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An Experimental Investigation of Wing-Tip Vortex Decay in Turbulence

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An experimental investigation of wing-tip vortex decay in turbulence

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Particle image velocimetry measurements were conducted for a wing-tip vortex decaying in free-stream turbulence. The vortex exhibited stochastic collapse with free-stream turbulence present, with the breakdown initiating earlier for higher levels of turbulence. An increased rate of decay of the vortex tangential velocity was also observed, increasing with increasing free-stream turbulence. The decay of the vortex tangential velocity without the free-stream turbulence was well represented by viscous diffusion, resulting in an increase in the core radius and decrease in the peak tangential velocity. With the addition of free-stream turbulence, the rate of decay of the peak tangential velocity of the vortex increases, whereas the rate of increase of core radius remains unchanged. The circulation of the vortex decayed in time when immersed in free-stream turbulence, whereas it remained approximately constant when free-stream turbulence was not present. This decay in circulation was found to be almost entirely due to a decrease in circulation of the vortex core, caused by the relative decrease in the peak tangential velocity without a corresponding increase in the core radius. The scaling of the radial profiles of velocity and circulation was also examined, and it was found that, regardless of the free-stream condition, the core was scaled by the peak tangential velocity and core radius. The region outside the core did not scale with these quantities, and an alternative scaling for circulation is proposed which results in improved collapse of the profiles. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4979133>]

I. INTRODUCTION

Vortices are ubiquitous in fluid flows and are important contributors towards, for example, vehicle aerodynamics, mixing processes, weather systems, and structural loading. Depending on the context, these vortices may be beneficial or detrimental to a system or process. One commonly occurring detrimental example is the vortex produced by a finite lifting surface as a by-product of lift production, often referred to as a wing-tip, or simply tip, vortex. Tip vortices are formed by the roll-up of the shear layer shed downstream of the lifting surface. These vortices reduce efficiency, cause drag, introduce additional unsteady structural loading, and may pose a safety hazard in the case of tip vortices produced by a large aircraft. As these vortices are very stable and have a slow decay rate, this hazard can persist for long periods of time. This persistence can be mitigated by external influences, such as through velocity shear and turbulence.³¹

Using a turbulent boundary layer analogy, Phillips²⁵ argued that a vortex should consist of a core region, logarithmic region, and defect region. The core region is bounded by a localized maximum in the tangential velocity. Although the core and logarithmic regions of a vortex should be self-similar, the defect region is highly dependent on the initial and boundary conditions of the vortex. This proposed similarity in the core and logarithmic regions has been observed in both measured tangential velocity^{3,28} and circulation.^{1,8,28} External turbulence can also affect the vortex formation process, and hence its structure. Although the initial vortex formation

process is not grossly affected by the presence of turbulence,³ the peak tangential velocity of the vortex has been observed to be reduced by 10% when the vortex forms in the presence of free-stream turbulence.¹⁵ Note that no such reduction was observed in a more recent study by van Jaarsveld *et al.*³³ Once the vortex forms, the rate of decay of the circulation contained within the vortex core has been found to be both independent of external turbulence³³ and dependent on it.^{2,5,20,30} Furthermore, despite introducing unsteadiness and decay in the vortex core,^{10,11,13,15,18} the self-similarity for the mean tangential velocity^{1,3} and circulation^{1,9} appears unaffected by the presence of external free-stream turbulence, at least in the near field.

Spalart³¹ discusses two different possible modes of vortex evolution and decay. In the first mode, referred to as predictable decay, the strength of vortex decreases gradually and at a predictable rate. In the second mode, referred to as stochastic collapse, the vortex circulation experiences an initial slow decay, followed by a sudden rapid decrease, culminating in a vortex breakup initiated by instability within the vortex or vortex system. When immersed in external turbulence, it is still unclear whether the external turbulence acts to enhance diffusion of the vortex or initiate collapse by enhancing instabilities.

It has been suggested that a vortex under conditions of moderate turbulence intensity will produce secondary vortical structures, i.e., in turbulence with kinetic energy sufficiently high to influence the vortex, yet insufficient to destroy the vortex through excessive deformation, coherent structures form through an interaction between the vortex and external turbulence. The secondary structures form azimuthally around the primary vortex and have been observed in both

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experiments^{5,6} and simulation,^{17,22,23,27,32} and it has been hypothesized that they form via stretching of azimuthally aligned vorticity around the primary vortex. The secondary vortices tend to become axisymmetric within two or three revolutions of the primary vortex,²⁴ form at a radial position twice the radius of the primary vortex, and have a longitudinal scale in the same order as the core radius.³²

The presence of these secondary vortical structures can potentially impact vortex decay in several ways. Holzäpfel *et al.*¹⁷ argued that the primary vortex transfers rotational energy to the secondary vortices by tilting and stretching them during their formation stage. Bandyopadhyay *et al.*⁵ observed intermittent exchange of a fluid between the vortex core and the organized vortical structure surrounding the vortex core, suggesting that this “vortex stripping” mechanism can be a more direct means for the transport of vorticity from the core. This vortex stripping has been attributed to a strong local radially aligned strain rate induced by the rotation of the secondary structures.^{21,22} Weaker secondary structures, however, are expected to induce a bending wave instability in the primary vortex without introducing vortex stripping.^{21,23,32}

Thus, although a vortex is generally considered to be highly stable and persisting, it is clear that the evolution of a vortex is impacted by the presence of external turbulence. However, most prior experiments were necessarily limited to the near field, in relatively close proximity to the vortex generator. There is little information available regarding the long-time evolution of a vortex in the presence of turbulence, from formation to breakdown. Examining the decay process over an extended time period is therefore the objective of the current research. To do this, towing tank experiments were conducted in which a tip vortex was generated within approximately homogeneous, isotropic grid turbulence and the time-dependent velocity field produced by the vortex examined in detail.

