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# Developing a Best Practice Model for Forecasting Annual Franchise Fee Revenue: The Case of the Lexington-Fayette Urban-County Government

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# Developing a Best Practice Model for Forecasting Annual Franchise Fee Revenue

## The Case of the Lexington-Fayette Urban-County Government

***Executive Summary:*** The LFUCG currently forecasts their revenues internally and has their forecasts validated by the Center for Business and Economic Research (CBER) at the University of Kentucky. However, it does not have a well-developed method of forecasting franchise fee revenue. They are not alone, as the literature on revenue forecasting that finds that between 50 and 75 percent of local governments rely on informal, judgmental approaches to forecast revenue instead of more formal, quantitative techniques. However, the literature also indicates that these judgmental approaches are less accurate.

Inspired by a study of St. Petersburg, Florida by Gianakis et al., and in an effort to find the best forecasting method for Lexington's franchise fee revenue, this capstone analyzes three different forecasting strategies: unsophisticated methods, Holt-Winters multiplicative method, and multiple regression using robust standard errors.

Results showed that a simple 12 month lag was the most consistently accurate method, while multiple regression showed promising results, especially for years where there were no unexpected shocks to the system. The results for multiple regression were hindered by a small number of observations and a missing forecast for February 2012. It is recommended that the LFUCG use a simple 12 month lag, revising using projections about natural gas prices and weather trends. Suggestions for future studies include developing a model to predict natural gas prices, and heating- and cooling-degree-days.

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## **Background and Research Question**

Many state governments use econometric modeling to forecast state revenues. These models vary in their complexity and methods, and are generally believed to be more reliable and accurate than other extrapolative methods like using moving averages (Grizzle & Klay, 1994). During my interview with the Lexington-Fayette Urban-County Government (LFUCG) Director of Budgeting, R. Barrow, he indicated that the current procedures and methods for forecasting employee withholdings, insurance, and business returns are mature and well developed, while corresponding procedures for franchise fees are relatively new less developed (personal communication, October 23, 2012). Franchise fees are the fees which utility and media companies pay the city for permission to operate and install infrastructure within the city limits. Considering franchise fees accounted for about 6.5% of revenue in 2011, this is a fairly substantial portion of revenue – in fact it was the fourth largest source of revenue in 2011. The relationship between forecast accuracy and productivity/efficiency has been studied by various scholars with consistent results showing that less accurate forecasts can adversely affect productivity (Cirincione, Gurrieri, & van de Sande, 1999; Klein, 1984; Rodgers & Joyce, 1996). Given the underdeveloped nature of the forecasting methods for franchise fees, the relevant research question to the Lexington-Fayette Urban-County Government (LFUCG) becomes the following: what would be considered a method of best practice in the context of forecasting franchise fee revenue for the LFUCG?

## **Forecasting Methods: Research Review**

There has been much interest recently in studying revenue forecasting because of the fiscal stress caused by multiple economic downturns. In general, according to the literature, local governments opt to utilize a judgmental approach to revenue forecasting. In fact, “a finding from a national survey of 290 local finance officers found that upwards of 75 percent of local governments do

not utilize formal forecasting” (Beckett-Camarata, 2006). This finding, from a study performed by McCullough and Frank, also found that for time horizons that exceed 6 months econometric techniques outperform time series methods like moving averages and exponential smoothing. In her review of the current literature on revenue forecasting, Beckett-Camarata cites a study by Bretschneider, Bunch and Gore (1992) which showed that while cities do a generally good job of forecasting taxes and tax revenue, they perform much more poorly in their forecasts of other revenue streams and intergovernmental revenues (Beckett-Camarata, 2006). This study supports the opinion of both the former Director of Budgeting for the LFUCG, Mr. Barrow, and the Director of Revenue, William O’Mara. When interviewed, Mr. O’Mara indicated that it is very difficult to forecast franchise fee revenue in particular, and that the LFUCG does not currently have a quantitative model to estimate this revenue stream. Instead it relies on simple averages, current events and news, as well as prognostications about weather patterns (personal communication, February 1, 2013).

Beckett-Camarata’s study of revenue forecasting in Ohio local government found that formal forecasting (using quantitative methods) is more accurate than informal methods (judgmental approach) by comparing the forecast accuracy between Summitt County and the city of Canton. Canton, which used formal forecasting techniques, had much more accurate forecasts than did Summitt County, which relied on a mainly judgmental process of forecasting. Additionally, the multiple regression method used by Canton proved to be the most accurate. However, in general, the author points out that prior research studies have suggested that methods like exponential smoothing and the Box-Jenkins method can be more accurate than regression techniques because they put more weight on the time periods closest to the forecast (Beckett-Camarata, 2006).

In summary, formal, quantitative methods outperform methods that rely on human judgment. Another interesting insight posed by Beckett-Camarata is the following: “The city of Canton uses a

variety of methods such as multiple regression and time series analysis, depending on both the revenue source and the quality, quantity, and mix of the data available. This has some unique advantages over a strict adherence to a single approach” (Beckett-Camarata, 2006). She goes on to further point out that different methods have different strengths that can be maximized by matching different methods to their appropriate revenue streams. This is precisely what this study has set out to do – find an approach that is best suited to forecast franchise fees for the LFUCG.

### **Forecasting Franchise Fees – A Study of St. Petersburg, Florida**

Although much of the research mentioned thus far is helpful as a foundation to analyzing the issue of forecasting LFUCG’s franchise fee revenue the localities under study do not derive much of their revenue from franchise fees as does the city of Lexington. As previously mentioned franchise fees are the 4<sup>th</sup> largest stream of revenue for the LFUCG, amounting to \$18.14M during FY 2011. Given this importance, it is beneficial to review another study by Gerasimos Gianakis and Howard Frank that specifically analyzes franchise fee revenue in St. Petersburg, Florida.

Gianakis et al. had seven years of continuous revenue data prior to the 1990 fiscal year which they used as inputs in their forecasts of 1990 revenues. They analyzed intergovernmental revenues, utility tax revenues and franchise fee revenues. . The authors discovered that franchise fee revenues “are influenced by population trends, weather changes, price increases, and payment changes negotiated with the city” (Gianakis & Frank, 1993). Moreover, the authors noted that the data for franchise fees showed seasonality, trends over time, and some degree of randomness.

They tested seven forecasting techniques (regression, moving average, Holt technique, single exponential smoothing, Box-Jenkins technique, general adaptive filtering, and Winters technique) paired with varying preceding data streams of 24, 48, and 72 months. For instance, St. Petersburg’s 1990 franchise fee revenue was forecasted using single exponential smoothing and the prior 24, 48, and 72

months of data as inputs. Further, each revenue type was tested with each technique–stream pair. They also aggregated the revenue types and tested whether level of aggregation affected accuracy of forecasts

The authors used the mean absolute percentage error (MAPE)<sup>1</sup> as a metric to gauge accuracy. This method uses a simple percentage error between the forecast value and the actual value and then averages the absolute values across n forecasts. Upon testing the data for St. Petersburg using their seven forecasting techniques, the authors found that the MAPE varied according to the particular combination of utility franchise fee and forecast technique. That is, for each individual utility’s revenue stream, a different method proved to be most accurate.

Table 1

<b>Aggregated Franchise Fee Revenue Forecast Error (MAPE)</b>			
<b>Technique</b>	<b>Data Stream</b>		
	<b>24 Months</b>	<b>48 Months</b>	<b>72 Months</b>
Box-Jenkins (12)	*	27.56	9.7
General Adaptive Filtering (9)	59.48	1.55	12.55
Holt (16)	20.16	100	54.06
Moving Average (7)	17.4	17.4	17.4
Regression (8)	19.73	1.42	32.38
Single Exponential Smoothing (13)	27.71	27.71	27.71
Winters (20)	100	100	54.06
<b>Legend</b>	<b>Gold (First)</b>	<b>Silver (Second)</b>	<b>Bronze (Third)</b>

\* excluded from 24-month data stream

Source: (Gianakis & Frank, 1993)

The author notes that most revenue sources tested exhibited trend and seasonality, which should point to the Winters, Holt and ARIMA methods as being the most likely to be the best performers. However, upon aggregating all franchise fee revenue sources together, regression, general

<sup>1</sup>  $MAPE = \frac{100\%}{n} \cdot \sum_{t=1}^n \left| \frac{F_t - A_t}{A_t} \right|$ , where n = number of forecasts,  $A_t$ = actual forecast at time t, and  $F_t$ =forecast value at time t.

adaptive filtering and moving average seemed to be the most accurate methods. These (aggregated revenue) results are outlined in table 1 above.

