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## THE PRIMORDIAL HELIUM ABUNDANCE: TOWARD UNDERSTANDING AND REMOVING THE COSMIC SCATTER IN THE $dY/dZ$ RELATION

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### ABSTRACT

We present results from photoionization models of low-metallicity H II regions. These nebulae form the basis for measuring the primordial helium abundance. Our models show that the helium ionization correction factor (ICF) can be nonnegligible for nebulae excited by stars with effective temperatures larger than 40,000 K. Furthermore, we find that when the effective temperature rises to above 45,000 K, the ICF can be significantly negative. This result is independent of the choice of stellar atmosphere. However, if an H II region has an [O III]  $\lambda 5007$ /[O I]  $\lambda 6300$  ratio greater than 300, then our models show that, regardless of its metallicity, it will have a negligibly small ICF. A similar, but metallicity-dependent, result was found using the [O III]  $\lambda 5007$ /H $\beta$  ratio. These two results can be used as selection criteria to remove nebulae with potentially nonnegligible ICFs. Use of our metallicity-independent criterion on the data of Izotov & Thuan results in a 20% reduction of the rms scatter about the best-fit Y-Z line. A fit to the selected data results in a slight increase of the value of the primordial helium abundance.

*Subject headings:* galaxies: abundances — galaxies: ISM — H II regions — ISM: abundances

### 1. INTRODUCTION

An accurate measurement of the primordial helium abundance would be an important test of standard big bang nucleosynthesis (Olive, Steigman, & Skillman 1997), and would also constrain the values of the photon-to-baryon ratio and  $\Omega_b$  (Olive, Steigman, & Walker 1999). The traditional procedure to measure the primordial helium abundance is to make use of the correlation between the helium mass fraction ( $Y$ ) and metal abundance ( $Z$ ). This correlation is then extrapolated to zero metallicity to estimate the primordial mass fraction of helium,  $Y_p$ . Spectroscopic observations of bright, low-metallicity extragalactic H II regions provide the data for these studies (e.g., Olive & Steigman 1995, Olive, Steigman, & Skillman 1997, Izotov, Thuan, & Lipovetsky 1994, 1997, Izotov & Thuan 1998, Torres-Peimbert, Peimbert, & Fierro 1989, Skillman, Terlevich, & Terlevich 1998).

To be cosmologically useful the value of  $Y_p$  has to be determined to better than 5%. Fortunately, abundance determination from measurements of line ratios is relatively straightforward (Peimbert 1975; Benjamin, Skillman, & Smits 1999) and can, *in theory*, give the desired accuracy. However, to reach the needed level of precision, any systematic errors involved with target selection, observations, and data analysis must be identified and corrected. Many such systematic errors have already been identified (Davidson & Kinman 1985; Dinerstein & Shields 1986; Pagel et al. 1992; Skillman et al. 1994; Peimbert 1996; Izotov et al. 1997; Steigman, Viegas, & Gruenwald 1997; Skillman et al. 1998), but any errors resulting from the so-called ionization correction factor (ICF) have so far been assumed to be small. The ICF corrects for the fact that some amount of atomic (i.e., unseen) helium might be present in ionized regions of hydrogen (Osterbrock 1989; Peimbert 1975). This correc-

tion traditionally has been assumed to be zero because measurements of the primordial helium abundance employ observations of bright extragalactic H II regions. These regions are excited by clusters of young stars with effective temperatures greater than 40,000 K. Calculations by Stasinska (1990) and Pagel et al. (1992) showed that the helium ICF should be negligibly small for these H II regions. As a result, recent determinations of  $Y_p$  have assumed that the helium ICF is small.