II. EXPERIMENTAL ARRANGEMENT

All measurements in this study were carried out in a 3.96 m long water-filled towing tank with a cross sectional area of $0.43 \times 0.38 \text{ m}^2$ (width $\times$ depth) made of 2.54 cm thick transparent polycarbonate. The experimental configuration, consisting of a towed wing and grid, as well as a fixed particle image velocimetry (PIV) system, is presented schematically in Fig. 1. The coordinate system used here is arranged with x aligned in the towing direction, y toward the wing root, and z in the direction of lift. The PIV system was arranged to measure the y and

z components of velocity, $U_y(y, z, t)$ and $U_z(y, z, t)$, at a single y - z plane of the tow tank.

The wing used to produce the tip vortex was a square-tipped, rectangular planform, NACA0012 finite-span wing, set at an angle of attack of 8° to the towing direction. The full wing had 38.1 cm of span and a chord length of $C = 0.10 \text{ m}$. However, only 17.8 cm of the wing span was fully submerged in the water, such that its tip was positioned near the centerline of the tow tank, a vertical distance of 15.24 cm from the bottom of the tank. In order to minimize the formation of a leading edge vortex, the suction surface was equipped with a boundary layer trip made of sandpaper located near the leading edge.

A turbulence-generating grid was towed at a fixed distance of 38.1 cm upstream of the wing and was used to produce approximately homogeneous, isotropic turbulence. Two different grids with a solidity ratio of 0.43 were used, producing different length scales and turbulence intensities. The grids were made of an aluminum sheet of thickness 0.63 cm with a mesh size $M = 25.4 \text{ mm}$ and $M = 35.6 \text{ mm}$, and will be referred to as the “small-grid” and “large-grid” cases, respectively. These grids were selected following a series of preliminary investigations which indicated that they produced a relatively high turbulence intensity in the towing facility, which was also approximately homogeneous and isotropic. The baseline condition, consisting of measurements without a grid, will be referred to as the “no-grid” case. Both the grid and the wing were towed with a velocity of $U_t = 0.120 \text{ m/s}$ corresponding to a Reynolds number of $U_t C / \nu = 12\,000$, where ν is the kinematic viscosity.

Due to the relatively slow towing velocity, surface waves were found to be small and no adverse effects were observed from these waves. Furthermore, following Ref. 14, to dissipate any waves forming in the tow tank, two vertically placed grids of mesh size 3 cm and 1.5 cm, separated by 7 cm distance were located 12 cm from the end of the towing tank (a distance of 235 cm from the measurement plane). Note also that effects caused by deceleration of the wing and grid are also not expected to impact the results. For the small-and large-grid cases, all data presented were acquired prior to deceleration initiating. For the no-grid case, deceleration of the wing began after approximately 75% of the data were acquired, and no adverse affects were observed in the data which could be correlated with this point in the data.

Particle image velocimetry was used to measure the 2-D velocity field normal to the towing direction at approximately the mid-point of the tow tank. The PIV system consisted of a laser, a digital charged coupled device (CCD) camera with lens, and an external timing controller. Glass spheres of diameter 9–13 μm were used to seed the water in the tow tank. The density of the glass spheres at $25^\circ \text{ centigrade}$ was 1.1 g/mL, which is close to the density of water. The glass spheres were evenly mixed throughout the towing tank prior to each measurement run, and the tank was allowed to return to rest prior to each measurement run. A Solo PIV Nd:YAG 50 mJ dual-head laser from New Wave Research, Inc., was used to produce a double-pulsed 532 nm wavelength light sheet by passing the laser through a cylindrical lens. It was estimated that a laser light thickness of approximately 5 mm was enough to prevent the out-of-plane loss of particles from the measurement plane

FIG. 1. Experimental configuration.

for the laser pulse separation and U_t used. Particle images were captured with a digital 12-bit Imperx LYNX:IPX-4M15-L CCD camera set in the dual-frame mode. The image sensor had a resolution of 2048×2048 pixel² and a pixel diameter of $7.5 \mu\text{m}$. The camera was equipped with an AF-S VR Micro-Nikkor lens having a focal length of 50 mm and the f -stop set to 2.8. PIV timing was controlled with a custom multi-camera, multi-laser timing controller in concert with a secondary pulse-delay generator which was used to amplify the trigger signals sent to the camera and laser. The delay between the two pulses was 6.0 ms and repeated at 7.5 Hz. PIV image pairs were acquired in sets of 200, 120, and 60 for the no-grid, small-grid, and large-grid cases, respectively, corresponding to measurement periods of 26.6 s, 16 s, and 8 s.

The analyzed image pairs were limited to $t > 0$, where $t = 0$ corresponds to the instant (± 0.07 s) at which the wing trailing edge exited the laser sheet plane. The processing of particle images was done using LaVision DaVis 8.1 software. Interrogation areas of 32×32 pixel² were used, and two-pass adaptive cross correlation was applied with 50% overlap. This interrogation area size was selected to produce a particle image density of approximately 15 particles per interrogation region, following the work of Keane and Adrian.¹⁹ The post-processing of each image pair resulted in 128×128 vectors with a spatial resolution of $\Delta y = \Delta z = 2.2$ mm. The resulting velocity fields were vetted based upon the correlation value, the allowable velocity range, and comparison of nearest neighbors. Approximately 2% of the vectors were rejected by the vetting process and were replaced by interpolation.