One promising result that the authors tested and confirmed was that it is possible to find a best technique for a given source, though it may require significant investment in trial and error experimentation. Further, a two-way ANOVA yielded no significant interactions between length of data stream and either technique or source, indicating that once a technique has been matched with a given source, the technique's accuracy should remain largely constant across time. Another interesting finding was that simple regression recorded the fewest most accurate scores as well as the second fewest top three scores across all revenue types and yet it remains the most frequently used method among local government forecasters. However, for franchise fee revenue specifically, as seen in table 1 above, regression was the second best method.

In summary, for forecasts relying on less input data (the 24 month stream above) moving average and regression seemed to outperform the other methods used. However, as the input data was increased, general adaptive filtering and Box-Jenkins seemed to take the lead in accuracy. The moving average technique also performed admirably across all data streams. Somewhat surprisingly, the Winters and Holt methods were not as accurate as the authors predicted. Using a simple ranking system where first place gets 1 point and last place gets 7 points, the moving average was ranked as the best option, with regression placing second, and general adaptive filtering placing third. Although the Box-Jenkins method was the fourth best technique in this ranking system its ranking suffers because it cannot be used with the 24 month data stream. If looking at only the 48 and 72 month data stream, the Box-Jenkins method ties general adaptive filtering for the best model.



# Forecasting LFUCG Franchise Fee Revenue

## Methods

The methods tested in this study come from Gianakis' study of St. Petersburg, theoretical arguments, and my own attempt to replicate methods currently used by the city of Lexington. Gianakis' study indicated that regression was one of the best performing methods and so it was included. Theory suggests that methods which account for both trend and seasonality perform better, naturally, when the data exhibit both of these features. Therefore, based on the seasonality of the franchise fee revenue, the Holt-Winters multiplicative method was used (Chatfield & Yar, 1988).

Multiple measures of accuracy are reported to compare different methods. MAPE, forecast error, and the absolute forecast error rate are all reported. The MAPE was discussed earlier, and forecast error is simply the difference between the forecast and the actual revenue. The forecast error rate was chosen because it indicates how the forecast performed relative to the actual change from period to period instead of measuring relative to the base value of the previous year. This method puts a higher burden on the forecaster because it neglects to use the annual revenue as a base. Using actual annual revenue in the denominator, like the MAPE, artificially inflates the accuracy of the method by using a large denominator. As forecasters, we are interested in the direction and the magnitude of change that will occur from the previous period to the current period. The absolute forecast error rate only uses changes, which is precisely our interest. The formula for the forecast error rate is below:

$${}^2\text{Absolute Forecast Error Rate} = \left| \frac{\text{predicted annual change} - \text{actual annual change}}{\text{actual annual change}} \right|$$

$${}^2\text{AFER}_{2010} = \left| \frac{[(\text{FFR}_{2010}^{\text{pred}} - \text{FFR}_{2009}) - (\text{FFR}_{2010} - \text{FFR}_{2009})]}{[\text{FFR}_{2010} - \text{FFR}_{2009}]} \right|$$

### Current Methods

In an attempt to replicate methods that may currently be in use, I developed three different unsophisticated methods. The first is a simple 12 month lag whereby it is assumed that the franchise fee revenue in the current month will be exactly equal to the franchise fee revenue in the same month in the previous year. The second method, which will be referred to as the double-lag model hereafter, uses a combination of a 12 month lag of franchise fee revenue and a 4 quarter lag of intra-quarter % change in franchise fee revenue. This was used because the data showed a pattern of intra-quarter trends that seemed fairly consistent. The third method was a simple average of the first two methods.

The double-lag model predicts the revenue of the first month of each quarter by using two different lagged values. To forecast the first month of each quarter a 12 month lag is increased by the average of the three most recent quarter-to-quarter percent changes for each respective quarter.<sup>3</sup> Then the last month of each quarter is predicted by increasing the prediction for the first month by an average of the previous three years' intra-quarter growth rates. The middle month of each quarter is simply an average of the predictions for the first and last month of each respective quarter. This model is designed to capture any recurring trends both within quarters and across time. Equations for this model can be found in the appendix.

### Holt-Winters Method

The equations for the Holt-Winters forecast method can be found in the appendix. This method is a form of exponential smoothing which is characterized by its three smoothing parameters:  $\alpha$ ,  $\gamma$ , and  $\delta$ . Each of these parameters can take on values between zero and one. They are used for updating the mean level, trend and seasonality index respectively at every time period. As a starting value for the mean level, a monthly average of the initial years' data was used. The seasonal indices

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<sup>3</sup> Because of limited data, for FY 2006 only the previous years' growth rate was used. For FY 2007 only the previous two years' growth rates were used.

were also derived from the initial or base years' data. Each month's seasonal index was simply the franchise fee revenue for that month divided by the total franchise fee revenue for that year. The seasonal indices were not normalized after the first year. The parameters as well as the initial trend value were varied for each fiscal-year/technique pair and were determined through the minimization of three separate measures of forecast error. Accuracy was measured from FY 2006 through FY 2012.

All three optimization techniques relied on the solver add-in for Microsoft Excel which minimizes an objective function subject to constraints that the user can define. Under all methods the only constraints that were defined were that the three parameters,  $\alpha$ ,  $\gamma$ , and  $\delta$  must be within the range  $[0, 1]$ , and the initial trend be within the range  $[-100,000, 100,000]$ . The first optimization technique minimized the average annual absolute forecast error rate for all years leading up to the forecast year. The second minimized the average monthly (or quarterly) absolute forecast error rate for all months (quarters) leading up to the forecast year. The last minimized the cumulative sum of absolute forecast errors for all months (quarters) leading up to the forecast year. Each optimization technique attempted to fit the historical data leading up to the forecast year as closely as possible to the actual historical values. Then, the annual forecasts (the sum of the forecasts for the twelve months of the forecast year) were measured for accuracy. The forecast year is the year for which predictions were calculated.

#### *Multiple Regression with Robust Standard Errors*

Initially, an autocorrelation plot was produced and analyzed to check for a severe autocorrelation problem. The data exhibited some degree of autocorrelation with a 12 and 24 month lag, but not much. Next, a Dickey-Fuller test was performed in STATA to determine if the data had a unit root process. The results were such that I rejected the presence of a unit root and the series was assumed to be stationary. This allowed for the use of the original franchise fee revenue data and exempted me from being forced to use a first or second difference. Therefore I proceeded to use STATA

to regresses monthly franchise fee revenues on heating-degree-days (HDD), cooling-degree-days (CDD), monthly precipitation, average monthly retail price of electricity to ultimate consumers in Kentucky, average natural gas citygate price in Kentucky, both monthly GDP and population for the city of Lexington, as well as time in the form of a simple counter from 1 to 112. Thirteen dummy variables were also included – one for each month and an additional dummy to gauge the effect of having incomplete data for six observations. It was assumed that the data exhibited heteroskedastic errors, which was confirmed by a simple plot of franchise fee revenue against time.

The next phase involved predicting the franchise fee revenues for specific varying forecast years. Forecasts were made for FY 2008 – FY 2012 by using the fitted values generated by regressing varying amounts (seven, six, five, four, and three years prior to the forecast year) of historical data on the corresponding explanatory variables. Sixteen different sets of predictions were yielded – one for each combination of forecast year and input data stream length. For example, FY 2010 franchise fee revenue was forecasted three different times. Data streams of length five, four, and three years were used to test whether the amount of input data affected the accuracy of the prediction. Once forecasts were calculated they were compared to actual historical values and judged for their accuracy.

## **Data**

Various data were collected in order to perform the multiple regression analysis. My response variable was partial franchise fee revenue and the Director of Revenue of the LFUCG furnished the data. The data is partial because the media/telecom portion of the franchise fee was omitted on the basis of its high autocorrelation and presumed ease of forecast. According to Bill O'Mara, this is because in 2006 the media/telecom utility sector transitioned to be state regulated as opposed to locally regulated. Since then all utility companies pay their fees to the state which then distributes the revenue to each locality based on population and usage (Personal communication, February 1<sup>st</sup>, 2013).

I mined data from excel files used to track franchise fee revenue and aggregated the data to form one dataset. The utilities which pay franchise fees to the LFUCG are Blue Grass Energy, Delta Gas, Columbia Gas, Clark Energy, Kentucky Utilities, and Kentucky American Water. Kentucky Utilities, Kentucky American Water, and Delta Gas all pay quarterly while the others pay monthly. Beginning in 2006 the revenue department began recording the franchise fees according to the month in which they were generated instead of the month in which they were received. Therefore, I corrected the data prior to 2006 by shifting each payment back one month (or one quarter) to match the current recording practices so that the data series maintained consistency across its entirety.

