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Excited 0^+ states in ^{160}Gd have been examined with the $(n,n'\gamma)$ reaction at incident neutron energies up to 2.8 MeV. Gamma-ray excitation functions and angular distribution measurements allow the confirmation of the existence of 0^+ states at 1379.70 keV and 1558.30 keV, but we reject the assignments of additional previously suggested 0^+ candidates. Limits on the level lifetimes of the observed 0^+ states permit an evaluation of the collectivity of these states.

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I. INTRODUCTION

The nature of excited 0^+ states remains an open challenge to our understanding in nuclear structure physics [1–6]. The review by Heyde and Wood [1] summarizes the difficulties that have emerged in understanding 0^+ states, both from experimental and theoretical viewpoints. The assertions in Ref. [1] are that a complete characterization of 0^+ states requires the measurements of transfer cross sections and $E0$ transition probabilities, in addition to the knowledge of level energies and absolute transition probabilities. Data on 0^+ states had been sparse until recent (p,t) studies established many excited $K^\pi = 0^+$ states in deformed nuclei [7–13]. Theoretical efforts [14–22] abound and continue to offer possible interpretations of these low-lying excitations in deformed nuclei.

In well-deformed regions of the nuclear landscape, excitations built on a deformed ground state have traditionally been described in terms of quadrupole excitations, leading to the decades-old classification of the first excited 0^+ bands as single-phonon β -vibrational bands. Newer interpretations include the possibility of phase changes at the onset of deformation (for example, at $N = 90$ and $Z = 64$) and the application of new symmetries to describe these nuclei [3,23–27]. Another explanation for the nature of 0^+ bands was given in terms of shape co-existence, where a competing shape is not the lowest favored shape but occurs low in excitation of a given nucleus. Other work [28] expanded on the original description of β vibrations and provided some guidelines to the clear identification of $K^\pi = 0^+$ bands as β vibrations if $B(E2; 2^+_{\beta} \rightarrow 0^+_{\text{g.s.}})$ values are in the range of 2.5–6 W.u., small two-nucleon transfer strengths, and large $E0$ values connecting them to the ground state. In the IBM [29–31], the first excited 0^+ and 2^+ bands are members of the same representation in the $SU(3)$ limit and they are only weakly (theoretically forbidden) connected to the ground-state band. Another recent development describes nuclei at or near the

onset of deformation within the Bohr Hamiltonian in the limit of rigid prolate axial symmetry with confined β -soft potentials [16,17,19]. These studies and others [32–34] on the nature of $K^\pi = 0^+$ bands in deformed nuclei show widely varying levels of collectivity for the first excited 0^+ states. Recent experiments have also shown enhanced collectivities in transitions connecting even higher excited states to the first excited 0^+ state [10,35–38].

The goal of this work is to investigate and characterize $K^\pi = 0^+$ bands in ^{160}Gd . In recent years, high-resolution (p,t) reactions on stable nuclei have been used to identify many 0^+ excitations in deformed nuclei; however, this reaction is not possible for ^{160}Gd as the required target nucleus, ^{162}Gd , is unstable ($T_{1/2} = 8.4$ min). The $^{158}\text{Gd}(t,p)^{160}\text{Gd}$ reaction has been performed [39] with the identification of a previously known 0^+ state at 1382 keV and a tentative candidate at 2236 keV. In the present work, we examine the known information on 0^+ states in ^{160}Gd and provide new limits on the collectivity of these excitations.

II. EXPERIMENT

We have measured γ -ray excitation functions and angular distributions using the $^{160}\text{Gd}(n,n'\gamma)$ reaction at the University of Kentucky Accelerator Laboratory (UKAL). Neutrons were produced by the $^3\text{H}(p,n)$ reaction. The scattering sample was 29.456 g of 98.12% enriched $^{160}\text{Gd}_2\text{O}_3$ contained in a thin-walled polyethylene cylinder 3.1 cm in height and 2.3 cm in diameter. The emitted γ rays were detected with a $\sim 50\%$ HPGe detector with time-of-flight gating for background reduction and an annular BGO shield for active Compton suppression [40]. A spectrum is shown in Fig. 1.