To account for the wandering, or meandering, of the vortex in the measurement plane,^{2,4,7,11} the coordinate system origin was centered on the vortex axis for each measured instance of a vector field. To identify the center of the vortex, it was assumed, following the work of Takahashi *et al.*,³² that the axial vorticity $\Omega_x(y, z, t) = \partial U_z(y, z, t)/\partial y - \partial U_y(y, z, t)/\partial z$ followed a bi-normal Gaussian distribution, such that

$$\Omega_x(y, z, t) \approx B \exp \left[- \left(\frac{(y - \mu_y)^2}{2\sigma_y^2} - \frac{(z - \mu_z)^2}{2\sigma_z^2} \right) \right]. \quad (1)$$

The optimum fit of Eq. (1) for each vorticity field returned the estimated center position of the vortex, μ_y , μ_z , as well as an estimate of the spatial extent of Ω_x through standard deviations σ_y and σ_z , and an estimate of the peak amplitude of axial vorticity through the coefficient, B .

Vortex wandering, as well as the mean trajectory of the vortex during shear layer rollup, also introduces slight misalignment of the vortex axis with the normal direction of

measurement plane due to the bending of the vortex at any instant of time. This could lead to an apparent decrease in the tangential velocity of the vortex, introduce artificial asymmetry in the velocity field, and cause an apparent increase in the apparent size of vortex core.² However, analysis of the estimated vortex trajectory from the measured data suggests that the mean vortex trajectory deviated from being perpendicular to the measurement plane by less than $\pm 2^\circ$. Furthermore, the additional deviation of the trajectory due to vortex wandering was also small due to the relatively long wavelength of the motion, and the corresponding deviation of the vortex axis from being perpendicular to the measurement plane is estimated to be $\pm 1.6^\circ$.

To determine average statistics, each test condition was repeated 10 times, allowing ensemble averaging to be performed. Ensemble-averaged quantities will be denoted by $\langle \rangle$ and were determined following re-centering the origin of the co-ordinate system on the vortex axis at each instant of time. Each ensemble-averaged value was found from the 10 samples measured for each time, t , relative to the wing passing through the measurement plane at $t = 0$.

III. RESULTS AND ANALYSIS

A. Grid turbulence characteristics

To characterize the turbulence produced by the small and large grids, a series of measurements were conducted without the wing in place. The normal Reynolds stresses, $\langle u_y^2(t) \rangle$ and $\langle u_z^2(t) \rangle$, in the y and z directions, respectively, were found by calculating the variance of $u_y(y, z, t)$ and $u_z(y, z, t)$ within each y - z plane and then ensemble-averaging these variances. To estimate the turbulent kinetic energy, isotropy was assumed so that the normal Reynolds stress in the x direction could be approximated by $\langle u_x^2 \rangle = (\langle u_y^2 \rangle + \langle u_z^2 \rangle)/2$.

Homogeneous isotropic grid turbulence lacks the mean shear necessary for turbulence production and hence the turbulence kinetic energy $k = \frac{1}{2}(\langle u_x^2 \rangle + \langle u_y^2 \rangle + \langle u_z^2 \rangle)$ decays in time. The turbulent kinetic energy decay downstream of the grid is shown in Fig. 2(a). This decay is expected to follow the power-law decay²⁶ such that

$$\frac{k}{U_t^2} = A \left(\frac{U_t t - x_0}{M} \right)^{-n}, \quad (2)$$

where $U_t t$ corresponds to the distance from the grid in the x direction. Here, x_0 is a virtual origin, and A and n are arbitrary coefficients corresponding to the power-law fit.

FIG. 2. (a) Ensemble-averaged turbulent kinetic energy decay and corresponding increase of (b) the integral length scale for both small- and large-grid cases without the wing in place.

The corresponding fit of Eq. (2) to the results is also shown in Fig. 2(a) and was found to match well for $U_t t/M < 70$. The values of A and n from the fitted power law were found to be 0.035 and 1.152, respectively, with the obtained value of n within the range of $1.15 < n < 1.45$ typically observed for grid turbulence,²⁶ indicating that the measured grid turbulence followed the expected behavior for nearly homogeneous isotropic turbulence decay. In addition, it was found that $\langle u_y^2 \rangle / \langle u_z^2 \rangle \approx 1 \pm 0.06$ for all the ranges of $U_t t/M$ considered for both the small and large grids providing support for homogeneity and isotropy. Note that, for $U_t t/M > 70$, the turbulent fluctuations no longer follow the expected power law decay. However at this location, the fluctuation level falls to approximately 1.4% of U_t . As the PIV pulse delay and interrogation region size were optimized for the large-scale vortex rotation, the value of particle displacement at this level is approximately 10% of a pixel. Hence, we believe that the deviation from the power law for $U_t t/M > 70$ can be attributed to the turbulent fluctuations decaying below the noise floor of the PIV system, as opposed to the presence of non-decaying background turbulent kinetic energy.

To characterize the size of the energy-containing scales of the turbulence, the longitudinal integral length scale in a particular direction was calculated following

$$\mathcal{L}_i(t) = \int_0^\infty R(\Delta x_i, t) dx_i, \quad (3)$$

where $R(\Delta x_i, t)$ is the spatial auto-correlation of fluctuating velocity components u_i found from

$$R(\Delta x_i, t) = \frac{\langle u_i(x_i, t) u_i(x_i + \Delta x_i, t) \rangle}{\langle u_i(x_i, t)^2 \rangle}. \quad (4)$$

The temporal development of ensemble-averaged integral length scale $\langle L(t) \rangle$ is presented in Fig. 2(b) for both the small- and large-grid cases. To obtain $\langle L(t) \rangle$, $\mathcal{L}_i(t)$ was calculated for each row and column of vectors in the y and z directions within each y - z plane and then averaged to obtain $L(t)$. These values were then ensemble-averaged over the 10 measurement runs to obtain $\langle L(t) \rangle$. The results presented in Fig. 2(b) show that the integral length scales increased from approximately $0.2M$ to $0.5M$ as the turbulent kinetic energy decays. This increase roughly followed a power law growth of $L/M \sim (U_t t/M)^b$ with b being 0.2 for the small-grid case and 0.4 for the large grid case. Note that the integral length scale of the turbulence was the same order of magnitude as the radius of the vortex core.