Monthly heating-degree-days (HDD) and cooling-degree-days (CDD), as well as monthly precipitation, were collected from the National Climatic Data Center (National Climatic Data Center, 2013). Heating-degree-days and cooling-degree-days are measures of daily variation from a base temperature of sixty-five degrees Fahrenheit. I used cumulative monthly heating-degree-days and cooling-degree-days to match my dataset. Heating-degree-days are typically larger in the winter months and cooling-degree-days are larger in the summer months (National Weather Service). These two measures were included because it was assumed that as the heat or cold becomes more extreme, the energy demands also become more extreme because people seek to maintain a comfortable indoor climate despite the outdoor climate. A positive coefficient is expected for both of these variables. Precipitation was included based on a theory that the city may use less water during periods of high rainfall. It is anticipated that activities such as watering city golf courses, keeping public pools full, watering city landscaping, watering residential landscaping, etc. will decrease if monthly rainfall amounts are sufficient to diminish the needs for such activities.

Natural gas and electricity prices were collected from the Energy Information Administration (United States Energy Information Administration, 2004-2012). They are each thought to affect the franchise fee revenue because as input prices increase for public utilities, they must increase prices for

electricity and heat, which will generate more revenues for them and thus result in more revenues for the city of Lexington via franchise fees. This is because the electric and gas companies pay a franchise fee of 3% of their sales. If sales go up, so should franchise fee revenues (Kentucky Public Service Commission, 2013).

The Kentucky citygate natural gas price was chosen among various measurements of natural gas price. According to the American Gas Association (AGA) the citygate price is the “sales price of the natural gas at [the point where natural gas is transferred from an interstate or intrastate pipeline to a local natural gas utility]: the price reflects the wholesale/wellhead price as well as the cost of transporting the natural gas by pipeline to the citygate. Citygate prices can show tremendous variation between regions, often reflecting regional usage patterns, weather trends and the number of competing interstate pipelines serving each region” (American Gas Association). Using the citygate price should capture much of the true variation in the natural gas prices that customers face. The only costs left out would be the costs in operating the utility and delivering it to the local customers. Those operating and delivery costs are likely less variable and constituted mainly of fixed costs that only change long-term due to investments in infrastructure.

Lexington Gross Domestic Product (GDP) and population were included in the model to control for changes in the base amount of public utility customers as well as any changes in income of those customers (United States Census Bureau, 2012), (Bureau of Economic Analysis, 2013). GDP is a measure of economic production of goods and services commonly reported on a national, regional and local basis. Only annual population data was available so a simple linear extrapolation was used to estimate the monthly data between each annual report of population. Additionally, because these population and GDP data had not been reported for portions of FY2012 and FY2013, those missing values were estimated. A simple regression (R-squared >0.99) on time was used to produce fitted values for population. A regression (R-squared >0.99) on time, national GDP, and Lexington monthly employment

was used to produce fitted values for Lexington GDP. It is expected that as population increases, there will be a corresponding positive increase in franchise fee revenues. It is also assumed that there will be a positive relationship between Lexington GDP and franchise fee revenue.

January serves as the omitted dummy variable against which the coefficients for the other eleven months should be compared. February – December were each included as dummy variables. It is expected that the coefficients on most other months will be negative given that the peak of the seasonality of the data is generally in January or February. December and February may not be statistically significantly different than January's amount. Lastly, the incomplete dummy variable is expected to be negative, because incomplete data implies a smaller amount than normal for those months.

## **Results**

### *Initial Analysis*

Upon a first look at the data for franchise fee revenues and the explanatory variables, one can see in figure 2 that there seems to be a slight upward trend in the data stream with a high degree of seasonality for the first six fiscal years until FY 2010<sup>4</sup>. After that point there seems to be a fundamental change in the pattern of the data. The upward trend deteriorates and there seems to be less variation which suggests that heteroskedasticity may be present. The peaks occur in the winter months of December-February and the troughs usually occur in June. After FY 2009, there also appear to be troughs in October.

Upon investigating the fundamental change in the data that occurs between FY 2009 and FY 2010 a distinct relationship between franchise fee revenues and natural gas prices was discovered. As can be seen in figure 3 there appears to be a dramatic rise in gas prices in FY2008 and then an equally dramatic fall in FY2009. Natural gas prices can be seen to flatten out after the sharp decline. From FY

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<sup>4</sup> A graph of franchise fee revenue on a quarterly basis can be found in the appendix.

2010 to FY 2013, franchise fee revenue is visibly lower and the slight upward trend has diminished greatly, if not completely leveled. The strong relationship between these two variables will be discussed in greater detail later.

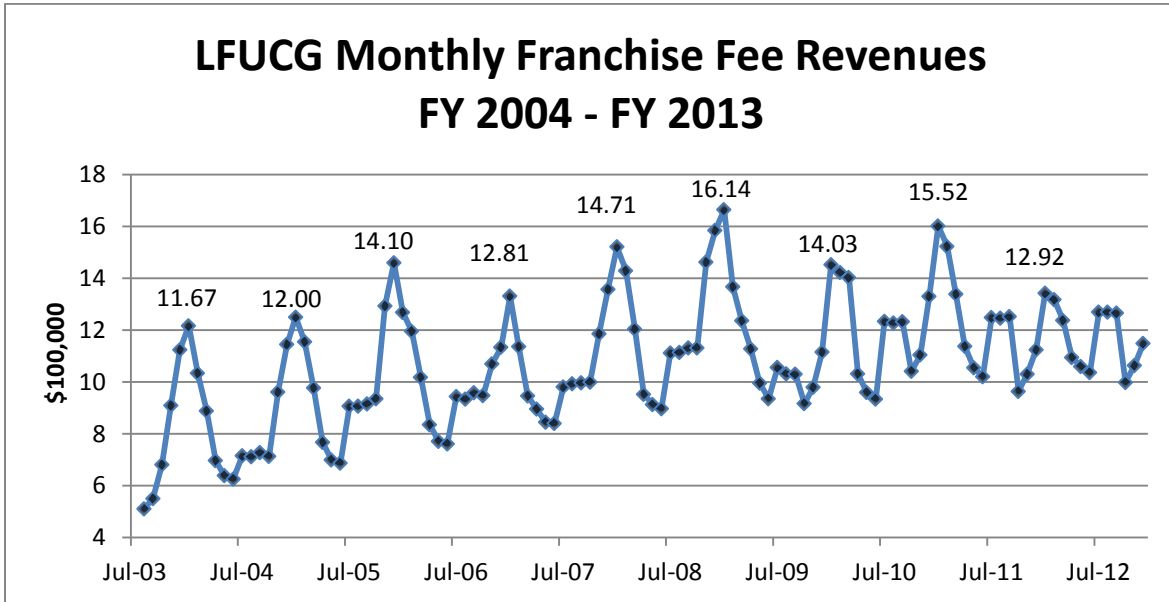


Figure 1

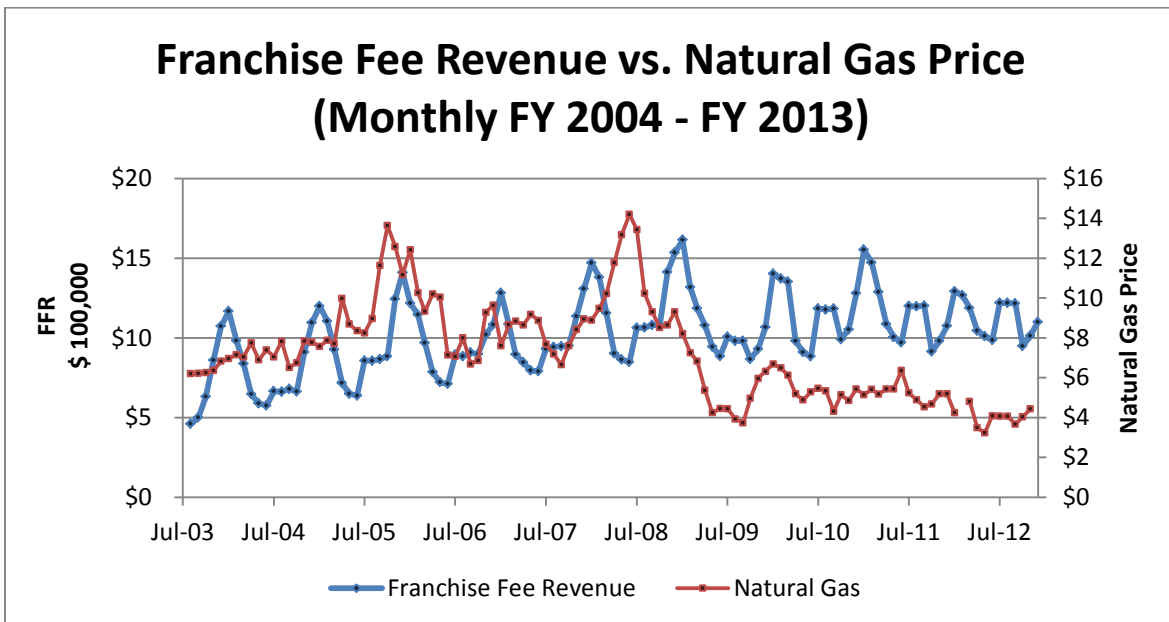


Figure 2



### Current Methods

My analysis shows that the simple twelve month lag method has many benefits. The first and perhaps strongest benefit of this method is that it generally underestimates the annual franchise fee revenue each year and prevents the LFUCG revenue staff from thinking they will have more revenue than they actually will for the upcoming fiscal year. However, this could be problematic as it reduces efficiency in the use of public funds and therefore the quality of public services. For instance, if revenues are perpetually under-forecast then the LFUCG will be operating below their capacity, and goods or services will be withheld when they could have been delivered given a more accurate revenue forecast. On the other hand, this method also guarantees that the forecast will have a forecast error equal to the difference from one year to the next. If the franchise fee revenue changes drastically from one year to the next, the 12 month lag method will perform very poorly – the larger the change, the worse this method becomes all else equal. Given a dataset that has a generally increasing trend, like the one being analyzed, a 12 month lag will also under estimate the revenues for most years. In fact, from FY 2006 to FY 2012, the 12 month lag underestimated four out of seven years.