An excitation function measurement, performed from $E_n = 1.5$ to 2.8 MeV in 0.08- or 0.1-MeV steps, with the detector at 90° with respect to the incident beam, provided yields of γ rays as a function of neutron energy. This measurement allowed the placement of γ rays to levels based on thresholds. Gamma-ray angular distribution measurements were performed at incident neutron energies of 1.5, 2.0 and 2.8 MeV at ten angles over

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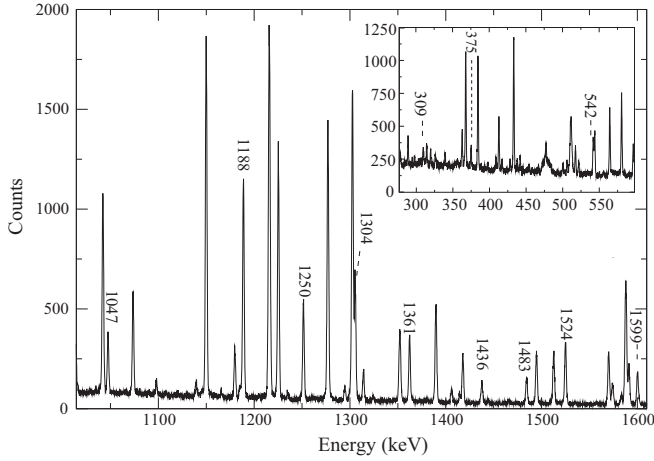


FIG. 1. Portion of the prompt γ -ray energy spectrum from the $^{160}\text{Gd}(n,n'\gamma)$ reaction with 2.0 MeV neutrons, recorded at 90° with respect to the beam axis. The γ -ray energies are given in keV and are located above the peak. The energies shown are for the γ rays associated with the 0^+ bands discussed in the text.

a range of 40° to 150° . The neutron energies were chosen to reduce feeding to the levels of interest to obtain the most accurate lifetimes. The yields of the γ rays, $W(\theta)$, were fitted with even-order Legendre polynomials,

$$W(\theta) = A_0[1 + a_2 P_2(\cos \theta_\gamma) + a_4 P_4(\cos \theta_\gamma)], \quad (1)$$

where a_2 and a_4 depend on the multiplicities and mixing amplitudes of the transition. These results may also be compared with statistical model calculations using CINDY [41] to determine or restrict spin possibilities. The angular

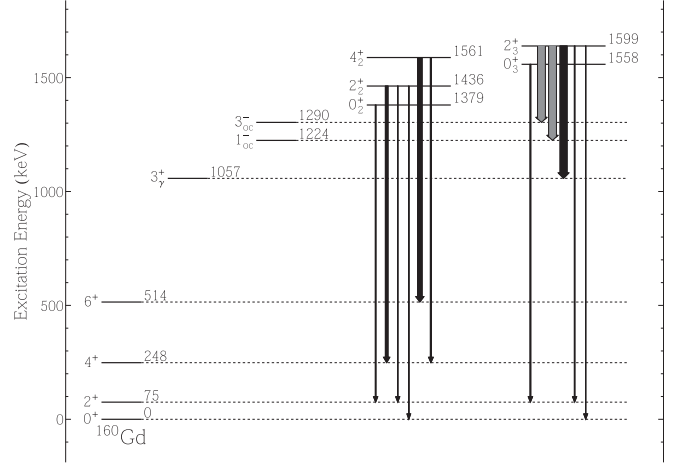


FIG. 2. Partial level scheme for the 0^+ bands discussed in this work. The solid black arrows are $E2$ transitions and the grey arrows are $E1$ transitions. The upper limits of the transition probabilities are given in Table I.

