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Similitude theory applied to plates in vibroacoustic field: a review up to 2020

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Abstract

Similitude methods are a set of tools which allow the design of scaled-up or scaled-down models of a full-scale structure called a prototype. In this way, the financial and temporal costs of experimental tests, and the problems associated with the set-up of too large (or small) test articles, may be overcome. This article provides a brief review of similitude methods applied to plates in a vibroacoustic field. Particularly, it is dedicated to a thorough analysis of similitude conditions and scaling laws for uncovering commonalities and differences, and physical interpretations, obtained from applying different scaling methods.

Keywords: Similitude; Scaling laws; Plates; Structural dynamics; Vibro-acoustics

Nomenclature

A	plate area
A_{mn}	coefficient of the solution of the characteristic equation
A_{rs}	EDA energy coefficients
$A_{Q_jQ_k}$	joint acceptance between the j-th and the k-th modes
a	plate length
b	plate width
C	distortion constant
c_L	longitudinal wave speed
D_{ij}	laminate flexural stiffnesses
E	Young's modulus of isotropic materials
E_{ij}	laminate Young's moduli
h	plate thickness
F	force amplitude
M	total mass
m	number of half waves along the horizontal direction
N_{ij}	in-plane force per unit length/area
N_M	number of modes
N_p	number of plies of laminated structures
n	number of half waves along the vertical direction
P_{IN}	input power
p	integer index in Galerkin procedure
q	integer index in Galerkin procedure
q_{ij}	sum of external forces per unit length/area
R	radiation resistance matrix
S	statistical modal overlap factor
S_{ff}	power spectral density of the force
S_p	power spectral density of wall pressure distribution due to a turbulent boundary layer
S_w	power spectral density of displacement
S_{wm}	mean value of displacement power spectral density over the spatial domain
T_r	kinetic energy
U_c	convective velocity
u	displacement along x direction
u_τ	friction velocity
V_m	mean velocity
v	displacement along y direction
w	displacement along z direction
X_w	cross-spectral density of plate displacement
x	coordinate along x direction
x_F	dimensionless excitation point coordinate
x_S	dimensionless acquisition point coordinate
y	coordinate along y direction
z	coordinate along z direction

Greek symbols

β	parameter linked to the number of plies
Γ_{jj}	auto-modal power mobility of j-th mode
Γ_{jk}	cross-modal power mobility between j-th and k-th mode

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δ turbulent boundary layer thickness
ε ASMA scale factor for dynamic scaling
ε_x Strain along *x* direction
ε ASMA scale factor for dynamic scaling
ζ damping ratio
η_j modal damping loss factor of j-th mode
θ modal density
Λ ratio between the natural frequencies of structure and fluid
λ_g scale factor of parameter g
μ modal overlap factor
ξ_x streamwise spatial separation in Corcos model
ξ_y spanwise spatial separation in Corcos model
ρ mass density
σ ASMA scale factor
σ_x stress along *x* direction
φ_j j-th mode shape
ψ_{jk} cross-mode participation factor between j-th and k-th mode
Ω nondimensional frequency
ω radial frequency
ω^* dimensionless radial frequency
ω_j j-th radial natural frequency

Other symbols

AR aspect ratio
ϵ small quantity describing the effect of added mass

Introduction

In their seminal work on structural similitudes [1], Simitses and Rezaeepazhand reported the details of final static tests of a Lockheed C-141A airlifter which gives an example of how much experimental effort is needed when validating products. In their example, the final phase required eight wing tests, 17 fuselage tests, and seven empennage tests.

It is easy to imagine how costly full-scale tests can be in both temporal and financial terms, especially when the experiments require the structure to be damaged, e.g. crashworthiness tests, or when repetition is required due to errors or uncertainties and other unforeseen events. Last, but not least, the testing itself may not always be straightforward to preform when specimens or objects have very large or small dimensions.

To bypass many of the problems listed in the foregoing, it is useful to test scaled-up or scaled-down variations (called *models*) of full-scale structures (called *prototype*). Actual savings of money and time were proven by Holmes and Sliter [2] in which the estimated savings were between one-third and one fourth of the cost of the full-scale object and its testing.

Increasingly, the complexity of current engineering systems usually demands experimental testing of the fundamental steps of the design of a product to check if all the reliability, safety and performance requirements have been satisfied. As a consequence, similitude methods have received great attention in recent years because they enable design models to be used that reconstruct the behavior of prototypes.

The aim of this article is to provide a brief review on the application of similitude theory in vibroacoustics, thereby offering readers with an overview of important parameters to scale when studying vibrational and sound transmission phenomena. As a matter of fact, noise and vibration propagation are subjects of intense research, affecting passengers' comfort in vehicles and people nearby, as well as addressing industrial topics such as structural failure and machinery fatigue.

The study of vibroacoustics is suitable for other interesting applications of similitude methods via numerical simulations. For example, it is well known that the Finite Element Method (FEM) is computationally prohibitive when analyses of high frequency ranges are required; even so, FEM is one of the most used numerical tools for structural analyses. In FEM, the spatial meshes are frequency dependent and can be greatly decreased as frequencies are increased. Furthermore, the Nyquist Theorem demands that the analyses must be pushed to at least twice the highest frequency range of interest, also leading to greater computational times. In such a scenario, similitude theory provides interesting tools allowing geometrical dimension changes, and concomitant decreases in mesh and eigensolutions.

This review focuses on one particular structural element, a plate, that is widely applied in many engineering fields such as plates in propeller blades, vibration adaptors, upper and lower skins or surfaces in aircraft wings, panels and soundproofing partitions.

Each section herein is dedicated to a similitude method and its application to plates in vibroacoustics. After the review, another section will assess the methods, their commonalities and the main physical aspects highlighted. In the end of each section, conclusions are drawn. Due to the wide range of applications and the resulting high number of parameters involved, readers may need to refer to the nomenclature at the beginning of the article.

Dimensional analysis

Dimensional Analysis (DA) relies on the derivation of dimensionless terms governing the phenomenon under investigation, and is definable by means of Buckingham's Π Theorem [3]. It is useful for complex systems which lack governing equations but requires an experienced analyzer to derive a set of dimensionless groups to be chosen carefully. Moreover, the dimensionless groups may be not unique, leading to a trial-and-error procedure for the determination of the right set, and not all of them are physically meaningful.

