THE DESIGN OF P200: MENSURAL QUANTITY AND PROPORTION IN THE EARLY VERSION OF THE ART OF FUGUE

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THE DESIGN OF P200:
MENSURAL QUANTITY AND PROPORTION
IN THE EARLY VERSION OF THE ART OF FUGUE

THESIS

A thesis submitted in partial fulfillment
of the requirements for the degree of Master of Arts
in the College of Fine Arts
at the University of Kentucky

By
Joel Thomas Runyan

Cincinnati, Ohio

Director: Dr. Karen Bottge, Professor of Music

Lexington, Kentucky

2013

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ABSTRACT OF THESIS

THE DESIGN OF P200:
MENSURAL QUANTITY AND PROPORTION
IN THE EARLY VERSION OF THE ART OF FUGUE

The intended structure of Bach's ostensibly unfinished Die Kunst der Fuge has been debated since the work's publication in 1751. This study examines the proportional design of an early version of the work, subsisting in the autograph manuscript (P200) of the early 1740s. As the only complete source extant, P200 and its revisions provide substantial insight into Bach's intentions for the later version of the Art of Fugue.

KEYWORDS: The Art of Fugue, Die Kunst der Fuge, Bach, Proportion, Phi

Joel Thomas Runyan

May 2, 2013
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April 24, 2013
For Lily
I am indebted to:

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I. Sources & Mensural Quantities

The extant sources for *Die Kunst der Fuge* are two: the autograph manuscript of the early 1740s, Deutsche Staatsbibliothek. ms. autogr. Bach P200; and the printed edition of 1751, published a year after Bach's death under the supervision of his sons. The main portion of P200 was at some point appended with three pieces of later dates, each of which went on to appear in the printed edition. It is probable that Bach at one time considered P200 to represent a finished composition, and only later, perhaps for submission to the Mizler Society, initiated the process of its revision and publication. The transition that occurred between P200 and the printed edition was thus subtle and deliberate: many of the pieces were carried over unchanged, or with only alternate forms of mensuration and prolation. The thrust of the revision occurs in its additions: four fugues, two of which are of spurious inclusion; two canons, each entirely new; and a single chorale harmonization, added by the publishers as a recompense for the great unfinished fugue.
The first edition is taken as the primary source for nearly all modern editions and performances; however, it is only partially complete. It holds a number of logical discrepancies in ordering and content that can be attributed to Bach's poor health and death during the last segment of its engraving. Some of these discrepancies are quite obvious, such as the fugue titled *Contrapunctus a 4*: it appears copied note for note from P200 on page 45 of the printed edition, yet is also duplicated 16 pages prior as *Contrapunctus 10*, only with 22 new measures tacked to the beginning. Other errata make it more difficult to draw a clear line between the presence and absence of the composer's supervision. The unfinished *Fuga a 3 Soggetti*, a piece now nearly synonymous with the Art of Fugue, is an unavoidable example. C.P.E. Bach's written account upon the last page of the Unfinished Fugue's source manuscript, bound as Appendix 3 to P200, states that his father died before the piece could be finished. For many years this testimony elevated the work to mythical status as Bach's ultimate testament; yet the paper on which it is written, paired with Bach's steady handwriting, date it at very least a year previous to his death. Recent research has thus begun to question if the piece was perhaps finished and then lost, and even if it was meant to be included in
Bach's final version of *Die Kunst der Fuge* at all.

Figure 1 is a general concordance of the Art of Fugue's extant components. The first column indicates the order and title of each component as they appear in the printed edition of 1751, hereafter known as 1Ed. The second column numbers the order of those pieces originating in the autograph manuscript. The third column indicates the current musicological consensus as to each component's inclusion in an “ideal” Art of Fugue, as would have been intended by Bach. The fourth column shows the revisions incurred in the transformation from P200 to 1Ed; operations to x and y represent changes to (m)ensuration and (p)rolation, additions represent measures of (c)oda or (i)ntruction. Thus, the most extensive revision, that to the double fugue at the tenth:

\[ 2x(m) \& y(p)/2 + 22(i) \]

indicates a doubling of total measure numbers, a halving of note values, and an addition of 22 measures (in the revised units) to the beginning of the piece. Labels for the pieces referenced in this work appear in the right-most column\(^1\).

It is noteworthy that the pieces of contentious

\(^1\) See Appendix: Labeling Conventions, pg. 41
Figure 1. The Art of Fugue’s Components

