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Sustainable Production Through Balancing Trade-Offs Among Three Metrics in Flow Shop Scheduling

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Sustainable production through balancing trade-offs among three metrics in flow shop scheduling

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Abstract

In sustainable manufacturing, inconsistencies exist among objectives defined in triple-bottom-lines (TBL) of economy, society, and environment. Analogously, inconsistencies exist in flow shop scheduling among three objectives of minimizing total completion time (TCT), maximum completion time (MCT), and completion time variance (CTV), respectively. For continuous functions, the probability is zero to achieve the objectives at their optimal values, so is it at their worst values. Therefore, with inconsistencies among individual objectives of discrete functions, it is more meaningful and feasible to seek a solution with high probabilities that system performance varies within the control limits. We propose a trade-off balancing scheme for sustainable production in flow shop scheduling as the guidance of decision making. We model trade-offs (TO) as a function of TCT, MCT, and CTV, based on which we achieve stable performance on min(TO). Minimizing trade-offs provides a meaningful compromise among inconsistent objectives, by driving the system performance towards a point with minimum deviations from the ideal but infeasible optima. Statistical process control (SPC) analyses show that trade-off balancing provides a better control over individual objectives in terms of average, standard deviation, C_p and C_{pk} compared to those of single objective optimizations. Moreover, results of case studies show that trade-off balancing not only provides a stable control over individual objectives, but also leads to the highest probability for outputs within the specification limits. We also propose a flow shop scheduling sustainability index ($FSSI$). The results show that trade-off balancing provides the most sustainable solutions compared to those of the single objective optimizations.

Keywords: Flow shop scheduling; Multi-objective optimization; Statistical process control; Trade-off balancing.

1. Introduction

Flow shop scheduling problem arises where a set of jobs on one or multiple machines must be sequenced in order to optimize a given objective function. Permutation flow shop is a special type of flow shop in which the processing order jobs is identical on all machines. Permutation flow shop has been the subject of a massive body of literature [1]. Maximum completion time (MCT) and total completion times (TCT) are two fundamental performance measures in flow shop scheduling, driving many other performance measures such as utilization, work in process (WIP) inventories, and material flow [2]. For an n-job m-machine flow shop, MCT is the completion time of the last job n on the last machine m, and TCT is the sum of completion times of all jobs on the last machine. Average completion time $ACT=TCT/n$ also known as flow time is the average time a job spends in the system. Minimization of ACT is equivalent to minimization of TCT for a fixed n. Minimization of MCT and TCT have been proven to be NP-complete for flow lines with the number of machines $m \geq 3$ and $m \geq 2$, respectively [3, 4]. Therefore, it is extremely difficult to find an optimal solution to a general n-job m-machine problem within a given computation time. Completion times variance (CTV) is another important performance measure for flow shop scheduling. CTV measures the variability of flow time. In service systems, CTV present service uniformity [5, 6], i.e, the variability of the time a job/customer spends in the system. Minimization of CTV also has been proven to be NP-hard [7].

Since MCT, TCT and CTV are all important performance measures in the production scheduling, it is necessary to develop multi-objective optimization models to optimize system performance [2]. Although MCT, TCT and CTV are all functions of completion times, minimizing one does not necessarily minimize another [8, 9]. Given NP-hardness and inconsisten-

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cies, it is necessary to balance the trade-offs among them for sustainable production.

The impact of production has been extended to the triple bottom lines (TBL) of economy, environment and society [10]. Although production impacts on environment and on economy have been reported in the literature such as carbon-efficient scheduling [11] and energy-efficient scheduling [12], sparse studies on flow shop scheduling considered all three aspects of TBL, because of the inconsistencies among objectives in TBL, Akbar and Irohara [14] provided a recent review of scheduling for sustainability.

Jawahir et al. [13] defined sustainable manufacturing as "sustainable manufacturing at product, process, and system levels must demonstrate reduced negative environmental impacts, offer improved energy and resource efficiency, generate minimum quantity of wastes, provide operational personnel health while maintaining and/or improve the product and process quality with the overall life-cycle cost benefits".

Lu et al. [15] proposed the Process Sustainability Index ($ProcSI$) in which all aspects of TBL were taken into consideration. $ProcSI$ consists of six clusters including: manufacturing cost, energy consumption, waste management, operational safety and personnel health. Each cluster is then divided into several sub-clusters to address specific areas of impact within each cluster. The sub-clusters are then divided into individual metrics that measure single and specific aspects of process sustainability [16]. Once the individual metrics are identified and measured, a bottom-up approach is implemented to aggregate the metrics into $ProcSI$. $ProcSI$ assigns a scalar score on a scale of 0 to 10 to the studied process, however, given the number of factors involved in TBL, such as people, planet, and profit, which are evaluated as metrics, optimization objectives in sustainability can be inconsistent with each other, and consequently the trade-offs between metrics need to be systematically balanced.

