waveforms by deconvolution. A better criterion is to observe the synthetic production blast waveform and the measured one.

(2) The ground medium, which is non-homogeneous and anisotropic, will result in varying single-hole waveforms. Furthermore, the ground medium will also make every production blast with the same design have different ground vibration waveforms after the interaction of all the various single-hole vibrations.

(3) Therefore, cross-correlation should also not be the indicator when comparing the synthetic production blast waveform with the measured production blast waveform. In practice, PPV and dominant frequency range are more important parameters in blast engineering for safety reasons. As long as the synthetic waveforms match the measured data by an envelope in the time domain and the dominant frequency ranges are similar in the frequency domain, the synthesis study of both single-hole and production blast vibrations can be set free from the restriction of cross-correlation.

(4) With the cross-correlation among single-hole waveforms is varying, there is little essential difference when comparing the synthetic single-hole vibration waveform with three measured single-hole vibration waveforms or only one measured single-hole vibration waveform.

4.5 Convolution or superposition?

The preliminary deconvolution analysis in this chapter is based on the convolution model described by Equation (8) to (11) and in Figure 2-5 and Figure 2-6. The main characteristic
of the convolution model is the assumption of the identical waveform for each blast hole, and thus the deconvolution procedure only gives one synthetic single-hole vibration waveform.

For the 6-hole blast, both spectral division deconvolution and Wiener filtering deconvolution work very well. The synthetic single-hole vibration waveform can be viewed as a normalized single-hole vibration waveform. The synthetic single-hole waveforms have different amplitudes than the measured single-hole vibration waveforms, but they can reconstruct the synthetic production blast waveform similarly to the measured data. There may be two reasons why it works for the 6-hole blast:

(1) The blast design is simple with 6 blast holes and a delay interval of 5ms. Under this situation, the interaction among single hole vibrations is not complicated. Even if each single hole waveform has variance, it is still not significant enough to influence the results.

(2) The delay interval of 5ms ensures that the magnitude spectra of the comb function and the synthetic single-hole vibration waveform are not influenced by the zero points within the dominant frequency domain. When observing the magnitude spectra, it is shown that the magnitude of the synthetic single-hole vibration waveform is roughly one-sixth of the magnitude of 6-hole blast. That makes the 6-hole blast look like a simultaneous blast.

When the number of holes increases, the spectral division deconvolution will no longer work because the first valley will appear within the desired frequency range as shown in Figure 4-25. Also, the interaction among single hole vibrations will also become complex when a mining blast has many holes and longer delay interval than 5ms, which makes hole-to-hole variance start to be significant. This is why the two deconvolution methods did not work well for the other four case studies with more blast holes.

Additionally, the comb function \(d[n]\) in Equation (8) produced by electronic detonators should be sufficiently accurate. If each blast hole produces an identical waveform, the Wiener filtering deconvolution methods proposed in this dissertation should work for any number of holes, not just 6-hole blast. So, the lack of success in deconvolution for more complex cases in this dissertation research proves the assumption that each blast hole generates different vibrations, and the production blast vibrations are the linear
superposition of the vibrations generated by each blast hole. The related illustration has been shown in Figure 2-7 and Figure 2-8. Further ideas about synthesizing single hole vibrations should be based on the superposition assumption rather than the stricter assumption of convolution.

To explain the differences between superposition and convolution: superposition is a more general concept of linear system and is described by Equation (5a) and (5b) and convolution is a mathematical operation of two functions, and it is a specific case of linear superposition. When a phenomenon cannot be expressed by a certain function, it is no longer suitable to use convolution to make an analysis.

Certainly, there are more advanced deconvolution methods which may be able to synthesize one compromised and normalized single hole waveform which can match the measured data by an envelope. However, when it is used to predict future blasts, random factors will still be added in. It is advantageous to find a method that directly generates a series of single hole waveforms that already contain randomness and are ready for use in a prediction.

The next step of the dissertation research is to develop a methodology which can solve the inverse problem of a superposition model, and synthesize a series of different single hole vibration waveforms which represent \( g_1[n] \), \( g_2[n] \), \( \ldots \), \( g_D[n] \). In this document, this procedure is called “statistical waveform synthesis”.

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Chapter 5 INVERSE PROBLEM OF SUPERPOSITION MODEL OF BLAST VIBRATIONS

5.1 Introduction

5.1.1 Phase and waveform shape

As analyzed in Chapter 4, the main problem of Wiener filtering deconvolution is the synthetic single hole waveform and synthetic production blast waveforms did not resemble the measured data very well, especially the tail part. The primary task of this Chapter is to find a way for the envelope of synthetic waveforms to be controlled.

Researchers have shown that the energy distribution in time domain of a signal is related to its phase. In Robinson’s study on high-resolution digital filters (Robinson, 1978), wavelets were classified into three types: minimum-delay, mixed delay and maximum delay (shown in Figure 5-1). The three types of wavelets have a common frequency magnitude spectrum and a common time duration. Here, the word “delay” means phase delay. The minimum-delay wavelet has a phase delay characteristic which is less than the phase delay characteristic of the maximum-delay wavelet. So, a minimum delay wavelet is also usually called a minimum phase wavelet. In geophysical exploration, a minimum delay wavelet is usually used for deconvolution. However, blast-induced ground vibrations are usually not minimum delay signals. Instead, they are generally mixed delay (Hawman, 2004), and the envelope of blast ground vibrations varies under different situations. In Figure 5-1, it is clearly shown that the influence of phase on waveform shape even if the magnitude spectrum keeps the same.

![Waveform Shapes](image)

Figure 5-1 Different waveforms with the same frequency content (adapted from Robinson, 1978)
Even though phase is important to the envelope of a ground vibration waveform, most Fourier analysis of blast-induced ground vibrations are mainly focused on the magnitude spectrum. Little research has paid attention to the phase of Fourier transform. Similarly, it was only the Fourier magnitude spectrum that was presented in the previous chapter when observing the deconvolution results. The phase spectrum was left without any considerations.

When conducting Fourier transform on any signals, two pieces of information are provided: magnitude and phase. As it is known, any function or signal can be decomposed into a series of sinusoids with different frequencies. Magnitude spectrum states how energy is distributed with frequency, and phase spectrum states the relative locations of the sinusoids. Phase does not affect the frequency energy distribution, but it is vital to the waveform in time domain.

Therefore, it is very important to study the phase characteristics of blast vibrations. Then, it is possible to develop a new methodology to synthesize single-hole vibration waveforms by operating on both magnitudes and phases. As shown in Figure 2-7 and Figure 2-8, each single-hole vibration waveform is different and contains random factors. So, the new methodology should also be statistical, which means the magnitude and phase spectrum should be of variance expressed by the standard deviation.

