INVESTIGATING THE IMPACT OF INTERACTIVE APPLETS ON STUDENTS’ UNDERSTANDING OF PARAMETER CHANGES TO PARENT FUNCTIONS: AN EXPLANATORY MIXED METHODS STUDY

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INVESTIGATING THE IMPACT OF INTERACTIVE APPLETS ON STUDENTS’ UNDERSTANDING OF PARAMETER CHANGES TO PARENT FUNCTIONS: AN EXPLANATORY MIXED METHODS STUDY

DISSERTATION

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Education at the University of Kentucky

By

Robin Rudd McClaran

Lexington, KY

Co-Directors: Dr. Margaret J. Mohr-Schroeder, Associate Professor of Mathematics Education and Dr. Carl W. Lee, Professor of Mathematics

Lexington, KY

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ABSTRACT OF DISSERTATION

INVESTIGATING THE IMPACT OF INTERACTIVE APPLETS ON STUDENTS’ UNDERSTANDING OF PARAMETER CHANGES TO PARENT FUNCTIONS: AN EXPLANATORY MIXED METHODS STUDY

The technology principle in the Principles and Standards for School Mathematics (NCTM, 2000) states that technology plays an important role in how teachers teach mathematics and in how students learn mathematics. The purpose of this sequential explanatory mixed methods study was to examine the impact of interactive applets on students’ understanding of parameter changes to parent functions. Students in the treatment classes were found to have statistically significantly higher posttest scores than students in the control classes. Although the data analysis showed a statistically significant difference between classes on procedural understanding, no statistically significant difference was found with regard to conceptual understanding. Student and teacher interviews provided insight on how and why the use of applets helped or hindered students’ understanding of parameter changes to parent functions.

KEYWORDS: Technology, Mathematics Education, Procedural Understanding, Conceptual Understanding, Algebra

Robin Rudd McClaran

April 12, 2013
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April 12, 2013
I would like to dedicate this dissertation to the most important people in my life – my husband, my mother and daddy, and my brother. Your love, support, and encouragement helped make my dream a reality. I love you.
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Chapter I: Introduction

The technology principle in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) states, “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ understanding” (p. 24). The goal of every mathematics teacher is to enhance their students’ understanding of mathematics. The use of single purpose applications such as interactive applets allows students to explore mathematical properties and relationships in ways that can potentially enhance students’ understanding. Single purpose applications have dynamically connected representations and mathematical objects that students can manipulate. Single purpose applications assist students with visualization of concepts. Single purpose applications provide a setting in which students can discover mathematical properties and relationships on their own. In working with single purpose applications, students are actively engaged in learning and in constructing their own knowledge.

Other organizations have taken a stance on the role technology should play in teaching and learning mathematics. The International Society for Technology in Education’s (ISTE) National Education Technology Standards for Teachers (NETS•T) has among its five standards one on teachers using their knowledge of content, pedagogy, and technology to “facilitate experiences that advance student learning”, and one on designing and developing learning experiences that utilize technology “to promote student learning” (ISTE, 2008, p. 1).

The Common Core State Standards for Mathematics (CCSSM) has among its eight standards for mathematical practice one on using “appropriate tools strategically” (CCSSO, 2010, p. 7). The CCSSM calls for students to use technology tools such as
single purpose applications “to explore and deepen their understanding of concepts” (CCSSO, 2010, p.7). Although the state of Texas has not adopted the CCSSM, the Texas Education Agency (TEA) has technology application standards for teachers that include the ability to “plan, organize, deliver, and evaluate instruction for all students that incorporates the effective use of current technology” (TEA, 2003, p. 1).

Calls have been made at the national and state level for technology to be used in the teaching and learning of mathematics as evidenced by the NCTM technology principle, the ISTE technology standards, the CCSSM standards for mathematical practice, and the TEA technology applications standards for teachers. The expectations have been set for teachers to use technology in the teaching and learning of mathematics. The critical question is, How can technology be used *effectively* in the teaching and learning of mathematics? This study attempts to address one aspect of this question.

The content focus for this study is transformations of parent functions. In the Texas Essential Knowledge and Skills (TEKS) for Algebra I, students are expected to “investigate, describe, and predict” the effects of changes in the parameters $a$ and $c$ on the graph of $y = ax^2 + c$ (TEA, 2005, p. 42). In the TEKS for Algebra II, students are expected to “investigate, describe, and predict” the effects of changes in parameters $a$, $h$, and $k$ on the graph of $y = a(x – h)^2 + k$ (TEA, 2005, p. 45). Precalculus TEKS call for students to apply transformations to parent functions including $a(f(x))$, $f(x) + d$, $f(x - c)$, and $f(b(x))$ to parent functions such as $y = x^n$ (TEA, 2005, p. 52). The CCSSM call for students to “experiment with cases and illustrate an explanation of the effects on the graph using technology” of transformations to parent function including $f(x) + k$, $k(f(x))$, $f(k(x))$, and $f(x + k)$ (CCSSO, 2010, p. 70). The CCSSM stress both procedural and
conceptual understanding with the intent that students learn mathematics rather than having students continue the cycle of memorizing enough to pass tests without truly understanding the content (CCSSO, 2010).

**Statement of the Problem**

The NCTM technology principle also states, “When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving” (NCTM, 2000, p. 24). For far too long, students have perceived that the focus of learning mathematics was memorizing formulas, rules, and procedures and getting correct numerical answers to problems. With technology, students can explore mathematical relationships, reason mathematically about what they observe, and make sense of mathematical situations. In doing so, students have the opportunity to gain a true perspective of what learning mathematics is all about. Given the emphasis placed on using technology in the teaching and learning of mathematics as outlined in the NCTM’s *Principles and Standards of School Mathematics* (2000) and the CCSSM, it is imperative that the mathematics education community gain more insight into how technology can be used to positively impact student understanding.

Much of the current research on using technology to teach mathematics has focused on using technology for visual representation purposes (Rakes & Ronau, 2010). Although this use of technology is beneficial to students, there is more that technology could be used for in the teaching and learning of mathematics. Other studies have focused on the long-term use of one particular technology intervention such as software or an online curriculum with animations (Gorman, Brown, & Slate, 2008; Harskamp, Suhre, & Van Streun, 2000). Although these studies attempted to show that the long-term use of the intervention had some effect on student achievement or understanding, without
examining why the impact was positive, negative, or non-existent the results cannot influence future development and implementation of technology for teaching mathematics.

Mathematics educators need to find ways to leverage the potential that technology has to offer by investigating the impact of technology in focused, specific ways so that educators gain a deeper understanding of what specific aspects of technology use have a positive effect as well as what aspects have a negative effect. The understanding gained from focused investigations, such as this research study, can then be parlayed into changes in how technology is used on a much broader scale. Knowledge of effective uses will enable mathematics educators to develop and implement mathematics curriculum and instructional practices that continue to capitalize on the technology.

**Purpose of the Study**

The purpose of the study was to examine the impact – positive, negative, or nonexistent – of the use of single purpose applications such as interactive applets has on students’ conceptual understanding of parameter changes to parent functions and to explain why. The NCTM technology principle states, “In the mathematics classroom envisioned in *Principles and Standards for School Mathematics*, every student has access to technology to facilitate his or her mathematics learning under the guidance of a skillful teacher” (NCTM, 2000, p. 25). The use of single purpose applications such as interactive applets along with activity sheets to guide the exploration has the potential to make this vision a reality.
Research Questions
This mixed methods study addressed the following research questions:

1. To what extent does the use of single purpose applications for teaching parameter changes to parent functions impact a student’s transition from procedural understanding to conceptual understanding?

2. From the teacher’s perspective, to what extent does the use of single purpose applications impact a student’s transition from procedural understanding to conceptual understanding of parameter changes to parent functions? To what does the teacher attribute the single purpose applications impacting students’ transition from procedural to conceptual understanding of parameter changes to parent functions?

3. From the student’s perspective, what additional insights on parameter changes to parent functions are gained by using a single purpose application? To what does the student attribute the single purpose applications providing additional insights on parameter changes to parent functions?

Significance of the Study
As stated previously, much of the research on using technology to teach mathematics has focused on using technology for visual representation purposes and on the long-term use of specific technology interventions such as software and online curricula. This study did not focus on such broad use but rather focused specifically on how single purpose applications were used in teaching parameter changes to parent functions to Algebra 2 students and what impact the use had on students’ conceptual understanding. This study went beyond the limited use of technology for visual representation purposes and extended to students interacting with the technology in an
environment that promoted investigation, conjecturing, reasoning, and sense making. Although this study had a narrow focus, the results of this study provided valuable insight into effective and ineffective ways of using technology in the teaching and learning of mathematics. This knowledge of effective and ineffective ways to use technology to teach mathematics could potentially impact teachers’ future use of technology on a broader scale. As stated previously, the vision is that students have access to technology in their mathematics classes to promote learning through exploration facilitated by their teachers (NCTM, 2000).

**Delimitations**

Due to the nature of the study, the researcher believed the use of existing classes would best serve the purpose of the study – to determine the impact that the use of single purpose applications such as interactive applets had on students’ understanding of parameter changes to parent functions. Although the use of random samples in quantitative research is critical, random assignment of students to treatment and control classes would have been impossible for this study. Only one Algebra 2 teacher at each of two different high schools agreed to participate in the study. Each teacher had to meet with Algebra 2 students that were assigned each period throughout the day. Within one class period, teachers could not accommodate students in both the treatment and control setting. Although students were not randomly assigned to treatment and control classes, the classes taught by participating teachers were randomly assigned as treatment and control classes. By using existing classes rather than randomly assigning students to treatment and control classes, the results of this study are not generalizable to the population of all high school mathematics students.
Limitations

Since it was not possible to test all high school Algebra 2 students in northeast Texas on all concepts related to functions, this study was limited to testing 97 students in two northeast Texas public high schools on parameter changes to parent functions. Although five high schools initially agreed to participate in the study, three schools declined when it was time to gather data for the first phase of the study. The reasons given by these schools for declining to participate included a change in campus leadership and the need to focus on meeting state assessment standards. This small sample size, along with the limited number of high schools choosing to participate, limits the generalizability of the findings.

Another limitation to the study involved the timing of events – pretests, use of applet applications for treatment classes and instruction for control classes, posttests, and interviews. The two teachers participating in the study did not have computers in the classroom for students to use and had to schedule time in advance to go to the computer lab or get a laptop cart for their classes. This led to minor inconsistencies in time between pretests, guided explorations or instruction, and posttests between the two high schools. Specifically, there was a one-day difference between pretest and guided explorations or instruction for the two high schools, which led to a one-day difference in time from pretest to posttest for the two schools.

Due to the constraints of the researcher’s current teaching job along with the need to interview students on two different campuses, the time between posttests and interviews was not be the same for all student participants. The researcher conducted all interviews within a two-week period beginning one week after the completion of the posttest. Thus, some students were interviewed within a week of completing the posttest
while others were interviewed within three weeks of completing the posttest. These differences in times could not be controlled by the researcher and affected the qualitative data as evidenced by a few students needing to look over the applets and activities to be reminded of what they completed earlier. One other limitation to the study involved test-retest issues due not only to the time interval between pretest and posttest but also to having the same questions on the pretest and posttest.

**Assumptions**

1. All student participants in the experimental classes actively participated in each guided exploration using the designated applet.
2. All student participants answered questions on the pretest and posttest to the best of their ability.
3. All student participants and participating teachers answered interview questions honestly.

**Definition of Terms**

The following definitions are provided for terms associated with this mixed methods study.

*Applet*: An application independent of the platform that is designed to perform a specific task and run within another application such as a web browser (Boese, 2010). An applet is interactive and provides a means to capture user input that results in changes to the graphical, numerical, and algebraic content displayed on the applet. For this study, the applets were web-based.

*Conceptual understanding*: Understanding that is based on having an “integrated and functional grasp of mathematical ideas” (National Research Council, 2001, p. 141).
Understanding that is based on knowing not only “how to” but “why” (Bosse & Bahr, 2008).

Procedural understanding: Understanding that is based on knowing rules for solving problems and how to perform step-by-step processes to get answers (Bosse & Bahr, 2008; Hiebert & Lefevre, 1986).

Single purpose application: A technology application, such as an interactive applet, that was developed with a particular instructional focus and provides students the capability to manipulate the mathematical objects built into the applet without being able to edit the content.

Organization of the Study

The study is organized into five chapters. Chapter 1 provides an introduction to the study including the statement of the problem, the purpose, and the significance of the study along with the research questions. Chapter 2 provides a review of literature relevant to the study. Chapter 3 provides a description of the research design and the methodology for conducting the research. Chapter 4 provides an explanation of the data analyses and the results. Chapter 5 provides a discussion of conclusions drawn by the researcher and recommendation for future research along with implications for future technology implementation in mathematics classrooms.
Chapter II: Review of Literature

Schools have been inundated with technology. Many believe technology is the answer to any educational problem that needs to be addressed. So often teachers are given technology with no research-based plan for how to effectively use the technology to impact their instruction and their students’ learning. Mathematics educators need to continually examine how the technology affects the curriculum, how it changes the learning goals, and how it impacts instruction and the learning experiences for students. As such, it is important to find research-based ways to effectively use technology to teach mathematics. Knowledge of effective uses of technology in the mathematics classroom will enable mathematics educators to develop and implement the mathematics curriculum and instructional practices that continue to capitalize on the technology.

In a systematic review of the literature on integrating instructional technology in teaching mathematics, Ronau, Rakes, Niess, Wagener, Pugalee, Browning, Driskell, and Mathews (2010) found a low percentage of research studies, and most of these studies were qualitative studies. Some of the research studies found were quantitative studies, but very few were mixed methods studies (Ronau et al., 2010). Ronau et al. (2010) also identified potential gaps that existed in the literature, and this included research on the use of online technology such as interactive applets.

Technology Used to Teach High School Mathematics

Rakes and Ronau (2010) defined educational technology as “digital tools used for teaching”, and these tools include calculators, computer software, and the internet. According to Rakes and Ronau (2010), graphing calculators are the main type of calculators used to teach high school mathematics. The computer software used to teach
high school mathematics included dynamic geometry and algebraic software as well as graphing software, and internet-based tools included virtual manipulatives and online applets (Rakes & Ronau, 2010).

**Calculators.** The use of graphing calculators (CAS and non-CAS) in high school mathematics classes is widespread (Forster, 2004). CAS graphing calculators utilize a computer algebra system (CAS) to produce symbolic results (Texas Instruments, 2013). CAS and non-CAS graphing calculators are currently allowed on national assessments including Preliminary Scholastic Aptitude Test (PSAT), Scholastic Aptitude Test (SAT), Advanced Placement (Calculus, Statistics, Chemistry, Physics), and Praxis, but American College Testing (ACT) only allows the use of non-CAS graphing calculators (Texas Instruments, 2012). CAS and non-CAS graphing calculators are currently permitted to be used on a majority of states’ assessments (Texas Instruments, 2012). It is not evident that graphing calculators (CAS and non-CAS) are used regularly in mathematics classes in states that allow graphing calculators to be used on their assessments.

**Attitudes and Beliefs.** The use of graphing calculators in high school mathematics classes has been shown not only to have a positive effect on students’ attitudes toward mathematics but also at times to have little or no effect on students’ attitudes toward mathematics (Ellington, 2003; Hembree & Dessart, 1992). In a meta-analysis of the effects of calculators, Ellington (2003) found that students who used a calculator during their mathematics class had a better attitude toward mathematics than students who did not use a calculator. Ellington (2003) added that students’ attitudes toward mathematics showed the most improvement after students had used calculators for at least two months.
**Student Achievement and Understanding.** The use of graphing calculators in high school mathematics classes has been shown not only to have a positive effect on students’ achievement and understanding but has also been shown to have little or no effect (Burrill, Allison, Breaux, Kastberg, Leatham, & Sanchez, 2002; Ellington, 2003). In a study comparing algebra classes using a reform curriculum that incorporated graphing calculators to traditional algebra classes that did not incorporate graphing calculators, Thompson and Senk (2001) found that students in the reform algebra classes performed significantly better than students in the traditional classes. Although the results of this study are positive, Thompson and Senk (2001) noted that it was not possible to attribute the difference solely to the use of graphing calculators since the classes also utilized a different curriculum. In another study investigating the impact of graphing calculators on student achievement in algebra, Currie (2006) found that students who used a graphing calculator in their algebra class significantly outperformed students that did not use a graphing calculator. However, Hatem (2010) found that the use of graphing calculators in college algebra and pre-calculus classes did not improve student achievement.