In this paper, given an n-job m-machine permutation flow shop, we first show the inconsistencies among objectives of minimizing MCT, TCT, and CTV. Second, we propose a trade-off balancing scheme that provides better control for flow shop scheduling. Third, by extending the scheme proposed by Lu (2015) [16], we propose a generic model for balancing trade-offs between inconsistent performance metrics in flow shop scheduling, which can be extended to balance trade-offs between any number of inconsistent objectives, in terms of linear regression models in statistics, such as $z = CX + d$, as $z = f(y_1, y_2, ..., y_O)$, and $y_o = CX + d_o$, for $o = 1, 2, ..., O$, where y_o is subject to $AX \leq B$.

The rest of this paper is organized as follows: the problem description is provided in Section 2, results of empirical case studies are discussed in Section 3, and conclusion and future works are presented in Section 4.

2. Problem description

The following formulations provide the mathematical descriptions of MCT, TCT and CTV in an n-job, m-machine per-

mutation flow shop. The processing time of job j on machine i is defined as $p_{j,i}$, $j = 1, 2, ..., n$ and $i = 1, 2, ..., m$. $C_{j,i}$ is the completion time of job j on machine i. Since all jobs are ready to be processed at the time 0, there is no idle time on the first machine. Equation 1 represents the completion time of job j on machine 1. Equation 2 represents the completion time of the first job on each machine. Therefore, we are able to calculate the completion time of each job on each machine by equation 3. MCT is the completion time of the last job on the last machine and is shown by equation 4. TCT is the sum of completion times of all jobs on the last machine and is presented by equation 5. Completion times variance is calculated by equation 7.

$$C_{j,1} = \sum_{h=1}^{j} p_{h,1} \quad j = 1, 2, ..., n \quad (1)$$

$$C_{1,i} = \sum_{i=1}^{m} p_{1,i} \quad (2)$$

$$C_{j,i} = \max(C_{j-1,i}, C_{j,i-1}) + p_{j,i} \quad (3)$$

$$j = 2, 3, ..., n$$
$$i = 2, 3, ..., m$$

$$MCT = C_{n,m} \quad (4)$$

$$TCT = \sum_{j=1}^{n} C_{j,m} \quad (5)$$

$$ACT = TCT/n \quad (6)$$

$$CTV = \frac{1}{n} \sum_{j=1}^{n} (C_{j,m} - ACT)^2 \quad (7)$$

Given an instance s with n jobs, $s \in \{1, 2, ..., S\}$, there are $n!$ different sequences in s. Let $\pi \in \{1, 2, ..., n!\}$ denote a sequence of n jobs, normalized deviation of objective k generated by sequence π can be defined as $ND(x_{\pi,s}^{[k]}) = \frac{x_{\pi}^{[k]} - \text{MIN}(x_s^{[k]})}{\text{MAX}(x_s^{[k]}) - \text{MIN}(x_s^{[k]})}$, with $k = 1$ for $\min(TCT)$, $k = 2$ for $\min(MCT)$, and $k = 3$ for $\min(CTV)$, and $x_{\pi}^{[k]}$ is the performance of sequence π on objective k. $\text{MAX}(x_s^{[k]})$ and $\text{MIN}(x_s^{[k]})$ are the worst (maximum) and the best (minimum) solutions for objective k in instance s. Let ω_k be as the decision maker's preference to objective k and $\Omega = [\omega_1, \omega_2, \omega_3]$, we propose a trade-off function represented by equation 8.

$$TO_{\pi,s}^{[\Omega]} = \sum_{k=1}^{3} \omega_k ND(x_{\pi}^{[k]}) \quad (8)$$
$$\sum_{k=1}^{3} \omega_k = 1,$$

This linear model can be extended to balance trade-offs in problem settings for machining or for TBL, which is beyond the scope of this paper. Let $\pi_s^*(\Omega)$ be the sequence that minimizes $TO_s^{[\Omega]}$, the arithmetic mean of normalized deviations over all instances is defined by $\overline{AND}(\Omega) = \frac{1}{3S} \sum_{s=1}^{S} \sum_{k=1}^{3} ND_{\pi^*,s}^{[k]}$. We utilize $\overline{AND}(\Omega)$ as a new performance indicator along with TCT, MCT, and CTV to evaluate the performance of each solution. Equation 8 explicitly integrates the decision maker's preference into the trade-off function by assigning a weight (ω_k) to each objective. With dynamics in production, decision makers' preferences might change as the process reveals its performance over the time. $\min(TO)$ is precisely equivalent to minimizing the deviations from an ideal point at which all the objectives are at their optimum values.