5.1.2 Fourier transform of linear superposition

The linear superposition of time-lagged varying single-hole vibration waveforms (shown in Figure 2-7 and Figure 2-8) into a complete production blast is expressed as:

\[ y = g_0(t - t_0) + g_1(t - t_1) + \cdots + g_D(t - t_D) \]  

(42)

Where \( y \) represents the measured production blast waveform;

\( g_i \) (\( i = 0, 1, \ldots, D \)) are individual single hole waveforms from \( D+1 \) blast holes,

\( t_i \) (\( i = 0, 1, \ldots, D \)) are firing times of the \( D+1 \) sequential blast holes, and \( t_i \) is 0.

The firing time sequence can also be represented as a Kronecker comb function.

Taking the Fourier transform of Equation (42), the frequency-domain representation of the superposition is obtained.
\[ Y(\omega) = G_0(\omega) + G_1(\omega) \cdot e^{-j\omega t_1} + \cdots + G_D(\omega) \cdot e^{-j\omega t_D} \]  

(43)

Where \( Y(\omega) \) is Fourier transform of measured production blast vibrations;

\( G_i(\omega) (i = 0, 1, \ldots, D) \) is Fourier transform for each single hole waveform.

Equation (43) tells us the production blast is a superposition of phase-delayed Fourier spectra of all the blast holes. Furthermore, by Fourier transform, a real function of time is transformed into a complex function of frequency (\( \omega \)). According to Euler’s formula,

\[ e^{jx} = \cos x + j \cdot \sin x \]  

(44)

Rewriting Equation (43) in terms of magnitude and phase by using Equation (44),

\[ A_Y e^{j\theta_Y} = A_G e^{j\theta_G_0} + A_G e^{j\theta_G_1} \cdot e^{-j\omega t_1} + \cdots + A_G e^{j\theta_G_D} \cdot e^{-j\omega t_D} \]

\[ = A_G e^{j\theta_G_0} + A_G e^{j(\theta_G_1 - \omega t_1)} \cdots A_G e^{j(\theta_G_D - \omega t_D)} \]  

(45)

Where \( A_Y \) is the magnitude of production blast;

\( A_G(i = 0, 1, \ldots, D) \) is the magnitude of each single hole waveform;

\( \theta_Y \) is the phase of production blast;

\( \theta_G_i - \omega t_i (i = 0, 1, \ldots, D) \) is the phase of each single hole waveform. There is a linear phase shift component added to the original phase.

5.1.3 Data used in the analysis

The data used in this chapter are the same as those used in Chapter 4, which come from Figure 4-2, Figure 4-4, Figure 4-6, Figure 4-8 and Figure 4-10. Specifically, the transverse component for each blast case is adopted, and they are they are numbered by Blast 1, Blast 2, Blast 3, Blast 4 and Blast 5 for the cases of 83 holes, 62 holes, 47 holes, 27 holes and 6 holes, respectively. When analyzing the phases and group delays, instead of using the simplest 6-hole data, the most complex 83-hole data are directly selected for the previous deconvolution methodology that did not work for complex blasts. If the developed new methodology works for the most complex case, it should also be capable of dealing with the simpler cases. However, in the section of case studies, all the data from the five cases will be analyzed, and all the results will be presented.
5.2 Magnitude spectrum of blast ground vibrations

5.2.1 Comparison of magnitude between measured production blast waveforms and single-hole vibration waveforms

In Equation (45), the magnitude of production blast waveform, $A_Y$, and a single-hole vibration waveform, $A_{Hn}$, can be known provided the seismograph recordings are available. In the inverse problem of linear superposition, a single-hole vibration waveform is the unknown function to solve. It is benefit to first compare the measured production blast vibrations and single-hole vibration waveform from the same blast so that to reveal a relationship between them (shown in Figure 5-2).

Figure 5-2 Comparison between production blast vibrations and single-hole vibrations from 5 blasts in the transverse direction

In Figure 5-2, there are five production blasts with decreasing number of delays from blast 1 to blast 5. The continuous red curves are the production blasts magnitude spectra, and the blue dash curves are corresponding magnitude spectra of the single-hole vibration waveforms. All the magnitude spectra are the main parts within the effective frequency limit as mentioned in Section 4.3.3. In general, the magnitude spectra of production blasts get more jagged and with higher values. The magnitude spectra of single-hole vibration waveforms appear smoother and with lower values. As the number of delays increases, the disparity is getting more significant. Even so, they still have similar dominant frequency and frequency-magnitude distribution.
5.2.2 Magnitude spectrum model of the single-hole vibration waveforms

The aforementioned comparison implies that an initial estimate can be made of the magnitude spectrum of a single-hole vibration waveform by multiplying a correction factor to a smoothed production blast magnitude. The magnitude correction is shown in Equation (46).

\[ A'_G = A_{YS} \cdot c_{ym} \] (46)

where \( A'_G \) is as a corrected magnitude spectrum for single-hole vibration waveform.

\( A_{YS} \) is the smoothed magnitude spectrum of measured production blast vibrations. The smoothing can be conducted by computation of moving average.

Assume a function \( y_1 \) to be smoothed to \( y_2 \) by moving average. With the span for the moving average being 5, the computation procedure is (Proakis and Manolakis, 1995):

\[
\begin{align*}
    y_2(1) &= y_1(1) \\
    y_2(2) &= (y_1(1) + y_1(2) + y_1(3))/3 \\
    y_2(3) &= (y_1(1) + y_1(2) + y_1(3) + y_1(4) + y_1(5))/5 \\
    y_2(4) &= (y_1(2) + y_1(3) + y_1(4) + y_1(5) + y_1(6))/5 \\
    &\vdots
\end{align*}
\]

The term \( c_{ym} \) is a magnitude correction factor which is a scalar. The range of \( c_{ym} \) is (0,1). This correction factor can be modified by validating the synthetic production blast waveform with the measured data.

Again, our assumption here is that each blast hole produces a different single-hole vibration waveform. That is, \( A_{G_i} \) and \( \theta_{G_i} \) vary for different holes. Equation (45) is an ill-posed problem, which cannot give a unique solution. Without loss of generality, \( A_{G_i} \) and \( \theta_{G_i} \) can be assumed to possess certain randomness, and they are the realization of an ensemble magnitude \( A_G \) and phase \( \theta_G \).

For the magnitude \( A_{G_i} \), it’s synthesis can be expressed as

\[
\bar{A}_{G_i} = A'_G \cdot c_{G_i} \] (47)

Where \( A'_G \) is taken as an average magnitude which can be determined by Equation (46).

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The term $c_{G_i}$ is a random factor which is assumed to follow a certain distribution which has a mean value of 1. The distribution can be simply assumed as a normal distribution with a standard deviation of $\sigma_{G_i}$, that is, $c_{G_i}$ follows $N(1, \sigma_{G_i})$.

