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MEASUREMENT OF THE $d\mu d$ QUARTET-TO-DOUBLET MOLECULAR FORMATION RATE RATIO ($\lambda q : \lambda d$) AND THE $\mu d$ HYPERFINE RATE ($\lambda q d$) USING THE FUSION NEUTRONS FROM $\mu^-$ STOPS IN D$_2$ GAS

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MEASUREMENT OF THE $d\mu d$ QUARTET-TO-DOUBLET MOLECULAR FORMATION RATE RATIO ($\lambda_q : \lambda_d$) AND THE $\mu d$ HYPERFINE RATE ($\lambda_{qd}$) USING THE FUSION NEUTRONS FROM $\mu^-$ STOPS IN $D_2$ GAS.

DISSertation

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Arts and Sciences at the University of Kentucky

By
Nandita Raha
Lexington, Kentucky

Director: Dr. Tim Gorringe, Professor of Physics
Lexington, Kentucky 2015

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MEASUREMENT OF THE $d\mu d$ QUARTET-TO-DOUBLET MOLECULAR FORMATION RATE RATIO ($\lambda_q : \lambda_d$) AND THE $\mu d$ HYPERFINE RATE ($\lambda_{qd}$) USING THE FUSION NEUTRONS FROM $\mu^-$ STOPS IN D$_2$ GAS.

The MuSun experiment will determine the $\mu d$ capture rate ($\mu^- + d \rightarrow n + n + \nu_e$) from the doublet hyperfine state $\Lambda_d$, of the muonic deuterium atom in the 1S ground state to a precision of 1.5%. Modern effective field theories (EFT) predict that an accurate measurement of $\Lambda_d$ would determine the two-nucleon weak axial current. This will help in understanding all weak nuclear interactions such as the stellar thermonuclear proton-proton fusion reactions, the neutrino reaction $\nu + d$ (which explores the solar neutrino oscillation problem). It will also help us understand weak nuclear interactions involving more than two nucleons — double $\beta$ decay — as they do involve a two-nucleon weak axial current term.

The experiment took place in the $\pi E3$ beam-line of Paul Scherrer Institute (PSI) using a muon beam generated from 2.2 mA proton beam — which is the highest intensity beam in the world. The muons first passed through entrance scintillator and multiwire proportional chamber for determining their entrance timing and position respectively. Then they were stopped in a cryogenic time projection chamber (cryo-TPC) filled with D$_2$ gas. This was surrounded by plastic scintillators and multiwire proportional chambers for detecting the decay electrons and an array of eight liquid scintillators for detecting neutrons.

Muons in deuterium get captured to form $\mu d$ atoms in the quartet and doublet spin states. These atoms undergo nuclear capture from these hyperfine states respectively. There is a hyperfine transition rate from quartet-to-doublet state — $\lambda_{qd}$ along with $d\mu d$ molecular formation which further undergoes a fusion reaction with the muon acting as a catalyst (MCF). The goal of this dissertation is to measure the $d\mu d$ quartet-to-doublet rate ratio ($\lambda_q : \lambda_d$) and $\mu d$ hyperfine rate ($\lambda_{qd}$) using the fusion neutrons from $\mu^-$ stops in D$_2$ gas. The $d\mu d$ molecules undergo MCF reactions from the doublet and the quartet state with rates $\lambda_d$ and $\lambda_q$ and yield 2.45 MeV mono-energetic fusion neutrons. Encoded in the time dependence of the fusion neutrons are the $d\mu d$ formation rates $\lambda_d$, $\lambda_q$ and hyperfine rate $\lambda_{qd}$. Consequently, the investigation of the fusion neutron time spectrum enables the determination of these kinetics parameters that are important in the extraction of $\Lambda_d$ from the decay
electron time spectrum. The final results of this work yield $\lambda_q : \lambda_d = 85.51 \pm 3.25$ and $\lambda_{qd} = 38.49 \pm 0.21 \, \mu s^{-1}$.

KEYWORDS: Two-nucleon Weak Axial Current, Fusion Neutron Analysis
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CONTENTS

Acknowledgments ........................................................................ iii

Contents ................................................................................... iv

List of Figures ............................................................................. vii

List of Tables ................................................................................ xv

Chapter 1 Introduction .................................................................. 1
  1.1 Other Experiments using Muon as a Fundamental Tool .......... 3
  1.2 Weak Interactions ................................................................. 5
  1.3 Physics Motivation ................................................................. 8
  1.4 Comparison of Previous Experiments and Theory .......... 11

Chapter 2 Experimental Design and Strategy ............................. 14
  2.1 Experimental Technique ...................................................... 14
  2.2 Muon Chemistry and Kinetics in Deuterium ....................... 16
    2.2.1 Hyperfine States ......................................................... 16
    2.2.2 Muon-Catalyzed Fusion .............................................. 18
    2.2.3 Resonant and Non-Resonant Molecular Formation ....... 20
    2.2.4 Overall Picture ........................................................... 21
  2.3 Analytical Solution for Population of States ....................... 22
    2.3.1 Effect of Changing Various Parameters on $A_1 : A_2$ .... 26
  2.4 Optimal Target Conditions ................................................. 28
  2.5 Essential Observables and Overall Experimental Setup ....... 29

Chapter 3 Experimental Setup .................................................... 31
  3.1 Experimental Overview ...................................................... 31
  3.2 Beamline ............................................................................ 32
    3.2.1 Electrostatic Kicker ..................................................... 33
    3.2.2 Separator ................................................................. 35
  3.3 Muon Entrance Detectors .................................................... 36
    3.3.1 $\mu$SCA ................................................................. 36
    3.3.2 $\mu$SC ................................................................. 36
    3.3.3 $\mu$PC ................................................................. 37
  3.4 TPC .................................................................................. 37
    3.4.1 Gas Purification and Recirculation System ................ 39
    3.4.2 Cryogenic System ..................................................... 41
    3.4.3 TPC Vessel ............................................................. 41
  3.5 $\mu$SR ............................................................................. 42
  3.6 Electron Detectors ............................................................. 43
LIST OF FIGURES

1.1 The Feynman diagrams of free muon decay (left panel) and muon’s nuclear capture in proton (right panel). The initial-to-final states run from the bottom to the top of the page. ................................. 2
1.2 The Feynman diagram of muon capture in deuterium. ................................. 3
1.3 A muon interacting with a photon. This shows the leading order QED contribution (also called the Schwinger term [14]), representing the underlying cause of the deviation of $a_{\mu}$ from 2 due to QED interactions. ................................. 4
1.4 Feynman diagram of the $\beta$ decay. The left panel shows the case using Fermi theory of point interaction and the right panels shows the same using Standard Model. At low energies the W boson is replaced by a point, which effectively looks like a point interaction as shown in the left panel. Thus we have generalized the Fermi theory to a V-A effective field theory of the weak interaction in the low-energy regime. ................................. 7
1.5 The two-nucleon interaction with an axial weak vector — (a) for long ranges using pion exchange current and (b) short ranges using a point interaction. Image credit [34]. ................................. 9
1.6 The effective vertex $d^R$ for several two-nucleon weak interactions. ................................. 10
1.7 The figure summarizes experimental and theoretical results of $\Lambda_{d}$. The light blue band around the Cargnelli experiment shows the anticipated 1.5% (6 s$^{-1}$) precision of MuSun experiment. ................................. 11

2.1 Lifetime Technique. The dashed line denotes the positive muon decay rate and the solid line denotes the negative muon disappearance rate in the gas target. The difference gives us the muon capture rate in deuterium gas. ................................. 15
2.2 Muon Kinetics in deuterium. The branching ratio of muon sticking to He (called sticking probability) is 0.1206 which is negligible and so these side branches can be ignored. Image credit: [1]. ................................. 17
2.3 Variation of $\lambda_{qd}$ with temperature. Image credit: [1]. ................................. 18
2.4 Variation of $\lambda_q$ and $\lambda_d$ with temperature. Here $\lambda_{dd\mu}$ represents the $dd\mu$ molecular formation rate from any hyperfine state. Image credit: [1]. ................................. 19
2.5 Resonant and non-resonant $d\mu d$ molecular formation rates versus kinetic energy of the $\mu d$ atom. Image credit: [77]. ................................. 21
2.6 Plots showing the doublet state populations in linear and log scale with no recycling. This shows the initial population $n_{1/2}(0) = \frac{1}{2}$. ................................. 23
2.7 Plots showing the quartet state populations in linear and log scale with no recycling. This shows the initial population $n_{3/2}(0) = \frac{2}{3}$. ................................. 23
2.8 Plots showing the time distribution of the quartet state $n_{3/2}(t)$ (in solid red) and the quartet state to total population $\frac{n_{3/2}(t)}{n_{1/2}(t) + n_{3/2}(t)}$ (in dashed blue) in linear and log scale. ................................. 26
2.9 Plots showing the variation of the effect of changes in rates $\lambda_{qd}$, $\lambda_d$ and $\lambda_q$ with the amplitude ratio $A_1 : A_2$ .................................................. 27

2.10 Time distributions of all relevant states under the experimental conditions of MuSun. Image credit: [1] ................................................................. 28

2.11 Histograms of all charged particle (top is energy spectrum and bottom is the time distribution) after muon stop from a previous experiment [90] with conditions similar to MuSun at $T=45.3$ K and $\phi = 0.0524$ and nitrogen impurity level is 41 ppb. Impurity capture background is represented by a dotted line. Image Credit - MuSun Proposal 07 .................................. 30

3.1 A cutaway view of the MuSun detector. The muon beam propagates from the left through the $\mu$SC and $\mu$PC before entering the TPC via a beryllium window. The muon is tracked by the TPC, and decays to an electron that is tracked and detected by two projection wire chambers (ePC1 and ePC2) and a segmented scintillating hodoscope (eSC). Image credit: [47] .................. 32

3.2 Left Panel: Cockcroft-Walton proton accelerator. Right Panel: Ring Cyclotron accelerates protons to about 80% of the speed of light. The magnets are shown in turquoise and the four acceleration cavities in dark gray. Image credit: [53] ......................................................... 33

3.3 The actual picture of the fully assembled experiment at PSI. Image credit: [48] ................................................................. 34

3.4 Four kicker cabinets with MOSFET stacks for fast power switching. Only two are in working condition for MuSun. Electrostatic deflector plates shown within the beamline, with the top plate at +12.5 kV, and the bottom at -12.5 kV, deflect $\approx 99\%$ electrons and muons from the upstream detectors. Image credit [62]. ................................. 35

3.5 Time distribution of the kicker signal, showing the kicker on and off times. When the kicker is on the beam is blocked as shown. This plot shows an extinction factor of about 100. .................................................. 36

3.6 CryoTPC - layout 1 - beryllium window, 2 - heater, 3, 10 - heat exchangers, 4 - shell, 5 - cathode, 6 - source, 7 - dividing resistor, 8 - cathode HV feed-through, 9 - main flange, 11 - anode HV feed-through, 12 - flat signal cable, 13 - support, 14 - grounding terminal, 15, 22 - brackets, 16 - shielding grid frame, 17 - grid, 18 - anode (pad plane), 19 - field-shaping wires, 20 - MACOR stand, 21 - grid insulator, 23 - guide. Image credit: [71] ................................. 38

3.7 Shielding grid and pad plane - Left: Shielding grid 1 - Front bracket, 2 - Kovar frame, 3 - Tungsten wires, 4 - Connection holes for pad structure, 5 - Rear bracket, 6 - Insulator, 7 - Adjusting screw, 6 - Fixing screw. Right: Pad structure 1 - 50-pin connectors, 2 - Flat cables, 3 - Bar, 4 - Pad plane, 5 - Pin, 6 - Connecting hole. Image credit: [71] ................................. 39

3.8 Conceptual design for the cooling system. Image credit: [1]. ................................. 40

3.9 (a) The CHUPS modified for MuSun. (b) The compressor shell of the CHUPS. Image credit: [60]. ................................. 40
3.10 A view of the electron detectors along the beam axis. The two cylindrical
centric multiwire proportional chambers, ePC1 and ePC2 are shown.
The electron scintillators (eSCs) and PMTs surround the ePC. They are
installed radially at positions numbered 1 to 16 in this figure. Image credit:
[60]. ................................................................. 43

3.11 Wire chamber ePC1 showing the anode and cathode strips. Image credit:
[60]. ................................................................. 44

3.12 Left Panel: Map of Neutron array surrounding the TPC. Right Panel:
The Bicron Neutron counter - BC501A ........................................ 46

3.13 A single mu metal sheet around the PMT and scintillator cell, extending
up to the “Gondola” e− scintillator detector, provided optimal shielding
against the stray field of the μSR magnet. ................................. 47

3.14 A schematic diagram showing front end electronics and data acquisition.
 ................................................................. 48

3.15 Data accumulation of production Run 4. The vertical axis shows the
number of μ events collected. Image credit: [69] .......................... 50

4.1 Effect of baseline restorer. The left panel shows the acoustic noise in a
channel of the TPC before using a baseline restorer. The right panel shows
the effect of using the baseline restorer which eliminates the noise. Image
credit: [69]. ................................................................. 53

4.2 An afterpulse in the neutron counter caused by an earlier signal electron
that deposits a large energy. .................................................. 53

4.3 An auto-correlation time distribution for the raw μSC hits is shown in
black. A software deadtime window of 29 ns shown in gray / blue is
applied to get rid of the afterpulses. Image credit: [62]. .................. 55

4.4 μPC-xy clusters. The x-y coincidences producing a profile of the incoming
muon beam. Image credit: [71]. .................................................. 56

4.5 A TPC pulse generated by drift electrons. An ADC count is equivalent to
4 mV. Image credit: [60] ...................................................... 57

4.6 A TPC pulse fitted by a template (in thick red) and a Gaussian (in thin
green or dotted gray) is shown in the left panel. The right panel shows
the χ² distribution from both these fits - Gaussian is shown in red (dotted
gray) line and the pulse template in solid black line. Image credit: [69]. 57

4.7 Right panel shows the plot of energy versus range in units of pad length.
The fractional distance the muon travels in the stopping relative to the
pad length is d. Left panel shows the variation of S-energy for d ranging
from 0.1 to 0.9 ................................................................. 59

4.8 Plot showing the variation of energy deposited in the stopping pad E0 and
just before the stopping pad E1. The straight line represents the S-energy
of this plot. ................................................................. 60

4.9 Energy deposited in the stopping pad E0 (in blue / gray) and just before
the stopping pad E1 (in red). The black shows the well peaked S-Energy.
Image credit: [69] ................................................................. 60
4.10 The fiducial volume cuts for the muon stop definition. Left panel shows the X-Z pad plane and the right panel shows the drift time spectrum for cuts along Y - direction. Image credit: [64] ................................. 61

4.11 Clusters and muon stops. Left panel shows these on the pad plane with pad numbers and the right panel uses the event display to show these. Image credit: [64] ................................................................. 62

4.12 A muon track in the TPC that eventually stops. Left bottom shows the Bragg energy distribution of the pulses generated on each anode pad as the muon is stopped. The right panel shows the amplitude and energy distribution on each pad (that increases continuously till Bragg peak is attained at the stop position). The X - axis of the bottom right plot shows the drift time of the ionized electrons. [65] ................................. 63

4.13 Online display showing the raw energy spectrum of all gondolas for run 4 data. The four plots in eSC segment shown in blue, green, red and black corresponding to the four channels IU (inner upstream), ID, OU and OD respectively. Image credit: [63] ................................................................. 64

4.14 The temporal and spatial coincidence windows used for track definition from clusters are based on these histograms. The left panel plots time difference, the middle panel plots the difference in $\phi$ coordinates and the right panel plots the difference in z coordinates for ePC1 and ePC2 clusters. Image credit: [66] ................................................................. 66

5.1 The dotted red plot is the result of interpolation using cubic spline which is superimposed on the raw pulse shown in a solid blue line. ................. 68

5.2 Time windows showing definitions of total and tail areas relative to peak time (left panel) and half time (right panel). The windows of total and tail areas shift a little relative to the half-height time. ......................... 70

5.3 Variation of PSD ratio with total area along with PSD cuts for counter NU3. The neutrons, gamma rays and after pulses are labeled in the plot. 73

5.4 A small slice of energy (total area) is plotted for the PSD ratio plot for counter NU3. TSpectrum object of ROOT was used to find the peaks shown. ................................................................. 74

5.5 Figure of Merit using peak time (left) and half-height time (right) for detector NU3 ................................................................. 74

5.6 Variation of PSD ratio with total area for all eight neutron counters. 75

5.7 Pseudo time distribution (within a clock cycle) of detector NU3 (left) and its true time determined from normalized running integral of pseudo times (right). ................................................................. 76

5.8 Template of counter NU3 for neutrons (solid red line) and gamma rays (dashed blue line) superimposed. 77

5.9 A neutron pulse first fitted with a neutron template (left) and then a gamma template (right), using MINUIT fit. 77
5.10 Finding a minimum of \( D(t) \) in the interval \((a,c)\) using Brent’s minimization. \( a_1 \) and \( c_1 \) are the initial points selected and \( b_1 \) is a point in between them corresponding to the minimum. A point \( x_1 \) is chosen either between \( a_1 \) and \( b_1 \) or \( c_1 \) and \( b_1 \). Since \( D(c_1) > D(a_1) \) \( x_1 \) is replaced by \( c_2 \), \( a_1 = a_2 \) and \( b_1 = b_2 \) as this is the second step. The figure shows four steps and a final set of points \( a_4 \), \( b_4 \), and \( c_4 \) and the process continues till tolerance is achieved.

5.11 A neutron pulse first fitted with a neutron template (left) and then a gamma template (right), using Brent’s minimization.

5.12 A gamma pulse first fitted with a neutron template (left) and then a gamma template (right), using Brent’s minimization.

5.13 Interpolation for bin \( i = 125 \) (in this case). The fraction \( X \) is the modulo of the raw pulse in this bin. The interpolated value is \( y = y_i + X(y_{i+1} - y_i) \).

5.14 Comparison of fit parameters obtained from MINUIT and Brent’s Minimization. The leftmost panel shows the comparison of areas from MINUIT and Brent’s method respectively. The center and right panels show the same for the times and the pedestals from both these methods.

5.15 Three energy dependent neutron templates (left) and gamma ray templates (right). These are normalized relative to their peak values. The neutron templates show a very small energy dependence and the gamma ray template is independent of energy.

5.16 Plot of chi squared difference between neutron and gamma ray templates with pulse energy (total area).

5.17 Two Extremely close pulses forming a double pulse on an island.

5.18 Residues of each bin in fit range 5 to 55 for neutron pulses fitted with a neutron template - all detectors.

5.19 Residues of each bin in fit range 5 to 55 for Gamma ray pulses with a Gamma ray template - all detectors.

5.20 Residues of fitted function and pulses for \( n \)th and \( (n+1) \)th bin for neutrons (left) and Gamma rays (right).

5.21 Neutrons were defined using this plot.

5.22 Comparing figure of merits of PSD from the two methods - tail total and \( \chi^2 \) for detector NU3.

5.23 Typical pulses that were misinterpreted as neutrons by the tail total method of PSD.

5.24 Left panel shows the decay scheme and photo peaks of \( ^{60}\text{Co} \). Image credit: [76]. and the right panel shows the same for \( ^{137}\text{Cs} \). Image credit: [82].

5.25 A plot of Klein Nishina formula in black solid line along with the fit function which is shown by the blue dotted line. The fit function is a convolution of Klein Nishina formula with a Gaussian distribution of sigma 20 keV. This is an example of Compton scattering from \( ^{137}\text{Cs} \) source with a photo peak of 667 keV and a Compton edge of 480 keV.
5.26 Energy distribution of the data from counter NU14 with source $^{60}$Co. The blue dotted line represents Klein Nishina distribution. The magenta dashed line is the fit function i.e. convolution of Klein Nishina cross section with the Gaussian. The red solid line is the fitted region. .............................. 92

5.27 Calibration procedure illustrated for counter NU3. Plot of channels Vs. keV$_{ee}$ for maximum kinetic energy for sources $^{60}$Co and $^{137}$Cs respectively. The slope of this plot gives the gain. ........................................ 93

5.28 Comparing the energy spectrum of all detectors using $^{137}$Cs source. ... 94

6.1 Neutron multiplicity distribution of all neutron detector pulses for one Midas run having $3.7 \times 10^6$ muon events. ................................................................. 96

6.2 Time distribution of all pulses seen by the neutron counters. This plot shows an estimate of the number of pulses detected by the neutron counters in the flat region (shown in cyan) which is about five times greater than the rest of the time dependent region (shown in blue). .............................. 97

6.3 Left panel shows the time distribution of all pulses detected by the neutron counters and the right panel shows the time distribution of neutrons after applying a neutron PSD cut. ....................................................... 98

6.4 The solid blue line is time distribution of all pulses detected by the counters, the dashed red line is that of neutrons only and the dotted dashed magenta line is that for neutrons with muon stops in the fiducial volume of the TPC. ................................................................. 98

6.5 Left panel: time distribution of electrons with respect to muon entrance time. Right panel: time distribution of electrons with respect to neutrons. 99

6.6 Distribution of the coordinates of the point of closest approach of electrons with respect to the beam axis. ................................................................. 99

6.7 Time distribution of fusion neutrons (in thick red / gray line) in log scale shows evidence of two lifetime components a prompt one and a delayed one. This is overlaid on the time distribution of neutrons with electron coincidences without an energy cut in blue (black). ......................... 100

6.8 Energy spectrum of fusion neutrons for neutron counter NU3. .......... 101

6.9 Delay time t0 and sigma for all neutron detectors using a finely binned histogram with a bin size of 1 ns. ................................................................. 102

6.10 Plot shows the fusion neutron time distribution before (in dashed red line) and after (in solid blue line) aligning the delay time t0 for all detectors. 103

6.11 Time distribution of fusion neutrons in black (blue) line and the distribution obtained from the pre electron condition in gray (red) line. .......... 104

6.12 The efficiency function denotes the probability of finding a neutron in the pre electron window. ................................................................. 105

6.13 The final background distribution (in gray / red) overlaid on the fusion neutron distribution. The left panel shows that the flat background from 7000 ns to 15000 ns overlays completely with the fusion neutron distribution. The right panel shows the details of the initial part of these spectra. 106
6.14 Left panel shows the efficiency function and right shows the final background distribution obtained after systematically shifting the efficiency function by ±20 ns relative to the original alignment. 106

6.15 The errors on the final distribution after propagating the errors. 107

7.1 Effect of including early times in fit range. The fit range in the left panel is from -100 to 2000 ns. The fit range in the right panel is from 100 to 2000 ns. The reduced χ² is much better for the right panel. 110

7.2 Start time scan of the rate corresponding to the fast lifetime. Start time of the fit was scanned from 100 ns to see the effect of the early time region on the fit. 111

7.3 Preliminary fit results of two data sets at a temperature of 34 K and a density of 6% of liquid hydrogen. 112

7.4 Plot showing the variation of the amplitude ratio A₁ : A₂ with rate ratio λ₁ : λ₂ with values of A₁ and A₂ obtained from fit results corresponding to our experimental conditions i.e. a temperature of 34 K and φ = 6 % of LH₂. 113

7.5 Systematic errors for rate corresponding to fast lifetime (upper panel), amplitude ratio (middle panel) and χ²/ndf (lower panel). The dotted (green) line denotes the fit values from the original cuts to get the final results. All cuts produce stable results that agree within the error bars of the original cut. 116

7.6 Systematics for muon stop cuts. The top panel shows the fiducial volume cuts, the middle panel shows the S-Energy cuts and the bottom panel shows the cuts for stop in Z-pads respectively for the mustop condition. 117

7.7 Left panel: Fit results of all nine data sets at a temperature of 34 K and a density of 6% of liquid hydrogen. Right panel: The plot of residues does not show any form of inconsistency. 119

7.8 Fit results for all nine data sets. 120

7.9 Start time scan results for the fit parameters and χ². 122

7.10 Stop time scan results for the fit parameters and χ². 123

8.1 Variation of the relative population of the doublet state (in dashed magenta), the quartet state (in dotted dashed blue) and total population (in solid black) with time. 124

8.2 Variation of the population of the quartet state (left panel) for later times. Right panel shows the same for the doublet state at later times to show the effect of recycling. 125

8.3 Variation of the population of the quartet state (left panel) for three values of λ₂. The same for the doublet state is shown in the right panel. These are plotted for a small range of time where the difference due to different values of λ₂ is noticeable. 126
8.4 Variation of the population of the quartet state (left panel) for the values including experimental errors in $\lambda_q : \lambda_d, \lambda_1, \lambda_2$. The same for the doublet state is shown in the right panel. These are plotted for a small range of time where the difference due to the errors is noticeable.

8.5 Variation of $N_d(t_0)/N(t_0)$ with time. At later times it is $< 1$ which is due to recycling of muons.

8.6 Error range when $\lambda_{qd}$ is changed in the left panel and the right panel shows the same for $\lambda_q : \lambda_d$.

7 Systematic effects of changing the pre-electron definition cuts.

8 Systematic effects of changing the energy cuts.

9 Systematic effects of changing the threshold energy (minimum energy) of neutron definition.
LIST OF TABLES

1.1 The table shows an example of each type of weak interaction. .................. 6

2.1 The table shows the $d\mu d$ binding energies for various rotational and vibrational states, taken from [51] ................................. 20

2.2 The table shows the values of physical parameters used in the calculation of the muon chemistry kinetic parameters. .......................... 25

4.1 Table summarizing the dimensions (X, Y and Z coordinates) of the various fiducial volume cuts. ................................................. 61

4.2 The table shows the cuts applied for forming ePC clusters. ................. 65

4.3 The table shows the cuts applied for forming tracks from clusters. These cuts are based on Fig. 4.14 and taken from the module GlobalElectronAnalysis.C .................................................. 66

5.1 The table lists various attributes of a waveform pulse. ......................... 69

5.2 The table lists various definitions of tail areas spanning from an initial to a final time window of the pulse after interpolation. ................. 71

5.3 The table shows the energy equivalent range of PSD for all neutron detectors, ‘D’ stand for downstream, ‘U’ for upstream, and the number stands for the position in $\phi$. It also shows the maximum and minimum value of PSD ratio for each range of energy ............................................. 72

5.4 The table shows the energy equivalent range of PSD for all neutron detectors, ‘D’ stand for downstream, ‘U’ for upstream, and the number stands for the position in $\phi$. .................................................. 73

5.5 The table shows the range of the ratio of neutron chi square to Brent area cuts for all energy ranges of all neutron detectors using template fit method of PSD, ‘D’ stand for downstream, ‘U’ for upstream, and the number stands for the position in $\phi$. .................................................. 86

5.6 The table shows the energy range of all neutron detectors using template fit method of PSD, ‘D’ stand for downstream, ‘U’ for upstream, and the number stands for the position in $\phi$. .................................................. 88

5.7 The table shows the photo peaks and Compton edges for $^{60}$Co and $^{137}$Cs sources ................................................................. 90

5.8 The table shows the procedure for calculating gains from the Compton Edges. This data was taken after shielding the neutron counters. ....... 93

5.9 The table shows the gains before installing $\mu$SR magnet and just after installing it. The last column shows the finally calibrated gains after successfully shielding the counters ........................................... 95

6.1 The table shows the delay times for each detector. .............................. 103
7.1 Table showing the rate ratio obtained from the amplitude ratio for three different values of $\lambda_d$. .................................................. 113

7.2 Table summarizing the results of all systematic studies. The original cuts are mentioned in brackets in the first column and the Fig. 7.2.1.2 shows the results of the original fit. .................................................. 115

7.3 Table summarizing the systematic errors associated with each cut. ........ 119

7.4 Table summarizing the criteria for creating data sets used for comparison of consistency of results. The HV of ePC1 is same for all data sets. .... 120

7.5 The table lists the elements of the covariant matrix which shows the correlation between the fit parameters. .................................................. 121

8.1 The table shows the correlation between the new fit parameters. ............ 128

8.2 The table compares results from various previous experiments with this work. The experiments are listed in chronological order ................. 129
Chapter 1

Introduction

A muon is an elementary particle classified as a lepton in the Standard Model. Leptons are fermions with spin 1/2 that interact via weak, electromagnetic and gravitational forces but do not undergo strong interactions. A muon is similar to an electron, but is \( \approx 206 \) times heavier than the electron. The negative muon interacts via the electroweak force to either undergo a decay or be captured by matter. It first undergoes an atomic capture in any matter, where it replaces an electron of the atom and forms a muonic atom that is around 200 times smaller than the original atom. Subsequently it undergoes a nuclear capture. Positive muons (antiparticle of a muon) are repelled by a nucleus so that it can only undergo a decay. According to CPT invariance \([1]\), both muons and their antiparticles have the same lifetime in vacuum. They decay via the reactions given by Eq.\((1.1)\) with a lifetime of 2.2 \(\mu s\).

\[
\begin{align*}
\mu^- & \rightarrow e^- + \bar{\nu}_e + \nu_\mu \\
\mu^+ & \rightarrow e^+ + \nu_e + \bar{\nu}_\mu.
\end{align*}
\] (1.1)

The simplest form of nuclear capture of the muon in proton is given by

\[
\mu^- + p \rightarrow n + n + \nu_\mu.
\] (1.2)

The Feynman diagrams of free muon decay and muon’s nuclear capture in proton is shown in the left and right panel of Fig. 1.1 respectively. Since this is a weak interaction involving a charged lepton, it is mediated by a charged weak force carrier the \(W^\pm\) boson. Muons can also be captured by any subsequent heavier elements like deuteron, He, etc.

The MuSun experiment is a new generation high precision experiment that aims to find the muon capture rate \(\Lambda_d\) in deuteron to a precision better than 1.5\%. This is governed by the reaction

\[
\mu^- + d \rightarrow n + n + \nu_\mu.
\] (1.3)

The Feynman diagram of the interaction described by Eq.\((1.3)\) is shown in Fig. 1.2. This is a two-nucleon weak interaction and the measurement of \(\Lambda_d\) will determine the poorly known two-nucleon weak axial current. Effective field theory (EFT) calculations need a low energy constant (LEC) to understand any two-nucleon weak interaction. The experimentally determined \(\Lambda_d\) can be parametrized to obtain this
LEC which will in turn help understand all two-nucleon weak interactions such as the stellar pp fusion reaction and the neutrino reaction $\nu + d$ (which explores the solar neutrino oscillation problem). It will also help us understand weak nuclear interactions involving more than two nucleons (for e.g. double beta decay) as they also involve a two-nucleon weak axial current term. All this is covered in great detail in Sec. 1.2.

In this chapter, we briefly discuss other high precision experiments with muons and its importance as a tool in studying the Standard Model. Further we study the theory of weak interactions and the motivation of MuSun experiment. Finally we compare the results of previous experiments and summarize the theoretical calculations of $\Lambda_d$. 

Figure 1.1: The Feynman diagrams of free muon decay (left panel) and muon’s nuclear capture in proton (right panel). The initial-to-final states run from the bottom to the top of the page.
1.1 Other Experiments using Muon as a Fundamental Tool

The muon being a light particle with a comparatively long lifetime (but not too long) allows very precise measurement of its properties. Also, it is much heavier than the electron that makes it particularly sensitive to new types of virtual particles [2], that could help probe new physics beyond the Standard Model (BSM) [3]. Thus, it is a very efficient instrument in understanding several fundamental aspects of particle physics [3]. The muon decay reaction 1.1 is very precisely understood within the Standard Model. Besides this, ejecting an electron that is easily detectable makes it even more suitable for studying several fundamental processes. These processes have been studied with the help of several high precision experiments.

All known particles experience weak interactions with a universal strength of a weak coupling constant called the Fermi constant $G_F$ [4]. Thus, $G_F$ includes all weak interaction effects in the low-energy EFT [6]. The free muon decay is a suitable weak decay reaction to find $G_F$ as it is a purely leptonic decay [7]. The relation between the muon lifetime $\tau_\mu$ and $G_F$ is given by the perturbative expansion of the expression [6, 7],

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} \left(1 + \sum_i \Delta q^{(i)}\right)$$

(1.4)

where $m_\mu$ is the muon mass and $\Delta q^{(0)}$ denotes the phase-space, $\Delta q^{(1)}$ the first-order QED correction, $\Delta q^{(2)}$ the second-order QED correction etc. These theoretical calculation were evaluated by van Ritbergen and Stuart (vRS) [6] which established a relation between $\tau_\mu$ and $G_F$ in a sub-ppm level after applying the two-loop QED corrections. The theoretically evaluated value of $G_F$ by vRS is $1.16637 \pm 0.00001 \times 10^{-5}$ GeV$^{-2}$. This work was the motivation for the MuLan experiment. The MuLan experiment measured the positive muon lifetime to a precision of 1 part per million.
which in turn helped in the accurate determination of the Fermi constant \( G_F = 1.1663788(7) \times 10^{-5} \text{ GeV}^{-2} \) \[8\] that determines the strength of weak interactions. A comparison of \( G_F \) from different processes such as muon and tau decays tests the universality of the weak interactions, which agree up to a level of ±0.1% \[9\].