In order to develop a comprehensive the flow shop scheduling sustainability index ($FSSI$), by extending the scheme proposed by Lu et al. [16], we propose a top to bottom decomposition followed by a bottom to top aggregation. At the decomposition phase, we divide $FSSI$ to three clusters covering the

Table 1: Average normalized deviations for all 66 weights

	(1,0,0)	(0,1,0)	(0,0,1)	(0.9,0.1,0)	(0.8,0.2,0)	(0.7,0.3,0)	(0.6,0.4,0)	(0.5,0.5,0)	(0.4,0.6,0)	(0.3,0.7,0)	(0.2,0.8,0)
TCT	**0.000**	0.187	0.489	0.003	0.012	0.026	0.043	0.062	0.080	0.100	0.119
MCT	0.245	**0.000**	0.176	0.185	0.133	0.092	0.060	0.037	0.022	0.011	0.005
CTV	0.368	0.180	**0.000**	0.347	0.328	0.310	0.292	0.274	0.257	0.238	0.223
$\overline{TO}$	0.204	0.122	0.222	0.178	0.158	0.143	0.132	0.124	0.120	0.117	0.116
	(0.1,0.9,0)	(0.9,0,0.1)	(0.8,0,0.2)	(0.7,0,0.3)	(0.6,0,0.4)	(0.5,0,0.5)	(0.4,0,0.6)	(0.3,0,0.7)	(0.2,0,0.8)	(0.1,0,0.9)	(0.0,0,0.9)
TCT	0.139	0.001	0.007	0.020	0.052	0.106	0.178	0.263	0.348	0.423	0.458
MCT	0.001	0.221	0.197	0.172	0.145	0.123	0.107	0.103	0.119	0.145	0.138
CTV	0.207	0.344	0.313	0.275	0.216	0.150	0.092	0.046	0.017	0.004	0.002
$\overline{TO}$	0.116	0.189	0.172	0.155	0.138	0.126	0.125	0.137	0.161	0.191	0.200
	(0,0.2,0.8)	(0,0.3,0.7)	(0,0.4,0.6)	(0,0.5,0.5)	(0,0.6,0.4)	(0,0.7,0.3)	(0,0.8,0.2)	(0,0.9,0.1)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.3,0.6)
TCT	0.424	0.389	0.355	0.323	0.294	0.266	0.238	0.216	0.389	0.354	0.320
MCT	0.107	0.080	0.057	0.039	0.025	0.014	0.006	0.001	0.106	0.076	0.052
CTV	0.007	0.016	0.029	0.044	0.061	0.082	0.107	0.131	0.010	0.021	0.035
$\overline{TO}$	0.180	0.162	0.147	0.135	0.127	0.121	0.117	0.116	0.168	0.150	0.136
	(0.1,0.4,0.5)	(0.1,0.5,0.4)	(0.1,0.6,0.3)	(0.1,0.7,0.2)	(0.1,0.8,0.1)	(0.2,0.1,0.7)	(0.2,0.2,0.6)	(0.2,0.3,0.5)	(0.2,0.4,0.4)	(0.2,0.5,0.3)	(0.2,0.6,0.2)
TCT	0.286	0.257	0.229	0.200	0.170	0.313	0.281	0.249	0.219	0.189	0.162
MCT	0.033	0.020	0.010	0.003	0.001	0.079	0.051	0.031	0.018	0.009	0.004
CTV	0.053	0.073	0.097	0.125	0.159	0.029	0.045	0.066	0.089	0.118	0.151
$\overline{TO}$	0.124	0.117	0.112	0.109	0.110	0.140	0.126	0.115	0.109	0.105	0.106
	(0.2,0.7,0.1)	(0.3,0.1,0.6)	(0.3,0.2,0.5)	(0.3,0.3,0.4)	(0.3,0.4,0.3)	(0.4,0.3,0.3)	(0.3,0.5,0.2)	(0.3,0.6,0.1)	(0.4,0.1,0.5)	(0.4,0.2,0.4)	(0.4,0.4,0.2)
TCT	0.133	0.233	0.205	0.177	0.150	0.112	0.125	0.107	0.156	0.134	0.094
MCT	0.003	0.064	0.039	0.023	0.014	0.027	0.010	0.010	0.065	0.041	0.022
CTV	0.194	0.062	0.084	0.111	0.144	0.173	0.182	0.217	0.111	0.139	0.210
$\overline{TO}$	0.110	0.120	0.109	0.104	**0.103**	0.104	0.106	0.111	0.111	0.104	0.108
	(0.4,0.5,0.1)	(0.5,0.1,0.4)	(0.5,0.2,0.3)	(0.5,0.3,0.2)	(0.5,0.4,0.1)	(0.6,0.1,0.3)	(0.6,0.2,0.2)	(0.6,0.3,0.1)	(0.7,0.1,0.2)	(0.7,0.2,0.1)	(0.8,0.1,0.1)
TCT	0.085	0.088	0.073	0.065	0.062	0.040	0.038	0.039	0.018	0.021	0.008
MCT	0.021	0.083	0.057	0.045	0.039	0.109	0.083	0.068	0.134	0.109	0.160
CTV	0.237	0.176	0.207	0.236	0.257	0.241	0.262	0.279	0.287	0.300	0.321
$\overline{TO}$	0.114	0.115	0.112	0.115	0.119	0.130	0.128	0.129	0.146	0.143	0.163

three aspects of *TBL* including economy, environment, and society. Each cluster is then divided into sub-clusters. Each sub-cluster covers a specific area of impact of its cluster. Accordingly, each sub-cluster is then divided to individual metrics that specifically measure a single performance indicator. Once the top-bottom structure is developed and all the individual metrics are measured, a bottom-up aggregation approach including normalization and weighting is utilized to calculate *FSSI*.