Although DA is the oldest and most used similitude method (especially in impact engineering [4]), an interesting application to plates in vibroacoustics is the work by Ciappi et al. [5]. They defined dimensionless parameters based on scaling laws for excitation frequencies and the Power Spectral Density (PSD). In fact, such a dimensionless representation could provide a universal expression for the structural response of systems excited by Turbulent Boundary Layers (TBL) in which induced vibrations in elastic structures are the source of the major noises in transport engineering involving naval, aerospace, and automotive applications.

The test article under analysis was a thin, flat elastic plate without prestresses wetted over one face by a stationary TBL. The flow was incompressible and without pressure gradient. By applying DA, the authors identified 11 governing dimensionless parameters, including:

$$\frac{S_w U_c}{h^3}, \rho\sqrt{\frac{U_c^3 h}{S_p}}, E\sqrt{\frac{h}{S_p U_c}}, \frac{\omega h}{U_c}, \frac{u_\tau}{U_c}, \frac{a}{h}, \frac{b}{h}, \frac{\delta}{h}, \frac{\xi_x}{h}, \frac{\xi_y}{h}, \eta . \quad (1)$$

By means of a thorough analyses of this set, these authors demonstrated that not all the groups were useful. In fact, the ratios a/h and b/h related to the structural modes could be assumed large because only the thin plate model (a Kirchhoff's one) was used. The ratio δ/h, representative of the fluid-structure coupling, was greater than one. Furthermore, because the order of magnitude of both displacement and thickness were the same, structural vibrations did not affect the fluid domain. Also, both the ratios ξ_x/h and ξ_y/h were negligible because the diagonal terms of the Cross Spectral Density (CSD) matrix were assumed to provide the major contributions to plate responses; the velocity ratio, u_τ/U_c, was not very important because of its small variations. Finally, een the damping η was not considered because it can be assumed constant for all the plate models under analyses.

Therefore, the only dimensionless ratios important for deriving a nondimensional form of the displacement PSD were those involving the pressure distribution PSD. In particular, using the following form of nondimensional frequency,

$$\omega^* = \frac{\omega h}{U_c}, \quad (2)$$

the dimensionless descriptions of the displacement PSD could be written as a function of Eq. (2):

$$\frac{S_w}{S_p}\left(\frac{E}{h}\right)^2 = \frac{S_w}{S_p}\left(\frac{\rho}{h}\right)^2 = g_I(\omega^*)\,, \quad (3)$$

$$\frac{S_w}{S_p}\left(\frac{U_c}{h}\right)^2 \rho E = \frac{S_w}{S_p}\left(\frac{U_c}{h}\right)^2 \rho^2 c_L^2 = g_{II}(\omega^*)\,, \quad (4)$$

$$\frac{S_w}{S_p}\left(\frac{\rho}{h}\right)^2 U_c^4 = \frac{S_w}{S_p}\left(\frac{E}{h}\right)^2 \left(\frac{U_c}{c_L}\right)^4 = g_{III}(\omega^*)\,. \quad (5)$$

Moreover, another form of dimensionless displacement PSD was derived from energy considerations, given by:

$$\frac{S_{wm}}{S_p}\left(\frac{\eta\rho^2 c_L U_c^3}{h^2}\right) = g_{IV}(\omega^*)\,. \quad (6)$$

After deriving these expressions, the authors compared the results of four experimental tests that had been performed at different conditions in both a wind tunnel and a towing tank, and plotted them on reference axes reporting the values of the dimensionless frequency on the horizontal axis and one of Eqs. (3)–(6) on the vertical axis.

These comparisons showed that the experimental data sets collapsed very close to each other, proving that the proposed representations allowed an estimate of displacement PSD in the whole frequency range, revealing it to be very useful for preliminary predictive steps. Particularly, Eq. (6) linked the nondimensional frequency and acceleration with just one, simple equation, thereby providing a quick estimate of structural responses in the entire frequency range. Furthermore, Eq. (3) provided a good response estimate when damping was difficult to identify; for example, when the plate was part of a greater structure such as a ship or an airplane.

DA was used by He et al. [6] in an interesting article on the problem of middle-frequency regions. In fact, the SEA method, which is meant to give consistent results in high modal density regions, would not work; moreover, the phase information was lost. Instead, the FE approach was able to provide phase information but its application outside low modal density ranges became computationally inefficient. Therefore, the authors decided to apply the coupling within a FE-SEA hybrid method that combined the advantages of both DA and FE methods and was suitable for the middle-frequency range.

With the aim of studying the coupling between solar arrays and a satellite, the authors first tested an assembly of two plates. The subsystems with low modal density were modeled with the FE approach while those at high modal density were scaled with the SEA approach. In this system, the excited plate was the deterministic subsystem while the receiver plate was the statistical one; both were made of aluminum.

Three fundamental scaling laws were defined by assuming a consistent material and a broadband concentrated force. Then, the spectral density of the force scaled as:

$$\lambda_{S_{ff}} = \frac{\lambda_F^4}{\lambda_L^2}, \tag{7}$$

where it was assumed that the geometrical dimensions of the plates scaled in the same way as λ_L.

In contrast, the velocity response scaled differently according to the type of subsystem. For a FE subsystem, the velocity scaled as:

$$\lambda_{\lambda_{FE}} = \frac{\lambda_F^2}{\lambda_L^3 \lambda_E}, \tag{8}$$

while for SEA subsystems it scaled as:

$$\lambda_{\lambda_{SEA}} = \frac{\lambda_F}{\lambda_L^2 \lambda_{\eta_j}^{1/2}}. \tag{9}$$

Numerical analyses were performed with the assumption that $\lambda_{\eta_j} \approx 1$, a reasonable approach because if the internal loss factor was smaller than 0.1 the difference between the systems would be small. As a consequence, the predictions of the RMS responses were consistent but, since SEA introduced uncertainties, the reconstructions of the response became less precise at increasing frequencies. Furthermore, the excited plate, i.e. the FE subsystem, provided better predictions than the receiving plate, i.e. the SEA subsystem, because of power loss in the hybrid line connection.

Similitude theory applied to governing equations

The second, most used similitude method is STAGE (Similitude Theory Applied to Governing Equations), introduced for the first time by Kline [7]. STAGE is based on the introduction of scale factors (either dimensional or nondimensional) directly into the governing equations as well as their solutions [4] to determine similitude conditions and scaling laws between two systems. Thereby, the set of conditions derived becomes more specific because, being equation driven, they have greater possibility for having fundamental physical meanings. For STAGE, the scale factor is the ratio between a model parameter $\hat{g}$ and a prototype parameter g, as shown in the following:

$$\lambda_g = \frac{\hat{g}}{g}. \tag{10}$$

This method has been widely applied to analyzing free vibrations of laminated plates by Rezaeepazhand et al. [8–9] and Simitses [10]. In particular, they focused on deriving the similitude conditions and scaling laws for angle- and cross-ply laminated plates. The main purpose of the investigation was to reveal problems associated with the design of models, especially to what extent it was possible to cause distortions before prototype behavior was totally unrecoverable.