<table>
<thead>
<tr>
<th>1Ed</th>
<th>P200</th>
<th>Inclusion</th>
<th>Revisions</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contrapunctus 1</td>
<td>1</td>
<td>Certain</td>
<td>2x(m) + 4(c)</td>
<td>S1</td>
</tr>
<tr>
<td>Contrapunctus 2</td>
<td>3</td>
<td>“</td>
<td>2x(m) + 6(c)</td>
<td>S2</td>
</tr>
<tr>
<td>Contrapunctus 3</td>
<td>2</td>
<td>“</td>
<td>2x(m) + 2(c)</td>
<td>S3</td>
</tr>
<tr>
<td>Contrapunctus 4</td>
<td>“</td>
<td></td>
<td>N/A</td>
<td>S4</td>
</tr>
<tr>
<td>Contrapunctus 5</td>
<td>4</td>
<td>“</td>
<td>None</td>
<td>P1</td>
</tr>
<tr>
<td>Contrapunctus 6. a 4 in Stylo Francese.</td>
<td>7</td>
<td>“</td>
<td>None</td>
<td>P2</td>
</tr>
<tr>
<td>Contrapunctus 7. a 4. per Augment et Diminut:</td>
<td>8</td>
<td>“</td>
<td>None</td>
<td>P3</td>
</tr>
<tr>
<td>Contrapunctus 8. a 3.</td>
<td>10</td>
<td>“</td>
<td>y(p)/2</td>
<td>T1</td>
</tr>
<tr>
<td>Contrapunctus 9. a 4. all Duodecima</td>
<td>5</td>
<td>“</td>
<td>2x(m) &amp; y(p)/2</td>
<td>D1</td>
</tr>
<tr>
<td>Contrapunctus 10. a 4 all Decima.</td>
<td>“</td>
<td></td>
<td>2x(m) &amp; y(p)/2 + 22(i)</td>
<td>D2</td>
</tr>
<tr>
<td>Contrapunctus. 11. a 4.</td>
<td>11</td>
<td>“</td>
<td>y(p)/2</td>
<td>T2</td>
</tr>
<tr>
<td>Contrapunctus inversus a 4</td>
<td>13</td>
<td>“</td>
<td>y(p)/2</td>
<td>M1r</td>
</tr>
<tr>
<td>Contrapunctus inversus. 12 á 4.</td>
<td>14</td>
<td>“</td>
<td>y(p)/2</td>
<td>M1i</td>
</tr>
<tr>
<td>Contrapunctus inversus a 3.</td>
<td>15</td>
<td>“</td>
<td>y(p)/2</td>
<td>M2r</td>
</tr>
<tr>
<td>Contrapunctus a.3</td>
<td>16</td>
<td>“</td>
<td>y(p)/2</td>
<td>M2i</td>
</tr>
<tr>
<td>Contrap: a 4.</td>
<td>6</td>
<td>Redundant</td>
<td>(See D2)</td>
<td></td>
</tr>
<tr>
<td>Canon per Augmentation in Contrario Motu.</td>
<td>12, App. 1</td>
<td>Certain</td>
<td>2x(m) &amp; y(p)/2 + 21</td>
<td>C1</td>
</tr>
<tr>
<td>Canon alla Ottava.</td>
<td>9</td>
<td>“</td>
<td>None</td>
<td>C8</td>
</tr>
<tr>
<td>Canon alla Duodecima in Contrapunto alla Quinta.</td>
<td>“</td>
<td></td>
<td>N/A</td>
<td>C12</td>
</tr>
<tr>
<td>Canon alla Decima Contrapunto alla Terza.</td>
<td>“</td>
<td></td>
<td>N/A</td>
<td>C10</td>
</tr>
<tr>
<td>Fuga a 2. Clav:</td>
<td>App. 2</td>
<td>Spurious</td>
<td>Two Keyboard Arr. of M2</td>
<td></td>
</tr>
<tr>
<td>Alio modo. Fuga a 2. Clav:</td>
<td>App. 2</td>
<td>“</td>
<td>“</td>
<td></td>
</tr>
<tr>
<td>Fuga a 3 Soggetti</td>
<td>App. 3</td>
<td>Debated</td>
<td>-6(c)</td>
<td>Q</td>
</tr>
<tr>
<td>Choral. Wenn wir in hoechsten Noethen</td>
<td></td>
<td>Spurious</td>
<td>N/a</td>
<td></td>
</tr>
</tbody>
</table>
inclusion appear toward the end of the manuscript, implying that Bach died at a point in which the majority of planning and even engraving had been completed. Superfluous material was probably added to bolster sales of what was known to be an incomplete version of an already difficult to sell work. The only piece of currently debatable inclusion is the unfinished *Fuga a 3 Soggetti*, beyond which the components of the ideal *Art of Fugue* number thirteen fugues (plus two mirrors) and four canons.

It must be acknowledged that the printed edition is inextricable from the editorial influence of Bach's sons and its engravers, who through an unknowable combination of ignorance, material scarcity, haste, and perhaps gall inked what is now the primary source for the Art of Fugue. A great deal of effort has thus been placed into re-establishing the work's "definitive" order in light of the first edition's shortcomings. These efforts draw from two bodies of interdependent evidence for their assertions: the recorded history, especially that latent in the physical substance of the source manuscripts; and the substance of the music as composed, subsisting in the formally abstract.

The first body of evidence is aptly demonstrated in the work of musicologists such as Christoph Wolff and Gregory Butler. Their research has placed tangible boundaries
around the so-called “dispositional problem”, particularly Butler's elaborate detective work into the engraving of the first edition. Butler's research has yielded highly influential (if polemic) conclusions, the most widely accepted of which is the repositioning of the Augmentation Canon to the end of its respective group. This displacement, though primarily justified in an analysis of page numbers and engraving marks, has its germ in 1Ed's patent violation of one of the Art of Fugue's more obvious organizational principles: Increasing Complexity (IC). The augmentation canon is considered to be the most complex of the canons, yet occurs first among its typal group in 1Ed.

IC is one of two topical processes used by Bach to organize the Art of Fugue, the other being the delineation [and bifurcation] of typal groups. Bach employs six types of fugue and four types of canon; each type of fugue is treated at least twice, each type of canon only once. Figure 2 lists these typal groups, along with the subjects and salient techniques of each of their components. Components of tenuous inclusion have been removed for convenience; the order otherwise strives to adhere to 1Ed. Though every subject in the Art of Fugue is derived from the primary (α) subject, only those variations which appear in two or more fugues are indicated with “α” and subscript.
### Figure 2. Typal Groups and Component Subjects