Strong interactions described by the Lagrangian of QCD (quantum chromodynamics) satisfy an approximate chiral symmetry \[12\]. The chiral symmetry is spontaneously broken to give rise to hadronic masses and the explicit breaking of chiral symmetry associates non-zero mass to \( u \) and \( d \) quarks. The determination of proton’s pseudoscalar form factor \( g_P \) will test our understanding of this symmetry breaking of QCD or the approximate chiral symmetry of QCD in the low-energy regime \[12\].

The MuCap experiment \[10\] measures the muon capture rate in proton from the singlet state \( \Lambda_S \), and helps in determining \( g_P \). The final results extract the rate of the reaction \( 1.2 \times 10^{10} \mu^- \) decays and derive the proton’s pseudoscalar coupling \( g_P(q_0^2 = -0.88 m^2_{\mu}) = 8.06 \pm 0.55 \) \[10\], where \( q_0^2 \) is the momentum transfer. This agrees well with the theoretical value of \( g_P \) at \( q_0^2 \) which is 8.02 \[11\].

The muon g-2 experiment \[13\] aims to find the muon’s anomalous magnetic moment \( a_\mu \). The muon’s magnetic moment \( \mu_\mu \) is given by,

\[
\mu_\mu = g_\mu \frac{e}{2m_\mu} S
\]

where \( S \) is the spin of the muon, \( m_\mu \) is its mass, \( e \) is its charge and \( g_\mu \) is the Lande g-factor. This is found by measuring the precession of muons (or antimuons) in a very precisely known uniform magnetic field in a storage ring.

![Figure 1.3: A muon interacting with a photon. This shows the leading order QED contribution (also called the Schwinger term \[14\]), representing the underlying cause of the deviation of \( a_\mu \) from 2 due to QED interactions.](image)
In the absence of an internal structure $g_\mu \neq 2$ from loop effects originating due to both quantum electrodynamics (QED) and QCD. This slight deviation of $g_\mu$ from 2 is given by $a_\mu = \{g - 2/2\}$. Thus the muon’s anomalous magnetic moment $a_\mu$ deviating from 2 could be due to QED interactions, with strong interactions etc. The muon’s leading order contribution of the QED interaction is shown in Fig. 1.3. The first calculation of the anomalous magnetic moment of the electron was done by Schwinger in 1947 [14]. Thus, the leading order contribution of the QED interaction in the calculation of $a_\mu$ is called the Schwinger term.

The world average for $a_\mu$ disagrees by 3.6$\sigma$ with theoretical predictions [15]. The deviation of muon’s anomalous magnetic moment from the theoretically calculated value will account for explaining any new physics beyond the Standard Model. Additionally the g-2 experiment also intends to find the muons electric dipole moment (EDM) [18]. A very small non-zero EDM is predicted by the SM and a deviation from this would imply time and parity violation also indicating evidence of new BSM physics.

Each family of lepton in the SM obeys a lepton family conservation number (i.e. weak interactions conserve lepton family number). A charged lepton flavour violation (LFV) process would be an evidence of BSM physics. New generations of experiments studying such processes also involve the muon. For example the neutrino-less radiative muon decay process $\mu^+ \to e^+ \gamma$ violates lepton family number conservation and is studied at PSI called the MEG (mu-e-gamma) experiment [16]. Here the gamma ray accounts for the conservation of momentum and energy instead of the neutrino. In the data collected from 2009 and 2010 no evidence of this decay is found, which makes the upper limit of the branching ratio of this process to be $2.4 \times 10^{-12}$ [17]. Improved detector performance will result in a final sensitivity of about $10^{-13}$ for future data acquisition [17]. The $\mu2e$ experiment [19] at the Fermilab is also searching for charged a LFV in the process, $\mu^- N \to e^- N$, where N is the nucleus that helps in momentum and energy conservation instead of a neutrino. An observed charged LVF process with a sensitivity of $10^{-14}$ would be a clear indication of BSM physics [19]. Thus, the muon can be used in investigating new BSM physics.

1.2 Weak Interactions

Weak interactions are interactions between fermions mediated by massive vector bosons, usually causing a decay and operating at distances less than $10^{-15}$ m. Thus, all hadrons and leptons experience weak interactions but are suppressed by much more rapid strong and electromagnetic interactions except the $\mu$ and $\pi^\pm$ decay, which do not decay via strong or electromagnetic interactions. The weak interactions involving leptons conserve the lepton family number, violate intrinsic parity (P) and charge conjugation (C) but conserve CP — though CP is not a symmetry of the Standard Model. The lifetime of a particle is inversely proportional to the coupling strength of its decay reaction so that $\mu$ and $\pi^\pm$ decays have a longer life compared to the lifetime of a $\pi^0(\pi^0 \to \gamma \gamma)$ decay which involves an electromagnetic interaction.

Different types of weak interaction exist depending on the type of particles involved in a reaction. If an interaction or decay involves only leptons it is called a leptonic
interaction or decay. If it involves other particles along with leptons it is called a semileptonic interaction. If it does not involve any leptons it is called a nonleptonic interaction. Examples of these various types of weak interactions and decays are listed in table 1.1.

<table>
<thead>
<tr>
<th>Type of interaction</th>
<th>Illustrations of the interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leptonic decay</td>
<td>$\mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu$</td>
</tr>
<tr>
<td>Semileptonic interaction</td>
<td>$\mu^- + d \rightarrow n + n + \nu_e$</td>
</tr>
<tr>
<td>Nonleptonic decay</td>
<td>$K^+ \rightarrow \pi^+\pi^0, \pi^+\pi^- \pi^0, \pi^+\pi^0\pi^0$</td>
</tr>
</tbody>
</table>

Table 1.1: The table shows an example of each type of weak interaction.

The $\beta$ decay is a weak interaction that was first explained by Fermi. It is a transformation of a neutron to a proton (or vice versa) in an atomic nucleus [4]. A free proton cannot decay to a neutron spontaneously as that is energetically impossible, but a free neutron decays to a proton via the decay $n \rightarrow p e^- \bar{\nu}_e$ with an average lifetime of 886.3 s [20]. Fermi assumed this to be a point interaction as shown in the left panel of Fig. 1.4 with $J^{(e^+)}_\mu$ and $J^{(n)}_\mu$ being the weak currents for $e^+$ and the neutron respectively, given by the expressions,

$$J^{(e^+)}_\mu = \bar{u}_e \gamma_\mu u_e$$

$$J^{(n)}_\mu = \bar{u}_p \gamma_\mu u_n$$

Thus, the invariant amplitude for this decay is given by [5],

$$M = \frac{G_F}{\sqrt{2}} (J^{(n)}_\mu)(J^{(e^+)}_\mu)$$

(1.6)

where $G_F$ is the weak coupling constant called the Fermi constant. Originally, Pauli postulated the neutrino to explain the energy and momentum conservation of this decay (any $\beta$ decay) which was later understood by Fermi’s theory. Neutrinos experience only weak forces (and negligible gravitational force owing to their very small mass). Fermi assumed the vector - vector form of the weak interaction current yielding the $\gamma_\mu$ term in the expressions for weak currents.

Parity violation for a weak interaction was first observed by Wu et al. in 1957 [21] in a famous experiment that studied the $\beta$ decay of polarized $^{60}$Co nuclei. The electrons emitted had a preferential direction that was opposite to that of the gamma rays. This asymmetry in the emitted electrons proved parity violation. This was followed by another experiment by Garwin et al. 1957 [22] that showed violation of parity and charge conjugation in pion and muon decays. A series of experiments over several years proved that the spin and momentum vectors are anti-parallel for leptons (and parallel for antileptons). It was found that only left-handed neutrinos ($\nu_L$) and right-handed ($\bar{\nu}_R$) antineutrinos participate in weak interactions. The $(1 - \gamma^5)/2$ being the left projection operator gives rise to parity violation and must be included in the
weak current. Charge conjugation, C is also violated (as C transforms $\nu_L$ state to $\bar{\nu}_L$ state, whereas only $\bar{\nu}_R$ state is observed, which implies C violation).

But later, it was found that these weak interactions (but not quark weak decays like kaon decays) were invariant under the combined effect of CP i.e. $\gamma^\mu (1 - \gamma^5)/2$ was conserved. Thus $\gamma^\mu$ was replaced by $\gamma^\mu (1 - \gamma^5)/2$ in the expressions for weak currents, which explains the V-A (vector minus axial vector) nature of weak interactions. The V - A theory for weak interactions was postulated by Feynman and Gell-Mann [23] and a similar work was also done by Sudarshan and Marshak [24] in 1958. We now have the invariant amplitude for $\beta$ decay given by,

$$M = \frac{G_F}{\sqrt{2}} (\bar{u}_e \gamma^\mu (1 - \gamma^5) u_n)(\bar{u}_e \gamma^\mu (1 - \gamma^5) u_{\nu_e} + \bar{u}_e \gamma^\mu (1 - \gamma^5) u_{e^+})$$

(1.7)

where the factor $1/\sqrt{2}$ is purely conventional to retain the original definition of $G_F$ given by Eq.(1.6).

Figure 1.4: Feynman diagram of the $\beta$ decay. The left panel shows the case using Fermi theory of point interaction and the right panels shows the same using Standard Model. At low energies the W boson is replaced by a point, which effectively looks like a point interaction as shown in the left panel. Thus we have generalized the Fermi theory to a V-A effective field theory of the weak interaction in the low-energy regime.

Now the Standard Model predicts weak interactions to be mediated by $W^\pm, Z$ bosons and so the point interaction is replaced by a W boson as shown in the right
Thus, M takes the form,
\[ M = \left( \frac{g}{\sqrt{2}} \bar{u}_p \gamma^\mu \frac{1}{2} (1 - \gamma^\mu) u_n \right) \frac{1}{M_W^2 - q^2} \left( \frac{g}{\sqrt{2}} \bar{u}_\nu \gamma^\mu \frac{1}{2} (1 - \gamma^\mu) u_e \right) \] (1.8)

where \( g/\sqrt{2} \) is the dimensionless weak coupling and \( q \) is the momentum transferred to the weak boson W of mass \( M_W \approx 80 \text{ GeV} \). The factors \( 1/\sqrt{2} \) and \( 1/2 \) are inserted to retain the conventional definition of \( g \). For weak interactions in our case, \( q^2 \ll M_W^2 \) and so comparing Eq.(1.7) and Eq.(1.8) we get,
\[ \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \] (1.9)

Thus, the point interaction assumption of the weak force in Fermi theory of \( \beta \) decay is an excellent approximation as the energy scale of the \( \beta \) decay is much smaller than the mass of W boson (80 GeV/c\(^2\)) and the Fermi theory is an effective field theory of the weak interaction in the low-energy regime.

### 1.3 Physics Motivation

All two-nucleon (2N) weak interactions have one new unknown low energy constant (LEC) characterizing the short-distance 2N axial current [1]. This LEC is called \( L_{1A} \) in the \( \pi \)EFT (pionless effective field theory) [40] and \( \hat{d}^R \) in \( \chi \)PT (chiral perturbation theory) [35]. The muon capture in deuterium described by Eq.(1.3) is a two-nucleon weak interaction and determination of \( \Lambda_d \) will help us find this LEC essential to understand all two-nucleon weak interactions. Theoretically, a weak interaction of an isolated nucleon is well understood, but the same process for a bound multi-nucleon system is not. Muon capture in hydrogen is well understood and this rate is found very precisely by the MuCap experiment [10]. The muon capture rate in a three nucleon system for example, \(^3\text{He}\) is very accurately measured experimentally (though not very well understood theoretically as it is a multi-nucleon system). This is because the capture reaction \( \mu^- + ^3\text{He} \rightarrow \nu_\mu + t \) emits a charged particle, triton \( t \) which can be measured easily. Thus, the rate of this reaction is accurately measured from the rate of production of tritons. On the other hand MuSun’s 2N weak reaction \( \mu^- + d \rightarrow n + n + \nu_\mu \) does not emit any charged particles and we do not find the rate directly by measuring the neutron rate, as it is very difficult to do this. Instead we use the lifetime technique which measures the decay electron time distribution to find \( \Lambda_d \) (the details of this technique are discussed in the next chapter). Thus the 2N weak axial current is more difficult to find and so \( L_{1A} \) and \( \hat{d}^R \) for 2N systems is poorly known. Its determination relies on the accurate determination of \( \Lambda_d \) and thus on MuSun results. The models used to derive and understand \( L_{1A} \) and \( \hat{d}^R \) are based on \( \pi \)EFT and \( \chi \)PT respectively. In this section I try to explain all these models briefly.

Theoretically, \( \Lambda_d \) is evaluated using various approaches. Before the development of EFT’s, \( \chi \)PT’s, etc. the standard nuclear physics approach (SNPA) was used. This approach was based on the fact that a nucleus is a system of interacting nucleons
The two-nucleon interaction with an axial weak vector — (a) for long ranges using pion exchange current and (b) short ranges using a point interaction. Image credit [34].

[25]. The NN (nucleon - nucleon) interaction was understood by \( nn, pp \) and \( np \) interactions. The Hamiltonian in this case involves a phenomenological wave function that is model dependent and so the value of \( \Lambda_d \) could change depending on the wave function used to model the potential between the nucleons. The approximate weak nuclear amplitude of the muon capture process is a superposition of all individual single-body amplitudes. This is called the impulse approximation (IA) [26, 12]. Thus, the weak transition operator includes, the single nucleon contribution associated with \( p\mu^- \rightarrow n\nu_\mu \) (i.e. the IA [26]), meson-exchange currents (MEC) and currents arising from the excitation of \( \Delta \)-isobar degrees of freedom [43, 44]. This approach has several drawbacks, as it does not consider NN potentials (i.e. they are also phenomenological) derived from effective chiral Lagrangians and gets very complicated as the number of nucleons increase.

\( \chi \)PT is a low energy EFT that provides a good description of QCD at the hadronic level. This is because the chiral (handedness) symmetry is spontaneously broken for a system at low energy where QCD becomes non-perturbative [36]. This spontaneously broken symmetry gives rise to massless pseudoscalar bosons i.e. the Nambu-Goldstone bosons which are the massless pions in this case [27]. Due to non-zero \( u \) and \( d \) quark masses chiral symmetry is also mechanically broken which gives a finite mass to the pion \( m_\pi \). The four-momentum transfer for these interactions is a low energy scale that is sufficiently smaller than the chiral scale \( \Lambda_\chi \sim 1 \text{ GeV} \) [27]. Since \( m_\pi \ll \Lambda_\chi \),
in this low energy scale the quarks and gluons of QCD are replaced by pions as the
degrees of freedom to describe the interactions. These interactions are mediated by a
pion exchange as shown in Fig. 1.5(a). This can be further extended to a lower energy
scale, where the pion is replaced by a nucleon. The resulting theory is called heavy
baryonic chiral effective field theory (HBχEFT) and in this case $\Lambda_{\chi} \approx m_N$. Thus, we
can visualize the interaction as replaced by a point interaction vertex and the pion
degrees of freedom are integrated out (i.e. the pion exchange forces are replaced by a
point interaction). This is denoted by $\hat{d}^R$ as shown in Fig. 1.6 for the four interactions.
These four interactions correspond to the “MuSun experiment” — muon capture in
deuterium given by Eq.(1.2), the “Stellar pp fusion” given by $p + p \rightarrow d + \nu_e + e^+$,
the “SNO experiment” given by $\nu_x + d \rightarrow n + p + \nu_x$ and the “triton $\beta$ decay” given by
$^3H \rightarrow ^3He + \bar{\nu}_e + e^-$. All these reactions can be understood in a model independent
way with the help of $\hat{d}^R$ evaluated using one of the LEC. Such an effective field theory
that eliminates pions is called the $\pi$EFT (pionless effective field theory). This LEC
is parametrized using $\Lambda_d$. Thus, reactions such as the solar pp fusion that cannot
take place under terrestrial conditions can be understood with this LEC, regardless
of the framework chosen and so MuSun is used for calibrating the Sun and is of great

Figure 1.6: The effective vertex $\hat{d}^R$ for several two-nucleon weak interactions.
astrophysical significance.

1.4 Comparison of Previous Experiments and Theory

In this section we compare the values of $\Lambda_d$ from previous experiments and summarize the results of various theoretical calculations. A study of the decay electron time distribution will determine $\Lambda_d$ (explained in detail in the next chapter). This experimental technique called the lifetime technique was used in an experiment by Bardin et al. in Saclay using a liquid deuterium target in 1986 [37]. Another method for determining the muon capture rate employs the direct detection of capture neutrons and is called the neutron detection technique. Muon capture in deuterium yields a pair of neutrons along with neutrinos that are almost impossible to detect. This reaction being a three body system in the final state imparts a wide range of energy to the capture neutrons yielding a maximum energy distribution up to $\approx 53$ MeV (i.e. half the muon mass) that peaks around 2–3 MeV. It is extremely hard to precisely determine the energy of neutrons in such a situation. Besides, complicated processes

Figure 1.7: The figure summarizes experimental and theoretical results of $\Lambda_d$. The light blue band around the Cargnelli experiment shows the anticipated 1.5% ($6 \, s^{-1}$) precision of MuSun experiment.
like muon catalyzed fusions, capture in impurities etc. makes this procedure even more difficult. A huge unwanted background of electrons and gamma rays accompany these processes that have to be well distinguished from capture neutrons with a very high efficiency. Therefore, this method has several disadvantages compared to the lifetime technique. This technique was used in the Vienna PSI experiment in 1989 using reduced gas density and temperature of the target \( \text{D}_2 \) \[38\]. The results of these experiments are shown in Fig. 1.7. Apparently, the result of Bardin’s experiment differs by about two standard deviations from the theoretical predictions (as is evident from Fig. 1.7). The reason for this is not known and may be a statistical fluctuation.

Theoretical predictions of \( \Lambda_d \) are based on the above-mentioned theories. The \( \Lambda_d \) found by Tatara \[31\], Doi \[32\] and Adam \[33\] are based on SNPA and are shown in Fig. 1.7. The value of the weak axial vector current \( g_A \) used in these cases to do these calculations was 1.2605±0.075 \[28\]. The calculations performed by Tatara used only the leading-order term of MEC in the non-relativistic reduction of the two body contributions. For the IA they used the terms \( O(1) \) and \( O(p/m_N) \), where \( m_N \) is the nucleon mass and \( p \) is its momentum. The MEC contributed to a range of 30 to 32 \( s^{-1} \) depending on the \( NN \) potential used. The leading-order contribution of IA ranged from 360 - 364.5 \( s^{-1} \) again depending on the potential used. The order \( O(p/m_N) \) had a much smaller effect as it varied from 4.4 to 4.5 \( s^{-1} \) \[31\]. Doi et al. did not include any recoil terms for both MEC and IA. Leading-order of IA gives \( \Lambda_d = 365 \ s^{-1} \) and including the the MEC gives a contribution of about 37 \( s^{-1} \), finally yielding \( \Lambda_d = 402 \ s^{-1} \) \[32\]. Adam et al. in 1990, calculated \( \Lambda_d \) up to the order \( O(p/M^2) \), where \( M \) is the nucleon mass. The velocity dependent terms were found to reduce the value of \( \Lambda_d \) by \( \approx 3\% \) and the effect due to the weak vector MEC is \((10–12) \ s^{-1} \), which is about 4–5\%, thus having a net error of about 1.7\% \[33\].

The next set of results were obtained from calculations based on Hybrid EFT. Here a combination of EFT and phenomenological wave functions are used. The weak operators are derived using EFT but their matrix elements are evaluated from phenomenological wave functions obtained from conventional potentials \[43\] to evaluate \( \hat{d}^R \) and then to ultimately calculate \( \Lambda_d \). \( \Lambda_d \) found by Ando \[39\] used the Hybrid EFT approach. Marcucci’s \[42\] approach is also based on Hybrid EFT. Ando used next-to-next-to-leading order (N3LO) calculation for the two-nucleon Lagrangian and N2LO correction for one-body hadronic current and MEC \[39\]. Marcucci used chiral N3LO two-nucleon potential for obtaining the nuclear wave functions and an order of \( O(p^2/m^2) \) for calculation of the single-nucleon pseudoscalar charge operator and the pseudoscalar two-body term in the \( N\to\Delta \) transition axial current. The final result of \( \Lambda_d \) found by Marcucci was larger than that found by Ando, due to contributions of loop corrections and contact terms in the vector part of the weak current, which were neglected in Ando’s work \[43\].

\( \Lambda_d \) found by Chen \[40\] used the \( \hat{a}EFT \) approach. Ricci \[41\] uses a combination of SNPA and Hybrid EFT to find \( \Lambda_d \). The value of \( g_A \) used for these calculations was 1.2695±0.0029 \[29\], which increased by 0.5\% compared to the previous calculations based on SNPA.

Marcucci’s subsequent work used a full EFT. Full EFT means that the nuclear
potentials and charge-changing weak currents are both derived using $\chi$EFT \cite{43}. Calculations involving complete EFT’s are model independent as they eliminate the usage of the phenomenological wave functions. The two-nucleon potential in this case is calculated to an order of $O(p^4/\Lambda^4_{\chi})$ in chiral expansion \cite{45,46} and to N2LO $\pi N$ chiral Lagrangians and contact terms \cite{43}. The latest work includes that of Adam \cite{34} in 2012 that involves full EFT in its evaluation of $\Lambda_d$. The calculation employed the nuclear wave functions generated from $NN$ potential of N3LO of heavy-baryon chiral perturbation theory and the MEC operator derived within the same formalism and order. The value of $\Lambda_d$ is measured with an accuracy of 1.5\% in this work \cite{34}. The value of $g_A$ used for these calculations was 1.2767(16) \cite{30}, which increased by 1.2\% compared to the old calculations based on SNPA. These theory calculations clearly show an improvement with an inclusion of error bars due to the incorporation of EFT methods as is evident in the Fig. 1.7. After the precise measurement of $\Lambda_d$ a fully consistent $\chi$PT formalism can be applied. A lot of work in the theory regime has been done using the above-mentioned theories, which are consistent with each other and a further consistency check from experiment relies on the precise measurement of $\Lambda_d$ by the MuSun experiment.

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Chapter 2

Experimental Design and Strategy

The ultimate goal of the MuSun experiment is the unambiguous extraction of the muon capture rate in deuterium from the doublet state ($\Lambda_d$), independent of the complicated muonic atomic physics uncertainties, by nearly an order of magnitude higher than that achieved in previous experiments. The measurement of $\Lambda_d$ must achieve an overall precision of 1.5% ($6\sigma$) \cite{1}. All this can be achieved by first understanding the muon chemistry and kinetics in deuterium gas, and then suppressing these complicated effects as much as possible with the help of optimal target gas conditions and appropriate experimental design strategies. This chapter focuses on how we arrive at these strategies and plan the experiment effectively for achieving the ultimate goal of our desired precision.

2.1 Experimental Technique

A negative muon passing through any material loses energy in subsequent collisions with the atoms or molecules of the gas (deuterium gas in our case) ionizing it and ejecting an electron. It eventually stops in the gas and undergoes the process of muon capture. The muon and its antiparticle decay via the set of reactions given by Eq.(1.1).

According to CPT theorem \cite{1} both the positive and negative muons decay with the same rate i.e. $\lambda_\mu^+$ and $\lambda_\mu^-$ are the same. Thus, a negative muon entering the deuterium gas target either gets captured by the gas or decays in most of the cases with a free muon rate of $\lambda_\mu^- = \lambda_\mu^+$. If the capture rate in deuterium from doublet state is represented by $\Lambda_d$ and the muon disappearance rate is denoted by $\lambda_{dis}$, then we have $\lambda_{dis} = \lambda_\mu^+ + \Lambda_d$. The muon capture rate in deuterium, is thus given by $\Lambda_d = \lambda_{dis} - \lambda_\mu^+$. The basic experimental technique of MuSun relies on the above equation and is called the lifetime technique \cite{37}. The value of $\lambda_\mu^+$ can be taken from the best world data for muon lifetime (for example from the results of MuLan experiment) and can be further verified by measurements of the muon decay rate employing a positive muon beam in our experiment itself. The negative muon disappearance rate $\lambda_{dis}$ is measured by the decay electron time spectrum, which is nothing but the time of an electron detected by an electron detector with respect to the initial time of the muon entering the detector. These measurements are shown in Fig. 2.1. This method has several advantages over the neutron detection method (as discussed in Sec. 1.4). It helps in achieving the desired precision as it relies on detecting decay electrons. Electrons
can be detected with excellent time resolution using plastic scintillators. Besides this the acceptance of the electron detecting system is 75% whereas the neutron detectors have an acceptance of about 2% only.

\[ \lambda_{\mu^+} + \lambda_{\mu^-} = \lambda_{\text{dis}} + \Lambda_d \]

Figure 2.1: Lifetime Technique. The dashed line denotes the positive muon decay rate and the solid line denotes the negative muon disappearance rate in the gas target. The difference gives us the muon capture rate in deuterium gas.

The precision of our experiment also requires the accurate measurement of muon entrance timing and spatial positions. This is done with good accuracy using muon scintillators and a high resolution Time Projection Chamber (TPC) that precisely determines the stopping position and time of the muon in the target. To attain the desired precision several other factors have to be taken in account and carefully scrutinized.

The muon capture should unambiguously take place from one spin state (these states will be clearly explained in the next section) as much as possible. We cannot get completely rid of the two-spin states of the muon captured but the population of one state can be minimized with appropriate physical conditions of the gas (like pressure, density temperature etc.). The deuterium gas should be as clean as possible with an impurity level of the order of a part per billion, which is very challenging to achieve. Thus it was essential to have an ultra pure gas system with purity monitoring at 1 ppb level. Besides this it is essential to have a very high accumulation of muon decay events for the accurate statistical interpretation muon disappearance rate. This requires a high statistics of $10^{10}$ muon decay events to find this rate to an accuracy in a part per million.
To understand and plan the optimal physical conditions of the target gas suitable for our experiment, we need to understand the complicated muon kinetics in pure deuterium, which is our next topic of discussion.

2.2 Muon Chemistry and Kinetics in Deuterium

When a negative muon enters pure deuterium gas, it loses its kinetic energy in subsequent collisions with the molecules of the gas. The initial kinetic energy of the muon beam ranges from 5 - 20 MeV such that it is enough to penetrate the target and slow down to attain energies of the range of 10-20 eV (this is well within the range of the ionization potential of deuterium). This energy range is suitable for the muon to undergo atomic capture. Thus, it stops in the gas to dissociate a D$_2$ molecule (with binding energy of 15.5 eV) into two atoms and gets captured atomically with one atom in a high atomic orbital (in the range of 15 -20) [55] [56]. In the procedure of capture it tends to lose its polarization due to spin flip in arbitrary directions [55]. This occurs as the it cascades to the ground 1s state (n=1) and forms a muonic deuterium ($\mu$d) atom. Finally about 20% of the polarization is retained for the negative muon beam and thus it can be considered to be in an almost depolarized state [61], [57]. This resulting $d\mu$ atom has two-hyperfine spin states and is in a bound state for about a microsecond [49]. After this it undergoes any one of the following processes:

1. The muon decays ejecting a decay (or Michel) electron via 1.1 process.

2. The muon gets captured by the deuterium to a muonic deuterium atom $\mu$d in two-hyperfine states i.e. a doublet or a quartet state.

3. A nuclear capture occurs on the deuteron from either of the two-hyperfine states via 1.3 reaction.

4. The muon catalyzes a fusion by forming $d\mu d$ molecules that undergo fusion either by forming helium along with fusion neutrons or by forming a triton and a proton. The reactions for these are in Sec. 2.2.2.

These processes will be discussed in detail in the subsections below. This entire muon chemistry is depicted in Fig. 2.2.

2.2.1 Hyperfine States

After the deuteron atomically captures the muon it forms two-spin states with the deuteron, namely

1. muon spin aligned with deuteron which gives rise to a quartet state with a total spin 3/2 i.e. $d\mu(\uparrow\uparrow)$

2. muon spin antialigned with deuteron which gives rise to a doublet state with a total spin 1/2 i.e. $d\mu(\uparrow\downarrow)$
Figure 2.2: Muon Kinetics in deuterium. The branching ratio of muon sticking to He (called sticking probability) is 0.1206 which is negligible and so these side branches can be ignored. Image credit: [1].

The energy difference between these two states is 0.0485 eV. The ratio of the initial population of the hyperfine states of the \( d\mu \) atom is proportional to the ratio of the number of spins states in each hyperfine state. The quartet state has four spin states namely \( m_s = -3/2, -1/2, 1/2, 3/2 \) whereas the doublet state has two-spin states i.e. \( m_s = -1/2, 1/2 \). So the initial population of the quartet to doublet hyperfine states is in the ratio of 2 : 1.

The V-A nature of weak interaction (vector - pseudovector) favours the capture of the the \( d\mu \) atom from its doublet state. The capture rate from quartet state is 12 s\(^{-1}\) (denoted by \( \Lambda_q \)) as against the rate from doublet state that is 386 s\(^{-1}\) (denoted by \( \Lambda_d \)). These numbers are theoretically calculated and are taken from [1]. Thus, one major goal of this experiment was to increase the doublet population as much as possible. This is very well accomplished at a density of 5% liquid hydrogen and a temperature of 30 K [50]. In Fig. 2.2 the initial fraction of quartet state is shown by \( q = 2/3 \), and so the fraction of doublet state is given by \( 1-q = 1/3 \). Thus, the incoming muon initially gets captured forming a muonic deuterium in either of the two states. The population of muonic deuterium in the quartet state decays to muonic deuterium in the doublet state. This hyperfine transition rate is denoted by \( \lambda_{qd} \) in Fig. 2.2. It varies by a small amount from 35 - 40 \( \mu s^{-1} \) over a wide range of temperature from 30 to 300 K [77]. Its variation with temperature is shown in Fig. 2.3. It is more sensitive
to the density of the deuterium gas.

![Figure 2.3: Variation of $\lambda_{qd}$ with temperature. Image credit: [1].](image)

A hyperfine transition in the reverse order is also possible i.e. from doublet to quartet state. This rate is denoted by $\lambda_{dq}$ in Fig. 2.2. It depends on the temperature and density of the gas and exhibits a great dependence on temperature. After attaining thermal equilibrium, the two-hyperfine states simply obey Maxwell - Boltzmann distribution and thus the double to quartet hyperfine transition rate is given by [86],

$$\lambda_{dq} = 2e^{-\frac{\Delta}{kT}} \lambda_{qd}$$  \hspace{1cm} (2.1)

where $k$ is the Boltzmann constant, having a value of $8.6174 \times 10^5$ eV/K. At room temperature, the two rates ($\lambda_{qd}$ and $\lambda_{dq}$) are comparable and so there is not enough transitions from quartet to doublet states. This is not a desirable condition of our experiment. But at low cryogenic temperatures it is evident from Eq.(2.1) that the rate $\lambda_{dq}$ vanishes. Thus, in spite of the fact that the initial population of the quartet state being twice of that of the doublet state, it quickly disappears due to an effective unidirectional transition from quartet to doublet state. This explains one of the reasons for the low temperature required for MuSun.

### 2.2.2 Muon-Catalyzed Fusion

In this sub section we thoroughly discuss the formation of $d\mu d$ molecules that leads to their fusion (yielding either He or tritium) with the muon behaving as a catalyst. This phenomenon is called muon-catalyzed fusion. A $\mu d$ atom collides with $D_2$ molecule to form these $d\mu d$ molecules The formation of $d\mu d$ molecules can be from either of the two states i.e. the quartet or doublet state. The rate of formation of $d\mu d$
molecules from the quartet state ($\lambda_q$) and from the doublet state ($\lambda_d$), both depend on temperature and density of the target gas. The temperature dependence of $d\mu d$ molecule formation rate ($\lambda_{d\mu d}$) is shown in Fig. 2.4. At cryogenic temperatures (till around 300 K) $\lambda_q$ and $\lambda_d$ are quite different from each other, but at higher temperatures, $\approx 400$ K the two values coincide as shown in Fig. 2.4. Also, it should be noted that the molecular formation rate is much higher from the quartet state than the doublet state i.e. $\lambda_q > \lambda_d$.