A special case is when $\sigma_{G_i}$ is equal to 0, $c_{G_i}$ becomes to 1, and the magnitude of each single-hole vibration waveform becomes to the same as the average magnitude $A_G$. The special case can be used to control the randomness of magnitude and focus the main attention on the phase spectrum, but in practical computation, this randomness should be taken into account.

This magnitude model indeed cannot produce the exact magnitude spectrum as the measured single hole vibration waveform for two reasons: (1) any measured single-hole vibration waveform is only one sample of the single-hole vibration waveform ensemble due to random nature of ground vibrations. (2) The synthetic magnitude spectrum of single hole vibrations is based on that of the production blast which is more complex than the single hole’s magnitude spectrum. In practical inverse problems for synthesizing a single-hole vibration waveform, any recordings for a single-hole vibration waveform is not available. Thus the production blast vibration waveform is the only data to use. Therefore it is acceptable to synthesize a magnitude spectrum of single-hole vibration waveform with a similar dominant frequency range to the production blast magnitude.

Another thing to note is the effective range of magnitude spectrum introduced in Section 4.3.3. There is little contribution of the frequency contents beyond $f_{lim}$, so the main focus on synthesizing the magnitude spectrum of single hole vibrations is the frequency contents within the effective frequency range.

5.3 Phase spectrum of blast ground vibrations

With only the magnitude up to $f_{lim}$ studied, there is also only need to place emphasis on the corresponding phases within the same range. Initially, the phase angles are computed as values within the range $(-\pi, \pi)$ which are called principle values (Schafer, 1969). The principle phase angles from the same production blast data to Figure 4-11 are shown in Figure 5-3.
Figure 5-3 Typical principle phase angles

Figure 5-3 implies that the computed principle phase angle curve is not a continuous function. However, continuity is a necessity to many operations on phases, such as computation of complex cepstrum, phase prediction, and computation of group delay. A continuous phase curve can be obtained by unwrapping the principle phase angles.

5.3.1 Two unwrapping algorithms

The continuous version of the phase curve can be expressed as the principle phase angle vector plus a correction vector.

\[ \theta_k = \theta_k^P + c_k \]  

(48)

Where \( \theta_k \) is the corrected continuous phase spectrum,

\( \theta_k^P \) is the initially computed principle value of phase,

\( c_k \) is the correction vector for unwrapping the phase spectrum.

The problem of unwrapping is to define appropriate correction vector \( c_k \). In this context, two unwrapping algorithms are to be reviewed.

Algorithm 1

The first is for making any phase difference between successive phase angles always nonpositive (Rad and Virtanen, 2012). In this situation, the correction vector is defined as

\[ c_k = \begin{cases} 0 & \text{for } k = 0 \\ c_{k-1} & \text{for } k > 0 & \theta_k^P \leq \theta_{k-1}^P \\ c_{k-1} - 2\pi & \text{for } k > 0 & \theta_k^P > \theta_{k-1}^P \end{cases} \]  

(49)

Ohsaki (1979) implemented this algorithm in the Argand diagram by measuring phase angles clockwise so that \( \theta_k^P \leq \theta_{k-1}^P \) always holds.
Algorithm 2
The second unwrapping algorithm origins from Schafer’s research about the homomorphic system and cepstrum (Schafer, 1969). It allows positive phase differences between successive phase elements by adding multiples of $\pm 2\pi$ when absolute jumps between adjacent frequency elements are greater than $\pi$. This algorithm is widely adopted by many other researchers (Childers et al., 1977; Shrikhande and Gupta, 2001; Thráinsson et al., 2000) and also by the mathematical software MATLAB.

$$c_k = \begin{cases} 0 & \text{for } k = 0 \\ c_{k-1} + 2\pi & \text{for } k > 0 \text{ and } \theta_k^p - \theta_{k-1}^p < -\pi \\ c_{k-1} & \text{for } k > 0 \text{ and } -\pi \leq \theta_k^p - \theta_{k-1}^p \leq \pi \\ c_{k-1} - 2\pi & \text{for } k > 0 \text{ and } \theta_k^p - \theta_{k-1}^p > \pi \end{cases}$$

These two unwrapping algorithms generate different phase curves. Figure 5-4 shows the unwrapped phase curves from the production blasts and single-hole vibration waveforms in Figure 5-2 by the two algorithms respectively. Generally, the unwrapped phase spectra have a monotonically decreasing trend. In addition, by observing the phases produced by algorithm 2, there is a large angle between the phase curves of the production blast waveforms and single-hole vibration waveforms when the number of delays is large. Moreover, the angle becomes smaller when the number of delays decreases. The phase curves generated by algorithm 1 do not reflect this feature. However, there is still no quantitative conclusions about the angle which can relate the phase of production blast with that of single hole together.

Another characteristic observable between the two algorithms is there are considerable phase jumps on the phase curves generated by algorithm 1, which makes those phase curves are lower than those by algorithm 2 for the same blast event. This jump phenomenon is becoming more significant when the FFT length gets than the length of signals. In Figure 5-4, the single-hole vibration waveforms are all shorter than the signals of production blasts, but the FFT length is uniformly selected as 4 times the production blast signal length for each blast. That is why there is more significant phase jumps on the phase curves of single-hole vibration waveforms, which are shown as dashed lines. All the discussion above indicates that algorithm 1 will overly unwrap the phase.
In conclusion, algorithm 2 gives better phase unwrapping results and phase difference distribution. Therefore, all further analysis will be based on an unwrapped phase by algorithm 2.

There are also other unwrapping algorithms such as numerical integration of phase derivative (Tribolet, 1977). This type of unwrapping algorithms is not included in the scope of discussion in this dissertation research.

![Figure 5-4 Comparison of unwrapped phase curves](image)

5.3.2 Statistical characteristics of phase spectra

Ohsaki (1979) and Nigam (1982) concluded that the principle phase angles of a random process approximately follow a uniform distribution, which is reflected in Figure 5-5b and Figure 5-6b. The phase however, principle or unwrapped, is not completely random by uniform distribution for the blast vibrations under research. It is frequency dependent to some extent.

First, the phase of production blast from Blast 1 in Figure 5-4 is analyzed. Figure 5-5a and Figure 5-5b contain the scatter plot and histogram of recorded ground vibration. Then, uniformly distributed random numbers are generated with the same range as Figure 5-5a, both in frequency and phase (Figure 5-5e), and the corresponding histogram is also plotted in Figure 5-5f. Ohsaki and Nigam’s conclusion seems to be verified by the comparison of histograms. However, closer observation of the scatter plots of both generated phase and phase of recorded ground vibrations shows some difference. The phase of recorded data in Figure 5-5a seems to have certain permutation by frequency, while the generated phase scatter plot in Figure 6e appears more random. This difference is shown more clearly by
the unwrapped phase curves in Figure 5-5c and Figure 5-5g. Figure 5-5c shows the unwrapped phase curve of recorded ground vibration, which is generally a monotonous decreasing curve resembling a straight line. Moreover, its histogram in Figure 5-5d also retains some feature of uniform distribution. Theoretically, if the unwrapped phase is exactly a straight line, its histogram also exactly follows a uniform distribution. However, a lot of random factors in the earth system (heterogeneity along wave propagation path) and blast process introduce fluctuations into the ideal phase straight line, which results in the actual phase curves. On the contrary, when the generated principle phase is unwrapped, the result (Figure 5-5g) is far from a monotonous curve, and its histogram (Figure 5-5h) is more like a triangle rather than uniform distribution.