distribution measurements were also used to measure lifetimes of excited states shorter than 1 ps [42] via the Doppler-shift attenuation method (DSAM). The energies of the detected photons are

$$E_\gamma(\theta_\gamma) = E_{\gamma 0} \left[1 + \frac{v_0}{c} F(\tau) \cos \theta_\gamma \right], \quad (2)$$

where $E_{\gamma 0}$ is the unshifted γ -ray energy, v_0 is the recoil velocity of the center-of-mass frame, θ_γ is the angle of observation, and $F(\tau)$ is the experimental attenuation factor [42]. The average lifetime of a state, τ , is determined by

TABLE I. Energy levels, associated with $K^\pi = 0^+$ bands, observed in this work. E_L is the level energy found in this work with uncertainties; I_γ is the relative γ -ray intensity, normalized to 100 for the most intense transition from each level. Mean lifetimes, τ , shown here are the lower limits, therefore, the $B(E2)$ values are upper limits. All of the level lifetimes were obtained from the 2.0 MeV angular distribution data except for the 1379 keV level, in which the 1.5 MeV angular distribution data were used.

E_L (keV)	E_f (keV)	$J_i^\pi \rightarrow J_f^\pi$	E_γ (keV)	I_γ	$F(\tau)$ (fs)	τ (fs)	$B(E2)$ (W.u.)	$B(E1)$ $\times 10^3$ (W.u.)	Notes
$K^\pi = 0_2^+$									
1379.70(7)	75.25	$0_2^+ \rightarrow 2^+$	1304.46(5)	100	0.015 ± 0.014	>1350	<3.10		
1436.47(4)	248.64	$2^+ \rightarrow 4^+$	1187.81(5)	100(1)	0.062 ± 0.074	>340	<13.1		
	75.25	$2^+ \rightarrow 2^+$	1361.05(6)	36.4(4)	0.041 ± 0.068		<2.42		
	0.0	$2^+ \rightarrow 0^+$	1436.34(6)	13.5(2)	0.005 ± 0.070		<0.68		
1561.59(6)	515.10	$4^+ \rightarrow 6^+$	1046.67(6)	100(1)	0.051 ± 0.091	>320	<22		a
	248.64	$4^+ \rightarrow 4^+$	1313.03(6)	74.8(3)	0.040 ± 0.072		<5.4		
$K^\pi = 0_3^+$									
1558.30(7)	75.25	$0_3^+ \rightarrow 2^+$	1483.06(6)	100	0.004 ± 0.069	>590	<3.74		
1599.00(4)	1290.01	$2^+ \rightarrow 3^-$	309.32(6)	8.9(4)	-0.132 ± 0.538	>300		<1.37	b
	1224.33	$2^+ \rightarrow 1^-$	374.78(6)	14.8(3)	-0.205 ± 0.326			<1.3	b
	1057.60	$2^+ \rightarrow 3^+$	541.53(6)	36.8(3)	0.161 ± 0.205		<174		c
	75.25	$2^+ \rightarrow 2^+$	1523.59(6)	100(1)	0.056 ± 0.064		<2.68		
	0.00	$2^+ \rightarrow 0^+$	1598.85(6)	78.7(1)	0.055 ± 0.063		<1.66		

^aThis level may be assigned incorrectly, please see text.

^bThis transition was not used to calculate the level lifetime.

^cThis is a mixed $E2/M1$ transition, if a pure $M1$, the value is $B(M1) < 0.18\mu_N^2$.

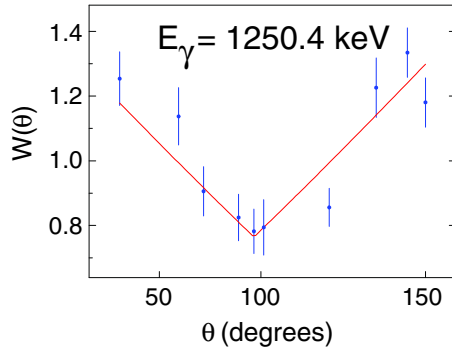


FIG. 3. (Color online) The γ -ray angular distributions for the 1250.4-keV γ ray, which is not isotropic and, therefore, is not from a 0^+ state.

examining the energy of the γ ray as a function of angle, extracting the $F(\tau)$ value, and comparing with the theoretical $F(\tau)$ curve calculated using the Winterbon formalism [43]. The lifetimes determined from each γ ray depopulating a level must match within experimental uncertainties, aiding in the assignment of γ rays to specific levels.