![Figure 2.4: Variation of $\lambda_q$ and $\lambda_d$ with temperature. Here $\lambda_{d\mu \mu}$ represents the $d \mu \mu$ molecular formation rate from any hyperfine state. Image credit: [1].](image)

There are two possible channels of this fusion reaction, which are as follows:

1. The $d\mu d$ molecule either fuses to form a $^3$He atom emitting a mono energetic neutron (we call this a fusion neutron) with an energy of 2.45 MeV as shown in the reaction below:

$$d\mu d \rightarrow ^3\text{He} + n + \mu$$  \hspace{1cm} (2.2)

or the $\mu$ adheres to the He atom forming a $\mu^3\text{He}$ atom also emitting a neutron.

2. The $d\mu d$ molecule fuses to form a tritium atom emitting a mono energetic proton with an energy of 3.02 MeV as shown in the reaction below:

$$d\mu d \rightarrow t + p + \mu$$  \hspace{1cm} (2.3)

or the $\mu$ sticks to the triton forming a $\mu^3t$ atom also emitting a proton.

The formation of $d\mu d$ molecule initiates a fusion reaction as discussed before. There is a specific mechanism for this process that was first suggested by Sakharov [52].
The $d\mu d$ molecule is analogous to the $D_2^+$ ion, except that it is 200 times smaller in size than the $D_2^+$ ion, which encloses the two deuterons in the $d\mu d$ molecule in a very small volume of about $\approx 500$ fm, which enables fusion to take place almost instantly, in about $\approx 1$ ns [83], even at ordinary temperatures. This happens due to the reduction of the repulsive Coulomb barrier at such small distances. Thus, the rate of fusion is much greater than the molecular formation rate. The two reaction channels given by 2.2 and 2.3 take place with almost equal probabilities in our experimental conditions i.e. the branching ratio $\beta$ is $\approx 50\%$. All this is shown in Fig. 2.2. There is a negligibly small probability of the muon sticking to the triton also, but this is not shown in Fig. 2.2 as the stripping probability is much higher [83]. The probability of the muon sticking to the helium atom is denoted by $\bar{\omega}$. Thus, effectively the fraction of muons sticking to the helium atom (or forming $\mu$He) is given by $\omega = \bar{\omega}\beta$. The sticking of muon to helium is an undesirable process which gives rise to a background. This should be minimized as much as possible. A lower temperature reduces $\lambda_d$, which in turn reduces the population of quartet state as the recycling of muons due to fusion is reduced. Consequently, the chances of formation of $\mu$He are also reduced.

### 2.2.3 Resonant and Non-Resonant Molecular Formation

In this section, I describe in detail the processes of $d\mu d$ molecule formation, thermalization of $d\mu d$ molecules, the stable resonant molecular formation and the effect of epithermal muons from non-resonant molecular formation [77], on the time distribution of the $d\mu d$ molecules formed.

A $\mu d$ atom colliding with $D_2$ molecule produces a compound molecule $[(d\mu d)dee]^*$ [86], which is in a very loosely bound state (suggested by Vesman [78]). Thus the energy is low enough to be absorbed by the rotational ($J$) and vibrational ($\nu$) states of this compound molecule. This is the resonant formation of the $d\mu d$ molecule. From the table 2.1, we notice the $J = 1$ and $\nu = 1$ state, has the lowest binding energy of 1.97 eV which is much lower than the ionization potential of $D_2$ (15.46 eV). This process occurs mostly for $\mu d$ atoms in the ground state and is always thermalized. Thus, the thermalized resonant formation of $d\mu d$ molecule is shown as,

$$\mu d + D_2 \rightarrow [(d\mu d)_{11}dee]^*$$  \hspace{1cm} (2.4)

where the subscript 1,1 denote the values of $J$ and $\nu$ respectively.

<table>
<thead>
<tr>
<th>$(J, \nu)$</th>
<th>Binding Energy (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>325.074</td>
</tr>
<tr>
<td>(0, 1)</td>
<td>35.844</td>
</tr>
<tr>
<td>(1, 0)</td>
<td>226.628</td>
</tr>
<tr>
<td>(1, 1)</td>
<td>1.974985</td>
</tr>
<tr>
<td>(2, 0)</td>
<td>86.494</td>
</tr>
</tbody>
</table>

Table 2.1: The table shows the $d\mu d$ binding energies for various rotational and vibrational states, taken from [51].
In all the other possible states shown in table 2.1, \( d\mu d \) molecular formation takes place via the non-resonant process, as the binding energy is greater than the ionization potential of \( D_2 \). These are in general represented by [77],

\[
\mu d + D_2 \rightarrow [(d\mu d)_{J\nu}de]^+ + e^- \tag{2.5}
\]

where \( J \neq \nu \). Collisions of thermalized \( \mu d \) atoms with \( D_2 \) forms non-resonant \( d\mu d \) molecule with a rate of \( 10^4 \text{ s}^{-1} \), whereas non-thermalized \( \mu d \) atoms form \( d\mu d \) molecules with a rate of \( 10^6 \text{ s}^{-1} \) [86]. Thus, the molecular formation rate depends on the temperature and hence the kinetic energy of \( \mu d \). The variation of all rates in all states (resonant and non-resonant) with the kinetic energy of \( \mu d \) is shown in Fig. 2.5.

![Figure 2.5: Resonant and non-resonant \( d\mu d \) molecular formation rates versus kinetic energy of the \( \mu d \) atom. Image credit: [77].](image)

Thus, before thermal equilibrium, the target deuterium gas is a complicated mixture of resonant and non-resonant state \( d\mu d \) molecules along with epithermal muons colliding with these molecules. This non-thermalized state is clearly a combination of several different rates as shown in Fig. 2.5 and so a fusion neutron time distribution would also reveal this complicated behaviour. For the optimized experimental conditions of MuSun, the \( \mu d \) atoms attain thermal equilibrium in about 100 ns [86]. This will be clearly seen in the fusion neutron time distribution.

### 2.2.4 Overall Picture

In an overall picture the complicated muon chemistry that goes on in the target after the muon stops, as depicted by Fig. 2.2 can be represented mathematically by sets of differential equations [83]. The populations of the doublet \( (n_{1/2}) \) and quartet \( (n_{3/2}) \)
states of the $\mu d$ atom can be found by solving the sets of coupled differential equations given below:

\[
\begin{align*}
\frac{dn_{1/2}}{dt} &= -(\lambda_{\mu} + \phi \lambda_d)n_{1/2} + \phi \lambda_{qd} n_{3/2} + \frac{1}{3} \lambda_f (1 - s) n_{dd} \\
\frac{dn_{3/2}}{dt} &= -(\lambda_{\mu} + \phi \lambda_{qd} + \phi \lambda_q) n_{3/2} + \frac{2}{3} \lambda_f (1 - s) n_{dd} \\
\frac{dn_{dd}}{dt} &= \phi \lambda_{d} n_{1/2} + \phi \lambda_q n_{3/2} - (\lambda_{\mu} + \lambda_f) n_{dd}
\end{align*}
\]  

(2.6)

where $s$ is half the sticking probability of He i.e. 0.0603 (effective sticking probability of He), $\lambda_f$ is the fusion rate, and $\phi$ is the density of the target.

### 2.3 Analytical Solution for Population of States

The analytical solutions of the population of $\mu d$ atoms in quartet and doublet states are necessary to extract useful kinetic parameters, that would in turn interpret and compare the experimentally obtained fit results. The populations of the doublet ($n_{1/2}$) and quartet ($n_{3/2}$) states of the $\mu d$ atom can be found by solving the sets of coupled differential Eqs. (2.6). The symbols and their values are listed in table 2.2. These sets of coupled differential equations have been solved in here with details in the appendix 8.3.

We first study the effect of switching off recycling of muons in the process. Next we include recycling to understand its effect on the fusion time distribution.

**Without muon recycling:**

To begin with we deal with a simple case of no muon recycling and see how the system behaves by plotting the solutions for doublet and quartet states. If the sticking probability of He is 1 then, the muon would always stick to the He atom which would eliminate the possibility of muon recycling. Thus, the last term in Eq.(2.6) becomes zero as $s = 1$. Also the third set of equation is zero as the number of $d\mu d$ molecules formed is negligibly small. Thus Eq.(2.6) now reduces to

\[
\begin{align*}
\frac{dn_{3/2}}{dt} &= -(\lambda_{\mu} + \phi \lambda_{qd} + \phi \lambda_q) n_{3/2} \\
\frac{dn_{1/2}}{dt} &= -(\lambda_{\mu} + \phi \lambda_{d}) n_{1/2} + \phi \lambda_{qd} n_{3/2}
\end{align*}
\]  

(2.7)

The eigenstate matrix of the above system of equations can be expressed as,

\[
\begin{pmatrix}
-\phi \lambda_q - \phi \lambda_{qd} - \lambda_{\mu} & 0 \\
\phi \lambda_{qd} & -\phi \lambda_d - \lambda_{\mu}
\end{pmatrix}
\]

The eigenvalues which represent the rate of each state $\lambda_1$ and $\lambda_2$ respectively are

\[
\lambda_1 = \phi \lambda_d + \lambda_{\mu}
\]

(2.8)
\[ \lambda_2 = \phi \lambda_q + \phi \lambda_{qd} + \lambda_U \]  

(2.9)

And the eigenvectors are, \( \{0, 1\}, \left\{-\frac{\lambda_d + \lambda_q + \lambda_{qd}}{\lambda_{qd}}, 1\right\} \)

For simplicity we used the following substitution,

\[ Y = -\frac{\lambda_d + \lambda_q + \lambda_{qd}}{\lambda_{qd}} \]  

(2.10)

\[ n_{1/2}(t) = \left(Y - \frac{2}{3}Y\right)e^{-\lambda_1 t} + \frac{2}{3}Ye^{-\lambda_2 t} \]  

(2.11)

Figure 2.6: Plots showing the doublet state populations in linear and log scale with no recycling. This shows the initial population \( n_{1/2}(0) = \frac{1}{3} \).

Figure 2.7: Plots showing the quartet state populations in linear and log scale with no recycling. This shows the initial population \( n_{3/2}(0) = \frac{2}{3} \).

The initial population is proportional to the total spin states of each state and so \( n_{1/2} \) and \( n_{3/2} \) is in the ratio of 1/3 : 2/3 initially. The final solution of Eq.(2.7) depicting the population of each state is given by, (refer appendix 8.3 for detailed steps)
The fusion distribution in general is the linear combination of the population of two states \( n_{1/2} \) and \( n_{3/2} \) and is thus given by,

\[
n(t) = \lambda_d \beta_d n_{1/2} + \lambda_q \beta_q n_{3/2}
\]  

(2.13)

where \( \beta_d \) and \( \beta_q \) are the probabilities of the formation of fusion neutrons and He from the doublet state and quartet state respectively. \( \lambda_d \) and \( \lambda_q \) are the \( d\mu d \) molecular formation rates from the doublet state and quartet state of the \( \mu d \) atom respectively.

The experimentally obtained fusion neutron time distribution should thus be fitted with a two lifetime fit function. This can be compared with the above Eq. (2.13) and is given by a theoretical fit function as,

\[
n(t) = A_1 e^{-\lambda_1 t} + A_2 e^{-\lambda_2 t}
\]  

(2.14)

\( A_1 \) and \( A_2 \) are the amplitudes corresponding to the two rates. Simplifying Eq. (2.13) we get the ratio of the amplitudes as,

\[
\frac{A_1}{A_2} = \frac{\lambda_d \beta_d (2 - Y)}{2(\lambda_d \beta_d + Y \lambda_q \beta_q)}
\]  

(2.15)

From Eq. (2.12) it is clear that the quartet state shows no recycling term and has just one rate. The ratio of the two amplitudes of the states is found to be, \( A_1 : A_2 \approx 53 : 1 \) using values of the constants and rates from table 2.2 [1]. It is also known that \( \beta_d \approx \beta_q \) [83].

**Including recycling of muons:**

It is known that fusion of \( d\mu d \) molecule takes place almost instantly in 1 ns time [85] and so the population of \( d\mu d \) molecules attain equilibrium very fast. In equilibrium the formation of \( d\mu d \) molecules is equal to its disappearance. Hence the rate \( dn_{dd}/dt \) is negligibly small and can be taken to be zero for practical purposes. Substituting \( dn_{dd}/dt = 0 \) in Eq. (2.6) we get,

\[
\phi \lambda_d n_{1/2} + \phi \lambda_q n_{3/2} = (\lambda_\mu + \lambda_f)n_{dd}
\]

\[
n_{dd} = \frac{\phi \lambda_d n_{1/2} + \phi \lambda_q n_{3/2}}{(\lambda_\mu + \lambda_f)}
\]  

(2.16)

Substituting the expression of \( n_{dd} \) in Eq. (2.6) and simplifying we get the eigenstate matrix of this set of differential equations as,

\[
\begin{pmatrix}
-\phi \lambda_d - \lambda_\mu + \frac{(1-s)\phi \lambda_d \lambda_f}{3(\lambda_f + \lambda_\mu)} & \phi \lambda_qd + \frac{(1-s)\phi \lambda_f \lambda_q}{3(\lambda_f + \lambda_\mu)} \\
\frac{2(1-s)\phi \lambda_q \lambda_f}{3(\lambda_f + \lambda_\mu)} & -\phi \lambda_q - \phi \lambda_qd - \lambda_\mu + \frac{2(1-s)\phi \lambda_f \lambda_q}{3(\lambda_f + \lambda_\mu)}
\end{pmatrix}
\]
In the term \( \frac{(1-s)\phi\lambda_d\lambda_f}{3(\lambda_f+\lambda_\mu)} \), \( \lambda_d \) is very small for our experimental conditions 2.2 and \( \lambda_f \gg \lambda_\mu \) and so we ignore this term. Thus the matrix reduces to
\[
\begin{pmatrix}
-\phi \lambda_d - \lambda_\mu + \frac{1}{3}(1-s)\phi \lambda_d & \phi \lambda_{qd} + \frac{1}{3}(1-s)\phi \lambda_q \\
\frac{2}{3}(1-s)\phi \lambda_d & -\phi \lambda_q + \frac{2}{3}(1-s)\phi \lambda_q - \phi \lambda_{qd} - \lambda_\mu
\end{pmatrix}
\]
The eigenvalues of the matrix give us the two lifetimes. If \( \phi \lambda_d \) is ignored compared to the other terms we get approximate expression for the short lifetime as,
\[
\lambda_1 \approx \frac{1}{3}(1+2s)\phi \lambda_q + \phi \lambda_{qd} + \lambda_\mu \tag{2.17}
\]
Similarly the long lifetime is found to be
\[
\lambda_2 \approx \lambda_\mu + \frac{1}{3}(2+s)\phi \lambda_d \tag{2.18}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value at 30 K</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fusion rate ( \lambda_f )</td>
<td>1000 ( \mu s^{-1} )</td>
<td>[85]</td>
</tr>
<tr>
<td>d( \mu )d formation rate from doublet state ( \lambda_d )</td>
<td>0.053(3) ( \mu s^{-1} )</td>
<td>[86]</td>
</tr>
<tr>
<td>d( \mu )d formation rate from quartet state ( \lambda_q )</td>
<td>3.98(5) ( \mu s^{-1} )</td>
<td>[86]</td>
</tr>
<tr>
<td>Density of the target ( \phi )</td>
<td>0.05 of LH(_2)</td>
<td></td>
</tr>
<tr>
<td>Hyperfine transition rate from quartet to doublet ( \lambda_{qd} )</td>
<td>37.0(4) ( \mu s^{-1} )</td>
<td>[86]</td>
</tr>
<tr>
<td>Muon decay rate ( \lambda_\mu )</td>
<td>0.455170(45) ( \mu s^{-1} )</td>
<td>[7]</td>
</tr>
</tbody>
</table>

Table 2.2: The table shows the values of physical parameters used in the calculation of the muon chemistry kinetic parameters.

Thus, the populations of each of these states in terms of our defined symbols is given by the equations
\[
n_{1/2} = \frac{X_1(2X_2-1) - \lambda_d t}{3(X_2-X_1)} e^{-\lambda_d t} + \frac{X_2(1-2X_1) - \lambda_\mu t}{3(X_2-X_1)} e^{-\lambda_\mu t} \tag{2.19}
\]
\[
n_{3/2} = \frac{(2X_2-1)}{3(X_2-X_1)} e^{-\lambda_d t} + \frac{(1-2X_1)}{3(X_2-X_1)} e^{-\lambda_\mu t} \tag{2.20}
\]
where,
\[
X_1 = \frac{\phi \lambda_q + \phi \lambda_{qd} - \frac{2}{3}(1-s)\phi \lambda_q - X}{\frac{4}{3}(1-s)\phi \lambda_d} \tag{2.21}
\]
\[
X_2 = \frac{\phi \lambda_q + \phi \lambda_{qd} - \frac{2}{3}(1-s)\phi \lambda_q + X}{\frac{4}{3}(1-s)\phi \lambda_d} \tag{2.22}
\]
and
\[
X = \sqrt{(\phi \lambda_q + \phi \lambda_{qd})^2 + \frac{4}{3}(1-s)\phi^2\lambda_q(1+\lambda_{qd})} \tag{2.23}
\]
where we again assumed $\lambda_{qd} \gg \lambda_d$ and thus ignored all terms involving $\lambda_d$ compared to $\lambda_{qd}$ in the numerator.

The time distribution for the ratio of the quartet population to that of the total population $\frac{n_{3/2}(t)}{n_{1/2}(t) + n_{3/2}(t)}$ is shown in the Fig. 2.8 in dotted blue line. The solid red line shows time distribution of the quartet population. Again from Eqs. 2.13 and

![Figure 2.8: Plots showing the time distribution of the quartet state $n_{3/2}(t)$ (in solid red) and the quartet state to total population $\frac{n_{3/2}(t)}{n_{1/2}(t) + n_{3/2}(t)}$ (in dashed blue) in linear and log scale.](image_url)

2.14, we get the ratio of the amplitudes as,

$$\frac{A_1}{A_2} = \frac{(\lambda_d X_1 + \lambda_q)(2X_2 - 1)}{(\lambda_d X_2 + \lambda_q)(1 - 2X_1)} \quad (2.24)$$

Substituting in the values of the parameters from table 2.2 and solving for $X_1$, $X_2$, we evaluate the ratio of $A_1 : A_2$ from Eq.(2.24) to be 46.56. This is slightly smaller than the ratio obtained with no recycling ($\approx 53$). In the subsection below we study the effect of changing the input parameters used in finding the amplitude ratio $A_1 : A_2$.

2.3.1 Effect of Changing Various Parameters on $A_1 : A_2$

A plot of the amplitude ratio vs. $\lambda_{qd}$ is shown in the top plot of Fig. 2.9. This reveals that there is a small dependence of this ratio with $\lambda_{qd}$.

This can be accounted due to the fact that at the cryogenic temperatures of MuSun, almost all $\mu d$ atoms in the quartet state quickly transit to the doublet state (as discussed in the muon chemistry section of experimental design), irrespective of the value of $\lambda_{qd}$.

A plot of the amplitude ratio vs. $\lambda_d$ is shown in the middle plot of Fig. 2.9. The population of $\mu d$ atoms in the quartet state would increase as $\lambda_d$ increases. Thus, the amplitude ratio decreases. But the effect is very prominent in this case. A slight rise in $\lambda_d$ decreases the value of the amplitude ratio. This could be attributed due to
Figure 2.9: Plots showing the variation of the effect of changes in rates $\lambda_{qd}$, $\lambda_d$ and $\lambda_q$ with the amplitude ratio $A_1 : A_2$
the fact that the formation of $d\mu d$ molecules occur predominantly from the quartet state. When doublet population increases, $d\mu d$ molecular formation is reduced and so recycling of the catalyzing muon is in turn inhibited, causing an enhanced decrease in quartet population. This reflects the sensitivity of $\lambda_q$ on the amplitude ratio and also on our data. Finally, a plot of the amplitude ratio vs. $\lambda_q$ is shown in the bottom plot of Fig. 2.9. The population of $\mu d$ atoms in the quartet state would obviously increase as $\lambda_q$ increases, which explains the increase in the amplitude ratio.

2.4 Optimal Target Conditions

To achieve the ultimate goal of the experiment i.e. precise determination of muon capture rate from the doublet state, we require a higher population of doublet state and a fast transition from quartet to doublet state i.e. a large value of $\lambda_{qd}$ and an extremely small value of $\lambda_{dq}$. The hyperfine transition rate is effectively given by $\phi\lambda_{qd}$ and so a larger value of density of the gas ($\phi$) would help in achieving a fast transition from quartet to doublet state. Thus the density was optimally set to 0.05% of LH$_2$ that is 5 times greater than the density used for the MuCap experiment.

![Figure 2.10: Time distributions of all relevant states under the experimental conditions of MuSun. Image credit: [1].](image)

The other physical quantity that controls the muon kinetics and hence the experiment is the temperature. Cryogenic temperatures around 30 K is suitable for optimizing our experiment due to lower value of $\lambda_{dq}$ and reduced background from $\mu$He as discussed in 2.2.2. The added advantage of lowering the temperature is that the $d\mu d$ molecular formation rates from doublet and quartet states i.e. $\lambda_d$ and $\lambda_q$ are drastically different at this temperature as shown in Fig. 2.4.
The fusion neutron time distribution that is the indirect study of the $d\mu d$ molecular formation rate would unambiguously display these two rates, which in turn gives us the population of the hyperfine states and the hyperfine transition rate $\lambda_{qd}$. This can be monitored using neutron detectors that have a very good time resolution. The time distributions of various states are shown in Fig. 2.10. The distribution of quartet population (in black) shows two lifetime components - a short and a long. This is because the quartet state decays to the doublet very promptly and also undergoes fusion, ultimately releasing the muon that is recycled back to the system. Thus, the short lifetime represents the hyperfine transition rate from quartet to doublet i.e. $\lambda_{qd}$ and the slow rate is predominantly the free muon decay rate. The doublet state is shown in red and the $\mu d$ distribution is shown in blue. The $\mu^3$He is shown in green.

From Fig. 2.3 it is evident that $\lambda_{qd}$ is not very well determined at high temperatures. The experimental uncertainty increases by 15% at 300 K [1]. Thus such high temperatures should be avoided if precision is the ultimate goal of MuSun. Besides this, from Eq.(2.1) we arrive at the conclusion that at cryogenic temperatures the hyperfine transition rate $\lambda_{dq}$ is negligible which again helps in reducing the quartet population as desired.

Finally, at a temperature of 30 K, five times higher density is permissible, allowing the pressure to be comparable to the MuCap conditions. In such a situation a high density, high resolution, cryogenic TPC different from the TPC of the MuCap experiment was essential. To summarize, the conditions $\phi=0.05$ and $T=30$ K shown in Fig. 2.10 were chosen as the optimal design of MuSun experiment.

### 2.5 Essential Observables and Overall Experimental Setup

The most important observables for a precision measurement of the muon decay from the doublet state are the muon and decay electron tracks. Besides this, fusion neutrons, capture neutron and impurity studies are also critically important.

Muon entrance counters and scintillators are used to detect muons and electron scintillators are used for detecting electrons. The high resolution cryogenic TPC, is used for generating muon tracks and ePC (multi wire proportional chambers) are used to construct electron tracks and electron vectors. The fusion neutron and capture neutron time and energy distributions are created using neutron detectors. The neutron detectors have a very high time resolution, which can generate a time distribution as shown in the bottom panel of Fig. 2.11. Fig. 2.11 shows the time and energy spectrum of all charged particles in the TPC after a muon is stopped in the target gas for a previous experiment [90] that had condition similar to our MuSun conditions. The range of all fusion products is such that they can easily be detected and tracked by the TPC. The $^3$He and $\mu^3$He have a range of 0.18 and 0.6 mm respectively and the recoil proton and triton have a range of 16 and 1 mm respectively. The corresponding energies are shown in Fig. 2.11 along with peaks corresponding to energies of $p\mu d$ fusion, $t+p$ branch of fusion.
Figure 2.11: Histograms of all charged particle (top is energy spectrum and bottom is the time distribution) after muon stop from a previous experiment [90] with conditions similar to MuSun at T=45.3 K and $\phi = 0.0524$ and nitrogen impurity level is 41 ppb. Impurity capture background is represented by a dotted line. Image Credit - MuSun Proposal 07

Thus, the big picture is that we require muon entrance counters—for recording their entrance timing, muon scintillators—for tracking muons, a cryo-TPC surrounded by electron and neutron detecting systems (radially surrounding the TPC to be able to detect all particles, in all directions). The details of the entire set up and detector sub systems will be discussed in the next chapter.
Chapter 3

Experimental Setup

The MuSun 2011 production run that took place in the πE3 area of PSI, Switzerland (Villigen), had a beam time spanning from June 20th to September 12th, was called Run 4. PSI runs several particle accelerators including the proton cyclotron generating a 2.2 mA proton beam, which is the highest intensity in the world, thus being a great source of versatile beamlines like protons, pions, neutrons and muons. This makes it highly suitable for all muon experiments. The MuSun experiment is based on MuLan and MuCap experiments that were also performed in PSI. The successful working of this experiment depended on the successful collaborative efforts of manufacturing, testing and installing various detector subsystems, all frontend electronics, data acquisition system (DAQ) developed by each collaboration institute. The cryogenic system required for the Time Projection System (TPC) and all TPC related subsystems underwent rigorous improvements between subsequent data acquisition periods (production runs) and all this was extremely well handled by the PNPI team at Gatchina. This chapter deals with a detailed explanation of the entire experimental setup that includes the beamline, muon entrance detectors, TPC, μSR Magnet, electron detectors, neutron detectors, frontend electronics and DAQ.

3.1 Experimental Overview

This section is intended to briefly describe the entire MuSun experiment. Emphasis is on the Run 4 data. The πE3 beamline produces a muon beam of momentum about 40 MeV/c that is controlled by an electrostatic kicker to cause a single muon to enter the gas target and thus prevent pile up in the experiment. The muon passes through muon entrance counters called μSC (a thin plastic scintillator) and μPC (multi wire proportional chamber) and then enters the TPC, which contains the target gas at our experimental conditions. The TPC is used for tracking the muon. In the TPC all complicated muon chemistry takes place that has already been discussed in the previous chapter. The muon decays into an electron and the tracking and time measurement of the electron is done with the eSC (electron scintillator), ePC1 and ePC2 (electron proportional chambers), as shown in Fig. 3.1. In brief the eSC is a segmented scintillating hodoscope surrounding the TPC. The ePC1 and ePC2 are two proportional wire chambers. The neutrons emitted by the capture process or during fusion are detected by a system of eight neutron detectors, arranged radially.
surrounding the TPC. All detectors and their subsystems will be elaborately explained in the subsequent sections.

Figure 3.1: A cutaway view of the MuSun detector. The muon beam propagates from the left through the \( \mu \text{SC} \) and \( \mu \text{PC} \) before entering the TPC via a beryllium window. The muon is tracked by the TPC, and decays to an electron that is tracked and detected by two projection wire chambers (ePC1 and ePC2) and a segmented scintillating hodoscope (eSC). Image credit: [47].

3.2 Beamline

The acceleration of the proton beam used to produce pions is a three-step process. The first step begins with the acceleration of protons to 810 keV by an electrostatic Cockcroft-Walton accelerator, as shown in Fig. 3.2. In the second stage the protons are transferred to the first of two ring cyclotrons, which increases the energy of the beam to 72 MeV, and then the second cyclotron that further boosts the energy by 590 MeV, finally generating the 2.2 mA proton beam with a velocity of 0.8c.

The protons collide with a graphite target producing pions that subsequently decay to muons. The proton beam penetrates the target, generating pions deeper inside the target. Positive muons produced from the decay of positive pions that are stopped closer to the target boundary are called “surface muons”. Negative pions which have enough energy to just leave the target before decaying to negative muons via \( \pi^- \rightarrow \mu^- \bar{\nu}_\mu \) are called “cloud muons”. Finally the muons produced have energies in the range from 0.5 keV to 60 MeV [54], which are highly polarized owing to their fixed helicity [60]. These muons were directed through secondary beamlines into our experimental area. For an adequate central stopping distribution of muons in the TPC, the beam momentum was determined to be approximately 40 MeV/c and was thus used in the experiment.

There is much greater number of positive pions than negative giving a greater flux of positive muon beam. Since a positive muon cannot be captured and neither does
it catalyze a fusion, so the positive muon beam was used to study room background for fusions and captures and also investigate muon stop data, which is free of these fusion and capture neutrons (that act as background for stop data).

The beamline area is enclosed within thick concrete shielding blocks. This is thick enough to stop not only the straying muons but also the relatively more penetrating electrons and gammas. A lead safety block is placed in front of the beamline, which is removed during data taking period. Following this is a group of dipole magnets (to essentially select the beam momentum), quadrupole magnets (to focus the beam), and slits (to collimate the beam). There is a control system that maintains vacuum in this region to minimize energy loss of the muon beam. The other two vital components are the electrostatic kicker to prevent pile up and the separator to stop electron contamination for our experiment, respectively described in the subsections below.

### 3.2.1 Electrostatic Kicker

The electrostatic kicker is inherited from the MuLan experiment and was originally designed by TRIUMF. The kicker introduces a time structure to the continuous muon beam as it was switched on at a regular interval. It is essentially a made up of 1.5 m long conducting deflecting plates, aligned horizontally to produce a vertical electric field normal to the muon beam. The plates are separated by 12 cm and there exists a potential difference of 25 kV when the kicker is on. The plates are completely charged
for 40 ns with the aid of four sets of fast switching stacks of MOSFET cards. The beam gets deflected when the kicker is on which prevents it from reaching the target. Thus, the beam goes undeflected into the target only when the kicker is off.

The kicker in Musun experiment works on “Muon-On-Request” (MOR) mode \cite{62}. Here the kicker is coupled with the muon entrance counter (\(\mu SC\)), which generates a signal as soon as a muon is detected by the \(\mu SC\) that in turn switches on the kicker deflecting the beam. For 25 \(\mu s\), the beam remains deflected which blocks the subsequent entrance of muons in this time window. There is a delay of approximately 600 ns between this detector signal to the deflection of the beam turning it off (this includes processing time in the electronics (\(\approx 350 \text{ ns}\)), signal speed (\(\approx 100 \text{ ns}\)), switching time of kicker (\(\approx 40-60 \text{ ns}\)) and time of flight of the undeflected muons (\(\approx 100 \text{ ns}\)).
The ratio of the unsuppressed muon beam during this time to suppressed muon beam when the kicker is on is called the extinction factor. The time distribution of the muon relative to the kicker signal is shown in Fig. 3.5. Each cycle of the kicker thus allowed one muon at a time to reach the target, which greatly reduced the possibility of the signal of one muon to interfere with another muon in the required time range. This interference or superposition of signals from successive muons is called pile-up.

Thus, the kicker in this mode helps reduce pile up to a great extent in the 25 \(\mu s\) time window. A precise pile up protection was further implemented in the software coding. The greatest advantage of this mode is that it enabled a much lower amount of data taking time required to accumulate our high statistical goal of \(10^{10}\) events.
3.2.2 Separator

The separator is located further downstream just after the kicker. It is a velocity selector, that uses crossed electric field (E) and magnetic field (B) and permits particles for which $qE = qvB$ to pass through undeflected [60]. The slow muons pass through it as the E and B field are tuned accordingly, whereas the fast electrons are diverted to a collimating slit. This produced a muon to electron ratio of 15:1 in the muon beam as measured by the $\mu$SC. The working voltage of the separator was 180 kV. This voltage was attained by increasing it gradually in small steps to prevent dangerous sparking as a precautionary measure.
Figure 3.5: Time distribution of the kicker signal, showing the kicker on and off times. When the kicker is on the beam is blocked as shown. This plot shows an extinction factor of about 100.

3.3 Muon Entrance Detectors

After exiting the beamline the muons initially pass through a 75 $\mu$m Mylar window and then encounter the entrance scintillator to precisely record the muon entrance timings and collimate the beam. Further downstream is a wire chamber that gives spatial resolution at the millimeter level. This enables a fine tuning of the beam position along the optical axis via the tuning magnets, slit widths, and slit positions.