For angle-ply configurations, the authors assumed that the prototype and model had similar mode shapes, implying that they could be well approximated by the same number of terms in the series obtained by applying Galerkin procedures for determining solutions of the characteristic equations. Hence, the scale factors associated with the integers characterizing this series were the same, i.e.:

$$\lambda_m = \lambda_n = \lambda_p = \lambda_q = \lambda_{A_{mn}} = 1 \tag{11}$$

Then, five scaling laws for the nondimensional frequency were derived:

$$\lambda_\Omega^2 = \frac{\lambda_{D_{11}}}{\lambda_{E_{22}} \lambda_h^3 \lambda_{AR}^4}, \tag{12}$$

$$\lambda_\Omega^2 = \frac{\lambda_{D_{12}}}{\lambda_{E_{21}} \lambda_h^3 \lambda_{AR}^2}, \tag{13}$$

$$\lambda_\Omega^2 = \frac{\lambda_{D_{22}}}{\lambda_{E_{22}} \lambda_h^3}, \tag{14}$$

$$\lambda_\Omega^2 = \frac{\lambda_{D_{16}}}{\lambda_{E_{22}} \lambda_h^3 \lambda_{AR}^3}, \tag{15}$$

$$\lambda_\Omega^2 = \frac{\lambda_{D_{26}}}{\lambda_{E_{22}} \lambda_h^3 \lambda_{AR}}. \tag{16}$$

These laws depended only on material properties and the total number of plies, not on the thicknesses of single plies. These authors also introduced an important parameter linked to the total number of plies, N_p, defined as:

$$\beta = \frac{3N_p^2 - 1}{N_p^3}. \tag{17}$$

To achieve complete similitude, Eqs. (12)–(16) must be satisfied simultaneously. For similitude to be satisfied when the plies had identical thicknesses, the condition of the same material properties and fiber orientation would require:

$$\lambda_{AR}^{-2} = 1 = \lambda_{AR}^2 = \frac{\lambda_\beta}{\lambda_{AR}} = \lambda_\beta \lambda_{AR}. \tag{18}$$

Eq. (18) leads to $\lambda_\beta = \lambda_{AR} = 1$. Thus, a true model was obtainable if the lengths and widths of the panel as well as the total number of plies scaled in the same way. The authors defined this scaling procedure as ply-level scaling. It is interesting to note that thickness was not directly involved in the scaling procedure and, thus, it was a free parameter.

Analogously, for cross-ply configurations, the dimensionless frequency scaling laws became:

$$\lambda_\Omega^2 = \frac{\lambda_{D_{11}} \lambda_m^4}{\lambda_{E_{22}} \lambda_h^3 \lambda_{AR}^4}, \tag{19}$$

$$\lambda_{\Omega}^2 = \frac{\lambda_{D_{12}}\lambda_m^2\lambda_n^2}{\lambda_{E_{21}}\lambda_h^3\lambda_{AR}^2}, \quad (20)$$

$$\lambda_{\Omega}^2 = \frac{\lambda_{D_{22}}\lambda_n^4}{\lambda_{E_{22}}\lambda_h^3}. \quad (21)$$

Complete similitude requires:

$$\lambda_m^4 = \lambda_m^2\lambda_n^2\lambda_{AR}^2 = \lambda_n^4\lambda_{AR}^4 , \quad (22)$$

which is satisfied if and only if $\lambda_m = \lambda_n$ and $\lambda_{AR} = 1$. Hence, the aspect ratio must be retained and the number of half waves m and n must scale in identical ways.

Tests on the prediction capability using partial similitudes highlight that angle-ply laminated plates are very sensitive to scaling procedures in contrast to ply-level scaling, and leads to adapting the value of scale factor λ_R to correct distortion which then often requires unsuitable design conditions. Instead, cross-ply construction tends to exhibit less sensitivity to changes in the number of plies and to less constrictions during the design phase.

The articles of Singhatanadgid and Ungbhakorn [11] and Singhatanadgid and Na Songhkla [12] also investigated using STAGE methods applied to laminated plates. In [11], the authors investigated free vibrations of antisymmetric cross- and angle-ply plates, and confirmed the results proposed in [8–10], especially the relevance of the stacking sequence. In particular, the fundamental role of flexural stiffness was elucidated. For example, when the flexural stiffness parameter was changed, greater distortions occurred than did in the cases where extensional or flexural-extensional stiffnesses were changed.

The work in [12] was totally devoted to experimentation that highlighted the difficulties arising in laboratory testing due to boundary conditions. In fact, among the many tests performed, it was the free plate configurations which exhibited the smallest discrepancies.

An interesting perspective of STAGE application was also provided by Luo et al. [13] so that the structure size interval of distorted models was determined in which the first order characteristics of natural frequencies and mode shapes of a prototype could be predicted with acceptable precision. For plates in partial similitude, they introduced a constant C inserted into the scaling laws, obtaining:

$$\lambda_\omega = \frac{\lambda_h}{C^2\lambda_b^2}, \quad (23)$$

$$\lambda_\omega = \frac{\lambda_h}{C\lambda_b^2}, \quad (24)$$

$$\lambda_\omega = \frac{\lambda_h}{\lambda_b^2}. \quad (25)$$

The determination of C then helped to identify the range of geometrical values satisfying a fixed value of precision between the prototype response and predictions from distorted models.