The double-lag method, on average, performed the worst of the three current methods over the observed timeframe (FY 2006 – FY 2012) with a forecast error rate of 189.83 percent. Whereas the single lag method usually under-forecasts, the double-lag model generally over-forecasts. From FY 2006 to FY 2012 the double-lag model overestimated three different years (2007, 2010, and 2012) by an average of \$2.06 million. Not coincidentally, those were all years for which total franchise fee revenue dropped from the previous year. When the data series changed directions the double-lag model performed the worst of the three models.

The lag-average model, which is a simple average of the two methods yielded interesting results. It boasts the lowest MAPE, but a much higher average annual forecast error rate. One weakness of this method was that it did not handle direction changes very well. In fact, the single-lag model

outperformed the lag-average method in each of the three years which saw a decrease in franchise fee revenues from the previous year. Although the lag-average model boasts a low MAPE, it is inferior when considering the forecast error rate, which is believed to be the superior judge of accuracy.

Table 2

Performance Summary: Current Methods						
Method		Actual Franchise Fee Revenue	Forecasted Revenue	Annual Absolute Forecast Error Rate	Annual Absolute Forecast Error	Annual MAPE
Double-Lag Model	Total	\$78,796,300	\$83,501,440		\$7,673,266	
	Average			189.83%	\$1,278,878	10.10%
	Std. Dev.			3.00	855,311	0.078
Lag-Average Model	Total	\$78,796,300	\$80,304,037		\$7,386,790	
	Average			144.92%	\$1,231,132	9.48%
	Std. Dev.			1.27	440,342	0.036
Single-Lag Model	Total	\$90,461,188	\$87,014,727		\$8,857,108	
	Average			100.00%	\$1,265,301	9.75%
	Std. Dev.			0.00	505,881	0.038
Legend	Gold (First)		Silver (Second)		Bronze (Third)	

It is important to note that for FY 2007 there were only two years of data available to calculate a three year average growth rate and a 3 year average intra-quarter growth rate. As a result, only two years of data was available and for FY 2007. Additionally, the single-lag model includes an additional observation (FY 2006) compared to the other two models. The double-lag and lag-average models did not have data for predictions for the first quarter of FY 2006. When excluding FY 2006 from analysis the single-lag model yields superior results across all measures of accuracy, including a MAPE of 8.86 percent, as seen in table 3 below. Using FY 2007 – FY 2012 for the single-lag model when comparing against the other models is likely more meaningful and fair because of the consistent time frame.

Table 3

Method		Actual Franchise Fee Revenue	Forecasted Revenue	Annual Absolute Forecast Error Rate	Annual Absolute Forecast Error	Annual MAPE
Single-Lag Model	Total	\$78,796,300	\$77,106,634		\$7,100,314	
	Average			100.00%	\$1,183,386	8.86%
	Std. Dev.			0.00	500,733	0.033

Holt-Winters Method

Both monthly and quarterly data were used for the Holt-Winters method. The monthly data were modeled first. The results using monthly data can be found below in table 4. The three different optimization techniques yielded very similar results whether monthly or quarterly data was used. However, it is clear that the monthly data structure outperformed the quarterly data structure for all three optimization techniques. With regard to average annual absolute forecast error rate none of the three methods performed better than the naïve 12 month lag when observing FY 2007 – FY 2012. However, the 2<sup>nd</sup> minimization technique using monthly data did perform better with regard to average annual absolute forecast error (\$1,002,731) and annual MAPE (7.88 percent).

Results for the quarterly data were also obtained and are recorded in table 5. Surprisingly, the quarterly data performed worse on average. I had imagined that allocating the revenue monthly would eliminate the noise in the model that was introduced by dividing the quarterly payments evenly among the months that constituted each quarter. I speculate that the lack of sufficient data, as well as the infrequency of the observations made it difficult for the quarterly model to match the seasonality component of the data. Because of the irregular and volatile seasonality of the data that occurred in the middle of the data, the quarterly model performed quite poorly with even the slightest change in the seasonal trend. All three methods share similar average forecast error rates and fiscal year 2007 was a bit of an outlier which brought the averages of all three methods down.

An interesting outcome of the monthly version of the Holt-Winters method was the large discrepancy in the forecast error rate and the MAPE, similar to what was observed with the lag-average

model previously. Upon further analysis, it is clear that the simple average forecast error for both of those methods is relatively small and on par with the single-lag model. The reason the average annual forecast error rates are so inflated for these methods is because of a subtlety with how the forecast error rate is calculated. Because the denominator is the actual difference in franchise fee revenue from year to year, if this change is small, like it was between FY 2006 and FY 2007, then the forecast error rate will be inflated.

The obvious method of choice among the six Holt-Winters versions is the combination of the monthly data structure with the 2<sup>nd</sup> minimization technique. It yielded an average annual forecast error rate of 134.40 percent, a MAPE of only 7.88 percent, and even has the lowest average absolute forecast error of \$1,002,731 per year. This compares favorably to the single-lag model. From among the six alternatives offered by the Holt-Winters family of methods, it merits consideration above the rest.

Table 4

Performance Summary: Holt-Winters Multiplicative Method (Monthly Data)						
Minimization Method		Actual Franchise Fee Revenue	Forecasted Revenue	Annual Absolute Forecast Error Rate	Annual Absolute Forecast Error	Annual MAPE
(1) Average Annual Absolute Forecast Error Rate	Total	\$78,796,300	\$86,608,663		\$8,591,496	
	Average			216.31%	\$1,431,916	11.16%
	Std. Dev.			2.89	963,922	0.078
(2) Average Monthly Absolute Forecast Error Rate	Total	\$78,796,300	\$81,581,592		\$6,016,385	
	Average			134.40%	\$1,002,731	7.88%
	Std. Dev.			1.65	733,134	0.060
(3) Average Monthly Absolute Forecast Error Rate	Total	\$78,796,300	\$83,387,309		\$6,863,318	
	Average			161.28%	\$1,143,886	9.04%
	Std. Dev.			1.99	795,952	0.065
Legend	Gold (First)		Silver (Second)		Bronze (Third)	

Table 5

Performance Summary: Holt-Winters Multiplicative Method (Quarterly Data)						
Minimization Method		Actual Franchise Fee Revenue	Forecasted Revenue	Annual Absolute Forecast Error Rate	Annual Absolute Forecast Error	Annual MAPE
(1) Average Annual Absolute Forecast Error Rate	Total	\$78,796,300	\$83,057,551		\$9,247,293	
	Average			205.15%	\$1,541,215	12.10%
	Std. Dev.			2.28	827,203	0.068
(2) Average Monthly Absolute Forecast Error Rate	Total	\$78,796,300	\$82,542,947		\$7,124,902	
	Average			161.11%	\$1,187,484	9.34%
	Std. Dev.			1.92	815,170	0.067
(3) Average Monthly Absolute Forecast Error Rate	Total	\$78,796,300	\$83,815,410		\$7,251,058	
	Average			175.42%	\$1,208,510	9.58%
	Std. Dev.			2.26	990,399	0.080
Legend	Gold (First)	Silver (Second)	Bronze (Third)			

Multiple Regression with Robust Standard Errors

The final method I applied is multiple regression. The regression output below shows that most of the explanatory variables were significant at a 5% level. The dummy variables for each month proved to be extremely significant. This is likely because of the distinct seasonality of the data. As expected, because January is generally the peak point in each seasonal cycle of the data, the coefficients for February-December are all negative indicating that, on average, the franchise fees for these months are less than those of January.

Further in line with my expectations were the coefficients for heating-degree-days and cooling-degree-days – both were positive indicating that having more heating- or cooling-degree-days in a given month leads to more franchise fee revenues for that month. Cooling-degree-days have both a higher coefficient and are nearly significant at the 1% level.