Harskamp et al. (2000) conducted a study examining the impact of graphing calculators on students’ understanding in a precalculus-level course and found that low-performing students benefitted the most from using a graphing calculator in the course. This benefit was greatest in classes where graphing calculators were used on a regular basis (Harskamp et al., 2000). Harskamp et al. (2000) also noted that students who used graphing calculators in their mathematics class were more apt to attempt to solve a
problem or apply what they knew to a new situation than students who did not use graphing calculators.

The use of graphing calculators has been shown to improve students’ ability to symbolically represent function (Ruthven, 1990; Schwarz & Hershkowitz, 1999). In a study involving students in precalculus-level courses, Ruthven (1990) found that students who used graphing calculators scored higher on test items that required students to write an equation given the graph than students who did not use graphing calculators. No significant difference was found on items requiring students to answer questions about a given graph (Ruthven, 1990).

Hollar and Norwood (1999) examined the impact of graphing calculators in an intermediate college algebra course and found that students who used graphing calculators not only performed better than students who did not use graphing calculators but were also more apt to examine functions from different perspectives. No difference was found between students who used graphing calculators and those who did not use graphing calculators on paper and pencil algebraic manipulations (Hollar & Norwood, 1999). In a similar study investigating the potential benefits of graphing calculators, Ford (2008) concluded that students who used graphing calculators had higher test scores and were more adept at using graphical and numerical representations to solve problems than students who did not use graphing calculators.

In several meta-analyses on the effects of calculator use, researchers reported that the use of calculators had a positive effect on students’ understanding and did not hinder students’ acquisition of paper-and-pencil skills (Hembree & Dessart, 1992; Smith, 1997). In a more recent meta-analysis on the effects of graphing calculators, Ellington (2003)
concluded that when calculators were used for instruction and testing, K-12 students improved “in operational skill as well as paper-and-pencil skills and the skills necessary for understanding mathematical concepts” (p. 456) although when calculators were used for instruction only students improved in operational skill.

**Computer Software.** Dynamic mathematical software used in teaching high school mathematics includes Geometer’s Sketchpad, GeoGebra, and TI–Interactive. With dynamic mathematical software students can explore mathematics by constructing and manipulating objects, exploring patterns and relationships, and making and testing conjectures (Hollebrands, 2007). Thus, dynamic mathematical software provides an environment where students can discover mathematical properties and relationship on their own. The dynamic software enables students to generate many variations of a concrete model, which allows students to move to the abstract representation and generalize their findings (Wiest, 2001).

**Attitudes and Beliefs.** Several studies focus on teachers’ and students’ attitudes concerning the use of computer technology for teaching and learning mathematics. Students’ perceptions about teachers using technology to teach mathematics were generally positive (Drickey, 2006; Hannafin, Burress, & Little, 2001; Mohd, Suncheleev, Shitan, & Mustafa, 2008). Hannafin et al., (2001) concluded that students believed the use of dynamic software made learning mathematics more enjoyable and more understandable. Although students believed that the use of technology enhanced their understanding of concepts and engaged them more with learning, little to no effect on their performance was found (Drickey, 2006; Hannafin et al., 2001; Mohd et al., 2008).
Gadowsky conducted a study on the meanings students construct when exploring function transformations with graphing software. Gadowsky (2001) reported the majority of students felt the investigative process of exploring function transformations using graphing software was a positive experience for them and that learning on their own through investigation was more beneficial. Although participants were initially apprehensive about participating in a technology-based exploration, participants successfully navigated through the exploration and commented positively about their exploratory learning experiences (Gadowsky, 2001).

Teachers are highly supportive of using technology, especially mathematical software and web-based curricula, to teach mathematics (Al-A’ali, 2008; Hannafin et al., 2001). Although most teachers see the benefit of computer technology in mathematics classrooms, some teachers believe that the computer is best used for drill and practice rather than as a tool for exploration and questioned the long-term understanding gained by students exploring mathematical properties using dynamic software (Hannafin et al., 2001).

**Student Achievement and Understanding.** Several studies focused on the impact of computer software on student achievement. In a study on the effects of using computer graphics as an instructional tool in a calculus class, Tiwari (2007) found that students who were taught concepts with computer graphics scored higher on both the conceptual and computation parts of the posttest than students who were not taught with computer graphics. This study, though, had many limitations including the researcher being the teacher of record for both the control and treatment groups as well as the uncertainty as to what extent technology was utilized by the treatment group (Tiwari, 2007). In a study
examining the impact that the implementation of mathematical software had on statewide benchmark scores, King (2004) found that scores improved for low performing elementary school students in Tennessee that used this software. Using computers to help students solve problems requiring critical thinking and analysis rather than using computers for drill and practice produced greater benefits for students (Wenglinsky, 2005). As Jones (2002) pointed out, though, when using computer software in the mathematics classroom, it is difficult to determine whether the measured learning gains were a direct result of using technology.

Internet. In their systematic review of the literature on technology integration in mathematics, Ronau et al. (2010) included virtual manipulatives, web-based courses and curriculum, and online applets as internet-based tools for teaching mathematics. According to Moyer, Bolyard, and Spikell (2002), a virtual manipulative is “an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (p. 373). Virtual manipulatives can also be thought of as “computer based renditions” of concrete manipulatives such as geoboards, algebra tiles, and pattern blocks (Dorward, 2002, p. 329). Although virtual manipulatives are designed to emulate concrete manipulatives, differences exist between the two. Virtual manipulatives are more readily available than concrete manipulatives and the dynamically connected representations possible with virtual manipulatives help students make connections between representations (Moyer, Bolyard, & Spikell, 2008). Also, virtual manipulatives give students the opportunity to explore and visualize concepts difficult to simulate with concrete models in an interactive environment that allows immediate feedback (Crawford & Brown, 2003). Virtual manipulatives can be
found at several websites including the National Library of Virtual Manipulatives website (nlvm.usu.edu), and NCTM’s Illuminations website (illuminations.nctm.org).

Web-based curricula such as Agilemind, offer teachers and students constant access to secondary mathematics content from Algebra 1 to Calculus. With web-based curricula such as Agilemind, teachers and students have access to teaching tools including topic introductions, in-depth explorations, formative and summative assessments along with a way for teachers and students to track progress (Agilemind, 2012). Teachers can use web-based curricula to teach content with animations, simulations, in-depth explorations, and discovery learning activities (Agilemind, 2012). Students can use web-based services to preview and review content as well as to assess their level of understanding (Agilemind, 2012).

Online applets can be defined as interactive, web-based dynamic representations that provide opportunities for exploring mathematics in much the same way that Moyer et al. (2002) defined virtual manipulatives. The difference is online applets are not modeled after concrete manipulatives. Online applets are typically designed with multiple representations and one instructional focus. In the case of this study, the online applets were designed to teach parameter changes to parent functions. Online applets can be found on the National Library of Virtual Manipulatives website (nlvm.usu.edu), NCTM’s Illuminations website (illuminations.nctm.org), Shodor’s Interactivate website (www.shodor.org/interactivate), GeoGebra’s website (www.geogebra.com), and ExploreLearning’s website (www.explorehlearning.com).

**Attitudes and Beliefs.** In a study investigating students’ attitudes and beliefs regarding the use of technology in the mathematic classroom, Kortering, DeBettencourt,
and Braziel (2005) concluded students wanted and needed “alternative formats for learning” and internet-based tools such as interactive applets provided an alternative to traditional ways of learning mathematics. Students believed the use of internet-based applets provided visual representations that helped them understand the mathematics (Heath, 2002; Pyzdrowski & Pyzdrowski, 2009). Also, students thought that not only did the use of virtual manipulatives assist them in learning, but that they were easy to use and made their learning experience enjoyable (Reimer & Moyer, 2005).

Many teachers maintained that virtual manipulatives or interactive applets should be used “during the core part of the instructional sequence in the lesson for student investigation and skill solidification” rather than merely being an add-on to the lesson (Moyer-Packenham, Salkind, & Bolyard, 2008, p. 209). While many educators believed technology use increases students’ ability to learn mathematical concepts, they also thought that in some cases using technology can have a negative effect on learning (Heath, 2002; Sedig, Klawe, & Westrom, 2001). If students must contend with learning how to use a mathematical software package while trying to learn mathematical content, it is possible that learning to use the technology could be the focus for students rather than learning the mathematics. Heath (2002) suggested that the use of applets would minimize the possible negative effect since applets “equip students with tools to immediately focus on concepts” (p. 1). With the use of single purpose applications, such as interactive applets, the focus is on learning mathematics and not on learning how to use the technology.

Teachers believed there were several obstacles to technology utilization that had to be overcome including: (a) availability of computers and (b) difficulty integrating
technology into the existing curriculum (Demetrius & Barbas, 2002; Pelgrum, 2001).

According to Cuban (2001), although computers were readily available at most schools, they were not being used much by mathematics teachers. Computers were mainly used for demonstrations as well as drill and practice (Guerrero, Walker, & Dugdale, 2004). The reason that many teachers gave for not utilizing computers in the mathematics classroom was that the use of computers took “away from real learning” (Li, 2007).

Student Achievement and Understanding. Internet-based classroom technology includes online curriculum and course support services such as Agilemind. Gorman et al. (2008) measured the impact of Agilemind as a teaching and learning tool in high school mathematics by comparing school districts using Agilemind to similar districts not using Agilemind. State assessment scores were gathered for over 5500 students over a three-year period. The results were inconclusive, and the researchers noted that a critical issue in the study was low usage rates for schools using Agilemind (Gorman et al., 2008).

Virtual manipulatives and applets are also included in internet-based classroom technology. Reimer and Moyer (2005) reported that the use of virtual manipulatives led to improvement in students’ test scores on conceptual knowledge. Students using both physical and virtual manipulatives showed more improvement from pretest to posttest than students using only one type of manipulative (Takahashi, 2002; Terry, 1995). Many believe for applet use to be most beneficial, applets must be used in a constructivist teaching/learning environment (Zhou et al., 2005).

Research on Teaching Transformations of Parent Functions

The common finding in studies of students’ understanding of transformations of functions is that students find horizontal transformations – translations, stretching and
shrinking – more difficult than vertical transformations (Baker, Hemenway, & Trigueros, 2000; Borba & Confrey, 1996; Eisenberg & Dreyfus, 1994; Kimari, 2008). Students had greater difficulty visualizing a horizontal translation than a vertical translation (Eisenberg & Dreyfus, 1994). When asked to explain the “counterintuitive behavior” of horizontal translations, students responded with memorized rules or generalizations from observed patterns (Zazkis, Liljedahl, & Gadowsky, 2003, p. 441).

Another common finding in studies of students’ understanding of transformations of functions is that students have memorized rules for transformations (Kimani, 2008; Ninness, Rumph, McCuller, Vasquez, Harrison, Ford, Capt, Ninness, and Bradfield, 2005; Zazkis et al., 2003). According to Zazkis et al. (2003), students memorized rules for vertical and horizontal translations and were more concerned with remembering the rules than with understanding the behavior. Zazkis et al. (2003) found that the majority of teachers who were interviewed for their study referred to a “rule for horizontal translations” when asked how they explained the counterintuitive behavior of horizontal translations to students (p. 442). Students’ reliance on memorized rules or procedures when answering questions regarding transformations of functions led Kimani (2008) to conclude that “students do not have a conceptual understanding of function transformations” (p. 231).

In a study conducted by Ninness et al. (2005), students’ instruction on transformations of functions included a lecture followed immediately by computer-assisted instruction where students completed review modules when responses to test items were incorrect. Students were taught rules for transformations of functions such as adding a positive constant outside parentheses results in a vertical shift upward, adding a
negative constant outside parentheses results in a vertical shift downward, subtracting a positive constant inside parentheses results in a horizontal shift right, and subtracting a negative constant inside parentheses results in a horizontal shift left (Ninness et al., 2005). The conclusion was that students were able to paraphrase rules for transformations of functions but that knowing the rules may not be sufficient for correctly applying them (Ninness et al., 2005). Some students were able to apply rules to graph functions with a single transformation but had difficulty applying rules to graph functions with multiple transformations (Ninness et al., 2005).

Gadowsky (2001) conducted a study where participants used graphing software to investigate the relationship between the algebraic and graphical representations of quadratic functions. The focus of the study was to determine the extent to which participants made generalizations about function transformations. According to Gadowsky (2001), technology-based explorations of transformations of functions helped students “conceptualize and construct mathematical meaning about function transformations” (p. 110) and helped students make connections between algebraic and graphical representations.

Gadowsky (2001) also noted that students presented these connections among representations as observations of patterns rather than by providing mathematical justification for these connections. Students were not inclined to think about “why a particular algebraic alteration of the equation affected the graphical output in a particular way” (Gadowsky, 2001, p. 80). All participants in the study were able to identify transformations of the parent quadratic function and were able to demonstrate the ability to transform graphs of the parent quadratic function using these transformations.
(Gadowsky, 2001). However, this understanding of transformations of quadratic functions did not carry over to transformations of the parent absolute value function as evidenced by the inability of all participants to identify a vertical translation of the absolute value function (Gadowsky, 2001).

**Advantages of Using Applets**

**Constructivism and Discovery Learning.** Discovery learning activities using single purpose applications such as interactive applets align with the constructivist way of teaching. The basic premise of constructivism is that students actively create their own knowledge (Van de Walle, 1999). Essential to constructing new knowledge is the active engagement of students in the learning process and the use of existing knowledge to give meaning to new ideas they are developing (Inch, 2002). The connection between existing knowledge and new knowledge results in the development of a cognitive schema of related ideas, which leads to greater understanding by students (Van de Walle, 1999). Manipulating representations in a single purpose application should assist students with schema construction, and it is through the construction and automation of schema that students develop expertise (Van Merriënboer & Ayers, 2005). Students who do not connect new knowledge to existing knowledge have an overabundance of unrelated ideas rather than a cognitive network of related ideas.

Discovery learning activities using single purpose applications engage students in the learning process and provide an arena in which students can think and communicate mathematically as well as make and test conjectures. The dynamically linked representations provide students the means to visualize a concept and make connections between representations. Discovery learning activities require detailed handouts and
teacher guidance so that students can use the single purpose application to reach the desired results (Bos, 2009). Care must be taken when developing handouts since providing too much guidance in discovery learning activities has been shown to be detrimental to students’ later use of supposed gained knowledge (Kirschner, Sweller, & Clark, 2006).

Current theories on learning mathematics state that mathematical knowledge must be constructed by students as they attempt to make sense of mathematical situations and identify patterns (Battista, 2001). Single purpose applications efficiently facilitate structured mathematical reasoning and sense-making endeavors that allow students to make conjectures and then test their ideas (Wiest, 2001). This structure enables students of all mathematical ability levels to manage their exploration and learning. Once students have discovered a mathematical property or relationship on their own using single purpose applications, students have a basis for a more formal consideration of the mathematics (Rubin, 1999).

**Multiple Representations.** A mathematical representation is a representation that includes verbal, concrete, numerical, graphical, pictorial, or symbolic components (Piez & Voxman, 1997; TEA, 2005). Stated concisely, a mathematical representation is a graphical, numerical, algebraic, or analytical representation. According to Tripathi (2008), a mathematical representation is a “mental or physical construct that describes aspects of the inherent structure of a concept and the interrelationships between the concept and other ideas” (p. 438).