3.3.1 $\mu$SCA

The $\mu$SCA is a 1 mm thick scintillating veto counter, thus acting as an anti scintillator. It has a 35 mm diameter circular hole in its center [71]. It is triggered, when the muon is sufficiently divergent from the optimal axis of the beamline and would fail to reach the TPC. On its downstream side it has a lead lining that is thick enough to absorb these low energy divergent muons. This causes the disappearance rate of unwanted divergent muons at a much faster rate compared to muon decay rate. This is due to the fact that the capture rate in lead is $12 \times 10^6$ s$^{-1}$ [74]. Thus the $\mu$SCA reduced background, monitored backgrounds on the upstream side of the target and helped in collimating the beam. An optimal beam width tuning kept the $\mu$SCA-detected losses at a percent level or below.

3.3.2 $\mu$SC

The next detector following downstream is the $\mu$SC, which is a 0.25 mm thick scintillator. It provides the 'start time’ or 'entrance time’ signal for the muon lifetime
measurements. It has precision at the nanosecond scale with discriminators, and sub-nanosecond scale when fitting digitized signals. Times in all detector sub systems (e.g. muon stops in the TPC, electron detector hits, and neutron events etc.) are all measured with respect to the signal from the μSC. Besides this, the μSC provides a trigger for the electrostatic kicker as discussed earlier. This in turn helps define a pile up free clean muon entrance.

3.3.3 μPC

The μPC is a multiwire muon proportional chamber just downstream of the μSC. It covers 5 x 5 cm area that provides the spatial resolution of the beam, X being the horizontal position and Y the vertical position of the beam coordinates. It employs a 24 x 24 grid of anode wires perpendicular to the beam axis. These anode wires have a separation of 2 mm and are sandwiched between four cathode planes. At the entrance and exit of this detector are located two 50 μm thick Mylar windows to minimize the amount of scattering introduced by the μPC. It operates at -2.5 kV. The wire chamber is filled with a mixture of 49.9% Argon, 49.9% C₂H₆, and 0.2% Freon gases, maintained at a pressure of 1 bar [60]. Thus it monitors the effects of steering and focusing magnets, which helps in fine-tuning the beam’s optical axis with respect to the scintillators, TPC and other detector subsystems.

3.4 TPC

The Cryogenic Time Projection Chamber (TPC) is an ionization chamber maintained at a very low temperature filled with ultra-pure deuterium at a pressure corresponding to a density $\phi = 0.05$ relative to liquid hydrogen. There is a 2.5 mm thick hemispherical beryllium window of radius 3.25 cm on the upstream face of the TPC pressure vessel, thin enough to minimize the scattering of muons and yet thick enough to hold the deuterium gas safely at the desired pressure and temperature. The muons enter the TPC through this window and undergo the chemistry in the target deuterium gas as described in Sec. 2.2.

The TPC essentially consists of two components - a drift region and an ionization chamber. The drift region is defined by a cathode plane made of aluminium at the top and a Frisch grid composed of 50 μm tungsten wires, each separated by 1 mm, at the bottom. The shape of the cathode is chosen to maintain a uniform electric field between the cathode and the first wire of the grid. The grid is mounted on 4 ceramic insulators at a distance of 1 mm from the bottom of the ionization chamber that has an anode pad plane. The active area of the pad plane is around 105×120 mm$^2$, and the height of the drift region is about 100 mm. The upper cathode plane is provided with a potential of -80 kV (in order to provide a uniform drift field) and the grid is has a voltage of -3.5 kV (for field shaping) [67] during operation. These potentials produce an electric field of 8 kV/cm in the drift region, and the electrons drift towards the anode pad plane at the bottom of the TPC with a speed of $\approx 0.5$ cm/μs due to the electric field between the grid (at -3.5 kV) and anode plane that is grounded.
The aluminium cathode plane itself rests on four insulating MACOR supports which have grooves for fitting 1.5 mm stainless steel field-shaping wires. These wires are connected in pairs by 10 kΩ resistors in series with the topmost wire connected to the cathode and bottommost to the ground which creates a gradual but smooth potential difference over the volume of the TPC. This provides a uniform field on the volume of the TPC fluctuating by about 5%.

The grid is composed of a massive Kovar frame weighing 400 gm, cut from a 5 mm plate and wires soldered along the x direction (dimensions as mentioned above). It provides an electrical shielding to positive charge and permits the drift electrons to move from the cathode to the anode pad plane. An alpha source with a rate of about 30 emissions per second was placed on the cathode to calibrate the TPC. For a further improvement of the cathode plane performance a 100 µm silver layer was laminated on the cathode for Run 4 data taking. Silver being a high-Z material has a very short muonic lifetime of 87 ns and muons stopping in cathode could be easily distinguished from muons stopping in the center of the TPC.

The anode pad plane was extracted from a 2.5 mm MACOR base, with 48 copper plated pads each having a dimension of 17.5 mm x 15 mm. A thin layer of gold was chemically plated on the top of each pad along with a new set of improved flat Kapton cables for signal readout was employed in Run 4. This avoided confusion with muons stopping in the anode pads due to the short lifetime of gold muonic capture.
Figure 3.7: Shielding grid and pad plane - Left: Shielding grid 1 - Front bracket, 2 - Kovar frame, 3 - Tungsten wires, 4 - Connection holes for pad structure, 5 - Rear bracket, 6 - Insulator, 7 - Adjusting screw, 6 - Fixing screw. Right: Pad structure 1 - 50-pin connectors, 2 - Flat cables, 3 - Bar, 4 - Pad plane, 5 - Pin, 6 - Connecting hole. Image credit: [71].

[69]. The subsequent sub sections describe the gas purification subsystem, cryogenic subsystem, condenser, TPC structure etc. in more detail.

3.4.1 Gas Purification and Recirculation System

A major requirement of this experiment is the maintenance of gas purity to a level of ppb, which is similar to that of the MuCap experiment. This was accomplished using the CHUPS (Continuous Hydrogen Ultrahigh Purification system) for MuSun’s experimental requirements [58]. It was extended from MuCap and hydrogen was replaced by deuterium gas with other desirable modification specific to MuSun. This was extremely successfully developed and implemented by the PNPI Gatchina collaboration. Due to impurities (mainly \(N_2\), \(O_2\) and water) building up during previous runs, maintaining chemical purity at the ppb level requires continuous filtration. This was the main purpose of CHUPS.

The CHUPS is made up of three major parts: the compressor, the purifier, and the automated control system. Continuously circulating the gas at a sufficiently high
rate prevented buildup of impurities. This was efficiently done by the compressor that continuously pumped the deuterium, from the TPC to the purification filters, using mass flow controllers connected in series to the TPC vessel. It also acts as a cryopump, by first cooling the gas, and then warming it with electric heaters for extraction. The gas is absorbed using activated carbon that facilitates heat exchange. The gas is condensed with the help of liquid nitrogen and cooled carbon to precipi-
tate impurities. These impurities are filtered using synthetic Zeolites filters. A high pressure (100 bar) reserve volume (15 L) was maintained to stabilize possible pressure differences, and electric heaters were controlled remotely to regulate the temperature. Safety valves were included as a precaution to prevent adverse consequences due to undesirably high pressure, as the TPC is a highly sensitive device and operates at a very high voltage. Isotopic impurity was done using the cryogenic separation facility (Deuterium Separation Unit). Pure deuterium was produced with less than 1 ppm protium contamination with the help of this unit [1].

3.4.2 Cryogenic System

A target temperature of 30 K was accomplished by circulating 27 K liquid neon from a cold head and a massive copper condenser through a series of copper pipes surrounding the TPC vessel [71]. The condenser has several vertical sheets in order to increase the heat transfer surface and is connected to the cold head with a thin indium foil to ensure better heat conduction. The condenser temperature is controlled by an electric heater, which helps in fine tuning and optimally adjusting the temperature. This entire cooling system is connected to a 40 L reservoir and filled with neon up to a pressure of 5 bar at room temperature. Liquid neon drains to the bottom of the cell, exiting through two copper bellows that surround the TPC within the vacuum shell.

Neon evaporates as soon as it reaches the TPC head exchanges (that contain deuterium) and rises back into the condensing cell via the end of the copper pipes, in gaseous state. This enables a natural circulation of neon in the cryosystem. This entire system is enclosed by a high vacuum (of $2-4 \times 10^{-7}$ mbar) insulation vessel, with 3-4 layers of aluminized Mylar shielding to reflect infrared radiation. All this had to be done inevitably to prevent and insulate the system from the outer heat transfer. It takes 12 hours to cool the entire system from room temperature.

3.4.3 TPC Vessel

The TPC Vessel is a custom built cylindrical shell with 2.5 mm thick aluminium walls sufficient to withstand 15 bar pressure under cryogenic conditions and thin enough to permit the passage of outgoing decay electrons with a minimal energy loss. The upstream face has a beryllium window for muon entrance and the downstream face has a stainless steel flange. It has an outer diameter of 20.2 cm and an approximate length of 36 cm. The end caps were connected to the shell by indium seals to provide dismountable connections between different materials and also avoids leakage under cryogenic conditions.

There are receptacles in the downstream flange for connecting the detector’s signal cables and voltage feedthroughs that provide an electron drift field [71]. The signal cables are made of Kapton that connect the pad plane of the detector to an external preamplifier for readout signals from the TPC. A port at the bottom of the flange enables the flow of deuterium from the CHUPS system.
Both the aluminium shell, the beryllium window were carefully tested in vacuum through a range of temperatures and pressures in accordance with PSI safety standards. In general, the TPC could withstand a maximum stress value at 2-3 times the 10 bar MuSun running conditions, which was a safe enough range for the entire experiment, including a wide range of pressure, temperature and density variation. This was essential to check as it is desirable to have data that scans over a wide range of temperature, pressure and density for impurity studies and other details of muon catalyzed fusions, etc.

3.5 $\mu$SR

According to Sec. 2.2, the negative muons get captured by matter and eventually lose their polarization. But on the other hand nuclear matter cannot capture positive muons. Instead around 60% of $\mu^+$ are captured by an unpolarized electron and subsequently form a bound state called a muonium (generally denoted by Mu) that behaves as a light radioactive hydrogen isotope. The $\mu^+$ behaves as a light proton with the electron around it thus, mimicking a hydrogen atom. In a Mu the spins of the electron and muon are either aligned or anti aligned forming a triplet or a singlet state respectively. In the triplet state the Mu precesses around the external magnetic field. In the singlet state the muon rotates around the electron, ignoring the external field. This is due to the fact that the magnetic moment varies inversely as mass and since the electron is much lighter than the muon, the magnetic field of the electron is much larger compared to the external field, where as in the triplet state the electron and muon are locked together and so the muon cannot be separated from the electron. The remaining muons that do not form Mu undergo a free muon decay. Eventually all muons would decay, but the effects due to the formation of Mu and retention of polarization must be taken into account.

To systematically control the precession, Run 4 of MuSun installed a $\mu$SR magnet which had a magnetic field of about 50 Gauss in the fiducial volume of the TPC, vertically perpendicular to the beam axis, for $\mu^+$ measurements. A steady current of 125 A was applied to maintain an approximately uniform magnetic field. Again to be consistent this magnet was installed and effective throughout the experiment both for $\mu^+$ and $\mu^-$ beams. This would remove the possibility of retaining any residual polarization of $\mu^- d$ atoms. The $\mu$SR magnet was essentially a paired saddle-coil magnet with extremely light windings to reduce scattering of electrons, located just outside the TPC. Each pair was made up of eight hollow aluminum tube windings, with cooling water flowing through them. The $\mu$SR magnet’s power supply was filtered using a 160 mH inductor and a 40 mF capacitor. To ensure safety, precautions were taken to make sure that the power supply was interlocked with the water pump providing the cooling water, and temperature sensors were placed on both magnets. To keep a check on the external conditions, two Hall probes, a humidity sensor, and four temperature sensors, were placed at different locations on and around the magnet, and were monitored by the Data Acquisition system (DAQ).
3.6 Electron Detectors

One of the major tasks of MuSun is to have an excellent electron detecting and tracking system. This detector system was inherited from the MuCap experiment with further improvements enabling a more proficient tracking system. The electron detector system essentially consists of a barrel hodoscope surrounding the TPC - two multiwire proportional chambers, the ePC1 and ePC2 for tracking the trajectory of the electrons and a segmented scintillator eSC for the time measurement of the electrons. Owing to the cylindrical symmetry of this detector system, measurements were made in terms of the azimuthal angle $\phi$ and $z$-coordinates as shown in Fig. 3.11.

![Figure 3.10: A view of the electron detectors along the beam axis. The two cylindrical concentric multiwire proportional chambers, ePC1 and ePC2 are shown. The electron scintillators (eSCs) and PMTs surround the ePC. They are installed radially at positions numbered 1 to 16 in this figure. Image credit: [60].](image)

The analysis uses coincidences from all these three systems to reconstruct the electron track and information of electrons originating in all directions from the TPC. Since the upstream and downstream parts were not covered by the cylindrical electron detector system, it has a solid angle coverage of about $3\pi$ for electrons emanating from the TPC. The maximum kinetic energy carried by these electrons is half the muon mass and the thickness of the TPC vessel was designed in such a way that there was a minimum loss of energy of these electrons. The loss in coverage with respect to $\approx 75\%$ may be due to various factors like detector inefficiencies, inappropriate TPC thickness causing the electrons to stop or scatter.
3.6.1 Electron Wire Chambers

The wire chambers were adopted from the MuCap experiment. The chambers are multiwire proportional chambers, ePC1 and ePC2 with radii of 19.2 cm and 32.0 cm, respectively. They consist of helical cathode strips with a spacing of 6 mm on each side wrapped with opposite helicity. Sandwiched between the cathode strips is anode plane that has wires running parallel to the beam axis, thus having a constant \( \phi \). The inner anode layers serve to resolve the azimuthal angle \( \phi \) whereas the temporal coincidences between anode and the criss cross cathode strips are used to find the position \( z \) in the cylindrical coordinate system. The space between the anode wires is 2 mm.

![Diagram of the ePC1 wire chamber](image)

Figure 3.11: Wire chamber ePC1 showing the anode and cathode strips. Image credit: [60].

This configuration determines a unique spatial position of an electron by forming temporal coincidences within 30 ns resolution of both cathodes and anodes which in turn aids in determining the intersection of \( \phi \) and \( z \). The anode of ePC1 has 512 wires and both the cathode planes have 192 strips. The anode of ePC2 has 1024 wires and both the cathode planes have 320 strips. Both these chambers are filled with a mixture of 49.9% Argon, 49.9% \( \text{C}_2\text{H}_6 \), and 0.2% Freon gases (the same as the \( \mu\text{PC} \)). The voltage across the cathode wires range from 2.5 and 3 kV.

Several tests and improvements were made on the ePCs for Run 4 [71]. The wire chambers were reassembled and were tested to full high voltage up to 3 kV employing a dense target and a \( \mu^+ \) beam along with cosmic particles, in February 2010 to make them further suitable and efficient for the MuSun experiment.

It was essential to take care of noise in the chamber, especially in the cathode strip readout. Since there are two planes of criss cross cathode strips, the signal to noise ratio is much worse than on the anodes. The cathode amplifiers detected an additional unwanted periodic signal from the digitizers which further enhance the noise. Increasing the discriminator thresholds above this pickup did not help much as the
noise still persisted. Further modifications were essentially incorporated before the run to get rid of this.

There was a problem of oscillation of the connectors in the readout circuits for the cathodes, when not connected. Due to aging of the connectors, the cable connection pins did not make good contact, causing oscillations. Thus, output pins on these circuit boards were replaced which drastically reduced the oscillations and greatly improving the number of stable, active channels [69].

### 3.6.2 Electron Scintillators

The electron scintillating counters (eSC) have a 38.6 cm radius and surround the ePCs. There are 16 eSC segments (or “gondolas”), numbered counter-clockwise from 1 to 16 for easy reference (numbered in Fig. 3.10). Each segment has two scintillating layers (an inner layer and an outer layer with respect to the TPC). Each gondola is 5 mm thick, and each inner and outer pair are placed in such a way so that outgoing electrons pass through both of them. This determines precise time coincidence up to a resolution 1.25 ns. Further each gondola has a light guide and photomultiplier tubes (PMT’s) attached to its two ends the upstream and downstream with respect to the beam direction, which gives them a shape of a gondola and hence the name (shown in Fig. 3.12). Thus there are four readout channels for each segment (or gondola). For convenience they were referred as IU (inner upstream), ID (inner downstream), OU (outer upstream) and OD (outer downstream). So the eSCs had 64 readout channels, IU, ID, OU and OD corresponding to the 16 gondolas.

These two inner and outer gondolas having four PMTs (corresponding to their upstream and downstream ends). When they all coincide they form what is ideally called a “4-fold coincidence”. Such a “4-fold coincidence” helps in accurately determining the electron’s track. The 16 gondolas thus needed 64 readout channels, which were connected to a threshold discriminator and further coupled to TDC (time to digital converter) modules forming a highly precise read out system. A fine tune calibration of high voltages (HVs) was set for each of these read out channels individually owing to the the response of their respective PMTs. A range of 1.3 - 1.7 kV was found to be an optimal value for best performance.

### 3.7 Neutron Detectors

The MuSun experiment used eight liquid scintillator neutron detectors for detecting all neutrons, which were installed in four pairs forming an array around the electron detector system in specific locations (labeled as 3, 6, 11 and 14 in Fig. 3.10) in the array of eSC (or gondolas). Thus, they were named NU3, ND3, NU6, ND6, NU11, ND11, NU14 and ND14, where ‘U’ means upstream and means ‘D’ downstream. They were placed in derlin holders and installed in their respective slots i.e. positions 3, 6 11 and 14 as shown in the left panel of the Fig. 3.12. Their distance from the center of the TPC is around 41.9 cm, thus contributing a solid angle coverage of about 4.5%. Finally owing to detector efficiencies, gamma rays interference (as mentioned above) and various other factors, only 1% of neutrons were detected. The
neutron counters are essentially cylindrical cells of 13 cm diameter and 13 cm depth and contain approximately 1.2 liters of BC501A organic liquid scintillator (which is a hydrocarbon called xylene) connected to a photo-multiplier tube (PMT). An example of this detector is shown in the right panel of Fig. 3.12. But NU11 and ND11 were home made detectors build by our collaborators at Regis University.

To protect the PMTs of the neutron counters from stray magnetic fields the best option was to shield them appropriately using mu-metal. Mu-metal is nickel-iron alloy composed of 77% nickel, 16% iron, 5% copper and 2% chromium or molybdenum. It has very high permeability and coercivity, as a result of which the surrounding magnetic field passed though it easily and quickly saturated [59]. Several desktop experiments were done to find the best configuration that would restore the initial gains of the detectors and retain their original ability to perform a good PSD. Mu-metal sheets about 0.16 mm thick were used for shielding the detectors. There was no space to wrap the neutron counter with mu-metal and then insert them into the delrin holders. So we had to wrap the delrin holders instead and put them in the neutron counter slots. Measurements were made to find the right size of the mu-metal sheets required to wrap the holders and these sheets were then cut according to the measured dimensions. Immense care had to be taken in doing all this, as the edges of these sheets were extremely sharp. The configuration of shielding discussed above is illustrated in Fig. 3.13. Thus, care was taken to make sure that the data set analyzed was after the installation of \( \mu \)SR magnet and the detectors were all shielded with mu-metal with HV reset for appropriate performance.
Figure 3.13: A single mu metal sheet around the PMT and scintillator cell, extending up to the “Gondola” $e^-$ scintillator detector, provided optimal shielding against the stray field of the $\mu$SR magnet.
## 3.8 Front End and Electronics

Each detector system was read out by their respective frontend electronics and PCs and then saved in data banks in backend PCs before finally saving data in hard drives for mass storage. This flow diagram is shown in Fig. 3.14. Listed below is each detector system readout by the corresponding electronic system:

1. \( \mu SC / \mu SCA \): The signal from these counters goes into a NIM crate, where it is actively split and routed to 8-bit 500 Hz waveform digitizers (WFD) and discriminators (CAEN TDCs). The WFD stores the waveform pulse shapes of the \( \mu SC \) and \( \mu SCA \), with the single time value output from the discriminated signal [60]. The discriminator is triggered below the amplitude for muons, but above that of electrons. This output is read by the TDCs, and is used as a final signal input for other parts of the experiment. Multiple copies of \( \mu SC \) are created to compare against one another as an integrity check.

![CryoTPC, DAQ](image1.jpg)

![Frontend electronics](image2.jpg)

![Slow Control PC](image3.jpg)

Figure 3.14: A schematic diagram showing front end electronics and data acquisition.

2. \( \mu PC \): The signals from this is fed to the discriminator and read by 48 dedicated CAEN TDC channels for the 24 anode wires corresponding to the X and Y planes respectively.

3. TPC: The drift electrons in the TPC get accumulated on a 6 x 8 segmented pad plane and produce capacitively charged signals that are read by a 3-card preamplifier, located at the base of the TPC vacuum flange. This output is then fed into three 16-channel shaping amplifiers, increasing an average muon pulse from 5-10 mV to about 80-120 mV [60]. This is then digitized by the WFDs that were originally designed for the MuLan experiment, with a sampling speed...
of 25 MHz. A typical waveform of WFD islands consisted of 88 samples, with 25 presamples from the region before the trigger signal. [69].

4. ePC: Both the ePCs are processed with the help of the custom-built multi-hit TDC modules developed by UC-Louvain and referred to as compressors [61]. Each ePC wire is connected to these TDCs. Electrons passing through the ePCs trigger multiple hits in each of its three layers, that results in production of enormous data in a very short interval of time. The TDCs are customized to efficiently compress these hits before they are passed on to the DAQ for storage [69].

5. eSC: Signals from the 64 PMTs are passively split between an array of discriminators recorded by the CAEN TDCs and digitized by the MuLan WFDs. A 500 MHz clock frequency records waveforms with 2 ns time bins, providing high time resolution pulse shapes.

6. Neutron Detectors: The neutron counter signals were digitized using eight 12-bit FADC channels (Flash Analog to Digital Convertors). Each FADC channel had a maximum capacity of 11-bits and the last bit was used to determine an overflowing signal. The data sampling speed of these FADCs was 170 MHz. These were customized WFDs originally developed by the Berkeley and Louvain groups for the MuCap experiment.

3.9 Data Acquisition System

The frontend systems described above collect all digitized output of each detector system and are connected to dedicated front end computers located in a barrack in the vicinity of the beamline. This is the central data acquisition system of MuSun, which relies on the standard PSI DAQ framework, with enhanced customization for MuCap and is called the Maximum Integration Data Acquisition System (MIDAS). Data was saved in MIDAS run files of size 1.6 GB, and each run takes about 5 minutes at optimal running conditions. The completed sets of MIDAS run files are then imported to other offline machines for a quick on-site analysis and diagnosis. The data is backed up on local PSI archives, and then saved on 2 TB hard disk cradles were connected to the backend PC over an eSATA interface. Later, the files were transferred to mass storage servers at the Lonestar supercomputing facility in Texas [69]. The DAQ had user-friendly control interfaces to access various detectors and other components from the barrack. It could also help control systems for the separator and beamline magnets, and voltages for the neutron detectors, wire chambers, PMT etc. The TPC conditions, namely temperature and pressure, are also actively monitored via a remote terminal in the barrack, which was developed by the PNPI group.

3.10 Run Summary

This thesis is based on the analysis of Run 4 where $5 \times 10^9 \mu^-\text{ events}$ were collected, which is about half of our overall statistical goal of $10^{10}$ events. About $5.6 \times 10^8 \mu^+$
events were collected in this run. The beam was tuned separately for $\mu^-$ and $\mu^+$ data runs. A total of 31.6 TB of data was collected in the entire run period, with

![Accumulated Statistics for R2011](image)

Figure 3.15: Data accumulation of production Run 4. The vertical axis shows the number of $\mu$ events collected. Image credit: [69]

approximately 23.7 TB of good quality data [69]. Nearly 21.2 TB was $\mu^-$ data and the rest was $\mu^+$ data. The accumulated statistics is shown in the Fig. 3.15. Efforts were made to ensure that the gas was both chemically and isotopically pure within the systematic limits. More details about the quality and analysis of the data of this run will be investigated thoroughly in the next chapter.
Chapter 4

Data Analysis of Muons and Electrons

This chapter deals with the data collection and analysis of the MuSun experiment. An overview of the experiment and software analysis of all detector systems, except the neutron detectors will be covered in this chapter. I have dedicated separate chapters on neutron data analysis as that forms an integral part of my research. The ultimate goal of the software analysis discussed in this chapter is to find the muon disappearance rate. Combining muons and electrons detected by their respective detector systems forms muon-electron pairs and then the muon decay time spectrum is fitted to find this disappearance rate.

4.1 Software Overview

Data was collected via the frontend electronics and various detector systems discussed in the previous chapter. The software analysis was done in two stages. The first stage was called MU and the second stage was called MTA.

In the first stage i.e. MU level of the software three files were generated from the Midas input file which were as follows
(i) two files (≈ 0.05 GB each) containing histograms of all the detector systems for diagnostic purposes,
(ii) the file (≈ 1.6 GB) containing ROOT tree files corresponding to each detector system along with important attributes (like peak time, energy etc.) of pulses from each detector.

The data collected was stored in blocks of about 2 ms each called a MIDAS event in MIDAS banks for each detector system at the MU level. A parameter called block time was defined for each detector system to refer to the time of occurrence/arrival of a particle (like muon entrance) relative to the particular block (or MIDAS event). The root tree file generated from the output of MU was used as the input for the analysis in the second stage i.e. MTA. The major difference between the MU and the MTA level is that data is processed event by event in MU, and saved in event trees for MTA to process it muon by muon. Thus MU converts data organized in 'events' to 'muon' hits. Digitized pulses are assigned with attributes like hit time, block time, amplitude, pedestal, etc. at the MU level.

It was ensured that a single muon enters the TPC in a time window of ± 25000 ns (pile up protection). The TDC data is compressed in the MU stage after fitting and
removing unwanted pulses and so redundant data is lost in this level. Clusters and tracks for TPC pulses and electrons are formed in the MTA level.

The online and offline data analysis were needed to monitor that good quality data was taken for the entire production period. The offline analyzer was essentially the same as the online, except that the online analyzer were operated for supervising only a small part of the entire offline data that was collected - just to ensure that every detector system was running smoothly.

4.2 Software Blinding

The clock signal for the experiment is derived from a 500 MHz Agilent synthesizer called the master clock. The output is split, routed to both the high frequency scintillator WFDs and the master logic in the MuSun barrack. Excepting the WFDs (both TPC and eSC), all other electronics clock signals are derived from this master logic. TPC digitizers reduces the signal to get an effective 25 MHz clock frequency [60]. Data was collected at a sampling rate of 170 MHz derived from the same master clock for the neutron detectors. The master clock frequency was blinded within a range of 498.5 MHz to 499.5 MHz by a PSI scientist who was not involved in the MuSun experiment, and was checked by the same scientist every week [72]. This purpose of this blinding is to analyze data and perform studies on systematic errors without any bias.

4.3 Experimental Overview

This section briefly describes the entire Run 4 of the MuSun experiment. The first few weeks were spent preparing the DAQ, various detector systems, taking calibration runs, background runs, etc. Actual production data taking started at the beginning of July [69]. A few problems encountered and taken care of either during the run or prior to the run for better performance are listed below.

TPC:

The cold head of the TPC had a tendency to produce significant acoustic vibrations. In the 2011 run, each channel of the shaping amplifiers was equipped with a baseline restorer so that they do not pick up these acoustic signals [69]. Consequently, the acoustic noise was remarkably reduced and the energy resolution increased by factor of 1.5 as shown in Fig. 4.1.

A smart trigger was designed by the Boston group. This was a sophisticated firmware to address the effect of noise on the trigger, without changing the signal itself. This helped in minimizing the effect of any residual acoustics, by running the samples through a real-time digital filter, which eliminated oscillations in the frequency region of 0-7000 Hz [69].
Figure 4.1: Effect of baseline restorer. The left panel shows the acoustic noise in a channel of the TPC before using a baseline restorer. The right panel shows the effect of using the baseline restorer which eliminates the noise. Image credit: [69].

**Neutron Detectors:**

Some large energy pulses that appeared to be very noisy and distorted in shape were observed in some neutron counters. These were always preceded by decay electrons and were perceived to be afterpulses in detectors that were old [58]. As a result

Figure 4.2: An afterpulse in the neutron counter caused by an earlier signal electron that deposits a large energy.
of aging, the vacuum in them got worse causing a small leakage of molecules that eventually got ionized producing electron ion pairs. The electrons were detected as the preceding gamma rays and the ionized molecules give rise to these strange pulses or afterpulses in these detectors. An example of an afterpulse is shown in Fig. 4.2. The old Bicron detectors (NU3, ND3 and NU6) and the home made detectors (NU11 and ND11) suffered more from these afterpulses. These were eliminated in the software analysis by applying appropriate energy cuts.

Period of beam shut down:

There was a beam shut down period towards the end of the run. During this shut down the muonic X-ray detector was installed which again had a negative impact on other detector systems. The hits in the entrance wire chamber increased along with occasional data losses encountered by the WFD used to read eSC data. All this was resolved by re-cabling and well grounding the X-ray detector. During the shut down lots of neutron/gamma source data was collected for calibration of the neutron detectors and for investigation of neutron / gamma pulse shape discrimination (PSD). Also a lot of cosmic ray data was taken for effective background studies. Data was also collected with variations in density and temperature of deuterium gas for investigating the effect of impurities on the signal [69].

4.4 Beam

The proton beam for run 4 was quite stable ranging from \( \approx 1.7 \text{ mA} \) to \( 2.3 \text{ mA} \), which subsequently caused a fluctuation in muon rate ranging from \( \approx 21 \text{ kHz} \) to \( 25 \text{ kHz} \). A good kicker extinction was achieved and maintained during the experiment. An average extinction of \( \approx 90 \) was achieved for \( \mu^- \) beam at a rate of \( 25 \text{ kHz} \). For the \( \mu^+ \) beam at a rate of \( \approx 30 \text{ kHz} \) the average extinction was \( \approx 35 \). The kicker was turned on in both cases for \( 25 \mu s \).

4.5 Muon Detector Analysis

A muon beam passes through the \( \mu \text{SC} \) and \( \mu \text{PC} \) before entering the TPC. Since the beam detectors and entrance counters were inherited from the MuCap experiment the software controlling these were also inherited from MuCap. Detailed analysis can be found in MuCap theses of Tom Banks [61], Steve Clayton [47] and Brenden Kiburg [62]. The muon entrance time \( t_\mu \) is determined by the hit time in the \( \mu \text{SC} \) counter. A window corresponding to a \( 25 \mu s \) time interval following a muon entrance was the main measurement period that was pile up protected.

Procedure to determine muon entrance times:

An artificial deadtime (i.e. no other hits in that detector are accepted for 29 ns after a hit) of 29 ns [62] was imposed on the \( \mu \text{SC} \) to prevent afterpulses. A temporal auto-correlation plot (a plot of the muon signal with itself is called an auto-correlation plot)
for the raw $\mu$SC signal (in black) and the signal after applying the deadtime (in gray) is shown in Fig. 4.3. This helped in cleaning the signal from the initial afterpulses. All coincidence and pile up protection conditions are applied to this clean signal.

![Figure 4.3: An auto-correlation time distribution for the raw $\mu$SC hits is shown in black. A software deadtime window of 29 ns shown in gray / blue is applied to get rid of the afterpulses. Image credit: [62].](image)

Since the $\mu$PC wires are 2 mm apart a muon passing will hit several adjacent wires. To prevent afterpulses a software deadtime 300 ns is imposed [60]. A $\mu$PC cluster is defined as muon hits that have a temporal coincidence window of $\pm 229$ ns and spatial coincidence of wires separated by $\leq 1$ (i.e. if the hits were on adjacent wires or had only one wire gap in between). These coincidences are defined for both $\mu$PC-x and $\mu$PC-y. In case of a gap of two or more wires for a muon hit two distinct clusters were constructed. The cluster time was found from the first hit in the cluster and the average position of all the wires involved in the hit determined the cluster position. Finally $\mu$PC-x and $\mu$PC-y clusters were paired using a temporal coincidence window of $\pm 190$ ns. The distribution of these clusters the muon beam is shown in Fig. 4.4 for run4 [71].