In all measurements, ^{226}Ra and ^{152}Eu standard sources were used out-of-beam for energy and efficiency calibrations. A ^{60}Co source placed near the detector during the angular distribution measurements was used as a continuous check for gain shifts. At higher neutron energies, an additional in-beam ^{24}Na source was employed for accurate energy identification. NaCl rings were irradiated off-line with neutrons from a ^{252}Cf source to produce ^{24}Na , which emits 1368.63- and 2754.03-keV γ rays. These methods and techniques are described in greater detail in other publications [40,44,45].

Lifetime and angular distribution measurements were compared with previous data. There are very few lifetimes known in the range of our measurements; however, we were able to compare our measured lifetime ($\tau = 21 \pm 2$ fs) of the 1^- level at 1224.28 keV with the evaluated value of $\tau = 22 \pm 6$ fs [46] which exhibits excellent agreement. The angular distribution were normalized to the known 0^+ γ -ray energy at 1304.46 keV.

III. RESULTS AND DISCUSSION

The results from these experiments are summarized for all of the 0^+ states in Table I. We confirm the 0^+ state at 1379.70 keV [39,47] and the former tentative assignment of a 0^+ state at 1558.30 keV [48]. We reject the former

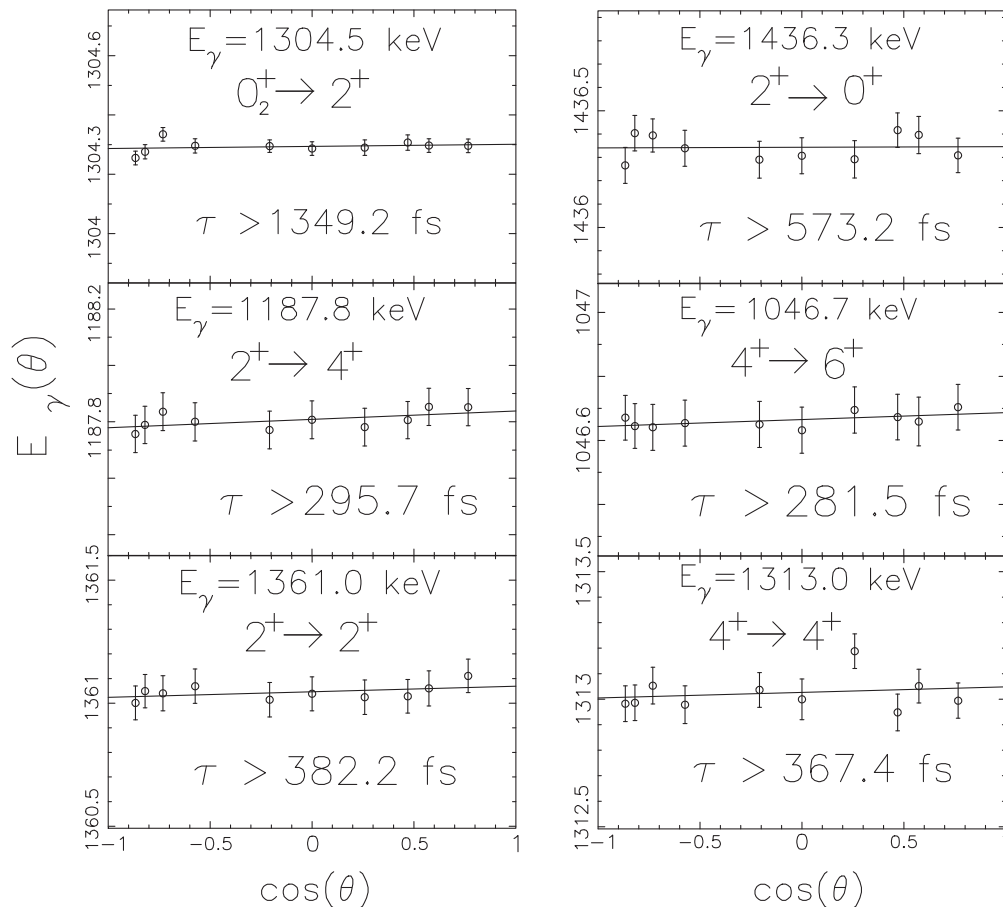


FIG. 4. $E_\gamma(\theta)$ (in keV) versus $\cos\theta$ for selected γ rays in ^{160}Gd . Each of the 0^+ , 2^+ , and 4^+ levels in the 0^+ bands are shown. The lifetime for each γ -ray transition is averaged (weighted for branching ratio) for the level lifetime. The γ rays exhibit small energy shifts and, therefore, only limits on the lifetimes are obtained.

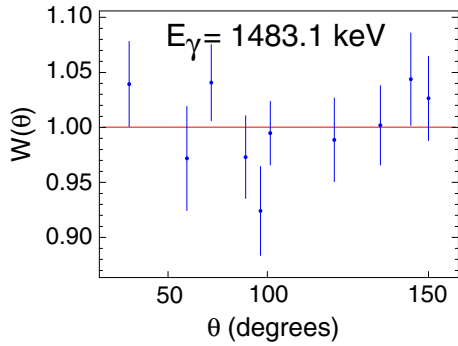


FIG. 5. (Color online) The γ -ray angular distribution of the 1483 keV γ ray from the 1558-keV 0^+ state, $a_2 = 0.053 \pm 0.038$.