Worth mentioning is a variation of STAGE proposed by Coutinho et al. [14]. Here, STAGE was applied to derive the similitude conditions and scaling laws from sets of basic equations from elasticity theory, force and moment resultants, stress-strain and strain-displacement relations, and displacement fields. This approach enabled scaling laws to be assembled according to the phenomena under investigation in addition to deriving the laws. Applying this STAGE method to numerical analyses of a stiffened pinned isotropic plate considered as a free body, the derived similitude conditions and scaling laws were:

$$\lambda_x^{plate} = \lambda_y^{plate} , \quad (26)$$

$$\lambda_u^{plate} = \lambda_v^{plate} = \left(\lambda_w \frac{\lambda_z}{\lambda_x}\right)^{plate} , \quad (27)$$

$$\lambda_{\sigma_x}^{plate} = \lambda_{\varepsilon_x}^{plate} = \left(\frac{\lambda_u}{\lambda_x}\right)^{plate} , \quad (28)$$

$$\lambda_N^{plate} = \left(\lambda_{\sigma_x}\lambda_z\right)^{plate} = \left(\frac{\lambda_u}{\lambda_x}\lambda_z\right)^{plate} , \quad (29)$$

$$\lambda_q^{plate} = \frac{\lambda_N^{plate}}{\lambda_x^{plate}} = \left(\frac{\lambda_u}{\lambda_x^2}\lambda_z\right)^{plate} . \quad (30)$$

Eq. (26), although expressed in terms of coordinates x and y, clearly relates to the condition already expressed in Eqs. (8)–(10), i.e. $\lambda_R = 1$. Hence, it is important to consider that when plates are examined it is necessary to keep aspect ratios independent from material properties.

A similar set of equations can be derived for a beam:

$$\lambda_x^{beam} = \lambda_z^{beam} , \quad (31)$$

$$\lambda_u^{beam} = \lambda_w^{beam} , \quad (32)$$

$$\lambda_{\sigma_x}^{plate} = \lambda_{\varepsilon}^{plate} = \left(\frac{\lambda_u}{\lambda_x}\right)^{plate} , \quad (33)$$

$$\lambda_N^{beam} = \left(\lambda_{\sigma_x}\lambda_y\lambda_z\right)^{beam} = \left(\frac{\lambda_u}{\lambda_x}\lambda_y\lambda_z\right)^{beam} = \left(\lambda_u\lambda_y\right)^{beam} , \quad (34)$$

$$\lambda_q^{beam} = \frac{\lambda_N^{beam}}{\lambda_x^{beam}} = \left(\frac{\lambda_u}{\lambda_x}\lambda_y\right)^{beam} . \quad (35)$$

However, because the test article under investigation is a stiffened plate, the equations given in (26)–(30) and (31)–(35), respectively, must be related by imposing continuity of displacements and internal forces at interfaces.

Finally, keeping in mind that for equivalent points the same material properties must hold, the following conditions were derived:

$$\lambda_x = \lambda_y = \lambda_z \,, \quad (36)$$

$$\lambda_u = \lambda_v = \lambda_w \,, \quad (37)$$

$$\lambda_{\sigma_x} = \frac{\lambda_w}{\lambda_x}, \quad (38)$$

$$\lambda_N^{plate} = \frac{\lambda_N^{beam}}{\lambda_y} = \lambda_w, \quad (39)$$

$$\lambda_q^{plate} = \frac{\lambda_q^{beam}}{\lambda_y} = \frac{\lambda_w}{\lambda_x} \quad (40)$$

In Eq. (39) through (40), the factors without superscripts indicate that the scale factors were applied to the whole stiffened plate and that geometrical dimensions scaled in the same way for both plate and stiffeners.

Eqs. (36)–(37) originate from the condition of continuity in displacements, while Eqs. (38)–(40) originate from the continuity of internal forces. These sets of equations are quite restrictive because they link geometrical dimensions to displacements, stresses, and internal and external forces. Also, Eq. (36) shows that thickness was not a free parameter when examining simple stiffened plates.

The simulations on plates under complete similitude and subjected to uniform pressures were capable of providing perfect predictions in terms of transverse displacements. In other words, these assessments pushed further the applications of STAGE to other more complex structural fields such as thermal deformations or acoustics, thereby enabling the analyses of more complex problems.

Asymptotic scaled modal analysis

The Asymptotic Scaled Modal Analysis (ASMA) method was conceived to deal with issues related to spatial meshing typical within dynamic analyses executed using FEM. A formal definition of ASMA was provided by De Rosa et al. [15] that relied on formal justification by means of Energy Distribution Analysis (EDA) [16]. Basically, in ASMA all parameters not related to energy transmission (like geometrical parameters) are scaled down with a factor $\sigma < 1$; this scaling moves the eigensolutions to higher frequencies. As a result, damping effects also need to be scaled to retain similitude in energy levels of scaled models and prototypes.

From the point of view of computational time, ASMA offers three benefits:

- Identical number of degrees of freedom and eigensolutions: the response of the model can be representative even at higher frequencies without computational advantages.
- Identical number of degrees of freedom and eigensolutions: if the dynamic response is evaluated at least in the same frequency range of the prototype, then computational times are lessened.
- Reduced number of degrees of freedom and eigensolutions: if the frequency response is within identical frequency ranges then an appreciable computational advantage occurs.

ASMA provides very good global estimates but local estimates may be impaired by artificially increased damping; moreover, it can be implemented in any finite element solver.

De Rosa et al. [15, 17] provided an interesting application of ASMA to the assembly of two plates in which one was the driver, i.e. excited one, and the other plate was the receiver. The lengths and widths of both plates were scaled as follows:

$$\hat{a} = \sigma a \,, \quad (41)$$

$$\hat{b} = \sigma b \,. \quad (42)$$

The scaled-down equations (41) and (42) implied that the j-th natural frequency scaled as:

$$\hat{\omega}_j = \frac{\omega_j}{\sigma^2}. \quad (43)$$

It is necessary to implement dynamic scaling when using ASMA to ensure that similar modal characteristics are retained. For this purpose, another scale factor $\varepsilon < 1$ was introduced and damping was scaled as:

$$\hat{\eta}_j = \frac{\eta_j}{\varepsilon}. \quad (44)$$

Scaling EDA parameters must also be taken into account such that spatial correlations between two modes must not change; in other words, the cross-mode participation factors remain the same:

$$\hat{\psi}_{jk} = \psi_{jk} \,. \quad (45)$$

For frequency characterizations, the auto-modal power mobilities scale as

$$\hat{\Gamma}_{jj} = \varepsilon\sigma^2\Gamma_{jj} \,. \quad (46)$$

The cross-modal power mobilities can assume large values when modes overlap, or small values when modes do not overlap. Thereby, different functions of the scale factors σ and η are involved, which requires different scaling procedures. Particularly:

$$\hat{\Gamma}_{jk,large} = \varepsilon\sigma^2\Gamma_{jk,large} \,, \quad (47)$$

$$\hat{\Gamma}_{jk,small} = \frac{\sigma^2}{\varepsilon}\Gamma_{jk,small} \,. \quad (48)$$

The large terms scale in the same way of the auto-modal power mobilities; the small terms are the only ones which would not scale according to the rule $\sigma^2\varepsilon$.