Figure 5-5 Phase statistics of production blast for blast 1
The observations above are more obvious for the phase of the single-hole vibration waveform of Blast 1, as shown in Figure 5-6, in which case the FFT size increases a lot compared to signal length.

If looking at the histogram of principle phase of the recorded single-hole vibration waveform (Figure 5-6b), it still looks similar to the histogram of generated uniform distributed phase (Figure 5-6f). However, their corresponding phase-frequency scatters plot are very different. The principal phase of the recorded single-hole vibration waveform (Figure 5-6a) is permuted by inclined columns which contrasts sharply with the scatter plot of purely uniform random phase (Figure 5-6e). The unwrapped phase curves are also very different. The recorded single-hole vibration waveform still has an unwrapped phase curve (Figure 5-6c) with a monotonous trend, while the generated one (Figure 5-6g) is also a nondescript. Their histograms are both triangle like (Figure 5-6d and Figure 5-6h), but the one from recorded unwrapped phase tends to contain some feature of uniform distribution.
All the analysis in this section implies that the phase of blast-induced ground vibrations is not completely uniform random, and it has a dependence on frequency. Therefore, the phase (principle or unwrapped) cannot be represented just by uniform random numbers.

5.3.3 Phase spectrum model of the single-hole vibration waveforms

According to previous assumption and Equation (45), the phase of single-hole vibration waveform, \( \theta_{G_i} \), varies hole by hole. As concluded in the previous sections, it is hard to synthesize the phase of a single-hole vibration waveform directly from the phase of production blast. Also, a uniform distribution cannot be used to simulate the random factor in phase spectrum. An alternative method is to compute phase by the cumulative integral of its derivative:

\[
\theta_{G_i} = \int_0^\omega \tau_{G_i}(\omega) \, d\omega, \quad \omega \in [0, f_{lim}]
\]

(51)

where \( \tau_{G_i} \) is the phase derivative for an individual single-hole vibration waveform, \( \omega \) is angular frequency and its range is \([0, f_{lim}]\).

By Equation (51) the statistical analysis is transferred from phase itself to its derivatives. Thus, it is useful to investigate the characteristics of group delay.

5.4 Group delay of blast ground vibrations

5.4.1 Group delay – phase derivatives

The derivative of phase concerning frequency is also called “group delay”. The continuous form of group delay is defined as below (Gouriet, 1958):

\[
\tau(\omega) = -\frac{d\theta}{d\omega}
\]

(52)

where \( \tau(\omega) \) is group delay which is a function of frequency;

\( \theta \) is the unwrapped phase;

\( \omega \) is angular frequency.

However, seismographs measure discrete signals, thus there must be a discrete form of group delay definition. By symmetric derivative theorem, the group delay can be expressed as the following limit (Aull, 1967).
\[ \tau(\omega) = -\lim_{\Delta \omega \to 0} \frac{\theta(\omega + \Delta \omega) - \theta(\omega + \Delta \omega)}{2\Delta \omega} \]  

(53)

where \( \Delta \omega \) is a unit increment of frequency.

If \( \Delta \omega \) is small, the central difference (Sheppard, 1899) in Equation (53) can be used as an approximation of the derivative, that is

\[ \tau(\omega) = -\frac{\theta(\omega + \Delta \omega) - \theta(\omega + \Delta \omega)}{2\Delta \omega} \]  

(54)

The unit frequency increment in Equation (53) and (54) is calculated by

\[ \Delta \omega = \frac{2\pi}{L} \]  

(55)

where \( L \) is the length of Discrete Fourier transform.

If written in the discrete form, \( \omega = n\Delta \omega \), Equation (54) becomes to

\[ \tau(n) = -\frac{\theta(n + 1) - \theta(n - 1)}{2\Delta \omega} \]  

(56)

Equation (56) shows group delay and phase difference are negatively correlated with a factor of \( \frac{1}{2\Delta \omega} \), which means both variables are able to measure the phase change rate. However, the following two sections will explain why group delay is a better choice for the problem studied in this dissertation research.

5.4.2 Phase difference

Fourier transforms are computed as discrete series on computers. By taking the difference between adjacent phase elements of the unwrapped phase curve, the phase’s difference sequences are given. Avoiding repetition, the phase differences are analyzed from Blast 1 in Figure 5-4. Figure 5-7 shows the phase difference-frequency scatter plot and corresponding histograms of phase differences by the two aforementioned unwrapping algorithms respectively. The phase difference by algorithm 1 lies within \( (2\pi, 0] \), and the counterpart by algorithm 2 ranges within \([-\pi, \pi]\). However, the phase difference of single-hole vibration waveform is more concentrated around zero than the production blast.

For algorithm 1, when a phase element is slightly greater than the previous one, a multiple of \( 2\pi \) is still added to it. This causes the concentration of distribution to be around \(-2\pi\). Values around \(-2\pi\) are more like outliers, making the distribution skewed severely and
even tend to make the distribution bimodal. In contrast, the distribution of phase difference by algorithm 2 is unimodal and more symmetric. The second unwrapping algorithm proves to be better from this perspective as well. One thing to note is in Shrikhande and Gupta’s paper (Shrikhande and Gupta, 2001), the second unwrapping algorithm was used, but the unwrapped results still range from \(-2\pi\) to 0, which conflicts with the adopted unwrapping algorithm.

Figure 5-7 Histograms of phase difference from a measured blast vibration signal

Figure 5-7 also shows that the phase difference histogram has a bell-shaped distribution. Ohsaki (1979) used a normal distribution to fit the bell shape, and pointed out that the location and scale of the distribution are closely related to the waveform of the corresponding measured ground vibration signal. Unfortunately, there is no quantitative relationship for the resemblance between the distribution and the waveform.