tentative assignments of 0^+ for states at 1325.73 keV [49] and 2236 keV [39]. A partial level scheme is shown in Fig. 2 including the confirmed 0^+ bands. A detailed discussion of the measurements by level energies follows below.

In this experiment we did not attempt to assign new spin-0 states but to confirm the existing levels and obtain level lifetimes. The angular distributions and excitation functions aided the outcome in different ways. First, excitation functions supported the γ rays as depopulating a given level by their

appearance thresholds. This also led to the identification of any additional γ rays from the level by matching energy and excitation function thresholds.

In order to confirm the 0^+ assignment of a level, it is required that the γ -ray angular distributions from these 0^+ states be isotropic, i.e., $a_2 = 0$ in Eq. (1). The angular distribution data were also used to extract level lifetimes. A search for the in-band decays of the 2^+ members of the 0^+ bands was unsuccessful due to absorption in our thick sample and internal conversion, we are generally unable to observe <100 keV γ rays. Table I lists the 0^+ states of ^{160}Gd confirmed in this work and the spectroscopic information obtained in these experiments. As internal conversion electron data are not available, the $E0$ decays to the ground states are not taken into account. Specific details follow for each level.

A. 1325.73-keV level

The level at 1325.73 keV was formerly given a tentative 0^+ assignment based on energy considerations by Berzin *et al.* [49] in $(n,n'\gamma)$ measurement using reactor neutrons. Gover *et al.* [48] disagreed, stating that the transition is not isotropic and placing the $E_\gamma = 1250.42$ keV as a decay from a $J^\pi = 4^-$, 1498.87 keV level. From our measurement, the 1250.42 keV γ -ray transition is not isotropic (see Fig. 3). The