When scaling the response of a system, or a directly excited subsystem belonging to a generic assembly, any value $0 < \sigma < 1$ can be chosen. Furthermore, it can be demonstrated that assuming $\sigma = \varepsilon$ usually provides acceptable results. Under these hypotheses, the energy terms of interest such as input power, kinetic energy, EDA energy coefficients and mean squared velocity scale as:

$$\hat{P}_{IN}^{(s)} = P_{IN}^{(s)} \,, \quad (49)$$

$$\hat{T}^{(r)} = \varepsilon\sigma^2 T^{(r)} = \sigma^3 T^{(r)} \,, \quad (50)$$

$$\hat{A}_{rs} = \varepsilon\sigma^2 A_{rs} = \sigma^3 A_{rs} \,, \quad (51)$$

$$\hat{V}_m^2 = \varepsilon V_m^2 = \sigma V_m^2 \,, \quad (52)$$

where the superscripts indicate a source plate (s) and a receiver plate (r).

According to Eq. (49), the input power is not scaled. This result, analytically, is due to scale factors erasing each other; nonetheless, it is coherent with energy redistribution among the modes that are imposed on the energy terms (EDA coefficients, damping, etc.) through the scale factors.

Computational savings can be obtained by scaling the number of modes as $N_M\sigma^2$, which is the number of modes required in the scaled response. This choice is motivated by the decreasing modal density when the plate area is reduced. However, $N_M\sigma^2$ is the minimum number of required modes. In fact, one can choose not to scale such a number and push the dynamic analyses of the model to additional frequencies. In this regard, the scale factors act as frequency modulators that control the width of the frequency window in which the model yields acceptable predictions.

The application of ASMA to the assembly has shown reasonable predictive capabilities when the modal overlap factor μ is high enough to expect a global response, i.e. in the same range of validity as that of SEA. On the one hand, the response of a source plate would be well reconstructed because it is dominated by the power input, which is related to the auto-modal power mobility; ASMA is very applicable to this condition. On the other hand, the predictions on the receiving system have exhibited some approximations but a noticeable computational saving has been obtained; for example, with 400 points in a source plate and 320 in a receiver plate, the prototype assembly could be modelled with to 60 and 48 points, respectively.

These results were strengthened by the results in [18] in which ASMA was applied to an aluminum flexural plate; it highlights how the ASMA method can reproduce responses only after averaging on all the acquisition and excitation points.

ASMA has also been applied to the case of a plate under TBL [19] and again underlined that the method is very useful for predicting global responses but not local responses. Nevertheless, comparisons of ASMA with FEM results showed how FEM diverged above the coincidence frequency due to spatial aliasing while the ASMA method remained accurate. This result was a consequence of the fact that the minimum scaled flexural wavelength was smaller than the TBL correlation length. Therefore, ASMA revealed to be again a cost-efficient approach to predict the mean square velocity response and radiated sound power over broad frequency ranges.

Finally, ASMA was applied also to assemblies of three plates in [17, 20] and Li elaborated on this research using ASMA in [21–22]. In the first paper [20], the main idea was to extend ASMA to predicting ensemble statistics by scaling the structural parameters of an original system and then testing whether ensemble statistics of high frequency vibrations could be recovered from a scaled model.

Assuming that the natural frequencies of the system formed a Gaussian Orthogonal Ensemble (GOE) and that the mode shapes were random, the expected value of the mean square velocity was invariant when:

$$\frac{\lambda_F^2\lambda_\theta}{\lambda_\eta\lambda_M^2} = 1 \,, \quad (53)$$

while its variance was invariant when:

$$\lambda_\eta\lambda_\theta = 1 \,. \quad (54)$$

In other words, Eqs. (53) and (54) were the scaling laws of mean value and variance of mean square velocities.

The system considered was a thin flat plate with randomly placed masses attached, for which Eqs. (53) and (54) assumed the following specific form:

$$\frac{\lambda_F^2}{\sqrt{\lambda_E\lambda_\rho^3}\lambda_\eta\lambda_A\lambda_h^3} = 1 \,, \quad (55)$$

$$\lambda_\eta = \sqrt{\frac{\lambda_E}{\lambda_\rho}\frac{\lambda_h}{\lambda_A}} \,. \quad (56)$$

Modal density was used as the control factor for the scaling procedure, and it was determined to be scaled-down to reduce computational costs. From Eq. (54) and Eq. (56), the following condition must hold:

$$\lambda_\theta = \sqrt{\frac{\lambda_\rho}{\lambda_E}\frac{\lambda_A}{\lambda_h}} < 1 \,. \quad (57)$$

However, auxiliary requirements had to be added to condition (57) to assure the reduction in modal density and a boosting in the frequency analyses. These requirements involved a statistical modal overlap factor, S ($\lambda_S = 1$), and a small quantity, ϵ, representative of the added mass ($\lambda_\epsilon = 1$). Applying the method to the plate under harmonic excitation, the mean square velocity ensemble statistics were well predicted and, as a consequence, the model produced ensemble characteristics the same as the original system. The conditions and scaling laws derived in [22] were also identical but auxiliary requirements did not need to be considered because the test article was a simple plate without added masses.

The idea that motivated Li's research was to scale material or geometrical properties of a plate while preserving the dynamics at high frequency in the same manner as SEA methods. This meant that the prototype and model must lead to the same results when SEA and ASMA were applied. In this case, the modal density acted as a control factor and it must decrease by some small amount to enable the SEA results to still be representative of the model dynamic behavior in the frequency range of interest. The SEA assumptions on the modal density were satisfied in the model if they were also satisfied in the prototype. The approach worked well for both local and global responses, and accurately estimated when the control factor $\lambda_\theta \cong 1$, i.e. when the modal density of the prototype was close to the modal density of the model.

Similitude and asymptotic models for structural-acoustic research applications

The Similitude and Asymptotic Models for Structural-Acoustic Research Applications (SAMSARA) method was formally introduced by De Rosa et al. [23]; its aim was to define similitude conditions for acoustic-structural systems, and employed an approach that increased the number of parameters to consider when deriving similitude conditions and scaling laws.

When a plate is geometrically scaled in all the dimensions, it is called a *replica*. However, it will be shown (even though already demonstrated in Eqs. (8)–(11)) that a true plate model does not require a complete geometrical scaling; it is, however, important to keep their aspect ratios constant. As such, these plates are called *proportional sides* whereas distorted models are instead called *avatars*.