Let $\gamma \in \{1, 2, 3\}$ denote the index of clusters with 1 for economy, 2 for environment, and 3 for society. Z_γ is the number of sub-clusters in each cluster, therefore, $\zeta_\gamma \in \{1, 2, ..., Z_\gamma\}$ denotes sub-cluster ζ in cluster γ. Let $K_{\gamma,\zeta}$ denote the number of individual metrics of sub-cluster ζ in cluster γ, i.e. $k_{\zeta,\gamma} \in \{1, 2, ..., K_{\gamma,\zeta}\}$.

Recalling $ND(x_\pi^{[k]})$, at the metric level, we use $M_{\pi,\zeta,\gamma}^{[k]} = 10(1 - ND(x_\pi^{[k]}))$ to calculate the sustainability score of sequence π for metric k. $M_{\pi,\zeta,\gamma}^{[k]} \in [0, 10]$ normalizes each metric to a scale of 0 to 10, where 0 is the worst performance and 10 is the best performance in terms of sustainability. $M_{\pi,\zeta,\gamma}^{[k]}$ attributes the sustainability of an individual metric to its normalized deviation. For example, *MCT* directly affects the production cost; therefore, a solution with $\min(ND(x_\pi^{[2]}))$ generates the highest sustainability score for production cost.

Once the top-bottom structure is developed and all the individual metrics are measured, a bottom-up aggregation approach by equations 9 to 14 is utilized to calculate *FSSI*. Equation 9 is the aggregation of individual metrics sustainability score to the sub-cluster sustainability index (*SCL*), where $\omega_k^{[\zeta,\gamma]} \in [0, 1]$ is the weight assigned to the metric k of sub-cluster ζ in cluster γ. Equation 10 imposes that the sum of all weights must be equal 1. Equation 11 is the aggregation of sub-cluster sustainability indices to the cluster sustainability index (*CL*), where $\omega_\zeta^{[\gamma]}$ is the weight of sub-cluster ζ in cluster γ. Equation 12 indicates that the sum of sub-cluster weights must be 1. Equation 13 aggregates cluster sustainability indices into *FSSI*, where, $\omega^{[\gamma]}$ is the weight of cluster γ. Equation 14 indicates that the sum of

cluster weights is equal to 1.

$$SCL_{\pi,\gamma,\zeta} = \sum_{k=1}^{K} \omega_k^{[\zeta,\gamma]} M_{\pi,\zeta,\gamma}^{[k]} \qquad (9)$$

$$\sum_{k=1}^{K} \omega_k^{[\zeta,\gamma]} = 1 \qquad (10)$$

$$CL_{\pi,\gamma} = \sum_{\zeta=1}^{Z_\gamma} \omega_\zeta^{[\gamma]} SCL_{\pi,\gamma,\zeta} \qquad (11)$$

$$\sum_{\zeta=1}^{Z_\gamma} \omega_\zeta^{[\gamma]} = 1 \qquad (12)$$

$$FSSI_\pi = \sum_{\gamma=1}^{3} \omega^{[\gamma]} CL_{\pi,\gamma} \qquad (13)$$

$$\sum_{\gamma=1}^{3} \omega^{[\gamma]} = 1 \qquad (14)$$

3. Case studies

To show the inconsistencies among objectives of $\min(TCT)$, $\min(MCT)$, and $\min(CTV)$ and to verify the effectiveness of balancing trade-offs among inconsistent objectives, we carry out a series of case studies. The number of jobs n changes from $j = 5, ..., 10$, resulting in six choices, number of machines m changes from 3 to 19 ($m = 2l + 1$, $l = 1, 2, ..., 9$), yielding nine choices. This configuration results in 54 combinations. For each combination, 100 instances are randomly generated. The processing times are randomly drawn from a uniform distribution in [1,99]. Therefore, in total we have 5400 instances. Given ω_k changes from [0.0: 0.1: 1.0], we have 66 combinations of three weights with $\sum_{k=1}^{3} \omega_k = 1$, i.e. $\Omega \in \{\Omega_1, \Omega_2, ..., \Omega_{66}\}$. The 66 minimization functions of $\min(TO^{[\Omega]})$ cover the three single-objective minimizations of $\min(TCT)$, $\min(MCT)$ and $\min(CTV)$ with a weight equal to [1, 0, 0], [0, 1, 0], and [0, 0, 1], respectively. Since the number of jobs is relatively small ($n \leq 10$), we are able to use enumeration to find $\text{MAX}(x^{[k]})$ and $\text{MIN}(x^{[k]})$ for all k as well as $\min(TO^{[\Omega]})$ for each weight. Table 1 shows the average normalized deviations for all 66 weights. As it was intuitively expected, weights of [1, 0, 0], [0, 1, 0], and [0, 0, 1] (single-objective optimization) generate no deviations on $\min(TCT)$, $\min(MCT)$ and $\min(CTV)$, respectively, but large deviations on the other objectives. Weight of [0.3, 0.4,

0.3] generates the minimum value of $\overline{AND}(\Omega)$, and it also provides a relatively wider variation range for each individual objective which makes it easier to control the process.