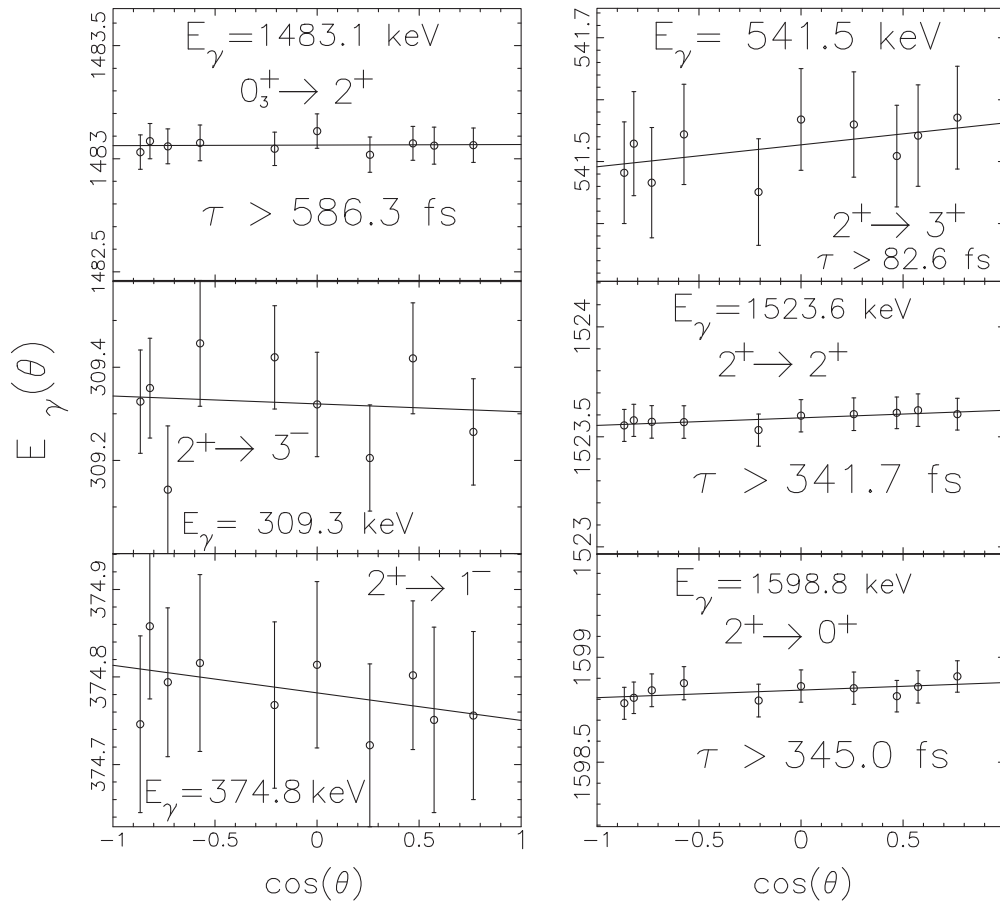


FIG. 6. $E_\gamma(\theta)$ (in keV) versus $\cos\theta$ of the 0_3^+ band in ^{160}Gd . Each of the 0^+ and 2^+ levels are shown. The lifetime for each γ -ray transition is averaged (weighted for branching ratio) for the level lifetime. The γ rays exhibit small energy shifts and, therefore, only limits on the lifetimes are obtained.

angular distribution has a nonzero a_2 value of 0.48 ± 0.10 , and therefore agree with Ref. [48] in excluding a spin and parity assignment of 0^+ at 1325.73 keV. The excitation function threshold of 1.5 MeV suggests two possible placements of this γ ray. One is the current placement at 1325 keV with decay to the 2_g^+ state; the second is at 1498 keV with decay to the 4_g^+ state. Since the threshold of the γ ray has a low intensity at $E_n = 1.5$ MeV, the placement at 1498 keV is favored. The CINDY calculations do not aid our ability to narrow down the spin possibilities. Our data, therefore, supports the placement of the 1250-keV γ ray at the 1498-keV level, but we cannot comment on the spin assignment made by Ref. [48], especially since the 441.51-keV γ ray was not observed.

B. $K = 0_2^+$ band

This band was observed in $^{158}\text{Gd}(t, p)$ reaction [39] and first assigned in early $(n, n'\gamma)$ work [47]. We have observed a single γ ray ($E_\gamma = 1304.46$ keV) from the 0_2^+ level at 1379.70 keV. The energy threshold of the excitation function supports this placement. We were able to extract a lifetime limit of > 1350 fs for this level, which was used to calculate a $B(E2; 0_2^+ \rightarrow 2_2^+)$ limit of < 3.10 W.u. as shown in Fig. 4.