The first application of SAMSARA was described in [23], in which a flexural plate was coupled with an acoustic room. It was first assumed that the global mode shapes remained unaffected, i.e. $\hat{\varphi}_j = \varphi_j$. This condition implied that the mode shapes did not need to be posed in similitude and, furthermore, ensured that the excitation and measurement points, which had the same dimensionless coordinates in both prototype and model, had similar behavior. Consequently, as seen in ASMA with Eq. (45), the cross-mode participation factors also did not scale, i.e. $\hat{\psi}_{jk} = \psi_{jk}$. The auto-power mobilities scaled as:

$$\hat{\Gamma}_{jj} = \frac{1}{\lambda_\eta \lambda_\omega^2} \Gamma_{jj} \,. \tag{58}$$

The cross terms involved different dependencies from the scaling parameters and thereby could not be posed in direct similitude. It was then necessary to separate large terms from small ones by using the following:

$$\hat{\Gamma}_{jk,large} = \frac{1}{\lambda_\eta \lambda_\omega^2} \Gamma_{jk,large} \,, \tag{59}$$

$$\hat{\Gamma}_{jk,small} = \frac{\lambda_\eta}{\lambda_\omega^2} \Gamma_{jk,small} \,. \tag{60}$$

The introduction of scale factors of the parameters directly involved in the equations made it relatively simple to provide a physical explanation. On the one hand, large values of damping would help to increase overlap between modes and small cross-power mobilities were proportional to the damping. On the other hand, when the modes already overlapped, to increase such overlap required a negligible amount of damping. However, damping is fundamental when the resonant response of each mode and the cross-modal power mobility must be reduced. Large cross-modal power mobilities must be inversely proportional to damping. More generally, the role of the cross-modal terms was not well reproduced when $\lambda_\eta \neq 1$. Thereby, a complete similitude approach necessarily required this scale factor to be equal to unity.

The spectral density force function or excitation, and other EDA parameters, already agreed with ASMA in Eqs. (49)–(52) and scaled as:

$$\hat{S}_f = \frac{\lambda_F}{\lambda_M} S_f \,, \tag{61}$$

$$\hat{P}_{IN} = \frac{\lambda_F^2}{\lambda_\omega \lambda_M} P_{IN} \,, \tag{62}$$

$$\hat{T}^{(r)} = \frac{\lambda_F^2}{\lambda_M \lambda_\eta \lambda_\omega^2} T^{(r)} \,. \tag{63}$$

The test article was a panel coupled with a parallelepiped filled with air; thus, the natural frequencies had to be determined for both the structural and fluid operators and then somehow linked.

The condition for the plate in complete similitude was to maintain constant aspect ratio scale factors. Then, the natural frequencies of the plate and the fluid volume uncoupled and retained correct material properties; they scaled as:

$$\lambda_\omega^{plate} = \frac{\lambda_h}{\lambda_a^2} \,, \tag{64}$$

$$\lambda_{\omega}^{fluid} = \frac{1}{\lambda_a}, \tag{65}$$

in which all the sides of the parallelepiped cavity scaled as the sides of a plate.

The coupling can be expressed by a ratio, Λ, between the natural frequencies of the same mode order and of the structure and fluid. Not altering the relative distribution of modes implied retaining the scale factor of this ratio to $\lambda_{\Lambda} = 1$. As a result, the application of these similitude conditions and scaling laws allowed models to be in complete similitude and to provide a near perfect reconstruction of the prototype behavior.

In [24], the interest moved to analyses of avatars. The plates in the avatars were simply supported such that their natural frequencies were in complete similitude scale as defined by Eq. (65). Simultaneously, the scaling law of the velocity frequency response was given by:

$$\hat{V}(x_F, x_S;\ \lambda_\omega \omega) = \frac{\lambda_F}{\lambda_M \lambda_\omega} V(x_F, x_S;\ \omega)\ , \tag{66}$$

where the acquisition and excitation points were expressed in nondimensional coordinates that must not change.

The original response can be recovered through a procedure in two steps, including a first step that aligns the resonance peaks by means of the frequency remodulation, $\lambda_\omega \omega$, and then a second step that adjusts the amplitude level of the term $\lambda_F / \lambda_M \lambda_\omega$. While satisfying the similitude conditions has led to very good predictions, working with avatars enables the reconstruction of prototype responses with limited discrepancies if the distortions are small.

In [24], it was also proposed to the define metric representatives of the similitude degree. In this first-ever attempt, the scale factor of modal density was used:

$$\lambda_\theta = \frac{r_a r_b}{r_h}, \tag{67}$$

and is actually Eq. (57) rewritten under the assumption of unchanged material properties and an explicit area scale factor. The results were coherent when avatars were considered and the scale factor increased as the distortion increased. However, they were totally incoherent with complete similitudes. For example, a replica and a proportional sides model that are quite close in terms of similitude degree exhibited values of λ_θ equal to 0.33 and 8, respectively. In other words, modal density was not a good estimator of degree of similitude.

Interesting insights into plates in similitude were also given also in [25] in which the test article was a cantilever plate; similitude conditions and scaling laws were the same as that of a simply supported plate (Eq. (64)). Numerical analyses that were accomplished with a constant damping ratio of $\zeta = 0.005$ and experimental analyses revealed that the both analyses gave complete similitude.

However, new numerical tests were accomplished because the assumption of constant damping was not compliant with the experimental results. In particular, simulations with damping ratios equal to 0.001 and 0.0075 were completed, and the results showed that, while the experimental results of the prototype were closer to the numerical simulations with $\zeta = 0.001$, those provided by the replica were closer to the numerical results when $\zeta = 0.0075$. This result may be linked to added damping due to the boundary conditions which sum to the classical internal dissipation mechanisms of the plate.

Furthermore, experimental tests results were closer to the numerical ones in regions with low modal density. Apparently, the size of the plates affected the response with high modal densities. This difference led to other tests with two different proportional sides models, one of which had dimensions twice those of the prototype (proportionally large) and the other with dimensions one-half of the prototype (proportionally small). The comparisons between the responses showed that similitudes work well when the frequency range contained enough poles among which the energy could be distributed; this fact was highlighted by the predictions for the proportionally large model were quite close to the reference one, while those provided by the proportionally small model were not. Tested with the same frequency range of the prototype, the smaller model exhibited fewer poles and larger discrepancies. Finally, the Hausdorff distance was tested as a similitude metric with encouraging results.