$\mu$PC clusters are combined with $\mu$SC in a temporal coincidence window given by $-140$ ns $< (t_{\mu PC} - t_{\mu SC}) < 120$ ns [60] to form a muon entrance object. The muon entrance time is given by $t_u = t_{\mu SC}$ as the $\mu$PCs have a poor time resolution compared to the $\mu$SC. The $\mu$PCs define the position of the muon entrance object. An array constructed from the logical OR of the $\mu$SC, $\mu$SCA, $\mu$PC-x clusters and $\mu$PC-y clusters was used to enforce pileup protection [62]. Only one muon hit was permitted in the pileup array within a pileup protection window of $\pm 25$ $\mu$s. Finally only the muons with all the above-mentioned conditions were considered as muon entrance objects.
4.6 Time Projection Chamber - Analysis

The basic TPC structure was inherited from MuCap (except the wire and strips were replaced by a pad plane structure for MuSun). Thus the framework of the analysis modules were subsequently inherited but modified to be compatible with the improved readout system. MuCap used custom built time-to-digital converters TDC400 for digitization and readout of pulses from the TPC, which were replaced by BU 8-bit waveform digitizers (WFDs) in MuSun. MuSun has full analog readout, whereas MuCap read just the hit times due to the huge data rate of the TDCs. Thus in MuSun the topology of the readout plane differs, and charged fusion products in the deuterium environment present a source of signals correlated in time with the muon signal [71].

4.6.1 TPC Pulse Finding and Fitting Algorithm

A more robust analysis was developed for muon identification and muon stops. The muon stop signals in deuterium had to be clearly distinguished from the charged fusion products (protons, tritons and $^3$He). Thus TPC pulses were formed from signals read by the pad plane. Fitted pulses were combined into clusters and tracks to finally identify muon stops and fusion products. A muon entering the TPC, ionizes the deuterium gas producing electrons that the TPC anode pad plane detects as a signal. The WFDs digitize these analog signals as samples (or ADC counts) such that each ADC count corresponds to 4 mV and stores them. Successive samples called

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Figure 4.4: $\mu$PC-xy clusters. The x-y coincidences producing a profile of the incoming muon beam. Image credit: [71].

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Figure 4.5: A TPC pulse generated by drift electrons. An ADC count is equivalent to 4 mV. Image credit: [60]

Figure 4.6: A TPC pulse fitted by a template (in thick red) and a Gaussian (in thin green or dotted gray) is shown in the left panel. The right panel shows the $\chi^2$ distribution from both these fits - Gaussian is shown in red (dotted gray) line and the pulse template in solid black line. Image credit: [69].

islands comprising of about 88 clock ticks (clock sampling speed of the WFDs was 25 MHz) is long enough to capture a muon pulse.

If an island had 3 or more consecutive samples above a certain threshold ADC count it was considered to be a pulse [68]. Such a pulse is shown in Fig. 4.5. A pulse is characterized by its time (on the island), pedestal, amplitude (pedestal subtracted height of the pulse), and sigma (width of the pulse). The pedestals and thresholds
are adjusted for each run. The pedestal is calculated from the mean of the first five samples [60]. The TPC had a much greater signal to noise ratio compared to other detector systems and so the threshold is selected to be 5-8 ADC counts above the pedestal to avoid noisy pulses. A detailed study about noise, thresholds, triggers, pulse finding and pulse fitting etc. is well explained in the thesis of Justin Phillips [60].

After a pulse was identified, a least square fit was performed with a custom pulse shape using a gradient based, chi-squared minimization procedure [60]. The custom pulse shape was a pulse template formed an average of a large number of pulses scaled to unit area and then centered about a common pulse time using a very fine time resolution [69].

This procedure was developed by the muon g-2 experiment at BNL and is also used for fitting the neutron detector pulses. The resulting template shape is not a Gaussian although a Gaussian distribution function can also be used as the fit function. But the pulse template yielded a better chi-squared distributions as shown in the right panel of Fig. 4.6. The left panel shows a pulse fitted by a pulse template (in thick red) and a Gaussian (in thin green) overlaid for comparison. The parameters returned by the fit function were the time, pedestal, amplitude and width or sigma of the pulse. If the fit failed to find a minimum in the specified tolerance than the pulse was tagged and if it is was successful, the relevant properties of the pulse like time, pedestal, amplitude, $\chi^2$, energy were saved.

The pulse fitting algorithm has some disadvantages. It has not been optimized for the separation of overlapping pulses or double pulses, which could be generated in case of fusions when the recoil $^3$He is produced promptly after the muon stop, such that the two signals overlap and look like double pulses. It also fails to fit the highly asymmetric pulses produced by protons. However, the clearly separated late $^3$He and triton signals can be well understood by the fit parameters of this pulse template [69]. The $^3$He signals being monoenergetic (0.82 MeV) were used for the energy calibration and the measurement of the energy resolution of the TPC.

4.6.2 Muon Tracking

Muon tracks were created from clusters which were a collection of pulses within 2 pads in the beam direction (Z), 1 pad along X - direction and 1 cm vertically (Y - direction). The drift time of electrons corresponding to a distance of 1 cm vertically was 2 $\mu$s [64] as their drift velocity 5 mm/$\mu$s. Thus the spatial cuts for creating a cluster can be summarized as follows:

$$\Delta X \leq 1; \Delta Z \leq 2; \Delta Y \leq 2\mu s(\equiv 1cm)$$ (4.1)

No energy cuts besides the hardware and software thresholds were applied to clusters. A track was defined as a cluster of length three or more pads along the beam direction.

The stopping power (energy loss per unit length = $-\langle \frac{dE}{dx} \rangle$) of the ionization energy given by (Bethe-Bloch equation) [70] for the non-relativistic classical limit can
Figure 4.7: Right panel shows the plot of energy versus range in units of pad length. The fractional distance the muon travels in the stopping relative to the pad length is $d$. Left panel shows the variation of S-energy for $d$ ranging from 0.1 to 0.9.

be approximately given by,

$$-\left\langle \frac{dE}{dx} \right\rangle = k' \frac{1}{\beta^2} = k' \frac{1}{E} \quad (4.2)$$

where $k'$ is approximately a constant in the classical limit for our energy range and $\beta = v/c \propto \sqrt{E}$, if the energy of the muon passing through the gas is totally its kinetic energy. Thus for an entire range $R$ of the muon we have,

$$R = kE^2 \quad (4.3)$$

A plot of the range versus energy is shown in the left panel of Fig. 4.7, where the range is plotted in units of pad length. Thus for a fractional distance of $d$ relative to the pad length a muon travels in the stopping pad, the energy ($E_0$) deposited is, $E_0 \propto \sqrt{d}$ and the energy deposited on the pad previous to the stopping pad ($E_1$) is $E_1 \propto \sqrt{d+1} - E_0$. Thus, we find that the sum of $E_0$ and twice $E_1$ called the S-Energy ($E_0 + 2 \times E_1$) is $\propto 2\sqrt{d}+1 - \sqrt{d}$. This is plotted in the right panel of Fig. 4.7 for $d$ ranging from 0.1 to 0.9 and is analytically found almost to be a constant.

The energy deposited on the stopping pad $E_0$ is either greater than $E_1$, corresponding to the Bragg peak (which is a prominent peak on the distribution of stopping power with $x$) when $d$ is large or much less than than $E_1$ when $d$ is very small (energy corresponding to the tail of the Bragg distribution). Thus it depends on the stopping position of the muon on the pad and is not a constant. The variation $E_0$ versus $E_1$ from a data run is shown in Fig. 4.8 where the straight line represents the S-energy of this run. Thus the S-Energy is empirically found to be a constant, though $E_0$ and $E_1$ can take any arbitrary values as shown in Fig. 4.9.

Event topologies of muons traveling along the edge of the TPC pad plane is complicated and cannot be distinguished from muons traveling beyond the active region of the TPC. Muon tracks very close to the anode pad plane or cathode also
need to be ignored as they most likely hit the anode or cathode. Thus a fiducial volume cut was essential to get clean muon stops in the TPC. This cut allows us to eliminate muon captures from the high Z materials near the walls and surroundings of the TPC. Different areas or regions of the TPC pad plane was considered for the fiducial volume cuts and were called the region of interest (ROI), the better box and the golden box. The ROI was the entire TPC volume. The better box was obtained
Figure 4.10: The fiducial volume cuts for the muon stop definition. Left panel shows the X-Z pad plane and the right panel shows the drift time spectrum for cuts along Y - direction. Image credit: [64].

<table>
<thead>
<tr>
<th>Fiducial Volume Cuts</th>
<th>X (mm)</th>
<th>Z (mm)</th>
<th>Y (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region of Interest (ROI)</td>
<td>±48.75</td>
<td>±64</td>
<td>0 to 71</td>
</tr>
<tr>
<td>Better Box (Default)</td>
<td>±36</td>
<td>±47.4</td>
<td>10 to 61</td>
</tr>
<tr>
<td>Golden Box</td>
<td>±23.25</td>
<td>±36.4</td>
<td>20 to 51</td>
</tr>
</tbody>
</table>

Table 4.1: Table summarizing the dimensions (X, Y and Z coordinates) of the various fiducial volume cuts.

by avoiding a single layer of pads on edges of TPC (except the beam entrance pads as shown in the left panel of Fig. 4.10). In this figure, the red (or gray) pads are prohibited for the muon to stop in a better box. A cut ranging from 10 mm to 61 mm in Y direction was applied (dashed line in Fig. 4.10) for the better box (i.e. 10 mm from the top and the bottom of the TPC were ignored). The golden box was defined by avoiding two layers of pads on edges of TPC and 20 mm from its top and bottom. To summarize the dimensions of all the fiducial volume cuts are listed in table 4.1

The default fiducial volume cut was taken to be the better box. The spikes on either side of the drift time spectrum shown in Fig. 4.10 are due to muon tracks that hit the cathode or anode. Additionally an S-energy cut greater than 300 channels (430 keV) is also applied on a muon stop condition. Thus to summarize a muon stop is from a muon track that traverses at least 2 pads in Z - direction (before stopping), within the fiducial volume of the TPC and having an S-Energy greater than 430 keV.

It is important to note that even a single pulse can be a cluster as shown in Fig. 4.11. Examples of clusters and muon stops are shown in Fig. 4.11 and the right panel uses the TPC event display to show clusters and muon stops. The X- axis in the event display is drift time of the pulse (pulse time relative to the muon entrance time) and the Y - axis is the anode pad number.
Figure 4.11: Clusters and muon stops. Left panel shows these on the pad plane with pad numbers and the right panel uses the event display to show these. Image credit: [64]

The track of an ideal muon stop is shown in Fig. 4.12. Each drift electron deposits energy on successive anode pads along the beam direction $Z$ and so $\Delta Z = 0$. The energy deposited along $Z$ also increases successively till it reaches the Bragg peak and this muon is well within the fiducial volume of the TPC. The stop length in $Z$ pad is 5 which is $\geq 3$. Also $\Delta X = 0$ and the difference in drift time is lower than 2 $\mu s$. Thus it depicts an ideal muon stop.
4.7 Electron Detector Analysis

The eSC, ePC1 and ePC2 are used to accurately find the decay electron time and their tracks. This is very important for the experiment as the final results of the MuSun experiment heavily rely on the lifetime fit of these Michel electrons. The minimal electron definition is a hit detected by one eSC segment (i.e. inner and outer gondola hits). The wire chambers i.e. ePC1 and ePC2 help in determining the track and position of the electrons along with background suppression. A complete electron track information and definition requires a coincidence between eSC, ePC1 and ePC2.

4.7.1 eSC Clusters and Electron Time

The electron time is entirely determined by the eSC (or Gondolas). The energy distributions of the gondolas are shown in Fig. 4.13. Each gondola has four plots shown in blue, green, red and black corresponding to the four channels IU (inner upstream), ID, OU and OD respectively. The final electron analysis is done at the MTA level in a module called GlobalElectronAnalysis.C [66]. An artificial dead time of 50 ns is imposed to avoid afterpulses in the PMTs [60]. If a decay electron emanating
Figure 4.13: Online display showing the raw energy spectrum of all gondolas for run 4 data. The four plots in eSC segment shown in blue, green, red and black corresponding to the four channels IU (inner upstream), ID, OU and OD respectively. Image credit: [63]

from the TPC passes through the inner and outer scintillators within a coincidence temporal window of 25 ns then a 4-fold coincidence corresponding to the 4 PMT times are constructed for a single eSC segment. Since the dead time is greater than the coincidence window, a Gondola (eSC segment) can have a maximum of only four hits corresponding to each of its PMT (which is the 4-fold coincidence). Besides this we can have a single, a 2-fold and a 3-fold eSC hit. Each such group is stored as a cluster to form electron tracks later. 2-fold coincident clusters are very common as the decay electrons cling to the inner layer of the eSC and cannot reach the outer layer. A hit timing with respect to the other PMT channels is adjusted by up to $\approx 2$ ns (due to slightly different cable lengths, etc.) to obtain the final eSC time [47]. The electron time is finally defined as the average of these four individual PMT times ($\bar{t}_{eSC}$) [62]. Finally the ($\bar{t}_{eSC}$, $\phi_{eSC}$, $z_{eSC}$) coordinates of the eSC cluster are saved. These eSC clusters in temporal and spatial coincidence with the two ePC clusters (described in
the next subsection) finally form an electron track.

### 4.7.2 Definition of ePC Clusters

An electron track is formulated with the help of an electron vector in the 3 dimensional space on a cluster within a temporal coincidence of ±200 ns of neighbouring wires. A lot of work has been done by the MuCap experiment’s software development and have been inherited in MuSun too, as there is no difference in the ePCs and eSC used in MuSun. An elaborate discussion can be found in Clayton’s thesis [47] and details of the current description are based on Brendan’s thesis [62]. An electron traveling through the ePC wires produces hits in them and certain constraints on these individual hits are grouped together into a cluster. There exist the following coincidences for forming clusters in the ePCs:

1. 3-fold coincidence between the anode plane and both the cathode planes determines \( z \) and \( \phi \) coordinates of the ePC hit.

2. 2-fold coincidence between the anode plane and any one of the cathode planes also determines \( z \) and \( \phi \) coordinates of the ePC hit.

3. Only a hit the anode plane is considered which helps in finding only the \( \phi \) coordinate of the ePC hit.

Cuts are applied on the allowed time windows between successive hits, allowed gap between wires (the hits should be on consecutive wires allowing a maximum gap of 2 wires as listed in table 4.2), a dead time on each wire and maximum number of wires hit by the electron (if the electron hits a number greater than this it is considered to be a spark). The table 4.2 shows the constraints applied to form clusters for each of the ePCs. The values of the constraints are initialized and read from the file `Parameters.cpp`. After applying the cuts of table 4.2 clusters are formed and the

<table>
<thead>
<tr>
<th>Cuts</th>
<th>ePC1</th>
<th>ePC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Window (ns)</td>
<td>( \approx 300 )</td>
<td>( \approx 300 )</td>
</tr>
<tr>
<td>Dead Time (ns)</td>
<td>1000.0</td>
<td>1000.0</td>
</tr>
<tr>
<td>Allowed gap in anode (# of wires)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Allowed gap in cathode (# of wires)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Maximum Wires</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4.2: The table shows the cuts applied for forming ePC clusters.

\((t_{ePC}, \phi_{ePC}, z_{ePC})\) coordinates of the ePC clusters are saved for final formation of an electron track.

### 4.7.3 Algorithm for Electron Tracks

Applying temporal and spatial coincidence to all ePC1, ePC2 and eSC clusters forms the final electron track. Ninety percent of the times we can match 4-fold eSC coincidences with exactly one ePC1 - ePC2 pair (this match is considered to be a good
electron track, provided it obeys all coincident cuts) and single, 2-fold, 3-fold eSC hits paring with at least two ePC1 - ePC2 pairs occur only in ten percent of the cases. The ePCs form the following classes of electron tracks:

![Figure 4.14](image)

Figure 4.14: The temporal and spatial coincidence windows used for track definition from clusters are based on these histograms. The left panel plots time difference, the middle panel plots the difference in $\phi$ coordinates and the right panel plots the difference in z coordinates for ePC1 and ePC2 clusters. Image credit: [66]

1. Anode only: Hits recorded by just the anode, we ignore the cathode information here.

2. Cathode - OR: Any one cathode is coincident with the anode.

3. Cathode - AND: Both inner and outer cathodes are coincident with the anode.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{eSC} - T_{ePC1}$ (ns)</td>
<td>0 to 300</td>
</tr>
<tr>
<td>$T_{eSC} - T_{ePC2}$ (ns)</td>
<td>0 to 300</td>
</tr>
<tr>
<td>$\phi_{eSC} - \phi_{ePC}$ (rad)</td>
<td>-0.35 to 0.35</td>
</tr>
<tr>
<td>$z_{eSC} - z_{ePC1}$ (mm)</td>
<td>-300 to 300</td>
</tr>
<tr>
<td>$z_{eSC} - z_{ePC2}$ (mm)</td>
<td>-300 to 300</td>
</tr>
</tbody>
</table>

Table 4.3: The table shows the cuts applied for forming tracks from clusters. These cuts are based on Fig. 4.14 and taken from the module `GlobalElectronAnalysis.C`

In the case of more than one ePC1 - ePC2 pair for a given 4-fold eSC hit, the code optimizes the best match based on the best fitting temporal and spatial cuts. In case there are many ways of associating ePC1 - ePC2 pair with a 4-fold eSC hit, the cut in $\phi$-coordinate is made more stringent to finally consider these clusters to be a track. In an extremely rare case when the combinations of ePC1 - ePC2 pair and a 4-fold eSC hit are too many, we ignore the entire cluster to prevent ambiguity and save computational time. The temporal and spatial cuts are based on the time windows shown in Fig. 4.14. These are listed in table 4.3
4.8 Summary

Thus, we conclude this chapter by summarizing a MuSun event. A MuSun event comprises of a muon entering the TPC with its time stored as $t_\mu$ and then stopping in the fiducial volume to either undergo a decay, a capture or a fusion. A decay electron detected by the eSC have times stored as $t_e$ and we form electron muon pairs for each event finally finding the $t_e - t_\mu$ i.e. electron time distribution and fit this histogram to find the muon disappearance rate essential to find $\Lambda_d$. The next chapters are dedicated to neutron data preparation and analysis of neutrons emitted from fusions and capture processes.
Neutron Data Preparation

This chapter describes the preparation of data from the neutron detectors. The large number of muon decay causes an overwhelming gamma ray and electron background. Thus, it is essential to be able to cleanly discriminate neutrons from gamma rays and electrons (we call this Pulse Shape Discrimination - PSD). The ability to discriminate neutrons from gamma rays relies on the scrupulous study of the pulse shapes of each. Finally, the procedure of energy calibration of the neutron counters using radioactive gamma ray sources $^{60}$Co and $^{137}$Cs will also be discussed here.

Figure 5.1: The dotted red plot is the result of interpolation using cubic spline which is superimposed on the raw pulse shown in a solid blue line.

Data from the neutron counters was read out using 12-bit FADC digitizers. It was then, saved in a derived MIDAS databank called $NDET$. The raw neutron data was saved in islands of consecutive samples around 70 samples long (i.e. about 420 ns) with most islands consisting of one pulse (there could be seldom cases of double pulses too). Data sampling was done by 170 MHz clock. Various parameters like the
number of samples, the sample values, the pedestal, the peak time, the total area and various tail areas were assigned to each pulse which are listed in the table 5.1. All this was done in the MU level of the software in a module called MNeutronFadcC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel</td>
<td>An integer from 0 to 7 identifying the neutron counter.</td>
</tr>
<tr>
<td>Number of</td>
<td>The number of samples on a single island recorded by the digitizer.</td>
</tr>
<tr>
<td>Samples</td>
<td>A C++ vector containing raw samples.</td>
</tr>
<tr>
<td>Peak Sample</td>
<td>The largest sample on an island or raw pulse.</td>
</tr>
<tr>
<td>Pedestal</td>
<td>The pedestal was defined as the median of the first three samples of the pulse.</td>
</tr>
<tr>
<td>Peak Time</td>
<td>The time corresponding to the peak sample $t_p$.</td>
</tr>
<tr>
<td>Block Time</td>
<td>Data is collected in time segments (about 100 ms long) called blocks or MIDAS events. The time of the first sample in the island with respect to the block start time.</td>
</tr>
<tr>
<td>Total Area</td>
<td>The pedestal subtracted area of the entire waveform. Defined as a time window from $t_p - 60$ fine bins to $t_p + 400$ samples.</td>
</tr>
<tr>
<td>Various Tail Areas</td>
<td>The pedestal subtracted area of the tail of the pulse. Several time windows are defined for a tail. For e.g. tailArea_100_400 is defined as a time window from $t_p + 100$ fine bins to $t_p + 400$ fine bins (a fine bin is defined in Sec. 5.1).</td>
</tr>
<tr>
<td>Overflow</td>
<td>A Boolean returning true when the pulse is outside the 0 to 1.2 V range of the FADC input.</td>
</tr>
</tbody>
</table>

Table 5.1: The table lists various attributes of a waveform pulse.

The TMusunNeutronPulse module then saves all the properties described in table 5.1 in the MuSun Event Tree, which helps in forming ROOT trees saved in the tree files. This tree file is finally used as an input for the analysis in the next stage i.e. the MTA level (explained in Sec. 4.1), where the majority of neutron data analysis has been done. This TMusunNeutronPulse module, also saves some additional useful parameters like threshold energies (minimum energy of PSD), cut off energies (maximum energy for PSD) and energy gains for each neutron detector. Data preparation, production of various time, energy spectra, neutron/gamma pulse discrimination, definition/event selection of fusion neutrons, capture neutrons etc. were all done in this level of analysis.

5.1 Time Definitions

Raw pulses were interpolated using cubic spline resampled by a factor of 20. A cubic spline is constructed from piecewise third-order polynomials passing through a set of control points called knots [75]. The knots were chosen in such way that, the interpolation function was continuous across the boundaries of the knots. The polynomial was defined and evaluated using the TSpline3 object of ROOT. The clock
sampling speed was originally $\approx 6 \text{ ns}$ and this interpolation helped us assign sample values for 0.3 ns interval between the clock ticks, which was called a fine bin. An example of a raw pulse (solid blue line) and the resulting waveform after applying cubic spline (dotted red line) is shown in Fig. 5.1. Two times were used in our analysis relative to which the windows for total and tail areas of the pulse were defined. These were the peak time and half-height time defined as follows:

1. Peak Time: The finely binned time $t_p$, corresponding to the peak sample after interpolating the raw pulse using TSpline.

2. Half-Height Time: The finely binned time $t_h$, corresponding to half the height of the peak sample after interpolation.

Figure 5.2: Time windows showing definitions of total and tail areas relative to peak time (left panel) and half time (right panel). The windows of total and tail areas shift a little relative to the half-height time.

All definitions of tail areas listed in table 5.2 were defined with respect to the peak time $t_p$. New definitions of tail areas were redefined with respect to half-height time (due to the shift in windows). The left and right panels of Fig. 5.2 show these windows for peak time and the half-height time respectively. The windows used in table 5.2 got shifted for half-height time due to the shift in $t_h$ relative to $t_p$. 
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Time (Fine bins)</th>
<th>Final Time (Fine bins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>tailArea_60_400</td>
<td>$t_p + 60$</td>
<td>$t_p + 400$</td>
</tr>
<tr>
<td>tailArea_80_400</td>
<td>$t_p + 80$</td>
<td>$t_p + 400$</td>
</tr>
<tr>
<td>tailArea_100_400</td>
<td>$t_p + 100$</td>
<td>$t_p + 400$</td>
</tr>
<tr>
<td>tailArea_120_400</td>
<td>$t_p + 120$</td>
<td>$t_p + 400$</td>
</tr>
<tr>
<td>tailArea_140_400</td>
<td>$t_p + 140$</td>
<td>$t_p + 400$</td>
</tr>
</tbody>
</table>

Table 5.2: The table lists various definitions of tail areas spanning from an initial to a final time window of the pulse after interpolation.

5.2 Neutron/Gamma - Discrimination

The neutron counters see a large number of gamma rays and electrons due to muon decay. The neutrons are detected by the ionization of the elastically scattered recoil protons following neutron, proton scattering whereas gamma rays are detected by the ionization of the recoil electrons following Compton scattering. Since heavier particles have higher specific ionization, the protons produce more delayed fluorescence light compared to the electrons which results in a larger tail area for neutron pulses compared to gamma ray pulses [76].

This distinguishing feature is used for PSD. Various approaches employing this underlying principle were used to distinguish neutrons from other pulses detected by the neutron counters. Various tail areas and the total area of the pulse was used to perform a PSD based on the peak time and the half-height time.

5.2.1 PSD using Tail and Total Areas

In this method of PSD, the total area of the pulse was compared to the tail area to determine if the pulse was a neutron or a gamma ray. A parameter called PSD ratio was defined which is the ratio of the tail area of the pulse to its total area. Figure 5.3 shows a plot of the PSD ratio versus the total area. Since neutrons have a larger tail area the middle region in Fig. 5.3 corresponds to the neutrons and the lower region corresponds to gamma rays. Based on Fig. 5.3, three energy ranges were defined for which neutrons were extracted depending on the minimum and maximum value of the PSD ratio. This range was specific to each detector and table 5.3 lists the maximum and minimum limits of these values for various ranges of energy (or total area). The upper left region above the region corresponding to the neutrons in Fig. 5.3, denoted after pulses. The old Bicrons (NU3, ND3, NU6, NU11 and ND11) suffered more from these after pulses. The plot of total area versus PSD ratio for all eight counters are shown in Fig. 5.6. This clearly shows that the after pulses are prominent in the above mentioned detectors - NU3, ND3, NU6, NU11 and ND11.

5.2.2 Figure of Merit

To quantify the ability to distinguish neutrons from gamma rays a parameter called the figure of merit (FOM) was defined. The distribution of the PSD ratio for a very
<table>
<thead>
<tr>
<th>Counter</th>
<th>Energy Range</th>
<th>Min. PSD ratio</th>
<th>Max. PSD ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>NU3</td>
<td>1300 - 3000</td>
<td>0.103</td>
<td>0.222</td>
</tr>
<tr>
<td></td>
<td>3000 - 6000</td>
<td>0.079</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td>6000 - 25000</td>
<td>0.077</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>1600 - 3700</td>
<td>0.091</td>
<td>0.180</td>
</tr>
<tr>
<td>ND3</td>
<td>3700 - 6000</td>
<td>0.061</td>
<td>0.160</td>
</tr>
<tr>
<td></td>
<td>6000 - 25000</td>
<td>0.051</td>
<td>0.150</td>
</tr>
<tr>
<td>NU6</td>
<td>1300 - 3700</td>
<td>0.122</td>
<td>0.215</td>
</tr>
<tr>
<td></td>
<td>3700 - 6000</td>
<td>0.085</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>6000 - 25000</td>
<td>0.087</td>
<td>0.190</td>
</tr>
<tr>
<td></td>
<td>1500 - 2800</td>
<td>0.122</td>
<td>0.200</td>
</tr>
<tr>
<td>ND6</td>
<td>2800 - 6000</td>
<td>0.117</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td>6000 - 15000</td>
<td>0.115</td>
<td>0.165</td>
</tr>
<tr>
<td>NU11</td>
<td>1600 - 3000</td>
<td>0.079</td>
<td>0.177</td>
</tr>
<tr>
<td></td>
<td>3000 - 6000</td>
<td>0.058</td>
<td>0.160</td>
</tr>
<tr>
<td></td>
<td>6000 - 25000</td>
<td>0.061</td>
<td>0.140</td>
</tr>
<tr>
<td>ND11</td>
<td>1750 - 2800</td>
<td>0.128</td>
<td>0.213</td>
</tr>
<tr>
<td></td>
<td>2800 - 6000</td>
<td>0.127</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>6000 - 25000</td>
<td>0.124</td>
<td>0.190</td>
</tr>
<tr>
<td>NU14</td>
<td>1400 - 2800</td>
<td>0.122</td>
<td>0.212</td>
</tr>
<tr>
<td></td>
<td>2800 - 6000</td>
<td>0.118</td>
<td>0.190</td>
</tr>
<tr>
<td></td>
<td>6000 - 15000</td>
<td>0.117</td>
<td>0.180</td>
</tr>
<tr>
<td>ND14</td>
<td>1350 - 2800</td>
<td>0.14</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>2800 - 6000</td>
<td>0.105</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td>6000 - 15000</td>
<td>0.102</td>
<td>0.160</td>
</tr>
</tbody>
</table>

Table 5.3: The table shows the energy equivalent range of PSD for all neutron detectors, ‘D’ stand for downstream, ‘U’ for upstream, and the number stands for the position in $\phi$. It also shows the maximum and minimum value of PSD ratio for each range of energy.

A narrow energy range is as shown in Fig. 5.4. The resulting plot was fitted with a double Gaussian curve with two peaks, the larger one to the left corresponding to the gamma rays (its peak position designated as $X_g$) and the smaller right peak corresponding to neutrons (its peak position shown as $X_n$). The sigma of the Gaussian for gamma rays was called $\delta_g$ and that for neutrons was called $\delta_n$. It is evident that smaller values of FWHM (Full width at half maximum) would mean a better resolution and thus contribute to a better PSD. Also if the separation between the two peaks is large the discrimination is better. Thus the FOM was defined as,

$$FOM = \frac{X_n - X_g}{2.35(\delta_g + \delta_n)} \tag{5.1}$$

Fitting the double Gaussian function was done using MINUIT and the initial guesses for peak values and sigmas were assigned using ROOT’s TSpectrum object.
that searches for peaks in a spectrum as shown in Fig. 5.4. This was done for various definitions of tail area (i.e. tailArea_60_400, tailArea_80_400, tailArea_100_400, tailArea_120_400, and tailArea_140_400). The plot of FOM versus energy for all different PSD definitions using the peak time is shown in the left panel of Fig. 5.5 and the half-height is shown in the right panel. All these were superimposed to compare with each other. From these plots it was evident that the tailArea_100_400 from peak time method was a good choice, as it showed the largest FOM. Thus in our further analysis the tail area was chosen to be the tailArea_100_400 from the peak time method.

Each detector had a different energy equivalent range corresponding to an optimal

<table>
<thead>
<tr>
<th>Counter</th>
<th>Min Range(keVee)</th>
<th>Max Range(keVee)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NU3</td>
<td>212.42</td>
<td>4084.97</td>
</tr>
<tr>
<td>ND3</td>
<td>257.65</td>
<td>4025.77</td>
</tr>
<tr>
<td>NU6</td>
<td>209.39</td>
<td>4025.77</td>
</tr>
<tr>
<td>ND6</td>
<td>242.72</td>
<td>2427.18</td>
</tr>
<tr>
<td>NU11</td>
<td>242.06</td>
<td>3782.15</td>
</tr>
<tr>
<td>ND11</td>
<td>234.27</td>
<td>3346.72</td>
</tr>
<tr>
<td>NU14</td>
<td>324.07</td>
<td>3935.19</td>
</tr>
<tr>
<td>ND14</td>
<td>312.50</td>
<td>4491.73</td>
</tr>
</tbody>
</table>

Table 5.4: The table shows the energy equivalent range of PSD for all neutron detectors, ‘D’ stand for downstream, ‘U’ for upstream, and the number stands for the position in $\phi$. 

Figure 5.3: Variation of PSD ratio with total area along with PSD cuts for counter NU3. The neutrons, gamma rays and after pulses are labeled in the plot.
Figure 5.4: A small slice of energy (total area) is plotted for the PSD ratio plot for counter NU3. TSpectrum object of ROOT was used to find the peaks shown.

Figure 5.5: Figure of Merit using peak time (left) and half-height time (right) for detector NU3.

PSD. It was difficult to go too low or high in energy. Electronic noise prevented a good PSD at very low energies and pulse overflow prevented it at very high energies. The energy equivalent range of all eight detectors used in the experiment are listed in table 5.4.

5.2.3 Pulse Fitting using Templates

A pulse template is an average pulse shape constructed from many individual pulses of a given detector. Pulses were selected from the energy region of 2000 - 7000 (channels), where neutrons and gamma rays could be unambiguously distinguished and were ‘lined-up’ with the peak time corresponding to 14 clock tick (chosen for
convenient alignment of all raw pulses). Thus, the template had a much better time resolution than the raw pulse. The time, amplitude and areas of the raw pulses could be determined much more accurately, by least-square fitting with the template created. We defined a pseudo time $\tau$ given by the expression,

$$\tau = (5.88 \text{ ns}) \left[ m + \frac{2}{\pi} \tan^{-1} \left( \frac{S_m - S_{m-1}}{S_m - S_{m+1}} \right) \right] \tag{5.2}$$

where $m$ is the bin corresponding to the maximum sample $S_m$. This was chosen to be increasing with time within the clock cycle.