Very recently, Armour et al. (1999) presented calculations that showed that H II regions excited by stars with temperatures greater than 40,000 K can have nonnegligible ICFs. Armour et al. (1999) found that the ICFs were often negative (i.e., the helium ionized zone is *larger* than the hydrogen one; Stasinska 1980, 1982; Peña 1986) for the hardest stellar continua. These results were confirmed by Viegas, Gruenwald, & Steigman (2000). In this paper we follow up on the work of Armour et al. (1999), and develop observational diagnostics of when the He ICF is important and when it can be ignored. We then apply these diagnostics to the data of Izotov & Thuan (1998) to illustrate how our technique can improve the precision of the measurement of  $Y_p$ .

We describe our calculations in § 2, and our results in § 3. The main results are summarized in § 4.

### 2. DESCRIPTION OF CALCULATIONS

In order to investigate the effects of a nonnegligible ICF on the determination of the primordial helium abundance, we ran photoionization models of H II regions and extracted the ICF for each nebula. These calculations are very similar to ones presented by Armour et al. (1999) and Bottorff et al. (1998), and were made with the development version of CLOUDY, last described by Ferland et al. (1998).

Since we are modeling H II regions, our models use the ISM abundances and grain model that were used and described by Armour et al. (1999). However, we scaled both the metal and grain abundances to lower values because, in this case, we are most interested in lower metallicity

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nebulae. The scaling was implemented so that all metals and grains were varied together relative to hydrogen and helium, but the He/H ratio was held constant. We modeled nebulae at three different metallicities:  $O/H = 32, 64,$  and  $128$  parts per million (ppm) ( $Z = Z_{\odot}/23, Z_{\odot}/12,$  and  $Z_{\odot}/6,$  respectively). For each metallicity, 1936 models were computed for each of the following spectral energy distributions: the LTE plane-parallel atmospheres of Kurucz (1991); the non-LTE, wind-blanketed, solar abundance CoStar atmospheres of Schaerer et al. (1996a, 1996b); the earlier non-LTE atmospheres of Mihalas (1972); and, for completeness, blackbodies. We also ran models using the subsolar abundance CoStar atmospheres, which are spectrally slightly harder than the solar abundance ones. These models resulted in slightly more negative ICFs, but the values of the line-ratio cutoffs (§§ 3.2 and 3.3) were not changed from the ones calculated with a solar abundance atmosphere.

For each atmosphere we computed models with  $10 \text{ cm}^{-3} \leq n_H \leq 10^6 \text{ cm}^{-3}$ ,  $10^{-4} \leq U \leq 10^{-0.25}$ , and  $40,000 \text{ K} \leq T_{\text{eff}} \leq 50,000 \text{ K}$ , where  $U$  is the ionization parameter defined as in equation (4) of Armour et al. (1999). Giant extragalactic H II regions that are observed are generally excited by large clusters, so this range of parameters should cover all such nebulae. We modeled the nebulae as plane-parallel constant-density slabs, a simple way to characterize blister H II regions. Our proposed diagnostic indicators are the  $[O \text{ III}] \lambda 5007$  and  $[O \text{ I}] \lambda 6300$  lines (§§ 3.2 and 3.3), and these should be fairly independent of the assumed geometry (sphere, sheet, or evaporating blister). The  $[O \text{ III}] \lambda 5007/H\beta$  ratio represents the cooling per recombination (the Stoy ratio) and so is primarily sensitive to the stellar temperature (Stoy 1933; Kaler 1978) rather than geometry. Similarly, the  $[O \text{ I}] \lambda 6300/H\beta$  intensity ratio mostly measures the “softness” of the hydrogen ionization front (where the line forms; Netzer & Davidson 1979).

### 3. RESULTS

#### 3.1. Temperature Dependence

Figure 1 plots the ICF calculated from the Kurucz and CoStar models versus the stellar temperature. Note that we define the ICF such that an ICF of zero corresponds to zero correction:

$$\text{ICF} = \frac{\langle H^+/H \rangle}{\langle He^+/He \rangle} - 1, \quad (1)$$

where the angle brackets denote the volume mean ionization fraction. This definition takes into account the presence of any  $He^{+2}$  in the nebula. Figure 1 clearly shows that one can obtain a nonnegligible ICF for stars with temperatures greater than 40,000 K. The harder CoStar atmospheres give preferentially negative ICFs, which, if not taken into account, would result in an overestimate of the helium abundance. The same is true of the Mihalas atmospheres (not shown). These results agree with the calculations of Armour et al. (1999) and Viegas et al. (2000). The softer Kurucz atmospheres result in preferentially positive ICFs (i.e., the helium ionized zone is smaller than the hydrogen one), although, at temperatures greater than 45,000 K, they can also give negative ICFs. The blackbody atmospheres, the least realistic, are softer still, but even they can produce negative ICFs in some models at the highest temperatures.