B. Vortex tangential velocity

The ensemble-averaged tangential velocity $\langle U_\theta(y, z, t) \rangle$ was calculated from the ensemble-averaged velocities $\langle U_y \rangle$

and $\langle U_z \rangle$ following

$$\langle U_\theta(y, z, t) \rangle = \langle U_y(y, z, t) \rangle \cos \theta + \langle U_z(y, z, t) \rangle \sin \theta, \quad (5)$$

with

$$\theta = \tan^{-1} \frac{z}{y}. \quad (6)$$

The radial dependence of ensemble-averaged tangential velocity, $\langle U_\theta(r, t) \rangle$, where $r = (y^2 + z^2)^{0.5}$, was determined by bin-averaging $\langle U_\theta(y, z, t) \rangle$ using sixty equally spaced, radial bins. Sample radial profiles of $\langle U_\theta(r, t) \rangle$ are presented for the no-grid, small-grid, and large-grid cases at two different times in Figs. 3(a)–3(c). Although all cases have slight differences, they all show the expected vortex profile, with the tangential velocity increasing with radial distance to a peak tangential velocity, $\langle U_c \rangle$, located at $r = \langle r_c \rangle$, and decreasing monotonically with r for $r > \langle r_c \rangle$. In this way we define the vortex core as the region contained within $r < \langle r_c \rangle$. For all three free-stream conditions, the vortex exhibits the evidence of decay in the form of a decrease in the peak tangential velocity and an increase in r_c with time.

In Fig. 3 the tangential velocity has been normalized by $\langle U_c^* \rangle$, the ensemble-averaged value of the maximum peak tangential velocity measured for each run. Note that the value of $\langle U_c^* \rangle$ was approximately $0.34U_t$, $0.30U_t$, and $0.27U_t$ for the no-grid, small grid, and large-grid cases, respectively, reflecting a reduction of U_c imposed by the turbulence during vortex formation and roll-up. Thus, this normalization is intended to emphasize relative differences between the vortex evolutions in each case, revealing that the rate of decay of the vortex is dependent on the free-stream conditions.

This decay is summarized in contour plots of the evolution of $\langle U_\theta(r, t) \rangle / \langle U_c^* \rangle$, which are presented in Figs. 4(a)–4(c) for the no-grid, small-grid, and large-grid cases, respectively. Note that the range of time presented in Figs. 4(a)–4(c) is different for each case as it was found that the vortex would experience breakdown for the cases with free-stream turbulence. The vortex breakdown point was identified as the point in time where vortex center could not be identified by a fit to Equation (1), i.e., the vorticity does not follow bi-normal distribution that is distinguishable from the background turbulence. Since we were unable to find the center of the vortex once this occurs, we do not present any data beyond this point and we consider the vortex to have broken down.

The results indicate that the tangential velocity of the vortex decreased with time for all cases, with the rate of decrease increasing with free-stream turbulence; however, the details of the decay appear to be slightly different for each case. For the no-grid case, Fig. 4(a), in the outer region of $r > 0.15C$, the value of $\langle U_\theta(r) \rangle$ remains relatively constant in time until

FIG. 3. Comparison of radial profiles of $\langle U_\theta \rangle / \langle U_c^* \rangle$ at two different times for (a) no-grid, (b) small-grid, and (c) large-grid cases.

FIG. 4. Contour plots showing evolution of $\langle U_\theta(r, t) \rangle / \langle U_c^* \rangle$ for (a) no-grid, (b) small-grid, and (c) large-grid cases. The dotted black lines in (a), (b), and (c) indicate core radius $\langle r_c \rangle$ at different times.

$U_t t/C \approx 10$, after which it experiences slow decay. This consistency in the outer region is also evident in Fig. 3(a). For the no-grid case in Fig. 4(a), there is also evidence of the vortex roll-up and growth near $r = r_c$ for $U_t t/C < 1$. This roll-up is also evident for the small-grid case for $U_t t/C < 1$, as can be observed through the close inspection of Fig. 4(b). However for this case, the decay in the outer region is much more rapid than the no-grid case for $U_t t/C \lesssim 10$, followed by a period of slower decay as presented in Fig. 4(b). For the large-grid case shown in Fig. 4(c), the decay is rapid from vortex formation until its collapse at $U_t t/C \approx 10$.

Although there are apparent differences in the decay behavior in the outer region of the vortex, to more clearly discern the evolution of the vortex core, we compare the evolution of $\langle r_c \rangle$ and the corresponding value of $\langle U_c \rangle$. This is done in Figs. 5(a) and 5(b) and evolutions consistent with decay of the vortex core are observed for all cases. Given that the viscous

decay of an idealized line vortex results in $r_c \propto \sqrt{vt}$ (see, for example, Saffman²⁹), we can expect that if the decay observed in Fig. 5 is due solely to viscosity that $\langle r_c \rangle$ will follow a similar square-root dependence. This is indeed the case because, as shown in Fig. 5(a), the core radius growth is well represented by

$$r_c = 2.5\sqrt{\nu(t + t_0)}, \quad (7)$$

where $t_0 = 8.5s$ is a virtual origin. The rate of growth of $\langle r_c \rangle$ is therefore consistent with viscous diffusion of vorticity.

Interestingly, as shown in Fig. 5(a), the rate of core growth is independent of free-stream conditions. A similar observation was also made by Bailey and Tavoularis² although they observed that r_c remained approximately constant with distance from the wing. However the present experiments were also conducted at an order of magnitude lower Reynolds number; hence, we can expect an increase in the rate of decay due to the increased role of viscosity.