Identified in Ref. [49] as part of the band structure, a 2^+ state at 1436 keV is included in Table I. Again, we were unable to observe the < 100 keV intraband transition. A level lifetime limit of > 340 fs was established and $B(E2)$ upper limits were calculated. The angular distribution data from the 2.0-MeV experiment was used for the lifetime measurements and branching ratios, because of the higher statistical quality than in the 1.5-MeV data.

Gover *et al.* [48] assigned a 4^+ state at 1561.48 keV as part of this band. Our γ ray intensities from this level agree within uncertainties with these in Ref. [48]. According to the Alaga rules, the $B(E2; 4^+ \rightarrow 2_g^+)$ should have approximately the same value as the $B(E2; 4^+ \rightarrow 4_g^+)$, however, the decay to the 2_g^+ is not observed. This is an indication that the 1561 keV level is not a member of this 0^+ band.

C. $K = 0_3^+$ band

A previous $(n, n'\gamma)$ experiment assigned $J^\pi = 0^+$ to the level at 1558.30 keV by the intensity and the nearly isotropic nature of the angular distribution of the 1483.06-keV γ ray [48]. The angular distribution data do not exclude the level as a spin-zero state, $a_2 = 0.053 \pm 0.038$ (Fig. 5). We were able to obtain a lower lifetime limit of 590 fs which corresponds to a $B(E2)$ upper limit of 3.74 W.u. The 2^+ member of the band at 1599.00 keV decays to many levels including the 3^+ level of the γ -vibrational band. The $B(E2)$ values are given

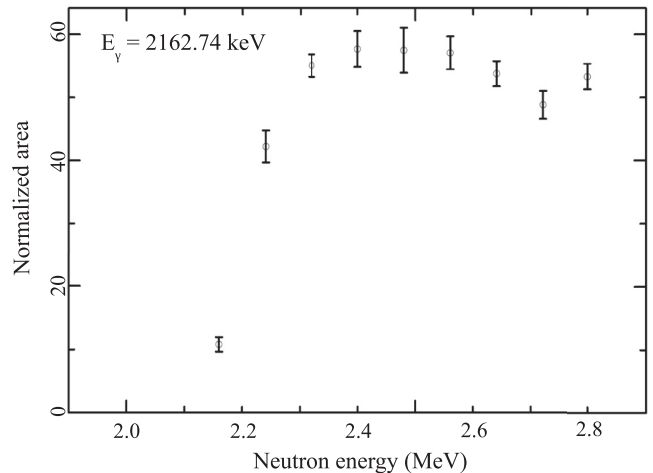


FIG. 7. Excitation function plot for the 2162.7-keV γ ray.

in Table I and the $F(\tau)$ plots are shown in Fig. 6. The lowest energy γ -ray decays have negligible shifts; these γ rays were not used in calculating the level lifetime.

D. 2236-keV level

One additional level was tentatively assigned as a spin-0 level in the (t, p) study [39]. We searched for γ rays from a possible level at this energy. A single γ ray, $E_\gamma = 2162.74$ keV, was observed, but it could not be assigned to this level, because the excitation function threshold of 2.16 MeV is too low (see Fig. 7) and the angular distribution is not isotropic.

IV. SUMMARY

We have studied the previously identified $K^\pi = 0^+$ states in ^{160}Gd with the $(n, n'\gamma)$ reaction. Gamma-ray excitation functions, angular distributions, and lifetime measurements were used to characterize the levels at 1325.73, 1379.70, 1558.30, and 2236 keV. Our results allow the confirmation of levels at 1379.70 keV and 1558.30 keV as 0^+ states and the measurements of level lifetimes for three and two states, respectively, built on these 0^+ states. Our angular distributions indicate that the levels at 1325.73 and the 2236 keV are not 0^+ states.

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