<table>
<thead>
<tr>
<th>Typal Group</th>
<th>Components</th>
<th>Subject(s)</th>
<th>Technical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S)imple</td>
<td>S1</td>
<td>$\alpha_1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>&quot;</td>
<td>Dotted rhythm</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>$\alpha_1, \alpha_2^i$</td>
<td>Inverted</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>$\alpha_1^i$</td>
<td>Inverted with Counter-subject, Stretto</td>
</tr>
<tr>
<td>(P)rolation</td>
<td>P1</td>
<td>$\alpha_2$</td>
<td>Counter-fugue</td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>$\alpha_2, \alpha_2^-$</td>
<td>Counter-fugue + Diminution</td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>$\alpha_2, \alpha_2^-, \alpha_2^+$</td>
<td>Counter-fugue + Diminution + Augmentation</td>
</tr>
<tr>
<td>(D)ouble</td>
<td>D1</td>
<td>$d_1 + \alpha_1$</td>
<td>Invertible at the 12th, dependent entrances</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>$d_2 + \alpha_2$</td>
<td>Invertible at the 10th, independent entrances</td>
</tr>
<tr>
<td>(T)riple</td>
<td>T1</td>
<td>$t_1 + t_2 + \alpha_3$</td>
<td>Dependent entrances</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>$t_1i + t_2i + \alpha_3^i$</td>
<td>Dependent entrances, All subjects of T1 inverted</td>
</tr>
<tr>
<td>(M)irror</td>
<td>M1</td>
<td>$m_1$</td>
<td>Completely invertible in 4 voices</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>$m_2$</td>
<td>Completely invertible in 3 voices</td>
</tr>
<tr>
<td>(C)anon</td>
<td>C+</td>
<td>$c^+$</td>
<td>Augmentation + Contrary Motion, Invertible at the 8ve</td>
</tr>
<tr>
<td></td>
<td>C8</td>
<td>$c_8$</td>
<td>Invertible at the 8ve</td>
</tr>
<tr>
<td></td>
<td>C10</td>
<td>$c_{10}$</td>
<td>Invertible at the 10th</td>
</tr>
<tr>
<td></td>
<td>C12</td>
<td>$c_{12}$</td>
<td>Invertible at the 12th</td>
</tr>
<tr>
<td>(Q)uadruple</td>
<td>Q</td>
<td>$q_{1-3} + \alpha_2^?$</td>
<td>Independent expositions</td>
</tr>
</tbody>
</table>
The remainder are labeled according to the component in which they appear. As elsewhere, the symbols “i”, “+”, and “-” indicate inversion, augmentation and diminution.

By comparison to the published order as displayed in Fig. 1, it can be seen that the only bifurcated group in 1Ed is the T-group (by the D-group). In P200, both the P-group (by the D-group) and the C-group (by the T-group) are divided. A comparison between the orders of the two source manuscripts is shown in Figure 3, with colors designating each typal group in order to clearly show the divisions in each source.

The complexity within and without typal groups can be seen to increase steadily for nearly the entirety of both sources. The greatest deviations occur at the aforementioned bifurcations, followed by the apparent misplacement of C+ in 1Ed. The fulfillment of IC within the C-group by the rearrangement of C+ then presents a contradiction: it implicitly accepts both the bifurcation of the T-group and the penultimate position of the C-group, in spite of both being strong violations of global IC. That Bach chose to blatantly violate the unilateral process of Increasing Complexity in two different orderings of the Art of Fugue suggests the presence of a third system of organization.
A system of this type was perhaps first demonstrated in Hans-Jörg Rechtsteiner's 1995 book *Alles geordnet mit Maß, Zahl und Gewicht: Der Idealplan von Johann Sebastian Bachs Kunst der Fuge*. Contrary to Butler, Rechtsteiner analyzes the work's musical structure contemporaneously with [and often before] any relevant historical knowns. The bulk of his analysis concerns the total measure numbers (hereafter...
Mensural Quantity, or MQ) of individual pieces and typal groups.

The impetus for this form of analysis is the discovery that, in 1Ed, the S-group, C-group, and T-group each total to exactly 372 measures. From this, Rechtsteiner eventually reasons to an “ideal” ordering that renders the entire work bilaterally symmetric. This proposed ordering hinges on two hypotheses: the unfinished fugue was intended to be 372 measures; and the canons were not meant to be bundled before Q, but interspersed throughout the work. These hypotheses are both compellingly argued in relation to physical evidence in the source manuscripts, but their theoretical basis is perhaps more telling. If the unfinished fugue contained four subjects, were 372 measures long, and still in the final position, it would form a conceptual mirroring of the S-group's four fugues and 372 measures at the beginning of the work. As for the canons, it is clear that Bach did not necessarily consider them to be a sequential chapter: they are not numbered like the Contrapuncti, and the C-group was in fact split once before in P200.

Figure 4 shows Rechtsteiner's proposed ordering, with the final fugue divided according to the expositions of its four possible subjects. This division illustrates the near
<table>
<thead>
<tr>
<th>Comp.</th>
<th>MQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>78</td>
</tr>
<tr>
<td>S2</td>
<td>84</td>
</tr>
<tr>
<td>S3</td>
<td>72</td>
</tr>
<tr>
<td>S4</td>
<td>138</td>
</tr>
<tr>
<td>P1</td>
<td>90</td>
</tr>
<tr>
<td>P2</td>
<td>79</td>
</tr>
<tr>
<td>P3</td>
<td>61</td>
</tr>
<tr>
<td>T1</td>
<td>188</td>
</tr>
<tr>
<td>C+</td>
<td>109</td>
</tr>
<tr>
<td>D1</td>
<td>130</td>
</tr>
<tr>
<td>C10</td>
<td>82</td>
</tr>
<tr>
<td>C12</td>
<td>78</td>
</tr>
<tr>
<td>D2</td>
<td>120</td>
</tr>
<tr>
<td>C8</td>
<td>103</td>
</tr>
<tr>
<td>T2</td>
<td>184</td>
</tr>
<tr>
<td>M1r</td>
<td>71</td>
</tr>
<tr>
<td>M1i</td>
<td>71</td>
</tr>
<tr>
<td>M2r</td>
<td>56</td>
</tr>
<tr>
<td>M2i</td>
<td>56</td>
</tr>
<tr>
<td>Q(1)</td>
<td>114</td>
</tr>
<tr>
<td>Q(2)</td>
<td>78</td>
</tr>
<tr>
<td>Q(3)</td>
<td>46...</td>
</tr>
<tr>
<td>Q(4)</td>
<td>?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>MQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>372</td>
</tr>
<tr>
<td>P</td>
<td>230</td>
</tr>
<tr>
<td>T</td>
<td>372</td>
</tr>
<tr>
<td>D</td>
<td>250</td>
</tr>
<tr>
<td>C</td>
<td>372</td>
</tr>
<tr>
<td>M</td>
<td>254</td>
</tr>
<tr>
<td>Q</td>
<td>372?</td>
</tr>
<tr>
<td>Total</td>
<td>2222</td>
</tr>
</tbody>
</table>