Natural gas price was highly statistically significant and had a fairly large coefficient of 14,084.15. This indicates that if natural gas price increases by one dollar per 1000 ft<sup>3</sup> of gas, franchise fee revenue increases by approximately \$169,010 annually, on average. Electricity price was significant

at a 10% level and had an extremely large coefficient. A one cent per kilowatt-hour increase in electricity price should increase franchise fee revenue by \$1.35 million annually, on average. This is not surprising given that a large percentage of the franchise fees for the LFUCG are generated from companies like Kentucky Utilities, Clark Energy, and Blue Grass Energy. Electricity prices were much less variable than natural gas prices. Electricity prices had a minimum price of 4.22 cents per kilowatt-hour and a maximum price of 7.6 cents per kilowatt-hour during the entire time period analyzed, whereas natural gas prices ranged from \$3.23 per 1000 ft<sup>3</sup> to \$14.2 per 1000 ft<sup>3</sup>. Even though the coefficient of electricity prices was substantially larger, the relative volatility of natural gas prices versus electricity prices makes the coefficient for natural gas effectively larger and more important provided the electricity prices remain stable, as they have historically.

A surprising finding was that population had a negative coefficient and was statistically significant at a 5% level. I find it hard to believe that an increase in population leads to a decrease in franchise fee revenue so this result is quite perplexing. Another confusing result was the insignificance and negative coefficient of GDP.

Although the coefficient estimates and associated t-stats are interesting and meaningful to this analysis, the main objective was to gauge how well the predictions of a multiple regression predict the next twelve months of franchise fee revenue. To that end, the multiple regression method performs admirably, especially when there are more than five years of data available.

It is important to note here that the number of observations for this method was limited, especially for the predictions which used longer input data streams. For instance, using the previous five years of data to make predictions for the forecast year only has three observations (FY 2010, FY 2011, and FY 2012). Using longer input data streams have even fewer observations available. However, when comparing the accuracy of the forecasts yielded by multiple regression for FY 2010 – FY 2012 against those of the other methods it is apparent that this method merits serious consideration.

Table 6

<b>Results from Multiple Regression with Robust Standard Errors</b>					
R-squared = 0.835; Observations = 112; F(20, 91) = 39.36					
<b>Explanatory Variable</b>	<b>Notation</b>	<b>Coefficient Estimate</b>	<b>Robust Standard Error</b>	<b>t-stat</b>	<b>P&gt; t </b>
Time	$\beta_1$	24,398.95	11,446.77	2.13	0.036 **
HDD	$\beta_2$	283.65	138.90	2.04	0.044 **
CDD	$\beta_3$	443.10	169.56	2.61	0.010 **
Precipitation	$\beta_4$	2,170.13	4,245.12	0.51	0.610
Natural Gas Price	$\beta_5$	14,084.15	6,431.70	2.19	0.031 **
Electricity Price	$\beta_6$	112,578.90	61,577.25	1.83	0.071 *
Population (Lex)	$\beta_7$	-60.76	28.40	-2.14	0.035 **
GDP (Lex)	$\beta_8$	-638.79	410.48	-1.56	0.123
Incomplete Data	$\beta_9$	16,823.22	37,308.05	0.45	0.653
February	$\beta_{10}$	-82,526.44	51,905.12	-1.59	0.115
March	$\beta_{11}$	-144,371.90	71,296.79	-2.02	0.046 **
April	$\beta_{12}$	-282,838.00	102,983.10	-2.75	0.007 ***
May	$\beta_{13}$	-357,009.00	123,765.20	-2.88	0.005 ***
June	$\beta_{14}$	-479,623.10	147,745.40	-3.25	0.002 ***
July	$\beta_{15}$	-343,021.80	156,152.30	-2.20	0.031 **
August	$\beta_{16}$	-348,464.50	162,494.40	-2.14	0.035 **
September	$\beta_{17}$	-211,616.00	137,656.90	-1.54	0.128
October	$\beta_{18}$	-277,448.40	104,106.60	-2.67	0.009 ***
November	$\beta_{19}$	-175,475.30	85,427.89	-2.05	0.043 **
December	$\beta_{20}$	-137,776.70	69,649.73	-1.98	0.051 *
_cons	$\beta_0$	17,100,000.00	7,930,752.00	2.16	0.034 **

Table 7

Performance Summary: Multiple Regression with Robust Standard Errors						
Input Data Stream (Years of Observation)		Actual Franchise Fee Revenue	Forecasted Revenue	Annual Forecast Error Rate	Absolute Forecast Error	Annual MAPE
All Available Data (1)	Total	\$12,086,422	\$12,513,321		\$426,899	
	Average			62.31%	\$426,899	3.53%
	Std. Dev.			-		
Preceding 7 Years (1)	Total	\$12,086,422	\$12,562,715		\$476,292	
	Average			69.52%	\$476,292	3.94%
	Std. Dev.			-		
Preceding 6 Years (2)	Total	\$26,330,677	\$26,044,398		\$1,105,183	
	Average			52.86%	\$552,592	4.14%
	Std. Dev.			0.10	202,430	0.011
Preceding 5 Years (3)	Total	\$39,060,706	\$41,122,176		\$3,952,247	
	Average			106.50%	\$1,317,416	10.19%
	Std. Dev.			0.43	818,596	0.065
Preceding 4 Years (4)	Total	\$53,320,568	\$55,732,373		\$7,258,659	
	Average			142.01%	\$1,814,665	13.57%
	Std. Dev.			0.37	751,655	0.055
Preceding 3 Years (5)	Total	\$66,149,070	\$51,908,565		\$24,259,721	
	Average			333.59%	\$4,851,944	35.67%
	Std. Dev.			3.58	5,572,488	0.389

Another point to address here is that the forecasts for FY 2012 exclude forecasts for February of that fiscal year because no natural gas price was supplied for that month. Therefore the forecast error rate and MAPE measures actually compare the 11 month predictions against the actual 11 month franchise fee revenue figures. It is also worth noting that February generally has the second highest monthly franchise fee revenue. The unavailability of predictions for February of FY 2012 is thus a serious omission that should be considered when comparing the accuracy of this method to other methods.

Because of these limitations in the regression forecasts, a different comparison proved to be helpful. For this analysis February 2012 was omitted for all calculations. Three different time periods within FY 2010 – FY 2012 were analyzed to compare the regression forecasts to the other worthy methods: the single-lag method and the Holt-Winters method using monthly data and the 2<sup>nd</sup> minimization technique. The results are in table 8 below. The results show that for FY 2010 – FY 2012,



the single-lag method seems to only slightly outperform the other two methods, although it is hard to distinguish much difference. This is an important time frame because it includes FY 2010, which is one of three years during which franchise fee revenue decreased from the previous year. Generally, most forecasting methods performed quite poorly during this year. The results for this time frame further indicate that the single-lag model performs marginally better when there are unexpected changes, or “shocks”, in the data.

Table 8

Performance Summary: FY 2010 - FY 2012*						
Fiscal Years Forecasted	Method	Franchise Fee Revenue	Predicted Revenue	Average Absolute Annual Forecast Error Rate	Average Absolute Forecast Error	Average Annual MAPE
FY 2010 - FY 2012	Single-Lag Model	\$39,060,706	\$39,761,423	100.00%	\$ 1,243,057	9.44%
	Regression (5 Years)		\$41,122,176	106.50%	\$ 1,317,416	10.19%
	Holt-Winters		\$41,058,434	99.38%	\$ 1,260,296	9.74%
FY 2011 - FY 2012	Single-Lag Model	\$26,330,677	\$25,501,561	100.00%	\$ 1,099,668	8.15%
	Regression (6 Years)		\$26,044,398	52.86%	\$ 552,592	4.14%
	Holt-Winters		\$26,064,423	75.08%	\$ 758,453	5.72%
FY 2012	Single-Lag Model	\$12,086,422	\$12,771,532	100.00%	\$ 685,110	5.67%
	Regression (7 Years)		\$12,562,715	69.52%	\$ 476,292	3.94%
	Regression (Full Data)		\$12,513,321	62.31%	\$ 426,899	3.53%
	Holt-Winters		\$12,711,747	91.27%	\$ 625,325	5.17%

\* All calculations exclude February 2012

Legend	Best Performer for time period
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However, for the FY 2011 – FY 2012 time period multiple regression seems to significantly outperform the other two methods. Similar results were also found for FY 2012. This particular analysis seems to show that multiple regression can be a competitive forecasting tool when more than five years of data are available and when there are no unexpected shocks to the system. However, because of the limited data, it is not clear if the improved performance is due to the availability of more data or to randomness. Therefore, it is not apparent if these results can be replicated for other time periods.