3.1. Inconsistencies among objectives

Figure 1 clearly shows that single optimization of min(TCT), min(MCT), and min(CTV) are inconsistent with each other, since there is no single point that yields the best value for all three objectives. In order to statistically confirm the inconsistency among min(TCT), min(MCT), and min(CTV), we perform Spearman's rank correlation analyses for the sequences generating min(TCT), min(MCT), and min(CTV). Table 2 shows the Spearman's rank correlation coefficient (ρ) between sequences. Small values of (ρ) confirms that the sequences generating minimum values for single objectives are not significantly correlated.

Table 2: Spearman's ρ among sequence for scheduling objectives

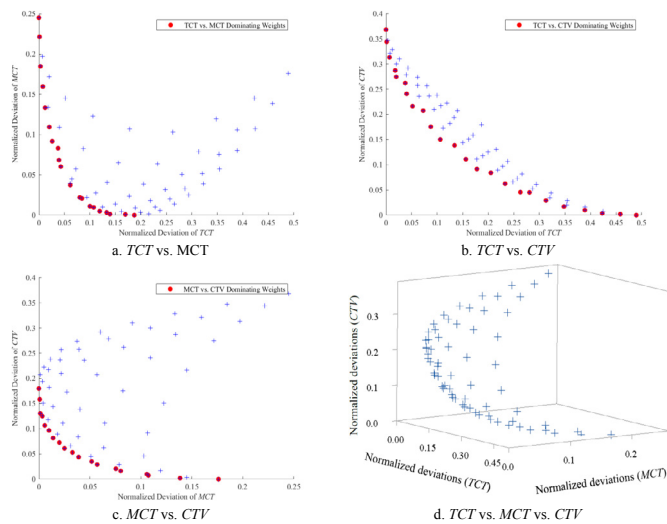|	*TCT* vs. *MCT*	*TCT* vs. *CTV*	*MCT* vs. *CTV*
Spearman's ρ	0.0823	0.2336	0.0528

Fig. 1: Scatter plots of normalized deviations of 66 weights, Pareto-dominant solution are shown by red markers

3.2. Pareto dominance

In the case of multi-objective optimization with inconsistent objectives, Pareto dominance is useful for decision making [17, 18]. For minimization problems, if $x_k^{[A]}$ and $x_k^{[B]} \in \mathbb{R}^K$ are two vectors that measure a positive attribute (k) such as the utility of decision A and B, respectively, decision A dominates decision B if the following conditions are satisfied:

$$x_k^{[A]} \leq x_k^{[B]}, \quad \forall \quad k \in \{1, 2, .., K\} \tag{15}$$

$$x_k^{[A]} < x_k^{[B]}, \quad \exists \quad k \in \{1, 2, .., K\} \tag{16}$$

Equation 15 states that decision A is not worse than decision B in any dimension, while equation 16 states that decision A is better than decision B at least in one dimension. Pareto optimal

outcome cannot be improved without sacrificing of at least one objective. Pareto dominant solutions are shown in figures 1 by red markers, each cross shows the data point obtained from one weight. It is observed that there is no Pareto-optimal solution when all three objectives are taken into consideration.

3.3. Process capability

Given inconsistencies among individual objective optimizations, and given different preferences on performance deviations, it is rare to have a unique Pareto-dominant solution, but a set of solutions, as shown in figure 1. Therefore, the question is left: what solution should be used? In order to answer this question, we utilize a series of statistical analyses including: SPC, and capability analyses(c_p,c_{pk}). Table 3 provides the summary of average performance and control limits ($\bar{x}$-R charts) of those weights with the best performance on single objectives and the weight with min($\overline{AND}(\Omega)$) i.e. [1,0,0],[0,1,0],[0,0,1], and [0.3,0.4,0.3] respectively, for the sake of brevity the SPC chart are not presented. Single objective optimization of min(TCT),