Thrónsson et al. (2000) used a beta distribution as the model of bell shape distribution. The beta distribution is on the interval [0,1], and the phase difference is bounded between
$-\pi$ and $\pi$. The probability density function (pdf) of a beta distribution is zero at points of 0 and 1; and similarly as shown in Figure 5-7 the phase difference by algorithm 2 at $-\pi$ and $\pi$ are also close to zero. It seems reasonable to use a beta distribution with normalization from $[-\pi, \pi]$ to $[0,1]$, but it is worth noting that the mean of the distribution is not on the center part of the whole range and the main part of distribution around mean is quite symmetric, leaving a longer tail towards $\pi$. If a beta distribution is used to fit the distribution for this situation, the pdf curve will be asymmetric and does not fit the data very well. That is why the authors need to shift the data to obtain a more unimodal and symmetrical distribution. However, the operation of shifting phase difference will change the signs and values of phase derivatives (group delays) for this shifted part and may further affect the conclusions which are based on group delays.

If the FFT size (number of sampled points) is large enough ($2^2$ times of the signal length), the histogram of phase difference becomes more symmetrical around the mean value, and the proportions of values around $-\pi$ and $\pi$ get smaller. That is, the phase differences around $-\pi$ and $\pi$ have less influence to the whole distribution. So, the phase difference can be viewed as following a normal distribution which is truncated at $-\pi$ and $\pi$. In addition, a normal distribution is easier to understand and use.

Phase differences divided by a unit increment of frequency result in group delays. So, the group delays have a similar distribution as that of phase differences except for dimension. Many researchers pay attention to phase differences and simulate earthquake ground motions by approximate distributions of the phase differences. The derivative of the phase difference, group delay, may still be a better measured to resemble the waveform of ground vibration. The main reason is the unit of group delay is time, which can be directly compared with the time-domain waveforms.

5.4.3 Statistical features of blast ground vibrations’ group delay
5.4.3.1 Frequency-dependent distribution
Group delays can be calculated by applying Equation (56). As an example, the group delays of both the production blast vibration waveform and the single-hole vibration waveform from Blast 1 in Figure 5 are calculated and presented in Figure 5-8. The green curves in Figure 5-8a and Figure 5-8c are the reference line of effective frequency limit ($f_{lim}$) which
is 49.1 Hz for Blast 1. There are also two dash curves in Figure 5-8a and Figure 5-8c respectively, which represent 3-standard-deviation (3σ) ranges for the group delay data. The 3σ curves indicate the variation tendency of group delays along with frequency, and are computed by Equation (57) to (59):

\[ \mu_\tau(n) = \frac{\sum_0^n \tau(m)}{n + 1} \]  
\[ \sigma_\tau(n) = \sqrt{\frac{\sum_0^n (\tau(m) - \mu_\tau(n))^2}{n + 1}} \]  
\[ 3\sigma \text{ range} = [\mu_\tau(n) - 3\sigma_\tau(n), \mu_\tau(n) + 3\sigma_\tau(n)] \]

where \( \mu_\tau(n) \) is a cumulative moving average of \( n \) group delay values; 
\( \sigma_\tau(n) \) is a cumulative standard deviation of \( n \) group delay values;

The 3σ range is the range which covers 99.7% of \( n \) group delay values.

By the aid of 3σ curves in Figure 5-8a and Figure 5-8c, the clouds of group delay data present tornado-like distributions. The wide range at the bottom of Figure 5-8a around 10 Hz is the result from the outliers in the statistical sense which are highlighted at the left bottom. It is easy to observe that the group delay is also frequency-dependent and has certain relatively compact distribution pattern below \( f_{lim} \), while beyond \( f_{lim} \) the values of group delay start to disperse and present similarly Gaussian random distribution. This demarcation of randomness beyond and below \( f_{lim} \) is clearer for the group delay of the single-hole vibration waveform. To clearly show the frequency dependence, two smoothed group delay spectra realized by moving average filters are plotted on the data of production blast waveform and the single-hole vibration waveform, respectively. The smoothed group delay spectra should be able to reflect the trends of most data along frequency. It is shown that the smoothed curves have more similarity than the scatter data plots. A obvious characteristic of the magnitude spectra in Figure 5-8b and Figure 5-8d is the energy mainly concentrates below \( f_{lim} \), and magnitude values diminish to almost zero beyond \( f_{lim} \).
Figure 5-8 A typical group delay plot of Blast 1. (a) $\tau_Y$ denotes group delay of production blast, (b) $A_Y$ denotes magnitude of production blast, (c) $\tau_G$ denotes group delay of single-hole vibration waveform, (d) $A_H$ denotes magnitude of single-hole vibration waveform.

Those observations of group delays and magnitudes imply the explanation of the phenomena. When a frequency has a relatively higher magnitude value, this frequency also has more contribution to the waveform of ground vibrations, and its corresponding group delay is more concentrated to the compact pattern. While a relatively low magnitude of a certain frequency, which has few contributions to a ground vibration waveform, tends to release its corresponding group delay far from the center part to an arbitrary value. In other words, higher magnitudes can constrain the group delays around the average, while lower magnitudes lose control of group delays which thus have higher scatters to the average and become more random. This is why the group delay distribution has a general watershed at $f_{lim}$. Even below $f_{lim}$, there are also large scattered values of group delay which are corresponding to small magnitude values. Also, the sharper the relative changes of magnitude spectrum is, the more scatters the group delay has.
5.5 Synthesis of single-hole vibration waveforms

5.5.1 The resemblance between group delay distribution and a waveform

As mentioned previously, a significant feature of group delay is its unit of time which is the same as that of ground vibration waveform. Investigations (Boore, 2003; Nigam, 1982; Sawada et al., 2000) have shown that histograms (or distributions) of group delays resemble the envelopes of ground vibration waveforms. The mean of distribution corresponds to the location of envelope’s peak, and the standard deviation relates to the duration of the waveform.

Figure 5-9 shows group delay histograms and waveforms of both the production blast and single-hole blast from Blast 1. Two histograms are presented in Figure 5-9a and Figure 5-9b, respectively. Histogram 1 covers the group delay data within a full frequency range [0, fs/2] which is [0 Hz, 512 Hz], for the sampling frequency in this paper is 1024 Hz. Histogram 2 represents the group delay data from 0 Hz up to \( f_{lim} \) which is 49.1 Hz in the case of Blast 1. Probability density functions (pdf) of the histograms are fitted by a normal distribution. The pdf (black solid curve) curves and the envelopes of waveforms (red and blue dash lines) all resemble a bell shape. And the waveform envelopes look like truncated pdf (black solid curve) curves at zero. There are two aspects of the resemblance to discuss: location and extent.

In regards to the relative locations among the curves, they can be observed by the peak point of each curve. Figure 5-9a shows the case of a production blast. The peaks of both pdf curves of histogram 1 and 2, which are also the averages of both distributions, roughly correspond to the peak of waveform’s envelope. However, it is not the case for the single-hole vibration waveform in Figure 5-9b. The pdf peak of histogram 2 coincides with the peak of waveform envelope very well, while the peak of histogram 2 deviates from the peak of waveform envelope about 0.3s. Therefore, Histogram 2 resemble the waveform envelope better in general.