The use of multiple representations in teaching mathematics has been of particular interest to mathematics educators. The focus initially was to determine whether the use of
multiple representations in teaching mathematics was appropriate. Then it shifted to finding ways multiple representations could be used to teach specific mathematics concepts. The research on multiple representations in recent years has focused more on visual representations than any other representation (Tripathi, 2008). According to Arcavi (2003) visualization accompanies symbolic development and is an essential component in reasoning and problem solving. Stylianou and Silver (2004) examined the similarities and differences in the use of visual representations in problem solving by expert and novice mathematicians and found that although both experts and novices perceived visual representations as an important part of problem solving, experts used the visual representations more frequently and on a wider variety of problems.

Mathematics is not a disjoint set of concepts to be learned or skills to be mastered. Mathematical concepts are interconnected through a variety of relationships, and learning a concept entails not just knowing the meaning of the concept but also of understanding the relationships that connect this concept to others. Multiple representations can be a powerful tool in facilitating students’ understanding of mathematical concepts, especially when connections are made between the representations (NCTM, 2000). As stated in the Principles and Standards for School Mathematics, “Representations should be treated as essential elements in supporting students’ understanding of mathematical concepts and relationships; and in recognizing connections among related mathematical concepts” (NCTM, 2000, p. 67).

Further, the NCTM (2000) articulated in the Principles and Standards for School Mathematics, “Different representations support different ways of thinking about and manipulating mathematical objects” (p. 360). Although Aspinwall and Shaw (2002) agree
that students’ conceptual understanding is enhanced by visualizations, they contend that students construct very different visual representations on their own leading to different understandings of the concept. Therefore, it is critical that teachers provide appropriate visual representations for students that would lead to an understanding of the concept (Aspinwall & Shaw, 2002). According to Piez and Voxman (1997), as technology becomes more available in the classroom the use of multiple representations, particularly numerical, graphical, and analytical representations to teach concepts will increase, and teachers will need to develop activities using multiple representations so that students gain flexibility in problem solving. Erbas, Ledford, Polly, and Orrill (2004) state that technology plays an important role in allowing students to explore multiple representations and the connections between those representations; and when supported by the teacher, students can use technology to investigate properties and relationships, explore, test conjectures, and make generalizations.

**Other Advantages.** The critical first step in teaching is to engage students. Investigations using single purpose applications such as interactive applets engage students in learning (Hannafin, 2004). Investigations are structured so students do not struggle with how to start the investigation or with what to do next. Most students find this type of learning experience not only engaging but also enjoyable (Rochowicz, 1996). The ease of use of single purpose applications enables students to focus on the mathematics rather than the technology; thus, students focus on mathematical reasoning and sense making rather than on operating the technology (Battista, 2001).

The use of single purpose applications assists students in making abstract concepts more concrete, which is the first step to better conceptual understanding (Bos,
Single purpose applications have interactive mathematics objects, use multiple representations, and allow students to manipulate the objects and representations so that patterns are observed and conjectures are made and tested (Bos, 2009; Cavanaugh, Gillan, Bosnick, Hess, & Scott, 2008; Connell, 2001). When mathematical concepts are taught to a deep level of understanding, the concepts are retained by students and can be used as the foundation for understanding other concepts (Sfard, 1991). Also, with the multiple representations provided by single purpose applications, difficult abstract concepts are made more concrete which is the first step to better conceptual understanding (Bos, 2009).

Students who have difficulty understanding the mathematics respond at times by disengaging from the learning process. The use of single purpose applications, though, makes certain mathematical topics more accessible to students by relieving students of many of the computations and symbolic manipulations they struggle with and extends the ability to visualize the mathematics to a much broader audience (Rubin, 1999; Wiest, 2001). As the single purpose application performs computations and symbolic manipulations that act as barriers to mathematical exploration for many students, students’ cognitive load is reduced allowing students to focus on exploring mathematical concepts and relationships (Wiest, 2001).

Another advantage of using single purpose applications such as interactive applets to teach mathematics relates to students’ perceptions and beliefs about learning mathematics. Single purpose applications enable students to experience learning mathematics as something other than memorizing formulas, rules, and procedures. Single purpose applications provide students multiple representations that can be used for
investigation and problem solving (Bos, 2009). When students are given the opportunity to explore a mathematical concept as well as make and test conjectures, they experience firsthand what learning mathematics encompasses. Also, the use of single purpose applications gives students access to mathematical topics once thought beyond their capabilities (Rubin, 1999).

**Disadvantages of Using Applets**

The focused exploration that defines single purpose applications limits students’ ability to explore ideas outside of the focus. With single purpose applications students are limited to the instructional focus built into the applet. Students can explore only aspects of the topic that are built into the application. This characteristic is an advantage with respect to providing a structured focus for students, but at the same time it is a disadvantage with respect to students’ ability to freely explore other aspects of the topic or other topics altogether.

There are ways to deal with the disadvantages associated with limited exploration. One possible solution is to broaden the exploration possible with the applet while maintaining the given instructional focus. For example, the applets designed for this study focused on parameter changes to quadratic functions. These applets could have been designed to focus on parameter changes to all parent functions and not just quadratic functions by giving students the option to select other parent functions with which to complete the guided exploration. This would allow for broader exploration while maintaining the instructional focus. Although this solution does not address the exploration of topics outside the instructional focus, it does expand the exploration possible for the built-in instructional focus.
Conclusions

Although many mathematics educators believe technology enhances mathematics teaching and learning, more research is needed not only to determine the impact technology has on students’ understanding but also to further investigate specifically how and why the technology had an impact. The assumption that the use of technology to teach mathematics is beneficial to students, enhances students’ understanding of mathematics, and increases student achievement needs more thorough investigation.

Past studies need to be replicated to build on the consistency of the findings. This may be challenging, however, due to the ever-changing state of technology. As the technology evolves, the methods for using the technology to teach mathematics will evolve. Studies to determine the most effective ways to harness the potential for new technology are greatly needed. Methods of using technology to teach mathematics will have to change to mirror the changes in technology. Researchers can utilize the knowledge from the past to narrow the search for effective methods. Schools will continue to invest in computer technology for the mathematics classroom because it is present in nearly every facet of society. As such, teachers will need guidance on how to effectively use technology to teach mathematics and potentially impact student learning.

Learning mathematics requires student engagement, and many instructional practices fail to actively engage students in their learning. Studies have indicated that computer technology engages and motivates students (Hannafin et al., 2001). If computer technology can impact engagement, then a critical question is how do mathematics teachers use computer technology to positively impact students’ understanding of mathematics? Computer technology has the potential not only to engage students in
learning mathematics but also to positively impact their understanding of mathematics. Mathematics educators must continue to examine current practices for teaching mathematics with technology to determine its effectiveness and to explore new ways to harness the potential that it brings as an instructional and learning tool.
Chapter III: Methodology

The purpose of this mixed methods study was to explain the effects of using single purpose applications, such as interactive applets, to teach parameter changes to parent functions. This study utilized an explanatory sequential mixed methods research design that begins with quantitative data collection and analysis followed by qualitative data collection and analysis (Creswell & Plano Clark, 2011). The quantitative phase of the study focused on examining the impact – positive, negative, or nonexistent - the use of single purpose applications had on students’ conceptual understanding, while the qualitative phase focused on investigating students’ and teachers’ beliefs regarding how and why the use of technology had the respective impact. The qualitative phase of this study was implemented for the “purpose of explaining the initial results more in depth” (Creswell & Plano Clark, 2011, p. 82).

The researcher believes that if the mathematics education community is to harness the potential technology has to offer as a teaching and learning tool it must continue to examine not only what the impact of using technology is but also why. Regardless of the quantitative results – positive, negative, or nonexistent impact - by having the qualitative follow-up, the researcher gained a deeper understanding of effective and ineffective ways technology can be used in the teaching and learning of mathematics.

Research Questions

This mixed methods study addressed the following research questions:

1. To what extent does the use of single purpose applications for teaching parameter changes to parent functions impact a student’s transition from procedural understanding to conceptual understanding?
2. From the teacher’s perspective, to what extent does the use of single purpose applications impact a student’s transition from procedural understanding to conceptual understanding of parameter changes to parent functions? To what does the teacher attribute the single purpose applications impacting students’ transition from procedural to conceptual understanding of parameter changes to parent functions?

3. From the student’s perspective, what additional insights on parameter changes to parent functions are gained by using a single purpose application? To what does the student attribute the single purpose applications providing additional insights on parameter changes to parent functions?

**Research Design**

Tashakkori and Teddlie (2006) define mixed methods research as research in which data is collected, data is analyzed, findings are merged, and inferences are drawn using both quantitative and qualitative methods in one study. The basic assumption in the definition of mixed methods studies is that the combination of both quantitative and qualitative research will enable the researcher to have a better understanding of the research problem than is possible with either research method alone (Creswell & Plano-Clark, 2007; Tashakkori & Teddlie, 2003). This study utilized the explanatory sequential mixed methods research design (see Figure 3.1).

The quantitative data that were collected for this study included Algebra 2 students’ scores on a pretest and posttest on parameter changes to parent functions. The pretest and posttest are described in more detail in the instrumentation section (phase one) and the validity section in this chapter. The pretest and posttest data were analyzed to determine the change in assessment scores for students that could be attributed to using
single purpose applications. The qualitative data included teacher and student interviews that were analyzed to determine specifically how and why the use of single purpose applications assisted, hindered, or had no effect on the student’s transition from procedural to conceptual understanding. The qualitative results were used to explain the quantitative results and together the two provided a better understanding of not only what was happening but also why it was happening than either could provide alone (Creswell, 2008).

Population and Sample

This study included participants from two public high schools in northeast Texas. One Algebra 2 teacher at each high school along with students in their respective Algebra 2 classes, six at one high school and two at the other, were included in the study. Initially,
a decision was made as to which classes would be assigned as treatment classes and control classes based on input from the participating teachers regarding availability of computers. Once a laptop cart was secured at one of the participating high schools, the issue of computer availability was eliminated and classes were then assigned randomly as treatment and control classes. A total of 126 students returned consent and assent forms, but only 97 of the students completed all activities associated with the study – pretest, technology intervention or instruction, and posttest. Of these 97 students, 49 were in treatment classes and 48 in control classes.

**Technology Intervention**

The technology interventions utilized in the study included two web-based interactive applets on parameter changes to parent functions that the researcher created using GeoGebra. The applets were uploaded to GeoGebraTube (www.geogebratube.org) so that students would need only internet access to utilize the applets. One applet focused on parameter changes of the form $g(x) = f(x) + c$ and was accessed using the web address www.geogebratube.org/student/m19019. The other applet focused on parameter changes of the form $g(x) = a*f(x)$. This applet was accessed using the web address www.geogebratube.org/student/m19021. The researcher verified that the applets would run on computers at the participating schools. Students in treatment classes were given an activity sheet with each applet to guide their exploration using the applets. Students in treatment classes completed the first guided exploration (see Appendix C) using the applet on parameter changes of the form $g(x) = f(x) + c$ while students in control classes were taught this transformation without the use of the applet. The following day students in the treatment classes completed the second guided exploration (see Appendix D) using
the applet on parameter changes of the form \( g(x) = a*f(x) \) while students in the control classes were taught this transformation without the use of the applet. Each guided exploration took one class period to complete.

Although it is impossible to duplicate exactly the utilization of the technology intervention on each campus since the researcher could not be present, participating teachers were trained by the researcher on specific expectations for utilizing the technology interventions so that the difference between campuses would be minimal. Due to scheduling conflicts, the training for the teachers was held after school rather than on Saturday as initially planned. During the training, the researcher shared the purpose of the research study, described the applets, and modeled how to use the sliders to change the representations on the applet. Specific guidelines (see Appendix G) were given to participating teachers on how to facilitate treatment classes completing the applet activities. Participating teachers were told to teach the two parameter changes to the control classes as they normally do. Neither teacher used computers or applets to teach parameter changes to parent functions. At the end participating teachers were given the opportunity to ask questions to clarify their role in the process.

**Instrumentation**

**Phase One.** The quantitative data collected in phase one were used mainly to address the first research question but were also used for specific interview questions that addressed the second and third research questions. The quantitative data collection instruments in phase one included a pretest and posttest assessment (see Appendix A) on parameter changes to parent functions that was given to Algebra 2 students in both treatment and control classes at the two participating high schools. The pretest and
posttest assessments were developed by the researcher using current state and national standards for Algebra 2 along with questions from released Algebra 2 end-of-course exams from various states. Pretest and posttest questions were the same. The pretest/posttest assessments were divided into four parts. Parts one and four addressed procedural understanding and parts two and three addressed conceptual understanding. The pretest consisted of 24 items ($\alpha = .72$), and the posttest consisted of 24 items ($\alpha = .84$). According to Nunnaly and Berstein (1994), an alpha of .70 can be used as a cutoff for reliability for newly developed instruments. For most basic research, though, an alpha of .80 should be used as a cutoff for reliability (Carmines & Zeller, 1979; Lance, Butts, & Michels, 2006; Nunnaly & Berstein, 1994). Thus, the posttest was found to be reliable. Although the pretest would not be considered reliable using .80 as the cutoff, the pretest would be considered reliable using .70.

The study took place in the fall semester when the participating teachers planned to teach parameter changes to parent functions. Students in both treatment and control classes were given a pretest prior to the time when these two transformations were to be taught. Students in the treatment classes completed the guided explorations using the applets while students in the control classes were taught the two transformations by their respective teacher. Students in the treatment classes took two days to complete the two guided explorations using applets while students in the controls classes received two days of instruction on parameter changes to parent functions without the use of applets. Students in both treatment and control classes were given a posttest upon completion of two days of exploration (treatment classes) or instruction (control classes). The pretest and posttest scores were used to determine the extent to which the students transitioned
from procedural understanding to conceptual understanding on parameter changes to
parent functions and whether the use of guided explorations with applets had an impact
on the transition.

The participating teachers were given a two-week timeframe for completing all
events associated with the study including the pretest, two applet activities (treatment
classes) or regular instruction (control classes), and the posttest. One week following the
pretest, three consecutive days were scheduled to complete all remaining events
associated with the study. One the first scheduled day, students in the treatment classes
completed the first applet activity on vertical translations while students in the control
classes received instruction on vertical translations from their teachers. The following
day, students in the treatment classes completed the second applet activity on vertical
stretching and shrinking while students in the control classes received instruction on
vertical stretching and shrinking from their teachers. The next day, students completed
the posttest on parameter changes to parent functions. Since the researcher was required
to administer the pretest and posttest, these assessments were not given on the same day
at each campus. Due to the fact that both participating teachers were scheduled to teach
parameter changes to parent functions at approximately the same time, the researcher was
able to give the posttest at each high school within the same week.

Phase Two. The qualitative data collected in phase two were used to address the
second and third research questions. The qualitative data collection instruments in phase
two included semi-structured interview protocols for teachers (see Appendix F) and
students (see Appendix E) that were developed by the researcher. The two participating
teachers were interviewed as were selected students from each high school. Using the
quantitative results from phase one, students in the treatment classes were categorized by their score decreasing, not changing, or increasing from pretest to posttest. At least one student per category per school was interviewed. The interviews took place on the respective school campuses, and each interview was approximately 30 minutes. The interviews were audio taped, but the researcher also took notes during the interviews. Interviews were transcribed and participants verified that the interview transcripts were correct. The interview questions were used to determine what each teacher and student attributed the impact of the use of single purpose applications to regardless of whether the impact of using single purpose applications was positive, negative, or nonexistent.