Table 3: Average performance and control limits of all objectives

	PI	min($x^{[1]}$)	min($x^{[2]}$)	min($x^{[3]}$)	min($x^{[4]}$)
	TCT	0.000	0.151	0.442	0.117
x-LCL	***MCT***	0.203	0.000	0.142	0.006
	CTV	0.327	0.147	0.000	0.117
	$\overline{AND}(\Omega)$	0.178	0.107	0.196	0.090
	TCT	0.000	0.187	0.490	0.150
$\bar{x}$	***MCT***	0.245	0.000	0.176	0.014
	CTV	0.368	0.180	0.000	0.144
	$\overline{AND}(\Omega)$	0.204	0.122	0.222	0.103
	TCT	0.000	0.223	0.537	0.183
x-UCL	***MCT***	0.287	0.000	0.211	0.021
	CTV	0.410	0.213	0.000	0.171
	$\overline{AND}(\Omega)$	0.231	0.138	0.248	0.116
	TCT	0.000	0.383	0.504	0.347
R-LCL	***MCT***	0.446	0.000	0.366	0.080
	CTV	0.441	0.348	0.000	0.288
	$\overline{AND}(\Omega)$	0.276	0.162	0.276	0.137
	TCT	0.000	0.603	0.796	0.547
R	***MCT***	0.702	0.000	0.576	0.126
	CTV	0.695	0.548	0.000	0.454
	$\overline{AND}(\Omega)$	0.435	0.256	0.435	0.216
	TCT	0.000	0.823	1.085	0.746
R-UCL	***MCT***	0.958	0.000	0.786	0.172
	CTV	0.949	0.748	0.000	0.619
	$\overline{AND}(\Omega)$	0.593	0.349	0.594	0.294

*LCL: Lower control limit, *UCL*: Upper control limit, $\bar{x}$: Average, *R*: Variation range

min(MCT), and min(CTV) generate no deviations on their intended objective but large deviations on the others. On the other hand, min(TO) not only provides the tightest bound on $\overline{AND}(\Omega)$ but also the second tightest on the other objectives. Let $B_{\min(x^{[k]})}^{[PI]}$ denote the difference between $UCL_{\min(x^{[k]})}^{[PI]} - LCL_{\min(x^{[k]})}^{[PI]}$ (the difference between upper and lower control limits), where PI is the performance indicator with 1 for TCT, 2 for MCT, 3 for CTV, and 4 for $\overline{AND}(\Omega)$. min($x^{[k]}$) represents the optimization objective with 1 for min(TCT), 2 for min(MCT), 3 for min(CTV), and 4 for min(TO). Also let $\mu_{\min(x^{[k]})}^{[PI]}$ denote the average performance of optimization objective min($x^{[k]}$) on per-

formance indicator *PI*. From Table 3 we can obtain the following inequalities which are also summarized in table 4:

$$\mu_{[1]}^{[1]} < \mu_{[4]}^{[1]} < \mu_{[2]}^{[1]} < \mu_{[3]}^{[1]} \tag{17}$$

$$B_{[1]}^{[1]} < B_{[4]}^{[1]} < B_{[2]}^{[1]} < B_{[3]}^{[1]} \tag{18}$$

$$\mu_{[2]}^{[2]} < \mu_{[4]}^{[2]} < \mu_{[3]}^{[2]} < \mu_{[1]}^{[2]} \tag{19}$$

$$B_{[2]}^{[2]} < B_{[4]}^{[2]} < B_{[3]}^{[2]} < B_{[1]}^{[2]} \tag{20}$$

$$\mu_{[3]}^{[3]} < \mu_{[4]}^{[3]} < \mu_{[2]}^{[3]} < \mu_{[1]}^{[3]} \tag{21}$$

$$B_{[3]}^{[3]} < B_{[4]}^{[3]} < B_{[2]}^{[3]} < B_{[1]}^{[3]} \tag{22}$$

$$\mu_{[4]}^{[4]} < \mu_{[2]}^{[4]} < \mu_{[1]}^{[4]} < \mu_{[3]}^{[4]} \tag{23}$$

$$B_{[4]}^{[4]} < B_{[2]}^{[4]} < B_{[3]}^{[4]} < B_{[1]}^{[4]} \tag{24}$$

Inequalities 24 and 23 state that $\min(TO)$ achieves minimum average and the tightest bounds on $\overline{AND}(\Omega)$, while inequalities 18 to 22, state that $\min(TO)$ achieves the second smallest average normalized deviation and the second tightest bounds on the normalized deviations of the other three objectives. This observation implies that minimizing trade-offs among inconsistent objective leads to the minimum deviation from the ideal point.

Table 4: Performance averages and bounds of objective functions

	min($x^{[1]}$)		min($x^{[2]}$)		min($x^{[3]}$)		min($x^{[4]}$)	
	μ	B	μ	B	μ	B	μ	B
TCT	0.000	0.000	0.187	0.072	0.490	0.095	0.150	0.066
MCT	0.245	0.084	0.000	0.000	0.176	0.069	0.014	0.015
CTV	0.368	0.083	0.180	0.066	0.000	0.000	0.144	0.054
$\overline{AND}(\Omega)$	0.204	0.053	0.122	0.031	0.222	0.052	0.103	0.026

In addition to SPC run charts, process capability indices, C_p and C_{pk}, are also good to compare different production schedules. Process capability index is the measure of the process capability to produce outputs that fall between the specification limits. Given μ and σ as the mean and standard deviation of the process outputs, $C_p = \frac{(USL-LSL)}{6\sigma}$ is the process capability index that measures if the process is capable of producing outputs that are centered around the center-line of the specification limits, LSL and USL denote lower and upper specification limits respectively. $C_{pk} = \min[\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}]$ is a performance indicator that measures if the mean value of process outputs falls between the specification limits [19]. Given LSL and USL, greater values of C_p and C_{pk} imply that a process generate outputs which are more centered with smaller variations.