The extent is as important as the location when to compare histograms to waveforms. It refers to the duration of a waveform. Observations show that the ground vibration waveforms in Figure 5-9a and Figure 5-9b contain two parts: (1) the part containing the particle velocity peak and the “essential” amplitudes of the waveform; (2) the part with
very small amplitudes (almost unessential amplitudes). The two parts can be named the main part and tail part, respectively. When a seismograph returns the raw seismic data, it is usually longer than what is needed, so the data needs truncation to appropriately contain the main part and the tail part. The main part is usually of a certain length, while the tail part can be of varying length depending on the actual truncation operation and specific demands.

The next question is how to quantitatively find the dividing point between the main part and the tail part. After analysis of different vibrations waveforms collected for this research, it was evident that 0.95 quantile of the group delay distribution of histogram 2 will separate the main and the tail part of the record. This is illustrated in Figure 5-10a and Figure 5-10b. In such figures, the lines of 2.02 s and 0.23 s correspond with the 0.95 quantile for the event waveform and the collected single-hole vibration waveform respectively. The equation representing the length of the main part of a general waveform can be written as:

\[ l_{\text{main}} = \mu + 1.65 \sigma \]  

(60)

On the contrary to the coincidence between the 0.95 quantile of group delay distribution and demarcation point of waveform’s main and tail parts for the case of Histogram 2, the span of Histogram 1 in both Figure 5-9a and Figure 5-9b is far beyond the duration of the main part and even beyond the entire duration of waveforms. These observations again prove the significance of effective frequency range. In conclusion, the statistical features of group delay up to frequency of \( f_{lim} \) are better parameters to resemble a ground vibration waveform than those up to the frequency of \( fs/2 \). The group delay contents beyond \( f_{lim} \) even bring some interference factors.
The discussion above also gives us several pieces of inspiration for synthesizing a time-domain waveform from frequency domain: (1) Only frequency contents of magnitudes and group delays below $f_{lim}$ are necessary to reconstruct phases and signals. Higher frequency contents may introduce noise and errors into the results. (2) As long as group delays can be simulated with proper means and standard deviations, so that the 0.95 quantile reference line properly separates main and tail of a waveform, it is possible to properly synthesize a signal waveform. (3) The former two conclusions apply to both the cases of a production blast.
blast and a single-hole blast. So, if some relationships can bridge the parameters of the production blast vibration waveform and the single-hole vibration waveform in Figure 5-9 and Figure 5-10, it is possible to synthesize a single-hole vibration waveform based on a measured production blast waveform.

As is shown in Figure 5-11, a production blast waveform \(y\) and single-hole vibration waveforms \(g_i\) are linked by Equation (42) with the timing sequence \(t_i\). To synthesize a single-hole vibration waveform from a production blast, it is first necessary to explore the relationship of group delay distribution between production blast waveforms and single-hole vibration waveforms. Based on Equation (42), Figure 2-8 and Figure 5-9, a schematic of production blast waveform \(y\) and the last single-hole vibration waveform \(h_D\) are shown in Figure 5-11. The dashed bell-shape curves represent both envelopes of waveforms and truncated pdf curves of group delay distributions with a frequency range of \([0, f_{lim}]\). By assuming every single-hole vibration waveform is of the same length, \(h_D\) can be used to represent any other single-hole vibration waveforms.

There are five duration parameters in Figure 5-11: (1) Duration of main part of production blast waveform, \(l_{y,main}\); (2) Duration of the main part of the single-hole vibration waveform, \(l_{g,main}\); (3) Duration of firing time sequence, which is also a comb function, \(l_{comb}\); (4) Duration of the entire production blast waveform, \(L_y\); (5) Duration of the entire single-hole vibration waveform, \(l_{g}\).

There are also four statistical parameters labelled in this figure: (a) Mean of production blast’s group delay distribution, \(\mu_y\); (b) Standard deviation of production blast’s group delay, \(\sigma_y\); (c) Mean of single-hole vibration waveform’s group delay distribution, \(\mu_g\), and (c) Standard deviation of single-hole vibration waveform’s group delay, \(\sigma_g\).
Figure 5-11 Duration and group delay’s statistics among the production blast vibration waveform, the single-hole vibration waveforms and the firing timing sequence

The parameters in Figure 5-11 form several relationships. The first one is about the duration of main parts of the production blast vibration waveform and the single-hole vibration waveform:

\[ l_{y,\text{main}} = l_{\text{comb}} + l_{g,\text{main}} \]  

(61)

Equation (61) indicates the difference between the duration of production blast’s main part and that of single-hole vibration waveform’s main part is just the duration of firing time sequence. When electronic detonators are used, the timing is accurate enough to determine \( l_{\text{comb}} \) as the delay time of the last detonator. After determining \( l_{y,\text{main}} \) by Equation (60), it is easy to find the value of \( l_{g,\text{main}} \).

Then a proportional relation of the statistical parameters can be devised from the ratio of duration parameters:

\[ \frac{\mu_y}{\mu_g} = \frac{\sigma_y}{\sigma_g} = \frac{l_{y,\text{main}}}{l_{g,\text{main}}} \]  

(62)

Equation (62) is used to find the value of \( \mu_g \) and \( \sigma_g \), provided all other parameters can be calculated based on collected data. In addition, \( \mu_y, \mu_g, \sigma_y \) and \( \sigma_g \) determine the relative
location and scale between the distributions of the production blast waveform and the single-hole vibration waveform.

Equation (61) and (62) can bridge the distributions of group delay between the production blast waveform and the single-hole vibration waveform, so it is possible to simulate random group delays of the single-hole vibration waveform. However, the duration of the main part cannot represent the entire waveform, and the tail part is also necessary to be included in a synthetic single-hole vibration waveform. As shown in Figure 5-11, the entire duration of production blast $l_y$ and timing sequence $l_{comb}$ are known, then entire duration of single-hole vibration waveform, $l_g$, can be computed by Equation (15) and written again as below:

$$l_y = l_{comb} + l_g$$

5.5.2 Synthesis of group delays from production blast

Based on the relationship between statistical parameters of group delay and waveform duration, as well as how they link production blast vibration waveforms and single-hole vibration waveforms together, a methodology will be developed to synthesize an ensemble of single-hole vibration waveforms, based on the group delay characteristics.

**Step 1:** Perform fast Fourier transform (FFT) of the measured blast event waveform and determine the effective frequency limit ($f_{lim}$) of the Fourier spectrum. This can be accomplished using the plot spectral magnitude vs frequency of the signal.

**Step 2:** Compute the group delay histogram and calculate the mean and standard deviation of the group delay data, $\mu_y$ and $\sigma_y$, for the measured production blast in the interval given by [0, $f_{lim}$].