A true time $t$ was defined as a function of the pseudo time, which was known to be independent of the phase of the clock tick as it corresponds to the particle arrival time [7]. The distribution $p(t)$ of true time has to be uniform and independent of the time within the clock cycle. Thus, by finding the running integral of pseudo time and normalizing it resulted in the true time. The true time as a function of pseudo time is given by the following equation,
Figure 5.7: Pseudo time distribution (within a clock cycle) of detector NU3 (left) and its true time determined from normalized running integral of pseudo times (right).

\[ t(\tau) = (5.88 \text{ ns}) - \frac{\int_0^{5.88 \text{ ns}} p(\tau')d\tau'}{\int_0^{5.88 \text{ ns}} p(\tau')d\tau'} \]  

(5.3)

A distribution of pseudo times and true times for the neutron counter NU3 is shown in the Fig. 5.7.

The neutrons and the gamma ray pulses were tagged from the previous PSD method based on areas of the tails of each pulse. A correction of true time was added to the clock tick corresponding to the peak sample’s bin to align the pulses on a sub-bin basis and interpolate points between bins of a clock tick. This term was subtracted from 14 and added to every clock tick (sampling time bin recorded by the FADC clock). We selected raw waveform pulses which had number of samples greater than 60. Finally template histograms were created by subtracting the pedestals and normalizing the sum of each pulse, to unit area. An illustration of a neutron template and gamma ray template both overlaid for counter NU3 is shown in Fig. 5.8.

5.2.4 Minimization

It is assumed that we have a single pulse on an island and the function that is minimized is given by,

\[ D(t) = \sum_{i \in \text{samples}} [S_i - P - Af_i(t)]^2 \]  

(5.4)

where \( S_i \) is the measured sample of a pulse corresponding to bin (or clock tick) \( i \) and \( f_i(t) \) is the average pulse template, \( t \) is the pulse time, \( A \) is the pedestal subtracted pulse area and \( P \) is the pedestal of the pulse. Each pulse was fitted using both a neutron and a gamma ray template to identify it [79].

**Fit Parameters:** Initially, a three parameter MINUIT least-square fit was used to
discriminate the pulses. The time $t$, pedestal subtracted area $A$ and pedestal $P$ of the pulse were used as the independent fit parameters which made the fit of many pulses very time consuming. Consequently, a more efficient single parameter fit function using Brent’s Minimization was used for fitting, which is described in detail later. Figure 5.9 shows a neutron pulse (defined using PSD ratio method) fitted by a neutron template and a gamma ray template. It shows the fit parameters obtained using MINUIT.

The parameter used for fitting using Brent’s method was $t$. As it is evident that the partial derivatives of pedestal $P$ and area $A$ are both linearly dependent
on the minimization function $D$, they can be solved analytically by evaluating the partial derivative of Eq.(5.4) with respect to $P$ and $A$ and setting them to zero at the minimum of $D$. Thus the partial derivatives are given by,

$$\frac{\partial D}{\partial P} = -2 \sum_{i \in \text{samples}} [S_i - P - Af_i(t)] = 0 \quad (5.5)$$

$$\frac{\partial D}{\partial A} = -2 \sum_{i \in \text{samples}} f_i(t)[S_i - P - Af_i(t)] = 0 \quad (5.6)$$

Solving Eq.(5.5) we obtain,

$$P = \frac{\sum_{i \in \text{samples}} [S_i - Af_i(t)]}{n} \quad (5.7)$$

where $n$ is the number of samples.

Substituting the value of $P$ from Eq.(5.7) in Eq.(5.5) we get an expression for area of pulse as,

$$A = \frac{n \sum_{i \in \text{samples}} S_i f_i(t) - \sum_{i \in \text{samples}} f_i(t)^2}{(n-1) \sum_{i \in \text{samples}} f_i(t)^2} \quad (5.8)$$
Figure 5.11: A neutron pulse first fitted with a neutron template (left) and then a gamma template (right), using Brent’s minimization.

Figure 5.12: A gamma pulse first fitted with a neutron template (left) and then a gamma template (right), using Brent’s minimization.

In the algorithm used, Eq.(5.8) was solved for every pulse iteratively to get the area first and then this area was plugged in Eq.(5.7) to find the pedestal. This ultimately reduces the minimization of $D$ to a single variable/parameter function of $t$, which was minimized using Brent’s method. The initial guess for $t$, was taken to be the true time of the peak sample evaluated from the method described previously. The ROOT class BrentMinimizer1D was used to implement Brent minimization. This class takes the function to be minimized with a single parameter and uses Brent’s parabolic minimization [80]. A time interval $(a_1,c_1)$ is selected as shown in Fig. 5.10 to minimize $D(t)$. An intermediate point $b_1$ is chosen such that $D(b_1)$ is less than both $D(a_1)$ and $D(c_1)$, which corresponds to the minimum to be found. A point $x_1$ is selected either between $a_1$ and $b_1$ or $c_1$ and $b_1$. If $D(c_1) > D(a_1)$ then $x_1 = c_1$ or vice versa. Thus, in Fig. 5.10 (first panel), $x_1 = c_1$ and the subscript 1 is changed.
to 2 for all the three points \(a, b\) and \(c\) as this denotes the second step and the new interval is now \((a_2, c_2)\). Now, the same procedure is repeated by choosing a point \(x_2\) either between \(a_2\) and \(b_2\) or \(c_2\) and \(b_2\). This process continues until the the golden selection rule [80] is satisfied at a sufficiently small interval. This interval is determined by the tolerance of the BrentMinimizer1D class. The tolerance and step size were selected such that the output was good enough to separate neutrons and gamma rays besides making the process time efficient.

![Figure 5.13: Interpolation for bin i = 125 (in this case). The fraction X is the modulo of the raw pulse in this bin. The interpolated value is \(y = y_i + X (y_{i+1} - y_i)\).](image)

### 5.2.5 Error Handling and Other Fit Details

Initially, the errors were set to unity for all bins including bins with a zero value (if any). This was done so that neutrons and gamma rays could be distinguished on the basis of the \(\chi^2\) values obtained by fitting each pulse with a neutron template and a gamma ray template. This was justified as it was assumed that there was no correlation between data from one bin to the other and uncertainties were independent of sample values. Overflowing samples were ignored in fitting which helped in recovering the overloaded pulses. Fit range used was from 5 samples to 55 samples i.e. 50 samples were used.

Since the pulses were fitted with a histogram (template) instead of a continuous function, it was important to interpolate between bins of the template histogram during fitting. The fit point of the pulse could lie anywhere within the bin of the discrete template. Let the fractional part of the point corresponding to the actual raw pulse be \(X\), for the \(i^{th}\) bin, with ADC count given by \(y_i\) then linear interpolation gives the ADC value \(y\) as,

\[
y = y_i + X(y_{i+1} - y_i).
\]  

(5.9)
This is shown in Fig. 5.13 for bin 125. (Note: bin size is 0.1 times the clock tick for the templates.)

Figure 5.14: Comparison of fit parameters obtained from MINUIT and Brent’s Minimization. The leftmost panel shows the comparison of areas from MINUIT and Brent’s method respectively. The center and right panels show the same for the times and the pedestals from both these methods.

Using the Brent’s minimization fit method instead of the three parameter root fit using MINUIT had several advantages. It was much faster than the MINUIT fit as the number of parameters reduced to one. Figure 5.11 shows the fit using Brent method on a neutron pulses fitted with neutron and gamma ray templates.

Next, comparisons of $t$, $A$ and $P$ were done from the output of both fit methods as a sanity check. The plots shown in Fig. 5.14 illustrates the comparison of $t$, $A$ and $P$ from the 3 parameter minimization using MINUIT and Brent’s methods respectively. The Brent areas and MINUIT fitted areas were comparable indicating that the template fit method could be used effectively for a good PSD. Looking at individual pulses that had different fit results from MINUIT and Brent minimization, it was found that they were either pulses due to electronic noise, after pulses, overloaded pulses, islands with small number of samples or pile up pulses (two pulses on one island). All this was encountered in later studies too and will be discussed in detail later.

5.2.6 Energy Dependence of Templates

The tails of neutron pulses depend on energy especially in the low energy region. Low energy pulses have a higher ionization density which results in a larger amplitude / area of the tail. Thus it was essential to test if the template shape had an energy dependence or not (especially for neutron templates). To test this, templates of three different energy ranges were created. The regions chosen were as follows:

1. 400 - 700 keVee corresponding to low energy
2. 700 - 1100 keVee corresponding to intermediate energy
3. 1000 - 1200 keVee corresponding to high energy
Figure 5.15: Three energy dependent neutron templates (left) and gamma ray templates (right). These are normalized relative to their peak values. The neutron templates show a very small energy dependence and the gamma ray template is independent of energy.

Figure 5.15 compares the templates for the three energy ranges for neutrons (left panel) and gamma rays (right panel). The three ranges are normalized to equal peak values for comparison. It is evident that the neutron templates are very slightly energy dependent. The gamma ray templates do not depend on energy. Thus, a single template for all energy ranges was adequate.

5.2.7 Chi Squared Distribution using Brent’s Algorithm

A least square fit of the raw pulses with the pulse templates would help detect a neutron or a gamma ray based on the $\chi^2$ value of the fit. Using Brent’s method, each pulse was fitted with a neutron template to obtain a $\chi_n^2$ and a gamma ray to obtain a $\chi_g^2$. A pulse was identified as a gamma ray or a neutron depending on which template gave a lower $\chi^2$ value. A plot of $\chi_n^2 - \chi_g^2$ versus energy is shown in Fig. 5.16.

A positive value of $\chi_n^2 - \chi_g^2$ corresponds to gamma rays and the negative value corresponds to neutrons. Additional cuts to get rid of the following were applied

1. Electronic noise that had no real pulse.
2. Pulses with island size smaller than the fit range.
3. Pileup pulses - i.e. two pulse on an island which were reasonably far apart and so could be handled.
4. Two pulse on an island which were extremely close and so could not be handled. An example of such a pulse is shown in Fig. 5.17

Pulses with small island size were rejected due to the fact that they did not have a tail. A correction for double pulses in an island (not pile up protected) was applied by changing the fit range and ignoring the second pulse on the island. Rejecting pulses with fitted time outside the island helped get rid of noise or unphysical pulses.
Correlation Between Bins

Initially statistical uncertainties of all bins were set to unity including empty bins, whereas it could be possible that pedestal regions could have different statistical uncertainties than peaks regions. The number of photo electrons is much less in the pedestal region compared to the peak regions, which could lead to statistical uncertainties based on the pulse height. Thus, the peak regions should have a higher statistical uncertainty compared to pedestal regions. In our effort to understand this, the first step was to find the difference between the ADC counts read by the pulses and the value of fit function at a point. This difference was called the residual. The distribution of residual of each bin is shown in Fig. 5.18 for neutrons and 5.19 for gamma rays. These plots show a larger fluctuation for peak regions compared to pedestal regions. We tried to use different errors for the pedestal and peak regions but found no significant change and so we finally decided to set all weights to unity. Also it was assumed that there was no correlation between bins. As the signal from each photo electron could be distributed in more than one bin, it could lead to correlation between bins. The distribution of residuals of consecutive bins is shown in Fig. 5.20.

After looking at the correlation plots and residual plots we concluded that there was no correlation between data - none for gamma rays and negligible for neutrons. The reason for this could be due to fact that a pulse is made up of several scintillating photons and not a single photon and so correlations probably get distributed and cancel out. In conclusion, it was found empirically that data is not correlated and the statistical uncertainties are almost uniform. So it was justified to have equal weight of uncertainties all set to unity for all bins and for all energy ranges.
5.2.9 PSD using Pulse Templates

The area, pedestal and time of the pulse yielding the best fit with a template was saved. The area was called Brent area (which was like the total area of the pulse). The distribution of $\chi_n^2$ versus Brent area for every pulse is shown in Fig. 5.21. A lower $\chi_n^2$ value indicated a better fit with a neutron template and so in this plot the pulses with low $\chi_n^2$ were selected to be neutrons and ones with higher values of $\chi_n^2$ were gamma rays. Cuts based on the ratio of $\chi_n^2$ values to Brent area for various energy ranges were thus applied to separate neutrons from gamma rays. Table 5.5
5.2.10 Comparison of PSD using Templates and Tail Total Method

Different neutron definitions were created and saved using the two methods i.e. pulse template method and tail total method. The PSD ratio in case of the template fit method was defined as the ratio of the tail area to the Brent area. A plot of Brent area versus PSD ratio was used to find the FOM using Eq.(5.1). A quantitative
study showing the FOM is shown in the Fig. 5.22 for detector NU3. It appeared from this figure that the threshold energy can be reduced a little more for the new pulse template fit method of PSD. For detector NU3 the threshold seemed to now be around 120 keVee instead of 200 keVee with the old tail total method of PSD. At higher energies too the FOM for this method looked better.

Several cuts were applied on both methods to get rid of the unwanted pulses like noise, double pulses, pulses with small island size etc. Finally we could not get rid of a few unwanted pulses like closely placed double pulses and overloaded pulses which were misidentified as neutrons using the tail total method but were eliminated from the template fit methods. Examples of these pulses are shown in Fig. 5.23. The left panel of this figure shows an example of an extremely close double pulse which could not be eliminated and could be misidentified as a neutron pulse. The right panel shows an overloaded pulse that was misinterpreted to be a neutron, by the old tail total method.

For consistency the same cuts which maximized the elimination of all undesirable pulses (after pulses or noisy pulses, double pulses etc.) were applied to both the methods of PSD. In the energy range of 2000 to 25000 channels, the tail total method showed some falsely identified neutrons due to its inability to distinguish overloaded pulses. About 65 percent of the falsely identified neutrons by the tail total method were these overloaded pulses and a handful of 10 percent were double pulses. Due to this misidentification the tail total method had around 10 percent more neutron data

<table>
<thead>
<tr>
<th>Counter</th>
<th>Energy Range</th>
<th>Chi square : Brent Area - Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>NU3</td>
<td>1116 - 3929</td>
<td>109.43 - 422.15</td>
</tr>
<tr>
<td></td>
<td>3929 - 25000</td>
<td>422.15 - 1639.45</td>
</tr>
<tr>
<td>ND3</td>
<td>1099 - 4766</td>
<td>99.41 - 448.63</td>
</tr>
<tr>
<td></td>
<td>4766 - 25000</td>
<td>448.63 - 1187.8</td>
</tr>
<tr>
<td>NU6</td>
<td>1100 - 3341</td>
<td>114.19 - 397.32</td>
</tr>
<tr>
<td></td>
<td>3341 - 25000</td>
<td>397.32 - 1706.36</td>
</tr>
<tr>
<td>ND6</td>
<td>1200 - 3743</td>
<td>149.51 - 421.09</td>
</tr>
<tr>
<td></td>
<td>3743 - 15000</td>
<td>421.09 - 1421.99</td>
</tr>
<tr>
<td>NU11</td>
<td>1498 - 3754</td>
<td>106.92 - 306.57</td>
</tr>
<tr>
<td></td>
<td>3754 - 25000</td>
<td>306.57 - 1104.17</td>
</tr>
<tr>
<td>ND11</td>
<td>1400 - 4325</td>
<td>204.63 - 419.91</td>
</tr>
<tr>
<td></td>
<td>4325 - 25000</td>
<td>419.91 - 1756.54</td>
</tr>
<tr>
<td>NU14</td>
<td>1200 - 1648</td>
<td>163.46 - 221.39</td>
</tr>
<tr>
<td></td>
<td>2800 - 17000</td>
<td>221.39 - 1572.54</td>
</tr>
<tr>
<td>ND14</td>
<td>1200 - 1832</td>
<td>137.56 - 211.09</td>
</tr>
<tr>
<td></td>
<td>1832 - 19000</td>
<td>211.09 - 1672.9</td>
</tr>
</tbody>
</table>

Table 5.5: The table shows the range of the ratio of neutron chi square to Brent area cuts for all energy ranges of all neutron detectors using template fit method of PSD, ‘D’ stand for downstream, ‘U’ for upstream, and the number stands for the position in $\phi$. 
than the template fit method, in the entire energy region. But in the energy range from 2000 to 15000 channels there was around 7.89 percent difference in neutron data. The energy range of each detector is shown in table 5.6.

To summarize the template method was better than the old tail total method for PSD for the following reasons:

1. It was able to handle noise, double pulses on island etc. better than the tail total method.
2. Fit method could take care of overflows by ignoring overloaded points.
3. A higher figure of merit was achieved.
4. PSD could be pushed to a little lower energy compared to the tail total method which increased the energy range.
Figure 5.23: Typical pulses that were misinterpreted as neutrons by the tail total method of PSD

<table>
<thead>
<tr>
<th>Counter</th>
<th>Min Range(keV&lt;sub&gt;ee&lt;/sub&gt;)</th>
<th>Max Range(keV&lt;sub&gt;ee&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NU3</td>
<td>182.28</td>
<td>4084.97</td>
</tr>
<tr>
<td>ND3</td>
<td>177.05</td>
<td>4025.77</td>
</tr>
<tr>
<td>NU6</td>
<td>177.13</td>
<td>4025.77</td>
</tr>
<tr>
<td>ND6</td>
<td>194.17</td>
<td>2427.18</td>
</tr>
<tr>
<td>NU11</td>
<td>182.28</td>
<td>3782.15</td>
</tr>
<tr>
<td>ND11</td>
<td>177.05</td>
<td>3346.72</td>
</tr>
<tr>
<td>NU14</td>
<td>177.13</td>
<td>3935.19</td>
</tr>
<tr>
<td>ND14</td>
<td>194.17</td>
<td>4491.73</td>
</tr>
</tbody>
</table>

Table 5.6: The table shows the energy range of all neutron detectors using template fit method of PSD, ‘D’ stand for downstream, ‘U’ for upstream, and the number stands for the position in φ.

The major disadvantage of this new method was that it took much longer time compared to the tail total method. After optimizing the most suitable parameters like step size and tolerance functions of the BrentMinimizer1D class of root we could achieve an optimized time of 24 minutes for a run as against 6 minutes for the same run with the tail total method. Thus the analysis got four times slower with this method. This was because each pulse had to be fitted twice, once with a neutron template and then with a gamma ray template. Thus, ultimately we used the PSD ratio method with tailArea_100_400 as the tail area for the final neutron gamma discrimination for our future analysis.

5.3 Energy Calibration of Neutron Detectors

Gamma ray sources undergo Compton scattering and so they were used to do the energy calibration of the neutron detectors. The specific energy of the Compton edge of the gamma ray, due to Compton scattering was compared with the number of channels read by the neutron counter corresponding to that energy. The pedestal
subtracted area of each pulse is proportional to the energy of the pulse and this area has arbitrary units of ADC counts which was converted/calibrated to units of energy in keV $ee$. The calibration procedure involved a suitable fitting of the data in the region of the energy spectrum that corresponds to the Compton scattering which gives rise to the Compton edge (or maximum kinetic energy of the electron). The fit function was a convolution of the theoretical distribution of differential scattering cross section $\frac{d\sigma}{d\theta}$ (given by Klein Nishina formula) and a Gaussian resolution function (owing to the energy resolution of the counters). The entire procedure is explained in detail in the following subsections.

5.3.1 Compton Effect

Compton effect is the scattering of a gamma ray photon with an electron in an atom which results in increasing the wavelength of the photon (or decreasing its energy). The expression for the change in scattering wavelength $\Delta \lambda$ is given by,

$$\Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta) \tag{5.10}$$

where $m_e$ is the mass of the electron and $\theta$ is the angle by which the incident gamma ray scatters. At the maximum scattering angle $\theta = 180^\circ$, the energy difference carried away by the scattered electron is its maximum kinetic energy $E_{Compton}$ given by the equation,

$$E_{Compton} = \frac{2E_i^2}{m_e c^2 + 2E_i} \tag{5.11}$$

as a function of the initial energy $E_i$ of the gamma ray [81].

The radioactive sources used during the experiment for calibration runs were $^{60}$Co and $^{137}$Cs. $^{60}$Co is an unstable isotope of Cobalt with a half life of about 5.27 years which undergoes a beta decay to an excited state of $^{60}$Ni, which in turn emits gamma rays with energies 1.17 and 1.33 MeV respectively. Since these spectral lines are in close proximity we took the average of these to be an equivalent spectral line emitted by this source, yielding an of energy 1.25 MeV. $^{137}$Cs undergoes a beta decay to $^{137}$Ba which emits a distinct photo peak of 662 keV, as shown in the right panel of Fig. 5.24.

The Compton edges corresponding to these photo peaks were calculated using the Eq. (5.11). The table 5.7 lists the Compton edge for the corresponding photo peaks of the two source for a quick and easy reference.

5.3.2 Klein Nishina Formula

Klein Nishina formula gives the expression for differential scattering cross section of photons scattered by single electron corresponding to the lowest order of quantum electrodynamics. Thus, we investigate this formula in detail and apply it for our
Figure 5.24: Left panel shows the decay scheme and photo peaks of $^{60}\text{Co}$. Image credit: [76]. and the right panel shows the same for $^{137}\text{Cs}$. Image credit: [82].

<table>
<thead>
<tr>
<th>Source</th>
<th>Photo Peak (keV)</th>
<th>Compton Edge (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{60}\text{Co}$</td>
<td>1170, 1330</td>
<td>960.29, 1115.67</td>
</tr>
<tr>
<td>$^{137}\text{Cs}$</td>
<td>662</td>
<td>480</td>
</tr>
</tbody>
</table>

Table 5.7: The table shows the photo peaks and Compton edges for $^{60}\text{Co}$ and $^{137}\text{Cs}$ sources.

calibration studies. The energy distribution of the differential cross section i.e. $\frac{d\sigma}{dE_T}$ is given by,

$$
\frac{d\sigma}{dE_T} = \frac{\pi r_e^2}{\epsilon h\nu} \left( 2 - \frac{2E_T}{\epsilon (h\nu - E_T)} \right) + \frac{E_T^2}{\epsilon^2 (h\nu - E_T)^2} + \frac{E_T^2}{h\nu (h\nu - E_T)}
$$

where $E_T$ is the kinetic energy of the recoil electron, $h\nu$ is the energy of the incident photon, $r_e$ is the classical electron radius and $\epsilon$ is given by $\epsilon = \frac{h\nu}{m_e c^2}$, $m_e$ being the rest mass of the electron [76].

The distribution of scattering cross section as a function of energy i.e. Eq.(5.12) is plotted in the Fig. 5.25 by the solid black line. This is shown for Compton scattering of electrons (having a Compton edge of 480 keV) with gamma ray photons emitted from $^{137}\text{Cs}$ (spectral line of 667 keV) as an example.

Selecting the Klein Nishina cross section as the theoretical model was reasonable enough as it could account for scattering of gamma rays and X rays. Besides it also considered the details of relativistic quantum mechanical effect in the high energy regime and so we expected it to give accurate results.

5.3.3 Fit Function

The convolution was done numerically using C++ and ROOT. The description of the algorithm used will be delineated briefly. The Klein Nishina cross section formula was
first plotted by the function defined by Eq. (5.12). A Gaussian function of suitable width \( w \), denoted by \( G(w) \) was superimposed, shifted by a bin over the Klein Nishina formula denoted by \( K \) (i.e. Eq. (5.12)) and finally integrated over the entire range of interest from \( E_1 \) to \( E_2 \). This procedure is illustrated by the equation,

\[
G \ast K \equiv \int_{E_1}^{E_2} G(w)K(E_T - w)dw
\]  

(5.13)

The range of interest was corresponding to the Compton edge of the spectrum. The resulting integrated function obtained was thus, the result of the convolution of the two which was our model for fitting the energy spectrum. The convolution function is shown in Fig. 5.25 as a dotted blue line. The input parameters of the fit function had a physical significance which were as follows:

1. The first parameter returned the channel corresponding to maximum kinetic energy, \( T_{max} \) or Compton edge.

2. The second parameter returned the \( \sigma \) of the Gaussian distribution used for convolution.
Figure 5.26: Energy distribution of the data from counter NU14 with source $^{60}$Co. The blue dotted line represents Klein Nishina distribution. The magenta dashed line is the fit function i.e. convolution of Klein Nishina cross section with the Gaussian. The red solid line is the fitted region.

3. The third parameter returned the number of counts, N in the channel or bin corresponding to the Compton edge.

4. The last parameter signified a flat time independent background, B that was subtracted to get a pedestal free background.

Figure 5.26 shows an example of the fit parameters and the fit range. In this figure the fit range is from 3800 to 6000 channels. Thus, the channel returning the $T_{\text{max}}$ could easily be read off from the first parameter of the fit function which was of vital importance for finding the gain of the counter. The example in Fig. 5.25 shows that the Compton edge for detector NU14 was 4001 channels at 1400 V for $^{60}$Co gamma source. The table 5.8 shows the channels for the Compton edges for all detectors with both the sources $^{60}$Co and $^{137}$Cs. Since the channels depend linearly the equivalent energy units (keV$_{ee}$), the two values of Compton edge were used as points to determine the gain. The equivalent $T_{\text{max}}$ for $^{60}$Co source was taken to be the average Compton edges corresponding to the two photo peaks i.e. 1038 keV. Thus, the gain was given by the slope of the line shown in the Fig. 5.27 which plots the average $T_{\text{max}}$ for both $^{60}$Co and $^{137}$Cs sources in channels (X axis) and in keV$_{ee}$ (Y axis) for detector NU3.
This can be written as,

\[ \text{Gain} = \frac{C_1 - C_2}{E_1 - E_2} \]  \hspace{1cm} (5.14)

where \( C_1 \) is the channel corresponding to the average Compton edge of \(^{60}\text{Co}\), \( C_2 \) is the channel corresponding to the Compton edge of \(^{137}\text{Cs}\), \( E_1 = 1038 \text{ keV}_{ee} \) (average of the Compton edges of the two photo peaks of \(^{60}\text{Co}\)) and \( E_2 = 480 \text{ keV}_{ee} \) (the Compton edge of \(^{137}\text{Cs}\)).

<table>
<thead>
<tr>
<th>Counters</th>
<th>Avg. ( T_{\text{max}} ) for (^{60}\text{Co} ) (ch)</th>
<th>( T_{\text{max}} ) for (^{137}\text{Cs} ) (ch)</th>
<th>Gain (ch/keV(_{ee}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>NU3</td>
<td>6428.12 ± 151.1</td>
<td>3013.16 ± 124.4</td>
<td>6.12 ± 0.49</td>
</tr>
<tr>
<td>ND3</td>
<td>6280.42 ± 20.1</td>
<td>2815.24 ± 31.5</td>
<td>6.21 ± 0.092</td>
</tr>
<tr>
<td>NU6</td>
<td>7265.34 ± 19.7</td>
<td>3800.16 ± 21.6</td>
<td>6.21 ± 0.083</td>
</tr>
<tr>
<td>ND6</td>
<td>6099.52 ± 10.2</td>
<td>2700.42 ± 50.0</td>
<td>6.18 ± 0.12</td>
</tr>
<tr>
<td>NU11</td>
<td>7962.36 ± 231.6</td>
<td>4237.98 ± 1.0</td>
<td>6.61 ± 0.46</td>
</tr>
<tr>
<td>ND11</td>
<td>5726.36 ± 119.9</td>
<td>1558.09 ± 153.0</td>
<td>7.47 ± 0.489</td>
</tr>
<tr>
<td>NU14</td>
<td>4001 ± 0.1</td>
<td>1521.8 ± 10.2</td>
<td>4.32 ± 0.02</td>
</tr>
<tr>
<td>ND14</td>
<td>4622 ± 51.0</td>
<td>2293 ± 55.0</td>
<td>4.17 ± 0.19</td>
</tr>
</tbody>
</table>

Table 5.8: The table shows the procedure for calculating gains from the Compton Edges. This data was taken after shielding the neutron counters.
5.3.4 Effect of Background

There was a horizontal room background independent of energy which is interpreted by the fourth parameter in the fit function. But besides this there also existed an exponentially decaying background and other noisy signals. Some detectors had a small background and others had a larger background. Figure 5.28 shows the energy spectrum of all counters which gives an idea of the nature and magnitude of the background of each counter. This is shown for $^{137}$Cs source for calibration data taken at the end of run. From this it is clear that the background also depended on the type of neutron counter used. For example NU6, ND6, NU14 and ND14 which were all new Bicron detectors showed a much lower background compared to the other detectors.

![Energy Spectrum of Different Detectors Using $^{137}$Cs Source](image)

Figure 5.28: Comparing the energy spectrum of all detectors using $^{137}$Cs source.

The background, could be attributed due to some or all of the following reasons:

1. Type of detector used (N3’s and N11’s show a larger background and a flatter peak which gives better results compared to N6’s which have a very prominent peak that make it a little hard to fit with the model chosen).

2. This may not be a perfect model owing to multiple scattering. A cascading effect due to multiple scattering, especially in the high energy region could cause an additional spread in the Compton edge region of the spectrum.
3. In the low energy region there are additional ambient background gamma rays (due to thermal neutrons) that does not depend on the source and could explain the exponentially decaying distribution in this energy region.

In some detectors (NU6 and ND6) there was an effect of this background but it did not effect the fit region of the spectrum. An exponentially decaying fit, so an exponentially decaying term was added to take care of the background. But this just made the fit worse. Nevertheless, since we are interested only in the Compton edge region of the spectrum and these backgrounds do not distort our region of interest, so the convolution function is a reasonable model for our detector calibrations. The calibration was done separately for two sets of data and gains were found to be slightly different for them. The first set of useful calibration data was taken after shielding the counters with mu-metal to protect them from magnetic field of the $\mu$SR magnet. There were other stray magnetic fields too that were effecting the counters. For example a magnet called ‘COBRA’ was often turned on and off that was used in a different experiment in the vicinity of the site of MuSun experiment which had a negligible effect on the gains. For a final check of consistency another set of calibration data was taken at the end of Run 4. The table below shows the calibration of all neutron detectors for both the cases using the fitting procedure and theory described above. The gains obtained from both the data sets agree within the range of uncertainties.

<table>
<thead>
<tr>
<th>Counters</th>
<th>Gain after shielding (ch/keV$_{ee}$)</th>
<th>Final Gain (ch/keV$_{ee}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NU3</td>
<td>6.12 ± 0.49</td>
<td>7.32 ± 0.70</td>
</tr>
<tr>
<td>ND3</td>
<td>6.21 ± 0.09</td>
<td>6.41 ± 0.27</td>
</tr>
<tr>
<td>NU6</td>
<td>6.21 ± 0.08</td>
<td>6.70 ± 0.46</td>
</tr>
<tr>
<td>ND6</td>
<td>6.18 ± 0.12</td>
<td>6.38 ± 0.09</td>
</tr>
<tr>
<td>NU11</td>
<td>6.61 ± 0.46</td>
<td>7.75 ± 0.72</td>
</tr>
<tr>
<td>ND11</td>
<td>7.47 ± 0.49</td>
<td>8.58 ± 0.63</td>
</tr>
<tr>
<td>NU14</td>
<td>4.32 ± 0.02</td>
<td>4.22 ± 0.09</td>
</tr>
<tr>
<td>ND14</td>
<td>4.23 ± 0.19</td>
<td>5.25 ± 0.90</td>
</tr>
</tbody>
</table>

Table 5.9: The table shows the gains before installing $\mu$SR magnet and just after installing it. The last column shows the finally calibrated gains after successfully shielding the counters.
Chapter 6

Fusion Neutron Event Selection

A study of the time distribution of fusion neutrons would give us the $\mu d$ chemistry kinetic parameters. The significance of these parameters were discussed in chapter 2. The distribution of the neutron peak time relative to the muon entrance time for a fusion neutron is the time distribution of fusion neutrons. To achieve this the following conditions were applied:

- a PSD cut was applied to separate neutrons from gamma rays
- neutrons with associated muon stops in deuterium (i.e. within the fiducial volume of the TPC) were selected
- neutrons accompanying electrons were selected.
- a neutron energy cut of 2.45 MeV was applied

Figure 6.1: Neutron multiplicity distribution of all neutron detector pulses for one Midas run having $3.7 \times 10^6$ muon events.
There were accidental background neutrons that leaked into this histogram and had to be removed to get a pure fusion neutron time distribution. After the extraction of the background subtracted fusion neutrons a least square fit of the time distribution was performed. This chapter describes the details of the fusion neutron event selection.

6.1 Fusion Neutron Definition

Several cuts were applied to ensure that fusion neutrons were unambiguously selected. It was ensured that data was pile up protected and the neutron multiplicity (i.e. number of neutron pulses was associated with each muon entrance) was one. A distribution of neutron multiplicities for a single run is shown in Fig. 6.1.

![Figure 6.1: Distribution of neutron multiplicities for a single run.](image)

Figure 6.1: Distribution of neutron multiplicities for a single run.