**Validity**

Since the researcher created the pretest and posttest assessments, a critical step in the instrument development process was assessing content validity. In developing questions for the pretest and posttest assessments, the researcher reviewed the Texas Essential Knowledge and Skills (TEKS) and the CCSSM so that assessment questions would be aligned with state standards and national recommended standards. According to the Algebra 2 TEKS, specifically 2A.7.B, students are expected to “use the parent function to investigate, describe, and predict the effects of changes in a, h, and k on the graph of \( y = a(x - h)^2 + k \)” (TEA, 2005, p. 45). According to the CCSSM students are expected to “experiment with cases and illustrate an explanation of the effects on the graph using technology” of transformations to parent function including \( f(x) + k \), \( k(f(x)) \), \( f(k(x)) \), and \( f(x + k) \) (CCSSO, 2010, p. 70).

Face validity of the pretest and posttest assessments was established using expert reviews. The assessments were initially reviewed by two mathematicians at two different
public research-intensive universities in south central United States who are considered experts in the field of mathematics. These reviews were followed by reviews from a team of mathematics professors from universities in northeast Texas. Also, colleagues that the researcher taught with over the years and considered to be expert high school mathematics teachers reviewed the assessments. Changes were made to the pretest and posttest based on the expert reviews.

Interview protocols were also reviewed by the researcher’s dissertation committee prior to the proposal defense. Changes to the interview protocols were initially made based on input from the dissertation committee during the proposal defense. Following the pilot study, more changes to the protocols were made based on the researcher’s experiences interviewing participants during the pilot study.

As discussed earlier in the limitation section of chapter one, history and testing were threats to validity. To minimize history effects, the researcher needed to shorten the length of time between pretest and posttest, but in order to minimize testing effects, the researcher needed to lengthen the time between pretest and posttest. It was not possible for the researcher to address both threats by simply adjusting the time between pretest and posttest. According to Pedhazur and Schmelkin (1991), one to two weeks between pretest and posttest is an appropriate time interval for maintaining test-retest reliability. For this study, the researcher found a statistically significant positive correlation between pretest and posttest \( (r = .59, p < .001) \); therefore, test-retest reliability is moderate. The researcher gave participating teachers a two-week window for completing assessments and applet activities, with applet activities beginning one week after the completion of the pretest. By doing this, the researcher minimized testing effects. Participating teachers
used three consecutive days for completing applet activities and the posttest - two days for applet activities for treatment classes and instruction for control classes followed by one day for posttest. Each participating teacher was trained on the expectations for facilitating the guided explorations and verified during their interview that the expectations were met. Since the unit on teaching parameter changes to parent functions started with the first of these three consecutive days, the researcher eliminated class time where these transformations can be discussed outside of the scope of the study. By doing this, the researcher minimized history effects.

Pilot Study

A pilot study was conducted in the spring of 2012 using existing precalculus classes at a northeast Texas high school. There were 21 students and one teacher that participated in the study pilot. Students who participated in the pilot study were in one of two precalculus classes taught by the participating teacher. One class was designated the treatment class and one the control class. Students in the treatment class had no major problems completing the two applet activities, but there were a few students who struggled initially to start the activities even though the activity sheets were intended to provide guidance for the exploration. Upon completion of the pretest, applet activities, and posttest, three students from the treatment class were selected and interviewed by the researcher. Following data collection and analysis, revisions were made to the assessments, scoring rubric, and interview protocol.

Changes on the assessments included deleting the original problems numbered seven and eight on parts one and two while adding a part four with four procedural questions to mirror the four conceptual questions on part three. With the original pretest
and posttest assessments, part one was procedural, part two was conceptual, and part three was both. The researcher discovered during the pilot study the difficulty in scoring part three in terms of procedural and conceptual understanding. By adding part four that focused on procedural understanding and mirrored part three, the scoring difficulty was alleviated. The scoring rubric was also reworded to alleviate inconsistencies in grading that the researcher discovered during the pilot study while scoring documents a second time. The researcher also made minor changes in the wording of some questions on interview protocols.

**Data Collection Procedures**

**Phase One.** For the quantitative portion of the mixed methods study, each student was given the pretest on parameter changes to parent functions during his or her Algebra 2 class. Students were given 45 minutes to complete the pretest and were not allowed to use calculators or the applets on the pretest. The pretest assessment was monitored and collected by the researcher. The participating teachers took two consecutive days to teach the two transformations to the students in their control classes while the students in their treatment classes completed the two applet activities. The next day following the completion of the guided explorations using the applets for treatment classes or teacher’s instruction for control classes, each student was given the posttest on parameter changes to parent functions. As with the pretest, students were given 45 minutes to complete the posttest and could not use calculators or the applets on the posttest. The researcher monitored and collected the posttest assessment.

**Phase Two.** The qualitative portion of the study included interviews with the two teachers participating in the study along with interviews from selected students.
participating in the study. Student selection was based on the following criteria: (a) there is change in score from pretest to posttest – positive, negative, nonexistent – and (b) each participating teacher would have at least three of his or her students interviewed. Teacher interviews began after the posttest had been given. Student interviews began once phase one data had been analyzed and students meeting the above criteria were selected.

Interviews took place on the respective high school campuses and took approximately 30 minutes each. On one campus, interviews took place in a conference room in the library. On the other campus, interviews took place in a conference room in the main office.

Interviews were audio taped and later transcribed. The researcher met with each teacher and student interviewed to verify the accuracy of the transcription and to get clarification if necessary.

**Scoring Rubric for Assessment Items**

A scoring rubric (see Appendix B) for the pretest and posttest assessments was developed by the researcher and reviewed by the researcher’s committee. After using the rubric to score pretests and posttests in the pilot study, revisions were made that simplified the conditions for awarding specific points and enabled the researcher to score assessments consistently.

The researcher was the only person who scored pretests and posttests. Before scoring assessments, the researcher randomly selected five participants and made copies of their pretests and posttests. To check the intra-rater reliability, the researcher scored the copies of the five participants’ pretests and posttests and compared the results to how the original pretests and posttests were scored. The average agreement rate was 97%.
Data Analysis

**Phase One.** The purpose of this phase of the study was to determine the impact the use of interactive applets had on students’ conceptual understanding of parameter changes to parent functions.

**Data Screening.** A total of 126 students from the two high schools returned consent and assent forms. Twenty-nine students were identified as having not completed all activities associated with the study – pretest, technology intervention or instruction, and posttest. The participating teachers reported that most of the absences were extracurricular absences due to school-sponsored activities or excused absences due to illness. These twenty-nine students were eliminated from the study for not completing all activities associated with the study. This reduced the sample size to 97 students.

After eliminating participants from the study who were missing pretest or posttest scores, the researcher examined the distribution of the dependent variable posttest_score and the covariate pretest_score. Descriptive statistics were run initially using SPSS to determine if the variables pretest_score and posttest_score were normally distributed. Since both variables were positively skewed, the researcher completed a log transformation of the data renaming the variables pretest_score_log and posttest_score_log. Descriptive statistics were run again, and the transformed variables for pretest and posttest scores were normally distributed.

**Descriptive Statistics.** Before conducting the ANCOVAs to determine whether the use of guided explorations using applets had an impact on students’ understanding, descriptive statistics for the transformed dependent variables (see Table 3.1) – posttest_score_log, post_procedural_log, and post_conceptual_log – and transformed
covariates – pretest_score_log, pre_procedural_log, and pre_conceptual_log – were calculated using SPSS. Skewness and kurtosis for the transformed dependent variables and transformed covariates were checked. Not only did the values for skewness and kurtosis for each transformed variable fall within the range of minus two standard errors to plus two standard errors, the absolute value of the skewness and kurtosis values for each transformed variable was less than one. Thus, the transformed dependent variables and transformed covariates were considered normally distributed.

Table 3.1
Descriptive statistics for transformed dependent variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Posttest score</td>
<td>1.166</td>
<td>.325</td>
<td>.106</td>
<td>-.520</td>
<td>-.126</td>
</tr>
<tr>
<td>Posttest procedural subscore</td>
<td>.930</td>
<td>.375</td>
<td>.140</td>
<td>-.565</td>
<td>-.400</td>
</tr>
<tr>
<td>Posttest conceptual subscore</td>
<td>.783</td>
<td>.359</td>
<td>.129</td>
<td>-.565</td>
<td>.013</td>
</tr>
</tbody>
</table>

Assumptions. Having a dependent variable that is normally distributed is one assumption for ANCOVA. Other assumptions for ANCOVA include homogeneity of variances, homogeneity of the regression slopes, the use of independent random sampling, and the existence of a linear relationship between the transformed dependent variable and the transformed covariate. The assumptions of ANCOVA have been met with the exception of random sampling. With this study as with most educational research, the use of existing classes is not only convenient but is also a necessity. In this study, the researcher was testing the impact of a technology intervention, and treatment and control classes were needed. Although students were not randomly selected for treatment or control classes, the Algebra 2 classes of participating teachers were
randomly assigned as treatment or control classes, thus providing some randomization. The researcher used ANCOVA to adjust for preexisting differences between the classes (Miller and Chapman, 2001; Warner, 2008). By using the pretest as a covariate, the researcher can determine if there was a statistically significant difference from pretest to posttest for students in the treatment classes as compared to students in the control classes.

**Phase Two.** The purpose of this phase of the study was to explain how and why the use of interactive applets had a positive, negative, or nonexistent impact on students’ understanding of parameter changes to parent functions.

**Selection of Interview Participants.** Once pretest and posttests were scored, the researcher identified students from both high school that showed either a positive change, negative change, or no change in score from pretest to posttest. At least one student in each of three categories – positive change, negative change, or no change – was identified at each high school. A total of 15 students were interviewed along with the two participating teachers. Since there were 97 student participants in the study, the researcher interviewed more than 10% of the participants.

**Transcribing and Coding Interview Data.** The researcher utilized a professional service to transcribe all interviews. Upon receiving the transcripts, the researcher read through each transcript noting places that were marked “inaudible” or places the researcher believed had been transcribed incorrectly. The researcher then listened to the recordings to verify what the student or teacher said in each place in question. In most cases, this process alleviated the problem. In a few instances, the researcher could not
determine what the student or teacher said and utilized her notes along with follow-up interviews to address the questionable areas.

The researcher also conducted a member check to verify the accuracy of the transcripts and the researcher’s initial findings. Transcripts were given to the participating teachers to distribute to students. Teachers were given instructions for completing the member check. Students were given their interview transcript along with a letter outlining the process and explaining the opportunity they have to correct errors and question interpretations they believe are not valid. Once transcripts were reviewed by interview participants, the researcher collected all transcripts and noted any changes that were made by participants. Few changes were made to the transcripts or findings. The researcher attributed this to restating students’ and teachers’ responses during the interview process and questioning the participant on the accuracy of the summary. This process of restating responses was utilized only on responses the researcher deemed critical to the study.

Since the researcher was the only person coding interview transcripts, the researcher needed to determine how consistent she was in coding transcripts. Intra-rater reliability was checked using a coding comparison in NVIVO. The researcher created a second profile for herself in NVIVO, recoded three randomly selected transcripts, and then ran a coding comparison to determine the agreement between the two profiles on each of the three interviews. One interview transcript from each of the three groups of students – scores increased, decreased, or did not change – was selected for recoding. The average rate of agreement was 95%.

The initial stages of the qualitative data analysis involved reading interview notes and transcriptions repeatedly as well as listening to interview recordings multiple times.
While reviewing the data, the researcher made notes of any ideas that might prove to be helpful later. The researcher organized qualitative data including audio recordings, transcripts, and notes by interviewees. Following organization, interview transcripts were coded using NVIVO. Given the specificity of some of the interview questions, the researcher believed that use of a priori categories was appropriate for this study. A priori categories included advantages from the teacher’s perspective, advantages from the student’s perspective, disadvantages from the teacher’s perspective, disadvantages from the student’s perspective, the student’s perspective on ways technology helped with understanding, and the student’s perspective on ways technology hindered understanding, to name a few. These categories were not only created by the researcher but were also organized in a hierarchy (see Figure 3.2). Given the nature and dynamics of the teaching and learning process in this study, emergent categories were also used (see Figure 3.2). Categories, both a priori and emergent, were separated into subcategories, and this process continued until the researcher felt there are no new categories or subcategories to be identified (J. Webb-Dempsey, personal communication, 2007). The researcher also identified patterns within and between categories and subcategories that were used to explain and support findings (Given, 2008).

The results of the qualitative phase were used to explain the quantitative results from phase one of the study regarding the impact – positive, negative, or nonexistent – of using single purpose applications. The researcher looked for findings that could be substantiated by both quantitative and qualitative data. Discrepancies in the quantitative and qualitative data, though, provided interesting discussion points such as students who
believed using single purpose applications enhanced their understanding but yet their test scores showed the use of single purpose applications had a negative impact on their

- **Advantages**
  - Student’s perspective
    - Positive change
    - No change
    - Negative change
  - Teacher’s perspective

- **Disadvantages**
  - Student’s perspective
    - Positive change
    - No change
    - Negative change
  - Teacher’s perspective

- **Helped**
  - Student’s perspective
    - Positive change
    - No change
    - Negative change
  - Teacher’s perspective

- **Hindered**
  - Student’s perspective
    - Positive change
    - No change
    - Negative change
  - Teacher’s perspective

- **Motivation and engagement**
  - Student’s perspective
    - Positive change
    - No change
    - Negative change
  - Teacher’s perspective

- **Retention of learning**
  - Student’s perspective
    - Positive change
    - No change
    - Negative change
  - Teacher’s perspective

- **Utilization scenarios**
  - Student’s perspective
    - Positive change
    - No change
    - Negative change
  - Teacher’s perspective

Figure 3.2 Hierarchy of nodes created by the researcher
understanding. Valuable insight was gained by the researcher not only from data that corroborate but also from data that do not.

**Research Bias**

It is important to discuss biases brought to the study. Not only do readers of the research need to be aware of the researcher’s biases, but it is also important that the researcher reflect on her biases. As a mathematics teacher for 28 years, the researcher’s focus has been on helping students understand mathematics. The researcher has provided support both inside and outside the classroom to help students experience success so they will have confidence in their ability to do mathematics. The researcher believes the use of technology has played an important role in that.

The researcher began teaching Advanced Placement (AP) Calculus in 1994, and although her students were successful on the AP exam each year, she was concerned their understanding of calculus was more procedural than conceptual. For example, some calculus students who could state the limit definition of a derivative and use the limit definition to find a derivative could not explain the derivation or meaning of the definition. The researcher realized that in order to address this problem her methods of teaching had to change. The researcher had been modeling concepts for students with traditional teaching tools such as a chalkboard or overhead projector, but these tools were ineffective. For example, with traditional tools the researcher could not effectively model what happens graphically to the secant line when you take the limit as $h$ approaches zero. Dynamic computer technology afforded the researcher ways to model the definition of a derivative and other concepts effectively, concepts that she had been previously unable to model effectively with traditional teaching tools. With computer demonstrations the
researcher created, her goal was to provide students with an animated model of a concept that would enhance their understanding of the concept. The researcher believes that her students have benefited from these demonstrations.

In 2007, the researcher was selected for the Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM) doctoral program. ACCLAIM was a National Science Foundation-funded Center for Learning and Teaching that aimed to improve the quality of instruction in rural Appalachia by increasing the mathematics teacher educator base in the Appalachian regions of Kentucky, West Virginia, Tennessee, and Ohio by providing a non-traditional doctoral program for mathematics teachers in rural Appalachia as well as professional development. With the third cohort of doctoral students, ACCLAIM accepted applications from mathematics teachers in rural schools across the United States, which led to the researcher being selected for the program. In reflecting on her experiences in ACCLAIM, the researcher realized these demonstrations could be adapted into more powerful teaching tools if she put the models into the hands of the students and transformed the demonstrations into guided discovery learning activities using single purpose applications. It is the belief of the researcher that single purpose applications such as interactive applets enable students to experience learning mathematics as something other than memorizing formulas, rules, and procedures. Due to the inherent focus of single purpose applications, the technology does not obscure the mathematics. The researcher believes with students actively engaged in learning using single purpose applications, students potentially can gain a deeper understanding of the concept, property, or relationship being explored.
Chapter IV: Results

The purpose of this explanatory sequential mixed methods study was to examine the impact – positive, negative, or nonexistent – of the use of single purpose applications, such as interactive applets, had on students’ conceptual understanding of parameter changes to parent functions and to explain the effects. To determine the impact the use of interactive applets had on students’ conceptual understanding of parameter changes to parent functions, the researcher conducted an analysis of covariance (ANCOVA). To explain why the use of interactive applets had this impact – positive, negative, or nonexistent – interview data were coded and analyzed.