**Figure 4.**

*Rechtsteiner's Ordering and Component MQs*

**Figure 5.**

*Typal Group MQs in 1Ed*
perfect potential symmetry of the work, with the only large variance being the 3:4 componential ratio between the P-group and M-group. The mensural quantities of pieces and typal groups across the axis of symmetry are also shown to be very similar. Figure 5 indicates the MQ of each group, and the potential global MQ: 2222. The axis of symmetry occurs between C10 and C12, with each half at falling at exactly 1111 measures.

As expounded by Rechtsteiner, the work's potential symmetry, and the many other obvious marks of proportional design throughout, seems well beyond the realm of coincidence. Yet the theory has garnered little sway since its publication. No references appear to have been made to it in the English literature, and no recordings have been made which abide by the proposed ordering. It is especially noteworthy that Butler's most recent article, published 15 years after Rechtsteiner, makes no mention of the criticisms lobbied against his previous argument.

This neglect of Rechtsteiner's work seems in part due to a prevalent brand of skepticism, espoused most prominently by Ruth Tatlow, in which the importance and even existence of proportional relations is logically undermined by the lack of evidence for the composer's intention or knowledge of them. This skepticism is no doubt
exacerbated by references to theological and metaphysical principles that often accompany the analysis of proportions, to say nothing of the lack of an accepted and rigorous system for the analyzing itself.

In the interest of brevity, it is here proposed that such skepticism might be sufficiently marginalized in regards to the present research. By any statistical measure, the coincidences and proportions that can be shown to subsist in the printed edition of the Art of Fugue quite simply cannot be the providence of chance or a “naturally” occurring trait of Bach's compositional style. This is because the relevant quantities are in no ways derived from conjecture—they are displayed very clearly in the incontrovertible measure totals of single pieces and their respective typal groups.

The most common method of analyzing proportion within a musical work is the assignment of structural importance to those points in a mensural or durational total which can be related to classically significant ratios. These ratios are typically 1:2, for bilateral symmetry; and 1:1.618 etc., for Phi or the golden ratio. This form of analysis has been argued against for its lack of substantive historical backing, but also for the lack of a qualitative formal hierarchy in many of the works to which it is applied. This
latter criticism is especially pertinent as concerns
Bachian fugue: in fugue, very few musical objects can be
shown to hold a significant formal prominence over any
other; the process is fundamentally organic. Proportional
analysis thus often requires some manner of confirmation
bias in order to award significance to whatever object lies
at a given point.

The structure of the Art of Fugue facilitates a form of
analysis distinctly different than the “intermensural”
system: its points of respective formal delineation, often
seen as the most subjective link in an analytical chain,
can be clearly marked by the beginning and ends of
components. As the total measure numbers of each component
[and consequently their typal groups] are inarguable, the
ratios between them necessarily follow. While the
proportions within a piece of music might accentuate a
specific facet of texture, form, or text; the proportional
divisions of entire pieces [and the groups to which they
belong] lends them mass, and accentuates their
participation in a temporal architecture.

It might be said that, if designed around such
proportions, the entire Art of Fugue mimics the process of
fugue itself. Each of its component pieces might function
as the entry of a subject, and where a fugue progresses by
employing its subject in increasingly complex combinations, the Art of Fugue employs its subject in increasingly complex fugues. The importance of the work's ordering is thus amplified: even by the most limited of definitions, the Art of Fugue is cyclical. The difficulty for Bach was surely not in the composition of many types of fugues on a single subject, but in the combination of these fugues in a coherent whole; thus, to build a structure commensurate with the gravity of fugal process, he may have looked no further than fugue itself. Fugue is often nothing more than the timely placement of a musical idea with that generated from it, and as such is restricted by neither the complexity of its content nor the space allotted to it. In most ways, it presents the ideal archetype for the large-scale monothematic exposition of a single form.

If the self-referential and organic properties so prevalent in Bach's fugues are somehow mimicked in a macro-fugue, they would necessarily require a greater measure in which parts could be quantified. As before, this is exactly what Rechtsteiner's analysis reveals: the mensural quantities of components can be reckoned as proportional units, as if the work were in fact one long fugue. Unfortunately, as Rechtsteiner's focus on mensural quantity is a means to deriving an "ideal plan", his groundbreaking
discoveries are at times mired in the controversiality of his conclusions. Rather, the sum total of Rechtsteiner's ideas appears to be too speculative. If the disparity between potential significance and the attention received is any indication, a link is missing between the formality of Butler and the transcendental picture witnessed in Alles Geordnet. This link possibly lies dormant in the only complete form of Bach's *Die Kunst der Fuge* extant—P200.

As P200 undoubtedly represents the intent of Bach in its entirety, it is devoid of the dispositional problems of the first edition. The nature of the original source and the revisions made to it might thus afford the best of all perspectives for understanding Bach's conception of the work. If it can be shown that the Art of Fugue's components were initially interdependent in accord with a proportional system, then the changes made to ordering, notation, and material must necessarily have occurred in conjunction with the system's rearrangement. What's more, the acuity of the modifications necessary to retain certain structural properties first employed in P200 might strongly imply that proportional design is not only a significant property of the Art of Fugue, but was the entire reason for its creation and revision.