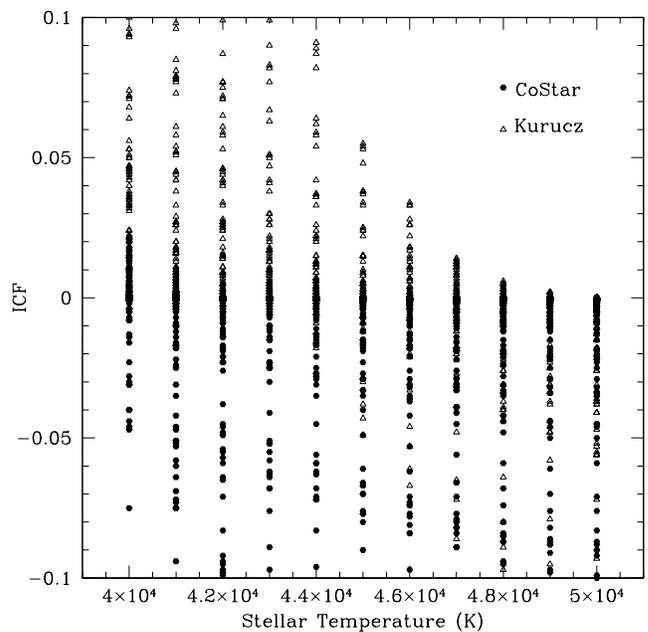


FIG. 1.—Plot of the He ICF values obtained from the CoStar and Kurucz photoionization models vs. stellar temperature of the atmospheres. Only points with ICF in the range  $\pm 10\%$  are plotted. There are non-negligible ICFs at all values of stellar temperature. The CoStar atmospheres generally result in negative ICFs, while the softer Kurucz atmospheres give positive ICFs for temperatures less than about 45,000 K. These models were run with a metallicity of  $(O/H) = 64$  ppm.

Therefore, negative ICFs seem to be found at high stellar temperatures *independent of the type of stellar atmosphere*.

#### 3.2. Metallicity-dependent Cutoff Criterion

A negative ICF occurs as the results of penetrating high-energy photons preferentially ionizing helium, because of its large photoionization cross section. This tends to be important for lower ionization parameter models, since these have significant regions where H and He are partially ionized. We expect these nebulae to be characterized by lower  $[O \text{ III}] \lambda 5007/H\beta$  ratios (a measure of excitation) and larger  $[O \text{ I}] \lambda 6300/H\beta$  ratios (since  $[O \text{ I}] \lambda 6300$  is formed in warm atomic regions).

The results presented in § 3.1 show that it is not appropriate simply to assume that the ICF is zero when a nebula is excited by a star with a temperature greater than 40,000 K. However, it would be important to develop observational diagnostics for when the ICF is important and when it is not. Figure 2 shows such a diagnostic. A plot of ICF versus  $[O \text{ III}] \lambda 5007/H\beta$  shows that beyond a line ratio of about 3–4 the ICF is negligible (for clarity, results are shown for Kurucz and CoStar atmospheres only; a plot for blackbody and Mihalas atmospheres is very similar). However, the value of the cutoff will depend on metallicity. We found very small ICFs for line ratios greater than the following cutoff:

$$([O \text{ III}] \lambda 5007/H\beta)_{\text{cutoff}} = (0.025 \pm 0.004)(O/H) + (1.139 \pm 0.306), \quad (2)$$

where  $O/H$  is measured in parts per million. H II regions which have an  $[O \text{ III}] \lambda 5007/H\beta$  ratio less than the cutoff for their metallicity might be subject to an ICF correction. Unless this correction can be made (and, in general, it cannot), these H II regions should be removed from the abundance analysis, as they will increase the scatter in the