Results

First Research Question. To what extent does the use of single purpose applications for teaching parameter changes to parent function impact a student’s transition from procedural understanding to conceptual understanding? As previously stated, the researcher conducted an ANCOVA to determine if there were statistically significant differences between posttest scores for treatment classes and control classes when controlling for differences in students’ mathematical ability by using pretest scores as a covariate. The independent variable class_type had two levels, treatment and control. Students in the treatment classes worked through the applet activities on parameter changes to parent functions while students in the control classes were taught the content by their teacher. Based on the results of the ANCOVA (see Table 4.1), $F(1,94) = 7.185$, $p = .009$, students in the treatment classes ($M = 1.24$, $SD = .29$) had statistically significantly higher scores on the posttest than students in the control classes ($M = 1.10$, $SD = .33$). The effect size (partial eta squared = .071) was a medium-to-large effect size since .06 is considered medium and .14 is considered large (Huck, 2012). This effect size
indicated of a moderate-to-strong relationship between class type (treatment and control) and posttest scores when pretest scores were used as a covariate. In order to rule out the teacher as a possible explanation for the difference in posttest scores between treatment and control classes, the researcher ran the original ANCOVA adding teacher as a blocking factor. Based on the results, $F(1,94) = .253, p = .616$, the difference in posttest scores cannot be attributed to the teacher.

Table 4.1
ANCOVA on transformed posttest scores with transformed pretest scores as covariate

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
<th>Partial Eta Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>4.034$^a$</td>
<td>2</td>
<td>2.017</td>
<td>31.050</td>
<td>&lt;.001</td>
<td>.398</td>
</tr>
<tr>
<td>Intercept</td>
<td>4.014</td>
<td>1</td>
<td>4.014</td>
<td>61.794</td>
<td>&lt;.001</td>
<td>.397</td>
</tr>
<tr>
<td>Classtype</td>
<td>.467</td>
<td>1</td>
<td>.467</td>
<td>7.185</td>
<td>.009$^*$</td>
<td>.071</td>
</tr>
<tr>
<td>Error</td>
<td>6.106</td>
<td>94</td>
<td>.065</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Corrected</td>
<td>10.139</td>
<td>96</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>142.065</td>
<td>97</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ R Squared = .398 (Adjusted R Squared = .385)

The researcher conducted a second ANCOVA to determine if there were statistically significant differences on posttest procedural subscores between treatment classes and control classes when controlling for differences in students’ procedural ability by using pretest procedural subscores as the covariate. Procedural subscores were obtained by adding the scores on parts one and four of the assessments. Based on the results of the second ANCOVA (see Table 4.2), $F(1, 94) = 21.684, p < .001$, students in the treatment classes ($M = 1.06, SD = .30$) had statistically significantly higher procedural subscores on the posttest than students in the control classes ($M = .80, SD = .37$). The effect size (partial eta squared = .187) was a large effect size and indicated a
strong relationship between class type (treatment and control) and posttest procedural subscores when pretest procedural subscores were used as a covariate. In order to rule out the teacher as a possible explanation for the difference in posttest procedural subscores between treatment and control classes, the researcher reran the ANCOVA adding teacher as a blocking factor. Based on the results, the difference in posttest procedural subscores cannot be attributed to the teacher.

Table 4.2
ANCOVA on transformed posttest procedural subscores with transformed pretest procedural subscores as covariate

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
<th>Partial Eta Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>6.158(^a)</td>
<td>2</td>
<td>3.079</td>
<td>39.516</td>
<td>&lt;.001</td>
<td>1</td>
</tr>
<tr>
<td>Intercept</td>
<td>3.771</td>
<td>1</td>
<td>3.771</td>
<td>48.399</td>
<td>&lt;.001</td>
<td>.340</td>
</tr>
<tr>
<td>Classtype</td>
<td>1.690</td>
<td>1</td>
<td>1.690</td>
<td>21.684</td>
<td>&lt;.001</td>
<td>* .187</td>
</tr>
<tr>
<td>Error</td>
<td>7.324</td>
<td>94</td>
<td>.078</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>97.409</td>
<td>97</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>13.482</td>
<td>96</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) R Squared = .457 (Adjusted R Squared = .445)

The researcher conducted a third ANCOVA to determine if there were statistically significant differences on posttest conceptual subscores across the independent variable class_type when controlling for differences in students’ conceptual understanding by using the pretest conceptual subscore as the covariate. Conceptual subscores were obtained by adding the scores on parts two and three of the assessments. Although students in the treatment classes (\(M = .79, SD = .35\)) had higher conceptual subscores on the posttest than students in the control classes (\(M = .78, SD = .37\)), the difference was not statistically significant (see Table 4.3). The effect size (partial eta squared = .000) indicated no relationship existed between class type (treatment and
control) and posttest conceptual subscores when pretest conceptual subscores were used as a covariate.

Table 4.3
ANCOVA on transformed posttest conceptual subscores with transformed pretest conceptual subscores as covariate

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
<th>Partial Eta Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>2.773(^a)</td>
<td>2</td>
<td>1.387</td>
<td>13.559</td>
<td>&lt;.001</td>
<td>.224</td>
</tr>
<tr>
<td>Intercept</td>
<td>7.514</td>
<td>1</td>
<td>7.514</td>
<td>73.474</td>
<td>&lt;.001</td>
<td>.439</td>
</tr>
<tr>
<td>Classstype</td>
<td>.003</td>
<td>1</td>
<td>.003</td>
<td>.025</td>
<td>.875</td>
<td>.000</td>
</tr>
<tr>
<td>Error</td>
<td>9.613</td>
<td>94</td>
<td>.102</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>71.776</td>
<td>97</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>12.387</td>
<td>96</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) R Squared = .224 (Adjusted R Squared = .207)

Since parts two and three of the pretest and posttest assessed conceptual understanding in different ways, the researcher decided to investigate the possibility that a statistically significant difference between posttest conceptual subscores for either part two or three existed across class type. The researcher conducted two more ANCOVAs but no statistically significant difference between posttest conceptual subscores was found for either part two or part three.

As stated previously, although the students in the treatment classes \((M = .79, SD = .35)\) had higher conceptual subscores on the posttest than students in the control classes \((M = .77, SD = .37)\), the difference was not statistically significant. Although most students believed that working through the applet activities helped them understand parameter changes to parent functions, many students were unable to correctly answer posttest questions on transformations of nonstandard functions (see problems 1 – 8 on part two of pretest/posttest in Appendix A). Many students believed that using the applet
helped them “see how the graph changes” when the parameter was changed but only felt “comfortable working with quadratics.” Many students were “confused when the questions went from quadratic to the other” as evidenced by students’ responses to questions on part two of the posttest but felt they would not have been as confused “had they worked more with other graphs.” The researcher considered allowing students to work with quadratics and other functions, particularly nonstandard functions, on the applets but decided to limit the use of other parent functions and nonstandard functions to the posttest as a way to evaluate students’ conceptual understanding of parameter changes to parent functions (see Figure 4.1).

The researcher made deliberate decisions concerning what should be put on the applets and what questions should be asked on the activity sheets. When asked by the researcher if the distances labeled $FA$ and $GA$ were used in attempting to understand not only what a particular parameter change does to the graph but why, most students stated they “did not notice the distances.” On the activity sheets that guided the applet explorations, students were expected to complete four tables. When asked by the researcher if the tables on the activity sheets were helpful in understanding what a particular parameter change does to the graph and why, most students responded that they could see “how the function changes, what the coordinates are, and how they are different from each other.” A few students noted that when using the applet, “you can look at the points as you change the slider and that helps you see how the y’s change.” The researcher concluded from the interviews that students using either the tables on the activity sheet or using the dynamic representations on the applet understood that for the same $x$-coordinate the points on the graphs of $f(x)$ and $g(x)$ would have $y$-coordinates
related by the given parameter and specified transformation. During interviews, the researcher specifically asked students to explain their reasoning on problems one, two, and three on part two of the posttest if they got the problems correct. If they missed the problems they were asked to try to answer the problems in light of what they said about “how the y’s change” on the quadratic (see Figure 4.2). The students who had gotten the problems correct were able to explain their reasoning. The students who missed the

8. Given the graph of \( f(x) = x^3 \), graph \( g(x) = x^3 - 5 \).

![Graph of f(x) and g(x)](image)

Figure 4.1 Part 2 #8 (pretest on top, posttest on bottom)
problems were still unable to answer the questions and stated “we studied quadratics and this isn’t quadratic.” These students also described the effects of the parameter changes they studied as “moving the graph up or down, making it more wide or narrow.” During interviews, the researcher asked students to explain why the graph was “more wide or narrow” and students typically responded by stating a rule they recalled from Algebra 1 and rediscovered while completing the activities. But even students who could state the rule could not explain why the rule worked, so their understanding was more procedural.

Using the graph of $f(x)$ shown below, graph $g(x)$ in problem 3.

![Graph of $f(x)$](image)

3. Graph $g(x) = 2f(x) + 1$.

![Graph of $g(x)$](image)

Figure 4.2 Part 2 #3 (Pretest on left, posttest on right)
Some students reported “having difficulty starting the activity.” Due to the expectations of participating teachers in treatment classes, students had to do the activities “on their own.” For students who have difficulty in mathematics, being asked to complete an activity, look for patterns in the tables, and relationships among the various representations was challenging. Some students stated that they “were not confident” that what they were doing in completing the guided exploration was correct. Although their teacher had students report their generalizations and summarized what students should have discovered, many students felt “the teacher should have introduced it first and interacted with us as we did the activity.” All students interviewed whose scores from pretest to posttest decreased stated during their interviews that they believed they would have had a better understanding if their teacher had taught or introduced the material first and then they completed the applet activities. On the other hand, almost all students whose scores from pretest to posttest increased stated “we should do the activities first” and believed having the “chance to think about it and figure it out on our own” helped with understanding (see Figure 4.3).

As stated previously, the difference in posttest conceptual subscores between classes was not statistically significant. However, there were many students in the treatment classes whose conceptual subscore from pretest to posttest improved. This increase in their conceptual subscores was due mainly to an improvement on part two. Most students commented that with the applets students not only could “see how the graph changes and compare it to the main graph” but students could “see a lot of different graphs quickly and can compare them a lot.” Without the use of applets, students are limited not only by how many different pairs of graphs they can compare based on what
their teacher can model for them but also by the fact that the models are not dynamic in most instances. According to the teachers participating the study, functions that they model for students are graphed either “on the graphing calculator or the whiteboard.”

3. Graph \( f(x) = 2x^2 - 3 \)

Figure 4.3 Part 1 #3 (Pretest on top, posttest on bottom)

Another factor related to the use of applets that students believed helped them understand was having “time to think about it – to think about what’s happening, to think
about why it’s happening” along with the fact that students “could control it.” Using the applet activities to teach parameter changes to parent functions gave students the opportunity to “figure it out before the teacher tells you.” The students the researcher interviewed believed that “whenever you have to think and figure something out on your own you remember it more than just having somebody just flat out tell you.” Some students, mainly those whose scores increased from pretest to posttest, discussed a what-if scenario that they used while completing the applet activities. The applet activities gave students “a lot of possibilities to see what if, what’s going to happen.” Although the activity sheets did not require students to do this, some students “made up a little theory” and tested it by “thinking about what might happen before I changed the value with the slider.” As one student said, “I’d think if $a$ were 5 this is what the graph looks like, then I could change $a$ to 5 and see.” The ability to control the applet along with having time to think about the effect the parameter has on the function and to have the opportunity to ask what-if questions helped students understand more conceptually.

Based on responses from student interviews, the students whose conceptual understanding improved from pretest to posttest were able to “pull out how it works and why it works” as evidenced by these comments. When asked about how he got problems 1 – 3 on part three of the posttest correct one student responded, “It’s not really different because it’s not quadratic, you still do it the same way” and was able to explain how he knew to change the $y$ coordinates. Another student shared “you had to realize it’s the distance that’s changing, when you realize that, all you have to do is apply the distance idea.” This student went on to explain that with functions of the form $g(x) = f(x) + k$ there is a “set distance between points with the same $x$-coordinate” on the two functions and
that with functions of the form \( g(x) = a \cdot f(x) \) the “distance from the \( x \) to points on the functions with the same \( x \)-coordinate” is such that “one distance is \( a \) times the other”. This student understands that “the distance idea would apply to all graphs.” Using the applets students “could see the \( x \) and \( y \) values and it shows what happens to the \( y \) value.” Another student explained, “I get that if you plug in a number for \( x \), the \( y \) values of \( f \) and \( g \) are different” and “if you keep on moving the point, the pattern stays the same like plus five or times two.” Another student stated it more succinctly by saying, when you change the parameter \( a \) in the function \( g(x) = a \cdot f(x) \) or the parameter \( k \) in the function \( g(x) = f(x) + k \) “that means \( y \) changes.”

Critical to students understanding parameter changes to parent functions conceptually is not only knowing what effect the parameter has on the function but also knowing why it has that effect. As stated earlier, some students were able to come to this understanding after working with applets that “let us change the graphs, let us see how the function changes, let us see what the coordinates are, how they are different from each other.” These students were able to make connections between the representations displayed on the applet as evidenced by this comment, “you can change it, numbers like \( a \) and \( k \), and then you see how the graph is changing, and on the chart you can see how the points are changing, and how the equation is changing” and it is “based on \( a \) or \( k \).”

**Second Research Question.** *From the teacher’s perspective, to what extent does the use of single purpose applications impact a student’s transition from procedural understanding to conceptual understanding of parameter changes to parent functions?*

As stated previously, although the students in the treatment classes \((M = .79, SD = .35)\) had higher conceptual subscores on the posttest than students in the control classes \((M = \)
.77, \(SD = .37\), the difference was not statistically significant. Both teachers participating in the study thought students would gain “a better conceptual understanding using the applets.” Teachers know that in order for students to learn, students must be engaged. The graphs, equations, tables, coordinates, and distances within the applet change dynamically as the slider is changed and “because it’s quick, it gets their attention.” Teaching parameter changes to parent functions with a graphing calculator means “you put in one graph, look at it, put in the next graph, look at it.” Using the graphing calculator is “a slow process” as compared to using the applets where you can see as many graphs as you want “quickly and easily.” Not only does the applet being “dynamic and quicker get their attention” but it “keeps their attention.” Teaching with the graphing calculator, “showing them a little bit at a time, some of them start getting distracted.” Drawing graphs on a whiteboard is as slow a process, if not slower, as using a graphing calculator to teach parameter changes to parent functions. With the applet, the “graphs change automatically” and on the whiteboard, the teacher would have to “change it and draw it” which takes a great deal of time as compared to the applets.

Another aspect of the interactive applets that the participating teachers believed helped their students with conceptual understanding was the “dynamically connected multiple representations.” In using graphing calculators to teach parameter changes to parent functions, teachers are limited with showing students or having students work with two representations at a time. With the interactive applets, students can see and manipulate the graph and points on the graph, equation, table, and distances that are labeled on the graph. Being able to “see all of the representations and how they change as the parameter is changed” helps students transition from procedural to conceptual
understanding. Students can see how changing the parameter $k$ to three changes the
graph, points labeled on the graph, equation, table, and distances labeled on the graph. By
“looking at the whole screen to see everything” students gain an understanding of “what
changes and why.” By completing the applet activities, students have the opportunity to
“make sense of the representations, how they are connected, and how they are affected by
the parameter changes.” Students would see that if you change the parameter $a$ or $k$ that
“it’s not just the graph that changes, everything changes – the equation, the table, the
points.” Both teachers agreed that this is difficult to make evident for students with a
graphing calculator or whiteboard.