A detailed study of P200's proportional design has yet
to be undertaken, and no English source exists at all for the type of data at its base. The implications of such a study are relevant not only to our understanding of the Art of Fugue in theory and performance, but to the understanding of the late compositional mind of Bach.
II. Proportions & Fibonacci Sequences

The analysis of proportion in P200 begins with the totaling of MQ for each component and group:

Figure 6.
Component MQs in P200

<table>
<thead>
<tr>
<th>Comp.</th>
<th>MQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>37</td>
</tr>
<tr>
<td>S3</td>
<td>35</td>
</tr>
<tr>
<td>S2</td>
<td>39</td>
</tr>
<tr>
<td>P1</td>
<td>90</td>
</tr>
<tr>
<td>D1</td>
<td>65</td>
</tr>
<tr>
<td>D2</td>
<td>49</td>
</tr>
<tr>
<td>P2</td>
<td>79</td>
</tr>
<tr>
<td>P3</td>
<td>61</td>
</tr>
<tr>
<td>C8</td>
<td>103</td>
</tr>
<tr>
<td>T1</td>
<td>188</td>
</tr>
<tr>
<td>T2</td>
<td>184</td>
</tr>
<tr>
<td>C+</td>
<td>44</td>
</tr>
<tr>
<td>M1</td>
<td>112/56</td>
</tr>
<tr>
<td>M2</td>
<td>142/71</td>
</tr>
</tbody>
</table>

Figure 7.
Group MQs in P200

<table>
<thead>
<tr>
<th>Group</th>
<th>MQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>111</td>
</tr>
<tr>
<td>P</td>
<td>230</td>
</tr>
<tr>
<td>D</td>
<td>114</td>
</tr>
<tr>
<td>T</td>
<td>372</td>
</tr>
<tr>
<td>C</td>
<td>147</td>
</tr>
<tr>
<td>M</td>
<td>254/127</td>
</tr>
</tbody>
</table>
In comparison with the previous figures, a few important characteristics of the revision are revealed. The notational alterations to mensuration and prolation bring each variation on the $\alpha$ subject into a consistent notational space: four measures long, with a half note receiving the beat. This consistency strengthens the thread of the subject: the changes to, and derivations of, $\alpha$ are more clearly now visible from page to page in score. Next, the P-group and M-group are the only groups to receive no revisions; consequently, they might occupy a certain position of primacy in the chronology of composition, whether first or last. One also sees that, while the notation of the the S-group and D-group was similarly altered, Bach added cadential material to the former and introductory material to the latter. These additions brought the MQ of the S-group up to 372, matching that quantity retained by the T-group. The C-group also receives substantial additions that bring it to 372 measures, however, only C+ was revised, leaving C8 at 103 measures. This might show that Bach began to prioritize a group MQ of 372 at a specific point in revision: after the composition of S4, as the additions to S1-S3 became necessary, but before the composition of C10 and C12, as C8 remained untouched.
The retention of three group MQs and the expansion of two other groups to meet 372 proves at the very least that Bach treated MQ as a manipulable compositional parameter. The manipulation of these quantities would seem to suggest that every group MQ (and each component MQ) in both sources was potentially derived, and not arrived upon incidentally in the act of composition. This hypothesis is solidified in the addition of the S-, D-, and T-groups in P200:

\[ 111 + 114 + 147 = 372 \]

none of which have adjacent components in the original ordering. That is to say, the groups solely possessing the MQs of 230 and 372 divide (and are divided by) three groups which combine to 372. In this way, the S-D-C “meta-group” mimics the bifurcation of its component groups.

While the meaning of the number 372 is of only auxiliary significance to the current study, it is of curious note that

\[ 12 \times 31 = 372 \]

where 12 is the number of fugues in P200, and 31 is the gematrian total of “J.S.B.” [10+19+2].

20
As it is the most recurrent quantity in both sources, the number 372 can be treated as embryonic to any proportional design in the Art of Fugue, and thus potentially the source from which the MQs of all other components are derived. Considering that the P-group's MQ of 230 is the next most significant quantity for its lack of revision, we might compare:

\[
\frac{230}{372} = 0.618 = \phi
\]

or

\[
372\phi = 230 = MQ(P)
\]

The totals of the T- and P-group, untouched in the transformation of P200 to 1Ed, are obviously intended to be in proportion. But the procession of phi from 372 continues, if outside of typal groups:

\[
\frac{372}{\phi^2} = 142 = MQ(M2)
\]

M2 is also notably unedited in the transformation to 1Ed, and the final piece in both sources. Other groups can be derived thusly:

\[
\frac{(372/2)}{\phi} = 114 = MQ(D)
\]
and

\[ \frac{372}{(\phi^2)} - 31 = 111 = MQ(S) \]

The latter of these, for repeat use of the number 31, might be considered a coincidence. However, the direct calculation of two out of three typal groups in the 372 meta-group allows the third to be produced by subtraction, or:

\[ 372 - MQ(D) - MQ(S) = 147 = MQ(C) \]

This subtraction might show us something of the work's chronology. If the MQ of the C-group is derived from its meta-group's "need" to total 372, the derivation possibly shows that the C-group was composed last, that is, composed to fit. This would explain the presence of the alternate version of C+ written at the end of the main portion of P200. If Bach wrote the canons last in order to fit a specific design, they may have been the weakest compositions, and the first to see revision when another larger-scale design was conceived.