A typical run had about $3.7 \times 10^6$ muon entrance events and about $6.8 \times 10^5$ pulses detected by the neutron counters. These pulses include neutrons, electrons, gamma rays, electronic noise, afterpulses and any other signal read by the neutron counters. Around $\approx 18\%$ of muons have pulses detected by the neutron counters. This large number could be due to the fact that the neutron counters were triggering on electron noise. Figure 6.2 shows that the number of pulses detected by the neutron counters in the flat region is about five times greater than the rest of the time dependent region which means that a large fraction of the pulses were due to electronic noise. Various time spectra were produced by analyzing a data set of about 1000 MIDAS runs. The time distribution of all pulses detected by the counters is shown in the left panel of Fig. 6.3 and that of neutrons (after PSD cuts) is shown in the right panel of this figure. This shows the overwhelming number of background and

![Figure 6.2: Time distribution of all pulses seen by the neutron counters.](image)

Figure 6.2: Time distribution of all pulses seen by the neutron counters. This plot shows an estimate of the number of pulses detected by the neutron counters in the flat region (shown in cyan) which is about five times greater than the rest of the time dependent region (shown in blue).
Figure 6.3: Left panel shows the time distribution of all pulses detected by the neutron counters and the right panel shows the time distribution of neutrons after applying a neutron PSD cut.

gamma rays $(\approx 4.56 \times 10^8$ are gamma rays and background out of $\approx 4.603 \times 10^8$ pulses detected by the neutron counters) that were removed after applying the PSD cuts. A kink around $\approx 1 \mu s$ in this figure is due to afterpulses produced in some detectors as discussed in chapter 4.

The next cut ensured the extraction of clean neutrons associated with muon stops in the fiducial volume of the TPC. This eliminates a very large number of neutrons produced by muon capture in the high Z materials of the surroundings. All these time distributions with different cuts defined above are overlaid in Fig. 6.4 to understand the effect of each cut.
Figure 6.5: Left panel: time distribution of electrons with respect to muon entrance time. Right panel: time distribution of electrons with respect to neutrons.

Figure 6.6: Distribution of the coordinates of the point of closest approach of electrons with respect to the beam axis.

Muons that stop in the TPC can produce both fusion and capture neutrons. A muon catalyzes the fusion process in deuterium and thus can later undergo a decay, resulting in a delayed Michel electron as opposed to a capture process. We took advantage of this property to distinguish a fusion neutron from a capture neutron. Thus an electron almost always (99.9 % of the times) followed fusion neutrons, whereas capture neutrons do have any electrons. The existence of an electron associated with a
Figure 6.7: Time distribution of fusion neutrons (in thick red / gray line) in log scale - shows evidence of two lifetime components a prompt one and a delayed one. This is overlaid on the time distribution of neutrons with electron coincidences without an energy cut in blue (black).

muon entering the TPC was determined based on finding an electron within a time window ranging from -12000 ns to +20000 ns of an electron time relative to the muon entrance time as shown in the Fig. 6.5. This time window was chosen to avoid electron pile up. It was further ensured that these electrons originate from the TPC and so cuts were applied to the distance of closest approach of the electron track relative to the beam axis. The X, Y and Z coordinates of the point of this closest approach were ±36 mm, ±47 mm and 0 to 79 mm respectively so that the point is within the fiducial volume of the TPC. The plots of Fig. 6.6 justify these cuts.

We made sure that the muon stopping in the TPC was also pile up protected (i.e. each muon track has one muon stop associated with it). Muon clusters, tracks and stops were all discussed and defined in chapter 4.

Finally a 2.45 MeV neutron energy cut was applied, as the fusion neutrons are monoenergetic having an energy of 2.45 MeV. This energy cut depends on the gain of each detector and was thus applied for each detector separately according to their individual calibration.

The final time distribution for fusion neutrons after applying the above-mentioned cuts is shown in Fig. 6.7 by the thick red line. The thin blue line is the time distribution with all above-mentioned cuts except the fusion energy cut. It is evident that the energy cut does not change the shape of the peak region of the histogram, but does suppress the background on either side of the peak.
Figure 6.8: Energy spectrum of fusion neutrons for neutron counter NU3.

6.2 Energy Spectrum of Fusion Neutrons

The energy of a neutron pulse is determined by its total area. Since each detector has a different energy gain, the energy spectrum is different for each of these. Thus, to investigate the energy spectrum properly, only one detector output is used. A fusion neutron recycles a muon which later decays giving an electron and thus a fusion neutron can be well identified with an electron following a neutron. We call this a delayed electron cut. To check this the energy spectrum of a neutron with a delayed electron cut is shown in Fig. 6.8 for neutron counter NU3. The energy of fusion neutrons (2.45 MeV) corresponds to about 700 to 800 keV, which is evident in Fig. 6.8. The time distribution of fusion neutron plot has an electron accompanying it and so an additional energy cut corresponding to 2.45 MeV neutron energy was essential to get the final fusion neutron time distribution.
6.3 Time Distribution of Fusion Neutrons

The neutron time distribution \( (t_n - t_\mu) \) for all neutron pulses from all channels was studied carefully. The incoming muon and subsequent neutron formed was in a pile up free time window of \(-25000 < t_n - t_\mu < 25000 \) ns. The time distribution was first aligned for different detectors and then the background was removed to get the final time distribution. All this is described in the subsequent subsections.

6.3.1 Time Alignment of Neutron Detectors

The different PMT high voltages and the time of flight of the neutrons in the neutron counter could contribute to different delay times \( t_0 \) in the time distribution for each detector. The resolutions of the detectors also effected the time distribution. All this was taken care by the convolution of the time resolution (which is a Gaussian function) with an exponential decay function i.e. an exponentially modified Gaussian function. This is given by,

\[
\int_{-\infty}^{\infty} e^{-(t-t')^2/2\sigma^2} e^{-t'/\tau} dt' = \sqrt{\frac{\pi}{2}} \frac{1}{\sigma \tau} e^{t/\tau} \left( 1 - \text{Erf} \left[ \frac{(\sigma^2 - (t+t_0)\tau)}{\sigma \tau \sqrt{2}} \right] \right) \tag{6.1}
\]
where \( \tau \) is the lifetime of the exponential decay and \( \sigma \) is the width of the Gaussian used. The details of the derivation of the equation are given in appendix 8.3. A finely binned histogram (of 1 ns bin size) was generated to evaluate the sigma \( \sigma \) and delay time \( t_0 \) for each detector. This was fitted with Eq.(6.1) in the very prompt region of the histogram from -300 ns to 100 ns to find the delay time and sigma of each detector. A fit result of all neutron counters is shown in Fig. 6.9.

<table>
<thead>
<tr>
<th>Counter</th>
<th>Delay Time ( t_0 ) (ns)</th>
<th>Sigma ( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NU3</td>
<td>161.6</td>
<td>7.5</td>
</tr>
<tr>
<td>ND3</td>
<td>168.2</td>
<td>5.89</td>
</tr>
<tr>
<td>NU6</td>
<td>166.0</td>
<td>6.63</td>
</tr>
<tr>
<td>ND6</td>
<td>165.8</td>
<td>5.53</td>
</tr>
<tr>
<td>NU11</td>
<td>136.1</td>
<td>6.22</td>
</tr>
<tr>
<td>ND11</td>
<td>141</td>
<td>8.39</td>
</tr>
<tr>
<td>NU14</td>
<td>163.4</td>
<td>6.07</td>
</tr>
<tr>
<td>ND14</td>
<td>162.7</td>
<td>6.76</td>
</tr>
</tbody>
</table>

Table 6.1: The table shows the delay times for each detector.

It was observed that all detectors had a comparable delay time and sigma (except for the two home made counters). The respective delay time of each detector was added in such a way that the peak time distribution was aligned to zero for consistency. Table 6.1 shows \( t_0 \) and \( \sigma \) from the fit results for each detector and Fig. 6.10 shows the comparison of the time distribution of fusion neutrons before and after aligning them. The electron time distribution also had an offset in time that was
subtracted to align the delay time to zero for consistency. Finally, we obtain a fusion neutron time distribution as shown in Fig. 6.11 (in a thick blue / black line).

![Graph showing time distribution of fusion neutrons and background neutrons.](image)

Figure 6.11: Time distribution of fusion neutrons in black (blue) line and the distribution obtained from the pre electron condition in gray (red) line.

### 6.3.2 Sources of Background

The fusion neutron time distribution obtained shows two kinds of backgrounds:

1. A time independent flat background - this originates due to room background, cosmic rays and due to accidental neutrons. Applying stringent PSD cuts minimized this.

2. A time dependent background - this originates due to the beam related background, which changes with time as the beam is off (blocked by the kicker) for 600 ns before the \( \mu \text{SC} \) signal triggers the kicker. It could also originate from the capture of the upstream muons (as the \( \mu \text{SC} \) does not fire) by the walls of the TPC and/or in deuterium giving rise to prompt (due to capture by high Z materials) and delayed (due to capture in deuterium) lifetime components in the background time distribution. They could also be due to misidentified gamma rays from bremsstrahlung’s due to electrons from muon decay and photo-neutrons from bremsstrahlung [83].

### 6.3.3 Removal of Background

A time distribution of background neutrons was first generated and then remove it from the fusion time distribution. Fusion neutrons are accompanied by delayed electrons. The time dependent and independent backgrounds are associated with accidental electrons. The fusion neutron time distribution we obtained with an electron
requirement $N_e$ has fusion neutrons and time dependent and independent background neutrons. Time dependent and independent backgrounds can be associated with associated with a pre electron condition on the neutron time distribution. Thus a histogram $N_{pree}$ with all above mentioned cuts (except replacing the electron requirement with a pre electron requirement) would contain time dependent and independent background neutrons only. The pre electron window ranges from -15000 to -100 ns of the electron time with respect to the neutron time (-15000 < (t_e - t_n) < -100 ns). In Fig. 6.11 the distribution in black (or blue) is the neutron distribution with any electron and the one in gray (or red) is with a pre electron condition applied.

![Distribution diagram](image)

Figure 6.12: The efficiency function denotes the probability of finding a neutron in the pre electron window.

A study of the time distribution of fusion neutron shown in black (or blue) in Fig. 6.11 shows the time independent background neutrons corresponding to the flat background on the right side of the peak time (from 10000 ns to 25000 ns). The region to the left of the peak time shows an exponentially decaying distribution. This is the time dependent part of the background distribution. The distribution obtained from the pre electron condition shown in gray in Fig. 6.11 was the raw distribution which had to be divided by the efficiency of finding the neutron with an electron in the pre electron window i.e. -15000 < (t_e - t_n) < -100 ns. This was obtained by finding the probability of a neutron leaking through this window. The number of favourable events for the neutrons leaking in this window was obtained by the area $A_p(i)$, of this window in the electron time distribution at that particular time (or time bin $i$ of the fusion neutron distribution). The sample space is just the fixed area $A_e$, of entire electron time distribution from -15000 to 20000 ns shown in Fig. 6.5. Thus this probability or efficiency function $\varepsilon$ is given by,

$$
\varepsilon = \frac{A_p(i)}{A_e} \quad (6.2)
$$
Figure 6.13: The final background distribution (in gray / red) overlaid on the fusion neutron distribution. The left panel shows that the flat background from 7000 ns to 15000 ns overlays completely with the fusion neutron distribution. The right panel shows the details of the initial part of these spectra.

where \( i \) is the bin number that ranges over the entire fusion neutron time distribution. The final output of the efficiency function for the pre electron window is shown in Fig. 6.12.

Figure 6.14: Left panel shows the efficiency function and right shows the final background distribution obtained after systematically shifting the efficiency function by \( \pm 20 \) ns relative to the original alignment.

The time distribution obtained from the pre electron condition (shown in gray
(red) in Fig. 6.11) was divided by this efficiency function to obtain the final background distribution shown in gray (red) in the left panel of Fig. 6.13. The right panel of Fig. 6.13 is an enlarged view of the left panel where we can clearly see a step function from 0 to 600 ns. Finally the background distribution was removed from the fusion neutron time distribution to obtain a background free distribution $N_f$, which we would ultimately fit. This is given by,

$$N_f = N_e - \frac{N_{pre_e}}{\varepsilon}$$  \hspace{1cm} (6.3)

### 6.4 Error Propagation of Fusion Neutron Time Distribution

The final signal $N_f$ obtained after background subtraction had bins with really low statistics (sometimes zero too) corresponding to later times greater than $\approx 10000$ ns. This resulted in no errors associated with these bins that could not be physically interpreted. This would also hinder the fitting procedure yielding incorrect results. Thus, the distribution was re-binned by a factor of 10 beyond 2000 ns. From Eq. (6.3) we calculated the errors after incorporating the correlation between $N_e$ and $N_{pre_e}$ which are given by,

$$\Delta N_f = \sqrt{\left(1 - \frac{1}{\varepsilon}\right)^2 N_{pre_e} + (N_e - N_{pre_e})}$$  \hspace{1cm} (6.4)

The errors from the very early times (0 to 600 ns) of the background distribution were not reliable as the distribution itself is not well understood. A systematic effect on error from this region due to alignment of the efficiency function is considered in the section below.

![Figure 6.15: The errors on the final distribution after propagating the errors.](image)

Figure 6.15: The errors on the final distribution after propagating the errors.
6.4.1 Effect of the Alignment of Efficiency Function on the Background Distribution

Here we systematically shift the efficiency function by ±20 ns relative to the original alignment and account for this error in our expression for the final error associated with the fusion distribution. The left panel of Fig. 6.14 shows the efficiency function generated by moving it by 20 ns to the right (in green) and to the left (in blue) respectively. The original efficiency function is shown in magenta. The right panel shows the final background distribution for these three cases and the corresponding errors. The average errors for the left and right shifted spectra relative to the original distribution were added in quadrature to Eq. (6.4) to account for this error. The final errors for the time region from 4000 ns to 10000 ns are shown in Fig. 6.15.
Chapter 7

Analysis of Fusion Neutron Time Distribution

In this chapter we perform a least square fit on the background subtracted fusion neutron time distribution with the theoretical fit function. We also see the effects of changing various cuts and factors that effect the final fit results and thereby evaluate the systematic errors. Finally we find quartet-to-doublet fusion ratio ($\lambda_q : \lambda_d$) and $\mu d$ hyperfine rate ($\lambda_{qd}$) including all errors.

7.1 Theoretical Lifetime Fit Function.

Muonic deuterium undergoes a hyperfine transition from quartet state to doublet state very quickly, which is evident in the early fast lifetime part of the fusion distribution i.e. black (blue) distribution of Fig. 6.11. This component may also include the $d\mu d$ molecular formation rate from the quartet state. The $d\mu d$ molecular formation rate from the doublet state follows this early fast transition and the catalyzing muon is recycled back which decays slowly with a muon decay. This is shown in the slow lifetime part of the fusion distribution in black (blue) in Fig. 6.11. Thus the fit function essentially is the combination of two exponentially decaying functions corresponding to these two lifetimes as given by Eq.(2.14). This equation was re arranged as follows,

$$n(t) = A_2 \left( \frac{A_1}{A_2} e^{-\lambda_1 t} + e^{-\lambda_2 t} \right)$$

such that first parameter is the amplitude $A_2$, the second parameter is the amplitude ratio $A_1 : A_2$, the third parameter is the rate of fast lifetime $\lambda_1$ and the forth parameter is the rate of slow lifetime $\lambda_2$.

7.2 Fitting the Histogram

The final background subtracted fusion histogram was fitted with the theoretical fit function given by Eq.(7.1). This gave us the rate for fast and slow lifetimes along with the amplitude ratio (ratio of amplitude of fast component to slow component as discussed in Sec. 2.3.1 of chapter 2). The difference in rates for fast and slow lifetime would ultimately help in evaluating the hyperfine transition rate $\lambda_{qd}$ and the amplitude ratio will help determine the rate ratio $\lambda_q : \lambda_d$ i.e. ratio of the rates of quartet to doublet population.
Figure 7.1: Effect of including early times in fit range. The fit range in the left panel is from -100 to 2000 ns. The fit range in the right panel is from 100 to 2000 ns. The reduced $\chi^2$ is much better for the right panel.

The measure of the goodness of fit is determined by the $\chi^2$ of the fit that is computed by comparing the data to the theoretical values obtained from the fit function for all fitted bins containing data. It is thus given by,

$$\chi^2 = \sum \frac{(y_i - y(t))^2}{\sigma_i^2}$$  \hspace{1cm} (7.2)

$$\Delta \chi^2 = \sqrt{2 \cdot ndf}$$  \hspace{1cm} (7.3)

where, for each bin $i$, $y_i$ is the data, $y(t)$ is the fit function value, $\sigma_i$ is the uncertainty and $ndf$ is the number of degrees of freedom. The statistically allowed uncertainty associated $\chi^2$, $\Delta \chi^2$, is a simple function of the $ndf$. A reduced (or normalized) $\chi^2$ is the $\chi^2$ for each degree of freedom and its value describes the quality of the theoretical model chosen. A perfect model has $\chi^2/ndf = 1$. A $\chi^2/ndf < 1$ may indicate there may be too many fit parameters and a $\chi^2/ndf > 1$ may indicate that the data requires more parameters than the fit function takes into account [87]. An allowed range of deviations in $\chi^2/ndf = 1 \pm 1/\sqrt{ndf}$.

7.2.1 Selection of Fit Range

The start and stop time of the fit range had to be appropriately selected. It was made sure that the fit should start from a region where the two lifetime theoretical fit function was applicable. Since we lacked in statistics at larger time regions we had to stop the fit where data was really sparse.

7.2.1.1 Effect of Epithermal Muonic Molecular Formation on Fit Range

Data was inconsistent with the theoretical function, in the very prompt region from 0 to about a 100 ns. Initially $d\mu d$ molecules are formed mostly via the resonant state
and sometimes via the non-resonant state, due to epithermal $\mu d$ atoms (due to hot muons before attaining thermal equilibrium). Several $d\mu d$ molecular formation rates [77] from various states overlap producing a very complicated fusion yield distribution within the first 100 ns and so it was decided to start the fit from 100 ns, instead of zero. This is discussed elaborately in Sec. 2.2.3. The effect of start time on the fit is shown in Fig. 7.2.

### 7.2.1.2 Effect of Ambiguous Early Time Background Distribution on Fit Range

It is evident that the final background distribution (shown in red / gray in Fig. 6.13) is not well understood in the early time region. This is the time dependent background that depends on the kicker activation signal received from the $\mu$SC signal and is thus ambiguous till 600 ns. To understand the effect of this early time the start time of the fit range was varied from 100 to 1050 ns and $\lambda_1$ was scanned as shown in Fig. 7.2. In this figure the red band is the allowed deviations in $\lambda_1$ and this clearly shows that value of $\lambda_1$ is inappropriate till 400 or 500 ns. So the fit start time was finally taken to be from 400 ns.

### 7.2.2 Preliminary Fit Results

The fit results for two data sets of about 2000 MIDAS runs are shown in Fig. 7.2.1.2. In this study we used a fit range from 400 ns to 8200 ns. Since we re-binned data, we used integral of function in a bin to fit data, instead of the value of the bin center in our fit options. The temperature of the gas for this data set was about 34 K at a pressure of 5.6 bar and density of 6.14% relative to liquid hydrogen. The fit
Figure 7.3: Preliminary fit results of two data sets at a temperature of 34 K and a density of 6% of liquid hydrogen.

parameters give the rate corresponding to the fast lifetime $\lambda_1$ and slow lifetime $\lambda_2$. The values of $\lambda_1$ and $\lambda_2$ are positive due to the form of the fit function 7.1 and it should be noted that $\lambda_1$ and $\lambda_2$ are now the positive values given by the pair of Eqs.(2.17) and (2.18). These (Eqs.(2.17) and (2.18)) give

$$\lambda_{qd} = \frac{1}{\phi} \left[ \lambda_1 - \lambda_2 - \frac{1}{3}(1+2s)\phi\lambda_q \right] \quad (7.4)$$

Here we ignored $\lambda_d$ and the value $\lambda_q$ is taken from [93]. Thus, we find that $\lambda_{qd}$ depends slightly on $\lambda_q$ (and the density $\phi$). Evaluating this for these two data sets gives, $\lambda_{qd} = 38.18 \pm 0.40 \mu s^{-1}$. At $\lambda_d=0.051 \mu s^{-1}$ [86], an amplitude ratio of $50.31 \pm 1.67$ (from fit result of Fig. 7.2.1.2) gives a rate ratio of

$$\frac{\lambda_q}{\lambda_d} = 81.82 \pm 2.87 \quad (7.5)$$

This rate ratio is found by plotting the amplitude ratio versus the rate ratio from Eq.(2.24), which is shown in Fig. 7.4. This was plotted for three different values of $\lambda_d$ (dotted dashed magenta for $\lambda_d=0.051 \mu s^{-1}$, solid blue for $\lambda_d=0.102 \mu s^{-1}$ and dashed green for $\lambda_d=0.026 \mu s^{-1}$) to check its consistency and dependency on $\lambda_d$. 
Figure 7.4: Plot showing the variation of the amplitude ratio $A_1 : A_2$ with rate ratio $\lambda_q : \lambda_d$ with values of $A_1$ and $A_2$ obtained from fit results corresponding to our experimental conditions i.e. a temperature of 34 K and $\phi = 6\%$ of LH$_2$.

<table>
<thead>
<tr>
<th>$\lambda_d$ (µs$^{-1}$)</th>
<th>$A_1 : A_2$</th>
<th>$\lambda_q : \lambda_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.106</td>
<td>50.31 ± 1.67</td>
<td>79.05 ± 2.68</td>
</tr>
<tr>
<td>0.051</td>
<td>50.31 ± 1.67</td>
<td>81.82 ± 2.87</td>
</tr>
<tr>
<td>0.027</td>
<td>50.31 ± 1.67</td>
<td>87.99 ± 3.31</td>
</tr>
</tbody>
</table>

Table 7.1: Table showing the rate ratio obtained from the amplitude ratio for three different values of $\lambda_d$.

It is evident that the results did not depend much on $\lambda_d$. This is shown in table 7.1. The results of a previous experiment [83] gives $\lambda_{qd} = 37.0^{+1.3}_{-1.5}$µs$^{-1}$ and $\lambda_q : \lambda_d = 79.5 \pm 8.0$ at a density of 4.8% liquid hydrogen. Our results agree very well within one standard deviation range when compared to previous results with similar experimental conditions of the temperature, pressure and density of the deuterium gas. At higher densities the collision between the atoms could cause a faster hyperfine transition from quartet to doublet state resulting in a larger value of $\phi \lambda_{qd}$. The reduced $\chi^2$ of the fit is 1.08, which is within the acceptable range.
7.3 Systematic Studies for Cut Optimization

In this section, various definition that contribute to the sources of errors have been changed systematically to study their effect on the fit results and $\chi^2$ of the lifetime distribution. This will help in eventually selecting the window or cut with best results and also help in determining the systematic errors and the stability of the fit results. For convenience all cuts applied on various sources (e.g. muon stop definitions, energy cuts etc.) and their results are listed in table 7.2 and shown in Fig. 7.5. It was observed that the new (changed) windows did not have a significant effect on the fit results or on the $\chi^2$. Most of the results agreed well within the error bars. The $\chi^2$ of the fits were also very stable in most of the cases as is evident from table 7.2.

The fusion neutron energy cut was varied by 10% as the energy calibration was known well within 10%. The lower limit of the pre electron cut was chosen to be -20000 ns to be completely in the pile up protection window. The upper limit was taken to be -10000 ns as above this limit the distribution is not flat and there is a small effect of the slow lifetime (muon decay) component. The neutron threshold cut was applied to test if gamma rays and other pulses were accidentally contaminating the distribution or not. For a cut less than the lower limit (threshold energy decreased by 100 channels) there could be no way distinguish between neutrons and other pulses. A further increase in the upper limit of the threshold energy from 100 channels will cut off many neutrons as is evident from Fig. 5.3 (one keVee $\approx$ 5 channels). The muon stop definition had the most prominent effect on the fit results and $\chi^2$ and so we discuss this in detail here and thereby justify the range of cuts applied. The effect of all other systematic errors are listed in table 7.2. A detailed report of all the fit results and plots for all systematic studies is discussed in appendix B A.2.

We also considered the effect of epithermal muons on the final results. The energy corresponding to epithermal muons is greater than 1 eV [86] and deuterium gas thermalizes after about 100 ns from Sec. 2.2.3. The $d\mu d$ formation rate for this energy is about four time greater than that at our experimental conditions (i.e. at 34 K). At this fusion rate before thermalization of the gas its population increases by 0.1% in about 100 ns. To be conservative we take a 1% additional population of this state which effects the amplitudes $A_1$ and $A_2$, but does not effect the rates $\lambda_1$ and $\lambda_2$. So we conclude that epithermal muons do not effect $\lambda_{qd}$ but change $\lambda_q : \lambda_d$. From Eq.(2.24) and plot 7.4 we get an error of 0.48 in $\lambda_q : \lambda_d$ due to epithermal muons.

7.3.1 Effect of the Muon Stop Definition

An efficient and robust muon stop definition is essential to isolate fusion neutrons. It could be possible that the muon appears to stop in the TPC, but actually stops elsewhere that could lead to a flawed muon stop definition. The details of the muon stop definition were discussed in Sec. 4.6.2. Here we systematically change various parameters of the muon stop definition and check its effect on the fusion neutron time distribution and the fit results. The parameters changed are the fiducial volume, S-energy and stop in Z - pad.
Table 7.2: Table summarizing the results of all systematic studies. The original cuts are mentioned in brackets in the first column and the Fig. 7.2.1.2 shows the results of the original fit.

1. **Fiducial volume cut:** We increased the volume to be greater than the volume corresponding to the ROI (defined in Sec. 4.6.2). The coordinates of this increased volume are given by $X = \pm 52.0$ mm, $Z = \pm 72.4$ mm and $Y$ ranges from 0 to 72.0 mm. The coordinates of the ROI are $X = \pm 48.75$ mm, $Z = \pm 64.0$ mm and $Y$ ranges from 0 to 71.0 mm. Next we reduced the fiducial volume to coordinates $X = \pm 42$ mm, $Z = \pm 62$ mm and $Y$ ranging from 5 to 75 mm. A large change in fiducial volume changes the results and $\chi^2$ drastically proving the sensitivity of the fiducial volume cut and its importance on the muon stop condition. This cut allows us to eliminate muon captures from the high Z materials near the walls and surrounding of the TPC. Thus care was taken not to be in the vicinity of the walls of the TPC when we varied the fiducial volume.

2. **S-Energy cut:** The default S-energy cut for the muon stop definition was $> 300$ channels (430 keV). We could not decrease it and so took two values of S-energy $> 600$ channels and S-energy $> 900$ channels. The number of fusion neutrons was greater for S-energy $> 600$ compared to S-energy $> 900$ which was sensible. Increasing the S-energy beyond 900 was not sensible as this could lead to eliminate pulses due to muon stops with lower energy. The maximum S-energy deposited by a muon stop on a stopping pad is close to 1000 channels [89] and so we selected 900 channels as the upper limit of the S-energy.

3. **Stop in Z - pad:** The default value was $\geq 3$ (thus we could not decrease it) and we increased it to $\geq 4$ (this number is also explained in Sec. 4.6.2) as a result of which the number of fusion neutrons decreased. A value $> 4$ could potentially include more muons that stop close to the walls of the TPC thereby contaminating the background distribution with muon captures from the high Z materials.
Figure 7.5: Systematic errors for rate corresponding to fast lifetime (upper panel), amplitude ratio (middle panel) and $\chi^2/\text{NDF}$ (lower panel). The dotted (green) line denotes the fit values from the original cuts to get the final results. All cuts produce stable results that agree within the error bars of the original cut.

materials near the walls. All results except $A_2$ (as the background is effected), agreed well within the error bars.

7.3.2 Final Systematic Uncertainties

The systematic uncertainties listed in table 7.2 associated with each source were considered to be independent of each other. The total systematic uncertainty was evaluated for $\lambda_{qd}$ (from Eq. (7.4)) and $\lambda_q : \lambda_d$ (from the plot in Fig. 7.4). The final results associated with each source are listed in table 7.3.

Evaluation of systematic uncertainty in $\lambda_{qd}$:

Two cuts different from the original cut was applied. The average of the difference of the results from these cuts and results of the original cut was the systematic error for
Figure 7.6: Systematics for muon stop cuts. The top panel shows the fiducial volume cuts, the middle panel shows the S-Energy cuts and the bottom panel shows the cuts for stop in Z-pads respectively for the mustop condition.
that particular source of error (e.g. fusion neutron energy cut). Finally errors from each source were considered independent of each and so the final systematic error was taken to be the quadrature of these errors. To illustrate the procedure used to evaluate the systematic uncertainties in \( \lambda_{qd} \), we take the an example of the fusion neutron energy cut (the first cut of table 7.2). When \( E < 2.45 \text{ MeV} \), we get the original value of the fit. Let \( \Delta \lambda_1 \) and \( \Delta \lambda'_1 \) be the uncertainties in \( \lambda_1 \) arising due to the cuts \( E < 2.7 \text{ MeV} \) and \( E < 2.2 \text{ MeV} \) respectively. From the plots in appendix B A.2 we get the values of \( \Delta \lambda_1 = |2.936 - 2.938| = 2 \times 10^{-3} \text{ } \mu s^{-1} \) and \( \Delta \lambda'_1 = |2.936 - 2.936| = 0 \text{ } \mu s^{-1} \) (where 2.936 \( \mu s^{-1} \) is the default value of \( \lambda_1 \) from the original fit).

The final error in \( \lambda_1 \) is the mean of these, \( \Delta \lambda_1 = 1 \times 10^{-3} \text{ } \mu s^{-1} \). Similarly we have \( \Delta \lambda_2 = 0.0005 \times 10^{-2} \mu s^{-1} \). Thus the total error in \( \lambda_{qd} \) by taking the partial derivative of Eq. (7.4) is given by,

\[
\Delta \lambda_{qd} = \frac{1}{\phi} \sqrt{(\Delta \lambda_1)^2 + (\Delta \lambda_2)^2 + \left(1 + 2s\right)^2 \phi^2 (\Delta \lambda_q)^2} \tag{7.6}
\]

The value of \( \Delta \lambda_q \) is taken to be \( 3.98(5) \text{ } \mu s^{-1} \) [1], [86]. This gives \( \Delta \lambda_{qd} = 1.6 \times 10^{-3} \mu s^{-1} \) for the energy cut of 2.45 MeV due to fusion neutrons. All errors for other cuts (sources) of \( \lambda_{qd} \) are listed in table 7.3.

**Evaluation of systematic uncertainty in \( \lambda_q : \lambda_d \):**

Based on a similar principle described above, we first found \( \left( \lambda_q : \lambda_d \right) \) corresponding to the cut \( E < 2.7 \text{ MeV} \) and \( \left( \lambda_q : \lambda_d \right)' \) corresponding to the cut \( E < 2.2 \text{ MeV} \) and took their differences from the original value of \( \lambda_q : \lambda_d \) (for \( A_1 : A_2 = 50.31 \)). The rate ratio was read from the amplitude ratio using the plot in Fig. 7.4. Thus we took the average of \( \Delta \left( \lambda_q : \lambda_d \right) \) and \( \Delta (\lambda_q : \lambda_d)' \) to find the final error \( \Delta (\lambda_q : \lambda_d) \) for this cut. All these errors are listed in table 7.3.

Finally, we analyzed about nine consistent data sets. The final plot of a sum of all the data sets is shown in Fig. 7.3.2. The results including the systematic and statistical errors for sum of all the data sets are as follows:

\[
\frac{\lambda_q}{\lambda_d} = 85.51 \pm 1.61_{\text{stat}} \pm 2.82_{\text{sys}}
\]

\[
\lambda_{qd} = 38.49 \pm 0.21_{\text{stat}} \pm 0.031_{\text{sys}} \text{ } \mu s^{-1}
\]

The effect of the changes in \( \lambda_{qd} \) is much less as it is a very strong signal and is not very sensitive to background effects and changes, where as \( \lambda_q : \lambda_d \) is very sensitive to background changes as it is a very weak signal (and has much lower statistics compared to \( \lambda_{qd} \)). The maximum contribution to the systematic uncertainties is from the fiducial volume as the muon stop definition and other background effects are very sensitive to this. We find the fit to be quite stable as is evident from the plot of residues as shown in the right panel of Fig. 7.3.2.