One of the participating teachers noted that when teaching parameter changes to
parent functions “I do a lot of modeling” and using the applets would “save time.”
Although our focus here is not one how using applets helps the teacher, students could
see many examples graphed on the applet rather than modeled by the teacher and this
saves learning time for the students. “The ease of looking at different graphs” is an aspect
of the applet that makes conceptual understanding possible. According to one
participating teacher, “a picture is worth a thousand words and this applet gives you
many, many pictures, and pictures help students go from procedural to conceptual
understanding.” Although it is possible to graph quadratic functions with a graphing
calculator or on a whiteboard, but it is impossible to graph as many different quadratic
functions as is possible with the applets.

Using the applets to teach parameter changes to parent functions helps the
students that “need more than one example” to understand. Both participating teachers
agree that students that need more time and more examples benefit greatly from using
applets. Also, since it is “hands-on and they did it, they will remember it more.” Being able to do the applet activities on their own, students “feel more successful and more confident” than if the teacher had told them what they needed to know.

The participating teachers agreed that most of their students are more interested in knowing procedures than understanding concepts. With respect to parameter changes to parent functions, some students have basically memorized rules for how parameters affect the graphs, and in some instances students have different rules for different functions. For example, changing the parameter $a$ in a linear function $g(x) = a*x + b$ makes the graph more flat or more steep, but changing the same parameter $a$ in a quadratic function makes the graph more wide or more narrow. Both teachers discussed how labeling the distances $FA$ and $GA$ on the graph could help students get away from the idea of that changing the parameter $a$ makes the quadratic function more wide or more narrow. Also, the participating teachers believed that focusing students’ attention on the $y$-coordinates when completing tables on the activity sheets possibly helped students answer the questions dealing with parameter changes to nonstandard graphs.

**Third Research Question.** From the student’s perspective, what additional insights on parameter changes to parent functions are gained by using single purpose applications? To what does the student attribute the single purpose applications providing additional insights? Students whose conceptual subscores improved from pretest to posttest indicated the realization that the parameter “changes were the same for all functions” was an insight gained from using the applets and completing the activities. This realization came from the focus on “the $y$ values” as the $x$ was changed using the slider along with the “distance” idea. During their interviews, many of these students
recalled how their Algebra 1 teacher taught parameter changes to a linear functions and quadratic functions differently. These students noted that they learned rules related to both. For linear, “bigger, positive number is steeper” and “smaller, positive number is flatter.” For quadratic, “bigger is narrower” and “smaller is wider.” From these responses, students have indicated that they were taught rules for how parameters affect a graph, but not in general. These students were taught separate rules for different functions. After using the applets and completing the activities, the students realized that the transformations were the same regardless of what the parent function was as evidenced by their answers to problem 12 on part three of the posttest.

Students whose conceptual understanding improved from pretest to posttest were able to see not only “how it works” but “why it works”, but these students are not the only students who gained additional insights from using the applet activities. Students whose scores from pretest to posttest decreased or did not change also gained additional insights. However, additional insights gained for these students were very different from insights gained by students whose scores increased. Many of the students whose scores decreased or did not change stated that in learning mathematics they typically tried to “memorize rules or steps” to get answers. These students, whose scores decreased or did not change, said that even though mathematics was hard for them, they “liked trying to figure it out using the applets” and believed they “would remember it better.” These students also believed that if they ever forgot what they learned about parameter changes to parent functions they “would know how to figure it out again.” These students pointed out that after using the applets and working through the activities that they knew they “could always find the different y’s that go with x.” By using the slider for x and looking
at the table, students were able to see that “if you plug in a number for $x$, the $y$ values of $f$ and $g$ are different.” Also, by using the slider and looking at the graph, students were able to see “the $y$’s change.” Although these students do not have posttest scores to show anything was gained from using the applets, their comments indicate otherwise.

**Additional Findings.** Nearly every student interviewed believed that working through the applet activities helped them understand parameter changes to parent functions although 10.2% of students in the treatment classes had scores that decreased from pretest to posttest and 6.1% had scores that remained the same. Also, students believed that the use of the applets would help them “remember things more” since they had “figured it out” on their own. As one student put it, “I remembered those two little bars when I was answering this question” (see Figure 4.4)

Students stated that not only were they motivated and engaged when completing the applet activities, but “everyone seemed to be engaged and having fun.” Students talked about how “difficult it is to stay focused when just listening to someone talk.” One student commented, “I stay more focused when I’m looking at the computer because I want to see what the difference is and I want to figure it out.” Another student added, “The questions on the activity sheet that asked what you noticed kept you from just clicking and filling in the blanks.” Although every student interviewed said they were motivated and engaged and enjoyed working with the applets on the computer, they agreed “doing this too much would make it just like working on paper” and then “it’s nothing special.”

The researcher was interested in what students believed would be the best way to incorporate interactive applets into their mathematics classes. When asked by the
researcher about three possible teaching scenarios, each student had a definitive response. The first scenario for teaching parameter changes to parent functions involved the teacher teaching the content first and then students completing the applet activities. The second scenario involved the teacher introducing the content first and then students completing the applet activities. The third scenario involved students doing the applet activities first and then the teacher reviewing and clarifying what students found. Three of the 15 students interviewed preferred the first teaching scenario. These three students believed that “it was hard because you’re there by yourself” and that “it would have been a lot

Using the graph of \( f(x) \) shown below, graph \( g(x) \) in problem 2.

![Graph of f(x)](image)

2. Graph \( g(x) = 0.5f(x) \).

![Graph of g(x)](image)

Figure 4.4 Part 2 #2 (pretest on the left, posttest on the right)
“easier” if the teacher had “taught it first.” Rather than “go into the activity blindfolded basically”, students “would have known” what they were doing when completing the applet activities if the teacher taught the content first. “Teach us and let us comprehend what was taught using the computer.” Five of the 15 students interviewed preferred the second teaching scenario. These five students believed that “if the teacher introduced it first” students would “get an idea of what this is going to be about” since the teacher would be laying “all the groundwork.” These students, though, wanted “time to think about it” and “time to work on the computer” since the applets “show you how everything works.” Seven of the 15 students interviewed preferred the third teaching scenario. These seven students believed that they would “understand better” if they had time to “use the computer to figure it out on their own at their own pace.” Then the teacher could teach to make sure students learned “everything they need to know.”

All students who were interviewed commented on the importance of “being able to see what changes” and “having time to think about it.” Visualization has been identified as a key aspect in the teaching and learning of mathematics. The students who were interviewed in this study reiterated the point. Teachers also know how important it is to give students time to think. With the applet activities, not only were students given time to think, they were given time to interact with the applet, look for patterns, and to make sense of the mathematics on their own.

**Summary of Results**

The purpose of this sequential explanatory mixed methods study was to examine the impact of interactive applets on students’ understanding of parameter changes to parent functions. Students in the treatment classes were found to have statistically
significantly higher posttest scores than students in the control classes. Although the data analysis showed a statistically significant difference between classes on procedural understanding, no statistically significant difference was found with regard to conceptual understanding. The participating teachers attributed students’ understanding mainly to the dynamically connected representations that helped students see the effect that the parameter change had on the different representations. Additional insights gained by students included: (a) an understanding that parameter changes are the same regardless of the function, and (b) a realization that they benefit from having the opportunity and time to not only think about mathematics but also to attempt to figure things out on their own.
Chapter V: Discussion, Conclusions, and Implications

Given the emphasis placed on using technology in the teaching and learning of mathematics at the state and national level, the need exists to gain more insight into how technology can be used to positively impact student understanding. This study examined how the use of single purpose applications such as interactive applets impacted students’ understanding of parameter changes to parent functions and whether the use helped students transition from procedural to conceptual understanding. The results of the study, although not generalizable to the population of all high school mathematics students, indicate that the use of interactive applets had a positive impact on students’ understanding of parameter changes to parent functions as evidenced by the difference in scores from pretest to posttest between treatment and control classes. This difference was not due to increased conceptual understanding—although students in the treatment classes did improve their conceptual understanding—but rather it was due to increased procedural understanding. Students whose scores increased, decreased, or did not change from pretest to posttest, indicated several aspects related to the applets and completing the applet activities that helped them understand parameter changes to parent functions. These aspects included the dynamic representations displayed on the applets that can be controlled and manipulated by the student and the time activities like this afforded students to think and figure things out on their own.

Discussion of Results

Overall Impact. After controlling for differences in treatment and control classes using the pretest scores as the covariate, treatment classes in this study were shown to have statistically significantly higher posttest scores than students in the control classes.
Other studies have shown that the use of appropriate technology positively impacts students’ understanding (Currie, 2006; Ellington, 2003; Ford, 2008; Harskamp et. al., 2000; Hollar & Norwood, 1999; Reimer & Moyer, 2005). These findings, although important to mathematics education, have limited value if one does not examine why the use of technology had an overall positive impact. During the qualitative phase of this study, the researcher investigated why the use of applets had a positive overall impact by having students identify aspects of the applets that they believed assisted them with understanding. Aspects of the applets that students identified included the following: (a) the ability to control the manipulation of objects with the slider, (b) being able to see how all the representations change as the slider is manipulated, and (c) having coordinates and distances labeled on the graph. Another aspect that students identified is one that is not built into the applet but is directly related to the applet activities in this study and that was having time to think and make sense of the mathematics on their own.

**Conceptual Versus Procedural.** Although the researcher expected the use of interactive applets to help students in treatment classes gain a better conceptual understanding than students in control classes, the findings of this study indicate the difference was not statistically significant. Past studies have indicated that some teachers are reluctant to use technology to teach mathematics due to the concern that students will lose procedural fluency if they are allowed to use technology (Schmidt & Callahan, 1992; Drier, 2001). The results of this study suggest that students can improve their procedural understanding by using interactive applets as evidenced by the statistically significant difference on posttest procedural subscores for the treatment and control classes. This was an unexpected result but one the researcher believes can have a long-term effect on
the implementation of technology in the mathematics classrooms since teachers are concerned that the use of technology could potentially lead to students losing procedural fluency and understanding.

Several studies have focused on the how students learned parameter changes to parent functions. These studies indicate that students not only memorize rules for transformations but in many instances are not able to correctly recall their rules when needing to do a transformation (Kimani, 2008; Ninness et al., 2005; Zazkis et al., 2003). For example, when a student is asked to graph a function like $g(x) = f(x) + 5$ given the graph of $f(x)$ or when a student is asked to state how the graph of $g(x) = x^2 + 5$ is a transformation of the parent function $f(x) = x^2$, results from previous studies have shown that students try to recall rules. Students are thinking about whether the plus 5 means up five or down five or left five or right five. By having students do applet activities like the ones in this study, even those students who have only a procedural understanding will have a way to rediscover the rules if they happen to forget them as many students indicated during their interviews.

In reflecting on interviews with students whose scores from pretest to posttest decreased or did not change, the researcher was surprised at the number of students who could explain how the graph of the function $y = x^2$ would change if they had to graph the function $y = x^2 + 3$ but could not explain how the graph of the function $f(x)$ would change on the first problem on part two of the posttest if they had to graph $g(x) = f(x) + 3$ (see Figure 5. 1). To address the problem of students not understanding that a particular parameter change affects the graph of any function the same way it affects the graph of a quadratic function, the researcher suggests adding different parent functions to the applet
along with the graphs of nonstandard functions so that students will have the opportunity to see that a particular parameter change affects all graphs the same way.

![Graph](image)

Using the graph of \( f(x) \) shown below, graph \( g(x) \) in problems 1.

1. Graph \( g(x) = f(x) + 3 \).

Figure 5.1 Part 2 #1 (pretest on the left, posttest on the right)

**Positive, Negative, and Nonexistent Impact.** As stated previously, students in all three groups – positive impact, negative impact, and nonexistent impact – believed that the use of the applet activities helped them learn and understand parameter changes to parent functions. A critical question then is, what made the outcome on posttest scores
different for students in the three groups? Although students in all three groups shared
during their interviews that they could “see the pattern of what was happening” and that it
made a difference that they could control the manipulation of objects on the applet using
the slider, there were several key differences between students whose scored increased
from pretest to posttest as compared to students whose scored decreased or did not change.

Students whose scores increased from pretest to posttest utilized the multiple
representations provided on the applet while exploring parameter changes to parent
functions. These students noticed how color was used to distinguish one function from
the other (blue equation went with the blue graph, black equation went with the black
graph). These students also noted that coordinates of points were labeled, as were
distances (distance between the two points, distance each point was from the \( x \)-axis). By
utilizing the various representations and the attributes labeled on the graphical
representation, students whose scores increased from pretest to posttest had the means to
not only see what was happening but to also understand why. Students whose scores from
pretest to posttest decreased or did not change did not mention the other representations
or the use of color. When asked specifically about the table, students whose scores
decreased or did not change replied that they “did not see the table.”

Students whose scores increased from pretest to posttest explored parameter
changes to parent functions more in depth than the activity sheet required. These students
thought through what-if scenarios. Students would think about what would happen if they
changed the parameter to a certain value. Many described “seeing the graph” mentally.
Then these students would check to see if their thinking was correct by actually changing
the value of the parameter with the slider. By repeating the what-if process, these students were able to not only strengthen their understanding but also to build confidence. After testing several what-if scenarios, these students were confident that their thinking was correct. Students, whose scores from pretest to posttest decreased or did not change, did not think through what-if scenarios. These students limited their exploration of parameter changes to parent functions by only changing the parameters as specified on the activity sheet.

The fact that these groups used the applets differently and noticed different things on the applets made a difference in how scores changed from pretest to posttest. The question that the researcher must answer is, What can be done to alleviate this difference? The researcher believes that many of the students whose scores increased from pretest to posttest are naturally inquisitive and thinking through what-if scenarios is quite instinctive for them. This is not the case for the students whose scores decreased or did not change from pretest to posttest. The researcher is confident that the students whose scores did not increase from pretest to posttest can be taught to use what-if scenarios to make sense of mathematics. By allowing students to do more discovery learning activities similar to the applet activities in this study, students will begin to develop the what-if way of thinking. The activity sheets should be changed, though, to incorporate more questions that lend themselves to students needing to think what-if. The activity sheets, in their current state, have students set the slider to a certain value and then complete a table, and this is repeated three times for three different values for the parameter. The last question on the activity sheet is a what-if question. The researcher believes that the more students have the opportunity to discover mathematical
relationships with activities such as the applet activities used in this study as long as more what-if questions are added to the activity sheets, the more students should develop the what-if strategy for making sense of mathematics. Thus, this difference between the groups can be eliminated.

The other difference that existed between the groups pertained to what students in each group noticed on the applet. With this study, students were expected to complete the applet activities on their own with no teacher input. Inevitably, students noticed different things on the applets. The solution for eliminating this difference that existed between the groups is quite simple. Before students begin the applet activities, teachers should show students the different representations and discuss the use of color and labeling. This will ensure that students begin the activities knowing exactly what they have available to them to use for making sense of the mathematics. The researcher believes had students whose scores did not increase from pretest to posttest actually utilized more of the representations and the attributes labeled on the graphical representation, these students would have gained a better understanding of parameter changes to parent functions, which would have been evidenced by scores that increased from pretest to posttest.