**Step 3:** Determine the duration of the main part of the production blast waveform, $l_{y,main}$, by Equation (60). Then, find the main duration part of single-hole vibration waveform ($l_{g,main}$) by Equation (61). In Equation (61), $l_{comb}$ is the duration of the firing sequence of the blast event.
Step 4: Compute mean and standard deviation of group delay, $\mu_g$ and $\sigma_g$, for the single-hole vibration waveform by Equation (62). Thus, $\mu_g$ and $\sigma_g$ are estimated parameters, and the corresponding distribution is used as a reference of location and scale for synthetic group delays. The pdf is shown as a blue curve in the top graph of Figure 5-12.

Step 5: Obtain a smoothed group delay spectrum, $\tau'_y$, of the blast event (production blast) with a moving average filter (Figure 5-12). It will be evident that the standard deviation of smoothed group delay, $\sigma'_y$, is smaller than $\sigma_y$.

Step 6: Scale $\tau'_y$ by the ratio $\left(\frac{\sigma_g}{\sigma_y}\right)$. Then relocate the result to $\mu_g$. A generated group delay spectrum can be expressed as:

$$\tau'_g = \left(\tau'_y - \mu'_y\right) \frac{\sigma_g}{\sigma_y} + \mu_g \quad (63)$$

At this point, as mentioned before, it is necessary to highlight that $\tau'_g$ represents the relationship between the group delay (derivative of the phase characteristic) and the frequency of the “synthetic” single-hole vibration waveform. In this sense, $\tau'_g$ can be seen as a seed of the relationship between the group delay and the frequency. If a random component is introduced to $\tau'_g$, it is possible to generate a set of varying group delay spectra and then generate a set of diverse synthetic single-hole vibration waveforms.

Step 7: Add random factors to the seed group delay spectrum ($\tau'_g$), including changes in the sign, the scale ($a$) and the shifting of the delay spectrum ($b$). The random generated group delay spectrum will be denoted by $\bar{\tau}_g$. The operation is expressed as:

$$\bar{\tau}_g = \text{sign} \cdot a \cdot \tau'_g + b \quad (64)$$

In Equation (64), a negative sign (-1) indicates to flip the group delay spectrum around the frequency axis. The value of -1 and 1 appear randomly by the probability of 0.5 respectively. Similar to step 5, the standard deviation of seed group delay, $\tau'_g$, is given by $\sigma'_g$, and should be smaller than $\sigma_g$, so the random scale factor ($a$) should be such that the extend of the synthetic group delay changes between $\sigma'_g$ and $\sigma_g$. So, the value of $a$ is between 1 and $\sigma_g/\sigma'_g$. Finally, the shifting factor ($b$), will move the synthetic group delay
within the range of the reference distribution, $b$ can be determined as a Gaussian number following $N(0, \sigma_g)$.

5.5.3 Synthesizing the ensemble of single-hole vibration waveforms

By combining Equations (46), (47), (51), (60)-(64), one can synthesize single-hole vibration waveforms from the measured waveforms of production blasts. Several parameters, including magnitude correction factor, shift factor of synthetic group delay, and span of moving average filter, are optimized after several trial-and-error computations.

**Step 1.** Generate a series of magnitude spectra with Equations (46) and (47). For the magnitude correction factor $c_{\gamma m}$, select an initial value of 0.3. The random factor $c_{G_i}$ can be first determined as 0.1.

**Step 2.** Using the generated synthetic single-hole vibration waveform group delay functions, it is possible to generate a series of unwrapped phases by Equation (51).

Step 4. Back-calculate the event waveform by applying the synthetic single-hole vibration waveforms into Equation (42).

Step 5. Compare the synthetic and measured event waveform. If the waveform envelope does not match very well, shift parameter b in Equation (64) needs adjusting. If the peak values are not close, then the magnitude correction factor in Equation (46) must increase or decrease. Repeat the procedure from Step 1 to Step 5 until the envelope and peak particle velocity are satisfactory.

5.6 Case studies
5.6.1 Illustration of computation
The transverse component of 62-hole blast data is used to illustrate the computation steps so as to show how the single-hole vibration waveforms are synthesized. The 62-hole waveform is shown below:

![62-hole waveform](image)

Figure 5-13 62-hole waveform

Step 1. Perform FFT and find the effective frequency limit.

![FFT results](image)

Step 2. Compute group delay and fit its histogram by a normal distribution.

![Group delay histogram](image)
Step 3. Determine main part of single-hole vibration waveform.

![Graph showing group delay vs. frequency]

Step 4. Compute mean and standard deviation of group delay distribution for the single-hole vibration waveform.

\[
\mu_g = \mu_y \cdot \frac{l_{y,\text{main}}}{l_{y,\text{main}}} = 0.944 \cdot \frac{0.706}{1.897} = 0.351
\]

\[
\sigma_g = \sigma_y \cdot \frac{l_{y,\text{main}}}{l_{y,\text{main}}} = 0.579 \cdot \frac{0.706}{1.897} = 0.215
\]

Step 5. Obtain a smoothed group delay of blast event.
**Step 6.** Obtain seed group delay

**Step 7.** Add random factors to the seed group delay of signature.

**Step 8.** Synthetic magnitudes and phases used for simulating single-hole vibration waveforms

Step 10. Simulated production blast.

5.6.2 Synthesis results
By the methodology developed above, the synthesized single-hole vibration waveforms and production blast waveforms are presented as follows.
Figure 5-14 Synthetic results vs measured data for radial component of 6-hole blast

Figure 5-15 Synthetic results vs measured data for transverse component of 6-hole blast
Figure 5-16 Synthetic results vs measured data for vertical component of 6-hole blast

Figure 5-17 Synthetic results vs measured data for radial component of 27-hole blast
Figure 5-18 Synthetic results vs measured data for transverse component of 27-hole blast

Figure 5-19 Synthetic results vs measured data for vertical component of 27-hole blast
Figure 5-20 Synthetic results vs measured data for radial component of 47-hole blast

Figure 5-21 Synthetic results vs measured data for transverse component of 47-hole blast
Figure 5-22 Synthetic results vs measured data for vertical component of 47-hole blast

Figure 5-23 Synthetic results vs measured data for radial component of 62-hole blast
Figure 5-24 Synthetic results vs measured data for transverse component of 62-hole blast

Figure 5-25 Synthetic results vs measured data for vertical component of 62-hole blast
Figure 5-26 Synthetic results vs measured data for radial component of 83-hole blast

Figure 5-27 Synthetic results vs measured data for transverse component of 83-hole blast
5.7 Discussion on statistical waveform synthesis method

The results have shown that this statistical waveform synthesis method is effective for all the five cases with different delay intervals and number of holes. To explain this methodology clearer, this section gives some discussion about the procedures and results.