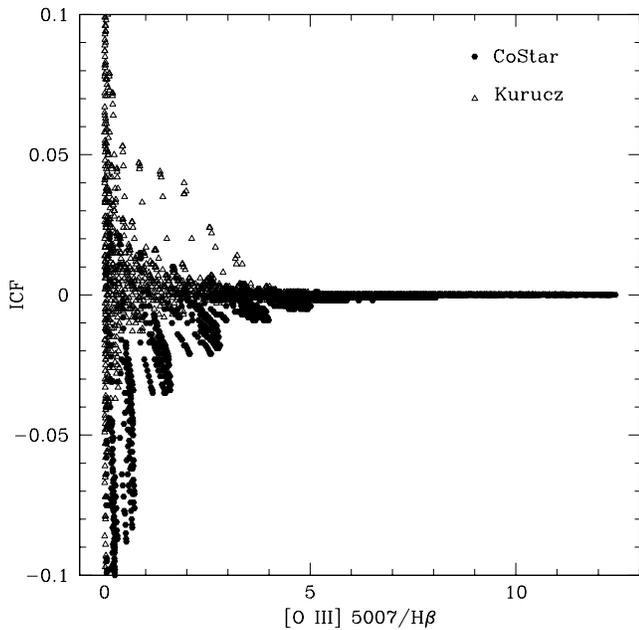
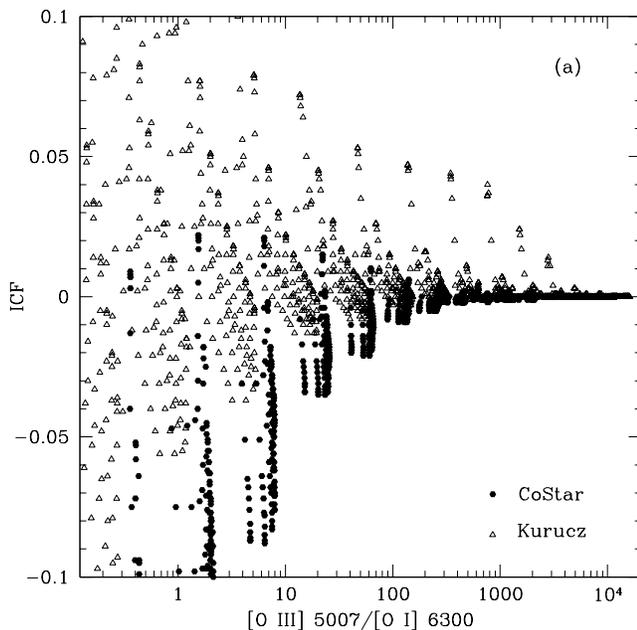


FIG. 2.—Plot of He ICF vs.  $[\text{O III}] \lambda 5007/\text{H}\beta$  using data from the CoStar and Kurucz grids run at  $(\text{O}/\text{H}) = 64$  ppm. Only points with ICF in the range  $\pm 10\%$  were plotted. At this metallicity the ICF-related cutoff is at a relative line strength of about 3–4. See the text for discussion on how cutoff varies with metallicity.

$dY/dZ$  relation used to determine the primordial helium abundance.

### 3.3. Metallicity-independent Cutoff Criterion

One can improve the above result by finding emission-line ratios that should be independent of metallicity. Figure 3 plots the helium ICF versus the  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  ratio. The figure shows that the ICF is negligible for a line ratio greater than about 300. Not surprisingly, this result is *independent of metallicity*.