Although the size of the core radius does not appear to be dependent on free-stream conditions, Fig. 5(b) shows that the rate of decay of $\langle U_c \rangle$ clearly increases with the intensity of the free-stream turbulence. By continuing with the analogy of the viscous decay of a line vortex, one can expect that $\Gamma_c = 2\pi r_c U_c$ should be constant. The U_c evolution corresponding to Eq. (7) is presented in Fig. 5(b) using the average value of Γ_c found for the no-grid case. This curve therefore represents the decay of U_c which could be expected due to viscous diffusion. The strong agreement between the evolution of $\langle U_c \rangle$ measured for the no-grid case and this curve suggests that the decay observed in the no-grid case is consistent with viscous decay.

However, the decay of $\langle U_c \rangle$ is accelerated with increasing turbulence, in contrast to the growth rate of $\langle r_c \rangle$, which remains unaffected. The different responses of $\langle U_c \rangle$ and $\langle r_c \rangle$ to turbulent free stream conditions indicate that the core circulation was not conserved during the decay process when free-stream turbulence was present. The constancy of r_c suggests that the decay in U_c was through a mechanism other than through turbulent diffusion and supports the existence of a more stochastic process of vortex decay. In other words, although the vortex decays, the geometric size of the vortex is not impacted by external turbulence; vorticity does not diffuse radially outward at an increased rate as turbulence increases. This thus suggests that the circulation lost from the core is due to either destruction of vorticity, or through re-orientation of the vorticity into azimuthal or radial components.

FIG. 5. (a) Ensemble-averaged increase of $\langle r_c \rangle/C$ and $\langle r_v \rangle/C$ along with (b) the corresponding decay of $\langle U_c \rangle / \langle U_c^* \rangle$ for no-grid small-grid and large-grid cases. Solid line indicates vortex core evolution corresponding to Eq. (7) and assuming constant core circulation.

C. Vortex circulation

To better examine the decay of the vortex circulation throughout the entire measurement domain, we obtain circulation $\Gamma(S, t)$ through integration of $\Omega_x(y, z, t)$ over the domain S , a selected area of integration in the y - z measurement plane. Here, we let $S = \pi r^2$ and investigate the r dependence of the vortex circulation, $\Gamma(r, t)$. The time evolution of the circulation of the vortex is presented as contour plots of $\langle \Gamma(r, t) \rangle$ in Figs. 6(a)–6(c) for the no-grid, small-grid, and large-grid cases, respectively. To isolate the influence of free-stream turbulence on the downstream evolution, for each free-stream condition the ensemble-averaged circulation has been normalized by $\langle \Gamma_0 \rangle = \langle \Gamma(0.5C, 0) \rangle$.

For all three cases, the circulation distributions indicate that there is an initial concentration of circulation towards the core, which can be attributed to roll-up of the shear layer. This initial roll-up occurs for $U_t t/C < 5$ for the no-grid case, but for the small-grid and large-grid cases, this roll-up is limited to $U_t t/C < 1$. Following roll-up, the circulation at fixed r/C decreases with t for all three cases. However, as indicated by the slope of the contour lines, with increasing turbulence intensity the rate of decay increases. Note that the slope of the circulation isocontours for the no-grid case closely follows the growth in $\langle r_c \rangle$, indicating that the circulation in the core remains constant in this case. However for the small-grid and large-grid cases, the previously observed loss of circulation within the core appears as different slopes of the iso-contour lines of $\langle \Gamma \rangle$ and the line indicating $\langle r_c \rangle$.

To compare the impact of free-stream conditions on the decay of circulation more quantitatively, we examine the evolution of the circulation contained within three separate regions. First, $\langle \Gamma_t \rangle$ estimates the total circulation within the

measurement plane, defined as $\langle \Gamma_t \rangle = \langle \Gamma(0.5C, t) \rangle$, given that $r = 0.5C$ is the largest radius around the vortex axis continually contained within the measurement plane. The second is $\langle \Gamma_v \rangle = \langle \Gamma(r_v, t) \rangle$ where we have introduced $\langle r_v \rangle$, which describes an intermediate radius of the mean rotational flow field using the normality assumption. This estimate is obtained through the fit of the vorticity distribution to Eq. (1) such that $\langle r_v \rangle = 3 \max(\{\sigma_y, \sigma_z\})$. The evolution of $\langle r_v \rangle$ is provided in Fig. 5(a), and it can be observed that, as with $\langle r_c \rangle$, the growth of this radius is also independent of the free-stream conditions. Finally, we also examine the evolution of $\langle \Gamma_c \rangle = \langle \Gamma(r_c, t) \rangle$, describing the core circulation. The dependence of these quantities on time for each of the free-stream conditions is presented in Fig. 7(a). Also shown in Fig. 7(b) is the corresponding difference between $\langle \Gamma_t \rangle - \langle \Gamma_c \rangle$ and $\langle \Gamma_v \rangle - \langle \Gamma_c \rangle$.

Assuming that $\langle \Gamma_t \rangle$ contains essentially all the vorticity within the vortex, we see that initially approximately 55% of the vorticity of the vortex is contained within the vortex core, and 80% contained within the region described by $\langle \Gamma_v \rangle$. As the vortex evolves, $\langle \Gamma_t \rangle$ remains approximately constant (within $\pm 0.1 \langle \Gamma_0 \rangle$) at all t for the no-grid case, confirming that circulation is conserved. However, for the small-grid and large-grid cases, there is a noticeable decay in $\langle \Gamma_t \rangle$, reflecting a loss of circulation of the vortex due to the free-stream turbulence.

Interestingly, whereas for the no-grid case $\langle \Gamma_v \rangle / \langle \Gamma_0 \rangle$ increases in time with the growth of $\langle r_v \rangle$, such that it eventually becomes equal to $\langle \Gamma_t \rangle / \langle \Gamma_0 \rangle$, for the small-grid and large-grid cases $\langle \Gamma_v \rangle$ remains at approximately 80% of $\langle \Gamma_t \rangle$ for the lifespan of the vortex, suggesting that the concentration of vorticity far from the core is being inhibited by the turbulence. Hence, the vorticity in the wake remains in the outer region of the vortex in a non-axisymmetric state.