## Costs, Benefits, and Limitations

A major interest of this study is to analyze which method would be best suited for Lexington's revenue department. Each method has its own costs and benefits. Obviously, the forecasting methods labeled in this study as the "current methods" are the least complex which required only basic analytical skills. Both the Holt-Winters and the multiple regression techniques are far more complex and involved. The costs of using the more complex methods are substantial and would require access to and proficiency using relevant software. In this case, STATA and Microsoft Excel (and the Solver add-in) were both used. Additionally, proficiency in concepts like ordinary least squares regression and exponential smoothing would be beneficial.

This study indicates that the best performing methods from each of the three categories (current methods, Holt-Winters, and multiple regression) were, respectively, the single-lag method, the Holt-Winters method using monthly data and minimizing the average monthly absolute forecast error rate, and multiple regression using six years of data. However, because the multiple regression method had limited observations, it is hard to tell exactly how well this method performed prior to FY 2010 and whether it's improved performance after FY 2009 can be replicated in the coming years.

One limitation of this study is that the forecasting methods used for the "current methods" collection of techniques was not strictly based on actual practices of the LFUCG – they were my best attempt to approximate the type of analysis that might be currently used by the LFUCG. This limits the applicability and practicality of the study because actual practices were not included in any comparisons.

Many of the limitations of this study involve the data. For example, the population data was only reported annually and had to be linearly extrapolated to get monthly data. This manipulation of the data yields only approximate monthly population figures. Also, Kentucky Utilities, Kentucky American Water, and Delta Gas all pay franchise fees quarterly while the other companies pay monthly. As a result, the revenue from the companies which pay quarterly had to be divided evenly among the three

months that make up each quarter. This resulted in an inaccurate representation of the actual monthly generation of franchise fees for those companies. These manipulations of the data introduce noise into the data and results.

It should be noted again that a major limitation of the validity of the multiple regression model to predict future values of franchise fee revenues is that I had the luxury of knowing the values of the explanatory variables. If accurate methods of estimating the explanatory variables could be developed, multiple regression may be the overall preferred method. The estimation for these variables may be quite easy for variables like population, GDP, and electricity price that are either highly auto-correlated or have a distinct seasonality component. However, natural gas prices, heating-degree-days, and cooling-degree-days are likely quite difficult to predict beforehand because of their variability and high degree of randomness.

### ***Recommendations***

The main problem with forecasting this particular data stream is unexpected “shocks” to the system. As long as the series is “nice” and maintains a constant trend and seasonality, most of the methods perform well enough. However, when exogenous factors like natural gas prices or cooling-degree-days suddenly change, most methods suffer a great deal. Therefore, an important thing to consider when recommending a forecasting policy is how the method performs when these “shocks” are present. Also important would be the variability of each method, its ease of use, and of course its accuracy across time. Lastly, the values and needs of the LFUCG should be considered.

Starting with my last point, my recommendation for the LFUCG would depend partially on what the revenue staff and the city council values the most. Is it more important to forecast the next twelve months of this revenue stream accurately a high percentage of the time or is it more important to avoid being extremely wrong in years where unexpected changes occur? For instance, the best performing

variation of the Holt-Winters method using monthly data was more accurate than the naïve forecast four out of six years, but was very inaccurate during the remaining two years. Similarly, multiple regression using six or more years of data outperformed all other methods with regard to all three measures of accuracy for FY 2011 and FY 2012. If the accuracy of a forecast of franchise fee revenue is not important to annual planning and only desired for cash-flow management throughout the year, using the Holt-Winters method or multiple regression may be okay provided there are adequate revision procedures in place. If, instead, accuracy is desired for annual planning and certain expenses depend on having the forecast be accurate at the very beginning of the fiscal year, I would suggest using the single-lag method because it handles shocks to the system better and has less variability in forecast error, limiting the chance for an extremely large forecast error.

For years where there are no shocks, it seems that multiple regression (with more than five years of data) does perform the best, if the analysis of FY 2010 – FY 2012 is to be trusted. Holistically, and for years when there are shocks to the system, the single-lag model outperforms all other models on average. It should be noted that the Holt-Winters method did provide accurate forecasts for years without unexpected shocks across the entire dataset. However for FY 2011 and FY 2012, both years where there were no unexpected shocks, multiple regression outperformed the Holt-Winters method. So, the question becomes the following: “Does the multiple regression method have enough benefits to overcome its many costs?”

If the multiple regression method is to be considered, it would be prudent for the revenue staff to focus on natural gas prices and heating- and cooling-degree-days in their attempts to forecast this particular revenue stream. If it is possible to predict these three variables with any accuracy by using things like the futures markets for natural gas or some long-term weather forecast then multiple regression would likely become a powerful forecasting method. However, it does require not only a monetary cost (to purchase an adequate software package), but also a time cost. Although this study

does begin to cover some of those costs, much time and analysis would likely be required before a valid multiple regression model could be implemented. Developing a feasible model of estimation for the explanatory variables would require another study similar to this one. There is also the question of whether being slightly more accurate at forecasting a revenue stream that makes up only 6.5% of total revenues would be worth the significant investments of time and money. These are questions that only the LFUCG can answer.

All things considered, my recommendation for the LFUCG would be to use a simple naïve 12 month lag and revise it up or down according to forecasts about natural gas prices as well as temperatures. This is because this method has minimal costs and accuracy that has proven to be unbeatable during the time period under study here. Until a feasible model of estimating natural gas prices, heating-degree-days, and cooling-degree-days is developed, the costs would likely be too large, and the benefits too small, to warrant the investment of time and money into using multiple regression to forecast franchise fee revenue. However, this study has indicated that future studies aimed at further developing a multiple regression model may be beneficial in increasing forecast accuracy for this particular revenue stream.

#### Revision Analysis & Potential Future Studies

This study, or one like it, could be performed again in a few years when more data is available so that more could be learned about the accuracy of the multiple regression method. Because only eight full fiscal years of monthly data were available, only two observations were available for testing the accuracy of multiple regression using six fiscal years of input data. Only one observation was available for multiple regression using seven fiscal years of input data. This is unfortunate because having a longer input data stream was shown to be more accurate in this study.

One issue that was not discussed in this paper but would most certainly be beneficial to the LFUCG revenue staff is how best to revise an estimate mid-year. This would essentially test which model adjusts to “shocks” in the system the quickest. It is likely that some form of an ARIMA (Autoregressive Integrated Moving Average) method would be a superior revision tool because of its emphasis on recent values and trends. ARIMA models, which relate a time series dataset to its own past values as well as past values of random “shocks” to the system, would likely be able to adjust to the changes in the data fairly quickly. However, a simple moving average or weighted moving average may perform well and also be more likely to be implemented.

Another area of further study would be to actually predict values of explanatory variables for use in the regression model instead of using actual known values to test how accurate the model is. With the current research design the accuracy of the multiple regression method is likely inflated because of its use of explanatory variables that would be unknown to the forecaster in a real-world setting.

Lastly, another interesting study would be to combine predictions from multiple methods and then take an average of those predictions and see how well different combinations of forecasts perform.

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## Appendix

### Holt-Winters Methodology and Equations

$t = \text{time period}$

$p = \# \text{ of observations per seasonal cycle}$

$$\text{Mean level} = L_t = \alpha \left[ \frac{X_t}{I_{t-p}} \right] + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$\text{Trend} = T_t = \gamma(L_t - L_{t-1}) + (1 - \gamma)T_{t-1}$$

$$\text{Seasonal index} = I_t = \delta \left[ \frac{X_t}{L_t} \right] + (1 - \delta)I_{t-p}$$

$$\text{Forecast} = X_t^f(k) = (L_t + kT_t)I_{t-p+k} \quad k = 1, 2, \dots, p$$

### Double-Lag Model Methodology and Equations

FFR = Franchise Fee Revenue; AGR = Average Growth Rate; IQGR = Intra-quarter Growth Rate

$$FFR_{Q1M1}^{FY2009} = FFR_{Q1M1}^{FY2008} * [1 + AGR_{Q1}^{FY2009}]$$

$$AGR_{Q1}^{FY2009} = \left[ \frac{\{GR_{Q1}^{FY2008} + GR_{Q1}^{FY2007} + GR_{Q1}^{FY2006}\}}{3} \right]$$

$$GR_{Q1}^{FY2008} = \frac{FFR_{Q1}^{FY2008} - FFR_{Q1}^{FY2007}}{FFR_{Q1}^{FY2007}}$$

$$FFR_{Q1M3}^{FY2009} = FFR_{Q1M1}^{FY2009} * IQGR_{Q1}^{FY2009}$$

$$IQGR_{Q1}^{FY2009} = \left[ \frac{IQGR_{Q1}^{FY2008} + IQGR_{Q1}^{FY2007} + IQGR_{Q1}^{FY2006}}{3} \right]$$

$$IQGR_{Q1}^{FY2008} = \frac{FFR_{Q1M3}^{FY2008} - FFR_{Q1M1}^{FY2007}}{FFR_{Q1M1}^{FY2007}}$$

$$FFR_{Q1M2}^{FY2009} = \frac{FFR_{Q1M1}^{FY2009} + FFR_{Q1M3}^{FY2009}}{2}$$

### Robust Multiple Regression Model

$$FFR = \beta_0 + \beta_1 \text{time} + \beta_2 \text{HDD} + \beta_3 \text{CDD} + \beta_4 \text{Precip} + \beta_5 \text{NatGasPrice} + \beta_6 \text{ElecPrice} + \beta_7 \text{LexPop} \\ + \beta_8 \text{LexGDP} + \beta_9 \text{Incomplete} + \beta_{10} \text{Feb} + \dots + \beta_{20} \text{Dec} + \varepsilon$$