The scaling of radiated acoustic power of a simply supported panel was studied attempted by Robin et al. [26]. Here, a main difficulty experienced was due to the scaling of the radiation resistance matrix, *R*, which the authors assumed to remain unchanged between the prototype and model. This constancy is ensured when:

$$\lambda_\omega \lambda_d = 1\ , \tag{68}$$

where d is the distance between two elementary radiators. A new frequency scale factor was also proposed that was not derived from equations. The new law was:

$$\lambda_\omega = \frac{2}{\lambda_a + \lambda_b}, \tag{69}$$

where the 2 at the numerator leads to the scaling laws of a replica when $\lambda_a = \lambda_b$.

Experiments with a proportional sides model using Eqs. (68)–(69) showed resonance peaks of the radiated power that were well reconstructed over a range of 100–1000 Hz but either underestimated or overestimated depending on which plate was used as reference at frequencies above 1000 Hz.

SAMSARA was applied by Franco et al. [27] in analyzing the response of a plate under TBL excitation described by Corcos model. The whole engineering problem was transformed into a new domain by scaling the excitations, structural and transmitted vibrations, and the structural noise.

Firstly, it was assumed that the damping did not change, which is an acceptable assumption if the material and the boundary conditions do not change between prototype and model. Then, the spatial dependence of the analytical mode shapes was not modified (i.e. $\lambda_\varphi = 1$), and the reduced frequency was the same in both *x* and *y* directions:

$$\frac{\lambda_x \lambda_\omega}{\lambda_U} = \frac{\lambda_y \lambda_\omega}{\lambda_U} = 1 \,. \tag{70}$$

This condition ensured that the ratios $\omega\xi_x/U_c$ and $\omega\xi_y/U_c$ remained constant between the reference and scaled solutions. Moreover, according to Eq. (70), the correlation area scaled as $\lambda_x\lambda_y$ which was the same scale factor of the structure. The basic constraint for complete similitude for models with proportional sides models was expressed in terms of joint agreement between the two modes:

$$\hat{A}_{Q_jQ_k} = \frac{A_{Q_jQ_k}}{\lambda_x \lambda_y} , \tag{71}$$

which were derived under the assumption of similar auto-spectral densities between prototype and model. Eq. (71) actually linked the geometry with the flow speed and, for a simply supported plate, the cross spectral densities of displacements scaled according to:

$$\frac{\hat{X}_w}{\hat{S}_p} = \frac{\lambda_x \lambda_y}{\lambda_h^4} \frac{X_w}{S_p} , \tag{72}$$

which is valid under the assumption of constant Corcos coefficients. Furthermore, Eq. (71) is valid for all the TBL models using the same coherence functions. In fact, Eq. (72) descends directly from Eq. (71), in which the joint acceptances are ruled by the choice of coherence function of the TBL model. However, when the material was changed, Eq. (72) would become:

$$\frac{\hat{X}_w}{\hat{S}_p} = \frac{1}{\lambda_x \lambda_y \lambda_h^2 \lambda_\omega^2 \lambda_\rho^2} \frac{X_w}{S_p} . \tag{73}$$

The predictions were compared in terms of the ratio of auto-spectral densities of acceleration with respect to the auto-spectral density of wall pressure distributions due to TBL versus frequency. Using Eq. (72) for aluminum plates and Eq. (73) for steel alloy and acrylic cast plates leads to perfect overlaps of the curves, demonstrating that the approach is suitable for materials involving different properties.

It was further demonstrated that, because the TBL load acts as an uncorrelated pressure field for increasing frequency, the solution was less affected by acceptance integrals. Hence, the condition expressed by Eq. (70) is no more a mandatory constraint at high frequencies. The auto-spectral densities measured experimentally confirmed the analytical results. Although the cross-spectral measurements were more noisey, it was is independent of the similitude method but rather due to experimental uncertainties and measurement processing.

Numerical analyses were also performed for more complex configurations, such as composite plates. In this regard, Eq. (70) should be rewritten as:

$$\frac{\lambda_x \lambda_{\omega_{mn}}}{\lambda_U} = \frac{\lambda_y \lambda_{\omega_{mn}}}{\lambda_U} = 1 \,. \tag{74}$$

Working with laminated plates underlines that the eigensolution sequences were not affected if thickness and, most importantly, the stacking sequence were unchanged. In fact, proportional sides models provided perfect matches even when the thickness was changed. Conversely, modifying the lay-out generates distortions.

Sensitivity analysis

Recent research has included Sensitivity Analysis (SA) in similitude applications and opens the way to analyses of the effects of multiple parameters on the similitude process, i.e., the amount of distortion associated with varying particular parameters. For example, Luo et al. [28] derived a set of four principles which supported the application of STAGE in deriving the exponent of scale factors and their relationship with frequency scale factors. Their four laws were aimed at deriving approximate, distorted scaling laws that returned errors lower than 5%.

Adams et al. [29] applied Local Sensitivity Analysis (LSA) in which the response of the model was given by the product between the prototype response and a set of scale factors, with each factor weighted by an exponent calculated by applying Buckingham's Π Theorem. In this manner, SA enabled the determination of sensitivity-based conditions that were derivable without having any prior knowledge of the scaling behavior of the system and with small effort instead of deriving similitude-based conditions; this method can also be implemented easily in an algorithm. However, the method is not lacking of drawbacks. First, it was sensitivity-based and did not rely on physical insights. Moreover, complex systems may incur prohibitive computational cost.

It is interesting to notice that, in [29], the exponents obtained by means of SA led to a frequency scaling law for a simply supported plate with the form:

$$\lambda_\omega = \lambda_a^{-2} \lambda_h^1 \,. \tag{75}$$

Eq. (75) is the same frequency scaling law obtained with STAGE and SAMSARA.

SA was applied to simply supported Kirchhoff plates and to a Mindlin-Reissner plates. The latter case may be the most interesting, and highlighted the pitfalls of SA originating from its mathematical foundations. In particular, the sensitivity analysis tended to overestimate both natural frequencies and Mean

Squared Transfer Admittance (MSTA) because the method did not consider the influence of thickness. This example is an important reminder of problems that may arise when an approach lacks any link to actual physical behaviors of a plate.

Some remarks

This brief review shows that even though DA and STAGE have been the most used approaches [4], their application has not been broadly used in vibracoustic fields. The main reason lies in the complexity of the physical phenomena governing the field that involve many parameters for which the derivation of similitude conditions and scaling laws is not a straightforward and requires an experienced analyzer. For example, the work by Ciappi et al. [5] clearly demonstrated that the investigation of a typical noise problem in transport engineering can lead to defining 11 dimensionless terms; although this number can be reduced, it is only accomplished by means of a thorough analysis of physical meanings and effects.