More importantly, it can also be observed that $\langle \Gamma_c \rangle$ follows the general trend of $\langle \Gamma_t \rangle$, with the no-grid case approximately constant following an initial roll-up. For the turbulence cases there is a monotonic decrease in $\langle \Gamma_c \rangle$ which increases with free-stream turbulence intensity. The difference between the circulation of the vortex core and the total circulation is displayed in Fig. 7(b). Although there is a decrease in the difference between $\langle \Gamma_t \rangle$ and $\langle \Gamma_c \rangle$ for the no-grid case, corresponding to a slight decrease in $\langle \Gamma_t \rangle$ in time, for the grid cases $\langle \Gamma_t \rangle - \langle \Gamma_c \rangle$ remains approximately constant in time. This indicates that the 20% loss in $\langle \Gamma_t \rangle$ which occurred for the cases with free-stream turbulence before breakdown arose from a loss of axial vorticity within the vortex core, whereas the net axial vorticity outside the core remained largely unaffected by the presence of turbulence. Furthermore, the vorticity is not simply transported to the outer region of the vortex, but is lost through another mechanism. This is supported by Fig. 7(b), which shows the difference between $\langle \Gamma_v \rangle$ and $\langle \Gamma_c \rangle$. This figure shows that the difference between these values remains approximately constant in time for the cases with free-stream turbulence, indicating that the circulation decrease observed in $\langle \Gamma_v \rangle$ also comes from the vorticity lost from the vortex core.

As noted previously, the mechanism for the vorticity loss from the core acts through a reduction in $\langle U_c \rangle$ without causing an increase in $\langle r_c \rangle$ beyond that which can be attributed to viscous decay, indicating that the additional increase in decay of

FIG. 6. Contour plot of normalized ensemble-averaged vortex circulation $\langle \Gamma \rangle / \langle \Gamma_0 \rangle$ between normalized radius r/C and time $U_t t/C$ for (a) no-grid, (b) small-grid, and (c) large-grid cases. The dotted black lines in (a), (b), and (c) indicate the core radius $\langle r_c \rangle$ at different times.

FIG. 7. Downstream variation for no-grid, small-grid, and large-grid cases of normalized ensemble averaged: (a) $\langle \Gamma_t \rangle / \langle \Gamma_0 \rangle$, $\langle \Gamma_v \rangle / \langle \Gamma_0 \rangle$, and $\langle \Gamma_c \rangle / \langle \Gamma_0 \rangle$; (b) corresponding differences $(\langle \Gamma_t \rangle - \langle \Gamma_c \rangle) / \langle \Gamma_0 \rangle$ and $(\langle \Gamma_v \rangle - \langle \Gamma_c \rangle) / \langle \Gamma_0 \rangle$.

circulation due to external turbulence is a non-diffusive process. This supports a non-diffusive mechanism such as the formation of secondary vortices in the azimuthal direction, which can potentially cause vortex decay due to the initiation of instabilities,²² transfer of energy from the primary vortex to secondary vortices,¹⁷ and spontaneous ejection of vorticity from the core.⁵ Holzäpfel *et al.*¹⁷ showed that secondary vortices developed when the turbulence intensity, $(2/3k)^{1/2}$ was approximately $0.05U_c$ for cases with both isolated vortices and vortex pairs. However, when the turbulence intensity was high, equivalent to $0.24U_c$, the ambient turbulence directly deformed the primary vortex and caused its rapid destruction. Also, Pradeep and Hussain²⁷ had found that for an initial turbulence intensity of $0.15U_c$, the core was surrounded by secondary vortices after 120 rotations. In the present experiment the turbulence intensity was $(0.14 \pm 0.005)U_c$ and $(0.09 \pm 0.01)U_c$ for large- and small-grid cases, respectively. Hence, the conditions appear amenable to the formation of secondary structures which, in turn, could contribute to the non-diffusive decay observed.

To investigate the possible presence of these secondary structures in further detail, a series of PIV measurements were taken in the x - z plane, with the y location approximately bisecting the vortex core. Due to wandering of the vortex, these measurements cannot be analyzed quantitatively as the position of the measurement plane relative to the vortex center cannot be known with precision. However, qualitatively, they support the presence of secondary azimuthal vortices and ejection of flow from the vortex core. Example snapshots of cross-plane vorticity are presented in Figs. 8(a)–8(c) for the no-grid, small-grid, and large-grid cases, respectively. Also shown in Figs. 8(d)–8(f) is the corresponding in-plane velocity magnitude. The core is clearly apparent in the in-plane velocity

magnitude as a region of enhanced velocity due to the wake-like streamwise velocity. There is also a signature of the core in the vorticity due to the corresponding contribution to Ω_y , from the streamwise velocity gradients within the core. However, also evident outside the core are bundles of cross-plane vorticity, believed to be the signature of secondary structures. Although relatively weak for the no-grid case, these patches are larger, and more distinct for the small- and large-grid cases, suggesting intensification of these structures when free-stream turbulence is present. Also evident in Figs. 8(d)–8(f), and most noticeable in the large-grid case, are fingers of higher velocity magnitude stretching out from the core region. Although difficult to tie to the azimuthal structures, it is believed that this is evidence of core fluid being ejected from the core. The increased prevalence of these instances for the grid cases supports the hypothesis that the increased rate of decay observed for the grid cases is caused by increased occurrence of the exchange of core and free-stream fluid.