The use of phi to derive numbers from 372 might insinuate a good deal of quasi-mathematical tinkering on the part of Bach. It is thus noted that "phi" is the ratio tended toward in the Fibonacci sequence, that where each
number is the sum of the previous two. The aforementioned MQs can then be expressed more clearly as part of a generalized Fibonacci sequence:

142:230:372:(602:974)

It is probable that this sequence was used by Bach to conceive proportions between groups, as it possesses little of the inexactitude of operations with phi. This sequence is hereafter referred to as the “P200FS”. The large numbers in parenthesis, while part of the sequence, do not exist as simple wholes in P200. Rather, in the same manner as the S-D-C-group, they subsist in the addition of smaller components. It is important to note that such totals are often necessarily the case by virtue of the Fibonacci sequence: because totals of 230 and 372 are present, the next number in the sequence, 602, exists recursively, or can exist dependently upon its components. This is to say that the recursive ratio of 602:372, where the components forming the smaller quantity take part in the larger, is no more significant in the sequence than 372:230, unless the division indicates something significant about the ordering. It does not ostensibly appear to do so. However, 372 appears twice in the totals of group MQs, generating
the next number in the sequence, 974, and a second truly divided proportion of 602:372. This is especially remarkable because the components of each meta-group are almost perfectly split, rendering the S-P-D-C-group in proportion with the T-group.

The bifurcation of the C-group, and specifically the placement of C+ after the T-group, is now greatly elevated in significance: C+ alone prevents the ordered segmentation of typal group proportions. If both components of the C-group occurred before the T-group, adjacent or otherwise, the 602:372 proportion would be undivided. That this division was made at such a high level, and clearly echoes the division of typal and meta-groups, shows a very specific design principle at work. Even if each typal group occurred sequentially, the proportions between them would be entirely inaudible in performance—especially one that abides by the written ordering. Thus, Bach's decision to divide the groups shows that the concealment of their derivative form was quite a high priority. It remains unclear whether this concealment was in order to further imbed some numerological meaning within the Art of Fugue's structure, or simply to embody a particular aesthetic.

In any case, if the division of ostensible groups is taken as an intended property of P200's ordering, then it
is very likely that Bach would have reckoned the MQs of individual components across the lines of bifurcation. Many numerical relationships are revealed in the analysis of component MQs, only some of which can be determined as certainly meaningful. The level of self-reference required to link each relationship to any degree of compositional intent is prohibitively large; so it remains that many of the relationships are simply an incidental effect of so many numbers in one place. However, one might ascribe a hierarchical priority to any number which exists in, or can be directly produced from, the P200FS. With this in mind, the sequence might be extrapolated backward further from the observed totals.


Again in the realm of numerological coincidence, one notices the number 14, being both the number of components in P200 and the oft-found gematrian total of “Bach”. Strangely enough, this number can be multiplied by the gematrian total of “J.S. Bach”:

14 x 43 = 602
to produce another number in the P200FS. Here, our process of derivations becomes somewhat circular in its obfuscation of source. The question of numerical origin, though seemingly important as explication of “372”, is here taken as ancillary to the structure in which each number is held. One is left feeling that, even if the origins of the P200FS were properly defined, the analysis of the the Art of Fugue's structure had really yet to begin. And so, while mutually inclusive, a bold line might be drawn between issues of historical and theoretical concern.

Via proximity, the next most significant MQ is 88. No individual component holds this number, but two components—S2 and D2—do total to it. Both of these components are last in their respective [and undivided] groups. Furthermore, D2 is in the center of the combined P- and D-groups. The components that surround this combination—S2 and C8—total to 142, the number previous to 88 in the descending sequence. This nesting of proportions between components of different typal groups appears to be the next step downward in Bach's scheme of group bifurcation. The hierarchy of MQ strength might be listed thus:

1. Whole Groups
2. Bifurcated Groups
3. Meta-groups

4. Individual Components

5. Component Groups

The T-group is the only other whole group “contained” by another. If this indicates a possible nesting of proportion (as with the D-group), a component group with an MQ of 230 probably surrounds it. This is found in the combination of the M-group and C8, with C+ once again appearing in a position to disrupt clean segmentation of the proportional scheme.

All relationships relative to the P200FS are shown in Figure 8, wherein braces are used to contain entire sections, and rounded brackets are used to contain only those components to which they point. A container that is attached to another in line adopts the properties of the inferior container. The 602 meta-group, for instance, contains all components from the beginning of the work to the rounded bracket containing the two components of the C-group. In this way, one sees very clearly the bifurcation of those groups and meta-groups which add to quantities in the P200FS.

It is noteworthy that the smaller manifested quantities, 88 and 142, seem to center the expanding
Figure 8. Proportional Relations Based on the P200 Fibonacci Sequence
sequence on D2, or very near the center of the 974 meta-group. The M-group is separated from the larger sequence by belonging to its own smaller (and recursive) rendition of the P200FS. This separation receives a peculiar marking in the score. Though Bach writes out each canon in single voice and solution form, the resolved form of C+ is altered from its written dux: four additional measures marked Finale are added to the end of the otherwise perpetual canon.

Figure 9. 'Finale' Marking in the First Version of the Augmentation Canon

In its current position, this “finale” acts as the divider between the mirror canons and the rest of the work, signaling the end of the 974 meta-group. No other markings
of this type appear in P200, and the “Finale” marking is removed in the ensuing revisions of C+.

If the P200FS is extended backward into negatives, one arrives at the MQ of M1 and P1, 56:90, in a ratio expectedly close to phi:


This ratio's position in an unlikely section of the otherwise straightforward sequence implies that Bach may have derived MQs according to an incredibly diffuse system. However, it seems he may also have approximated them from a very limited set of foundational quantities. There is an unusual similarity between MQs within and without groups: 35, 37, and 39 in the S-group; 184, and 188 in the T-group; 111, 112, and 114 totals for the S-group, M1, and the D-group. Each of these totals could be general approximations of a few phi-based relationships. It thus seems beneficial to calculate the instances of phi occurring between components and component groups, the lower members in the hierarchy of Mensural Quantity types. Figure 10 shows those phi-based relationships that do not subsist in the P200FS. As before, the braces on the right of the figure contain component groups, while the rounded brackets on the left
These proportions seem to gain strength in proximity and/or shape. Rather, the adjacent relationships between D2-P2 and P3-C8 seem particularly strong, especially because they bridge the bifurcations of typal groups. Likewise, the adjacent pairing of T2 with the last three components might explain the placement of C+. The remaining relationships seem convoluted in comparison because of
their asymmetrical extension. Nonetheless, a great deal of similarity is displayed in the tripartite ratios between S3, M1, and P1; and M2, the D-group, and the T-group. No components are shared between these relationships. In fact, it appears that though all components can be included in a phi-based proportion, there is practically no overlap or ambiguity among them. The relationship between S3 to M1, for example, divides the S-group and previously mentioned “last three” component group. Yet the combination of S1 and S2 forms a relationship with P2 and P3, also split from a singular component of their group; and C+ and M2 are directly in proportion. The split component, P1, relates right back to M1... and so on. In combination with the P200FS proportions, it can be seen that the MQ of every single component can be derived from all others in a sublimely circular fashion.
III. Revisions