To perform process capability analyses, we first need to define the specification limits of each performance indicator. Equations 25 and 26 represent the LSL and USL of performance indictor *PI* using the performance of $\min(x^{[4]})$. This definition not only provides a tight specification limits with only one standard deviation but also drives the specification limits towards 0 which is desirable for minimizing the deviation from the best value for each *PI*. Table 5 shows the specification limits used in this study.

$$LSL^{[PI]} = \max[0, \mu_{\min(x^{[4]})}^{[PI]} - \sigma_{\min(x^{[4]})}^{[PI]}] \tag{25}$$

$$USL^{[PI]} = \mu_{\min(x^{[4]})}^{[PI]} + \sigma_{\min(x^{[4]})}^{[PI]} \tag{26}$$

Table 5: Specification limits of performance indicators

	min($x^{[4]}$)		Specification limits	
	μ	σ	LSL	USL
TCT	0.1504	0.1267	0.0237	0.2771
MCT	0.0138	0.0231	0.0000	0.0377
CTV	0.1439	0.1142	0.0297	0.2581
$\overline{AND}(\Omega)$	0.1027	0.0495	0.0532	0.1522

Given the specification limits shown by table 5, we calculate C_p and C_{pk} for each objective $\min(x^{[k]})$. The results of the capability analyses are shown by table 6.

Table 6: Capability results for trade-off balancing

	PI	min($x^{[1]}$)	min($x^{[2]}$)	min($x^{[3]}$)	min($x^{[4]}$)
	TCT	inf	0.32	0.22	0.34
C_p	**MCT**	0.19	inf	0.23	0.56
	CTV	0.30	0.32	inf	0.32
	$\overline{AND}(\Omega)$	0.22	0.32	0.20	0.34
	TCT	-inf	0.23	-0.37	0.34
C_{pk}	**MCT**	-0.47	-inf	-0.31	0.39
	CTV	-0.22	0.23	-inf	0.32
	$\overline{AND}(\Omega)$	-0.16	0.20	-0.22	0.34

It is observed that the single objective optimizations poorly perform in terms of C_p and C_{pk}. $\min(x^{[4]})$ outperforms all three single objective optimizations. The outputs of $\min(x^{[4]})$ not only centered around the average value but also provide greater values of C_{pk} that means the process is better under control compared to the single objective optimizations. In order to further evaluate the capability of different solutions, we have provided the percentage of observations that fall above the upper specification limits (i.e. $\% > USL$) in table 7. Lower values of $\% > USL$ demonstrate that a process has greater capability relative to the upper specification limit. We use $\% > USL$ to evaluate the capability of solutions, because in a minimization problem the objective is to minimize the deviations from the best value. Therefore, those observations that fall above the USL show large deviations from the best solution and are of extreme importance in decision making. Table 7 shows the performance of all objective functions in terms of $\% > USL$ for all performance indicators, where $Max - Min$ for each objective function shows the difference between the maximum and the minimum values of $\% > USL$.

Table 7: Percentage of observations greater than USL ($\% > USL$)

	PI	min($x^{[1]}$)	min($x^{[2]}$)	min($x^{[3]}$)	min($x^{[4]}$)
	TCT	0.00	21.71	86.37	15.33
	MCT	91.91	0.00	82.48	15.57
	CTV	73.66	24.33	0.00	16.54
$\% > USL$	$\overline{AND}(\Omega)$	70.76	35.32	77.41	15.68
	Min	0.00	0.00	0.00	15.33
	Max	91.91	35.32	86.37	**16.54**
	Max-Min	91.91	35.32	86.37	**1.21**

Table 7 indicates that balancing trade-offs (i.e. $\min(x^{[4]})$) generates the most uniform results among all 4 optimization functions in terms of $\% > USL$ for all 4 performance indictors. Balancing tarde-offs drives the system performance to a point that the deviations from the best value for all performance indictors are fairly and uniformly small with the value of $Max - Min = 1.21\%$. On the other hand, single objective op-

timizations poorly perform with large values of $Max - Min$. $\min(x^{[1]})$ shows the worst performance among all objectives with $Max - Min = 91.91\%$.

3.4. Sustainability index

Following the scheme proposed in section 2, we propose a basic decomposition structure for this case study represented by table 8. Since in this case study our purpose is to show the proof of concept, we only consider the most basic elements in the decomposition structure.