D. Self-similarity of mean vortex properties

The different response of $\langle U_c \rangle$ and $\langle r_c \rangle$ to free stream conditions, the former decaying at an increased rate with increasing turbulence intensity and the latter remaining unaffected by it, suggests that their behavior may not be self-similar. However, the velocity profile itself, when normalized by these quantities, has been hypothesized to be self-similar through an analogy to the turbulent boundary layer.^{16,25} Based on this analogy, a vortex can be divided into three regions: the vortex core region, the logarithmic region, and the defect region. Phillips²⁵ suggested that the vortex remains self-similar within the core and logarithmic regions when scaled by U_c and r_c for $r/r_c \leq 1.2$. The proposed semi-empirical equation describing

FIG. 8. Contour plot of y component of vorticity, Ω_y , normalized by the ensemble-averaged peak vorticity of the vortex in the x direction, $\langle \Omega_x^* \rangle$, at time $U_t t / C = 2.55$ for (a) no-grid, (b) small-grid, and (c) large-grid cases. Corresponding in-plane velocity magnitude is shown normalized by towing velocity in (d), (e), and (f), respectively. Note that the vertical z axis has been emphasized to better view the features in the radial direction.

the velocity within this self-similar region is

$$\frac{U_\theta}{U_c} = \frac{r_c}{r} \left[1.772 \left(\frac{r}{r_c} \right)^2 - 1.0467 \left(\frac{r}{r_c} \right)^4 + 0.2747 \left(\frac{r}{r_c} \right)^6 \right] \quad (8)$$

for the region, $r/r_c \leq 0.92$, where viscous forces are non-negligible and

$$\frac{U_\theta}{U_c} = \left[\ln \left(\frac{r}{r_c} + 1 \right) \right] \frac{r_c}{r} \quad (9)$$

for $0.92 \leq r/r_c \leq 1.2$ which is the analogous region to the logarithmic overlap region in boundary layer turbulence.

As shown in Fig. 9(a), the scaling holds in the present experiment, with $\langle U_\theta \rangle / \langle U_c \rangle$ following Eqs. (8) and (9) for $r/\langle r_c \rangle \leq 1.2$ for all free-stream conditions. This is despite the likely presence of high levels of unsteadiness in the vortex core, which was observed in the instantaneous vector fields, as well as being observed in other studies.^{10,11,13,15,18} Self-similarity of a vortex for the same range of normalized radius in the presence of turbulence was also observed by Bailey *et al.*³ and Ahmadi-Baloutaki *et al.*,¹ although these previous measurements were limited to the velocity field closer to the wing than in the present case.

To evaluate how the measured velocity profiles compare to other models for vortex velocity, we also compare the results to the Lamb-Oseen vortex²⁹

$$U_\theta = \frac{\Gamma_t}{2\pi r} \left(1 - \exp \left(-1.2526(r/r_c)^2 \right) \right) \quad (10)$$

FIG. 9. Comparison of measured profiles of (a) $\langle U_\theta \rangle / \langle U_c \rangle$ and (b) $\langle \Gamma \rangle / \langle \Gamma_c \rangle$ for no-grid, small grid, and large-grid cases incorporating all profiles up to $U_t/C = 32, 19.2, 9.6$, respectively.

and the Burnham-Hallock vortex¹²

$$U_\theta = \frac{\Gamma_t}{2\pi r} \left(\frac{r^2}{r^2 + r_c^2} \right). \quad (11)$$

The comparison is performed in Fig. 9 and was made by choosing Γ_t to ensure that $U_\theta/U_c = 1$ at $r = r_c$. In both cases, the models match the velocity and circulation in the core with the same level of agreement as the Phillips model. However, outside the vortex core, the Lamb-Oseen model provides the best agreement with the no-grid case, consistent with the viscous decay observed in Fig. 5. However, for the cases with free-stream turbulence present, the Burnham-Hallock model appears to provide a better agreement.

The turbulent boundary layer analogy suggests that the region $r/r_c > 1.2$ is analogous to the outer region in the turbulent boundary layer, and that in this region viscous forces become negligible and there is increasing dependence on free-stream conditions. Specifically, Hoffmann and Joubert¹⁶ argued that the effect of boundary conditions would be confined to the outer region.

In the present experiments, this dependence on free-stream conditions appears as increased data scatter in Fig. 9(a). The scatter was higher in the presence of free-stream turbulence but is not random, as shown in Fig. 10, which shows the time evolution of the scaled velocity profiles. All three cases present a slightly different behavior for $r/\langle r_c \rangle > 1.2$, with the spatial extent of the decrease of $\langle U_\theta \rangle / \langle U_c \rangle$ from unity to zero outside the core increasing with increased turbulence.

FIG. 10. Evolution of ensemble-averaged tangential velocity when normalized by $\langle U_c \rangle$ and $\langle r_c \rangle$ for (a) no-grid, (b) small-grid, and (c) large-grid cases. Note that as $\langle r_c \rangle$ increases, the range of $r/\langle r_c \rangle$ contained within the measurement plane decreases. The edge of the measurement region is indicated by the boundary of the contour plot.

FIG. 11. Comparison of (a) ensemble-averaged scaled circulation $((\Gamma_t) - \langle \Gamma \rangle) / \langle \Gamma_c \rangle$ based on the turbulence boundary layer analogy from Hoffmann and Joubert¹⁶ and (b) ensemble-averaged modified scaled circulation $((\Gamma_v) - \langle \Gamma \rangle) / (\langle \Gamma_v \rangle - \langle \Gamma_{avg} \rangle)$ based on the turbulence boundary layer analogy from Zagarola and Smits³⁴ with normalized radius $r / \langle r_v \rangle$ for no-grid, small-grid, and large-grid cases in the defect region of a vortex. Only data points up to $U_t / C = 9.6$ are shown for clarity.

This broadening reflects an enhanced decay rate of $\langle U_c \rangle$ in the presence of free-stream turbulence. Note that the observation that the vorticity loss is confined to the core implies that the outer region decays at a rate commiserate with the no-grid case. Hence as the core decays, the scaled velocity in the outer region will appear to increase relative to the no-grid case when core quantities r_c and U_c are selected as scaling parameters.