(1) In the results from Figure 5-14 to Figure 5-28, both measured single-hole waveforms and production blast waveforms are included to compare with the synthetic waveforms. The measured single-hole waveforms are generally shown enveloped within the grey area formed by the generated single-hole waveforms. The synthetic production blast waveforms also have similar wave shape and PPV to the measured data. In addition, all the Fourier magnitude spectra have similar frequency distribution and dominant frequency between the measured data and synthetic results.

(2) The key step in synthesizing group delays is step 7. In this step, the scale parameter (a) and shift parameter (b) influence the location and spread of the group delay histogram, which means they influence the location and spread of synthetic single-hole vibration waveforms' envelopes. However, the scale parameter (a) is a uniformly distributed value between 1 and $\sigma_g/\sigma_g^*$, and the shift parameter (b) is a normally distributed variable following $N(0, \sigma_g)$. So, the generated group delay has a random location and spread. This
cannot guarantee a satisfactory synthetic production blast waveform from synthetic single-hole waveforms. During practical operations, some constrains should be applied to the mean and standard deviation of the generated group delay, according to the comparison between synthetic and the measured production blast waveform. Let $\tilde{\mu}_g'$ and $\tilde{\sigma}_g'$ denote the mean and standard deviation of generated group delays. The constrain condition can be expressed as

$$\mu_g + b_1 \cdot \sigma_g < \tilde{\mu}_g < \mu_g + b_2 \cdot \sigma_g$$

$$\tilde{\mu}_g' + 3 \cdot \tilde{\sigma}_g \leq \mu_g + a_1 \cdot \sigma_g$$

(65)

Where $b_1$ and $b_2$ can be any real number, but have to make $\mu_g + b_1 \cdot \sigma_g < \mu_g + b_2 \cdot \sigma_g$;

$a_1$ is also a real number.

The range $(\mu_g + b_1 \cdot \sigma_g, \mu_g + b_2 \cdot \sigma_g)$ determines the location of the generated ground delay histograms, and the upper limit $\mu_g + a_1 \cdot \sigma_g$ constrains the spread of the generated group delay histogram. As mentioned previously, $\mu_g$ and $\sigma_g$ are corresponding to the reference distribution in Figure 5-12. That is, the histograms of synthetic group delays should be within or around this reference distribution most of the time. An extreme situation where there are no constrains is

$$\mu_g - 3 \cdot \sigma_g < \mu_g < \mu_g + 3 \cdot \sigma_g$$

$$\tilde{\mu}_g' + 3 \cdot \tilde{\sigma}_g \leq \mu_g + 3 \cdot \sigma_g$$

In actual practice, if the synthetic production blast waveform has significant values in the tail part compared to the measured waveform, the range can be narrowed $(\mu_g + b_1 \cdot \sigma_g, \mu_g + b_2 \cdot \sigma_g)$ and moved to the left, and at the same time reduce the spread limit $\mu_g + a_1 \cdot \sigma_g$. In contrast, if the front part of the synthetic production blast waveform has significant contents compared to the measured waveform, the range must be moved $(\mu_g + b_1 \cdot \sigma_g, \mu_g + b_2 \cdot \sigma_g)$ to the right. This is the importance of the reference distribution.
6.1 Conclusions

(1) Synthesis of single hole vibration waveforms belongs to a type of inverse problem in blasting engineering. It is of great interest to synthesize single hole vibration waveforms from a mining blast, and high possibilities exist when electronic initiation is used.

(2) Electronic detonators have high accuracy and precision for firing times compared to pyrotechnic detonators. This provides enough practical support for the assumption of the research in this dissertation that the delay timing sequence is viewed as known.

(3) Based on the assumption of known timing sequence, preliminary analysis of two deconvolution methods has been conducted. The spectral division deconvolution is simple in theory, easy to understand, but limited for practical applications (large production shots). It is only applicable for blast with short delay interval and a small number of holes, for example, 5ms of delay interval and 6 blast holes in this dissertation research. In other words, for a typical large production blast in mining engineering, this deconvolution method is not applicable. However, it does show the timing comb function cannot be taken as the input with energy to the ground system. The single hole vibrations also cannot be assumed as impulse response of the ground system.

In theory, Wiener filtering deconvolution can be applied to the data from any scales of blasts. However, in this research, it still only works well for the 6-hole case and not effective when dealing with the other four cases. This method produces an even longer signal by convolving the Wiener filter and a production blast waveform, which makes it necessary to obtain a single hole waveform with a shorter length by truncation. However, this operation usually cannot give a good envelope of the resulted waveform, especially on the tail part. The optimum-lagged Wiener filter did not appear to work better than the zero-lagged Wiener filter when dealing with complex blast data.

(4) It is not necessary to pursue high cross-correlation coefficients among single-hole vibration waveforms. The 6-hole case study has already proved that the synthetic production blast waveform can still have a good coincidence with the measured data even
if the synthetic single hole waveforms did not have high cross-correlation coefficients. Moreover, when the number of blast holes is increasing, waveform interactions among holes gets complicated. It will also be hard to get a high cross-correlation coefficient for the synthetic production blast waveforms.

Instead of focusing on the cross-correlation, it is more meaningful and practical to pay attention to the peak particle velocity, the general envelope, and the frequency content distribution of the synthetic waveforms.

(5) The histogram or pdf curve of a signal’s group delays resembles its waveform envelope to a large extent. The time-domain waveform can be changed when modifying the statistics of group delay in the frequency domain. Based on this characteristic of group delay, the statistical waveform synthesis methodology has been developed.

This methodology can successfully synthesize a series of single hole waveforms instead of only one for the deconvolution methods. Then, the synthetic production blast waveforms have PPVs close to that of the measured data, and the general envelope the synthetic waveform also coincides with the measured data. Also, the frequency domain energy distribution coincides with the measured data.

6.2 Novel contributions

The research on inverse problems of ground vibrations is nearly a virgin territory in the mining blast engineering area. This dissertation presents some meaningful exploratory work on the synthesis of single-hole vibration waveforms.

Using the statistical waveform synthesis method (group delay), it is possible to assess a different waveform to each hole. The methodology applied to the collected data from the current production blast will result in a calibrated waveform for each hole, and those waveforms can be used for prediction of future blasts.

6.3 Recommendations for Future Work

(1) In the current stage, only the designed timing between explosives charges was considered in the impulse train function. In future developments, the travel time between the source and the point of interest will be included.
(2) More fields tests are required to further validate the proposed methods and identify their application conditions.

(3) For the deconvolution methods, more advanced deconvolution methods, like the theories of blind deconvolution may be considered.
REFERENCES


Presented at the Proceedings of the Thirty-seventh Annual Conference of Explosives and Blasting Techniques, International Society of Explosives Engineers, Cleveland, Ohio, San Diego, California.


Levinson, N., 1947. The Wiener RMS (root mean square) error criterion in filter design and prediction.


Wilhelm, E., 1926. Electric time fuse for blasting cartridges. US1570733A.


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