There are some Kurucz and blackbody models that result in a nonnegligible ICF at line ratios larger than this cutoff. These models had low stellar temperatures (40,000–42,000 K), and were found over a narrow range in both  $\log U$  ( $-1.5$  to  $-2.25$ ) and  $\log n_{\text{H}}$  (1.0–3.0). Their positive ICF is a result of combining the low stellar temperatures and the softness of their atmospheres; their large  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  ratio is a result of combining the fairly high ionization parameter with the low density. The CoStar and Mihalas models, which are considered more “realistic,” result in only very small ICFs in this region.

Therefore, we find that any H II region, regardless of its metallicity, that has an  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  ratio less than about 300 might be subject to an ICF correction and should not be used to determine the primordial helium abundance.

### 3.4. Application to Real Data

To see how these new results affect the determination of  $Y_p$ , we applied the metallicity-independent rejection criterion to the data of Izotov & Thuan (1998), and the results are shown in Figure 4. There are a number of points to note from this figure:

1. Our criterion rejects points over the entire range of metallicity, so there is no metallicity bias. The rejected points also fall evenly over the range of  $Y$ -values, which implies that there is no correlation between  $Y$  and the  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  ratio.

2. At an  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  cutoff of 300 there is a 20% reduction in the weighted rms scatter about the best-fit line. This is encouraging evidence that part of the scatter was due to the ICF, and the situation has indeed improved by the implementation of the cutoff.

3. The negative slopes predicted by using a cutoff  $\geq 300$  result from weighted fits to the small number of data that remain after applying the cutoff, and are probably not rea-

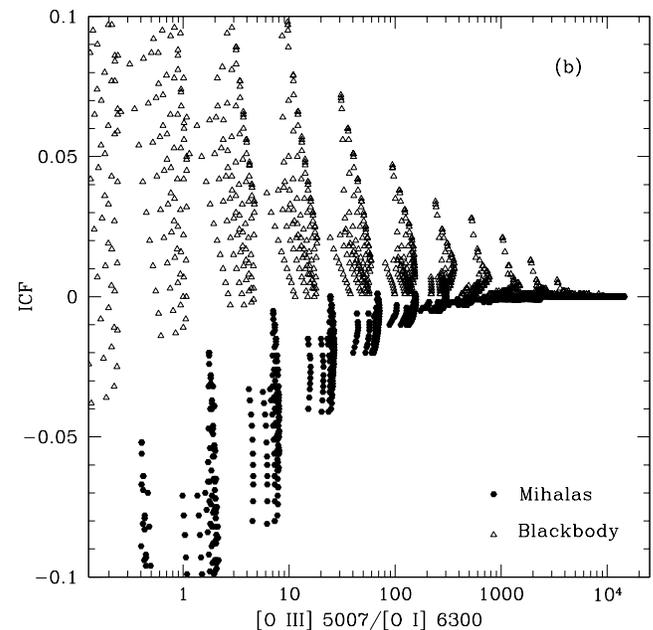


FIG. 3.—(a) Plot of He ICF vs.  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  with data obtained from the CoStar and Kurucz model grids run at  $(\text{O}/\text{H}) = 64$  ppm. The ideal ICF-related cutoff here is at a line ratio of 300. This cutoff is *independent* of metallicity. The Kurucz models that have positive ICFs in the allowed zone are models with a particular combination of parameters (see text). (b) As in (a), but for Mihalas and blackbody atmospheres.