There was a discrepancy between the experimentally obtained value of \( \lambda_2 \) and its calculated value. To understand this, we studied the effect of impurities in our final
Figure 7.7: Left panel: Fit results of all nine data sets at a temperature of 34 K and a density of 6% of liquid hydrogen. Right panel: The plot of residues does not show any form of inconsistency.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta \lambda_1$ ($\mu s^{-1}$)</th>
<th>$\Delta \lambda_2$ ($\mu s^{-1}$)</th>
<th>$\Delta \lambda_{qd}$ ($\mu s^{-1}$)</th>
<th>$\Delta (A_1 : A_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Cut</td>
<td>0.001</td>
<td>0.0005</td>
<td>0.0016</td>
<td>0.257</td>
</tr>
<tr>
<td>Pre Electron</td>
<td>0.003</td>
<td>0.004</td>
<td>0.0051</td>
<td>0.675</td>
</tr>
<tr>
<td>Neutron threshold energy</td>
<td>0.0035</td>
<td>0.0018</td>
<td>0.0042</td>
<td>0.539</td>
</tr>
<tr>
<td>Fiducial volume</td>
<td>0.027</td>
<td>0.0079</td>
<td>0.028</td>
<td>2.382</td>
</tr>
<tr>
<td>S-Energy change</td>
<td>0.0025</td>
<td>0.0004</td>
<td>0.0027</td>
<td>0.161</td>
</tr>
<tr>
<td>Stop in-Z pads</td>
<td>0.011</td>
<td>0.0006</td>
<td>0.012</td>
<td>1.109</td>
</tr>
<tr>
<td>Epithermal</td>
<td></td>
<td></td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>Total Uncertainty</td>
<td>$\pm 0.029$</td>
<td>$\pm 0.009$</td>
<td>$\pm 0.031$</td>
<td>$\pm 2.82$</td>
</tr>
</tbody>
</table>

Table 7.3: Table summarizing the systematic errors associated with each cut.

Results. The main source of impurities were $N_2$ and $O_2$ gases. The concentration of these impurities were found using chromatographic tests. The concentrations of $N_2$ and $O_2$ were found to be 1 ppb (parts per billion) and < 0.4 ppb respectively [94]. If these impurities caused the deviation of experimental value from theory then the deviation should yield the above mentioned concentration of $N_2$ and $O_2$ obtained from the equation below:

$$\Delta \lambda_2 = C_N \phi \lambda_{dN} + C_O \phi \lambda_{dO}$$  \hspace{1cm} (7.7)

where $C_N$ and $C_O$ are concentrations of $N_2$ and $O_2$ respectively and $\lambda_{dN}$ and $\lambda_{dO}$ are transfer rate of deuterium with $N_2$ and $O_2$ respectively. We took $\lambda_{dN} = 14.5 \times 10^{10}s^{-1}$ and $\lambda_{dO} = 6.3 \times 10^{10}s^{-1}$ [1]. These yielded the concentrations of $N_2$ and $O_2$ to be a few ppm which did not match the chromatography results and were thus rejected to be the cause of this deviation.
7.4 Consistency Between Data Sets

A data set is a collection of about 1000 MIDAS run, which had some common properties and enough statistics to analyze the fusion neutron lifetime distribution. They were created based on several criteria like the operating high voltages (HV) of the ePC’s, momentum of the muon beam and shielding of neutron detectors with mu-metal. Nine data sets were selected for our analysis and comparison and these were all shielded. The other properties are listed in table 7.4. Some data sets that had all the same criteria were grouped in chronological order containing about 1000 runs (example data sets 4 - 6 in table 7.4).

The fit range was chosen to be from 400 to 8200 ns for all the data sets. The results agreed well within the error bars. The fit results for all data sets are shown in Fig. 7.8. The left panel shows the results for $\lambda_{qd}$ and the right panel shows $\lambda_q : \lambda_d$. The straight line indicates the result of all data sets added together. Data set 7 and 8 had much fewer runs compared to other data sets which accounts for the large error bars in their results.

<table>
<thead>
<tr>
<th>Data Sets</th>
<th>ePC1 - HV (V)</th>
<th>ePC2 - HV (V)</th>
<th>Beam Momentum Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2630</td>
<td>2830</td>
<td>-1.45</td>
</tr>
<tr>
<td>2,3</td>
<td>2630</td>
<td>2800</td>
<td>-1.45</td>
</tr>
<tr>
<td>4-6</td>
<td>2630</td>
<td>2850</td>
<td>-1.45</td>
</tr>
<tr>
<td>7,8</td>
<td>2630</td>
<td>2850</td>
<td>-1.44</td>
</tr>
<tr>
<td>9</td>
<td>2630</td>
<td>2800</td>
<td>-1.44</td>
</tr>
</tbody>
</table>

Table 7.4: Table summarizing the criteria for creating data sets used for comparison of consistency of results. The HV of ePC1 is same for all data sets.

Figure 7.8: Fit results for all nine data sets.
7.5 Correlation Between the Fit Parameters

The fit parameters of our fit function i.e. amplitude ratio, $\lambda_1$ and $\lambda_2$ are correlated with each other as shown in Table 7.5. It shows a high correlation between $A_2$ and $\lambda_2$ and an anticorrelation (i.e. if one quantity increases the other decreases and vice versa) between $A_1 : A_2$ and $\lambda_2$.

<table>
<thead>
<tr>
<th></th>
<th>$A_2$</th>
<th>$A_1 : A_2$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_2$</td>
<td>1</td>
<td>-0.91897</td>
<td>0.66077</td>
<td>0.9394</td>
</tr>
<tr>
<td>$A_1 : A_2$</td>
<td>-0.91897</td>
<td>1</td>
<td>-0.33019</td>
<td>-0.88537</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.66077</td>
<td>-0.33019</td>
<td>1</td>
<td>0.56589</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.9394</td>
<td>-0.88537</td>
<td>0.56589</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7.5: The table lists the elements of the covariant matrix which shows the correlation between the fit parameters.

This also shows that $\lambda_1$ does not have a great effect on the amplitude ratio whereas $\lambda_2$ does. The negative sign indicates an anticorrelation between the parameters.

7.6 Consistency of Fit Range

In this section we study the effect the stability of the fit range on the fit parameters. We changed the start time keeping the stop time constant (and vice versa) and found these parameters to be quite consistent. The scan of the start or stop time is a subset of the entire fixed start and stop time and so a set-subset variance of the allowed errors on the plots showing this scan had to be considered and evaluated correctly. This is because each subset of fit is highly correlated with the entire set as it is obtained by successively excluding one time bin as we walk our way through the entire scan range. These allowed deviations are evaluated from the difference of the variance of the subset range and the complete set range [88]. This is given by the equation below:

$$\langle (x_{1i} - x_{2i})^2 \rangle = \sigma_{2i}^2 - \sigma_{1i}^2$$

(7.8)

where $x_{1i}$ is the value of the $i^{th}$ fit parameter for the entire range of data (denoted by 1) and $x_{2i}$ is the value of the $i^{th}$ fit parameter for the subset range of data (denoted by 2) at later start or stop times. Similarly $\sigma_{1i}$ and $\sigma_{2i}$ are the variances of the $i^{th}$ fit parameter of the entire range and the subset range of data respectively. The errors in reduced $\chi^2$‘s are determined using Eq.(7.3).

7.6.1 Start Time Scan

The start time of the fit range was scanned from 400 ns to a maximum of 1300 ns keeping the stop time fixed at 8200 ns, to see its effect on the fit results and $\chi^2$‘s. The variation of the rate corresponding to the fast lifetime and the amplitude ratio with fit start time are shown in the upper left and right panels of Fig. 7.9. The solid red line in all these plots indicates an envelop of one standard deviation band allowed
Figure 7.9: Start time scan results for the fit parameters and $\chi^2$.

for a “random walk” of the fit parameters ($\lambda_1$ and $A_1 : A_2$) [88], evaluated using Eq.(7.8) centered around the initial value (which is arbitrary). The errors in reduced $\chi^2$’s plotted for this scan is shown in the bottom right panel of Fig. 7.9. The ndf for our data set is around 100 which allow a deviation of $1/\sqrt{\text{ndf}} \approx 0.1$ in the values of reduced $\chi^2$. From the bottom right panel of Fig. 7.9 the range of acceptable values of reduced $\chi^2$ for start time scan is from 400 to 1200 ns. From the other panels of this figure we find that the fit parameters are also stable and in the allowed band for this time range.

7.6.2 Stop Time Scan

The stop time of the fit range was scanned from 4200 ns to a maximum of 8200 ns keeping the start time fixed at 400 ns, to study its effect on the fit results and $\chi^2$’s. The variation of the fast lifetime and the amplitude ratio with fit stop time are shown in the left and right panels of Fig. 7.9. The plots showing the variation of the fit parameters with fit stop time have an envelop of one standard deviation allowed “random walk” centered around the final value in this case. This figure shows that
the fit parameters and the $\chi^2$s are stable and in the allowed band for this time range.
Chapter 8

Conclusion

An important goal of this dissertation is to determine the time-integrated relative population of the $\mu d$ atom in the doublet state, in the experimental conditions of the MuSun experiment. We define the time-integrated population of a state as the integral of the population of that state (e.g. doublet state $n_{1/2}$) starting from the fit start time $t_0$ given by,

$$N_d(t_0) = \int_{t_0}^{\infty} n_{1/2}(t)dt$$  \hspace{1cm} (8.1)

where $n_{1/2}$ is the populations of doublet at any instant of time given by the Eq.(2.19). Similarly $N_q$ is the same for the quartet state with $n_{3/2}$ given by Eq.(2.20). $N_d(t_0)/N(t_0)$ is extremely important to extract the muon capture rate in deuterium from the doublet state i.e. the ultimate goal of the MuSun experiment. This is obtained from the fusion neutron analysis that was discussed in the previous chapter. Here we plot the population of all states of the $\mu d$ atom and $N_d(t_0)/N(t_0)$ with time, using

Figure 8.1: Variation of the relative population of the doublet state (in dashed magenta), the quartet state (in dotted dashed blue) and total population (in solid black) with time.
the experimental results in the theoretical Eqs. (2.19) and (2.20). Detailed study of $N_d(t_0)/N(t_0)$ within the error ranges of $\lambda_{qd}$, $\lambda_q : \lambda_d$ will be done owing to the importance of $N_d(t_0)/N(t_0)$ in MuSun results. Finally, we compare our results with the results of the previous experiments.

### 8.1 Population of the $\mu d$ Atom in Various States

To recall, I plot the variation of populations of the two states of the $\mu d$ atom in Fig. 8.1, using values of $\lambda_q : \lambda_d$ and $\lambda_{qd}$ from our fit results. The plots are same as the plots corresponding to the analytical solutions. In the plot 8.1 we assumed $\lambda_d = 0.051 \text{ } \mu s^{-1}$ [90]. To understand and show the effect of recycling more clearly, (which would mean a non-zero relative population of quartet state and a relative population slightly less than unity for the doublet state at a later time) we plot the time distributions of both these states. This is shown in Fig. 8.1 with quartet state distribution in the left panel and doublet state distribution in the right panel.

Next we plot the time distribution of $n_{1/2}$ and $n_{3/2}$ by varying the value of $\lambda_d$. We use three values of $\lambda_d$, 0.102 $\mu s^{-1}$, 0.051 $\mu s^{-1}$ and 0.025 $\mu s^{-1}$ to see its effect on our data by plotting the relative populations of the $n_{1/2}$ and $n_{3/2}$ in the left and right panels of Fig. 8.3 respectively. It is evident that even such a very large variation in $\lambda_d$ does not change the population of the state considerably.

![Figure 8.2](image-url)

**Figure 8.2:** Variation of the population of the quartet state (left panel) for later times. Right panel shows the same for the doublet state at later times to show the effect of recycling.

Finally we plot the time distribution of $n_{2/3}$ and $n_{1/3}$ with our experimental errors on $\lambda_q : \lambda_d$, $\lambda_1$, $\lambda_2$. This is shown in Fig. 8.4, left panel showing the quartet and the right panel showing the doublet state population respectively. The solid black line shows the original experimental values (with no errors), the magenta dashed line shows the distribution with errors added to the original experimental values and the blue dotted dashed line shows the distribution with errors subtracted from the original experimental values.
8.2 Time-integrated Relative Population of the Doublet State

The time-integrated relative population of the doublet state \((N_d(t_0)/N(t_0))\) using the definition of \(N_d\) given by Eq. (8.1), can be written as,

\[
\frac{N_d(t_0)}{N_d(t_0) + N_q(t_0)} = \frac{\int_{t_0}^{\infty} n_{1/2}(t)dt}{\int_{t_0}^{\infty} n_{1/2}(t)dt + \int_{t_0}^{\infty} n_{3/2}(t)dt} \tag{8.2}
\]

This is shown in Fig. 8.5. This distribution is extremely important for finding the doublet capture rate \(\Lambda_d\). It shows the effect of recycling more prominently compared to other plots. At much later times around 5 \(\mu s\) the fraction of doublet population is 0.9992 which is slightly < 1 due to recycling.
Figure 8.5: Variation of $N_d(t_0)/N(t_0)$ with time. At later times it is $< 1$ which is due to recycling of muons.

**Error propagation of time-integrated relative population of the doublet state:**

Due to the correlation between the fit parameters of our fit function as discussed in the previous chapter and shown in table 7.5, we study the effect of various fit parameters on the error propgation of $N_d(t_0)/N$. The populations of quartet and doublet states are highly correlated and so the error propagation of expression 8.2 was avoided. To understand the effect of the errors due to the fit parameters we investigate the effect of the fit results $\lambda_q : \lambda_d$ and $\lambda_{qd}$ on $N_d(t_0)/N$. Table 7.5 does not show the correlation of $\lambda_{qd}$ with the amplitude ratio. Thus we redefine our fit function with $\lambda_{qd}$ as a fit parameter and find the correlations between the new fit parameters. This is listed in table 8.1. It is evident that the correlation between $\lambda_{qd}$ and $A_1 : A_2$ is negligibly small (correlation coefficient of 0.012) and so we can consider the effect of $\lambda_{qd}$ and $A_1 : A_2$ separately in the distribution of $N_d(t_0)/N$.

**Effect of $\lambda_{qd}$ on $N_d(t_0)/N$:**

We compare the plots of $N_d(t_0)/N$ with $\lambda_{qd} = 38.28 \mu s^{-1}$ in a dotted dashed blue line, $\lambda_{qd} = 38.49 \mu s^{-1}$ in a solid black line and $\lambda_{qd} = 38.70 \mu s^{-1}$ in a dashed magenta line in the left panel of Fig. 8.6. As is evident there is no major difference between the minimum and maximum values. Even at early times the approximate range of difference between the minimum and maximum value is of the order of $10^{-4} \mu s^{-1}$. After 2.5 $\mu s$ the three plots almost coincide.
Effect of $\lambda_q : \lambda_d$ on $N_d(t_0)/N$:

The amplitude ratio $A_1 : A_2$ would have the same effect on $N_d(t_0)/N$ as $\lambda_q : \lambda_d$. Thus, we compare the plots of $N_d(t_0)/N$ with $\lambda_q : \lambda_d = 81.26$ in dotted dashed blue line, $\lambda_q : \lambda_d = 85.51$ in a solid black line and $\lambda_q : \lambda_d = 88.76$ in a dashed magenta line in the right panel of Fig. 8.6. In this case the difference between the minimum and maximum values is even less. Even at early times the approximate range of difference between the minimum and maximum value is of the order of $10^{-6}$. Thus we conclude that the errors are extremely small and the time-integrated relative population of the doublet state is measured with very good precision.

<table>
<thead>
<tr>
<th></th>
<th>$A_2$</th>
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<th>$\lambda_2$</th>
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<td>$A_2$</td>
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<td>0.9394</td>
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<tr>
<td>$A_1 : A_2$</td>
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<td>0.0119</td>
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<tr>
<td>$\lambda_{qd}$</td>
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<td>0.011913</td>
<td>1</td>
<td>0.21506</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.9394</td>
<td>-0.88537</td>
<td>0.21506</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 8.1: The table shows the correlation between the new fit parameters.

Figure 8.6: Error range when $\lambda_{qd}$ is changed in the left panel and the right panel shows the same for $\lambda_q : \lambda_d$.

8.3 Comparison of Our Results with Previous Experiments

I studied a few previous experiments and found the values $\lambda_q : \lambda_d$ and $\lambda_{qd}$ from our experiment is similar to that of these previous experiments in the similar physical condition of temperature and density. It is known that these values are density dependent and thus have been scaled by the density. The values of $\lambda_q : \lambda_d$ agree within a standard deviation and $\lambda_{qd}$ agrees within two standard deviations for similar experimental conditions. All this proves a good agreement of our results with the results of previous experiments. I have listed these values in table 8.2. We obtain quite stable results which would help us find the relative populations of $\mu d$ in quartet...
<table>
<thead>
<tr>
<th>Experiment</th>
<th>Year</th>
<th>$\lambda_{qd}$ ($\mu$s$^{-1}$)</th>
<th>$\lambda_q : \lambda_d$</th>
<th>Density (rel to LH2)</th>
<th>Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIN [83]</td>
<td>1983</td>
<td>37.0$^{+1.3}_{-1.7}$</td>
<td>79.5(8)</td>
<td>4.8%</td>
<td>34.7</td>
</tr>
<tr>
<td>PSI [91]</td>
<td>1987</td>
<td>36.89(0.8)</td>
<td>80.17(7.8)</td>
<td>4.83%</td>
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<tr>
<td>JINR [92]</td>
<td>1991</td>
<td>37.84(21)</td>
<td>65.51(0.59)</td>
<td>4.9%</td>
<td>53</td>
</tr>
<tr>
<td>PSI (PNPI) [93]</td>
<td>2011</td>
<td>37.1(3)</td>
<td>80.98(1.59)</td>
<td>5.14%</td>
<td>32.2</td>
</tr>
<tr>
<td>This work</td>
<td>2014</td>
<td>38.49(0.21)</td>
<td>85.51(3.25)</td>
<td>6.12%</td>
<td>34</td>
</tr>
</tbody>
</table>

Table 8.2: The table compares results from various previous experiments with this work. The experiments are listed in chronological order and doublet states under our experimental conditions. The relative population of the doublet state from this analysis will be further used by MuSun to ultimately find $\Lambda_d$. 

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Appendix A: Derivation of various formulae used the Analytical solutions

In this appendix part I will derive the details of a few formula used and also derive in
detail the coupled differential equations for the analytical solution of the kinematic
equations governing the muon chemistry in deuterium gas.

A.1 Derivation of EMG (exponentially modified Gaussian function)

The EMG was used to find the rise time of all neutron detectors. The \( \sigma \) is for the
Gaussian due to the detector’s resolution and \( \tau \) is the lifetime of the exponential decay
due to the time of flight of neutrons. I used mathematica to derive this function.

\[
\int_{-\infty}^{\infty} e^{-\frac{(t-t')^2}{2\sigma^2}} e^{-t'/\tau} dt'
\]

Evaluating the above integral (using mathematica) and further simplifying taking the
positive roots we get,

\[
\frac{e^{\frac{\sigma^2-2\tau t}{2\tau^2}} \sqrt{\frac{\tau}{\pi}}}{\sqrt{\frac{\tau}{\pi}}} \left( \frac{\tau}{\sigma^2 \tau^2} \left( \frac{(\sigma^2-(t+t_0)\tau)}{\sigma^2 \tau^2} \right)^2 + \frac{1}{\sigma^2 \tau^2} (-\sigma^2+(t+t_0)\tau) \text{Erf} \left[ \frac{\sqrt{\frac{(\sigma^2-(t+t_0)\tau)}{\sigma^2 \tau^2}}}{\frac{\sigma^2 \tau^2}{\sqrt{2}}} \right] \right) \right), \text{Re} \left[ \sigma^2 \right] > 0
\]

\[
= e^{\frac{\sigma^2-2\tau t}{2\tau^2}} \sqrt{\frac{\tau}{\sigma^2 \tau^2}} \left( \frac{\tau}{\sigma^2 \tau^2} \left( \frac{(\sigma^2-(t+t_0)\tau)}{\sigma^2 \tau^2} \right)^2 - \frac{1}{\sigma^2 \tau^2} \frac{(\sigma^2-(t+t_0)\tau)}{\sigma^2 \tau^2} \text{Erf} \left[ \frac{\sqrt{\frac{(\sigma^2-(t+t_0)\tau)}{\sigma^2 \tau^2}}}{\frac{\sigma^2 \tau^2}{\sqrt{2}}} \right] \right) \right)
\]

\[
= e^{\frac{\sigma^2-2\tau t}{2\tau^2}} \sqrt{\frac{\tau}{\sigma^2 \tau^2}} \left( 1 - \frac{1}{\sigma \tau} \frac{(\sigma^2-(t+t_0)\tau)}{\sigma \tau} \text{Erf} \left[ \frac{\sqrt{\frac{(\sigma^2-(t+t_0)\tau)}{\sigma^2 \tau^2}}}{\frac{\sigma^2 \tau^2}{\sqrt{2}}} \right] \right) \right)
\]

\[
= ke^{-\frac{t}{\tau}} \left( 1 - \text{Erf} \left[ \frac{(\sigma^2-(t+t_0)\tau)}{\sigma \tau \sqrt{2}} \right] \right)
\]

Just to recall, here \( t_0 \) is the rise time of each detector due to different HV’s.
A.2 Derivation of the Analytical solutions of population of states

For simplicity we assumed the coefficients of \( n_{1/2} \) and \( n_{3/2} \) in the above set of Eqs. (2.6) as a, b, c and d. Thus the problem now reduces down to solving the eigen values and eigen vectors of the matrix representation of the equation in terms of a, b, c and d as shown below

\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}
\]

The eigenvalues of this matrix is given by,

\[
\left\{ \frac{1}{2} \left( a + d - \sqrt{a^2 + 4bc - 2ad + d^2} \right), \frac{1}{2} \left( a + d + \sqrt{a^2 + 4bc - 2ad + d^2} \right) \right\}
\]

(3)

The eigen vectors of this matrix in general are given by

\[
\left\{ \frac{-a + d + \sqrt{a^2 + 4bc - 2ad + d^2}}{2c}, 1 \right\}, \left\{ \frac{-a + d - \sqrt{a^2 + 4bc - 2ad + d^2}}{2c}, 1 \right\}
\]

(4)

The above general equations will be used throughout to find eigenvalues and eigenvectors for different cases.

\[
a = -\phi \lambda_{qd} - \phi \lambda_q - \lambda_\mu
\]

\[
b = 0
\]

\[
c = \phi \lambda_{qd}
\]

\[
d = -\phi \lambda_d - \lambda_\mu
\]

The general solution to the set of coupled differential equation represented by Eq.(2.7) is given by,

\[
\begin{pmatrix}
n_{3/2} \\
n_{1/2}
\end{pmatrix} = a_1 e^{\lambda_1 t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + a_2 e^{\lambda_2 t} \begin{pmatrix} Y \\ 1 \end{pmatrix}
\]

Next we plug in the initial conditions to find \( a_1 \) and \( a_2 \). Initially at \( t = 0 \), the population of doublet and quartet states is proportional to \( 1/3 \) and \( 2/3 \) respectively,
owing to their relative possible spin states, as explained previously. This gives us,
\[
\begin{pmatrix}
\frac{2}{3} \\
\frac{1}{3}
\end{pmatrix}
= 
\begin{pmatrix}
Ya_2 \\
a_1 + a_2
\end{pmatrix}
\]

From the above we can readily see that,
\[a_1 = \frac{(Y - 2)}{3Y} \text{ and } a_2 = \frac{2}{3Y}\]

**Including recycling:** We find that the eigenstate matrix is given by,
\[
\begin{pmatrix}
-\phi \lambda_d - \lambda_\mu + \frac{1}{3}(1 - s)\phi \lambda_d & \phi \lambda_{qd} + \frac{1}{3}(1 - s)\phi \lambda_q \\
\frac{2}{3}(1 - s)\phi \lambda_d & -\phi \lambda_q + \frac{2}{3}(1 - s)\phi \lambda_q - \phi \lambda_{qd} - \lambda_\mu
\end{pmatrix}
\]

Again for simplicity the matrix elements were replaced by a, b c, and d given by the following equations:

\[a = -\phi \lambda_d - \lambda_\mu + \frac{1}{3}(1 - s)\phi \lambda_d \quad (5)\]

\[b = \phi \lambda_{qd} + \frac{1}{3}(1 - s)\phi \lambda_q \quad (6)\]

\[c = \frac{2}{3}(1 - s)\phi \lambda_d \quad (7)\]

\[d = -\phi \lambda_q + \frac{2}{3}(1 - s)\phi \lambda_q - \phi \lambda_{qd} - \lambda_\mu \quad (8)\]

Thus, from Eq.(3) we have,
\[
\lambda_1 = \frac{1}{2} \left( a + d + \sqrt{a^2 + 4bc - 2ad + d^2} \right) \quad (9)
\]

\[
\lambda_2 = \frac{1}{2} \left( a + d - \sqrt{a^2 + 4bc - 2ad + d^2} \right) \quad (10)
\]

Here again we assumed \( \phi \lambda_q \) can be ignored compared to \( \phi \lambda_{qd} \) and \( \lambda_\mu \) which sets \( c = 0 \) and so we have,
\[
\lambda_1 = d \approx -\frac{1}{3}(1 + 2s)\phi \lambda_q - \phi \lambda_{qd} - \lambda_\mu \quad (11)
\]

Similarly the slow lifetime is found to be
\[
\lambda_2 = a \approx -\lambda_\mu - \frac{1}{3}(2 + s)\phi \lambda_d \quad (12)
\]
The complete solution (without any approximation) gives,
\[
\lambda_1 = -\frac{1}{6} ((2+s)\phi \lambda_d + \phi ((1+2s)\lambda_q + 3\lambda_{qd}) + 6\lambda_\mu + \\
\sqrt{\phi^2 ((2+s)^2 \lambda_d^2 + (\lambda_q + 2s\lambda_q + 3\lambda_{qd})^2 + 2\lambda_d ((2+s(-13+2s))\lambda_q + 3(2-5s)\lambda_{qd}))}
\]
(13)

Similarly the second eigen value of the matrix is found to be
\[
\lambda_2 = -\frac{1}{6} ((2+s)\phi \lambda_d + \phi ((1+2s)\lambda_q + 3\lambda_{qd}) + 6\lambda_\mu - \\
\sqrt{\phi^2 ((2+s)^2 \lambda_d^2 + (\lambda_q + 2s\lambda_q + 3\lambda_{qd})^2 + 2\lambda_d ((2+s(-13+2s))\lambda_q + 3(2-5s)\lambda_{qd}))}
\]
(14)

Plugging in the values of a, b, c and d from the above equations without ignoring anything the approximate results for the eigen vectors in matrix form are as follows.
\[
\left( \begin{array}{c}
X_1 \\
1
\end{array} \right) \text{ and } \left( \begin{array}{c}
X_2 \\
1
\end{array} \right)
\]
where \(X_1\) and \(X_2\) are the first elements of the two eigen vectors given by the equations below:
\[
X_1 = \frac{a-d-X}{2c} = \frac{\phi \lambda_q + \phi \lambda_{qd} - \frac{2}{3}(1-s)\phi \lambda_q - X}{\frac{4}{3}(1-s)\phi \lambda_d} 
\]
(15)
\[
X_2 = \frac{a-d+X}{2c} = \frac{\phi \lambda_q + \phi \lambda_{qd} - \frac{2}{3}(1-s)\phi \lambda_q + X}{\frac{4}{3}(1-s)\phi \lambda_d} 
\]
(16)

where we again assumed \(\lambda_{qd} \gg \lambda_d\) and thus ignored all terms involving \(\lambda_d\) compared to \(\lambda_{qd}\) in the numerator. \(X\) is a variable designated for the long expression shown below
\[
X = \sqrt{(a-d)^2 + 4bc} 
\]
(17)
\[
= \sqrt{(\phi \lambda_q + \phi \lambda_{qd})^2 - \frac{4}{3}(1-s)\phi^2 \lambda_q (1+\lambda_{qd})}
\]

The symbols \(X_1, X_2\) and \(X\) have been used just for convenience and simplicity and have no other significance

**Initial conditions:**

The the solution to the set of coupled differential equation represented by Eq.(2.6) is given by,
\[
\left( \begin{array}{c}
\frac{n_1}{2} \\
\frac{n_3}{2}
\end{array} \right) = a_1 e^{-\lambda_1 t} \left( \begin{array}{c}
X_1 \\
1
\end{array} \right) + a_2 e^{-\lambda_2 t} \left( \begin{array}{c}
X_2 \\
1
\end{array} \right)
\]

133
where $a_1$ and $a_2$ are the amplitudes of each state determined from the initial conditions. Initially at $t = 0$, the population of doublet and quartet states is proportional to $1/3$ and $2/3$ respectively, owing to their relative possible spin states, as explained previously.

Thus, for $t = 0$ we have,

$$
\begin{pmatrix}
\frac{1}{3} \\
\frac{2}{3}
\end{pmatrix}
= 
\begin{pmatrix}
a_1 X_1 \\
a_1
\end{pmatrix}
+ 
\begin{pmatrix}
a_2 X_2 \\
a_2
\end{pmatrix}
$$

From the above we can readily see that,

$$a_1 = \frac{(2X_2 - 1)}{3(X_2 - X_1)} \quad \text{and} \quad a_2 = \frac{(1 - 2X_1)}{3(X_2 - X_1)}$$

which gives,

$$
\begin{pmatrix}
n_{1/2} \\
n_{3/2}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{(2X_2 - 1)}{3(X_2 - X_1)} e^{-\lambda_1 t} X_1 + \frac{(1 - 2X_1)}{3(X_2 - X_1)} e^{-\lambda_2 t} X_2 \\
\frac{(2X_2 - 1)}{3(X_2 - X_1)} e^{\lambda_1 t} X_1 + \frac{(1 - 2X_1)}{3(X_2 - X_1)} e^{\lambda_2 t} X_2
\end{pmatrix}
$$

Substituting the values of $\lambda_1$ and $\lambda_2$ from Eqs. 2.17 and 2.18 respectively and then solving the above matrix equation we get the simplified matrix equation as,

$$
\begin{pmatrix}
n_{1/2} \\
n_{3/2}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{X_1 (2X_2 - 1)}{3(X_2 - X_1)} e^{\lambda_1 t} + \frac{X_2 (1 - 2X_1)}{3(X_2 - X_1)} e^{\lambda_2 t} \\
\frac{2X_2 - 1}{3(X_2 - X_1)} e^{\lambda_1 t} + \frac{(1 - 2X_1)}{3(X_2 - X_1)} e^{\lambda_2 t}
\end{pmatrix}
$$

The above matrix gives the complete solution for the population of states from where we find the amplitudes $A_1$ and $A_2$ for the fusion neutron time spectrum.
Appendix B: Systematic studies

The individual fit to the fusion neutron lifetime spectrum employed for systematic studies after tweaking various cuts and definition are discussed in detail and shown in the figures of this section / appendix.

Figure .7: Systematic effects of changing the pre-electron definition cuts.

Figure .8: Systematic effects of changing the energy cuts.
B.3 Changing the pre-electron window

A change in the pre-electron definition would effect the background spectrum. The original cut ensured that the associated electron is from a single pile up protected muon to avoid double counting. A systematic change was applied to test and optimize the best pre-electron window. Fig. 7 shows the effect of changing this cut and its corresponding systematic errors.

B.4 Changing the neutron energy cuts

All fusion neutrons have an energy of 2.45 MeV, but we increased and decreased this cut to see its effect on the fit results. From the left panel of Fig. 6.8, it is evident that there are almost no neutrons with energy > 2.45 MeV in a neutron spectrum with coincident electrons (fusion neutron spectrum). Thus the fit results and number of fusion neutrons for an energy cut >2.45 MeV (for example I took an energy of 5 MeV) will be very close to results with 2.45 MeV. For example the left (E < 2.5 MeV) and rightmost panels (E < 5 MeV) of Fig. 7.2.1.2 have almost same number of fusion neutrons.

B.5 Changing the neutron definition

The PSD cuts to determine neutrons depended on the energy of the pulse. It was obscure to determine neutrons in the very low energy region and also for overflowing pulses. To check the accuracy of the threshold (minimum) and cut off (maximum) energies the energy range of the neutron definition is changed from the default values. We increased the and decreased the cut off and threshold energies and obtained consistent number of fusion neutrons as seen in Fig. 9.

Figure 9: Systematic effects of changing the threshold energy (minimum energy) of neutron definition
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