**Beliefs and Attitudes.** Students in all three groups – positive impact, negative impact, and nonexistent impact – believed that the use of the applets activities helped them learn and understand parameter changes to parent functions. Not all of these students showed improvement from pretest to posttest. Had the researcher only conducted a qualitative study, the quantitative evidence to the contrary would not have existed. Students in all three groups were “engaged and having fun” while working with the applets. Other studies have shown that students believed the use of technology made the
learning experience enjoyable and had a positive effect on their attitude about mathematics (Drickey, 2006; Ellington, 2003; Gadowsky, 2001; Hannafin et. al., 2001; Mohd et. al., 2008; Reimer & Moyer, 2005). Students in all three groups believed that “having time to think” about the mathematics was an important part of the learning process for them.

**The Case of Mary.** Although there were several students whose scores dramatically increased from pretest to posttest, Mary (pseudonym) was the student who stood out from the rest. Not only was her increase in score from pretest to posttest impressive, but her explanations of how and why she thought the applets helped her with understanding were explicit. Mary stated at the beginning of her interview that she “had never been good at graphing.” The researcher found that surprising given the impressive posttest score that came from what Mary was able to glean on her own using the applets. After completing the applet activities, Mary believed that “for the first time” she “understood how to graph a function.” As evidence by her posttest score, she was correct. Mary’s score on part one of the pretest was 1 but her score on part one of the posttest was 24. The maximum score possible for part one was 32 points.

The first thing Mary talked about was how the questions on the activity sheet helped her to “know what to focus on” when using the applet. For example, number four on the activity sheet (see Appendix C and D) has students use the slider for $x$ to change the $x$-coordinate of the corresponding points on $f(x)$ and $g(x)$. Students are expected to complete the table on the activity sheet, which can be done using the table provided on the screen or using the coordinates that are labeled on the graph. Mary stated that she “did not use the table on the computer” although that would have been an easy way to
complete the table on the activity sheet. Rather she used the “points F and G” labeled on
the graphs whose coordinates dynamically changed as she changed the value of the x-
coordinate using the slider for x. Mary said she focused on how each “x value got a y
value using the function.” Mary realized all she “needed to do to graph was to find y’s to
go with x’s.” This was Mary’s path to procedural understanding using the applets. From
the researcher’s perspective, this was a significant point to make since Mary was now
able to precisely graph functions by plotting points that before seemed completely foreign
to her (see Figure 5.2). Mary stated that in answering questions on the posttest, she
thought back to “using the slider and finding different y’s.” Another example of how the
questions on the activity sheet helped her to “know what to focus on” when using the
applets was questions two and three. While changing the specific parameter using the
slider, Mary looked to see what was “changing in the equation” of $g(x)$ and how this
related to the y-coordinates for points F and G.

Since Mary’s interview came early in the interview process, the researcher was
able to ask other students specifically what they used on the applet to create the table on
number four. Most of the students interviewed stated that they used the table that was
provided on the applet. Although some of these students were as precise as Mary in
plotting points to graph functions, most were not. This raises an interesting question for
the researcher – Should the table be included on the applet? Although the researcher
believes that students need to see the algebraic, graphical, and tabular representations so
they can make connections between the representations, it is possible, based on Mary’s
case, that students gain more procedural understanding if the table is not included on the
applet (see Figure 5.3).
Mary epitomizes everything this study is about for the researcher. She is a student that has difficulty at times learning mathematics. By using the applets, though, she was not only able to gain a better understanding of something she said she “had never been good at” before but was also able to gain some confidence in her ability to do mathematics. Not only do mathematics teachers want their students to learn the mathematics being taught but they also want their students to be confident in their
capability to do mathematics. Without this, mathematics will potentially act as a
gatekeeper for students, limiting their future educational trajectory.

11. Graph \( y = ax^2 + 1 \) for \( a = \frac{1}{2}, a = 1, a = 1\frac{1}{2} \) and \( a = 2 \) by completing the tables
below. How does each new graph compare to the previous graph as you increase
the value of \( a \) from \( \frac{1}{2} \) to 1 to \( 1\frac{1}{2} \) and finally to 2?

Figure 5.3 Part 4 #11 (Pretest on top, posttest on bottom)
**Implementation and Utilization.** Although the purpose of the study was to examine the impact – positive, negative, or nonexistent – the use of single purpose applications such as interactive applets had on students conceptual understanding of parameter changes to parent functions and to explain why, the researcher hoped that the results of the study would provide valuable insight into effective and ineffective ways to use technology to teach mathematics. The participating teachers and students who were interviewed shared their beliefs on the best way to use applets in the mathematics classroom.

Although both participating teachers thought the “applets were great”, they shared ideas about alternative ways to utilize them in the classroom. One teacher suggested that the applets should be used as discovery learning activities like they were used in the study but added one exception. The one change is that teachers should “show students how to use the sliders first” and also “point out what all students are given on the applet.”

Based on teacher and student interviews, there were a few students who struggled to start the activities. By showing students what they have on the applet and how to use it, this problem would be alleviated. Students’ time would be spent thinking about the mathematics rather than spend their time trying to figure out how to use the applet. The focus should be on the mathematics and not the technology.

The other participating teacher believes that students need to “understand procedures before they can understand concepts”, and as such suggested that the teacher teach the content first and then have students complete the applet activities. This would enable students to have an opportunity to clearly “see what the teacher was talking about.” The use of the applet activities after instruction would provide clarity to students’
understanding. This teacher also suggested that some students would benefit from completing the applet activities first and then receiving instruction the teacher. The use of instruction after completing the applet activities would also provide clarity to students’ understanding but the difference with this is students are adding clarity to what they discovered on their own.

Both participating teachers believed that the applets on parameter changes to parent functions could be used by the teacher for demonstration purposes only. One of the teachers pointed out that the applet could be manipulated by the teacher on the Smartboard to show students how changing the parameter affects the graph. How do students benefit from seeing it on the Smartboard as compared to seeing it and manipulating it own their own computer? Students benefit from the demonstration using the applets because they can see many more graphs than the teacher could draw or display with a graphing calculators. However, students watching as someone else manipulates the applet is not as effective as students manipulating it themselves. As several students commented, they “learn more by doing it” themselves. Given the availability of laptop carts and computer labs at the participating high schools, though, using the applets as demonstrations is a viable option to not using them at all.

Students suggested that having the teacher interact more with students while they are completing the activities would be beneficial to students. Each student would have their own computer to use in completing the applet activities but would also have a partner to work with while completing the activities. Partners could help one another in the following ways: (a) assist with starting the activity, (b) assist with the technology, (c) discuss their observations, and (d) explain their reasoning. Partners will have the
opportunity to examine “what’s different and what’s the same” in their observations. By having students work with a partner to complete applet activities like the ones used in this study, not only do students have the opportunity to discover mathematical properties and relationships on their own but they also have the opportunity to discuss mathematics with another person. High school mathematics teachers, who feel that “students are passive” in their approach to learning mathematics, should embrace utilizing applet activities in their classrooms. As a former high school mathematics teacher, the researcher wanted nothing more than to actively engage students in learning mathematics. Applet activities are a means to that end.

Another factor to consider when implementing the use of interactive applets into the mathematics classroom is how often should these activities be used. Students at both participating high schools liked using the applets and “it was fun.” Teachers thought the applet activities were a “break from lecturing” for the students. The researcher questions whether applet activities could be used too often. Is it possible that applet activities could be used so often that students become as bored with them as they are worksheets?

Conclusions and Future Implications

This study, although narrow in instructional focus, delineated aspects of interactive applets that students deem helpful with understanding the mathematics. The results of this study also showed that the use of technology did not cause students to lose procedural understanding. Also, participating students and teachers felt that with using the applets, students could “see a lot of different graphs quickly and can compare them a lot.” What does this mean for mathematics education? More specifically, what does this mean for high school mathematics teachers? The two excuses that most high school
mathematics teachers give as reasons why they do not use technology are that using technology to teach mathematics takes too much time and students will not understand procedures. This study suggests that these two excuses can be eliminated, thus making way for teachers to incorporate activities similar to the ones in this study in their classrooms. Since technology use in the teaching and learning of mathematics is not as widespread as mathematics educators would hope, the results of this study should help mathematics teachers see how technology can be used on a small-scale basis to impact student understanding. To avoid overwhelming teachers which results in no implementation, the implementation of technology such as interactive applets into the mathematics classroom needs to start small-scale with a few applets each six weeks or each unit. The researcher believes that similar studies need to be conducted to identify other ways that applets can be used effectively to teach mathematics and to identify other key aspects that assist students with learning and understanding. Also, the researcher believes that many teachers are reluctant to commit to using a technology intervention for the duration of their course, but those same teachers might be more willing to use technology to teach certain concepts.

Since these interactive applets can be made available to students and utilized outside of class as long as students have internet access, it is possible that students will spend even more time exploring mathematical concepts than the teacher allowed in class to complete applet activities. The students in this study indicated that having “time to think about it – to think about what’s happening” was important. How many times are students told what they need to know without being given the opportunity to think about it first? Many mathematics teachers complain that their students do not retain what they
supposedly learned from prior classes. How many memorized rules and procedures can students retain from year to year? If a student believes that they “learn better when I have to do it”, why should they not be given the opportunity to discover some things on their own? As students in this study noted, if they happened to forget what they discovered using the applets, they would at least have a means to rediscover it, whereas, if they memorized what the teacher told them and then happened to forget it, there would be no way to retrieve the information.

Based on the results of this study, the researcher believes similar studies are needed. Mathematics educators need to examine the impact of interactive applets on the same small-scale as this study but with concepts from other high school courses such as Algebra 1, Geometry, Precalculus, and Calculus. These studies should also have a narrow instructional focus similar to this study. The results of these studies could not only mean more effective uses of interactive applets to teach specific concepts in other high school mathematics courses but could also be the impetus to widespread use of interactive applets in high school mathematics classes. The researcher believes that technology implementation and use on a smaller-scale will eventually lead to implementation and use on a larger-scale. The researcher believes that for effective large-scale implementation of technology to take place, teachers must start with implementation and use on a much smaller scale such as using interactive applets to teach several concepts in their classes.

The researcher is also interested in conducting similar studies on using applets to teach parameter changes to parent functions examining the impact of the use of applets has on students’ understanding for specific student groups. These groups include ethnic groups, special education students, students on free and reduced lunch, students in other
mathematics courses that cover parameter changes to parent functions, and students in PreAP and regular mathematics classes. The following research questions are of interest to the researcher:

1. To what extent does the use of single purpose applications for teaching parameter changes to parent functions impact a student’s transition from procedural understanding to conceptual understanding for specific students groups including male, female, African-American, Caucasian, Hispanic, special education, students with limited English proficiency, and students on free and reduced lunch?

2. To what extent does the use of single purpose applications for teaching parameter changes to parent functions impact a student’s transition from procedural understanding to conceptual understanding for specific courses (Algebra 1, Algebra 2, and Precalculus) and course levels (PreAP and regular)?

With the questions listed above, the researcher would also like to examine whether the students improve their procedural understanding as well as what the impact would be if students worked with a partner. This involves many studies, but each would contribute to mathematics educators’ understanding of how to utilize technology effectively to meet the needs of students.

The researcher believes that many teachers, who would be hesitant to use applet activities similar to the ones used in the study, would use the applets for demonstration purposes. As such, there is a need to conduct a similar study where the treatment classes use the applet activities for discovery learning and in the control classes the teacher uses the applets for demonstration purposes only. The students in this study believed that “watching the teacher” demonstrate the changes with the applet would not have been as
beneficial as “interacting with it and changing it” themselves. As one student simply stated, “I learn by doing, not by listening.” Thus, it is important to determine whether applets are most effective when used for discovery learning or demonstration. The researcher believes teachers will be more apt to use the applets for demonstration purposes if evidence of a better way to use them is not provided.

For the researcher, this study is a small step in a never-ending journey to find effective ways to utilize technology in the teaching and learning of mathematics. It is the hope of the researcher that this research and similar research will impact the implementation and utilization of technology in mathematics classrooms. Technology has the potential to enhance not only how teachers teach mathematics but also how students learn mathematics. Students over the years have perceived that the focus of learning mathematics was memorizing rules and following procedures to get answers to problems. With technology, students can explore mathematical relationships on their own, reason mathematically about what they observe, make sense of mathematical situations, and have the opportunity to gain a true perspective of what learning mathematics is all about. This is the mathematics classroom that NCTM envisioned, and with similar research mathematics educators can continue to take steps in making this vision a reality for all students.
APPENDIX A

Pretest/Posttest - Parameter Changes to Parent Functions

Part 1

1. Graph \( f(x) = x^2 - 4 \)

2. Graph \( y = 0.5x^2 \)
3. Graph \( f(x) = 2x^2 - 3 \)

4. Complete the table for \( g(x) \) given \( g(x) = 4x^2 + 2 \).

<table>
<thead>
<tr>
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<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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</tbody>
</table>

<table>
<thead>
<tr>
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<th>( g(x) )</th>
</tr>
</thead>
<tbody>
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</tr>
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<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

5. Given the graphs of \( f(x) \) and \( g(x) \) below, write and equation for \( g(x) \) in terms of \( f(x) \) if only a translation is used.
6. Given the graphs of \( f(x) \) and \( g(x) \) below, write and equation for \( f(x) \) in terms of \( g(x) \) if only a translation is used.

\[ f(x) = g(x) \]

7. Given the graph of \( f(x) = x^2 \), graph \( g(x) = 3x^2 \).
8. Given the graph of \( y = x^2 \), graph \( y = \frac{5}{3}x^2 \).
Part 2

Using the graph of \( f(x) \) shown below, graph \( g(x) \) on problems 1 – 3.

1. Graph \( g(x) = f(x) + 3 \).
2. Graph \( g(x) = .5f(x) \).

3. Graph \( g(x) = 2f(x) + 1 \).
4. Complete the table for \( g(x) \) given \( g(x) = 4f(x) - 3 \).

<table>
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<tr>
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<td>-6</td>
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<table>
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<tr>
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</tr>
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<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

5. Given the graphs of \( f(x) \) and \( g(x) \) shown below, write an equation for \( g(x) \) in terms of \( f(x) \).
6. Given the graphs of \( f(x) \) and \( g(x) \) shown below, write and equation for \( f(x) \) in terms of \( g(x) \).

7. Given the graph of \( f(x) = x^3 \), graph \( g(x) = .25x^3 \).
8. Given the graph of \( f(x) = x^3 \), graph \( g(x) = x^3 - 5 \).
Part 3

9. Describe how the graph of \( y - 6 = x^2 \) relates to the graph of \( y = x^2 \) in terms of transformations and explain why.

10. Describe how the graph of \( y = 4x^2 \) relates to the graph of \( y = x^2 \) in terms of transformations and explain why.

11. You graph a family of functions of the form \( y = ax^2 + 1 \). How does each new graph compare to the previous graph as you increase the value of \( a \) from \( \frac{1}{2} \) to 1 to \( 1 \frac{1}{2} \) and finally to 2? Explain why.

12. Given the graph of \( f(x) = x \) and the graph of \( g(x) = 3x \) shown below. Explain how the graph of \( g(x) = 3x \) relates to the graph of \( f(x) = x \). Is this different from how the graphs of \( f(x) = x^2 \) and \( g(x) = 3x^2 \) relate? If yes, explain how it is different? If no, explain how it is the same.

![Graph of f(x) = x and g(x) = 3x](image)
Part 4 – To be completed after taking up parts 1 - 3

9. Graph \( y - 6 = x^2 \) and \( y = x^2 \) by completing the tables below. Describe how the graph of \( y - 6 = x^2 \) relates to the graph of \( y = x^2 \) in terms of transformations.