The partitioning of individual groups would seem to indicate that the total MQ of the Art of Fugue might have been as planned as that of its components. Unfortunately, the total is variable, as two groups can be seen to possess alternate MQs. The mirror fugues are written on top of each other on the page, and Bach even goes so far as to brace their respective staves together. This heightens the “mirror” effect, but also implies that the components have a dual function in MQ. This dual function is clearly manifest in the foregoing figures, where relationships can be drawn between both MQs for each mirror fugue.

The canons may also display this dual function, as they each contain a somewhat superfluous repeat. With repeats taken into account, C8 and C+ respectively total to 179 and 84. The number 84 does not appear to be in proportion with any other component or group, though it is coincidentally the product of 7 and 14, and the revised MQ of S2 in 1Ed. However,
179\phi = 111 \text{ or } MQ(C8) = MQ(S)/\phi

This dual function rendering of 179 for C8 would be questionable were it not for a few slight marks written at the beginning of D1:

Figure 11. Manuscript Revision to the Double Fugue at the Twelfth

Bach has indicated a revised prolation and mensuration (which D1 assumes in 1Ed), doubling MQ to 130. These marks are significant due to their rarity; similar marks were not made to components in P200 that were similarly revised. This shows that the changes made to D1 came very early in the process of revision, lending insight as to its purpose:
130 + 49 = 179

or

MQ(Drev.) = MQ(C8) = MQ(S)/φ

and

MQ(P+Drev.) = 409, 409φ = 254

or

MQ(P+Drev.) = MQ(M)/φ

Bach began to revise D1 in order to bring the D-group to 179 measures, but changed his mind in favor of revising D2 and/or bringing the D-group to 250 measures, as it appears in 1Ed. This alteration of the D-group disrupted the S-D-C-group MQ of 372. It is exceptional that Bach took the other two typal groups of this previous meta-group and added exactly enough material to make them each 372 measures... the very same number in which they participated together previously.

It is noteworthy that the S1-M2 Component group—that which bookends the entire work—also totals to 179. That the number 179 appears thrice in P200 and in a phi-based relationship with an entire typal group, implies that it might subsist in a Fibonacci sequence as the P200FS:

Though 43, the gematrian total of “J.S. Bach”, once again appears near the beginning of the sequence, the 11th number –1228—is rather more shocking. It is the total MQ of P200, sans the repeats of the canons; rather, it is the total number of measures that appear in score. This relationship is especially telling because a total MQ of 1228 can only be reached if C8 is not counted as 179; yet, 179 appears to gain most of its significance in a sequence that contains 1228. This interplay of proportional strata echoes both the manifold derivations of MQ in components and the bifurcation of typal groups (and their MQ) in ordering.

The manifoldness of MQ derivatives in P200 might be further explored in the confluence of early revision marks. The revision of D1 to facilitate its participation in the 179:111 ratio patently displays a middle period in the Art of Fugue’s design. Many of the changes made in this period were not assumed in 1Ed; principal among these is the second version of C+, written immediately after the M-group in the main portion of P200. This second version is the same as that which appears in 1Ed, sans altered prolation and mensuration. It thus appears in the same notation as the first version, but with an additional 10 measures. This brings its MQ to 54—perhaps unsurprisingly, the number
previous to 88 in the P200FS. The “Finale” mark is removed, as the piece no longer serves as the close to the 974 meta-group. Though the second version of C+ further divorces the S-D-C-group from an MQ of 372, it ties up the loose end left by the C-group's not belonging to the new relationship wrought between the S-group and revised D-group: the “middle period” total of the C-group becomes 157, and

\[
\frac{157}{\varphi} = 254
\]

or

\[
\text{MQ(Crev.)} / \varphi = \text{MQ(M)}
\]

The relationships between typal groups in the medial version of P200 are shown in Figure 10. These middle period proportions produce a picture of the Art of Fugue's design entirely different from that of both P200 and the published form of 1Ed. Phi clearly links alternating typal groups, in spite of their bifurcations. This linking probably occurred simultaneously with a global re-ordering, but the intended ordering for this version, if different from P200, is likely unknowable.

With Bach's abandonment of the P200FS in favor of more direct relationships between whole groups, the previous fundamental quantities dissolve, and the circular unity of
derivative MQs follows with them. However, the uniformity rendered by this “blocking” of groups lends them substantially more gravity. The segmentation of proportions between groups turns into a segmentation of global order in 1Ed, where, at least in the published ordering, only a single bifurcation occurs. The 372:230 ratio between the adjacent S-group and P-group at the outset of 1Ed clearly
exemplifies this drastic differences between the structure of P200 and 1Ed. 1Ed makes clear the direct relation of group MQs, and perfectly compensates with its homogenization of the written subject—what initially seemed to be the purpose of the notational revisions. The organic and interdependent system of P200, with its variable subject notation and interlocked component proportions, is rendered solid and architectonic in 1Ed by the segregation of typal groups and transformation of the subject into a single notational space. The subjects unifying thread, once a florid and variable manifestation of a single gene, now displays a linear [and calculated] evolution; it becomes stackable, nigh interchangeable, like brick.