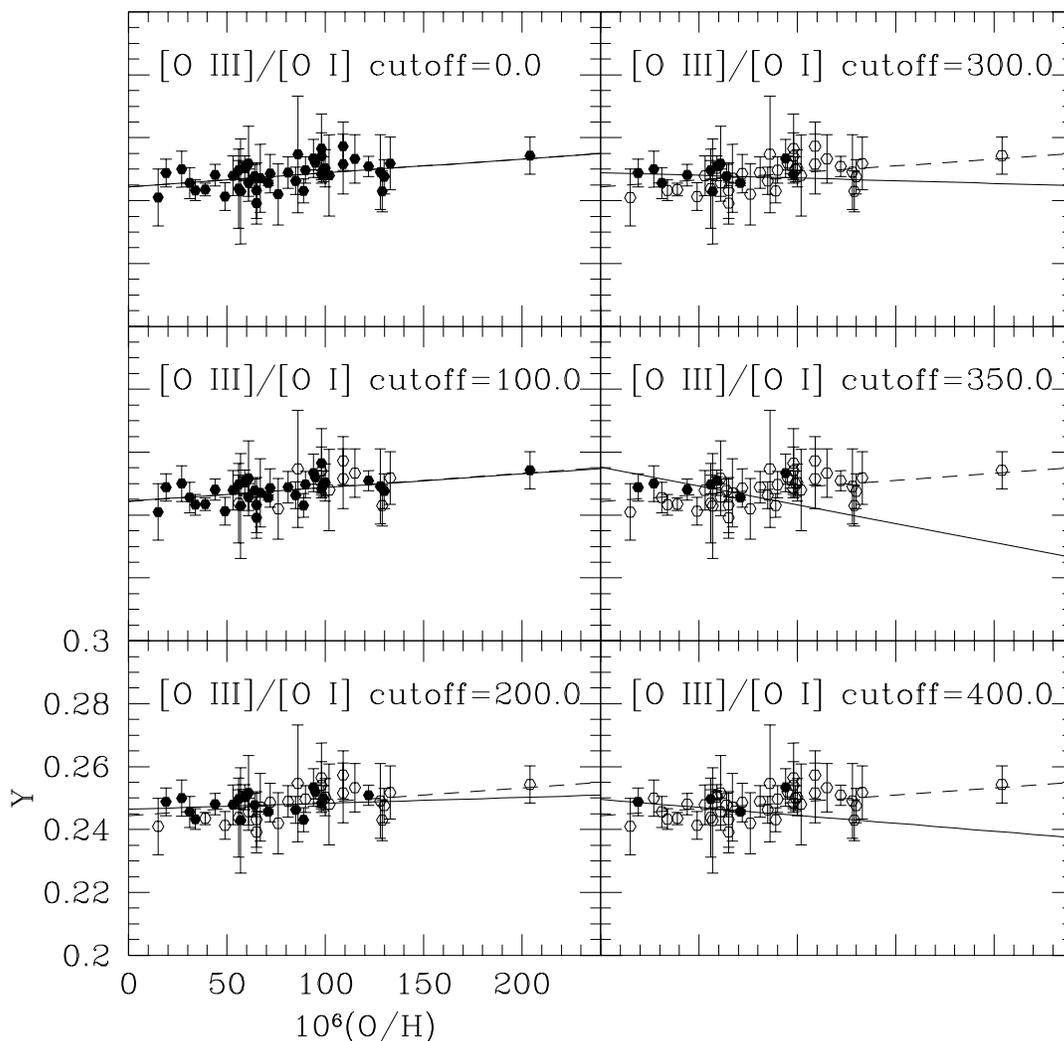


FIG. 4.—Effects of applying the metallicity-independent rejection criterion to the data of Izotov & Thuan (1998) are shown. The solid line is a weighted least-squares fit to the selected data points shown by the solid symbols. The open symbols are the rejected points. For reference, the dashed line is the fit with no cutoff applied. Note that as the  $[\text{O III}]\lambda 5007/[\text{O I}]\lambda 6300$  cutoff becomes larger, the scatter of the points about the best-fit line becomes smaller (there is a 20% reduction in the rms scatter when the cutoff is 300). This procedure also shows that the value of  $Y_p$  determined by Izotov & Thuan (1998) might be an underestimate.

listic. Given the current data, a cutoff value greater than 300 is too severe (not practical).