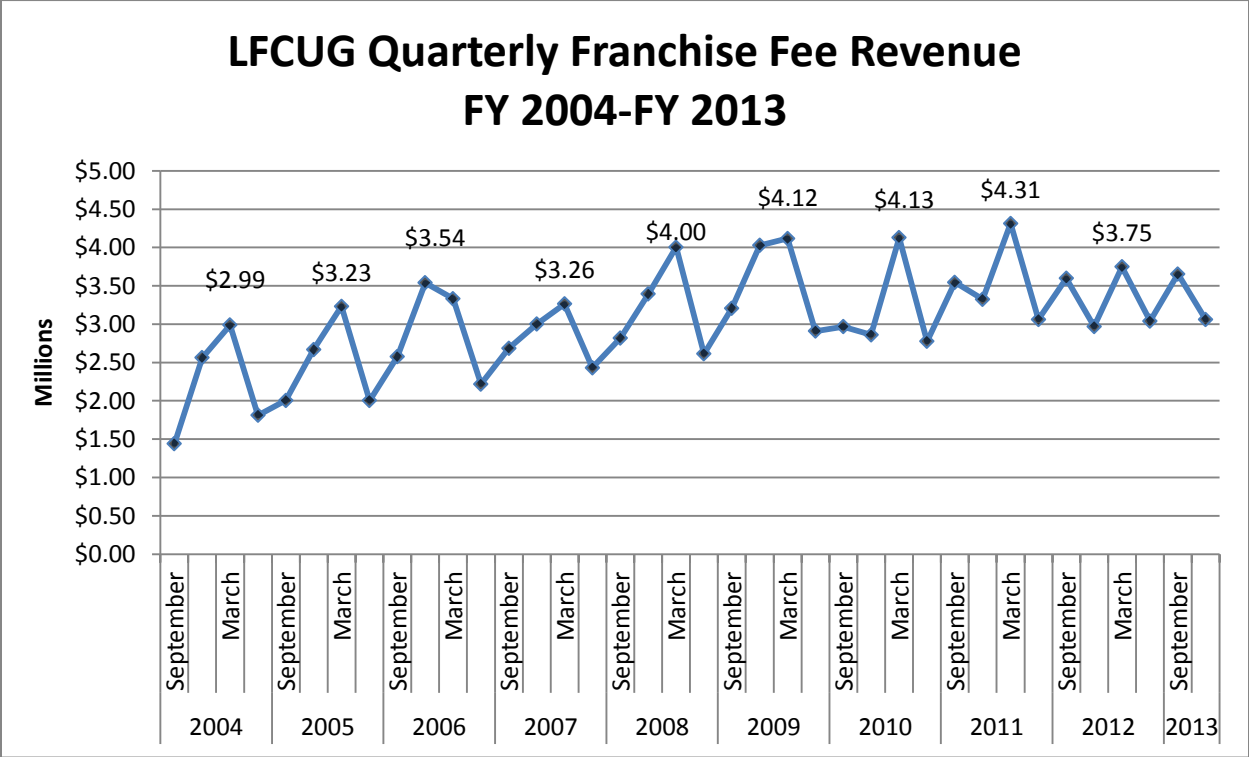


Figure 3

Table 9

<b>Performance Summary: Current Methods</b>					
<b>Double-Lag Model</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2006*</b>	\$11,664,888				
<b>2007</b>	\$11,379,098	\$13,342,920	687.16%	\$1,963,822	17.26%
<b>2008</b>	\$12,828,502	\$12,400,836	29.51%	\$427,666	3.33%
<b>2009</b>	\$14,259,862	\$14,014,462	17.14%	\$245,400	1.72%
<b>2010</b>	\$12,730,029	\$15,351,715	171.37%	\$2,621,687	20.59%
<b>2011</b>	\$14,244,255	\$13,433,258	53.56%	\$810,997	5.69%
<b>2012</b>	\$13,354,554	\$14,958,248	180.25%	\$1,603,695	12.01%
<b>Total</b>	\$78,796,300	\$83,501,440		\$7,673,266	
<b>Average</b>			189.83%	\$1,278,878	10.10%
<b>Std. Dev.</b>			2.54	855,311	0.078
<b>Lag-Average Model</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2006*</b>	\$11,664,888				
<b>2007</b>	\$11,379,098	\$12,503,904	393.58%	\$1,124,805	9.88%
<b>2008</b>	\$12,828,502	\$11,889,967	64.75%	\$938,535	7.32%
<b>2009</b>	\$14,259,862	\$13,421,482	58.57%	\$838,380	5.88%
<b>2010</b>	\$12,730,029	\$14,805,789	135.69%	\$2,075,760	16.31%
<b>2011</b>	\$14,244,255	\$13,081,643	76.78%	\$1,162,612	8.16%
<b>2012</b>	\$13,354,554	\$14,601,252	140.13%	\$1,246,698	9.34%
<b>Total</b>	\$78,796,300	\$80,304,037		\$7,386,790	
<b>Average</b>			144.92%	\$1,231,132	9.48%
<b>Std. Dev.</b>			1.27	440,342	0.036
<b>Single-Lag Model</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2006*</b>	<b>\$11,664,888</b>	\$9,908,094	100.00%	\$1,756,794	15.06%
<b>2007</b>	<b>\$11,379,098</b>	\$11,664,888	100.00%	\$285,789	2.51%
<b>2008</b>	<b>\$12,828,502</b>	\$11,379,098	100.00%	\$1,449,404	11.30%
<b>2009</b>	<b>\$14,259,862</b>	\$12,828,502	100.00%	\$1,431,360	10.04%
<b>2010</b>	<b>\$12,730,029</b>	\$14,259,862	100.00%	\$1,529,834	12.02%
<b>2011</b>	<b>\$14,244,255</b>	\$12,730,029	100.00%	\$1,514,227	10.63%
<b>2012</b>	<b>\$13,354,554</b>	\$14,244,255	100.00%	\$889,701	6.66%
<b>Total</b>	\$78,796,300	\$77,106,634		\$7,100,314	
<b>Average</b>			100.00%	\$1,183,386	8.86%
<b>Std. Dev.</b>			0.00	500,733	0.033

\* Excluded from averages

Table 10

<b>Performance Summary: Holt-Winters Multiplicative Method (Monthly Data)</b>					
<b>Minimization Method: Cumulative Average Annual Forecast Error Rate</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Forecasted Revenue</b>	<b>Annual Absolute Forecast Error Rate</b>	<b>Annual Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2006</b>	<b>\$11,664,888</b>				
<b>2007</b>	\$11,379,098	\$13,616,766	782.98%	\$2,237,667	19.66%
<b>2008</b>	\$12,828,502	\$12,648,534	12.42%	\$179,968	1.40%
<b>2009</b>	\$14,259,862	\$14,050,263	14.64%	\$209,599	1.47%
<b>2010</b>	\$12,730,029	\$14,669,566	126.78%	\$1,939,537	15.24%
<b>2011</b>	\$14,244,255	\$16,214,541	130.12%	\$1,970,285	13.83%
<b>2012</b>	\$13,354,554	\$15,408,993	230.91%	\$2,054,439	15.38%
<b>Total</b>	\$78,796,300	\$86,608,663		\$8,591,496	
<b>Average</b>			216.31%	\$1,431,916	11.16%
<b>Std. Dev.</b>			2.89	963,922	0.078
<b>Minimization Method: Cumulative Average Monthly Forecast Error Rate</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Forecasted Revenue</b>	<b>Annual Absolute Forecast Error Rate</b>	<b>Annual Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2006</b>	<b>\$11,664,888</b>				
<b>2007</b>	\$11,379,098	\$12,681,393	455.68%	\$1,302,295	11.44%
<b>2008</b>	\$12,828,502	\$12,197,091	43.56%	\$631,411	4.92%
<b>2009</b>	\$14,259,862	\$14,167,307	6.47%	\$92,555	0.65%
<b>2010</b>	\$12,730,029	\$14,994,011	147.99%	\$2,263,983	17.78%
<b>2011</b>	\$14,244,255	\$13,352,675	58.88%	\$891,580	6.26%
<b>2012</b>	\$13,354,554	\$14,189,114	93.80%	\$834,560	6.25%
<b>Total</b>	\$78,796,300	\$81,581,592		\$6,016,385	
<b>Average</b>			134.40%	\$1,002,731	7.88%
<b>Std. Dev.</b>			1.65	733,134	0.060
<b>Minimization Method: Cumulative Sum of Absolute Monthly Forecast Error</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Forecasted Revenue</b>	<b>Annual Absolute Forecast Error Rate</b>	<b>Annual Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2006</b>	<b>\$11,664,888</b>				
<b>2007</b>	\$11,379,098	\$12,935,670	544.66%	\$1,556,571	13.68%
<b>2008</b>	\$12,828,502	\$12,010,678	56.42%	\$817,824	6.38%
<b>2009</b>	\$14,259,862	\$13,941,531	22.24%	\$318,331	2.23%
<b>2010</b>	\$12,730,029	\$15,055,757	152.02%	\$2,325,729	18.27%
<b>2011</b>	\$14,244,255	\$14,568,903	21.44%	\$324,648	2.28%
<b>2012</b>	\$13,354,554	\$14,874,769	170.87%	\$1,520,216	11.38%
<b>Total</b>	\$78,796,300	\$83,387,309		\$6,863,318	
<b>Average</b>			161.28%	\$1,143,886	9.04%
<b>Std. Dev.</b>			1.99	795,952	0.065