The applications in which STAGE has been applied [8–14] have shown that the derivation of similitude conditions and scaling laws can become more problematic by simply adding a new degree of difficulty, such as changing the type of material, eg. for isotropic or composite plates. This difficulty is a natural consequence of the increase of the number of parameters describing the material, and only by assuming simplifying hypotheses it is possible to reduce the number of similitude conditions. A fresh approach may be a consequence of the modular approach proposed by Coutinho et al. [14].

Thus, it is not surprising if new methods have been introduced in the last years that attempt to overcome the excessive freedom of DA or the stiff procedure of STAGE; they strive to look at scaling problems from new perspectives, such as in energetic problems. One such new method is ASMA, based on an energy approach and by introducing two scale factors modulating the geometrical scaling and the distribution of energy in frequency, has been shown to be able to reproduce the global response of structural systems. At the same time, ASMA is an interesting solution to the infamous problem of the computational costs when applying FEM to high frequency dynamics problems.

SAMSARA is justified, as is ASMA, with EDA but significative differences exist between these methods. First of all, ASMA requires only two scale factors that can assume any value between 0 and 1; however, in general, their mutual relationship must be determined. On the other hand, SAMSARA uses as many scale factors as the number of parameters involved. Their evaluation is straightforward because it is provided by the direct ratio between the model and prototype parameter values. This relative simplicity allows similitude conditions and scaling laws to be derived in a classical fashion, as STAGE does. The work by Franco et al. [27] has confirmed the flexibility of the method that is able to deal with complex problems such as the response of a plate under TBL. Last, but not least, SAMSARA recovers both the global and local response with a very good accuracy.

SA, instead, opens the way to scaling laws derived out of any physical framework but with a mathematical approach. Even though this may be a drawback, it is undeniable that the possibility of automatizing the procedure with an algorithm is quite an interesting perspective.

Despite the quantity of methods and working principles, many of them comply with the similitude conditions for a true model. A most important condition is the aspect ratio which must scale equal to one. This restriction is governed by the equations [8–12], the modular approach [14] and SAMSARA [23–25] applied to the equation of the natural frequencies. It is through the SAMSARA method, indeed, that such results become easier to understand and why the aspect ratio has such a relevant effect—basically, length and width rule the modes succession. When aspect ratios are distorted, it is true that the frequency scale factor is no longer constant in the entire frequency range since each natural frequency would scale according to different factors. However, the scaling law, as shown by Eq. (64), provides a fixed value of frequency scale factor, and this directly affects Eq. (66) in which the velocity response is clearly linked to this scale factor.

The similitude conditions become more restrictive in [23], where not only the length and width of the plate must scale similarly but also the sides of the acoustic cavity. Moreover, the geometrical properties of both plate and cavity must scale in the same way because of fluid-structure coupling. Physically, the succession of modes is altered if the amount of distortion is significant. When a plate is excessively distorted, the remodulation procedure is incapable of replicating the mode swap, and causes inconsistency in both frequency and amplitude.

However, parameters such as thickness and isotropic material properties which like length and width affect the wave speed as well as the wavenumber, do not appear in the similitude conditions. Possibly, this happens because the thickness, Young's modulus and mass density are separable from the number of half waves (or nodal lines), while this separability is not possible for both length and width. Furthermore, the situation changes completely when new structural configurations are introduced like in stiffened plates where thickness is no more a free parameter (as Eq. (36) shows). In this regard, STAGE and SAMSARA also agree on the conclusion that ply-level scaling returns complete similitude [10, 27].

A common point shared by ASMA and SAMSARA concerns spatial scaling. Basically, it is necessary to ensure that homologous points—at the same nondimensional coordinates—are placed in similitude if the same excitations and responses are to be compared. Notably, this idea is at the base of the scaling of the correlation area in the Corcos model, and leads to the condition $\hat{\varphi}_j = \varphi_j$ in SAMSARA that, applied to EDA, returns $\hat{\psi}_{jk} = \psi_{jk}$. The same conclusion is drawn with ASMA. Moreover, in both methods, the auto-modal and large cross-modal power mobilities scale with the same law, which is different from that of the small cross-modal power mobilities.

However, significant differences also exist. For example, SAMSARA scales the input power according to Eq. (62) while it is not scaled at all in ASMA (Eq. (49)). While damping scaling is necessary in ASMA to balance the effect of geometrical scaling, in SAMSARA it must not change if complete similitudes are desired [23]. Nonetheless, such a condition is not easy to setup in experimental tests especially due to boundary conditions, as seen in [25].

Finally, similitudes provide important insights in experimental procedures. First, as noted in [27], remodulated results, which should coalesce into a single curve along with the prototype results, may provide information at hand about the quality of the measured data as well as discrepancies in some frequency ranges. Furthermore, accurate predictions of prototype behaviors may heavily affect actual experimental setups. As already explained in the work from Meruane et al. [25], scaling down a structure moves the natural frequencies to higher values. In this case, a good prediction is only possible if the model is tested in a frequency range wider than that of the prototype to enable consideration of a correct number of poles. In [27] this fact was underlined in scaled-down models, i.e. $\lambda_\omega > 1$, broadly affect the experimentation in terms of choice of sensors, sampling frequency, frequency windowing and signal dynamics.

Further research and conclusions

This brief review highlights the importance of similitude theory in vibroacoustics. Not only is it useful in reducing the complexity and costs of necessary laboratory experimentation, it also presents a synthesized form of equations governing the phenomena under investigation and highlighting fundamental parameters and their interactions.

Some similitude methods have been proven to be suitable for complex applications but all cannot deal with distorted models. Partial similitudes are not purely analytical because manufacturing limits and errors may prevent the production of models compliant with the similitude conditions. Therefore, it remains necessary to develop more fundamental insight into how to deal with these models and how to reconstruct prototype behaviors.

In this regard, two approaches seem to very interesting and informative. First, try to reconstruct a prototype response mode-by-mode. In this way it may been possible to understand and overcome assessment problems associated with distortions in natural frequencies, dynamic responses and radiated power. Furthermore, the explosion of machine learning techniques may prove helpful in finding implicit relationships between prototypes and distorted models.

Finally, it may be of great relevance to further the development of metrics for similitude degrees, like that attempted in [24–25]. In fact, defining these metrics may open the way to other, new approaches to similitudes.

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