Table 8: Top to bottom decomposition of $FSSI$

Index	Cluster	Sub-cluster	Metric	PI
FSSI	Economy	Manufacturing cost	Production cost	MCT
			Holding cost	TCT
	Environment	Waste management	CO_2 emission	MCT
	Society	Flow time	Flow time	TCT
			Flow time variance	CTV

We consider equal weights for individual metrics in each sub-cluster, and also for sub-clusters in each cluster. For the cluster weights, we change $\omega^{[\gamma]}$ from $[0.0:0.1:1]$, we have 66 combinations of weights with $\sum_{\gamma=1}^{3} \omega^{[\gamma]} = 1$. Table 9 shows the $FSSI$ value of $\min(x^{[1]})$, $\min(x^{[2]})$, $\min(x^{[3]})$, and $\min(x^{[4]})$ for all 66 combinations of $\omega^{[\gamma]}$.

Table 9: $FSSI$ for trade-off balancing

		$\min(x^{[1]})$	$\min(x^{[2]})$	$\min(x^{[3]})$	$\min(x^{[4]})$
FSSI	μ	8.16	9.08	7.49	**9.19**
	σ	1.10	0.67	1.29	**0.58**
	max	8.78	10.00	8.23	9.86
	min	7.55	8.16	6.67	**8.23**

$\min(x^{[4]})$ (i.e. balancing trade-offs) generates the highest average $FSSI$ of 9.19 with the smallest standard deviation of 0.58. It also generates the highest minimum of 8.23 for $FSSI$. These results indicate that trade-off balancing is the most sustainable scheme among all the objective functions that we reviewed.

4. Conclusion and future work

Given an n-job, m-machines permutation flow shop scheduling problem, we statistically showed that inconsistencies exist among three objectives of $\min(TCT)$, $\min(MCT)$, and $\min(CTV)$. With inconsistencies it is impossible to simultaneously optimize all three objectives. Therefore, we proposed a scheme of trade-off balancing that provides a meaningful compromise among inconsistent objectives by driving the system performance towards a point with minimum deviations from the best possible value of each objective. Statistical analyses showed that balancing trade-offs provides a better control over the individual objectives in terms of average, standard deviation, C_p and C_{pk} compared to those of single objective optimizations that implies our scheduling method can get good results to NP-complete scheduling problems. Moreover, based on normalized deviations, we developed a flow shop scheduling sustainability index ($FSSI$). Case study results showed that balancing trade-offs provides the most sustainable solutions for the flow shop scheduling problem. Our future work will be on developing a comprehensive $FSSI$ decomposition structure in order to address various aspects of flow shop scheduling problem.

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References

[1] Pinedo ML. Scheduling: theory, algorithms, and systems. Springer, 2016.

[2] Yenisey MM, Yagamahan B. Multi-objective permutation flow shop scheduling problem: Literature review, classification and current trends. OMEGA-Int J Manage S 2014;45:119–135.

[3] Garey MR, Johnson DS, Sethi R. The complexity of flowshop and jobshop scheduling. MATH Oper Res 1976;1.2:117–129.

[4] Hoogeveen JA, Kawagushi T. Minimizing total completion time in a two-machine flowshop: analysis of special cases. Math Oper Res 1999;24.2:887-910.

[5] Mosheiov G. Minimizing mean absolute deviation of job completion times from the mean completion time. Nav Res Log 2000;47.8:657–665.

[6] De P, Gosh JB, Wells CE. On the minimization of completion time variance with a bicriteria extension. Oper Res 1999;40.6:1148–1155.

[7] Kubiak W. Completion time variance minimization on a single machine is difficult. Oper Res Lett 1993;14.1:49–59.

[8] Li W, Dai H, Zhang D. The relationship between maximum completion time and total completion time in flowshop production. Procedia Manuf. 2015;1:146–156.

[9] Ye H, Li W, Abedini A. An effective and efficient heuristic for no-wait flow shop production to minimize total completion time, Comput Ind Eng 2017;108:57–69.

[10] Garbie HI. Sustainability optimization in manufacturing enterprises, Procedia CIRP 2015;26:504–509.

[11] Ding JY, Song S, Wu C. Carbon-efficient scheduling of flow shops by multi-objective optimization. Eur J Oper Res 2016;248.3:758–771.

[12] Mansouri SA, Aktas E, Besikci U. Green scheduling of a two-machine flowshop: Trade-off between makespan and energy consumption. Eur J Oper Res 2016;248.3:772–778.

[13] Jawahir IS, Badurdeen F, Rouch KE. Innovation in sustainable manufacturing education, Universitätsverlag der TU Berlin; 2015.

[14] Akbar M, Irohara T. Scheduling for sustainable manufacturing: A review. J Clean Prod. 2018;250:866–883.

[15] Lu T. A metrics-based sustainability assessment of cryogenic machining using modeling and optimization of process performance, PhD dissertation, University of kentucky, 2014.

[16] Lu T, Jawahir IS. Metrics-based sustainability evaluation of cryogenic machining, Procedia CIRP 2015;29:520–525.

[17] Voorneveld M. Characterization of Pareto dominance, Oper Res Lett 2003;31.1:7–11.

[18] Li W, Nault BR, Ye H. Trade-off balancing in scheduling for flow shop production and perioperative processes. Eur J Oper Res 2019;273.3:817–830.

[19] Montgomery DC. Statistical quality control, volume 7, Wiley New York; 2009.