4. Despite the increased errors in the slope and the intercept, we find that implementing our cutoff will result in a larger value of  $Y_p$ . Specifically, we find  $Y_p = 0.2489 \pm 0.0030$  when the cutoff is taken at a  $[\text{O III}]\lambda 5007/[\text{O I}]\lambda 6300$  ratio of 300. Izotov & Thuan (1998) find  $0.2443 \pm 0.0015$  (our fit gives  $0.2443 \pm 0.0013$  with no cutoff), so the two results are barely consistent at the  $1\sigma$  level. This result is in the opposite direction of the shift predicted by the Monte Carlo simulations of Viegas et al. (2000), but moves the value of  $Y_p$  closer to the theoretically predicted values (Olive & Steigman 1995).

There is always the possibility that systematic errors are introduced whenever data are rejected by a certain criterion. The above selection criterion preferentially selects H II regions that have large  $[\text{O III}]\lambda 5007/[\text{O I}]\lambda 6300$  ratios. This ratio is generally large whenever the  $[\text{O III}]\lambda 5007$  line is strong (this is consistent with our metallicity-dependent criterion). The  $[\text{O III}]\lambda 5007$  line is a major source of cooling in a nebula, and so by selecting H II regions with strong

$[\text{O III}]\lambda 5007$  lines we are selecting regions ionized by hotter stars. Since these large extragalactic H II regions are generally ionized by clusters, ones with strong  $[\text{O III}]\lambda 5007$  lines are preferentially younger. However, this is unlikely to introduce a systematic error in the primordial helium abundance determination, as young clusters can form at any metallicity. Indeed, Figure 4 shows that the points rejected by applying the cutoff span the entire range of metallicity.

There is also the possibility that physical conditions within the H II regions may bias our results. For example, in some nebulae the intensity of the  $[\text{O I}]\lambda 6300$  line could be enhanced as a result of shock heating. This would lower the measured  $[\text{O III}]\lambda 5007/[\text{O I}]\lambda 6300$  ratio and could move it below our selection criterion. However, shock heating would not change the ICF of the nebula, so even though application of our criterion might reject such a H II region, it will not bias the determination of  $Y_p$ .

Another potential situation in H II regions is that the nebula may be matter bounded (i.e., optically thin to the Lyman continuum) in certain solid angles or sectors. Because there will not be a hydrogen ionization front in such sectors, and therefore no  $[\text{O I}]\lambda 6300$  line emission, the

presence of such sectors will increase the measured  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  ratio, possibly pushing it above our selection criterion. However, these sectors will also have little or no neutral helium within them, and so will have no ICF. Therefore, although the other, ionization-bounded sectors of the H II region could have a nonnegligible ICF, this will be diluted by the matter-bounded sectors. We anticipate that even if such an H II region were shifted into our selected data, there ought not be a large effect on determining  $Y_p$ . According to Figure 3, to severely bias the results a number of points would have to shift to the right in  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  by a factor larger than 10; but such a large shift in the line ratio would probably result in a large dilution in the ICF. More modeling would be needed to quantify how little an impact matter-bounded sectors would have on the  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  ratio and ICFs.

#### 4. CONCLUSIONS

In this paper we have shown the following:

1. There can be a nonnegligible ICF correction for H II regions excited by stars with temperatures greater than 40,000 K. At temperatures higher than 45,000 K, the ICF is

preferentially negative. This result is independent of the atmosphere of the O star.

2. There is a simple procedure to determine whether an ICF correction needs to be made for a given H II region. If the  $[\text{O III}] \lambda 5007/[\text{O I}] \lambda 6300$  ratio is greater than 300, then no correction is needed. This criterion is independent of metallicity. If the  $[\text{O I}] \lambda 6300$  line cannot be measured, then there is a metallicity-dependent cutoff (eq. [2]) that can be used with the  $[\text{O III}] \lambda 5007$  line.

3. Applying the metallicity-independent criterion to the data of Izotov & Thuan (1998) results in reducing the rms scatter about the best-fit Y-Z line by 20%. This will help remove systematic errors relating to unrecognized ICF effects, and ought to improve the reliability of the  $Y_p$  determination. Furthermore, an analysis of the selected data gives a larger value of  $Y_p$  than was originally measured, which is closer to the theoretically expected value.

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