Table 11

<b>Performance Summary: Holt-Winters Multiplicative Method (Quarterly Data)</b>					
<b>Minimization Method: Cumulative Average Annual Forecast Error Rate</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
2006	\$11,664,888				
2007	\$11,379,098	\$13,229,331	647.41%	\$ 1,850,233	16.26%
2008	\$12,828,502	\$11,091,247	119.86%	\$ 1,737,255	13.54%
2009	\$14,259,862	\$14,621,032	25.23%	\$ 361,170	2.53%
2010	\$12,730,029	\$15,326,675	169.73%	\$ 2,596,647	20.40%
2011	\$14,244,255	\$13,488,489	49.91%	\$ 755,766	5.31%
2012	\$13,354,554	\$15,300,776	218.75%	\$ 1,946,222	14.57%
<b>Total</b>	<b>\$78,796,300</b>	<b>\$83,057,551</b>		<b>\$ 9,247,293</b>	
<b>Average</b>			<b>205.15%</b>	<b>\$ 1,541,215</b>	<b>12.10%</b>
<b>Std. Dev.</b>			<b>2.28</b>	<b>827,203</b>	<b>0.068</b>
<b>Minimization Method: Cumulative Average Monthly Forecast Error Rate</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
2006	\$11,664,888				
2007	\$11,379,098	\$12,900,555	532.37%	\$ 1,521,457	13.37%
2008	\$12,828,502	\$12,132,119	48.05%	\$ 696,383	5.43%
2009	\$14,259,862	\$13,996,109	18.43%	\$ 263,753	1.85%
2010	\$12,730,029	\$15,287,813	167.19%	\$ 2,557,784	20.09%
2011	\$14,244,255	\$13,515,264	48.14%	\$ 728,991	5.12%
2012	\$13,354,554	\$14,711,087	152.47%	\$ 1,356,533	10.16%
<b>Total</b>	<b>\$78,796,300</b>	<b>\$82,542,947</b>		<b>\$ 7,124,902</b>	
<b>Average</b>			<b>161.11%</b>	<b>\$ 1,187,484</b>	<b>9.34%</b>
<b>Std. Dev.</b>			<b>1.92</b>	<b>815,170</b>	<b>0.067</b>
<b>Minimization Method: Cumulative Sum of Absolute Monthly Forecast Error</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
2006	\$11,664,888				
2007	\$11,379,098	\$13,120,690	609.40%	\$ 1,741,591	15.31%
2008	\$12,828,502	\$12,261,364	39.13%	\$ 567,138	4.42%
2009	\$14,259,862	\$14,053,352	14.43%	\$ 206,510	1.45%
2010	\$12,730,029	\$15,427,485	176.32%	\$ 2,697,456	21.19%
2011	\$14,244,255	\$13,901,930	22.61%	\$ 342,325	2.40%
2012	\$13,354,554	\$15,050,590	190.63%	\$ 1,696,036	12.70%
<b>Total</b>	<b>\$78,796,300</b>	<b>\$83,815,410</b>		<b>\$ 7,251,058</b>	
<b>Average</b>			<b>175.42%</b>	<b>\$ 1,208,510</b>	<b>9.58%</b>
<b>Std. Dev.</b>			<b>2.26</b>	<b>990,399</b>	<b>0.080</b>

Table 12

<b>Performance Summary: Multiple Regression with Robust Standard Errors</b>					
<b>Input Data Stream: Preceding 7 years and 11 months</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2011</b>	\$14,244,255				
<b>2012</b>	\$12,086,422	\$12,513,321	62.31%	\$426,899	3.53%
<b>Total</b>	\$12,086,422	\$12,513,321		\$426,899	
<b>Average</b>			62.31%	\$426,899	3.53%
<b>Std. Dev.</b>			-		
<b>Input Data Stream: Preceding 7 years</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2011</b>	\$14,244,255				
<b>2012</b>	\$12,086,422	\$12,562,715	69.52%	\$476,292	3.94%
<b>Total</b>	\$12,086,422	\$12,562,715		\$476,292	
<b>Average</b>			69.52%	\$476,292	3.94%
<b>Std. Dev.</b>			-		
<b>Input Data Stream: Preceding 6 years</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2010</b>	\$12,730,029				
<b>2011</b>	\$14,244,255	\$13,548,524	45.95%	\$695,731	4.88%
<b>2012</b>	\$12,086,422	\$12,495,874	59.76%	\$409,452	3.39%
<b>Total</b>	\$26,330,677	\$26,044,398		\$1,105,183	
<b>Average</b>			52.86%	\$552,592	4.14%
<b>Std. Dev.</b>			0.10	202,430	0.011

\*Note for calculation: FY 2011 Franchise Fee Revenue (excluding February) = \$12,771,532

\*\*Note: February 2012 was omitted because no natural gas price was reported for that month.

Table 13

<b>Performance Summary: Multiple Regression with Robust Standard Errors</b>					
<b>Input Data Stream: Preceding 5 years</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2009</b>	\$14,259,862				
<b>2010</b>	\$12,730,029	\$14,985,984	147.46%	\$2,255,955	17.72%
<b>2011</b>	\$14,244,255	\$13,298,866	62.43%	\$945,389	6.64%
<b>2012</b>	\$12,086,422	\$12,837,325	109.60%	\$750,903	6.21%
<b>Total</b>	\$39,060,706	\$41,122,176		\$3,952,247	
<b>Average</b>			106.50%	\$1,317,416	10.19%
<b>Std. Dev.</b>			0.43	818,596	0.065
<b>Input Data Stream: Preceding 4 years</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2008</b>	\$12,828,502				
<b>2009</b>	\$14,259,862	\$11,836,435	169.31%	\$2,423,427	16.99%
<b>2010</b>	\$12,730,029	\$15,217,368	162.59%	\$2,487,339	19.54%
<b>2011</b>	\$14,244,255	\$15,577,629	88.06%	\$1,333,374	9.36%
<b>2012</b>	\$12,086,422	\$13,100,941	148.08%	\$1,014,519	8.39%
<b>Total</b>	\$53,320,568	\$55,732,373		\$7,258,659	
<b>Average</b>			142.01%	\$1,814,665	13.57%
<b>Std. Dev.</b>			0.37	751,655	0.055
<b>Input Data Stream: Preceding 3 years</b>					
<b>Fiscal Year</b>	<b>Franchise Fee Revenue</b>	<b>Predicted Revenue</b>	<b>Absolute Annual Forecast Error Rate</b>	<b>Absolute Forecast Error</b>	<b>Annual MAPE</b>
<b>2007</b>	<b>\$11,379,098</b>				
<b>2008</b>	<b>\$12,828,502</b>	\$8,228,971	317.34%	\$4,599,531	35.85%
<b>2009</b>	<b>\$14,259,862</b>	\$13,795,611	32.43%	\$464,251	3.26%
<b>2010</b>	<b>\$12,730,029</b>	\$17,071,409	283.78%	\$4,341,380	34.10%
<b>2011</b>	<b>\$14,244,255</b>	\$57,924	936.87%	\$14,186,331	99.59%
<b>2012</b>	<b>\$12,086,422</b>	\$12,754,650	97.54%	\$668,227	5.53%
<b>Total</b>	\$66,149,070	\$51,908,565		\$24,259,721	
<b>Average</b>			333.59%	\$4,851,944	35.67%
<b>Std. Dev.</b>			3.58	5,572,488	0.389
*Note for calculation: FY 2011 Franchise Fee Revenue (excluding February) = \$12,771,532					
**Note: February 2012 was omitted because no natural gas price was reported for that month.					