The revisions to P200 imply that 1Ed was meant to be a work of greater lucidity, breadth, and elegance. The assimilation of many original mensural quantities indicates that a despотically controlled system of derivation was not employed in 1Ed: the majority of quantities cannot be understood without relation to their earlier context in the autograph manuscript. For this reason, P200 may very well be the more complicated of the two sources, but further research into the transition between the medial version and the first edition is needed, especially as regards the dispositional problem. The proportions of P200 prove that
Increasing Complexity was only a single property of a much larger design, and cannot alone inform the ordering of components. This reinforces Rechtsteiner's proposal, at least in conception: Bach was far more concerned with the inner designs of his compositions, in proportion and numeration, than has ever been previously considered. Thus, any conjecture about the structure of the Art of Fugue that is not partially based in analysis of Mensural Quantity and proportion is heavily impaired. It is hoped that the research which can follow from the data first shown here might serve to fill those remaining gaps in a future solution to the dispositional problem, facilitating complete proportional analysis of the work and its first correct performance in over 250 years.
Appendix: Labeling Conventions

Component titles in the first edition of the Art of Fugue present two major problems for the analyst whose work involves their constant use. Firstly, the set of numbered pieces (all titled “Contrapunctus”) does not include three of the mirror fugues, the canons, or the unfinished fugue. This requires them to be addressed by either full title or technical description. Secondly, as they possess no contextual reference, the given numbers require that the reader be familiar with each fugue's respective typal group.

The most common approach to labeling is thus a mix of Contrapunctus number, full title/description, and sometimes BWV\(^2\) number. This approach lacks the consistency and concision essential for a rigorous exposition of the work's architecture, especially where titles are necessary symbols in a graphic system of structural relationships. A practical labeling system should A) include all relevant

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2 The BWV numbers take inclusion in the first edition (BWV1080) as a sole qualifier. Thus, works like the Contrapunctus a 4, an earlier (and redundant) version of a fugue that precedes it in the first edition, receive equal treatment in the ordination.
components, B) indicate typal context, and C) defer absolute order to alleviate numerical confusion.

I propose the following method with these traits. It requires first the naming of typal groups[^3]:

1. Simple Fugues (S-group)
2. Prolation Fugues (P-group)
3. Triple Fugues (T-group)
4. Double Fugues (D-group)
5. Mirror Fugues (M-group)
6. Canons (C-group)
7. Quadruple Fugue (Q)

These seven typal groups are the most unambiguous delineation of structure extant, as there is essentially no debate about their respective components. In addition, the only discrepancies in intra-group ordering from P200 to 1Ed occur within the S-group and C-group, both of which receive new components. Owing to this otherwise remarkable consistency, we may confidently number each component within its respective group, with few modifications: the

[^3]: The components of the second group are often referred to as Counterfugues. This title is of less use because it fails to indicate the one technique unique to this group: the alteration of the subject's prolation. While the first fugue in this group does not employ prolation technique, its relation to the other two fugues by subject and proximity in 1Ed is justification for inclusion. The lack of another clear letter designation for the Mirror and Canon groups should one employ "(M)ensuration" or "(C)ounter" for Group 2 is also a considerable restraint.
mirror fugues, being in essence two pieces in one, receive a single number followed by either an “r” for rectus or an “i” for inversus; and the canons, each employing a specific technique, are marked accordingly. The ordering from 1Ed, excluding the spurious works, is rendered thus:

<table>
<thead>
<tr>
<th>1st Edition Title</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Contrapunctus 1.</td>
<td>S1</td>
</tr>
<tr>
<td>Contrapunctus 2.</td>
<td>S2</td>
</tr>
<tr>
<td>Contrapunctus 3</td>
<td>S3</td>
</tr>
<tr>
<td>Contrapunctus 4.</td>
<td>S4</td>
</tr>
<tr>
<td>Contrapunctur [sic] 5.</td>
<td>P1</td>
</tr>
<tr>
<td>Contrapunctus 6. a 4 in Stylo Francese.</td>
<td>P2</td>
</tr>
<tr>
<td>Contrapunctus 7. a 4. per Augment et Diminut:</td>
<td>P3</td>
</tr>
<tr>
<td>Contrapunctus 8. a 3.</td>
<td>T1</td>
</tr>
<tr>
<td>Contrapunctus 9. a 4. all Duodecima</td>
<td>D1</td>
</tr>
<tr>
<td>Contrapunctus 10. a 4 all Decima.</td>
<td>D2</td>
</tr>
<tr>
<td>Contrapunctus 11. a 4.</td>
<td>T2</td>
</tr>
<tr>
<td>Contrapunctus inversus [sic?] a 4</td>
<td>M1r</td>
</tr>
<tr>
<td>Contrapunctus inversus. 12 à 4</td>
<td>M1i</td>
</tr>
<tr>
<td>Contrapunctus inversus a 3.</td>
<td>M2i</td>
</tr>
<tr>
<td>Contrapunctus a 3</td>
<td>M2r</td>
</tr>
<tr>
<td>Canon per Augmentation in Contrario Motu.</td>
<td>C+</td>
</tr>
<tr>
<td>Canon alla Ottava.</td>
<td>C8</td>
</tr>
<tr>
<td>Canon alla Duodecima in Contrapunto alla Quinta.</td>
<td>C12</td>
</tr>
<tr>
<td>Canon alla Decima Contrapunto alla Terza.</td>
<td>C10</td>
</tr>
<tr>
<td>Fuga a 3 Soggetti</td>
<td>Q</td>
</tr>
</tbody>
</table>

4 In 1Ed, Both parts of the first mirror fugue are erroneously labeled “inversus”. The designation of “rectus” for the first component is arbitrary. The second part of the second mirror fugue, the only not labeled, is likewise arbitrarily assumed to be the rectus form.
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CURRICULUM VITAE

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