10. Graph \( y = 4x^2 \) and \( y = x^2 \) by completing the tables below. Describe how the graph of \( y = 4x^2 \) relates to the graph of \( y = x^2 \) in terms of transformations.
11. Graph \( y = ax^2 + 1 \) for \( a = \frac{1}{2}, a = 1, a = 1 \frac{1}{2} \) and \( a = 2 \) by completing the tables below. How does each new graph compare to the previous graph as you increase the value of \( a \) from \( \frac{1}{2} \) to 1 to 1 \( \frac{1}{2} \) and finally to 2?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td>-2</td>
<td></td>
</tr>
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<td>-1</td>
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<tr>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

12. Given the graphs of \( f(x) = x \) and \( g(x) = 3x \) shown below. Complete the tables below to graph the functions \( f(x) = x^2 \) and \( g(x) = 3x^2 \). Explain how the graph of \( g(x) = 3x \) relates to the graph of \( f(x) = x \). Is this different from how \( f(x) = x^2 \) and \( g(x) = 3x^2 \) relate or is this the same?
**APPENDIX B**

Rubric for Pretest and Posttest

<table>
<thead>
<tr>
<th>Rubric for Pretest/Posttest Questions</th>
<th>Procedural Understanding – Parts 1 and 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 points</td>
<td>The answer is not correct.</td>
</tr>
<tr>
<td>1 – 2 points</td>
<td>The answer is partially correct.</td>
</tr>
<tr>
<td></td>
<td>Student does not show procedures. (1)</td>
</tr>
<tr>
<td></td>
<td>Student shows procedures. (2)</td>
</tr>
<tr>
<td>3 – 4 points</td>
<td>The answer is correct.</td>
</tr>
<tr>
<td></td>
<td>Procedures are not shown. (3)</td>
</tr>
<tr>
<td></td>
<td>Procedures are shown. (4)</td>
</tr>
<tr>
<td>/4 points</td>
<td>TOTAL POINTS FOR PROCEDURAL UNDERSTANDING</td>
</tr>
</tbody>
</table>

|                                      | Conceptual Understanding – Parts 2 and 3 |
| 0 points                             | The answer is not correct.               |
| 1 – 2 points                         | The answer is partially correct.         |
|                                      | Student does not state/describe transformation(s). (1) |
|                                      | Student states/describes transformation(s). (2) |
| 3 – 4 points                         | The answer is correct.                   |
|                                      | Student does not state/describe transformation(s). (3) |
|                                      | Student states/describes transformation(s). (4) |
| /4 points                            | TOTAL POINTS FOR CONCEPTUAL UNDERSTANDING|
| /8 points                            | TOTAL POINTS FOR THE PRETEST/POSTTEST QUESTION |
APPENDIX C

Parameter Changes to Parent Functions – Applet Activity #1

Please follow instructions carefully in completing the guided exploration.

1. Go to www.geogebratube.org/student/m19019. Click on the “Go to student worksheet” link. The applet should be titled Parameter Changes to Parent Functions #1.
2. You will see two graphs on the screen – the parent quadratic f(x) = x^2 and the transformed function g(x) = x^2 + k. The graph of f(x) and the equation for f(x) are in black while the graph and equation for g(x) are in blue. There is a table at the right that will allow you to see how the ordered pairs compare.
3. Use the slider labeled “k” to change the value of k in the function g(x) = x^2 + k. As you change the value of k, what do you notice about the graph of g(x)?
4. Use the slider for “k” to set k = 20. Now use the slider labeled “x” to change the value of x. As you change the value of x, what do you notice? Complete the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>equation for g(x)</th>
<th>y-coordinate for F</th>
<th>y-coordinate for G</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>g(x) =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>g(x) =</td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>g(x) =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>g(x) =</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Use the slider for “k” to set k = -10. Now use the slider for “x” to change the value of x. As you change the value of x, what do you notice? Complete the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>equation for g(x)</th>
<th>y-coordinate for F</th>
<th>y-coordinate for G</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>g(x) =</td>
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<td>-1</td>
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<td>1</td>
<td>g(x) =</td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>g(x) =</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. Use the slider for “k” to set $k = 5$. Now use the slider for “x” to change the value of $x$. As you change the value of $x$, what do you notice? Complete the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>equation for $g(x)$</th>
<th>y-coordinate for $F$</th>
<th>y-coordinate for $G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>$g(x) =$</td>
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<td>$g(x) =$</td>
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</tr>
<tr>
<td>2</td>
<td>$g(x) =$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. If the slider for $k$ could be set at $k = 500$, how would the graph of $g(x)$ relate to the graph of $f(x)$? Complete the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>equation for $g(x)$</th>
<th>y-coordinate for $F$</th>
<th>y-coordinate for $G$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$g(x) =$</td>
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<td>$g(x) =$</td>
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</tr>
<tr>
<td>2</td>
<td>$g(x) =$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. What generalization can you make about how the graph of $g(x) = x^2 + k$ relates to the graph of $f(x) = x^2$? Explain your reasoning.
APPENDIX D

Parameter Changes to Parent Functions – Applet Activity #2

Please follow instructions carefully in completing the guided exploration.

1. Go to www.geogebratube.org/student/m19021. Click on the “Go to student worksheet” link. The should be titled Parameter Changes to Parent Functions #2.
2. You will see two graphs on the screen – the parent quadratic \( f(x) = x^2 \) and the transformed function \( g(x) = a \cdot x^2 \). The graph of \( f(x) \) and the equation for \( f(x) \) are in black while the graph and equation for \( g(x) \) are in blue. There is a table at the right that will allow you to see how the ordered pairs compare.
3. Use the slider labeled “\( a \)” to change the value of \( a \) in the function \( g(x) = a \cdot x^2 \). As you change the value of \( a \), what do you notice about the graph of \( g(x) \)?
4. Use the slider for “\( a \)” to set \( a = 2 \). Now use the slider labeled “\( x \)” to change the value of \( x \). As you change the value of \( x \), what do you notice? Complete the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>equation for ( g(x) )</th>
<th>( y )-coordinate for ( F )</th>
<th>( y )-coordinate for ( G )</th>
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</thead>
<tbody>
<tr>
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<td>( g(x) = )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>( g(x) = )</td>
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<td></td>
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<td>( g(x) = )</td>
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</tr>
<tr>
<td>2</td>
<td>( g(x) = )</td>
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<td></td>
</tr>
</tbody>
</table>

5. Use the slider for “\( a \)” to set \( a = 3 \). Now use the slider for “\( x \)” to change the value of \( x \). As you change the value of \( x \), what do you notice? Complete the table below.

<table>
<thead>
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<th>( y )-coordinate for ( G )</th>
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<td>( g(x) = )</td>
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</tr>
<tr>
<td>2</td>
<td>( g(x) = )</td>
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</tbody>
</table>
6. Use the slider for “a” to set $a = 0.5$. Now use the slider for “x” to change the value of $x$. As you change the value of $x$, what do you notice? Complete the table below.

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<th>y-coordinate for $G$</th>
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<tbody>
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<tr>
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<td>$g(x) =$</td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>$g(x) =$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. If the slider for a could be set at $a = 500$, how would the graph of $g(x)$ relate to the graph of $f(x)$? Complete the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>equation for $g(x)$</th>
<th>y-coordinate for $F$</th>
<th>y-coordinate for $G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>$g(x) =$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>$g(x) =$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$g(x) =$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$g(x) =$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$g(x) =$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. What generalization can you make about how the graph of $g(x) = a \times x^2$ relates to the graph of $f(x) = x^2$? Explain your reasoning.
APPENDIX E

Semi-Structured Interview for Students

1. The teacher’s method for teaching parameter changes to quadratic functions.
   a. What do you think you were expected to learn from your teacher’s lesson?
   b. What are the advantages and disadvantages of this method for you personally?
      - Describe aspects of the lesson that helped or hindered your understanding of parameter changes to quadratic functions.
      - Tell me more about (selected response to previous question).
   c. Are there other advantages and disadvantages of this method for students in general that you have not already mentioned?
      - Describe aspects of the lesson that you believe helped or hindered other students’ understanding of parameter changes to parent function.
   d. Describe specific examples that you can give as evidence that you understand parameter changes to quadratic functions.
      - What specifically can you show or state to demonstrate your understanding?
      - Tell me more about (selected response to previous question).
   e. In reflecting on how being taught with this method helped or hindered your understanding of parameter changes to parent functions, what suggestions would you make for changing this lesson to better meet your needs as a learner?
      - Tell me more about (selected response to previous question).

2. Teaching parameter changes to quadratic functions using applets
   a. What do you think you were expected to learn from the applets activities?
   b. What are the advantages and/or disadvantages of using applets to teach parameter changes to quadratic functions for you personally?
      - Describe aspects of the lesson that helped or hindered your understanding of parameter changes to quadratic functions.
      - Tell me more about (selected response to previous question).
   c. Are there other advantages and/or disadvantages of this method for students in general that you have not already mentioned?
      - Describe aspects of the applets activities that you believe helped or hindered students’ understanding of parameter changes to quadratic functions.
   d. Describe specific examples that you can give as evidence that you understand parameter changes to quadratic functions.
      - What specifically can you show or state to demonstrate your understanding?
      - Tell me more about (selected response to previous question).
   e. In reflecting on how being taught with applets helped or hindered your understanding of parameter changes to parent functions, what suggestions
would you make for changing the applet activities to better meet your needs as a learner?
  • Tell me more about (selected response to previous question).

3. Compare and contrast the two methods for being taught parameter changes to quadratic functions
   a. Compare and contrast the two methods for teaching parameter changes to quadratic functions.
      • What was similar about the two methods for teaching parameter changes to quadratic functions?
      • What was different about the two methods for teaching parameter changes to quadratic functions?
      • Discuss the effectiveness and/or ineffectiveness of each method.
      • Tell me more about (selected response to previous question).
   b. Discuss your beliefs and attitudes about being taught parameter changes to quadratic functions with each method.
      • Discuss your engagement and motivation with each method.
      • Discuss other students’ engagement and motivation with each method.
   c. Discuss your beliefs about which method contributed to your understanding.
      • Describe the aspects of each teaching method that you believe contributed to your understanding.
      • Explain how you believe these specific aspects of the teaching method contributed to your understanding.
      • Tell me more about (selected response to previous question).
   d. What additional insights, if any, did you gain from using the applet activities?
      • Describe how the applet activities contributed to you gaining these insights.
      • Tell me more about (selected response to previous question).
   e. Summarize what you know about parameter changes to quadratic functions.
      • How does what you know about parameter changes to quadratic functions extend to other parent functions?
      • (Use responses to pretest/posttest questions #1-3 on part 2 and #12 on part 3 to examine this further)
   f. Which method do you think contributed most to your understanding of parameter changes to quadratic functions?
      • Explain how and why one method contributed more to your understanding than the other.
   g. Based on your pretest/posttest scores (conclusion from test data), to what would attribute this (positive, negative, non-existent) impact?
      • Tell me more about (selected response to previous question).
• (Use answers to certain pretest/posttest questions to explore how and why the applet activities had an impact)
APPENDIX F

Semi-Structured Interview for Teachers

1. The teacher’s method for teaching parameter changes to quadratic functions
   a. Describe how you teach parameter changes to quadratic functions.
      - Discuss what you expect students to learn in this lesson.
   b. What are your reasons for choosing this method?
      - Describe past experiences teaching this lesson especially with regard to what worked and what did not work, student successes and failures, and student misconceptions.
   c. What are the advantages and/or disadvantages of this method for the teacher?
   d. What are the advantages and/or disadvantages of this method for the students?
      - Tell me more about (selected response to previous question).
   e. Does this approach for teaching parameter changes to quadratic functions lead to a more procedural or conceptual understanding by students?
      - How does this method for teaching parameter changes to quadratic functions help students gain this understanding?
      - Tell me more about (selected response to previous question).
   f. Describe specific examples that you can give as evidence that a student’s understanding of parameter changes to quadratic functions is procedural or conceptual.
      - What can students specifically show or state to demonstrate their procedural or conceptual understanding?
      - Tell me more about (selected response to previous question).

2. Teaching parameter changes to quadratic functions using applets
   a. Explain why your original method for teaching parameter changes to quadratic functions did not incorporate the use of applets.
      - Discuss your experiences using applets as a teacher and/or a student.
   b. What are the advantages and/or disadvantages of using applets to teach parameter changes to quadratic functions for the teacher?
   c. What are the advantages and/or disadvantages of using applets to teach parameter changes to quadratic functions for the students?
      - Tell me more about (selected response to previous question).
   d. Does using applets to teach parameter changes to quadratic functions lead to a more procedural or conceptual understanding by students?
      - How does this method for teaching parameter changes to quadratic functions help students gain this understanding?
      - Tell me more about (selected response to previous question).
   e. Describe specific examples that you can give as evidence that a student’s understanding of parameter changes to quadratic functions is procedural or conceptual.
• What can students specifically show or state to demonstrate their procedural or conceptual understanding?
• Tell me more about (selected response to previous question).

3. Comparing and contrasting the two methods for teaching parameter changes to quadratic functions
   a. Compare and contrast the two methods for teaching parameter changes to quadratic functions.
      • What was similar about the two methods for teaching parameter changes to quadratic functions?
      • What was different about the two methods for teaching parameter changes to quadratic functions?
      • Discuss the effectiveness and/or ineffectiveness of each method.
      • Tell me more about (selected response to previous question).
   b. Discuss your beliefs and attitudes about teaching parameter changes to quadratic functions with each method.
   c. In your opinion, what are students' beliefs and attitudes about being taught parameter changes to quadratic functions with each method?
      • Discuss student engagement and motivation with each method.
   d. Discuss your beliefs about which method contributes to students' conceptual understanding.
      • Describe specifically the aspects of the teaching method that you believe attribute to students' conceptual understanding.
      • Explain how you believe these specific aspects of the teaching method contribute to students' conceptual understanding.
   e. In your opinion, what are students' beliefs about which method contributes to their understanding?
      • Describe specifically the aspects of the teaching method that students believe attribute to their conceptual understanding.
      • Explain how you believe these specific aspects of the teaching method contribute to students' conceptual understanding.
   f. What additional insights, if any, did students gain from using the applets?
      • Describe how the applet activities contributed to students gaining these insights.
      • Tell me more about (selected response to previous question).
   g. Based on the pretest/posttest scores of (selected student), to what would you attribute this (positive, negative, non-existent) impact of applets?
      • Describe what you think (selected student) knows about parameter changes to quadratic functions.
      • (Repeat this for different students)
   h. Discuss your preference of method for teaching parameter changes to quadratic functions in the future and the reasons for choosing this method.
APPENDIX G

Guidelines for Facilitating Applet Activities

1) In setting up the activity, make sure each student has a computer with internet access and an activity sheet to guide their exploration.

2) Introduce the activity and give students the following instructions:
   a) Complete the activity sheet using the applet.
   b) Work independently.
   c) Do not assist other students.
   d) Do your best to answer the question on the activity sheet.

3) Assist students with technology issues such as logging onto the network, accessing the internet, etc.

4) Do not assist students in answering questions on the activity sheet.

5) Encourage students as they work.

6) When all students have completed the activity, have students share their generalizations to wrap up the activity.
REFERENCES

Journal of Applied Sciences, 5(11), 1506-1517.


Mathematics Teacher, 95, 434-440.

functions. In H. Chick, K. Stacey, & J. Vincent (Eds.), Proceedings of the 12th
ICMI study conference on the future of the teaching and learning of algebra (Vol
1). University of Melbourne.

Battista, M. (2001). Shape makers: A computer environment that engenders students'
construction of geometric ideas and reasoning. In D. J. Tooke & N. Henderson
(Eds.), Using information technology in mathematics education (pp. 105-120).
New York: The Haworth Press, Inc.

Boese, E. (2010). An introduction to programming with java applets. Sudbury, MA:
Jones and Bartlett Publishers.

in a multirepresentational environment. Educational Studies in Mathematics,
31(3), 319-337.

the math classroom. Computers in the Schools, 26(2), 107-114.


(3232348).


cutoff criteria: What did they really say?. *Organizational Research Methods, 9*(2),
202 - 220.


Committee, Center for Education, Division of Behavioral Sciences and Education. Washington, DC: National Academy Press.


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