Noting also that the core parameters U_c and r_c represent inner-scaling parameters in the boundary layer analogy, we thus instead seek a more suitable set of parameters to scale the outer region. To address this question, we observe that Eqs. (8) and (9) can be re-expressed to describe the circulation

$$\frac{\Gamma}{\Gamma_c} = \left[1.772 \left(\frac{r}{r_c} \right)^2 - 1.0467 \left(\frac{r}{r_c} \right)^4 + 0.2747 \left(\frac{r}{r_c} \right)^6 \right] \quad r/r_c \leq 0.92, \quad (12a)$$

$$\frac{\Gamma}{\Gamma_c} = \left[\ln \left(\frac{r}{r_c} + 1 \right) \right] \quad 0.92 \leq r/r_c \leq 1.2. \quad (12b)$$

The measured results are presented as circulation profiles in Fig. 9(b) and re-iterate the success of this scaling for $\langle \Gamma \rangle / \langle \Gamma_c \rangle$ within the range $r / \langle r_c \rangle \leq 1.2$ for all free-stream conditions.

This defect scaling proposed by Hoffman and Joubert¹⁶ suggested scaling parameters U_c and r_t , where r_t is the radius where $\Gamma = 0.99\Gamma_t$ would be suitable for the outer region such that

$$\frac{\Gamma_t - \Gamma}{\Gamma_c} = f \left(\frac{r}{r_t} \right), \quad (13)$$

where f represents an arbitrary function. When applied to the present results as done in Fig. 11(a), it becomes evident that this defect law scaling is ineffective for the cases with free-stream turbulence, despite the circulation profile showing the expected decrease of circulation with radius.¹⁶ Not only is there dependence on the free-stream conditions but there is also dependence on t .

We therefore seek alternative scales to normalize the circulation profiles. Here, we continue to follow the turbulent boundary layer analogy but instead apply the outer layer scaling approach proposed by Zagarola and Smits³⁴ for pipe flows. In their scaling, the velocity in the outer region was found to scale with the area-averaged velocity, and the appropriate length scale was found to be the pipe radius. By analogy, we

suggest that the appropriate circulation and length scales will be the area-averaged circulation and the extent of the vortex will be represented of the size of the axi-symmetric portion of rotational region, as represented by r_v .

Using these scales, we propose a modified circulation scaling for the defect region of a vortex of

$$\frac{\Gamma_v - \Gamma}{\Gamma_v - \Gamma_{avg}} = f \left(\frac{r}{r_v} \right), \quad (14)$$

where Γ_{avg} represents area-averaged circulation up to r_v . Figure 11(b) demonstrates that this modified scaling significantly improves collapse of the circulation profiles, not only amongst the different free-stream conditions, but also with varying t . This is perhaps not unexpected, as Γ_{avg} will encompass the decay of the outer region better than Γ_c , which we have observed to decay at a different rate than the entire vortex.

IV. CONCLUSIONS

The objective of this study was to observe and document the effects of freestream turbulence on the temporal evolution of wing-tip vortex. To do this, a wing-tip vortex was generated by towing a NACA0012 wing oriented at an 8° angle of attack and nearly homogeneous and isotropic turbulence of different kinetic energies and length scales was produced upstream of the wing by towing one of the two turbulence-generating grids upstream of the wing. The ensemble-averaged velocity field was examined as a function of time and free-stream turbulence conditions.

For all free-stream conditions examined, the vortex exhibited evidence of decay in the form of a decrease in the tangential velocity with time, most notably in the core region, accompanied by a corresponding increase in the vortex core size. For the baseline case without free-stream turbulence, this decay was consistent with the expected decay due to viscous effects and the circulation of both the vortex and the vortex core remained approximately constant in time. Similar observations were made in the work of Bailey and Tavoularis;² however, the prior study required the reconstruction of the velocity field over a limited streamwise extent from multi-point measurements. In the present case, these observations are made from a direct measurement of the vortex velocity and cover a much larger streamwise distance from the wing, showing that this form of decay is robust.

When free-stream turbulence was present, there was a measurable decrease in the circulation of the vortex, leading to breakdown of the vortex. Increased free-stream turbulence leads to more rapid decay and earlier breakdown. The increased decay of the vortex in the presence of free-stream turbulence was epitomized in an increase in the rate of decay of the peak tangential velocity. However, despite the more rapid decay of the peak tangential velocity, there was not a corresponding increase in the rate of growth of the core radius. As a result, the vortex core lost approximately one third of its circulation prior to breakdown. It was found that the observed decrease in the total circulation of the vortex could largely be attributed to the decrease in circulation within the vortex core.

The lack of a corresponding increase in core radius suggests that this circulation loss cannot be attributed to a diffusive process. Hence, it is unlikely that this loss of circulation can be captured by eddy-viscosity-based turbulence models. It also suggests that the vorticity initially aligned in the streamwise direction is preferentially transferred to other components or asymmetrically destroyed by interaction with the turbulence. This mode of vortex decay is consistent with prior observations of vorticity stripping by secondary coherent structures which form azimuthally around the vortex and re-orient the vorticity into the azimuthal direction. Some evidence in support of this mechanism was provided in snapshots of the vorticity and velocity fields in a plane parallel to the vortex axis.

The scaling of the radial profiles of velocity was also investigated and it was found that the velocity within the core scaled on peak tangential velocity and core radius, regardless of free-stream conditions. These quantities were found to be ineffective at scaling the vortex velocity profile outside the core region. A strong contribution to this lack of scaling can be attributed to the increased decay of peak tangential velocity introduced by free-stream turbulence. Since the decay of circulation outside the vortex core was unaffected by the free-stream turbulence, this resulted in the core quantities being inappropriate for scaling the outer region. To account for this, an alternative scaling parameter is required. It was suggested that the average circulation of the vortex could serve as a suitable scaling parameter, and improved collapse was observed in the radial profiles of circulation in the outer layer when this parameter was incorporated into the scaling.

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